

# CBSE Class 12 Physics Question Paper 2023 with Solution PDF-Set 55/4/1



## Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

### General Instructions

- (i) This question paper contains **35** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
- (iii) In **Section A** – Questions no. **1 to 18** are Multiple Choice (MCQ) type questions, carrying **1** mark each.
- (iv) In **Section B** – Questions no. **19 to 25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C** – Questions no. **26 to 30** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D** – Questions no. **31 to 33** are Long Answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E** – Questions no. **34 and 35** are case-based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 3 questions in Section D and 2 questions in Section E.
- (ix) Use of calculators is **not** allowed.

Use the following values of physical constants, if required :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

### Section A (18 marks)

1. Two charges  $q_1$  and  $q_2$  are placed at the centres of two spherical conducting shells of radius  $r_1$  and  $r_2$  respectively. The shells are arranged such that their centres are  $d [ > (r_1 + r_2) ]$  distance apart. The force on  $q_2$  due to  $q_1$  is: [1 mark]

- (A)  $\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{d^2}$   
(B)  $\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{(d - r_1)^2}$   
(C) Zero  
(D)  $\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{[d - (r_1 + r_2)]^2}$

**Correct Answer:** (C) Zero

#### Solution:

##### Concept:

- Electrostatic shielding: a closed conducting shell in electrostatic equilibrium prevents an external charge from producing an electric field in its cavity.
- The electric force on a charge is  $\vec{F} = q\vec{E}$ , where  $\vec{E}$  is the field at the charge due to other charges, including induced surface charges.

**Step 1: Identify the shielded charge** Since  $d > r_1 + r_2$ , the shells do not overlap and  $q_1$  lies outside the shell containing  $q_2$ . Its field redistributes charge on the conducting surfaces. The resulting induced field cancels the external field inside the second cavity. A constant cavity potential contributes no electric field.

**Step 2: Apply the shielding condition** The field transmitted into the second cavity by the external charge is zero. The charge  $q_2$ , at the centre of its spherical cavity, also has no net force from its own symmetric induced inner-surface charge. A centre-to-centre Coulomb expression neglects the conducting shells. Replacing the distance by a surface gap, as in the other nonzero options, cannot account for electrostatic shielding either.

**Step 3: Conclusion** The force on  $q_2$  due to the external charge  $q_1$ , with the conducting shells present, is  $F = 0$ . Hence, the correct option is (C).

**Quick Tip to Solve:** For a charge enclosed by a closed conductor, check electrostatic shielding before substituting any distance into Coulomb's law.

2. An electron enters a uniform magnetic field with speed  $v$ . It describes a semicircular path and comes out of the field. The final speed of the electron is: [1 mark]

- (A) Zero
- (B)  $v$
- (C)  $\frac{v}{2}$
- (D)  $2v$

**Correct Answer:** (B)  $v$

**Solution:**

**Concept:**

- Magnetic force on a moving charge is  $\vec{F} = q(\vec{v} \times \vec{B})$ ; it is perpendicular to the instantaneous velocity.
- Work changes kinetic energy,  $W = \Delta K$ , while the instantaneous power is  $P = \vec{F} \cdot \vec{v}$ .

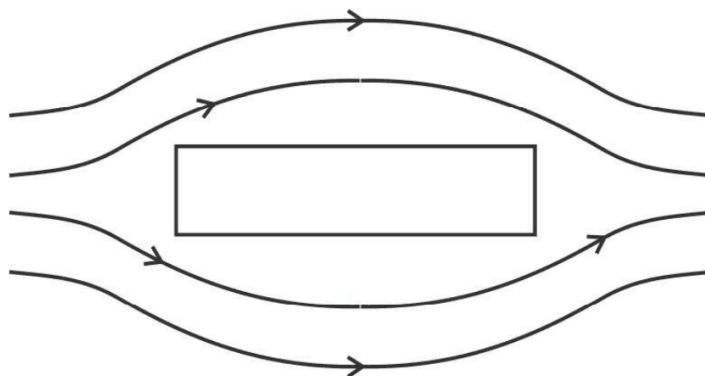
**Step 1: Determine the work done by the field** At each point of the semicircular trajectory the magnetic force is normal to the electron's displacement. It therefore acts as a centripetal force and does no work:  $\vec{F} \cdot \vec{v} = 0$ . The electron's negative charge determines the direction of bending, but does not change this perpendicularity.

**Step 2: Compare initial and final kinetic energies** Since the magnetic field does no work,  $\frac{1}{2}mv_f^2 = \frac{1}{2}mv^2$ . The velocity reverses direction after a semicircle, but its magnitude stays unchanged. Zero, half and double the initial speed would each require a change in kinetic energy, so none can result from this magnetic force alone.

**Step 3: Conclusion** The electron leaves the field with speed  $v_f = v$ . Hence, the correct option is (B).

**Quick Tip to Solve:** Distinguish speed from velocity: a magnetic field bends the velocity vector but cannot change the particle's kinetic energy.

3. The magnetic field lines near a substance are as shown in the figure. The substance is: [1 mark]



- (A) Copper
- (B) Iron
- (C) Sodium
- (D) Aluminium

**Correct Answer:** (A) Copper

**Solution:**

**Concept:**

- A diamagnetic substance acquires magnetisation opposite to an applied magnetic field, with magnetic susceptibility  $\chi_m < 0$ .
- Its relative permeability is  $\mu_r = 1 + \chi_m < 1$ ; field lines tend to avoid its interior, unlike a paramagnetic or ferromagnetic substance.

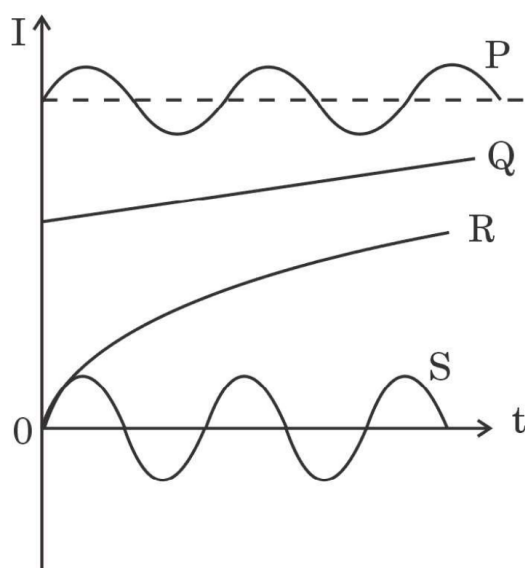
**Step 1: Interpret the field-line pattern** The lines bend outward around the rectangular specimen instead of concentrating inside it. This depicts expulsion of magnetic flux from the specimen and identifies a diamagnetic response. The drawing is schematic: the outward bending, rather than an exact line count, is the identifying feature.

**Step 2: Compare the listed materials** Copper is diamagnetic and has the required weak repulsion from an applied magnetic field. Iron is ferromagnetic and would concentrate the field strongly. Sodium and aluminium are paramagnetic and would weakly concentrate field lines inside the specimen, rather than give the outward bending shown.

**Step 3: Conclusion** The specimen is copper, a diamagnetic substance. Hence, the correct option is (A).

**Quick Tip to Solve:** Field lines bending away from a specimen indicate diamagnetism; lines crowding into it indicate a positive magnetic susceptibility.

4. The figure shows variation of current ( $I$ ) with time ( $t$ ) in four devices P, Q, R and S. The device in which an alternating current flows is: [1 mark]



- (A) P
- (B) Q
- (C) R
- (D) S

**Correct Answer:** (D) S

**Solution:**

**Concept:**

- An alternating current changes direction periodically, so its signed value takes both positive and negative values in successive intervals.
- In an  $I$ -versus- $t$  graph, direction reversal is indicated by crossing the zero-current time axis; variation in magnitude alone is insufficient.

**Step 1: Locate the zero-current axis** The horizontal line through the origin is  $I = 0$ . Curve S repeatedly rises above and falls below this line. Its sign therefore reverses periodically, indicating that charge flows alternately in opposite directions through the device.

**Step 2: Check the remaining curves** P oscillates about a dashed positive-current level and remains above the actual time axis. It has an oscillating component but does not reverse the total current. Q rises linearly while remaining positive, and R also stays positive as its magnitude increases. Thus P, Q and R represent unidirectional currents of varying magnitude.

**Step 3: Conclusion** Alternating current flows in device S. Hence, the correct option is (D).

**Quick Tip to Solve:** Test whether the curve crosses the actual  $I = 0$  axis; oscillation around a positive dashed line alone does not reverse current.

5. The electromagnetic waves used in radar systems are:

[1 mark]

- (A) Infrared waves
- (B) Ultraviolet rays
- (C) Microwaves
- (D) X-rays

**Correct Answer:** (C) Microwaves

**Solution:**

**Concept:**

- Radar locates objects by transmitting electromagnetic pulses and detecting their reflected echoes.
- Microwaves have sufficiently short wavelengths to form narrow directional beams with practical antennas. The echo delay  $\Delta t$  gives range  $R = c\Delta t/2$ , where  $c$  is the wave speed in air.

**Step 1: Match the wave to the application** Radar requires a directed beam that can travel to a distant target, reflect from it and return to a receiver. Microwaves are suitable for this purpose and are used in radar systems for aircraft navigation, detection and ranging. Their wavelength permits useful beam directionality with an antenna of practical size.

**Step 2: Eliminate the other spectral regions** Infrared radiation is associated with thermal imaging and remote controls. Ultraviolet rays are used for applications such as sterilisation, while X-rays are used in radiography. These are not the spectral region identified with the radar systems asked about here.

**Step 3: Conclusion** The electromagnetic waves used in radar systems are microwaves. Hence, the correct option is (C).

**Quick Tip to Solve:** Associate radar with directed microwave pulses and their returning echoes; the round-trip travel time gives twice the target range.

6. In a Young's double-slit experiment, the fringe width is found to be  $\beta$ . If the entire apparatus is immersed in a liquid of refractive index  $\mu$ , the new fringe width will be:

[1 mark]

- (A)  $\beta$
- (B)  $\mu\beta$
- (C)  $\frac{\beta}{\mu}$
- (D)  $\frac{\beta}{\mu^2}$

**Correct Answer:** (C)  $\frac{\beta}{\mu}$

**Solution:**

**Concept:**

- Young's double-slit fringe width is  $\beta = \lambda D/d$ , where  $\lambda$  is the wavelength,  $D$  the slit-to-screen distance and  $d$  the slit separation.
- On entering a liquid of refractive index  $\mu$ , light retains its frequency while its speed and wavelength become  $c/\mu$  and  $\lambda/\mu$ .

**Step 1: Identify the quantity changed by immersion** Immersing the entire apparatus changes the medium throughout both optical paths. The source frequency and geometrical distances  $D$  and  $d$  stay

fixed, but the wavelength in the liquid is  $\lambda' = \lambda/\mu$ . Therefore the fringe spacing must decrease for an ordinary liquid with  $\mu > 1$ .

**Step 2: Substitute the wavelength in the fringe-width formula** Using the same apparatus gives

$$\beta' = \frac{\lambda' D}{d} = \frac{\lambda D}{\mu d} = \frac{\beta}{\mu}.$$

The unchanged-width option ignores the wavelength change. Multiplication by  $\mu$  predicts the opposite trend, and division by  $\mu^2$  incorrectly introduces a second factor although only the wavelength changes.

**Step 3: Conclusion** The new fringe width is  $\beta' = \beta/\mu$ . Hence, the correct option is (C).

**Quick Tip to Solve:** For complete immersion of a fixed double-slit apparatus, replace only the wavelength by  $\lambda/\mu$ ; fringe width scales by the same factor.

**7. Photons of energy 3.2 eV are incident on a photosensitive surface. If the stopping potential for the emitted electrons is 1.5 V, the work function for the surface is:** [1 mark]

- (A) 1.5 eV
- (B) 1.7 eV
- (C) 3.2 eV
- (D) 4.7 eV

**Correct Answer:** (B) 1.7 eV

**Solution:**

**Concept:**

- Einstein's photoelectric equation is  $E_\gamma = \phi + K_{\max}$ , where  $\phi$  is the work function and  $K_{\max}$  the maximum emitted-electron kinetic energy.
- A stopping potential of magnitude  $V_s$  removes this maximum kinetic energy:  $K_{\max} = eV_s$ , where  $e$  is the elementary charge magnitude.

**Step 1: Convert the stopping potential to kinetic energy** The fastest photoelectrons are just stopped by  $V_s = 1.5$  V. An electron crossing one volt changes its energy by one electronvolt, so

$$K_{\max} = e(1.5 \text{ V}) = 1.5 \text{ eV}.$$

**Step 2: Subtract kinetic energy from photon energy** The incident photon supplies 3.2 eV, part of which frees the electron and the rest becomes kinetic energy. Hence

$$\phi = E_\gamma - K_{\max} = 3.2 - 1.5 = 1.7 \text{ eV}.$$

The value 1.5 eV is the kinetic energy, not the work function. Taking 3.2 eV leaves no kinetic energy, and 4.7 eV incorrectly adds the energies.

**Step 3: Conclusion** The work function of the surface is 1.7 eV. Hence, the correct option is (B).

**Quick Tip to Solve:** When photon energy is given in eV, subtract the numerical stopping potential in volts directly to obtain the work function in eV.

**8. Which of the following statements is not true for nuclear forces?**

[1 mark]

- (A) They are stronger than Coulomb forces.
- (B) They have about the same magnitude for different pairs of nucleons.
- (C) They are always attractive.
- (D) They saturate as the separation between two nucleons increases.

**Correct Answer:** (C) They are always attractive.

**Solution:**

**Concept:**

- The nuclear force is a strong, short-range interaction between nucleons and is approximately charge independent.
- It is attractive at typical internucleon separations but becomes strongly repulsive at very small separations, preventing collapse of the nucleus.

**Step 1: Test the claim of attraction at every distance** At separations of the order of a femtometre the attractive nuclear force binds protons and neutrons. At much smaller separations its repulsive core resists further compression. Thus the word “always” in option (C) contradicts the distance-dependent character of the force.

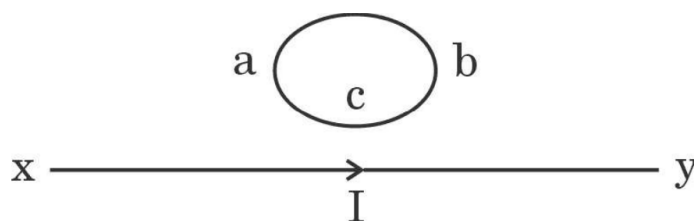
**Step 2: Compare the other stated properties** Within its effective range, the nuclear interaction is stronger than the Coulomb repulsion of protons, consistent with (A). Its approximate charge independence gives comparable strengths for different nucleon pairs, as in (B). Option (D) refers loosely to the short-range, saturating character: a nucleon interacts effectively with nearby neighbours, and the force becomes negligible at larger separations. Saturation should not be taken to mean that the force stays attractive at every distance.

**Step 3: Conclusion** Nuclear forces are not always attractive; they are repulsive at very small separations. Hence, the correct option is (C).

**Quick Tip to Solve:** In a nuclear-force question, test the word “always” against the repulsive core at extremely small internucleon separations.

9. The direction of induced current in the loop abc is:

[1 mark]



- (A) along abc if  $I$  decreases
- (B) along acb if  $I$  increases
- (C) along abc if  $I$  is constant
- (D) along abc if  $I$  increases

**Correct Answer:** (D) along abc if  $I$  increases

**Solution:**

**Concept:**

- For a straight wire, the right-hand thumb rule gives the magnetic-field direction around the current.
- Lenz's law says that the induced current opposes the change in magnetic flux through the loop.

**Step 1: Find the field through the loop** The current in the straight wire flows from left to right. At the loop above it, the magnetic field points out of the page. Increasing  $I$  therefore increases the outward magnetic flux.

**Step 2: Apply the opposing-flux rule** To oppose an increase in outward flux, the loop must produce a field into the page. This requires clockwise current, which follows  $a \rightarrow b \rightarrow c \rightarrow a$  in the figure. If  $I$  decreases, the induced current instead goes along  $acb$ , so (A) fails. Option (B) gives the wrong sense for increasing current. Constant current gives constant flux and no induced current, ruling out (C).

**Step 3: Conclusion** The induced current is along  $abc$  when  $I$  increases. Hence, the correct option is (D).

**Quick Tip to Solve:** First find the field at the loop, then oppose its change; do not oppose the field itself in every case.

10. An ac voltage  $v = v_0 \sin \omega t$  is applied to a series combination of a resistor  $R$  and an element  $X$ . The instantaneous current in the circuit is  $I = I_0 \sin \left( \omega t + \frac{\pi}{4} \right)$ . Then which of the following is correct?

[1 mark]

- (A)  $X$  is a capacitor and  $X_C = \sqrt{2}R$
- (B)  $X$  is an inductor and  $X_L = R$
- (C)  $X$  is an inductor and  $X_L = \sqrt{2}R$
- (D)  $X$  is a capacitor and  $X_C = R$

**Correct Answer:** (D)  $X$  is a capacitor and  $X_C = R$

**Solution:**

**Concept:**

- In a series RC circuit, current leads the applied voltage; in a series RL circuit, current lags it.
- For a series RC circuit,  $\tan \phi = X_C/R$ , where  $\phi$  is the current lead and  $X_C$  is the capacitive

reactance.

**Step 1: Identify the phase lead** The voltage has phase  $\omega t$ , whereas the current has phase  $\omega t + \pi/4$ . Thus current leads voltage by  $\pi/4$ , which identifies  $X$  as a capacitor. An inductor would give a lag, so (B) and (C) are excluded.

**Step 2: Use the phase-angle relation** Substitute the given phase difference into the RC relation:

$$\frac{X_C}{R} = \tan \frac{\pi}{4} = 1.$$

Option (A) would instead give  $\tan \phi = \sqrt{2}$ , inconsistent with the stated phase. The factor  $\sqrt{2}$  belongs to the impedance magnitude  $Z = \sqrt{R^2 + X_C^2}$  when the two components are equal, not to their ratio.

**Step 3: Conclusion**  $X$  is a capacitor and  $X_C = R$ . Hence, the correct option is (D).

**Quick Tip to Solve:** A positive phase added to current means a current lead; identify the capacitor before applying  $\tan \phi = X_C/R$ .

**11. A plane wavefront is incident on a concave mirror of radius of curvature  $R$ . The radius of the refracted wavefront will be:** [1 mark]

(A)  $2R$

(B)  $R$

(C)  $\frac{R}{2}$

(D)  $\frac{R}{4}$

**Correct Answer:** (C)  $\frac{R}{2}$

**Solution:**

**Concept:**

- A plane incident wavefront represents parallel rays, which a concave mirror brings to its principal focus in the paraxial approximation.
- For a spherical mirror, the focal-length magnitude is  $f = R/2$ , where  $R$  is the mirror radius of curvature.

**Step 1: Identify the outgoing wavefront** The printed word “refracted” is understood here as “reflected”, since the optical element is a mirror. Reflection of a plane wavefront produces a converging spherical wavefront centred on the principal focus.

**Step 2: Relate the wavefront radius to the focus** For the reflected wavefront just leaving the mirror near its pole, the distance to its centre of curvature is the focal length. Thus its radius is

$$r_{\text{wave}} = f = \frac{R}{2}.$$

The mirror radius  $R$  is not the outgoing wavefront radius. Neither  $2R$  nor  $R/4$  places the wavefront centre at the mirror focus. As the wave propagates towards the focus its radius decreases; the question uses the usual radius at reflection.

**Step 3: Conclusion** The reflected wavefront has radius  $R/2$  at the mirror. Hence, the correct option is (C).

**Quick Tip to Solve:** Locate the centre of the outgoing spherical wavefront at the focus, then use  $f = R/2$ , not the mirror radius itself.

**12. A proton and an alpha particle have the same kinetic energy. The ratio of de Broglie wavelengths associated with the proton to that with the alpha particle is:** [1 mark]

- (A) 1
- (B) 2
- (C)  $2\sqrt{2}$
- (D)  $\frac{1}{2}$

**Correct Answer:** (B) 2

**Solution:**

**Concept:**

- The de Broglie wavelength is  $\lambda = h/p$ , with Planck constant  $h$  and particle momentum  $p$ .
- For non-relativistic kinetic energy  $K$ ,  $p = \sqrt{2mK}$ ; at fixed  $K$ , wavelength varies as  $1/\sqrt{m}$ .

**Step 1: Write the wavelength ratio** Both particles have the same kinetic energy, so  $h$  and  $\sqrt{2K}$  cancel when taking their wavelength ratio:

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_p}}.$$

The alpha particle contains two protons and two neutrons, giving  $m_\alpha \simeq 4m_p$ .

**Step 2: Substitute the mass ratio** Using the standard nuclear-mass approximation gives

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{4} = 2.$$

Equal kinetic energy does not imply equal momentum, so (A) fails. The square-root dependence gives 2 rather than  $2\sqrt{2}$ , excluding (C); (D) reverses the requested ratio. The heavier alpha particle has the shorter wavelength.

**Step 3: Conclusion** The proton-to-alpha wavelength ratio is 2 : 1. Hence, the correct option is (B).

**Quick Tip to Solve:** At equal kinetic energy, put the other particle mass in the numerator under the square root when forming a wavelength ratio.

13. The potential energy of an electron in the second excited state in hydrogen atom is: [1 mark]

- (A)  $-3.4 \text{ eV}$
- (B)  $-3.02 \text{ eV}$
- (C)  $-1.51 \text{ eV}$
- (D)  $-6.8 \text{ eV}$

**Correct Answer:** (B)  $-3.02 \text{ eV}$

**Solution:**

**Concept:**

- Hydrogen energy levels have total energy  $E_n = -13.6/n^2 \text{ eV}$ , where  $n$  is the principal quantum number.
- For the Coulomb-bound electron, potential energy is  $U_n = 2E_n = -27.2/n^2 \text{ eV}$ , whereas kinetic energy is positive.

**Step 1: Identify the second excited level** The ground state is  $n = 1$ , the first excited state is  $n = 2$ , and the second excited state is  $n = 3$ . The phrase “second excited” must therefore not be replaced by  $n = 2$ .

**Step 2: Calculate the potential energy** Substitute  $n = 3$  into the potential-energy expression:

$$U_3 = -\frac{27.2}{3^2} \text{ eV} = -3.02 \text{ eV} \quad (\text{approximately}).$$

Option (C),  $-1.51 \text{ eV}$ , is the total energy of this level, not its potential energy. The values in (A) and (D) correspond respectively to the total and potential energies at  $n = 2$ , the first excited state.

**Step 3: Conclusion** The potential energy is approximately  $-3.02 \text{ eV}$ . Hence, the correct option is (B).

**Quick Tip to Solve:** Convert “second excited” to  $n = 3$ , then double the negative total energy to obtain the potential energy.

14. The difference in mass of  ${}^7\text{X}$  nucleus and total mass of its constituent nucleons is  $21.00 \text{ u}$ . The binding energy per nucleon for this nucleus is equal to the energy equivalent of: [1 mark]

- (A)  $3 \text{ u}$
- (B)  $3.5 \text{ u}$
- (C)  $7 \text{ u}$
- (D)  $21 \text{ u}$

**Correct Answer:** (A)  $3 \text{ u}$

**Solution:**

**Concept:**

- The binding energy of a nucleus is  $B = \Delta m c^2$ , where  $\Delta m$  is its mass defect and  $c$  is the speed of light.

- Binding energy per nucleon is  $B/A = (\Delta m/A)c^2$ , where the superscript mass number  $A$  counts all nucleons.

**Step 1: Read the printed mass defect and mass number** The question prints a mass difference of 21.00 u and the nucleus  ${}^7X$ , so use  $\Delta m = 21.00$  u and  $A = 7$ . The mass defect given is for the whole nucleus, not for each nucleon.

**Step 2: Divide the total binding energy by seven** The requested quantity is

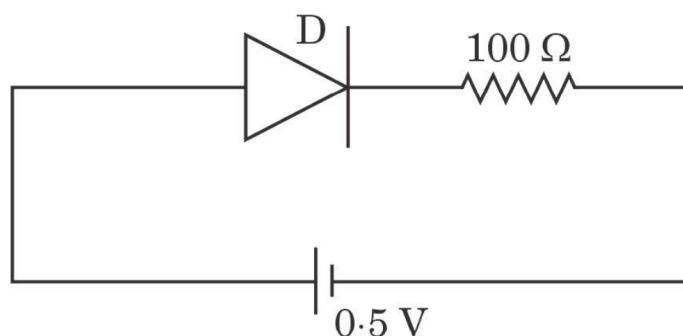
$$\frac{B}{A} = \frac{21.00 \text{ u}}{7} c^2 = (3.00 \text{ u}) c^2.$$

Thus the comparison is with the energy equivalent of a mass of 3 u. Option (D) uses the whole nuclear mass defect; (B) and (C) do not divide by the printed nucleon count. Although the printed mass defect is physically unrealistic for a seven-nucleon nucleus, the answer follows algebraically from the stated data.

**Step 3: Conclusion** Using the printed data, the binding energy per nucleon equals the energy equivalent of 3 u. Hence, the correct option is (A).

**Quick Tip to Solve:** When options give masses instead of energies, compare  $\Delta m/A$ ; the common factor  $c^2$  need not be evaluated.

**15. The threshold voltage for a p-n junction diode used in the circuit is 0.7 V. The type of biasing and current in the circuit are:** [1 mark]



- (A) Forward biasing, 0 A
- (B) Reverse biasing, 0 A
- (C) Forward biasing, 5 mA
- (D) Reverse biasing, 2 mA

**Correct Answer:** (A) Forward biasing, 0 A

**Solution:**

**Concept:**

- A diode is forward biased when its anode is connected towards the positive supply terminal and its cathode towards the negative terminal.
- In the threshold model, appreciable forward current begins only when the applied forward voltage reaches the threshold voltage.

**Step 1: Check the battery and diode polarities** The longer battery line on the left is the positive

terminal. It connects to the diode anode; the cathode, marked by the vertical bar on the right, returns through the resistor to the negative terminal. The diode is therefore forward biased, ruling out (B) and (D).

**Step 2: Compare the supply with the threshold** The supply is only 0.5 V, below the threshold 0.7 V. In the threshold approximation used here, the diode does not conduct and the current is zero. The calculation  $0.5/100 = 5 \text{ mA}$  in (C) incorrectly treats the diode as an already conducting ideal short circuit. Forward bias by itself does not guarantee appreciable current below threshold.

**Step 3: Conclusion** The diode is forward biased and the circuit current is approximately 0 A in the threshold model. Hence, the correct option is (A).

**Quick Tip to Solve:** Determine bias from terminal polarity first; then compare the source voltage with 0.7 V before using Ohm's law.

**16. Assertion (A):** When three electric bulbs of power 200 W, 100 W and 50 W are connected in series to a source, the power consumed by the 50 W bulb is maximum.

**Reason (R):** In a series circuit, current is the same through each bulb, but the potential difference across each bulb is different. [1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

**Correct Answer:** (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

### Solution:

#### Concept:

- For bulbs rated at a common voltage  $V_r$ , their resistances satisfy  $R = V_r^2/P_r$ , where  $P_r$  is rated power.
- In series the same current  $I$  passes through every bulb, so actual power is  $P = I^2 R = IV$ , with  $V = IR$  the drop across that bulb.

**Step 1: Evaluate the assertion** Using the usual common rated voltage and fixed-resistance bulb model, the resistances are

$$R_{200} : R_{100} : R_{50} = \frac{1}{200} : \frac{1}{100} : \frac{1}{50} = 1 : 2 : 4.$$

Since their series current is equal, actual powers are also in the ratio 1 : 2 : 4. The bulb rated 50 W therefore consumes the greatest actual power. Assertion (A) is true.

**Step 2: Evaluate the reason** The bulbs form one series path, so charge conservation requires the same current through each. Their resistances differ, giving different voltage drops according to  $V = IR$ . Reason (R) is therefore true, excluding options (C) and (D).

**Step 3: Connect the reason to the assertion** The common current and differing voltage drops explain the unequal powers:  $P = IV$ . The 50 W bulb has the largest resistance, hence the largest voltage drop and power at this common current. Thus the series-circuit property stated in the reason explains the

assertion; option (B) incorrectly denies this connection.

**Step 4: Conclusion** Both Assertion (A) and Reason (R) are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

**Quick Tip to Solve:** For equal rated voltage, the lowest rated wattage means the highest resistance; at a common series current that bulb dissipates the most power.

**17. Assertion (A):** A current carrying square loop made of a wire of length  $L$  is placed in a magnetic field. It experiences a torque which is greater than the torque on a circular loop made of the same wire carrying the same current in the same magnetic field.

**Reason (R):** A square loop occupies more area than a circular loop, both made of wire of the same length. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false and Reason (R) is also false.

**Correct Answer:** (D) Assertion (A) is false and Reason (R) is also false.

**Solution:**

**Concept:**

- The torque on a single current loop is  $\tau = IAB \sin \theta$ , where  $I$  is current,  $A$  is enclosed area and  $\theta$  is the angle between the area vector and magnetic field  $B$ .
- For equal wire lengths (equal perimeters), compare the enclosed areas; a circle encloses more area than a square.

**Step 1: Evaluate the assertion** A square made from wire of length  $L$  has side  $L/4$ , so  $A_s = L^2/16$ . A circle has radius  $L/(2\pi)$ , so  $A_c = L^2/(4\pi)$ . For the same orientation and the stated equal current and field,

$$\frac{\tau_s}{\tau_c} = \frac{A_s}{A_c} = \frac{\pi}{4} < 1.$$

Thus the square does not produce greater torque. The same comparison applies to maximum torques; the assertion is false.

**Step 2: Evaluate the reason** Since  $4\pi < 16$ , we have  $L^2/(4\pi) > L^2/16$ . The circular loop, rather than the square loop, encloses the larger area. The reason is therefore false.

**Step 3: Check the explanation and answer code** A false area comparison cannot explain the torque. Codes (A) and (B) require both statements to be true, while (C) requires the assertion to be true; none matches these results.

**Step 4: Conclusion** Both the assertion and the reason are false. Hence, the correct option is (D).

**Quick Tip to Solve:** With a fixed wire length, first calculate the enclosed areas; for equal orientations the torque ratio is exactly the area ratio.

**18. Assertion (A):** In 'n' type semiconductor, number density of electrons is greater than the number density of holes but the crystal maintains an overall charge neutrality.

**Reason (R):** The charge of electrons donated by donor atoms is just equal and opposite to that of the ionised donor. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).  
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).  
(C) Assertion (A) is true, but Reason (R) is false.  
(D) Assertion (A) is false and Reason (R) is also false.

**Correct Answer:** (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

**Solution:**

**Concept:**

- Pentavalent donor impurities supply mobile electrons to an  $n$ -type semiconductor; electrons are the majority carriers and holes are the minority carriers.
- Charge neutrality includes mobile carriers and fixed ionised impurities. An ionised donor carries charge  $+e$ , balancing the donated electron's charge  $-e$ .

**Step 1: Evaluate the assertion** Each ionised donor supplies an additional conduction electron. Thus the electron number density  $n$  exceeds the hole number density  $p$ . Nevertheless, the donor ion left in the lattice is positively charged, so an excess of mobile electrons does not imply a net negative charge of the entire crystal. The assertion is true.

**Step 2: Evaluate the reason** When a neutral donor releases one electron of charge  $-e$ , it becomes a fixed donor ion of charge  $+e$ . These charges have equal magnitudes and opposite signs, so their sum is zero. The reason is true.

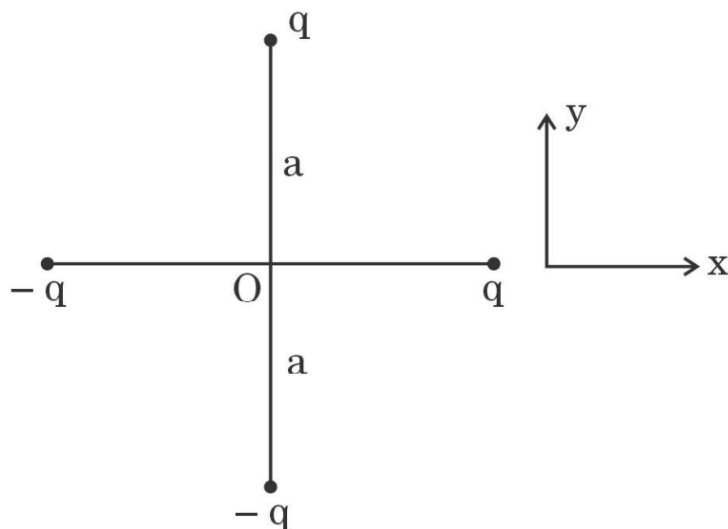
**Step 3: Connect the reason to the assertion** In a donor-doped crystal without acceptors, neutrality gives  $n = p + N_D^+$ , where  $N_D^+$  is the density of ionised donors. Hence  $n > p$  is consistent with zero total charge precisely because the positive donor ions balance the additional electrons. The reason explains the assertion; code (B) incorrectly denies this link, and (C) and (D) wrongly reject true statements.

**Step 4: Conclusion** Both statements are true, and the reason correctly explains the assertion. Hence, the correct option is (A).

**Quick Tip to Solve:** When checking neutrality in a doped semiconductor, count the fixed donor ions as well as the mobile electrons and holes.

### Section B (14 marks)

19. Two identical dipoles are arranged in x-y plane as shown in the figure. Find the magnitude and the direction of net electric field at the origin O. [2 marks]



#### Solution:

##### Concept:

- A point charge produces electric field of magnitude  $E = k|q|/r^2$ , where  $k = 1/(4\pi\epsilon_0)$ ,  $q$  is charge and  $r$  is distance from it. The field points away from a positive charge and towards a negative charge.
- Superposition gives the net field as the vector sum of all individual fields. Perpendicular components satisfy  $E = \sqrt{E_x^2 + E_y^2}$ .

**Step 1: Add the fields from the horizontal dipole** The identical dipoles have the same charge magnitude and separation  $2a$ ; O is the common midpoint shown. Each charge is therefore a distance  $a$  from O. At O, the field of the positive charge on the right points left, and the field of the negative charge on the left also points left. Thus

$$\vec{E}_x = -\frac{2kq}{a^2} \hat{i}.$$

Here  $q$  denotes the positive magnitude of either charge, and  $\hat{i}$  points along positive x.

**Step 2: Add the fields from the vertical dipole** The positive charge above O produces a downward field at O, as does the negative charge below O. With  $\hat{j}$  along positive y,

$$\vec{E}_y = -\frac{2kq}{a^2} \hat{j}, \quad \vec{E} = -\frac{2kq}{a^2} (\hat{i} + \hat{j}).$$

The equal negative components give a resultant along the bisector between the negative x- and negative y-axes. Its magnitude follows from Pythagoras, not by scalar addition of the two dipole fields.

**Step 3: Conclusion** The net field has magnitude  $E = \frac{2\sqrt{2}kq}{a^2} = \frac{2\sqrt{2}q}{4\pi\epsilon_0 a^2}$ . It points into the third quadrant, at  $225^\circ$  measured anticlockwise from the positive x-axis ( $45^\circ$  below the negative x-axis).

**Quick Tip to Solve:** At a dipole's midpoint, both charge fields point from positive to negative; add the two perpendicular dipole fields as vectors.

**20. Write two differences between the emf and terminal potential difference of a cell. What is the most important precaution that one should take while drawing current from a cell?** [2 marks]

**Solution:**

**Concept:**

- The emf  $\varepsilon$  of a cell is the energy supplied by its source per unit charge passing through it; terminal potential difference  $V$  is the energy transferred per unit charge between its terminals.
- For a cell supplying current  $I$  with internal resistance  $r$ ,  $V = \varepsilon - Ir$ . The term  $Ir$  is the internal potential drop.

**Step 1: Distinguish the physical meanings** The emf measures the work done by the cell's chemical source per unit charge in driving charge around the complete circuit. The terminal potential difference measures the energy available per unit charge to the external circuit between the two terminals. Both are measured in volts, but some of the energy supplied by the source is dissipated inside the cell when current flows.

**Step 2: Compare their dependence on current** For a cell under fixed physical and chemical conditions, its emf is a source property and equals its open-circuit terminal voltage. The terminal potential difference during discharge decreases as the drawn current increases, according to  $V = \varepsilon - Ir$ . Consequently  $V < \varepsilon$  when the cell supplies current, whereas  $V = \varepsilon$  when no current is drawn.

**Step 3: State the essential precaution** Never short-circuit the cell by directly connecting its terminals with a low-resistance wire. Use a suitable external resistance to limit the current. In a short circuit the current approaches  $\varepsilon/r$ , which can be very large and cause excessive internal heating and damage to the cell.

**Step 4: Conclusion** Emf is the source energy supplied per unit charge and equals the open-circuit voltage; terminal potential difference is the energy available per unit charge externally and drops by  $Ir$  during discharge. The essential precaution is to avoid short-circuiting the cell or drawing excessive current.

**Quick Tip to Solve:** Write  $\varepsilon = V + Ir$ : it separates the cell's energy supply into the useful external voltage and the internal voltage loss.

**21.** A small magnetised needle P is placed at the origin of x-y plane with its magnetic moment pointing along the y-axis. Another identical magnetised needle Q is placed in two positions, one by one.

Case 1: at  $(a, 0)$  with its magnetic moment pointing along x-axis.

Case 2: at  $(0, a)$  with its magnetic moment pointing along y-axis.

(a) In which case is the potential energy of P and Q minimum?

(b) In which case is P and Q not in equilibrium?

Justify your answers.

[2 marks]

### Solution:

#### Concept:

- The interaction energy of a magnetic moment  $\vec{m}$  in an external magnetic field  $\vec{B}$  is  $U = -\vec{m} \cdot \vec{B}$ , and its torque is  $\vec{\tau} = \vec{m} \times \vec{B}$ .
- For a short magnetic dipole of moment magnitude  $m$ , the axial field at distance  $a$  is  $2Cm/a^3$  along its moment, and the equatorial field is  $Cm/a^3$  opposite its moment, where  $C = \mu_0/(4\pi)$ .

**Step 1: Determine the energy and torque in Case 1** P has moment  $m\hat{j}$ . The position  $(a, 0)$  of Q is on P's equatorial line, so P's field there is  $\vec{B}_P = -Cm\hat{j}/a^3$ . Since Q has moment  $m\hat{i}$ , its moment is perpendicular to this field:

$$U_1 = -\vec{m}_Q \cdot \vec{B}_P = 0, \quad |\tau_Q| = \frac{Cm^2}{a^3} \neq 0.$$

Similarly, Q's field at P points along x and is perpendicular to P's moment. Both needles therefore experience a turning effect and this arrangement is not in rotational equilibrium.

**Step 2: Determine the energy and alignment in Case 2** Q at  $(0, a)$  is on P's axial line. P's field is  $\vec{B}_P = 2Cm\hat{j}/a^3$ , parallel to Q's moment. Therefore

$$U_2 = -\frac{2Cm^2}{a^3} < U_1, \quad \tau_Q = 0.$$

P's moment is likewise aligned with Q's field, so its torque is zero. At the specified separation, this parallel axial arrangement has minimum interaction energy and stable orientational equilibrium. These comparisons concern the needles' orientations at their stated positions.

**Step 3: Conclusion** (a) The potential energy is minimum in Case 2:  $U_2 = -\mu_0 m^2/(2\pi a^3)$ , compared with  $U_1 = 0$ . (b) Case 1 is not in rotational equilibrium because the magnetic moments are perpendicular to the respective fields and experience nonzero torques.

**Quick Tip to Solve:** First find the direction of one needle's field at the other; compare the second moment with that local field to decide both energy and torque.

**22(a).** What is a displacement current? How is it different from a conduction current?

[2 marks]

### Solution:

#### Concept:

- A changing electric flux produces a magnetic field through the displacement-current term  $I_d = \epsilon_0 d\Phi_E/dt$  in vacuum, where  $\Phi_E = \int \vec{E} \cdot d\vec{A}$  is electric flux through the chosen surface.
- Conduction current  $I_c = dQ/dt$  is the rate of transport of electric charge through a surface by

mobile charge carriers.

**Step 1: Define displacement current** Displacement current is the current-equivalent associated with a time-varying electric field or electric flux. In vacuum its value is  $I_d = \varepsilon_0 d\Phi_E/dt$ . Maxwell introduced it in the Ampere-Maxwell law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(I_c + I_d),$$

so that the magnetic field is accounted for even where no charge carriers pass across the surface.

**Step 2: Distinguish its origin and location from conduction current** Conduction current arises from the actual directed motion of free charge carriers, such as electrons in a metal wire, and requires mobile charges. Displacement current arises from changing electric flux and can exist in vacuum, without conduction charge crossing the gap. For example, during charging of a capacitor, conduction current flows in its connecting wires, while the changing electric field between vacuum-separated plates produces displacement current. In the ideal capacitor, their magnitudes are equal, maintaining continuity of the total current.

**Step 3: Conclusion** Displacement current is  $I_d = \varepsilon_0 d\Phi_E/dt$  in vacuum and comes from changing electric flux. Conduction current comes from the motion of free charges; displacement current can exist where no charges cross, such as the vacuum gap of a charging capacitor.

**Quick Tip to Solve:** For a charging capacitor, identify conduction current in the wires and displacement current in the gap; both contribute to magnetic-field production.

OR

**22(b). Write any two characteristics of an electromagnetic wave. Why are microwaves used in radar systems?** [2 marks]

**Solution:**

**Concept:**

- An electromagnetic wave consists of time-varying electric and magnetic fields that propagate together and transport energy.
- The diffraction of a wave depends on its wavelength relative to the transmitting aperture: a shorter wavelength permits a narrower directed beam.

**Step 1: State two characteristics** First, electromagnetic waves are transverse: the electric field  $\vec{E}$ , magnetic field  $\vec{B}$ , and direction of propagation are mutually perpendicular. The two fields oscillate in phase. Second, they need no material medium and propagate through vacuum with speed

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m s}^{-1}.$$

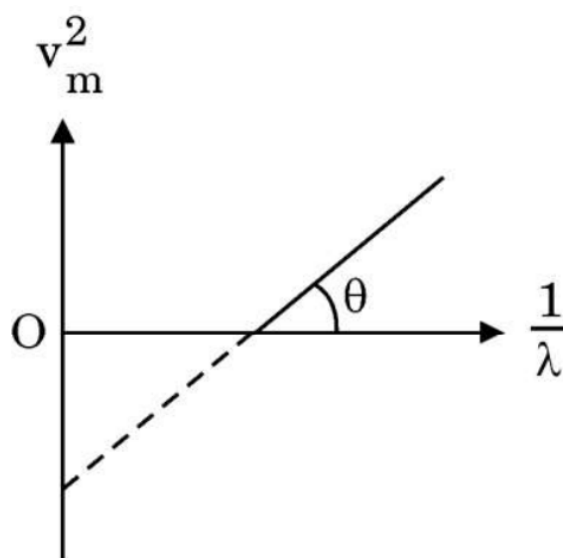
Here  $\mu_0$  and  $\varepsilon_0$  are the permeability and permittivity of free space.

**Step 2: Relate microwave wavelength to radar operation** Microwaves have short wavelengths, so a suitably sized antenna can transmit them as a narrow beam with little diffraction. This gives good directional resolution, allowing a radar to locate a target accurately. They are also reflected by conducting targets such as aircraft; the returning echo is detected to determine the target's position and distance.

**Step 3: Conclusion** Electromagnetic waves are transverse and can travel through vacuum at  $c$ . Microwaves are used in radar because their short wavelengths allow narrow, well-directed beams and detectable reflected echoes.

**Quick Tip to Solve:** Link the use of microwaves in radar directly to short wavelength, small diffraction, and accurate directional detection.

**23.** The figure shows  $v_m^2$  versus  $\frac{1}{\lambda}$  graph for photoelectrons emitted from a surface where  $v_m$  is the maximum speed of electrons and  $\lambda$  is the wavelength of incident radiation. Using this graph and Einstein's photoelectric equation, obtain the expression for Planck's constant and work function of the surface. [2 marks]



**Solution:**

**Concept:**

- Einstein's photoelectric equation is  $\frac{1}{2}m_e v_m^2 = h\nu - \phi$ , where  $m_e$  is electron mass,  $h$  is Planck's constant, and  $\phi$  is the work function.
- Since  $\nu = c/\lambda$ , plotting  $v_m^2$  against  $1/\lambda$  gives a straight line. Its slope and extrapolated vertical intercept identify the constants.

**Step 1: Put Einstein's equation in straight-line form** Substitute the incident photon energy  $hc/\lambda$  and rearrange:

$$\frac{1}{2}m_e v_m^2 = \frac{hc}{\lambda} - \phi, \quad v_m^2 = \frac{2hc}{m_e} \left( \frac{1}{\lambda} \right) - \frac{2\phi}{m_e}.$$

Comparing with  $y = Sx + b$ , the ordinate is  $y = v_m^2$ , the abscissa is  $x = 1/\lambda$ , and the slope is  $S = 2hc/m_e$ .

**Step 2: Read the slope and intercepts** Let  $S = \Delta v_m^2 / \Delta(1/\lambda)$  be the slope measured using the axis scales. It is represented by  $\tan \theta$  in the given graph. Thus  $h = m_e S / (2c)$ . If the extrapolated line cuts the vertical axis at  $-B$ , where  $B > 0$ , comparison gives  $B = 2\phi/m_e$ . Equivalently, if the horizontal intercept is  $x_0 = 1/\lambda_0$ , then  $v_m = 0$  there and  $\phi = hc x_0 = hc/\lambda_0$ . The dashed negative portion is only an extrapolation, not a physical negative squared speed.

**Step 3: Conclusion**

$$h = \frac{m_e S}{2c} = \frac{m_e \tan \theta}{2c}, \quad \phi = \frac{m_e B}{2} = \frac{hc}{\lambda_0}.$$

Here  $B$  is the magnitude of the negative vertical intercept, and  $S$  is the slope evaluated with the graph's axis scales.

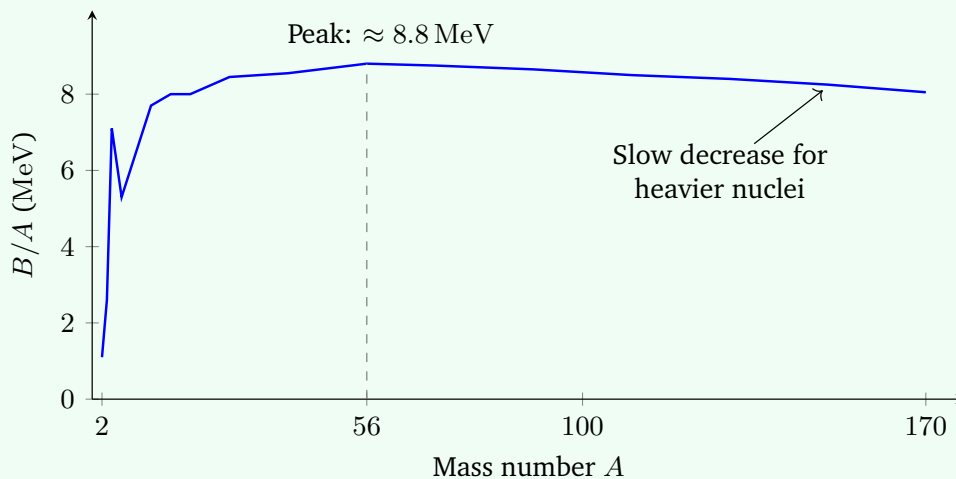
**Quick Tip to Solve:** Write the equation as ordinate equals slope times abscissa plus intercept; the work function uses the magnitude of the negative vertical intercept.

**24. Draw the graph showing the variation of binding energy per nucleon with mass number  $A$  of nuclei ( $2 < A < 170$ ). Use this graph to explain the release of energy in nuclear fission.** [2 marks]

**Solution:****Concept:**

- Binding energy per nucleon is  $B/A$ , where  $B$  is the total nuclear binding energy and  $A$  is the mass number. A larger value indicates a more tightly bound nucleus.
- Nuclear energy release equals the increase in total binding energy:  $Q = B_{\text{products}} - B_{\text{initial}}$ , equivalently the decrease in rest mass times  $c^2$ .

**Step 1: Draw and interpret the binding-energy curve** The schematic curve rises rapidly for light nuclei, with local irregularities, and reaches a broad maximum of about 8.8 MeV per nucleon near  $A \simeq 56$ . It then falls slowly towards larger mass numbers. The requested range  $2 < A < 170$  is shown below; the heavy-nucleus trend continues beyond this range.



*Schematic binding energy per nucleon for the requested mass-number range.*

**Step 2: Explain energy release in fission** In fission, a heavy nucleus splits into two medium-mass nuclei, usually with some free neutrons. The daughter nuclei lie closer to the broad peak and have greater binding energy per nucleon than the parent. The increase in their combined binding energy means that the total rest mass of the products is smaller. Hence

$$Q = (M_{\text{initial}} - M_{\text{products}})c^2 > 0.$$

All participating particles are included in these masses. The released energy appears mainly as kinetic energy of the fragments and neutrons, with some radiation.

**Step 3: Conclusion** The curve peaks near the iron region and declines slowly for heavier nuclei. Fission releases energy because the medium-mass products are more tightly bound and have larger total binding energy than the initial system.

**Quick Tip to Solve:** On the heavy side of the curve, move towards medium mass numbers: the rise in binding energy per nucleon explains why fission releases energy.

**25(a).** Obtain an expression for electrostatic potential energy of a system of three charges  $q$ ,  $2q$  and  $-3q$  placed at the vertices of an equilateral triangle of side  $a$ . [2 marks]

**Solution:**

**Concept:**

- Electrostatic potential energy is the work needed to assemble the charges from infinite separation, taking their initial potential energy as zero.
- For point charges, the total energy is the sum over distinct pairs:  $U = \sum_{i < j} kq_i q_j / r_{ij}$ , where  $k = 1/(4\pi\epsilon_0)$ . Charge signs must be retained.

**Step 1: Identify the three interactions** Write the charges at vertices  $A, B, C$  as  $q, 2q, -3q$ , respectively. Since the triangle is equilateral, every separation is  $a$ . There are exactly three distinct pairs:  $AB$ ,  $AC$ , and  $BC$ . The first pair is repulsive and contributes positive energy; each pair containing the negative charge contributes negative energy.

$$U_{AB} = \frac{2kq^2}{a}, \quad U_{AC} = -\frac{3kq^2}{a}, \quad U_{BC} = -\frac{6kq^2}{a}.$$

**Step 2: Add the pair energies once each** The total energy is

$$\begin{aligned} U &= U_{AB} + U_{AC} + U_{BC} \\ &= \frac{kq^2}{a}(2 - 3 - 6). \end{aligned}$$

No extra factor of one-half is needed because each pair has already been counted exactly once. In an assembly calculation, placing the first charge costs no work, adding the second costs  $2kq^2/a$ , and adding the third costs  $-9kq^2/a$ , confirming the same sum.

**Step 3: Conclusion**

$$U = -\frac{7kq^2}{a} = -\frac{7q^2}{4\pi\epsilon_0 a}.$$

The negative sign means the assembled system has lower potential energy than the charges at infinite separation.

**Quick Tip to Solve:** For three charges at equal separations, factor out  $k/a$  and add the three signed products; never replace the charges by their magnitudes.

**25(b).** Two small conducting balls A and B of radius  $r_1$  and  $r_2$  have charges  $q_1$  and  $q_2$  respectively. They are connected by a wire. Obtain the expression for charges on A and B, in equilibrium. [2 marks]

**Solution:**

**Concept:**

- Connected conductors attain a common electrostatic potential in equilibrium; charge flows only while their potentials differ.
- For a small conducting sphere of radius  $r$ , neglecting mutual influence, its surface potential is  $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$ , where  $Q$  is its charge.
- Total charge is conserved when the two balls and their connecting wire form an isolated system; the thin wire is taken to store negligible charge.

**Step 1: Apply the equilibrium condition** Let the final charges on balls A and B be  $Q_A$  and  $Q_B$ . In the usual small-sphere approximation, their potentials become equal:

$$\frac{Q_A}{4\pi\epsilon_0 r_1} = \frac{Q_B}{4\pi\epsilon_0 r_2}, \quad \frac{Q_A}{Q_B} = \frac{r_1}{r_2}.$$

Thus the charges redistribute in proportion to the radii; they need not become equal. Equality of charge follows only for equal radii.

**Step 2: Combine with conservation of charge** The initial total charge is  $q_1 + q_2$ , including the signs of both charges. Therefore

$$Q_A + Q_B = q_1 + q_2.$$

Writing  $Q_A = r_1 K$  and  $Q_B = r_2 K$ , the common proportionality constant satisfies  $K(r_1 + r_2) = q_1 + q_2$ . This also covers a negative total charge. If the initial total is zero, both final charges vanish in this approximation.

**Step 3: Conclusion** The equilibrium charges are

$$Q_A = \frac{r_1(q_1 + q_2)}{r_1 + r_2}, \quad Q_B = \frac{r_2(q_1 + q_2)}{r_1 + r_2}.$$

**Quick Tip to Solve:** Use equal potentials, not equal charges: first write  $Q_A/r_1 = Q_B/r_2$ , then conserve the signed total charge.

### Section C (15 marks)

**26.** Two circular loops A and B, each of radius 3 m, are placed coaxially at a distance of 4 m. They carry currents of 3 A and 2 A in opposite directions respectively. Find the net magnetic field at the centre of loop A. [3 marks]

#### Solution:

##### Concept:

- A circular loop of radius  $R$ , carrying current  $I$ , produces axial field  $B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$  at axial distance  $x$  from its centre.
- At the loop centre,  $x = 0$ , so  $B = \mu_0 I / (2R)$ . The right-hand rule gives its axial direction.
- Magnetic fields superpose as vectors. Coaxial loops with currents circulating in opposite senses have opposing axial fields at the specified point.

**Step 1: Field of loop A at its own centre** Both loop radii are  $R = 3$  m. For loop A,  $I_A = 3$  A. Choose the positive axis along the field of A at its centre. Its magnitude is

$$B_A = \frac{\mu_0 I_A}{2R} = \frac{4\pi \times 10^{-7} \times 3}{2 \times 3} = 2\pi \times 10^{-7} \text{ T.}$$

**Step 2: Field of loop B at the centre of A** The centre of A is  $x = 4$  m from the centre of B along the common axis. With  $I_B = 2$  A,

$$\begin{aligned} B_B &= \frac{\mu_0 I_B R^2}{2(R^2 + x^2)^{3/2}} \\ &= \frac{4\pi \times 10^{-7} \times 2 \times 9}{2(9 + 16)^{3/2}} \\ &= \frac{36\pi}{125} \times 10^{-7} \text{ T.} \end{aligned}$$

Here  $(9 + 16)^{3/2} = 125 \text{ m}^3$ . Because its current is oppositely directed, B's field points along the negative chosen axis.

**Step 3: Superpose the opposing fields** The field of A is larger, so the magnitude of the resultant is their difference:

$$\begin{aligned} B_{\text{net}} &= B_A - B_B \\ &= \left(2 - \frac{36}{125}\right) \pi \times 10^{-7} \\ &= \frac{214\pi}{125} \times 10^{-7} \text{ T.} \end{aligned}$$

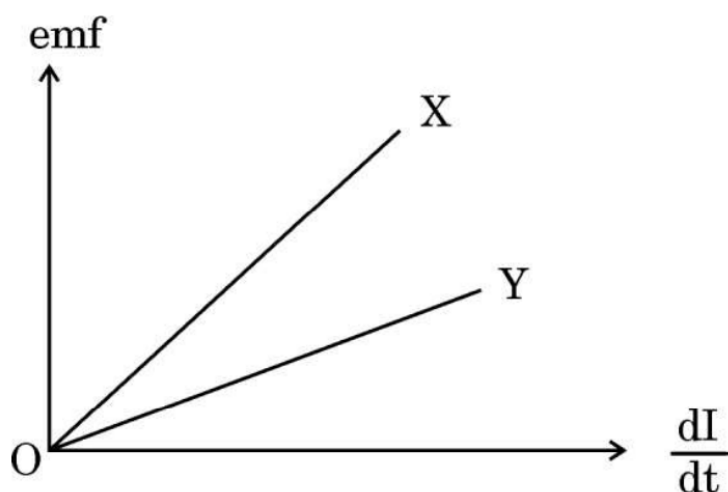
Subtraction is essential: the loops share an axis, but the opposite circulation of current makes their field vectors antiparallel.

**Step 4: Conclusion**  $B_{\text{net}} \approx 5.38 \times 10^{-7} \text{ T}$ , along the common axis in the direction of the field produced by loop A, as determined by the right-hand rule.

**Quick Tip to Solve:** Use the centre-field formula only for A; for B retain the axial separation  $x = 4$  m, then subtract the opposing fields.

27. (a) The figure shows the variation of induced emf as a function of rate of change of current for two identical solenoids X and Y. One is air cored and the other is iron cored. Which one of them is iron cored? Why?

(b) Obtain an expression for self-inductance of a long solenoid of length  $L$  and cross-sectional area  $A$  having  $N$  turns. [3 marks]



### Solution:

#### Concept:

- Self-induction obeys  $\mathcal{E} = -\mathcal{L} dI/dt$ , where  $\mathcal{L}$  is self-inductance and the minus sign expresses Lenz's law. The slope of a graph of emf magnitude against  $dI/dt$  gives  $\mathcal{L}$ .
- For a long solenoid, the nearly uniform internal magnetic field is  $B = \mu nI$ , where  $n = N/L$  is turns per unit length and  $\mu$  is the core permeability.
- Self-inductance is flux linkage per unit current:  $\mathcal{L} = N\Phi/I$ , with  $\Phi$  the magnetic flux through each turn. The symbol  $L$  here denotes solenoid length, not inductance.

**Step 1: (a) Interpret the two slopes** The printed graph shows a steeper straight line for X than for Y. Treating the plotted induced emf as its magnitude,

$$\frac{|\mathcal{E}|}{|dI/dt|} = \mathcal{L}, \quad \mathcal{L}_X > \mathcal{L}_Y.$$

For identical coil dimensions and number of turns, inductance is proportional to core permeability. Iron has a much higher permeability than air, increasing the magnetic flux linkage for the same current. Thus the greater slope identifies X as the iron-cored solenoid; Y is air-cored. The negative sign in the signed emf law specifies opposition to changing current and does not reverse this comparison of magnitudes.

**Step 2: (b) Calculate the flux linkage** Neglect end effects for the long solenoid of length  $L$ , area  $A$ , and  $N$  turns. Its internal field is

$$B = \mu \frac{N}{L} I.$$

Since the field is normal to the area of each turn, the flux through one turn is  $\Phi = BA = \mu NIA/L$ . Each of the  $N$  turns links approximately the same flux, giving

$$N\Phi = \frac{\mu N^2 AI}{L}.$$

**Step 3: (b) Identify the self-inductance** Comparing  $N\Phi = \mathcal{L}I$  with the flux linkage above gives  $\mathcal{L} = \mu N^2 A/L$ . For an air or vacuum core, take  $\mu = \mu_0$ ; for a linear magnetic core,  $\mu = \mu_0 \mu_r$ . The expression assumes a fixed geometry and approximately constant permeability over the current range considered.

**Step 4: Conclusion** (a) X is iron-cored, because its larger emf-versus-current-rate slope means greater self-inductance. (b) For air,

$$\mathcal{L} = \frac{\mu_0 N^2 A}{L}; \quad \text{with a core,} \quad \mathcal{L} = \frac{\mu_0 \mu_r N^2 A}{L}.$$

**Quick Tip to Solve:** Read inductance from the slope, then derive it by counting flux linkage  $N\Phi$ , not just the flux through one turn.

**28(a).** A resistor of  $30 \, \Omega$  and a capacitor of  $\frac{250}{\pi} \, \mu\text{F}$  are connected in series to a 200 V, 50 Hz ac source. Calculate (i) the current in the circuit, and (ii) voltage drops across the resistor and the capacitor. (iii) Is the algebraic sum of these voltages more than the source voltage? If yes, solve the paradox. [3 marks]

**Solution:**

**Concept:**

- Capacitive reactance is  $X_C = 1/(2\pi fC)$ , where  $f$  is supply frequency and  $C$  is capacitance. A series RC circuit has impedance magnitude  $Z = \sqrt{R^2 + X_C^2}$ .
- Using rms values,  $I = V/Z$ ,  $V_R = IR$ , and  $V_C = IX_C$ . The same current flows through both series components.
- The resistor voltage is in phase with current, while the capacitor voltage lags current by  $90^\circ$ . Their voltage phasors are perpendicular, so  $V^2 = V_R^2 + V_C^2$ .

**Step 1: (i) Determine the reactance and circuit current** Convert the capacitance to farads:  $C = (250/\pi) \times 10^{-6} \text{ F}$ . With  $f = 50 \text{ Hz}$ ,

$$X_C = \frac{1}{2\pi \times 50 \times (250/\pi) \times 10^{-6}} = 40 \, \Omega.$$

Hence, for  $R = 30 \, \Omega$ ,

$$Z = \sqrt{30^2 + 40^2} = 50 \, \Omega, \quad I = \frac{200}{50} = 4 \text{ A}.$$

The stated supply voltage is interpreted as rms, as usual for an ac source, so this current is also rms.

**Step 2: (ii) Evaluate the component voltage drops** Multiplying the common rms current by the resistance and reactance respectively gives

$$V_R = IR = 4 \times 30 = 120 \text{ V}, \quad V_C = IX_C = 4 \times 40 = 160 \text{ V}.$$

These are magnitudes of alternating voltage drops, whose maxima occur at different times because of their phase difference.

**Step 3: (iii) Resolve the apparent voltage paradox** Their arithmetic sum is  $120 + 160 = 280 \text{ V}$ , which exceeds the 200 V supply. However, rms voltage magnitudes with different phases cannot be added

arithmetically to obtain the supply magnitude. The correct phasor addition is

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{120^2 + 160^2} = 200 \text{ V}.$$

Kirchhoff's voltage law remains valid for instantaneous signed voltages:  $v(t) = v_R(t) + v_C(t)$ . It does not imply that rms magnitudes add algebraically when the component voltages are out of phase. Thus there is no physical contradiction.

**Step 4: Conclusion**  $I = 4 \text{ A}$ ,  $V_R = 120 \text{ V}$ , and  $V_C = 160 \text{ V}$ , all rms. Their arithmetic sum exceeds the supply voltage, but their perpendicular phasor sum equals 200 V.

**Quick Tip to Solve:** Recognise the 30-40-50 impedance triangle; use the same right-triangle rule for the resistor and capacitor voltages.

OR

**28(b).** A series LCR circuit with  $R = 20 \Omega$ ,  $L = 2 \text{ H}$  and  $C = 50 \mu\text{F}$  is connected to a 200 volts ac source of variable frequency. What is (i) the amplitude of the current, and (ii) the average power transferred to the circuit in one complete cycle, at resonance? (iii) Calculate the potential drop across the capacitor. [3 marks]

**Solution:**

**Concept:**

- At series resonance,  $X_L = X_C$ , so the impedance is  $Z = R$ , the phase angle is zero, and the power factor is unity.
- An unspecified ac voltage rating is its rms value. For sinusoidal current,  $I_0 = \sqrt{2}I_{\text{rms}}$ ; average power is  $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}} \cos \phi$ .
- The resonance angular frequency is  $\omega_0 = 1/\sqrt{LC}$ , and the capacitor rms voltage is  $V_C = I_{\text{rms}}/(\omega_0 C)$ .

**Step 1: (i) Calculate the current amplitude at resonance** Take the stated source voltage as  $V_{\text{rms}} = 200 \text{ V}$ . The inductive and capacitive reactances cancel in the series impedance, leaving only the resistance  $R = 20 \Omega$ . Thus

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{200}{20} = 10 \text{ A}, \quad I_0 = \sqrt{2} \times 10 = 10\sqrt{2} \text{ A}.$$

Current amplitude means peak current, so it must be distinguished from the rms current used in power calculations.

**Step 2: (ii) Find the power averaged over a complete cycle** At resonance, voltage and current are in phase. The resistor dissipates energy while the ideal inductor and capacitor return their stored energy during the cycle; their average powers are zero. Consequently,

$$P_{\text{av}} = V_{\text{rms}}I_{\text{rms}} = 200 \times 10 = 2000 \text{ W}.$$

The same result follows from  $I_{\text{rms}}^2 R = 10^2 \times 20$ .

**Step 3: (iii) Calculate the capacitor voltage** Convert the capacitance to SI units,  $C = 50 \times 10^{-6} \text{ F}$ .

With  $L = 2 \text{ H}$ ,

$$\omega_0 = \frac{1}{\sqrt{2 \times 50 \times 10^{-6}}} = 100 \text{ rad s}^{-1},$$

$$X_C = \frac{1}{100 \times 50 \times 10^{-6}} = 200 \Omega,$$

$$V_{C,\text{rms}} = I_{\text{rms}} X_C = 10 \times 200 = 2000 \text{ V}.$$

This exceeds the source voltage because the equally large inductor voltage is opposite in phase to the capacitor voltage; their phasor sum vanishes at resonance. The capacitor's peak voltage is  $2000\sqrt{2} \text{ V}$ .

**Step 4: Conclusion** (i) The current amplitude is  $10\sqrt{2} \text{ A} \approx 14.1 \text{ A}$ . (ii) The average power is  $2000 \text{ W}$ . (iii) The capacitor potential drop is  $2000 \text{ V rms}$ .

**Quick Tip to Solve:** At resonance use  $Z = R$  for current, but retain  $X_C$  when finding capacitor voltage; convert rms current to peak only for the requested amplitude.

- 29(a).** (i) In diffraction due to a single slit, the phase difference between light waves reaching a point on the screen is  $5\pi$ . Explain whether a bright or a dark fringe will be formed at the point.
- (ii) What should the width ( $a$ ) of each slit be to obtain eight maxima of two double-slit patterns (slit separation  $d$ ) within the central maximum of the single slit pattern?
- (iii) Draw the plot of intensity distribution in a diffraction pattern due to a single slit. [3 marks]

**Solution:**

**Concept:**

- For a slit of width  $a$ , the phase difference between waves from its edges is  $\delta = 2\pi a \sin \theta / \lambda$ . Dark fringes occur when  $a \sin \theta = m\lambda$ , where  $m$  is a nonzero integer.
- The single-slit intensity is  $I = I_0 (\sin u / u)^2$ , with  $u = \pi a \sin \theta / \lambda$ ; its central maximum is twice as wide as each subsidiary lobe.
- At screen distance  $D$ , the central diffraction width is  $W = 2D\lambda/a$ , while double-slit interference fringe spacing is  $\beta = D\lambda/d$ .

**Step 1: (i) Test the given phase difference** The printed phase difference is  $\delta = 5\pi$ , so the edge-to-edge path difference is

$$a \sin \theta = \frac{\delta \lambda}{2\pi} = \frac{5\lambda}{2}.$$

Divide the slit into five equal zones, with adjacent zones differing in path by  $\lambda/2$ . Four zones cancel in pairs, leaving a nonzero contribution from one zone. The point therefore lies in a weak bright subsidiary fringe, not at a dark minimum. Quantitatively,

$$\frac{I}{I_0} = \left( \frac{\sin(5\pi/2)}{5\pi/2} \right)^2 = \frac{4}{25\pi^2} \approx 0.0162.$$

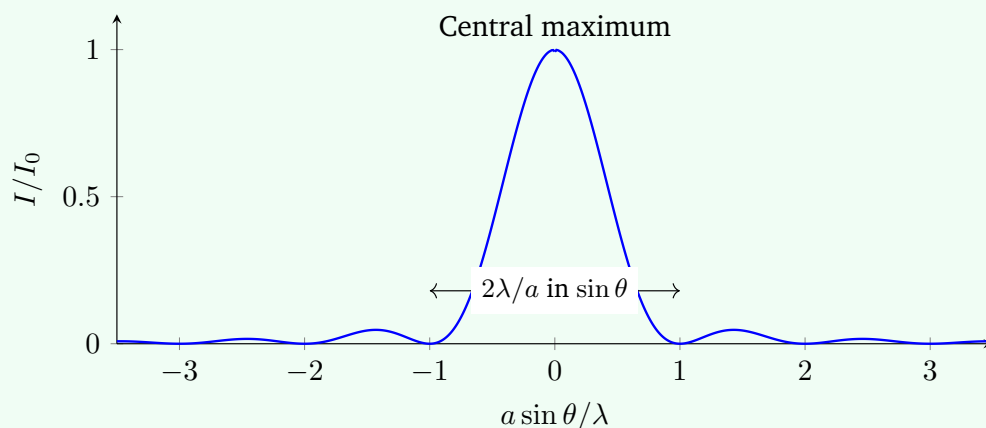
This is the usual approximate location of the second secondary bright fringe. Exact subsidiary peaks obey  $\tan u = u$ , so  $5\pi$  is not an exact peak condition.

**Step 2: (ii) Compare the central width with the fringe spacing** Using the standard fringe-width interpretation of the requested eight maxima, the central diffraction width contains eight interference fringe spacings:

$$\frac{W}{\beta} = \frac{2D\lambda/a}{D\lambda/d} = \frac{2d}{a} = 8 \Rightarrow a = \frac{d}{4}.$$

There is a counting subtlety in the wording: for a perfectly symmetric pattern, this width covers eight spacings, but only seven nonzero interference maxima lie strictly between the first diffraction minima. The orders  $m = \pm 4$  coincide with those minima and are suppressed. Thus  $a = d/4$  is the conventional width-ratio answer intended here; an exact count of eight symmetric bright orders is not possible.

**Step 3: (iii) Draw the single-slit intensity distribution** The central peak at  $\theta = 0$  has intensity  $I_0$ . Intensity falls to zero at  $a \sin \theta / \lambda = \pm 1, \pm 2, \dots$ ; the weaker side lobes decrease in height outward. The central lobe extends from  $-1$  to  $+1$  on the dimensionless horizontal axis, while each side lobe occupies one unit.



*Single-slit diffraction: normalized intensity versus  $a \sin \theta / \lambda$ .*

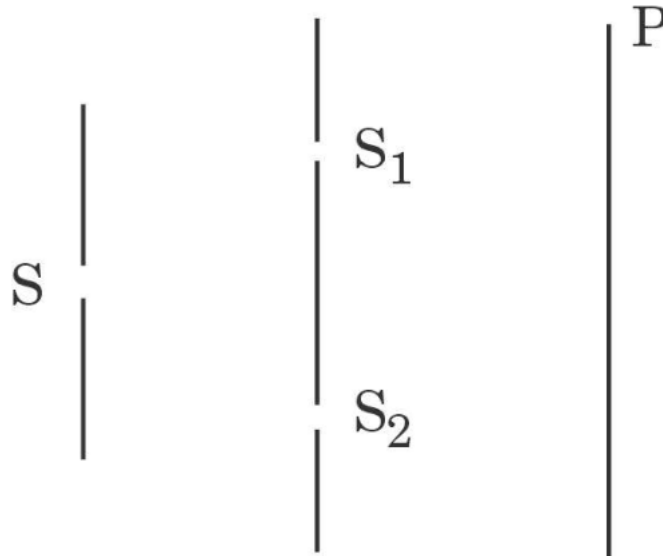
**Step 4: Conclusion** (i) A weak bright subsidiary fringe is obtained. (ii) The standard width-ratio result is  $a = d/4$ , with the exact-count qualification explained above. (iii) The graph shows a broad central maximum and diminishing secondary maxima.

**Quick Tip to Solve:** For single-slit darkness test an integer path difference across the entire slit; do not apply the two-wave odd-half-wavelength dark-fringe rule.

**29(b).** (i) In a Young's double-slit experiment  $SS_2 - SS_1 = \frac{\lambda}{4}$ , where  $S_1$  and  $S_2$  are the two slits as shown in the figure. Find the path difference ( $S_2P - S_1P$ ) for constructive and destructive interference at P.

(ii) What is the effect on the interference fringes in a Young's double-slit experiment, if the monochromatic source S is replaced by a source of white light?

[3 marks]



### Solution:

#### Concept:

- Interference depends on the total optical path difference from the common source through the two slits to the observation point, including unequal source-to-slit distances.
- For coherent light of wavelength  $\lambda$ , constructive interference requires total path difference  $n\lambda$ ; destructive interference requires  $(n + 1/2)\lambda$ , where  $n$  is an integer.
- In Young's experiment, fringe spacing  $\beta = D\lambda/d$  increases with wavelength;  $D$  is slit-to-screen distance and  $d$  is slit separation.

**Step 1:** (i) **Include the source-side path difference** Define the requested signed path difference by  $x = S_2P - S_1P$ . The total difference between the path through  $S_2$  and that through  $S_1$  is

$$\begin{aligned}\Delta &= (SS_2 + S_2P) - (SS_1 + S_1P) \\ &= (SS_2 - SS_1) + (S_2P - S_1P) \\ &= \frac{\lambda}{4} + x.\end{aligned}$$

Retaining this sign order is essential because the waves already have a phase difference on arrival at the slits.

**Step 2:** **Apply the constructive and destructive conditions** For constructive interference, set  $\Delta = n\lambda$ . Therefore,

$$S_2P - S_1P = x = \left(n - \frac{1}{4}\right)\lambda = \frac{(4n - 1)\lambda}{4}, \quad n \in \mathbb{Z}.$$

For destructive interference, set  $\Delta = (n + 1/2)\lambda$ . This gives

$$S_2P - S_1P = x = \left(n + \frac{1}{4}\right)\lambda = \frac{(4n + 1)\lambda}{4}, \quad n \in \mathbb{Z}.$$

For example, the zero-total-path bright fringe is at  $x = -\lambda/4$ ; the nearby dark fringe with  $\Delta = \lambda/2$  is at  $x = +\lambda/4$ .

**Step 3: (ii) Superpose the patterns of different colours** White light contains many wavelengths. At the point of zero total path difference, all wavelengths interfere constructively, giving a white central fringe. For this off-axis source, that point satisfies  $S_2P - S_1P = -(SS_2 - SS_1)$ , so it is displaced from the geometric screen centre. On either side, a few coloured fringes are visible. Violet, with the shorter wavelength, has closer fringes and lies nearer the white fringe in a given order; red lies farther away. At larger orders, maxima and minima of different wavelengths overlap, reducing contrast until distinct fringes disappear into nearly uniform illumination.

**Step 4: Conclusion** Constructive interference requires  $S_2P - S_1P = (n - 1/4)\lambda$ ; destructive interference requires  $S_2P - S_1P = (n + 1/4)\lambda$ , with integer  $n$ . White light gives a white zero-path-difference fringe, a few coloured fringes on either side, and then loss of distinct fringes.

**Quick Tip to Solve:** Write the complete source-to-screen path through each slit before subtracting; an off-axis source shifts the central fringe but preserves the interference conditions.

**30. Briefly explain Geiger-Marsden experiment. Show the variation of the number of particles scattered ( $N$ ) with scattering angle ( $\theta$ ) in this experiment. What is the main conclusion that can be inferred from this plot?** [3 marks]

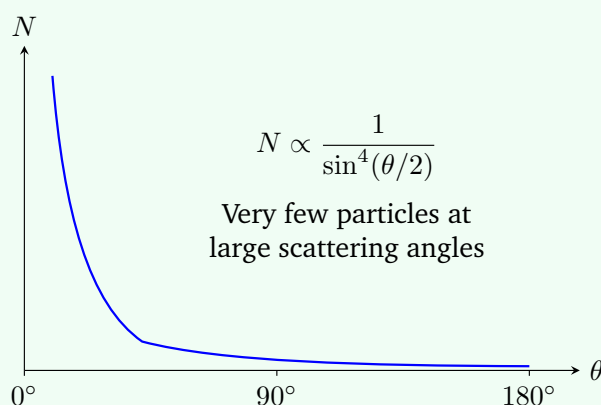
**Solution:**

**Concept:**

- An  $\alpha$ -particle is a positively charged helium nucleus. Its deflection by an atom probes the distribution of positive charge and mass inside that atom.
- For the same incident beam and equal detector solid angles, the Rutherford scattering law is  $N(\theta) \propto 1/\sin^4(\theta/2)$ , where  $N$  is the number detected in a fixed time at scattering angle  $\theta$ .

**Step 1: Describe the experimental arrangement** Geiger and Marsden directed a narrow beam of fast  $\alpha$ -particles from a radioactive source onto a very thin gold foil in an evacuated enclosure. Lead shielding and a slit collimated the beam. A movable zinc sulphide screen, viewed through a microscope, detected individual particles by their flashes (scintillations). Moving the detector around the foil allowed the number scattered through different angles to be counted in equal time intervals.

**Step 2: Relate the observed counts to the scattering angle** Most particles passed through the foil with practically no deflection. Some were deflected through small angles, while very few scattered through large angles; a tiny fraction even returned towards the source. Thus the count is very large near the forward direction and falls rapidly as  $\theta$  increases. The curve below is schematic: the ideal Rutherford law applies for nonzero angles and predicts a small but nonzero count even at  $180^\circ$ . The direct forward beam and finite angular resolution prevent interpreting the ideal divergence at zero as a physically infinite measured count.



*Schematic angular distribution of scattered  $\alpha$ -particles.*

**Step 3: Interpret the rare large deflections** The overwhelmingly undeflected beam shows that most of an atom is empty space. The rare large deflections and backscattering require a very strong repulsive force during close encounters with a small, heavy, positively charged centre. A diffuse positive charge throughout the atom could not produce such large changes in the direction of the massive fast particles.

**Step 4: Conclusion**  $N$  decreases steeply with increasing scattering angle. The main conclusion is that nearly all the atomic mass and positive charge are concentrated in a tiny nucleus, while most of the atom is empty space.

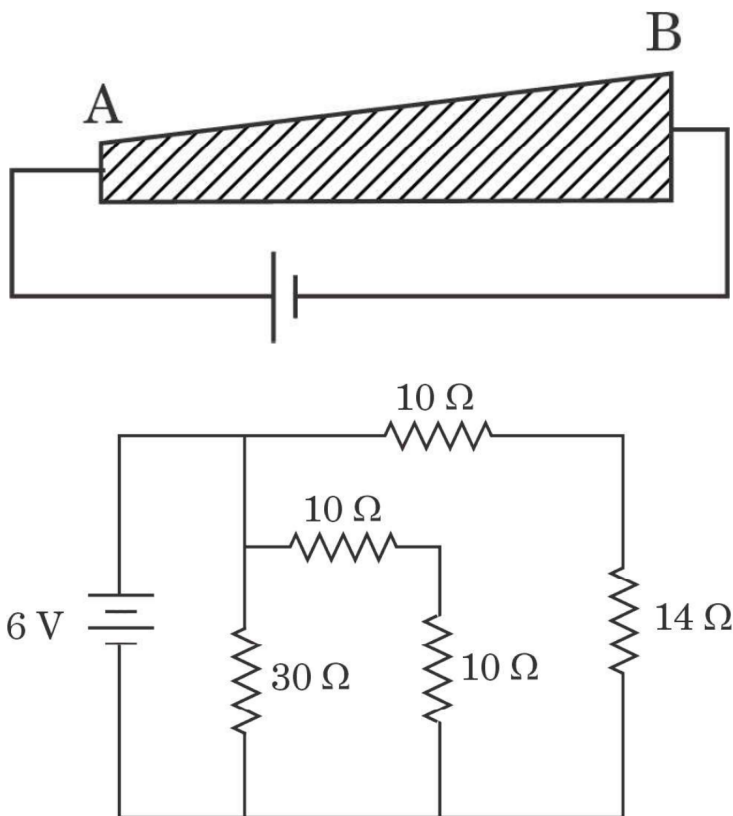
**Quick Tip to Solve:** Link the many forward particles to empty space and the few backward particles to a tiny massive positive nucleus; show both features on the falling curve.

### Section D (15 marks)

**31(a).** (i) Define mobility of electrons. Give its SI units.

(ii) A steady current flows through a wire AB, as shown in the figure. What happens to the electric field and the drift velocity along the wire? Justify your answer.

(iii) Consider the circuit shown in the figure. Find the effective resistance of the circuit and the current drawn from the battery. [5 marks]



#### Solution:

##### Concept:

- Electron mobility is the drift-speed magnitude per unit electric-field magnitude:  $\mu = v_d/E = e\tau/m_e$ , where  $e$  is the magnitude of electron charge,  $\tau$  is the relaxation time and  $m_e$  is electron mass.
- For a steady current in a homogeneous conductor,  $I = neAv_d$  and  $J = I/A = \sigma E$ . Here  $n$  is the free-electron number density,  $A$  is the local cross-sectional area and  $\sigma$  is conductivity.
- Resistances in series add. Parallel branches have the same potential difference, with  $1/R_{eq} = \sum 1/R_i$ ; the battery current is  $I = V/R_{eq}$ .

**Step 1: (i) Define mobility and obtain its SI unit** Mobility of electrons is the magnitude of the average drift velocity acquired per unit electric field applied to a conductor. Using magnitudes ensures that mobility is positive; the electron drift direction is opposite to the electric field. Thus

$$\mu = \frac{v_d}{E}, \quad [\mu] = \frac{\text{m s}^{-1}}{\text{V m}^{-1}} = \text{m}^2 \text{V}^{-1} \text{s}^{-1}.$$

**Step 2: (ii) Use continuity in the widening wire** The wire becomes wider from A to B, so its cross-sectional area increases in that direction. In a steady state, charge does not accumulate anywhere; hence the same current crosses every section. For the same material at constant temperature,  $n$ ,  $\sigma$

and  $\mu$  are constant. Consequently,

$$v_d = \frac{I}{neA}, \quad E = \frac{I}{\sigma A}.$$

Both the electric-field magnitude and the drift-speed magnitude therefore decrease from A to B, each varying inversely with the local area. The battery polarity shown drives conventional current and the electric field from A towards B; electrons drift from B towards A. The statement that drift speed decreases from A to B compares values at successive positions along the wire, rather than describing the electron's direction of motion.

**Step 3: (iii) Identify the three parallel branches** The upper wire connected to the positive battery terminal is one common node; the lower wire is the other. Between these nodes there are three separate paths. The left path contains  $30\ \Omega$ . The middle path contains two  $10\ \Omega$  resistors in series, since there is no branching at their junction. The outer right path contains  $10\ \Omega$  and  $14\ \Omega$  in series. Their branch resistances are

$$R_1 = 30\ \Omega, \quad R_2 = (10 + 10)\ \Omega = 20\ \Omega, \quad R_3 = (10 + 14)\ \Omega = 24\ \Omega.$$

**Step 4: Combine the branches and calculate the battery current** All three paths share both end nodes, so they are parallel across the  $6\ \text{V}$  battery:

$$\begin{aligned} \frac{1}{R_{\text{eq}}} &= \frac{1}{30} + \frac{1}{20} + \frac{1}{24} \\ &= \frac{4 + 6 + 5}{120} = \frac{1}{8}\ \Omega^{-1}, \\ R_{\text{eq}} &= 8\ \Omega, \\ I &= \frac{6}{8} = 0.75\ \text{A}. \end{aligned}$$

As a check, the branch currents are  $6/30 = 0.20\ \text{A}$ ,  $6/20 = 0.30\ \text{A}$  and  $6/24 = 0.25\ \text{A}$ ; their sum is  $0.75\ \text{A}$ .

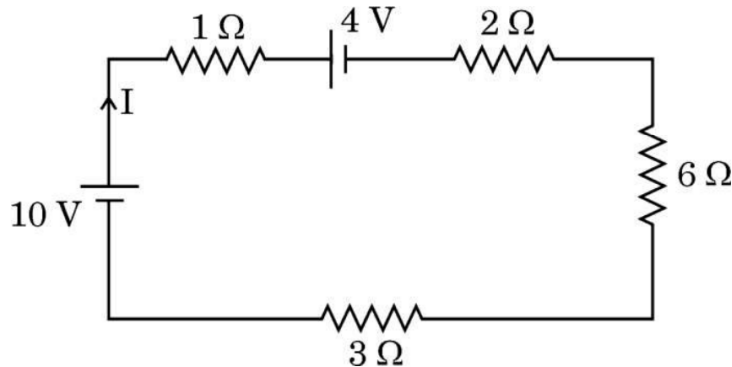
**Step 5: Conclusion** (i)  $\mu = v_d/E$ , with SI unit  $\text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$ . (ii) Electric-field magnitude and electron drift-speed magnitude both decrease from A to B. (iii) The effective resistance is  $8\ \Omega$ , and the battery supplies  $0.75\ \text{A}$ .

**Quick Tip to Solve:** First identify the two battery nodes: the circuit reduces immediately to parallel branches of  $30\ \Omega$ ,  $20\ \Omega$  and  $24\ \Omega$ . In the tapered wire, keep current constant, not current density.

31(b). (i) Define electrical conductivity of a wire. Give its SI unit.

(ii) High current is to be drawn safely from (1) a low-voltage battery, and (2) a high-voltage battery. What can you say about the internal resistance of the two batteries?

(iii) Calculate the total energy supplied by the batteries to the circuit shown in the figure, in one minute. [5 marks]



### Solution:

#### Concept:

- Conductivity  $\sigma$  measures current density produced per unit electric field:  $\mathbf{J} = \sigma \mathbf{E}$ . It is the reciprocal of resistivity  $\rho$ .
- A battery of emf  $\mathcal{E}$ , internal resistance  $r$ , and external resistance  $R$  gives  $I = \mathcal{E}/(R + r)$ . Internal resistance limits current and causes internal heating  $I^2 r$ .
- Kirchhoff's loop rule uses signed emfs. A battery supplies energy when current leaves its positive terminal; it absorbs energy when current enters its positive terminal. The energy magnitude is  $\mathcal{E}It$ .

**Step 1: (i) Define conductivity and obtain its unit** The electrical conductivity of the material of a wire is the reciprocal of its resistivity:

$$\sigma = \frac{1}{\rho} = \frac{J}{E}.$$

Thus it is the current density established for unit applied electric field, for an ohmic material at a specified temperature. Since the SI unit of resistivity is  $\Omega \text{ m}$ , that of conductivity is  $\Omega^{-1} \text{ m}^{-1}$ , equivalently  $\text{S m}^{-1}$ . For a uniform wire of length  $l$ , cross-sectional area  $A$ , and resistance  $R$ ,  $\sigma = l/(RA)$ .

**Step 2: (ii) Compare the internal resistances needed** For a low-voltage battery, the internal resistance should be very small. Otherwise the internal drop  $Ir$  consumes much of its small emf and the desired large current cannot be drawn. A small  $r$  also reduces internal heating for a specified current.

For a high-voltage battery, a comparatively larger internal resistance can be used to limit the current to a safe value; its larger emf can still drive a large current. Indeed, for a chosen current and external load,  $r = \mathcal{E}/I - R$ . A high-voltage source with extremely small total resistance could produce a dangerously large current. These are comparative design requirements: the actual safe current also depends on the load and battery rating.

**Step 3: (iii) Apply the loop rule with the printed battery polarities** The four resistors carry the same current and their total resistance is

$$R_{\text{total}} = 1 + 2 + 6 + 3 = 12 \Omega.$$

Following the indicated clockwise current, the 10 V battery is crossed from negative to positive, whereas the 4 V battery is crossed from positive to negative. Their emfs therefore oppose each other. Kirchhoff's loop rule gives

$$10 - 4 - 12I = 0, \quad I = \frac{6}{12} = 0.50 \text{ A}.$$

The positive result agrees with the indicated direction of current.

**Step 4: (iii) Account for energy supplied and absorbed** In one minute,  $t = 60 \text{ s}$ . The 10 V battery supplies

$$W_{10} = 10(0.50)(60) = 300 \text{ J}.$$

Current enters the positive terminal of the 4 V battery, so this battery absorbs energy:

$$W_{4,\text{absorbed}} = 4(0.50)(60) = 120 \text{ J}.$$

Consequently, the net energy supplied by the batteries together to the resistive circuit is the supplied energy minus the absorbed energy. The independent heating check is

$$I^2 R_{\text{total}} t = (0.50)^2 (12)(60) = 180 \text{ J},$$

consistent with conservation of energy. Adding the magnitudes of both battery energies would incorrectly treat the opposing battery as an energy source.

**Step 5: Conclusion** (i)  $\sigma = 1/\rho$ , with SI unit  $\text{S m}^{-1}$ . (ii) The low-voltage battery should have very small internal resistance; the high-voltage battery should have comparatively larger internal resistance to limit current safely. (iii) The current is 0.50 A clockwise and the net energy supplied is  $300 - 120 = 180 \text{ J}$  in one minute.

**Quick Tip to Solve:** Trace the long positive plate of each cell before adding emfs: here the 4 V battery absorbs energy, so subtract its energy from that supplied by the 10 V battery.

**32(a).** (i) Draw a ray diagram to show how the final image is formed at infinity in an astronomical refracting telescope. Obtain an expression for its magnifying power.

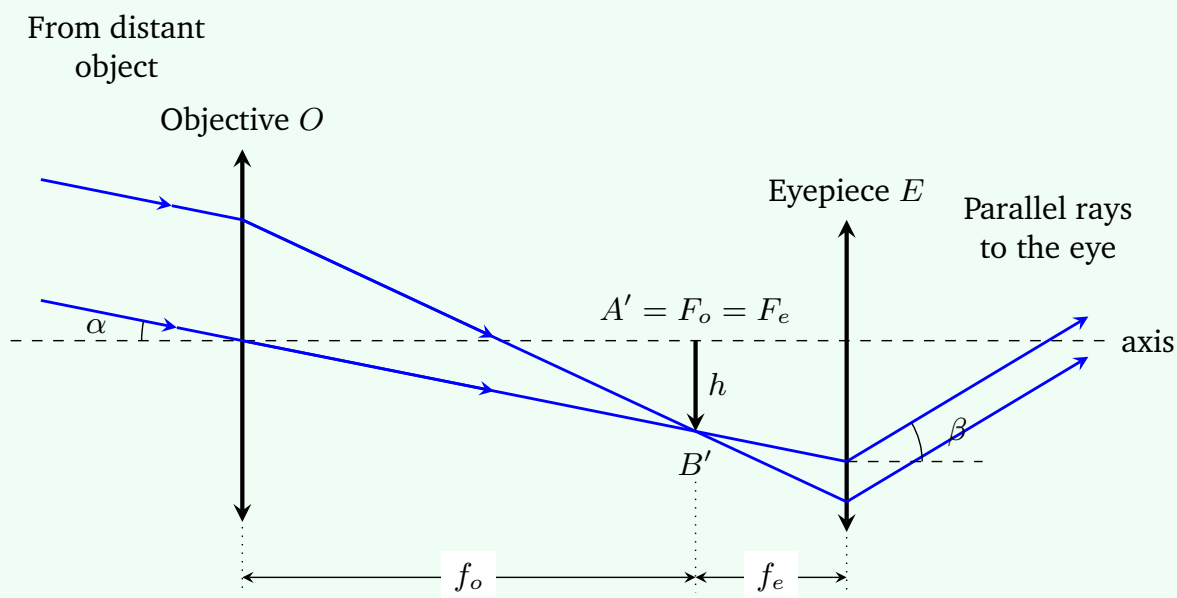
(ii) Two thin lenses  $L_1$  and  $L_2$ ,  $L_1$  being a convex lens of focal length 24 cm and  $L_2$  a concave lens of focal length 18 cm are placed coaxially at a separation of 45 cm. A 1 cm tall object is placed in front of the lens  $L_1$  at a distance of 36 cm. Find the location and height of the image formed by the combination. [5 marks]

**Solution:**

**Concept:**

- In normal adjustment of an astronomical telescope, the objective's real focal-plane image lies at the first focal plane of the eyepiece, so the final emergent rays are parallel and the final image is at infinity.
- Angular magnifying power is  $M = \beta/\alpha$ , where  $\alpha$  and  $\beta$  are the small angular sizes of the object and final image. The negative sign indicates inversion.
- For each thin lens use  $1/v - 1/u = 1/f$  and transverse magnification  $m = v/u$ . With light travelling left to right, rightward distances from the lens are positive; the magnification of two lenses is  $m_1 m_2$ .

**Step 1: (i) Form the image in normal adjustment** The objective  $O$ , of large focal length  $f_o$ , receives parallel rays from a distant object and produces a real, inverted intermediate image  $A'B'$ . Place the converging eyepiece  $E$ , of short focal length  $f_e$ , so that its first focal plane coincides with the objective's second focal plane. Thus  $OE = f_o + f_e$ . The intermediate image is at the eyepiece's focus, so each pencil of rays emerges parallel; the eye sees the final virtual image at infinity without accommodation. The diagram shows two rays from one distant off-axis object point.



*Astronomical telescope in normal adjustment:  $F_o = F_e = A'$ , and the rays leaving the eyepiece are parallel.*

**Step 2: (i) Derive the angular magnifying power** Let the magnitude of the intermediate image height be  $h = A'B'$ . In the paraxial approximation, the object subtends an angle of magnitude  $|\alpha| \simeq h/f_o$  at the objective, and the final image subtends  $|\beta| \simeq h/f_e$  at the eyepiece. Hence

$$|M| = \frac{|\beta|}{|\alpha|} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}.$$

Since the final image is inverted relative to the distant object, the signed angular magnifying power is  $M = -f_o/f_e$ . The angles indicated in the diagram are their magnitudes.

**Step 3: (ii) Find the image due to the first lens** For the convex lens  $L_1$ , take  $u_1 = -36$  cm,  $f_1 = +24$  cm and original object height  $h_0 = 1$  cm. Then

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{24} - \frac{1}{36} = \frac{1}{72},$$

$$v_1 = +72 \text{ cm}, \quad m_1 = \frac{v_1}{u_1} = \frac{72}{-36} = -2.$$

Thus, without  $L_2$ , the rays would meet 72 cm to the right of  $L_1$  at an inverted image of height  $h_1 = -2$  cm.

**Step 4: (ii) Use the converging rays as a virtual object for the second lens** The concave lens intercepts these converging rays only 45 cm to the right of  $L_1$ , before the first image can form. Their intended meeting point is  $72 - 45 = 27$  cm to the right of  $L_2$ . This is a virtual object for  $L_2$ , so  $u_2 = +27$  cm,

while  $f_2 = -18$  cm. Therefore

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{18} + \frac{1}{27} = -\frac{1}{54},$$
$$v_2 = -54 \text{ cm.}$$

The negative image distance means that the emergent rays diverge and their backward extensions meet on the left of  $L_2$ .

**Step 5: (ii) Calculate the final height and position relative to the first lens** The second-lens magnification and the total magnification are

$$m_2 = \frac{-54}{+27} = -2, \quad m = m_1 m_2 = (-2)(-2) = +4.$$

Hence  $h_f = mh_0 = +4$  cm. Measured from  $L_1$ , the final image position is  $45 - 54 = -9$  cm. The positive final height indicates an erect image relative to the original object; its virtual nature follows from the diverging emergent rays.

**Step 6: Conclusion** (i) For a telescope in normal adjustment,  $M = -f_o/f_e$ , with magnitude  $f_o/f_e$ , and the final image is at infinity. (ii) The combination forms a virtual, erect image 54 cm to the left of  $L_2$ , equivalently 9 cm to the left of  $L_1$ , of height 4 cm.

**Quick Tip to Solve:** Locate the image of the first lens on a common axis before assigning the second object distance: here it lies beyond  $L_2$ , so  $u_2 = +27$  cm, not a negative distance.

OR

**32(b). (i) Explain the working principle of an optical fibre with the help of a diagram. Mention one use of a light pipe.**

**(ii) A ray of light is incident at an angle of  $60^\circ$  on one face of a prism with the prism angle  $A = 60^\circ$ . The ray passes symmetrically through the prism. Find the angle of minimum deviation ( $\delta_m$ ) and refractive index of the material of the prism. If the prism is immersed in water, how will  $\delta_m$  be affected? Justify your answer.** [5 marks]

**Solution:**

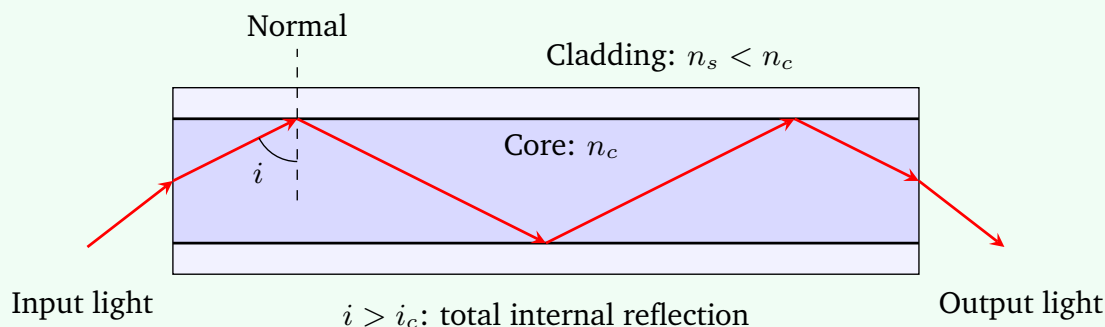
**Concept:**

- Total internal reflection occurs when light travels from a denser medium to a rarer medium and the angle of incidence exceeds the critical angle. For a fibre core and cladding,  $n_c > n_s$  and  $\sin i_c = n_s/n_c$ , where  $n_c$  and  $n_s$  are their refractive indices.
- For a prism of angle  $A$ , the internal refraction angles obey  $r_1 + r_2 = A$ , and the deviation is  $\delta = i + e - A$ , where  $i$  and  $e$  are the incidence and emergence angles.
- At minimum deviation, the ray travels symmetrically:  $i = e$  and  $r_1 = r_2 = A/2$ . The refractive index relative to the surrounding medium is  $\mu_{\text{rel}} = \sin[(A + \delta_m)/2] / \sin(A/2)$ .

**Step 1: (i) Explain guidance of light through the fibre** An optical fibre consists of a transparent core of refractive index  $n_c$ , surrounded by a cladding of smaller refractive index  $n_s$ . Light entering within the suitable acceptance range is refracted into the core. It then strikes the core-cladding boundary from the denser side at an incidence angle  $i > i_c$ , measured from the normal to that boundary.

The ray undergoes total internal reflection at each encounter with the boundary and therefore travels

along the core by repeated reflections. This guides light to the other end with small loss; a suitably bent fibre can guide it around bends while the total-internal-reflection condition remains satisfied. The sketch shows one guided ray and the normal at its first reflection.



*Light is guided by total internal reflection inside the higher-index core.*

**Step 2: State one use of a light pipe** A light pipe made from optical fibres is used in medical endoscopy. It carries light into an internal organ to illuminate the region being examined; an appropriate fibre bundle can also carry the image back for observation. This permits examination of otherwise inaccessible internal regions.

**Step 3: (ii) Apply the condition of symmetric passage** Since the ray passes symmetrically through the prism, the given incidence angle  $i = 60^\circ$  equals the emergence angle, and the prism operates at minimum deviation. With  $A = 60^\circ$ ,

$$e = 60^\circ, \quad r_1 = r_2 = \frac{A}{2} = 30^\circ,$$

$$\delta_m = i + e - A = 60^\circ + 60^\circ - 60^\circ = 60^\circ.$$

The symmetry condition is essential: an arbitrary ray through the same prism need not have this deviation.

**Step 4: Calculate the refractive index of the prism** Taking the surrounding medium initially as air, Snell's law at the first face gives the refractive index of the prism material:

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} \simeq 1.73.$$

Equivalently, substituting  $A = 60^\circ$  and  $\delta_m = 60^\circ$  in the minimum-deviation formula gives the same result. Refractive index is dimensionless.

**Step 5: Explain the effect of immersing the prism in water** Let  $n_w$  be the refractive index of water relative to air. The prism's refractive index relative to water is reduced to  $\mu_{\text{rel}} = \sqrt{3}/n_w$ , because water is optically denser than air. The new minimum deviation obeys

$$\sin \left( \frac{A + \delta_{m,w}}{2} \right) = \frac{\sqrt{3}}{n_w} \sin \frac{A}{2},$$

$$\delta_{m,w} = 2 \sin^{-1} \left( \frac{\sqrt{3}}{2n_w} \right) - 60^\circ.$$

As  $n_w > 1$ , the sine argument is smaller than it was in air, so the minimum deviation decreases. Physically, the smaller refractive-index contrast produces less bending at the prism faces. This compares the minimum deviations in the two media; the incidence angle for minimum deviation also

changes on immersion.

**Step 6: Conclusion** (i) An optical fibre guides light by repeated total internal reflection at the core-cladding boundary; one use of a light pipe is medical endoscopy. (ii) In air,  $\delta_m = 60^\circ$  and  $\mu = \sqrt{3} \simeq 1.73$ . On immersion in water, the minimum deviation decreases because the relative refractive index of the prism decreases.

**Quick Tip to Solve:** For symmetric passage through a prism, immediately set  $e = i$  and  $r = A/2$ . On changing the surrounding medium, use the relative refractive index in the minimum-deviation formula.

**33(a).** (i) A germanium crystal is doped with antimony. With the help of energy-band diagram, explain how the conductivity of the doped crystal is affected.

(ii) Briefly explain the two processes involved in the formation of a p-n junction.

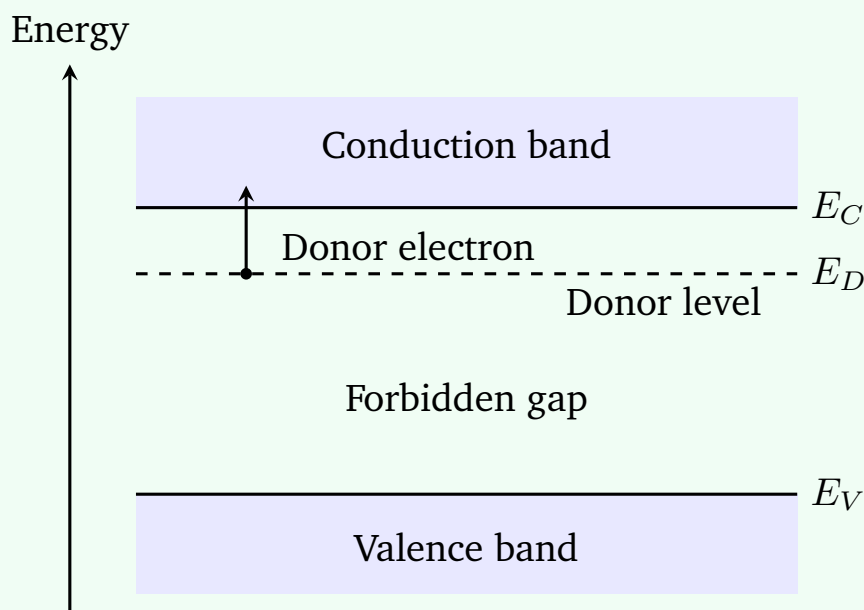
(iii) What will the effect of (1) forward biasing, and (2) reverse biasing be on the width of depletion layer in a p-n junction diode? [5 marks]

**Solution:**

**Concept:**

- Pentavalent donor doping of tetravalent germanium produces an n-type semiconductor: electrons are the majority carriers and holes are the minority carriers.
- Conductivity is  $\sigma = e(n\mu_e + p\mu_h)$ , where  $n, p$  are electron and hole number densities,  $\mu_e, \mu_h$  their mobilities, and  $e$  the magnitude of electronic charge.
- Diffusion is carrier motion due to a concentration gradient; drift is carrier motion under an electric field. Their currents balance in an unbiased p-n junction at equilibrium.

**Step 1: (i) Explain the effect of antimony doping** Germanium has four valence electrons, whereas antimony has five. When an antimony atom replaces a germanium atom, four of its electrons form covalent bonds with the neighbouring germanium atoms. Its fifth electron is weakly bound and can become a conduction electron using a small amount of thermal energy. The donor atom left behind is an immobile positive ion; the crystal as a whole remains electrically neutral. In the energy-band diagram, antimony introduces a donor energy level  $E_D$  just below the conduction-band edge  $E_C$ . Thus  $E_C - E_D$  is much smaller than the forbidden energy gap  $E_C - E_V$ , where  $E_V$  is the valence-band edge. At room temperature, donor electrons readily enter the conduction band without an equivalent creation of valence-band holes. Electron density therefore rises substantially and  $n \gg p$ . The increased supply of mobile electrons increases conductivity, producing n-type germanium.



*Energy bands of antimony-doped germanium (schematic, not to scale).*

**Step 2: (ii) Describe diffusion and depletion-layer formation** Immediately after a p-n junction forms, electrons diffuse from the n-side, where their concentration is high, into the p-side. Holes similarly diffuse from the p-side into the n-side. Near the junction these carriers recombine, leaving immobile positive donor ions on the n-side and negative acceptor ions on the p-side. This region, depleted of mobile majority carriers, is the depletion layer. The uncovered ions establish an electric field directed from the n-side towards the p-side and a potential barrier opposing further majority-carrier diffusion.

**Step 3: (ii) Describe drift and equilibrium** The junction electric field causes drift: an electron experiences a force opposite to the field, towards the n-side, while a hole experiences a force along the field, towards the p-side. Minority electrons in the p-region and minority holes in the n-region reaching the depletion layer are swept across it. This gives a drift current opposite to the conventional diffusion current. As the barrier builds, diffusion is reduced; at thermal equilibrium the drift and diffusion currents have equal magnitudes, so the net current through the unbiased junction is zero.

**Step 4: (iii) Compare forward and reverse bias** In forward bias, the p-side is connected to the positive terminal and the n-side to the negative terminal of the supply. The applied field opposes the junction field and lowers the potential barrier. Majority carriers are pushed towards the junction, reducing the extent of uncovered ions; hence the depletion-layer width decreases. In reverse bias, the p-side is connected to the negative terminal and the n-side to the positive terminal. The applied field supports the junction field and increases the barrier. Majority carriers are pulled away from the junction, exposing more fixed ions; hence the depletion-layer width increases.

**Step 5: Conclusion** Antimony supplies easily excited donor electrons and increases germanium's conductivity, making it n-type. Diffusion and drift govern p-n junction formation and balance at equilibrium. Forward bias narrows the depletion layer; reverse bias widens it.

**Quick Tip to Solve:** Place a donor level just below the conduction band. For biasing, track majority carriers: pushed towards the junction means a narrower depletion layer; pulled away means a wider layer.

**33(b).** (i) With the help of a circuit diagram, briefly explain the working of a full-wave rectifier using p-n junction diodes.

(ii) Draw V-I characteristics of a p-n junction diode. Explain how these characteristics make a diode suitable for rectification.

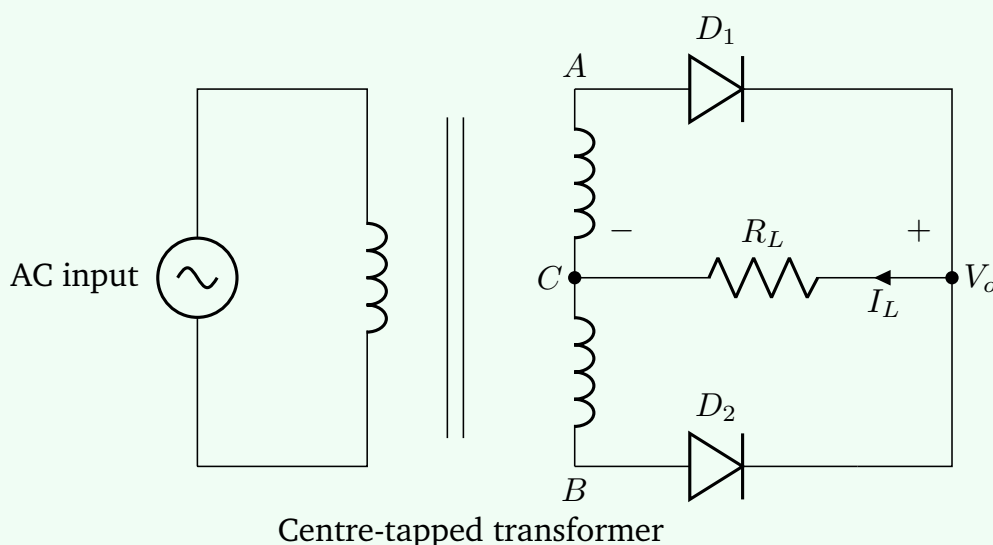
(iii) Carbon and silicon have the same lattice structure. Then why is carbon an insulator but silicon a semiconductor? [5 marks]

### Solution:

#### Concept:

- A p-n junction conducts appreciably in forward bias and offers very high resistance in reverse bias below breakdown. This directional conduction enables rectification.
- A centre-tapped secondary supplies equal alternating voltages of opposite polarity relative to its centre tap. Two diodes can therefore drive the same load during alternate half-cycles.
- The forbidden energy gap  $E_g$  separates the valence and conduction bands; thermal excitation across it creates mobile charge carriers.

**Step 1: (i) Connect a centre-tapped full-wave rectifier** Connect the anodes of diodes  $D_1$  and  $D_2$  to the secondary ends  $A$  and  $B$ . Join their cathodes at the positive output terminal. Connect the load  $R_L$  between this junction and the centre tap  $C$ , the negative output terminal. Apply the input alternating voltage to the transformer primary. The circuit below shows the conventional current direction through the load.



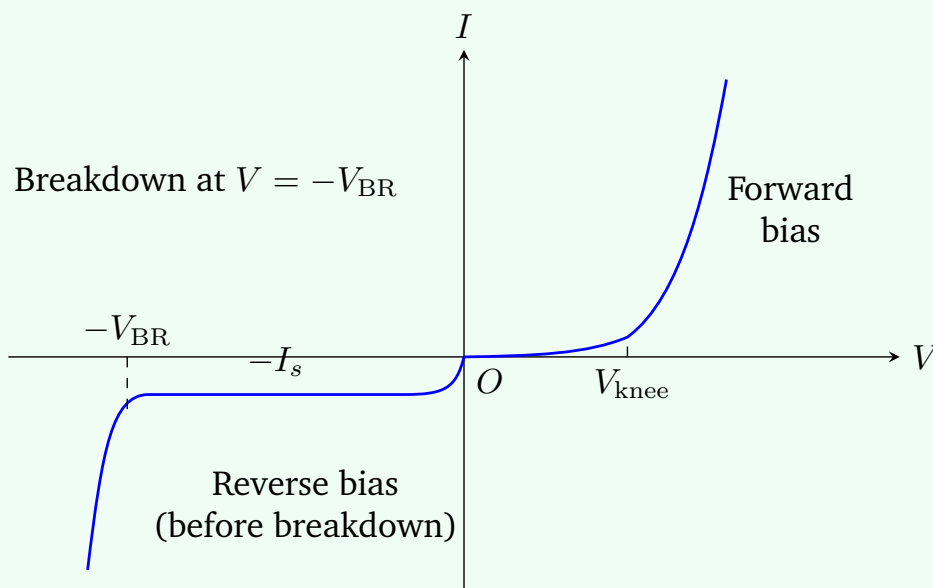
*Centre-tapped full-wave rectifier; both diode cathodes feed the positive output.*

**Step 2: (i) Follow the two half-cycles** When  $A$  is positive relative to  $C$ ,  $B$  is negative:  $D_1$  is forward biased and  $D_2$  is reverse biased. Current flows from  $A$ , through  $D_1$ , the load, and back to  $C$ . In the next half-cycle,  $B$  is positive and  $A$  negative, so  $D_2$  conducts and  $D_1$  blocks. Current then flows from  $B$ , through  $D_2$ , the same load, and back to  $C$ . Thus both half-cycles give load current in the indicated direction. Ignoring diode drops,  $v_o = |V_m \sin \omega t|$ , where  $V_m$  is the peak voltage of either half-secondary. The output is pulsating dc, with ripple frequency  $2f$  for an input frequency  $f$ ; a filter is needed to smooth it.

**Step 3: (ii) Read the forward characteristic** The characteristic plots diode current  $I$  against the

voltage  $V$  of its p-side relative to its n-side. For positive  $V$ , the diode is forward biased. Initially the current is small; beyond the knee voltage it rises rapidly for a small increase in voltage. The knee is approximately 0.7 V for silicon and 0.3 V for germanium. The conducting diode consequently offers a low effective forward resistance.

**Step 4: (ii) Read the reverse characteristic and relate it to rectification** For negative  $V$ , only a very small reverse saturation current flows until breakdown. The reverse current then rises sharply in magnitude; ordinary rectifiers are operated below this breakdown voltage. The graph uses different illustrative current scales for the forward and reverse branches. Its strong asymmetry means that a diode passes the appropriate half-cycle and blocks the opposite half-cycle. In the two-diode circuit, the diodes exchange these roles, giving full-wave rectification.



*Diode V-I characteristic (schematic; reverse current is shown on an enlarged scale).*

**Step 5: (iii) Compare carbon and silicon energy gaps** Here carbon means diamond, whose lattice structure is like that of silicon. Having the same crystal structure and four valence electrons does not imply the same band gap: the strength of covalent bonding and the band energies differ. Diamond has a large forbidden gap of about 5.5 eV, whereas silicon has a much smaller gap of about 1.1 eV. At ordinary temperature, thermal energy cannot excite an appreciable number of electrons across diamond's large gap, leaving its conduction band effectively empty. Silicon's smaller gap allows enough thermally generated electrons and holes to provide semiconductor conductivity. Graphite is a different form of carbon and is not the lattice comparison intended here.

**Step 6: Conclusion** The two diodes conduct on alternate half-cycles while the load current remains unidirectional. The diode's asymmetric forward and reverse characteristics permit rectification. Diamond is insulating because of its much larger band gap, while silicon is semiconducting because its smaller gap allows thermal carrier generation.

**Quick Tip to Solve:** Trace each half-cycle from a secondary end through its diode and the load back to the centre tap; the load-current arrow must stay unchanged.

### Section E (8 marks)

**34(a).** Electrostatics deals with the study of forces, fields and potentials arising from static charges. Force and electric field, due to a point charge is basically determined by Coulomb's law. For symmetric charge configurations, Gauss's law, which is also based on Coulomb's law, helps us to find the electric field. A charge/a system of charges like a dipole experience a force/torque in an electric field. Work is required to be done to provide a specific orientation to a dipole with respect to an electric field.

Answer the following questions based on the above :

(a) Consider a uniformly charged thin conducting shell of radius  $R$ . Plot a graph showing the variation of  $|\vec{E}|$  with distance  $r$  from the centre, for points  $0 \leq r \leq 3R$ . [1 mark]

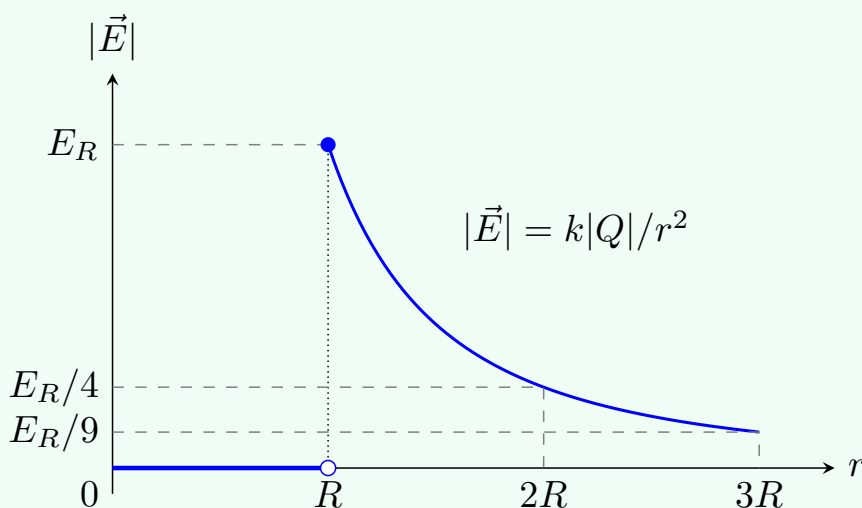
#### Solution:

##### Concept:

- Gauss's law gives zero electric field inside a uniformly charged spherical conducting shell and the point-charge field outside it.
- Write  $k = 1/(4\pi\epsilon_0)$ , let  $Q$  be the shell's charge, and define  $E_R = k|Q|/R^2$ , the field magnitude immediately outside its surface.

**Step 1: Find the field on either side of the shell** For  $r < R$ , a concentric Gaussian sphere encloses no charge, so  $|\vec{E}| = 0$ . For  $r > R$ , it encloses  $Q$ ; hence  $|\vec{E}| = k|Q|/r^2$ . The ideal thin surface has a discontinuity: the inner limiting value is zero and the outer limiting value is  $E_R$ .

**Step 2: Plot the specified interval** Keep the graph on the horizontal axis from the centre to just inside  $R$ . Immediately outside  $R$ , start the inverse-square curve at  $E_R$ , falling to  $E_R/4$  at  $2R$  and  $E_R/9$  at  $3R$ . The dotted vertical line indicates the surface jump; it is not a region of continuously increasing field.

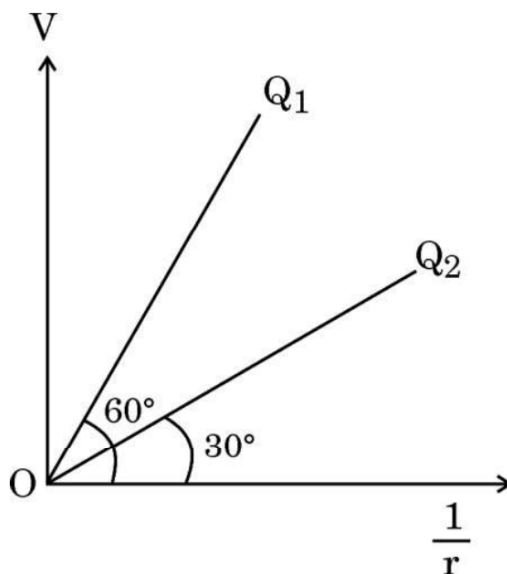


*Electric-field magnitude versus distance; the dotted line marks the discontinuity at the surface.*

**Step 3: Conclusion** The graph is zero inside the shell, jumps to  $k|Q|/R^2$  at the outer surface, and then decreases as  $1/r^2$  up to  $3R$ .

**Quick Tip to Solve:** Anchor the exterior curve at the three values  $E_R$ ,  $E_R/4$ , and  $E_R/9$ ; do not draw a straight decreasing line.

**34(b).** The figure shows the variation of potential  $V$  with  $\frac{1}{r}$  for two point charges  $Q_1$  and  $Q_2$ , where  $V$  is the potential at a distance  $r$  due to a point charge. Find  $\frac{Q_1}{Q_2}$ . [1 mark]



**Solution:**

**Concept:**

- The potential of a point charge is  $V = kQ/r$ , with zero potential at infinity and  $k = 1/(4\pi\epsilon_0)$ .
- For a graph of  $V$  against  $1/r$ , the straight-line slope is  $kQ$ . With common axis scales, the ratio of slopes equals the ratio of the charges.

**Step 1: Identify the two inclinations** Read each angle from the positive horizontal  $1/r$ -axis. The steeper line for  $Q_1$  makes  $60^\circ$ , and the line for  $Q_2$  makes  $30^\circ$ . Both slopes are positive, so both charges are positive. The larger slope corresponds to the larger charge.

**Step 2: Divide the slopes in the requested order** Since  $m_1 = kQ_1$  and  $m_2 = kQ_2$ , the common constant and graph scales cancel:

$$\frac{Q_1}{Q_2} = \frac{m_1}{m_2} = \frac{\tan 60^\circ}{\tan 30^\circ} = \frac{\sqrt{3}}{1/\sqrt{3}}.$$

The angles themselves cannot be divided directly, since slope depends on their tangents.

**Step 3: Conclusion**  $Q_1/Q_2 = 3$ ; the steeper line represents a charge three times as large.

**Quick Tip to Solve:** In a  $V$  versus  $1/r$  plot, use tangent ratios and preserve the numerator charge asked for in the printed question.

**34(c).** An electric dipole of dipole moment of  $6 \times 10^{-7} \text{ C}\cdot\text{m}$  is kept in a uniform electric field of  $10^4 \text{ N/C}$  such that the dipole moment and the electric field are parallel. Calculate the potential energy of the dipole. [2 marks]

**Solution:**

**Concept:**

- The potential energy of an electric dipole is  $U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$ , where  $p$  is its dipole moment,  $E$  is the uniform field and  $\theta$  is their angle.
- The conventional zero of potential energy is the perpendicular orientation,  $\theta = 90^\circ$ . Parallel alignment has the minimum potential energy.

**Step 1: Identify the orientation** The dipole moment is parallel to the electric field, so  $\theta = 0^\circ$  and  $\cos \theta = 1$ . Hence the required energy is obtained from  $U = -pE$ , retaining the minus sign from the dot-product formula.

**Step 2: Substitute in SI units** With  $p = 6 \times 10^{-7} \text{ C}\cdot\text{m}$  and  $E = 10^4 \text{ N/C}$ ,

$$U = -(6 \times 10^{-7})(10^4)(1) \text{ J}.$$

The product of the powers of ten is  $10^{-3}$ , and the units reduce to  $\text{C}\cdot\text{m} \times \text{N/C} = \text{N}\cdot\text{m} = \text{J}$ . The negative sign means this aligned state lies below the chosen zero-energy orientation.

**Step 3: Conclusion** The potential energy is  $U = -6 \times 10^{-3} \text{ J}$ , or  $-6 \text{ mJ}$ .

**Quick Tip to Solve:** Write the angle before substituting: parallel means  $0^\circ$ , so the dipole energy is negative.

OR

**34(c).** An electric dipole of dipole moment  $\vec{p}$  is initially kept in a uniform electric field  $\vec{E}$  such that  $\vec{p}$  is perpendicular to  $\vec{E}$ . Find the amount of work done in rotating the dipole to a position at which  $\vec{p}$  becomes antiparallel to  $\vec{E}$ . [2 marks]

**Solution:**

**Concept:**

- For a dipole in a uniform electric field,  $U(\theta) = -pE \cos \theta$ , where  $\theta$  is the angle between  $\vec{p}$  and  $\vec{E}$ .
- For slow rotation with no change in kinetic energy, the work supplied by an external agent is  $W_{\text{ext}} = \Delta U$ ; the field does the opposite work.

**Step 1: Calculate the initial potential energy** Initially the dipole moment is perpendicular to the field, so  $\theta_i = 90^\circ$ . Using the conventional zero at this orientation,

$$U_i = -pE \cos 90^\circ = 0.$$

The magnitudes  $p$  and  $E$  remain unchanged during the rotation.

**Step 2: Find the energy change during rotation** Antiparallel alignment means  $\theta_f = 180^\circ$ , for which  $\cos 180^\circ = -1$ . Thus

$$U_f = -pE \cos 180^\circ = +pE, \quad W_{\text{ext}} = U_f - U_i.$$

This rotation is against the field torque, which tends to restore parallel alignment. Consequently the agent supplies positive work and the electric field does negative work of the same magnitude.

**Step 3: Conclusion** The work required to rotate the dipole is  $W_{\text{ext}} = pE$ . The work done by the electric field is  $-pE$ .

**Quick Tip to Solve:** Use  $W_{\text{ext}} = pE(\cos \theta_i - \cos \theta_f)$ ; distinguish applied work from field work.

**35(a).** The lens maker's formula is useful to design lenses of desired focal lengths using surfaces of suitable radii of curvature. The focal length also depends on the refractive index of the material of the lens and the surrounding medium. The refractive index depends on the wavelength of the light used. The power of a lens is related to its focal length.

Answer the following questions based on the above :

(a) How will the power of a lens be affected with an increase of wavelength of light? [1 mark]

**Solution:**

**Concept:**

- For a thin lens in air,  $P = 1/f = (\mu - 1)(1/R_1 - 1/R_2)$ , where  $\mu$  is the lens refractive index and  $R_1, R_2$  are signed surface radii.
- In normal dispersion, the refractive index of ordinary lens glass decreases as wavelength increases.

**Step 1: Apply the wavelength dependence** Increasing the wavelength decreases  $\mu$ , and therefore decreases the factor  $\mu - 1$ . The surface curvatures are fixed, so only this refractive-index factor changes in the lens maker formula.

**Step 2: Interpret the change in lens strength** The magnitude  $|P|$  decreases and  $|f|$  increases. Thus a convex lens has a smaller positive power. A concave lens becomes less strongly diverging: its negative power moves towards zero, so its numerical signed value increases.

**Step 3: Conclusion** The lens becomes weaker as wavelength increases:  $|P|$  decreases. For a convex lens, its positive power decreases.

**Quick Tip to Solve:** Longer wavelength means smaller refractive index in normal dispersion and therefore weaker lens power.

**35(b).** The radius of curvature of two surfaces of a convex lens is  $R$  each. For what value of  $\mu$  of its material will its focal length become equal to  $R$ ? [1 mark]

**Solution:**

**Concept:**

- The lens maker formula in air is  $1/f = (\mu - 1)(1/R_1 - 1/R_2)$ , with signed radii measured in the direction of incident light.
- For an equiconvex lens, the radii have equal magnitudes but opposite signs:  $R_1 = +R$  and  $R_2 = -R$ .

**Step 1: Insert the correct surface signs** For light travelling from left to right, the first centre of curvature lies to the right of its surface and the second lies to the left. Hence

$$\frac{1}{f} = (\mu - 1) \left( \frac{1}{R} - \frac{1}{-R} \right) = \frac{2(\mu - 1)}{R}.$$

Using  $+R$  for both surfaces would incorrectly give zero power.

**Step 2: Apply the required focal length** Set  $f = R$  and multiply both sides by  $R$ :

$$1 = 2(\mu - 1), \quad \mu - 1 = \frac{1}{2}.$$

The refractive index is a dimensionless ratio, so no length unit remains.

**Step 3: Conclusion** The required refractive index is  $\mu = \frac{3}{2} = 1.5$ .

**Quick Tip to Solve:** Equal radii in a convex lens mean equal magnitudes, not equal signs: use  $+R$  and  $-R$ .

**35(c).** The focal length of a concave lens of  $\mu = 1.5$  is 20 cm in air. It is completely immersed in water of  $\mu = \frac{4}{3}$ . Calculate its focal length in water. [2 marks]

**Solution:**

**Concept:**

- For a thin lens in a surrounding medium,  $1/f = (\mu_{\text{lens}}/\mu_{\text{medium}} - 1)K$ , where  $K = 1/R_1 - 1/R_2$  depends only on its shape.
- A concave glass lens in air is diverging, so the stated focal-length magnitude corresponds to  $f_a = -20$  cm.

**Step 1: Compare the relative refractive indices** In air the refractive-index factor is  $1.5 - 1 = 1/2$ . In water it becomes

$$\frac{\mu_{\text{lens}}}{\mu_{\text{water}}} - 1 = \frac{3/2}{4/3} - 1 = \frac{9}{8} - 1 = \frac{1}{8}.$$

The lens retains the same surface radii on immersion, so  $K$  is unchanged.

**Step 2: Calculate the focal length using a ratio** Dividing the water and air lens maker equations cancels the curvature factor:

$$\frac{1/f_w}{1/f_a} = \frac{1/8}{1/2} = \frac{1}{4}, \quad f_w = 4f_a.$$

Substituting the signed air focal length gives  $f_w = 4(-20)$  cm. The lens is still diverging because its refractive index exceeds that of water, but the smaller index contrast reduces the magnitude of its power.

**Step 3: Conclusion** The focal length in water is  $f_w = -80$  cm; its magnitude is 80 cm.

**Quick Tip to Solve:** Compare relative refractive-index factors first; a fourfold power reduction gives a fourfold focal-length magnitude.

**35(c).** An object is placed in front of a lens which forms its erect image of magnification 3. The power of the lens is 5 D. Calculate the distance of the object and the image from the lens. [2 marks]

**Solution:**

**Concept:**

- The lens power is  $P = 1/f$  with  $f$  in metres. The thin-lens formula is  $1/f = 1/v - 1/u$  in the Cartesian sign convention.
- The transverse magnification is  $m = v/u$ . An erect image has positive magnification; for a real object in front of a converging lens it is virtual.

**Step 1: Determine the focal length and image relation** The positive power  $P = +5$  D identifies a converging lens. Therefore

$$f = \frac{1}{5} \text{ m} = +20 \text{ cm}.$$

Since the image is erect,  $m = +3$ , so  $v/u = 3$  and  $v = 3u$ . Both  $u$  and  $v$  will be negative for this virtual image.

**Step 2: Solve the lens equation** Substitute  $v = 3u$  into the lens formula:

$$\frac{1}{20} = \frac{1}{3u} - \frac{1}{u} = -\frac{2}{3u}, \quad u = -\frac{40}{3} \text{ cm}.$$

Then  $v = 3u = -40$  cm. The object lies inside the focal length, as required for an enlarged erect image formed by a convex lens. The negative image distance places the virtual image on the same side as the object.

**Step 3: Conclusion** The object is 13.3 cm in front of the lens and its virtual image is 40 cm from the lens on the object side ( $u = -40/3$  cm,  $v = -40$  cm).

**Quick Tip to Solve:** Erect magnification means  $m = +3$ , so use  $v = 3u$ , with a negative object distance in the lens equation.