

# CBSE Class 12 Physics Question Paper 2023 with Solution PDF-Set 55/4/2



## Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

### General Instructions

- (i) This question paper contains **35** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
- (iii) In **Section A** – Questions no. **1** to **18** are Multiple Choice (MCQ) type questions, carrying **1** mark each.
- (iv) In **Section B** – Questions no. **19** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C** – Questions no. **26** to **30** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D** – Questions no. **31** to **33** are Long Answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E** – Questions no. **34** and **35** are case-based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 3 questions in Section D and 2 questions in Section E.
- (ix) Use of calculators is not allowed.

Use the following values of physical constants, if required :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

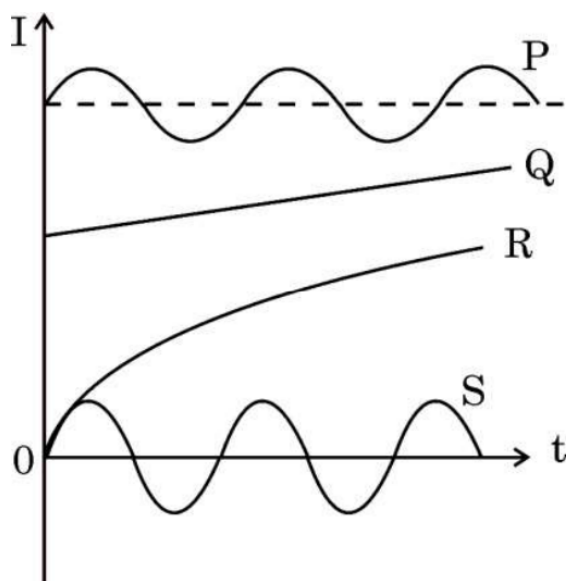
$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

### Section A (18 marks)

1. The figure shows variation of current ( $I$ ) with time ( $t$ ) in four devices P, Q, R and S. The device in which an alternating current flows is: [1 mark]



- (A) P
- (B) Q
- (C) R
- (D) S

**Correct Answer:** (D) S

#### Solution:

##### Concept:

- Alternating current reverses direction periodically; positive and negative values of  $I$  represent opposite directions.
- A time-varying current need not be alternating: direction reverses only when its sign changes.

**Step 1: Locate the zero-current line** The horizontal time axis is  $I = 0$ . Curve S repeatedly passes above and below this axis, so its current changes sign and reverses direction in successive half-cycles.

**Step 2: Test the other three curves** Curve P oscillates about a positive value but remains above zero; its variation alone does not make it alternating. Curves Q and R also remain positive while increasing with time. Thus P, Q and R all represent currents flowing in one direction over the interval shown.

**Step 3: Conclusion** Device S carries alternating current. Hence, the correct option is (D).

**Quick Tip to Solve:** Check whether the graph crosses the zero-current axis; waviness about a positive offset is insufficient.

2. The electromagnetic waves used in radar systems are:

[1 mark]

- (A) Infrared waves
- (B) Ultraviolet rays
- (C) Microwaves
- (D) X-rays

**Correct Answer:** (C) Microwaves

**Solution:**

**Concept:**

- Radar transmits electromagnetic pulses and detects their reflected echoes to locate objects.
- Microwaves have wavelengths suitable for forming narrow directional beams with practical antennas and obtaining strong echoes from aircraft and ships.

**Step 1: Identify the wave needed for radar** A radar beam must travel to a distant target, reflect from it and return to a receiver. Microwaves are commonly used for this purpose; the echo time gives the target distance through  $d = c\Delta t/2$ , where  $c$  is wave speed and  $\Delta t$  is the round-trip time.

**Step 2: Compare the remaining choices** Infrared waves are associated with thermal detection and remote controls. Ultraviolet radiation is used in germicidal applications, while X-rays are used in radiography. These are not the standard electromagnetic band specified for radar systems in this question.

**Step 3: Conclusion** Radar systems use microwaves. Hence, the correct option is (C).

**Quick Tip to Solve:** Associate radar with microwave echoes; do not confuse locating a target by an echo with detecting its infrared heat.

3. In a Young's double-slit experiment, the fringe width is found to be  $\beta$ . If the entire apparatus is immersed in a liquid of refractive index  $\mu$ , the new fringe width will be:

[1 mark]

- (A)  $\beta$
- (B)  $\mu\beta$
- (C)  $\frac{\beta}{\mu}$
- (D)  $\frac{\beta}{\mu^2}$

**Correct Answer:** (C)  $\frac{\beta}{\mu}$

**Solution:**

**Concept:**

- In Young's double-slit experiment, fringe width is  $\beta = \lambda D/d$ , where  $D$  is screen distance and  $d$  is slit separation.
- In a liquid of refractive index  $\mu$ , the wavelength is  $\lambda' = \lambda/\mu$ ; frequency remains unchanged.

**Step 1: Identify what immersion changes** Immersing the entire apparatus changes the light speed and wavelength, but does not change the geometrical distances  $D$  and  $d$ . Thus the new fringe width must change in the same ratio as the wavelength.

**Step 2: Substitute the new wavelength** Applying the fringe-width relation in the liquid gives

$$\beta' = \frac{\lambda' D}{d} = \frac{\lambda D}{\mu d} = \frac{\beta}{\mu}.$$

Option (A) neglects the wavelength change. Option (B) predicts an increase instead of a decrease, and option (D) introduces an extra factor of  $\mu$  even though no distance changes.

**Step 3: Conclusion** The new fringe width is  $\beta/\mu$ . Hence, the correct option is (C).

**Quick Tip to Solve:** When the whole double-slit apparatus enters a medium, divide the original fringe width by its refractive index.

**4. Photons of energy 4.3 eV are incident on a photosensitive surface of work function 2.3 eV. The stopping potential for photoelectrons is:** [1 mark]

- (A) 2.0 eV
- (B) 2.3 eV
- (C) 4.3 eV
- (D) 6.6 eV

**Correct Answer:**  $V_s = 2.0\text{ V}$ . Option (A) is numerically intended, but all printed options incorrectly use eV, an energy unit.

**Solution:**

**Concept:**

- Einstein's photoelectric equation is  $K_{\max} = E_\gamma - \phi$ , where  $E_\gamma$  is photon energy and  $\phi$  is work function.
- The stopping potential satisfies  $eV_s = K_{\max}$ , with  $e$  the magnitude of electron charge. Potential is measured in volts, not electron-volts.

**Step 1: Calculate the maximum kinetic energy** The work function is the energy needed to escape from the surface. Subtract it from the incident photon energy:

$$K_{\max} = (4.3 - 2.3)\text{ eV} = 2.0\text{ eV}.$$

Thus 2.3 eV is the escape energy, 4.3 eV is the incident energy, and 6.6 eV incorrectly adds the two.

**Step 2: Convert energy to stopping potential** Since  $1\text{ eV} = e \times 1\text{ V}$ , dividing the maximum kinetic energy by  $e$  gives

$$V_s = \frac{2.0\text{ eV}}{e} = 2.0\text{ V}.$$

The official paper prints eV in every option. None is dimensionally a potential, although the numerical value in (A) clearly corresponds to the intended result.

**Step 3: Conclusion** The stopping potential is 2.0 V. No option is correct with its printed unit; (A) is the intended numerical choice if eV is read as V.

**Quick Tip to Solve:** Subtract the work function in eV; the numerical difference gives stopping potential in volts, not in eV.

5. Which of the following statements is not true for nuclear forces?

[1 mark]

- (A) They are stronger than Coulomb forces.
- (B) They have about the same magnitude for different pairs of nucleons.
- (C) They are always attractive.
- (D) They saturate as the separation between two nucleons increases.

**Correct Answer:** (C) is the intended answer, but (D) also misstates saturation; the printed wording does not give a unique scientifically precise choice.

**Solution:**

**Concept:**

- Nuclear forces are strong, short-range forces between nucleons. They are attractive at typical nucleon separations but strongly repulsive at extremely small separations.
- Nuclear-force saturation means that each nucleon interacts effectively with only a limited number of nearby nucleons, not that the force becomes constant as separation increases.

**Step 1: Check attraction and strength** Statement (A) expresses that nuclear attraction can overcome proton-proton Coulomb repulsion within a nucleus. Statement (B) expresses approximate charge independence: the nuclear interaction has about the same strength for different nucleon pairs under comparable conditions. Statement (C) is false because the short-distance repulsive core prevents nucleons from collapsing into one another.

**Step 2: Examine the printed saturation statement** The nuclear force rapidly becomes negligible as nucleon separation increases beyond its short range. This decrease is not the definition of saturation. Consequently, the exact wording of (D), which says the forces saturate as separation increases, is also scientifically imprecise. If (D) was meant simply to assert the saturation property, (C) would be the expected single answer.

**Step 3: Conclusion** The intended choice is (C): nuclear forces are not always attractive. As printed, (D) also misdescribes saturation, so a unique strictly correct selection cannot be asserted without that caveat.

**Quick Tip to Solve:** Reject the word “always” in nuclear attraction: there is a repulsive core. Distinguish saturation from decreasing force with distance.

6. A point charge  $q$  is kept at a distance  $r$  from an infinitely long straight wire with charge density  $\lambda$ . The magnitude of the electrostatic force experienced by charge  $q$  is: [1 mark]

- (A) Zero  
(B)  $\frac{q\lambda}{2\pi\epsilon_0 r}$   
(C)  $\frac{q\lambda}{4\pi\epsilon_0 r}$   
(D)  $\frac{q\lambda}{\epsilon_0 r}$

**Correct Answer:** (B)  $\frac{q\lambda}{2\pi\epsilon_0 r}$

**Solution:**

**Concept:**

- Gauss's law states  $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$ , where  $Q_{\text{enc}}$  is enclosed charge.
- The force magnitude on a point charge is  $F = |q|E$ . An infinite line has cylindrical symmetry, so its field varies inversely with radial distance.

**Step 1: Apply Gauss's law to the wire** Choose a coaxial cylindrical Gaussian surface of radius  $r$  and length  $L$ . The field is radial, constant on the curved surface and perpendicular to the end-cap normals. Hence the end caps contribute no flux, and

$$E(2\pi rL) = \frac{|\lambda|L}{\epsilon_0}, \quad E = \frac{|\lambda|}{2\pi\epsilon_0 r}.$$

**Step 2: Relate field to force** Multiplication by the magnitude of the test charge gives

$$F = \frac{|q\lambda|}{2\pi\epsilon_0 r}.$$

With the charge symbols read as magnitudes, this is option (B). Option (A) neglects the wire's field, (C) has the wrong factor of two, and the printed (D) omits the factor  $2\pi$ . The arbitrary length  $L$  correctly cancels.

**Step 3: Conclusion** The force magnitude is  $|q\lambda|/(2\pi\epsilon_0 r)$ , represented by option (B). Hence, the correct option is (B).

**Quick Tip to Solve:** For an infinite line charge use the cylindrical area  $2\pi rL$ ; the resulting field varies as  $1/r$ .

7. The ratio of the magnitude of magnetic moment to that of angular momentum, of an electron revolving around the nucleus in a hydrogen atom is: [1 mark]

- (A) Zero
- (B)  $9.27 \times 10^{-24}$  C/kg
- (C)  $8.8 \times 10^{10}$  C/kg
- (D)  $6.6 \times 10^{-12}$  C/kg

**Correct Answer:** (C)  $8.8 \times 10^{10}$  C/kg

**Solution:**

**Concept:**

- An orbiting electron produces a current loop of magnetic-moment magnitude  $m = IA$ , where  $I$  is current magnitude and  $A$  is orbit area.
- For a circular orbit, angular-momentum magnitude is  $L = m_e v r$ , where  $m_e$ ,  $v$  and  $r$  are electron mass, speed and orbital radius.

**Step 1: Express the orbital magnetic moment** The orbital period is  $T = 2\pi r/v$ , so the current magnitude is  $I = e/T = ev/(2\pi r)$ . Multiplying by the area gives

$$m = \frac{ev}{2\pi r} \pi r^2 = \frac{evr}{2}.$$

**Step 2: Divide by angular momentum** The orbital factors cancel, giving the ratio

$$\frac{m}{L} = \frac{e}{2m_e} = \frac{1.6 \times 10^{-19}}{2(9.1 \times 10^{-31})} \simeq 8.8 \times 10^{10} \text{ C/kg}.$$

The ratio is nonzero. The value  $9.27 \times 10^{-24}$  recalls the Bohr magneton, which is a magnetic moment, not this ratio; the last option has the wrong magnitude. The electron's moment and angular momentum point oppositely, but their requested magnitudes have a positive ratio.

**Step 3: Conclusion** The ratio of magnitudes is  $8.8 \times 10^{10}$  C/kg. Hence, the correct option is (C).

**Quick Tip to Solve:** Use  $m/L = e/(2m_e)$  for an orbital electron; the orbit radius and speed both cancel from the ratio.

8. Which one of the following has negative value of susceptibility? [1 mark]

- (A) Nickel
- (B) Aluminium
- (C) Lead
- (D) Iron

**Correct Answer:** (C) Lead

**Solution:**

**Concept:**

- Magnetic susceptibility  $\chi_m$  relates induced magnetisation to magnetic-field intensity through  $M = \chi_m H$ .
- Diamagnetic materials have negative susceptibility because their induced magnetisation opposes the applied field; paramagnetic and ferromagnetic materials have positive susceptibility in the

usual classification.

**Step 1: Identify the required magnetic behaviour** A negative value of  $\chi_m$  means that the induced magnetisation is opposite to the applied magnetic field. We must therefore select the diamagnetic substance from the four listed materials.

**Step 2: Classify the options** Lead is diamagnetic and has a small negative susceptibility. Aluminium is paramagnetic, so its susceptibility is small and positive. Nickel and iron are ferromagnetic, with strong magnetisation along the applied field under ordinary conditions. Thus none of these three alternatives has the negative susceptibility asked for.

**Step 3: Conclusion** Lead has negative magnetic susceptibility. Hence, the correct option is (C).

**Quick Tip to Solve:** First translate negative susceptibility into diamagnetism; among lead, aluminium, nickel and iron, select lead.

**9. The difference in mass of  ${}^7X$  nucleus and total mass of its constituent nucleons is 21.00 u. The binding energy per nucleon for this nucleus is equal to the energy equivalent of:** [1 mark]

- (A)  $3 u$
- (B)  $3.5 u$
- (C)  $7 u$
- (D)  $21 u$

**Correct Answer:** (A)  $3 u$

**Solution:**

**Concept:**

- The mass defect  $\Delta m$  is the difference between the total free-nucleon mass and the bound nuclear mass; its energy equivalent is the binding energy  $B = \Delta mc^2$ .
- Binding energy per nucleon is  $B/A$ , where  $A$  is the mass number, the total number of protons and neutrons.

**Step 1: Identify the number of nucleons** The superscript in  ${}^7X$  is the mass number, so the nucleus contains  $A = 7$  nucleons. Use the given mass difference  $\Delta m = 21.00 u$  as printed.

**Step 2: Divide the total mass defect by the mass number** The requested quantity is the energy equivalent of the mass defect per nucleon:

$$\frac{B}{A} = \frac{\Delta m}{A} c^2 = \frac{21.00 u}{7} c^2 = 3.00 u c^2.$$

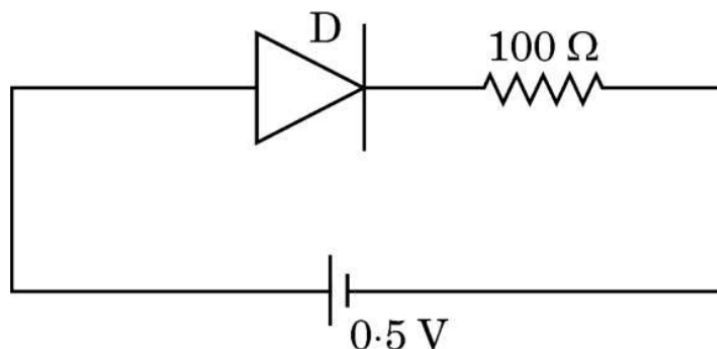
The value  $21 u$  corresponds to the whole nucleus; 7 is its nucleon count, not its mass defect per nucleon. Dividing by six would incorrectly give  $3.5 u$ .

**Step 3: Conclusion** The binding energy per nucleon is the energy equivalent of  $3 u$ . Hence, the correct option is (A).

**Quick Tip to Solve:** For binding energy per nucleon, divide the supplied mass defect by the superscript mass number before converting mass to energy.

10. The threshold voltage for a p-n junction diode used in the circuit is 0.7 V. The type of biasing and current in the circuit are:

[1 mark]



- (A) Forward biasing, 0 A
- (B) Reverse biasing, 0 A
- (C) Forward biasing, 5 mA
- (D) Reverse biasing, 2 mA

**Correct Answer:** (A) Forward biasing, 0 A

**Solution:**

**Concept:**

- A diode is forward biased when its anode (p-side) is connected to the positive supply terminal relative to its cathode (n-side).
- In the threshold-voltage model, appreciable forward current flows only when the applied voltage reaches the threshold  $V_\gamma$ .

**Step 1: Identify the battery and diode polarities** The longer battery plate on the left is the positive terminal. It connects to the left-hand anode of the diode, while the cathode bar on the right connects through the resistor to the negative terminal. Thus the connection is forward bias, regardless of whether appreciable current flows.

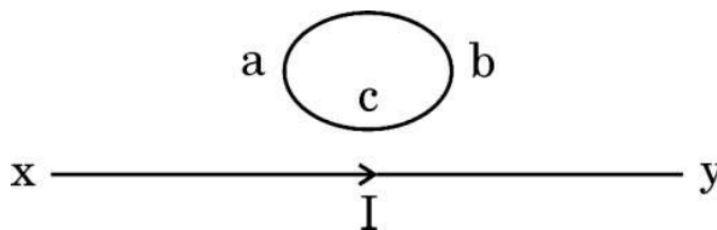
**Step 2: Compare the supply with the threshold** The supply is 0.5 V, smaller than the diode threshold of 0.7 V. The diode therefore remains effectively non-conducting in the stated model, so  $I \simeq 0$  A. The tempting value  $0.5/100 = 5$  mA ignores the diode threshold. Both reverse-bias options have the polarity wrong.

**Step 3: Conclusion** The diode is forward biased, but the circuit current is taken as 0 A. Hence, the correct option is (A).

**Quick Tip to Solve:** Determine bias from the terminal polarities first; then compare the supply voltage with the threshold before applying Ohm's law.

11. The direction of induced current in the loop abc is :

[1 mark]



- (A) along abc if  $I$  decreases
- (B) along acb if  $I$  increases
- (C) along abc if  $I$  is constant
- (D) along abc if  $I$  increases

**Correct Answer:** (D) along abc if  $I$  increases

**Solution:**

**Concept:**

- The right-hand thumb rule determines the field of a straight current-carrying wire: point the thumb along the current and curl the fingers along the magnetic field.
- Faraday's law gives  $\mathcal{E} = -d\Phi_B/dt$ ; Lenz's law makes the induced field oppose the change in magnetic flux  $\Phi_B$ .

**Step 1: Find the magnetic field through the loop** The current in wire  $xy$  flows to the right. At the loop, which lies above the wire, the magnetic field points out of the page. An increasing wire current therefore increases outward magnetic flux through the loop.

**Step 2: Apply Lenz's law and check the paths** To oppose increasing outward flux, the induced current must produce a field into the page, requiring a clockwise current. The path  $a \rightarrow b \rightarrow c \rightarrow a$  is clockwise in the figure. If  $I$  decreases, the induced current is anticlockwise, along  $acb$ , to support the decreasing outward flux. Constant  $I$  produces no induced current. Thus the directions stated in (A), (B) and (C) do not fit their conditions.

**Step 3: Conclusion** The induced current flows along  $abc$  if  $I$  increases. Hence, the correct option is (D).

**Quick Tip to Solve:** Above a wire carrying current to the right, the field is out of the page; increasing that current induces a clockwise loop current.

**12. An ac voltage  $v = v_0 \sin \omega t$  is applied to a series combination of a resistor  $R$  and an element  $X$ . The instantaneous current in the circuit is  $I = I_0 \sin \left( \omega t + \frac{\pi}{4} \right)$ . Then which of the following is correct?** [1 mark]

- (A)  $X$  is a capacitor and  $X_C = \sqrt{2}R$
- (B)  $X$  is an inductor and  $X_L = R$
- (C)  $X$  is an inductor and  $X_L = \sqrt{2}R$
- (D)  $X$  is a capacitor and  $X_C = R$

**Correct Answer:** (D)  $X$  is a capacitor and  $X_C = R$

**Solution:**

**Concept:**

- In a series RC circuit current leads applied voltage; in a series RL circuit current lags applied voltage.
- For a series RC circuit, the current lead angle  $\phi$  satisfies  $\tan \phi = X_C/R$ , where  $X_C = 1/(\omega C)$  is capacitive reactance and  $R$  is resistance.

**Step 1: Read the phase difference from the waveforms** The voltage phase is  $\omega t$ , while the current phase is  $\omega t + \pi/4$ . The positive additional phase means that current leads voltage by  $\pi/4$ , or  $45^\circ$ . Consequently the element is a capacitor; an inductor would give a lag, ruling out (B) and (C).

**Step 2: Determine the reactance to resistance ratio** Using the RC phase relation,

$$\frac{X_C}{R} = \tan \left( \frac{\pi}{4} \right) = 1.$$

Thus option (A), which gives  $X_C = \sqrt{2}R$ , would produce a different phase angle. The factor  $\sqrt{2}$  instead appears in the impedance magnitude  $|Z| = \sqrt{R^2 + X_C^2}$  when the two magnitudes are equal.

**Step 3: Conclusion**  $X$  is a capacitor and  $X_C = R$ . Hence, the correct option is (D).

**Quick Tip to Solve:** A plus phase in the current signals a lead: choose a capacitor, then use the tangent of the phase angle to find  $X_C/R$ .

**13. A plane wavefront is incident on a concave mirror of radius of curvature  $R$ . The radius of the refracted wavefront will be:** [1 mark]

- (A)  $2R$
- (B)  $R$
- (C)  $\frac{R}{2}$
- (D)  $\frac{R}{4}$

**Correct Answer:** (C)  $\frac{R}{2}$

**Solution:**

**Concept:**

- A plane wavefront has parallel normals, so its rays form a parallel beam incident on the mirror.
- For a spherical concave mirror in the paraxial approximation, the principal focal length has magnitude  $f = R/2$ , where  $R$  is the mirror's radius of curvature.

**Step 1: Interpret the outgoing wavefront** The printed word *refracted* refers here to the outgoing reflected wavefront: a mirror produces reflection. The incident parallel rays converge at the principal focus after reflection, so the emerging wavefront is a converging spherical wavefront whose centre is the focus.

**Step 2: Relate the wavefront radius near the mirror to the focal length** At the mirror, the radius of this reflected wavefront is the distance from the mirror to its focus. Using the mirror relation for an object at infinity gives  $1/f = 1/u + 1/v$  with  $1/u = 0$ , hence  $v = f$ . For a spherical mirror  $f = R/2$ . The alternatives  $2R$ ,  $R$ , and  $R/4$  do not equal the focal distance. As the wave propagates towards the focus its radius decreases; the question refers to the wavefront just after reflection.

**Step 3: Conclusion** The intended reflected wavefront has radius  $R/2$  near the mirror. Hence, the correct option is (C).

**Quick Tip to Solve:** For parallel incident rays, locate the focus first: the outgoing spherical wavefront is centred there, so its initial radius is  $f = R/2$ .

**14. A proton and an alpha particle have the same kinetic energy. The ratio of de Broglie wavelengths associated with the proton to that with the alpha particle is:** [1 mark]

- (A) 1
- (B) 2
- (C)  $2\sqrt{2}$
- (D)  $\frac{1}{2}$

**Correct Answer:** (B) 2

**Solution:**

**Concept:**

- The de Broglie wavelength is  $\lambda = h/p$ , where  $h$  is Planck's constant and  $p$  is momentum.
- For a non-relativistic particle with kinetic energy  $K$  and mass  $m$ ,  $p = \sqrt{2mK}$ , so  $\lambda = h/\sqrt{2mK}$ .

**Step 1: Use the equal kinetic energies** Because the two particles have the same  $K$ , their wavelengths vary inversely as the square root of mass. An alpha particle contains two protons and two neutrons, so its mass is approximately four times the proton mass:  $m_\alpha \simeq 4m_p$ .

**Step 2: Form the requested proton to alpha ratio** Cancelling the common factors gives

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_p}} = \sqrt{4} = 2.$$

Equal kinetic energies do not mean equal momenta, ruling out (A). The value  $1/2$  reverses the requested ratio. The alpha particle's charge does not enter at fixed kinetic energy, so the extra factor in (C) is unwarranted.

**Step 3: Conclusion** The ratio  $\lambda_p/\lambda_\alpha$  is 2. Hence, the correct option is (B).

**Quick Tip to Solve:** At equal kinetic energy use the inverse square-root mass rule; the lighter proton must have the longer wavelength.

15. The potential energy of an electron in the second excited state in hydrogen atom is: [1 mark]

- (A)  $-3.4$  eV
- (B)  $-3.02$  eV
- (C)  $-1.51$  eV
- (D)  $-6.8$  eV

**Correct Answer:** (B)  $-3.02$  eV

**Solution:**

**Concept:**

- The hydrogen energy levels are  $E_n = -13.6/n^2$  eV, where  $n$  is the principal quantum number.
- For a Coulomb-bound electron, potential energy  $U = -2K$  and total energy  $E = K + U$ ; therefore  $U = 2E$ .

**Step 1: Identify the quantum number** The ground state is  $n = 1$ , the first excited state is  $n = 2$ , and the second excited state is  $n = 3$ . Thus the total energy in the specified state is

$$E_3 = -\frac{13.6}{3^2} \text{ eV} \simeq -1.51 \text{ eV}.$$

**Step 2: Convert total energy to potential energy** The requested quantity is potential energy, so

$$U_3 = 2E_3 = -\frac{27.2}{9} \text{ eV} \simeq -3.02 \text{ eV}.$$

Option (C) gives the total energy instead. Options (A) and (D) correspond respectively to the total and potential energies at  $n = 2$ , which is only the first excited state.

**Step 3: Conclusion** The potential energy in the second excited state is approximately  $-3.02$  eV. Hence, the correct option is (B).

**Quick Tip to Solve:** Second excited means  $n = 3$ ; after finding  $-13.6/n^2$ , double it because the question asks for potential energy.

**16. Assertion (A):** A current carrying square loop made of a wire of length  $L$  is placed in a magnetic field. It experiences a torque which is greater than the torque on a circular loop made of the same wire carrying the same current in the same magnetic field.

**Reason (R):** A square loop occupies more area than a circular loop, both made of wire of the same length. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).  
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).  
(C) Assertion (A) is true, but Reason (R) is false.  
(D) Assertion (A) is false and Reason (R) is also false.

**Correct Answer:** (D) Assertion (A) is false and Reason (R) is also false.

**Solution:**

**Concept:**

- A planar loop carrying current  $I$  has magnetic moment magnitude  $m = IA$ , where  $A$  is its enclosed area.
- The torque magnitude is  $\tau = IAB \sin \theta$ , where  $B$  is the magnetic field magnitude and  $\theta$  is the angle between the loop normal and the field.

**Step 1: Evaluate the Assertion** For a square of perimeter  $L$ ,  $A_s = (L/4)^2 = L^2/16$ . For a circle of circumference  $L$ ,  $r = L/(2\pi)$ , giving  $A_c = \pi r^2 = L^2/(4\pi)$ . At the same orientation,

$$\frac{\tau_s}{\tau_c} = \frac{A_s}{A_c} = \frac{\pi}{4} < 1.$$

The square has a smaller torque for nonzero  $\sin \theta$ , and a smaller maximum possible torque. The asserted greater torque is false; changing orientation independently cannot establish the asserted general comparison.

**Step 2: Evaluate the Reason** The area ratio is  $A_s/A_c = \pi/4 < 1$ . A circle encloses more area than a square made from the same length of wire. The Reason states the opposite, so it is false.

**Step 3: Check the explanation and answer codes** A false area statement cannot explain the torque claim. Both statements are false. Options (A) and (B) incorrectly regard both statements as true, while (C) incorrectly treats the Assertion as true.

**Step 4: Conclusion** Both the Assertion and the Reason are false. Hence, the correct option is (D).

**Quick Tip to Solve:** For equal wire length compare  $L^2/16$  with  $L^2/(4\pi)$ ; the circle has the larger area and hence the larger maximum torque.

**17. Assertion (A):** In 'n' type semiconductor, number density of electrons is greater than the number density of holes but the crystal maintains an overall charge neutrality.

**Reason (R):** The charge of electrons donated by donor atoms is just equal and opposite to that of the ionised donor. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).  
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).  
(C) Assertion (A) is true, but Reason (R) is false.  
(D) Assertion (A) is false and Reason (R) is also false.

**Correct Answer:** (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

**Solution:**

**Concept:**

- In an  $n$ -type semiconductor, donor impurities provide mobile electrons; electrons are majority carriers and holes are minority carriers.
- Ionisation of a neutral donor produces one electron of charge  $-e$  and one immobile donor ion of charge  $+e$ , where  $e$  is the elementary charge.

**Step 1: Judge the assertion** Donor doping increases the electron number density  $n$  above the hole number density  $p$ . The extra mobile negative charge is balanced by positively charged donor ions fixed in the lattice. Thus  $n > p$  does not imply a net negative charge, and the assertion is true.

**Step 2: Judge the reason** Each donor that supplies an electron loses one negative charge and becomes a positive ion. The donated electron and the ionised donor therefore carry equal and opposite charges. The reason is true.

**Step 3: Test the explanatory link** For donor doping alone, charge neutrality is expressed as  $n = p + N_D^+$ , where  $N_D^+$  is the number density of ionised donors. This directly explains how excess electrons coexist with overall neutrality. Option (B) incorrectly denies this link; (C) and (D) incorrectly reject true statements.

**Step 4: Conclusion** Both the assertion and the reason are true, and the reason correctly explains the assertion. Hence, the correct option is (A).

**Quick Tip to Solve:** Count the fixed donor ions as well as the mobile carriers when checking the charge neutrality of an  $n$ -type crystal.

**18. Assertion (A):** When three electric bulbs of power 200 W, 100 W and 50 W are connected in series to a source, the power consumed by the 50 W bulb is maximum.

**Reason (R):** In a series circuit, current is the same through each bulb, but the potential difference across each bulb is different. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).  
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).  
(C) Assertion (A) is true, but Reason (R) is false.  
(D) Assertion (A) is false and Reason (R) is also false.

**Correct Answer:** (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

**Solution:**

**Concept:**

- For bulbs rated at the same voltage  $V_0$ , their resistances are compared using  $R = V_0^2/P_0$ , where  $P_0$  is the rated power.
- Series-connected bulbs carry the same current  $I$ ; the actual power dissipated by each is  $P = I^2R = IV$ , where  $V$  is its voltage drop.

**Step 1: Judge the assertion** Using the usual common rated voltage and resistance model for the bulbs, their resistance ratio is

$$R_{200} : R_{100} : R_{50} = \frac{1}{200} : \frac{1}{100} : \frac{1}{50} = 1 : 2 : 4.$$

As the current is the same in series, actual power is proportional to resistance. The 50 W bulb therefore dissipates the greatest power, so the assertion is true. Its rated power must not be confused with its actual power in this circuit.

**Step 2: Judge the reason** A series connection provides a single path for current. The bulb resistances differ, so their voltage drops  $V = IR$  also differ, in the ratio 1 : 2 : 4. Thus the reason is true.

**Step 3: Test the explanatory link** The common series current and unequal voltage drops explain the power comparison: the highest-resistance 50 W bulb has the largest drop and hence the largest  $IV$ . Thus (B) wrongly denies the explanation, while (C) and (D) wrongly reject true statements.

**Step 4: Conclusion** Both statements are true, and the reason correctly explains the assertion. Hence, the correct option is (A).

**Quick Tip to Solve:** At a common rated voltage, reverse the rated-power order to get the resistance order; series power then follows that resistance order.

### Section B (14 marks)

**19(a).** Obtain an expression for electrostatic potential energy of a system of three charges  $q$ ,  $2q$  and  $-3q$  placed at the vertices of an equilateral triangle of side  $a$ . [2 marks]

**Solution:**

**Concept:**

- Taking the energy of infinitely separated point charges as zero, the potential energy of a pair is  $U_{ij} = kq_i q_j / r_{ij}$ , where  $k = 1/(4\pi\epsilon_0)$ .
- The energy of several charges is the sum over all distinct pairs, counted once. In an equilateral triangle, all three pair separations equal its side  $a$ .

**Step 1: Identify the three interaction pairs** Let the charges at vertices A, B and C be  $q$ ,  $2q$  and  $-3q$ , respectively. The pairs are AB, AC and BC. The first pair has a positive product, while the other two products are negative because they contain unlike charges.

$$\begin{aligned}U_{AB} &= \frac{k(q)(2q)}{a} = \frac{2kq^2}{a}, \\U_{AC} &= \frac{k(q)(-3q)}{a} = -\frac{3kq^2}{a}, \\U_{BC} &= \frac{k(2q)(-3q)}{a} = -\frac{6kq^2}{a}.\end{aligned}$$

**Step 2: Add the pair energies** Assemble the system from infinity. Bringing in the first charge requires no work; bringing in the other two accounts for precisely these three interactions. Hence

$$U = U_{AB} + U_{AC} + U_{BC} = \frac{kq^2}{a}(2 - 3 - 6).$$

No extra factor of  $1/2$  is needed because each distinct pair has already been included only once.

**Step 3: Conclusion** The electrostatic potential energy is

$$U = -\frac{7q^2}{4\pi\epsilon_0 a}.$$

The negative sign indicates an energy lower than that of the infinitely separated charges.

**Quick Tip to Solve:** For three charges, write the three pair products first and retain their signs; a common separation can then be factored out.

**19(b).** Two small conducting balls A and B of radius  $r_1$  and  $r_2$  have charges  $q_1$  and  $q_2$  respectively. They are connected by a wire. Obtain the expression for charges on A and B, in equilibrium. [2 marks]

**Solution:**

**Concept:**

- Charge flows through the connecting wire until both conducting balls have equal electrostatic potential; equal potential does not generally mean equal charge.
- For a small spherical conductor treated as isolated,  $V = kQ/r$ , where  $Q$  is its charge,  $r$  its radius and  $k = 1/(4\pi\epsilon_0)$ . The total charge of the connected system is conserved.

**Step 1: Apply equality of potentials** Let the final charges on A and B be  $Q_A$  and  $Q_B$ . Use the usual approximation that the balls are sufficiently far apart compared with their radii, so mutual influence is negligible, and the thin wire holds negligible charge. In equilibrium,

$$V_A = V_B \Rightarrow \frac{kQ_A}{r_1} = \frac{kQ_B}{r_2}.$$

Thus the final charges are proportional to their respective radii. This relation follows from equal potential, not from equal surface charge density.

**Step 2: Combine this relation with charge conservation** No charge is supplied from outside, so

$$Q_A + Q_B = q_1 + q_2.$$

Write  $Q_A = Cr_1$  and  $Q_B = Cr_2$ , with the same constant  $C$ . Substitution into the conservation equation gives

$$C(r_1 + r_2) = q_1 + q_2, \quad C = \frac{q_1 + q_2}{r_1 + r_2}.$$

This algebra also covers a negative total charge or a zero total charge.

**Step 3: Conclusion** The equilibrium charges, in the small, widely separated sphere approximation, are

$$Q_A = \frac{r_1(q_1 + q_2)}{r_1 + r_2}, \quad Q_B = \frac{r_2(q_1 + q_2)}{r_1 + r_2}.$$

**Quick Tip to Solve:** First equate  $Q/r$  for connected spheres, then conserve total charge; their final charges divide in the radius ratio.

Answer the following giving reasons:

**20(a).** Do the magnetic field lines also represent the 'lines of force' on a moving charged particle at every point? [2 marks total]

**Solution:**

**Concept:**

- The tangent to a magnetic field line gives the direction of the magnetic field  $\vec{B}$  at that point.
- A charge  $q$ , moving with velocity  $\vec{v}$ , experiences magnetic force  $\vec{F} = q(\vec{v} \times \vec{B})$ .

**Step 1: Identify the direction represented by the line** A magnetic field line specifies the local field

direction. Its tangent depends on the field, whereas the force on a moving particle also depends on the particle's velocity and the sign of its charge.

**Step 2: Compare the force with the field** Whenever the magnetic force is nonzero, the cross product makes it perpendicular to both  $\vec{v}$  and  $\vec{B}$ , rather than along the field-line tangent. If the particle moves parallel or antiparallel to  $\vec{B}$ , the magnetic force is zero even though the magnetic field can be nonzero.

**Step 3: Conclusion** No. Magnetic field lines do not represent the lines of force on a moving electric charge: a nonzero magnetic force is perpendicular to the magnetic field, and its direction also depends on the velocity and charge sign.

**Quick Tip to Solve:** Use the cross product in  $\vec{F} = q(\vec{v} \times \vec{B})$ : the field-line tangent shows  $\vec{B}$ , not the magnetic force on the charge.

## 20(b). If magnetic monopoles existed, how would the Gauss's law of magnetism be modified?

**Solution:**

**Concept:**

- In the absence of magnetic monopoles, Gauss's law for magnetism is  $\oint_S \vec{B} \cdot d\vec{S} = 0$ , where  $S$  is any closed surface and  $d\vec{S}$  points outward.
- A magnetic monopole would act as a source or sink of magnetic field. Define its magnetic charge  $q_m$  as the SI magnetic pole strength, so the field of an isolated pole has magnitude  $B = \mu_0 |q_m| / (4\pi r^2)$ .

**Step 1: Allow a net magnetic source inside the surface** Without monopoles, magnetic field lines form closed loops and every line entering a closed surface also leaves it. A monopole enclosed by the surface would allow an excess of outward or inward field lines, so their net flux would depend on the enclosed magnetic charge.

**Step 2: Write the modified flux law** In the stated magnetic pole-strength convention, the modified law would be

$$\oint_S \vec{B} \cdot d\vec{S} = \mu_0 q_{m,\text{enclosed}}.$$

Here  $q_{m,\text{enclosed}}$  is the algebraic sum of the magnetic charges within  $S$ , positive for a north monopole and negative for a south monopole. Equal opposite charges still give zero net flux.

**Step 3: Conclusion** Gauss's law would equate the net magnetic flux through a closed surface to  $\mu_0$  times its enclosed magnetic charge; it would be nonzero whenever the enclosed net magnetic charge is nonzero.

**Quick Tip to Solve:** Replace the zero in Gauss's magnetic flux law by an enclosed magnetic-source term, and state the magnetic-charge convention used.

## 21(a). What is a displacement current? How is it different from a conduction current?

[2 marks]

**Solution:**

**Concept:**

- A changing electric field contributes to the magnetic field through Maxwell's displacement-current term. In vacuum,  $I_d = \epsilon_0 d\Phi_E/dt$ , where  $\Phi_E = \int \vec{E} \cdot d\vec{A}$  is electric flux.

- Conduction current is the rate of transport of free electric charge across a surface:  $I_c = dQ/dt$ , with  $Q$  the transported charge.

**Step 1: Define displacement current** Displacement current is the effective current associated with a time-varying electric flux, given in vacuum by

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}.$$

It is called a current because its contribution to the magnetic field enters the Ampere–Maxwell law alongside conduction current:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(I_c + I_d).$$

A steady electric flux produces no displacement current.

**Step 2: Contrast the physical origins** Conduction current requires the actual motion of free charge carriers, such as electrons drifting through a metal wire. Displacement current need not involve free charge crossing the region; it can exist in vacuum or in an insulating gap whenever electric flux changes. For example, while a capacitor charges, conduction current flows through its leads but not across the vacuum gap between its plates. The changing electric field in that gap supplies displacement current, equal to the lead current for an ideal charging capacitor.

**Step 3: Conclusion** Displacement current is  $I_d = \varepsilon_0 d\Phi_E/dt$ , caused by changing electric flux. Conduction current is caused by free-charge transport; displacement current can occur even across a vacuum gap without such transport.

**Quick Tip to Solve:** Use a charging capacitor to remember the distinction: moving charges in the leads, changing electric flux between the plates.

OR

**21(b). Write any two characteristics of an electromagnetic wave. Why are microwaves used in radar systems?** [2 marks]

**Solution:**

**Concept:**

- An electromagnetic wave consists of oscillating electric and magnetic fields which sustain propagation without a material medium.
- Diffraction becomes smaller when the wavelength is small compared with the aperture of the transmitting antenna.

**Step 1: State two characteristics** First, electromagnetic waves are transverse: the electric field and magnetic field are perpendicular to one another and both are perpendicular to the direction of propagation. Second, they can travel through vacuum, where their speed is  $c = 1/\sqrt{\mu_0\varepsilon_0}$ , approximately  $3 \times 10^8 \text{ m s}^{-1}$ . Thus no particles of a material medium are required to carry the disturbance.

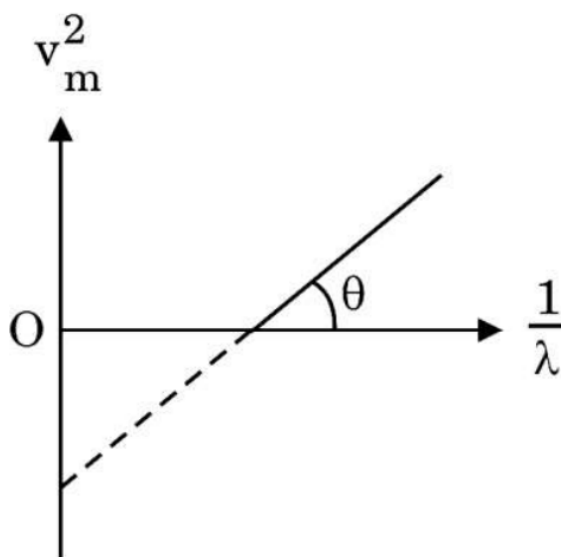
**Step 2: Relate microwave wavelength to radar operation** Microwaves have short wavelengths, so a suitably sized antenna can transmit them as a narrow, highly directional beam with little diffraction. The beam can be aimed at a target, and the reflected signal detected to determine its direction and distance. A narrow beam also helps distinguish nearby targets, which is essential for radar accuracy.

**Step 3: Conclusion** Electromagnetic waves are transverse and propagate through vacuum at speed  $c$ .

Microwaves are used in radar because their short wavelength permits narrow directional beams and useful reflections from targets.

**Quick Tip to Solve:** For radar, connect the short microwave wavelength to small diffraction and hence a narrow beam; merely saying that microwaves travel fast does not distinguish them from other electromagnetic waves.

**22.** The figure shows  $v_m^2$  versus  $\frac{1}{\lambda}$  graph for photoelectrons emitted from a surface where  $v_m$  is the maximum speed of electrons and  $\lambda$  is the wavelength of incident radiation. Using this graph and Einstein's photoelectric equation, obtain the expression for Planck's constant and work function of the surface. [2 marks]



**Solution:**

**Concept:**

- Einstein's photoelectric equation is  $\frac{1}{2}m_e v_m^2 = hc/\lambda - \phi$ , where  $m_e$  is electron mass and  $\phi$  is the surface work function.
- Comparing a straight-line equation with  $y = Sx + b$  identifies its slope  $S$  and signed vertical intercept  $b$ .

**Step 1: Put the photoelectric equation in graph form** Multiply Einstein's equation by  $2/m_e$ , and choose the plotted variables  $y = v_m^2$  and  $x = 1/\lambda$ :

$$v_m^2 = \frac{2hc}{m_e} \left( \frac{1}{\lambda} \right) - \frac{2\phi}{m_e}.$$

Consequently the measured slope is  $S = \Delta(v_m^2)/\Delta(1/\lambda) = 2hc/m_e$ . In the graph's slope notation this is represented by  $\tan \theta$ , with the scales of both axes accounted for.

**Step 2: Read the intercept to find the work function** The extrapolated straight line cuts the vertical axis below the origin. Let the magnitude of that negative intercept be  $a$ , so its signed value is  $b = -a$ . Comparing constants gives  $a = 2\phi/m_e$ . Alternatively, if the horizontal intercept is  $x_0 = 1/\lambda_0$ , the maximum kinetic energy there vanishes and  $\phi = hc x_0$ . The negative continuation is only an extrapolation; negative electron kinetic energies are not physical.

**Step 3: Conclusion** Planck's constant is  $h = m_e S/(2c)$ , conventionally  $h = m_e \tan \theta/(2c)$ . The work function is  $\phi = m_e a/2 = -m_e b/2$ , or equivalently  $\phi = hc/\lambda_0$ .

**Quick Tip to Solve:** Use the slope for Planck's constant and the magnitude of the negative vertical intercept for the work function; keep the factor of two from kinetic energy.

**23. Show that the density of a nucleus is independent of its mass number. Why are lighter nuclei more stable than heavier nuclei?** [2 marks]

**Solution:**

**Concept:**

- A nucleus of mass number  $A$  has approximately mass  $M = Am_N$  and radius  $R = R_0 A^{1/3}$ , where  $m_N$  is a nucleon mass and  $R_0$  is constant.
- Nuclear attraction is short ranged and saturates, whereas electrostatic repulsion between protons is long ranged.

**Step 1: Calculate the nuclear density** Treating the nucleus as a sphere, its volume is  $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$ . Therefore,

$$\rho = \frac{M}{V} = \frac{Am_N}{(4/3)\pi R_0^3 A} = \frac{3m_N}{4\pi R_0^3}.$$

Both mass and volume are proportional to  $A$ , so the factor  $A$  cancels. This establishes the approximately constant nuclear density within the empirical radius model.

**Step 2: Explain the reduced stability of very heavy nuclei** In a heavy nucleus each nucleon attracts mainly its near neighbours through the short-range nuclear force. However, every proton repels the other protons electrically. As the proton number increases, this growing long-range repulsion competes more strongly with the saturating nuclear attraction, making very heavy nuclei less tightly bound and more prone to decay or fission. Lighter nuclei have a smaller disruptive Coulomb repulsion. This is the intended comparison with very heavy nuclei; it is not a universal ordering by mass, since binding energy per nucleon is highest near iron and nickel, rather than for the very lightest nuclei.

**Step 3: Conclusion** Nuclear density is  $\rho = 3m_N/(4\pi R_0^3)$ , independent of  $A$ . Relative to very heavy nuclei, lighter nuclei suffer less proton-proton repulsion, while nuclear attraction remains effective over nearby nucleons.

**Quick Tip to Solve:** Cube  $R_0 A^{1/3}$  before dividing mass by volume; for stability, contrast the short range of nuclear attraction with the long range of Coulomb repulsion.

**24. Use Gauss's law to obtain the expression for electric field due to a uniformly charged infinite plane thin sheet.** [2 marks]

**Solution:**

**Concept:**

- Gauss's law states that the electric flux through a closed surface is  $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$ .
- Translational symmetry of a uniformly charged infinite sheet makes the field normal to the sheet and equal in magnitude on its two sides.

**Step 1: Choose a Gaussian pillbox** Let the uniform surface charge density be  $\sigma$ . Choose a small

cylinder passing through the sheet, with its axis perpendicular to it and with end faces of equal area  $S$  parallel to the sheet. Reflection symmetry gives the same field magnitude  $E$  at the two end faces. For positive charge the field points away from the sheet, parallel to each end face's outward area vector.

**Step 2: Evaluate the flux and apply Gauss's law** The field is parallel to the curved surface, so its flux through that surface is zero. The outward flux through each flat end is  $ES$ . The cylinder encloses the charge on the area  $S$  of the sheet, namely  $Q_{\text{enc}} = \sigma S$ . Therefore, for  $\sigma > 0$ ,

$$\oint \vec{E} \cdot d\vec{A} = ES + ES = 2ES,$$

$$2ES = \frac{\sigma S}{\epsilon_0}.$$

The end-face area cancels; the position of the end faces does not enter the result.

**Step 3: Conclusion** The electric field magnitude on either side is  $E = |\sigma|/(2\epsilon_0)$ , independent of distance. It is perpendicular to the sheet, directed away from a positively charged sheet and towards a negatively charged sheet.

**Quick Tip to Solve:** Count both flat faces of the pillbox: each contributes flux. Omitting one face incorrectly gives  $\sigma/\epsilon_0$  for an isolated thin sheet.

**25. Two identical cells, each of emf  $E$  and internal resistance  $r$ , are connected with a load resistance  $R$ , first in series and then in parallel. Obtain the condition under which the current through  $R$  is same in both cases.** [2 marks]

**Solution:**

**Concept:**

- The load current is the equivalent emf divided by the total resistance, including the battery's internal resistance.
- Identical cells in series aiding have equivalent emf  $2E$  and resistance  $2r$ ; in parallel with like terminals joined they have emf  $E$  and resistance  $r/2$ .

**Step 1: Write the load currents in the two arrangements** For the series connection, the emfs add and the same current passes through both internal resistances. In the parallel connection, the terminal voltage corresponds to one cell's emf, while two identical internal resistances act in parallel. Hence the currents through the external load are

$$I_s = \frac{2E}{R + 2r}, \quad I_p = \frac{E}{R + r/2} = \frac{2E}{2R + r}.$$

Here  $I_p$  is the total load current, not the current supplied by just one cell.

**Step 2: Equate the currents and simplify** For a nonzero cell emf, imposing  $I_s = I_p$  permits the common numerator  $2E$  to cancel. Thus the denominators must be equal:

$$R + 2r = 2R + r,$$

$$2r - r = 2R - R.$$

As a consistency check, substituting the resulting equal resistance values into either current expression gives the same value  $2E/(3r)$ , for positive internal resistance.

**Step 3: Conclusion** The currents are equal when the external load resistance equals the internal resistance of each cell:  $R = r$ . The common load current is then  $2E/(3r)$ .

**Quick Tip to Solve:** For identical cells, write both currents with numerator  $2E$ , then equate denominators. In parallel the internal resistance halves, while the emf stays  $E$ .

### Section C (15 marks)

**26(a).** A resistor of  $30\ \Omega$  and a capacitor of  $\frac{250}{\pi}\ \mu\text{F}$  are connected in series to a  $200\ \text{V}$ ,  $50\ \text{Hz}$  ac source. Calculate (i) the current in the circuit, and (ii) voltage drops across the resistor and the capacitor. (iii) Is the algebraic sum of these voltages more than the source voltage? If yes, solve the paradox. [3 marks]

**Solution:**

**Concept:**

- Capacitive reactance is  $X_C = 1/(2\pi fC)$ , where  $f$  is the supply frequency and  $C$  is the capacitance.
- For a series RC circuit,  $Z = \sqrt{R^2 + X_C^2}$  and the rms current is  $I = V/Z$ . The component rms voltages are  $V_R = IR$  and  $V_C = IX_C$ .
- The resistor voltage is in phase with current, whereas the capacitor voltage lags current by  $90^\circ$ ; their rms magnitudes therefore add as perpendicular phasors.

**Step 1: (i) Calculate the reactance and current** Treat the quoted ac source voltage as an rms value. Convert capacitance from microfarads to farads before substitution:

$$C = \frac{250}{\pi} \times 10^{-6}\ \text{F}, \quad X_C = \frac{1}{2\pi(50)(250/\pi)10^{-6}} = 40\ \Omega.$$

Hence the impedance magnitude and rms current are

$$Z = \sqrt{30^2 + 40^2} = 50\ \Omega, \quad I = \frac{200}{50} = 4.0\ \text{A}.$$

The same current flows through both components because they are connected in series.

**Step 2: (ii) Find the voltage across each component** Multiply the rms current by the resistance or reactance of the corresponding component:

$$V_R = 4 \times 30 = 120\ \text{V}, \quad V_C = 4 \times 40 = 160\ \text{V}.$$

These are the readings that ideal ac voltmeters connected across the resistor and capacitor would show.

**Step 3: (iii) Resolve the apparent voltage paradox** Their algebraic sum is  $120 + 160 = 280\ \text{V}$ , which exceeds  $200\ \text{V}$ . This does not violate Kirchhoff's voltage law: the two instantaneous voltage drops add to the instantaneous supply voltage, but their peaks occur a quarter-cycle apart. Consequently their rms magnitudes cannot be added algebraically. The phasor sum gives

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{120^2 + 160^2} = 200\ \text{V},$$

exactly the specified source voltage.

**Step 4: Conclusion** The rms current is 4.0 A; the resistor and capacitor voltage drops are 120 V and 160 V, respectively. Their magnitude sum exceeds the source voltage because the drops differ in phase by  $90^\circ$ , and their vector sum is 200 V.

**Quick Tip to Solve:** Recognise the 30-40-50 impedance triangle; use the same perpendicular-triangle relation for the rms voltages.

OR

**26(b).** A series LCR circuit with  $R = 20\ \Omega$ ,  $L = 2\ \text{H}$  and  $C = 50\ \mu\text{F}$  is connected to a 200 volts ac source of variable frequency. What is (i) the amplitude of the current, and (ii) the average power transferred to the circuit in one complete cycle, at resonance? (iii) Calculate the potential drop across the capacitor. [3 marks]

**Solution:**

**Concept:**

- A series LCR circuit resonates at angular frequency  $\omega_0 = 1/\sqrt{LC}$ . Then  $X_L = X_C$ , so the impedance is purely resistive:  $Z = R$ .
- An ac source voltage quoted without a peak qualifier is its rms value. Current amplitude is  $I_0 = \sqrt{2}I_{\text{rms}}$ , and average power is  $P = V_{\text{rms}}I_{\text{rms}}\cos\phi$ .
- At resonance  $\phi = 0$ . The capacitor's rms voltage is  $V_C = I_{\text{rms}}X_C$ ; it can exceed the supply voltage because the inductor and capacitor voltages cancel as phasors.

**Step 1: (i) Find the current amplitude at resonance** The inductive and capacitive reactances cancel at resonance. Thus only the resistance  $R = 20\ \Omega$  limits the series current. Taking 200 V as the supply rms voltage gives

$$I_{\text{rms}} = \frac{200}{20} = 10\ \text{A}, \quad I_0 = \sqrt{2} \times 10 = 10\sqrt{2}\ \text{A} \approx 14.1\ \text{A}.$$

The amplitude requested is the maximum instantaneous current, so it is larger than the rms current by  $\sqrt{2}$ .

**Step 2: (ii) Calculate the average power over a cycle** The power factor is unity at resonance. Ideal inductors and capacitors store and return energy, with zero net absorption over one full cycle; only the resistor dissipates energy:

$$P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos 0^\circ = 200 \times 10 = 2000\ \text{W}.$$

The equivalent check is  $I_{\text{rms}}^2 R = 10^2 \times 20 = 2000\ \text{W}$ .

**Step 3: (iii) Calculate the capacitor voltage** First use  $C = 50 \times 10^{-6}\ \text{F}$  to find the resonance frequency and capacitor reactance:

$$\omega_0 = \frac{1}{\sqrt{2(50 \times 10^{-6})}} = 100\ \text{rad s}^{-1},$$

$$X_C = \frac{1}{100(50 \times 10^{-6})} = 200\ \Omega.$$

Therefore  $V_{C,\text{rms}} = 10 \times 200 = 2000\ \text{V}$ . Its peak value, if required, is  $2000\sqrt{2}\ \text{V}$ . The equally large inductor voltage is opposite in phase, leaving the resistor voltage equal to the supply voltage.

**Step 4: Conclusion** At resonance, the current amplitude is  $10\sqrt{2}\ \text{A} \approx 14.1\ \text{A}$ , average power is 2000 W,

and the capacitor voltage drop is 2000 V rms (peak  $2000\sqrt{2}$  V).

**Quick Tip to Solve:** At resonance compute the rms current with  $V/R$ , then multiply by  $\sqrt{2}$  only when the question asks for amplitude.

- 27(a).** (i) In diffraction due to a single slit, the phase difference between light waves reaching a point on the screen is  $5\pi$ . Explain whether a bright or a dark fringe will be formed at the point.
- (ii) What should the width ( $a$ ) of each slit be to obtain eight maxima of two double-slit patterns (slit separation  $d$ ) within the central maximum of the single slit pattern?
- (iii) Draw the plot of intensity distribution in a diffraction pattern due to a single slit. [3 marks]

**Solution:**

**Concept:**

- For a slit of width  $a$ , the edge-to-edge phase difference is  $\delta = 2\pi a \sin \theta / \lambda$ . With  $u = a \sin \theta / \lambda$ , the intensity is  $I = I_0 [\sin(\pi u) / (\pi u)]^2$ .
- Diffraction minima occur at  $a \sin \theta = m\lambda$ , for nonzero integers  $m$ . Secondary bright fringes lie between successive minima and become weaker away from the centre.
- On a distant screen at distance  $D$ , the central diffraction width is  $W = 2D\lambda/a$ , while the double-slit interference fringe spacing is  $\beta = D\lambda/d$ .

**Step 1: (i) Classify the fringe for the printed phase difference** The official question gives  $\delta = 5\pi$ . Hence the path difference across the slit is

$$a \sin \theta = \frac{\delta \lambda}{2\pi} = \frac{5\lambda}{2}.$$

Divide the slit into five equal strips: adjacent strips differ in phase by  $\pi$ , so four strips cancel in two pairs while one remains uncompensated. The point is therefore in a weak bright fringe, not at a dark minimum. In the usual school approximation it is the second secondary maximum. More precisely,

$$\frac{I}{I_0} = \left[ \frac{\sin(5\pi/2)}{5\pi/2} \right]^2 = \frac{4}{25\pi^2} > 0.$$

Exact secondary maxima satisfy  $\tan(\delta/2) = \delta/2$ , so  $5\pi$  is the approximate maximum position rather than an exact stationary maximum.

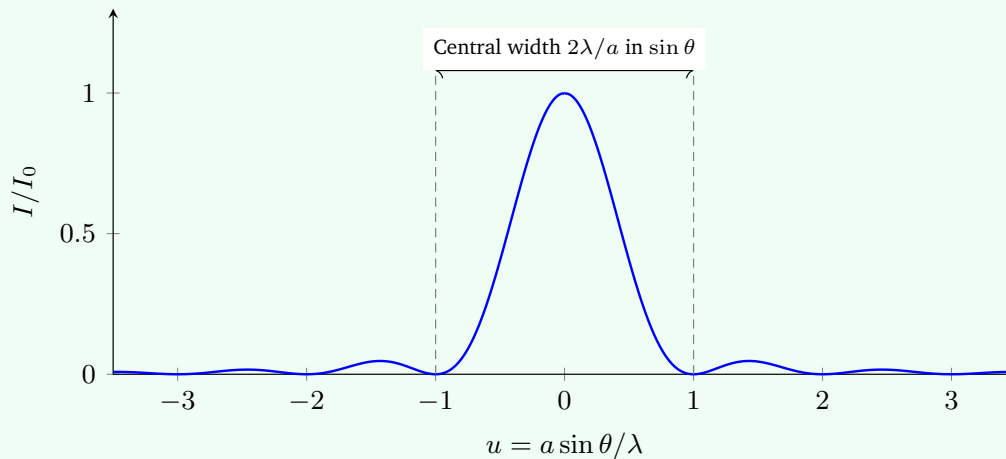
**Step 2: (ii) Relate the central width to eight fringe spacings** Using the standard width-ratio interpretation of the stated eight maxima, set the central diffraction width equal to eight interference fringe spacings:

$$\frac{2D\lambda}{a} = 8 \frac{D\lambda}{d} \Rightarrow \frac{2d}{a} = 8 \Rightarrow a = \frac{d}{4}.$$

This is the intended slit-width relation. Strictly, a symmetric central envelope contains an odd number of distinct surviving bright centres, including the central one. For  $d/a = 4$ , orders  $m = \pm 4$  coincide with diffraction zeros, leaving seven centres  $m = 0, \pm 1, \pm 2, \pm 3$ . Thus “eight” here uses the conventional fringe-width count, not a literal count of those surviving centres.

**Step 3: (iii) Show the single-slit intensity distribution** Plot the normalised intensity against  $u = a \sin \theta / \lambda$ . The central peak has intensity  $I_0$  and extends between the first zeros at  $u = -1$  and  $u = 1$ . It is twice as wide as each secondary lobe between adjacent zeros. The pattern is symmetric, with weak side maxima whose heights decrease away from the centre, as the plotted squared-sinc function

shows.



*Single-slit intensity: the central lobe spans twice the width of a side lobe.*

**Step 4: Conclusion** (i) A weak bright fringe is obtained for  $5\pi$ , near the second secondary maximum. (ii) The intended width-ratio result is  $a = d/4$ . (iii) The plot shows a broad central maximum, zeros at nonzero integer  $a \sin \theta / \lambda$ , and progressively weaker side maxima.

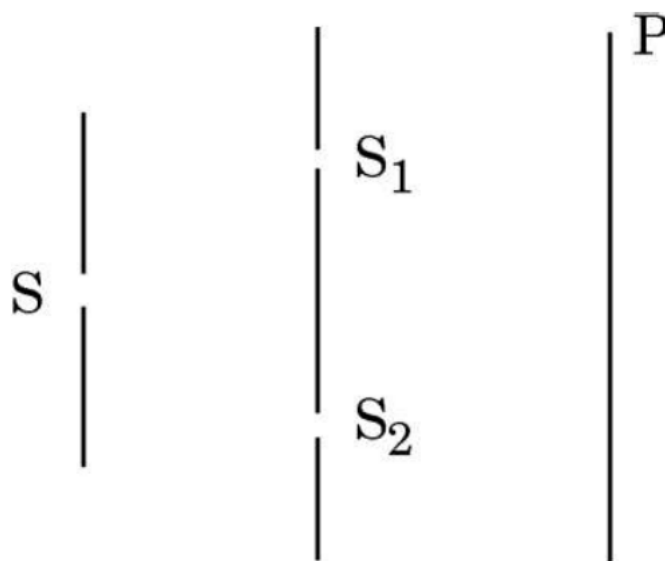
**Quick Tip to Solve:** For single-slit diffraction, an odd multiple of  $\pi$  across the whole slit is not a dark-fringe condition; minima need a nonzero multiple of  $2\pi$ .

OR

**27(b).** (i) In a Young's double-slit experiment  $SS_2 - SS_1 = \frac{\lambda}{4}$ , where  $S_1$  and  $S_2$  are the two slits as shown in the figure. Find the path difference ( $S_2P - S_1P$ ) for constructive and destructive interference at P.

(ii) What is the effect on the interference fringes in a Young's double-slit experiment, if the monochromatic source S is replaced by a source of white light?

[3 marks]



### Solution:

#### Concept:

- Interference depends on the total path difference from the common source to the observation point, including the unequal source-to-slit paths.
- Constructive interference requires total path difference  $n\lambda$ ; destructive interference requires  $(n + \frac{1}{2})\lambda$ , where  $n$  is any integer.
- In Young's experiment the fringe width is  $\beta = \lambda D/d$ , where  $D$  is the screen distance and  $d$  is the slit separation.

**Step 1: (i) Include the initial path difference** Put  $x = S_2P - S_1P$ . The two waves travel from S through their respective slits before reaching P, so their total path difference is

$$\begin{aligned}\Delta &= (SS_2 + S_2P) - (SS_1 + S_1P) \\ &= \frac{\lambda}{4} + x.\end{aligned}$$

Using only the slit-to-screen paths would omit the phase difference already present at the slits.

**Step 2: Apply the bright and dark conditions** For constructive interference, set  $\Delta = n\lambda$ . Thus

$$S_2P - S_1P = \left(n - \frac{1}{4}\right)\lambda.$$

For destructive interference, set  $\Delta = (n + \frac{1}{2})\lambda$ . This gives

$$S_2P - S_1P = \left(n + \frac{1}{4}\right)\lambda.$$

Here  $n \in \mathbb{Z}$ ; the signed path difference permits fringes on both sides of the zero-total-path position. Equivalently, the bright family can be written  $(4m + 3)\lambda/4$  and the dark family  $(4m + 1)\lambda/4$ , with integer  $m$ .

**Step 3: (ii) Superpose the colours of white light** White light contains many wavelengths. At the position where the total path difference is zero, all wavelengths interfere constructively together, producing a white central fringe. Since the source is off the perpendicular bisector, this position is shifted from the geometrical screen centre. On either side a few coloured fringes appear: violet is nearer the white fringe and red is farther away because fringe width increases with wavelength. At larger orders, maxima and minima of different colours overlap, so the fringes become indistinct and merge into nearly uniform illumination. The original monochromatic fringe width is therefore replaced by different widths for different colours.

**Step 4: Conclusion** The required path differences  $S_2P - S_1P$  are  $(n - \frac{1}{4})\lambda$  for constructive interference and  $(n + \frac{1}{4})\lambda$  for destructive interference. White light gives a white zero-total-path fringe, with a few coloured fringes on either side before overlap removes clear fringes.

**Quick Tip to Solve:** First add the source-to-slit path difference; then impose the bright or dark condition on the total path difference.

**28. (a)** Differentiate between ‘impact parameter’ and ‘distance of closest approach’ in alpha particle scattering experiment. Identify, situations when impact parameter is (i) minimum, and (ii) very large. **(b)** An electron cannot revolve around the nucleus in a hydrogen atom at an arbitrary distance. Explain. [3 marks]

**Solution:**

**Concept:**

- In Rutherford scattering, the impact parameter  $b$  specifies the incident trajectory, whereas the distance of closest approach specifies the minimum actual separation during the deflected motion.
- Coulomb repulsion bends an alpha particle’s trajectory; smaller impact parameters produce greater deflection for a fixed incident kinetic energy.
- Bohr’s quantisation condition is  $m_e v r = n\hbar$ , with  $n = 1, 2, 3, \dots$ ,  $m_e$  the electron mass and  $\hbar = h/(2\pi)$ .

**Step 1: (a) Distinguish the two distances** The impact parameter is the perpendicular distance between the initial straight-line direction of motion of the incident alpha particle and the centre of the nucleus. It is defined from the incident trajectory, before appreciable deflection. The distance of closest approach is the smallest distance between the alpha particle and the nuclear centre along its actual curved trajectory. Thus one describes the initial aim and the other describes how close the particle really gets under electrostatic repulsion. For a head-on approach with initial kinetic energy  $K$ , energy conservation gives  $r_{\min} = k(2e)(Ze)/K$ , where  $Ze$  is the nuclear charge and  $k = 1/(4\pi\epsilon_0)$ .

**Step 2: Identify the limiting impact parameters** (i) The impact parameter is minimum,  $b = 0$ , for a head-on approach directed exactly towards the nucleus. The particle momentarily stops at its closest approach and is scattered backwards through  $180^\circ$ . Its closest-approach distance remains nonzero despite zero impact parameter. (ii) The impact parameter is very large when the incident trajectory passes far from the nuclear centre. The Coulomb impulse is then small, and the particle passes almost undeviated, with scattering angle close to zero.

**Step 3: (b) Apply orbital quantisation in hydrogen** An allowed Bohr orbit must satisfy both the Coulomb centripetal-force condition and angular-momentum quantisation:

$$\frac{m_e v^2}{r} = \frac{k e^2}{r^2}, \quad m_e v r = n\hbar.$$

Eliminating  $v$  gives discrete radii,

$$r_n = \frac{n^2 \hbar^2}{m_e k e^2} = n^2 a_0, \quad n = 1, 2, 3, \dots$$

Equivalently, the electron’s de Broglie wave must form a standing wave:  $2\pi r = n\lambda_{\text{dB}}$ , with  $\lambda_{\text{dB}} = h/(m_e v)$ . An arbitrary radius does not satisfy this condition and therefore cannot represent an allowed stationary orbit in the Bohr model.

**Step 4: Conclusion** Impact parameter measures the initial sideways offset; closest approach measures the minimum actual nuclear separation. Head-on backscattering has  $b = 0$ , and almost undeviated scattering has very large  $b$ . Hydrogen permits only the quantised Bohr radii  $r_n = n^2 a_0$ .

**Quick Tip to Solve:** Keep the initial aiming distance separate from the actual turning distance; use the standing-wave condition to explain allowed atomic radii.

**29.** Two infinitely long parallel conductors A and B, separated by 2 cm, carry 6 A and 2 A currents in opposite directions respectively. Find:

(a) the net magnetic field at a point midway between A and B.

(b) the force acting per unit length on B.

[3 marks]

**Solution:**

**Concept:**

- An infinitely long straight conductor carrying current  $I$  produces magnetic field  $B = \mu_0 I / (2\pi r)$  at perpendicular distance  $r$ ; its direction follows the right-hand thumb rule.
- Magnetic fields superpose vectorially. Between parallel wires carrying opposite currents, the individual magnetic fields point in the same direction.
- The force per unit length between two long parallel conductors separated by  $d$  is  $F/L = \mu_0 I_A I_B / (2\pi d)$ ; opposite currents repel.

**Step 1: (a) Fix the midpoint distances and directions** The wire separation is  $d = 2 \text{ cm} = 0.020 \text{ m}$ , so the midpoint is at  $r = d/2 = 0.010 \text{ m}$  from each wire. By the right-hand rule, reversing the current in the second wire makes its field between the wires parallel to the first wire's field. Their magnitudes must therefore be added. The question specifies opposite currents but no absolute orientation, so the field direction is stated relative to a chosen orientation below.

**Step 2: Calculate the net field** With  $I_A = 6 \text{ A}$ ,  $I_B = 2 \text{ A}$  and  $\mu_0 / (2\pi) = 2 \times 10^{-7} \text{ T m A}^{-1}$ ,

$$\begin{aligned} B_A &= \frac{2 \times 10^{-7} \times 6}{0.010} = 1.2 \times 10^{-4} \text{ T}, \\ B_B &= \frac{2 \times 10^{-7} \times 2}{0.010} = 4.0 \times 10^{-5} \text{ T}, \\ B_{\text{net}} &= B_A + B_B = 1.6 \times 10^{-4} \text{ T}. \end{aligned}$$

The net field is perpendicular to the plane containing the wires. For example, with A on the left carrying current upwards and B on the right carrying current downwards, both midpoint fields are into the page. Reversing both currents reverses this direction.

**Step 3: (b) Use A's field at the position of B** The force on B is due to the magnetic field of A evaluated at the full separation  $d$ , not the midpoint distance and not B's own field. Therefore

$$\begin{aligned} \frac{F_B}{L} &= I_B \frac{\mu_0 I_A}{2\pi d} \\ &= \frac{2 \times 10^{-7} \times 6 \times 2}{0.020} \\ &= 1.2 \times 10^{-4} \text{ N m}^{-1}. \end{aligned}$$

Since the currents are in opposite directions, the conductors repel. The force on B is directed away from A, perpendicular to B within the plane of the two conductors.

**Step 4: Conclusion** (a) The midpoint field is  $1.6 \times 10^{-4} \text{ T}$ , normal to the plane of the wires in the common right-hand-rule direction. (b) The force per unit length on B is  $1.2 \times 10^{-4} \text{ N m}^{-1}$ , directed away from A.

**Quick Tip to Solve:** Add the fields between wires with opposite currents; use half the separation for the midpoint field and the full separation for their force.

**30(a).** Two coplanar concentric circular loops of radii  $R$  and  $r$  ( $r \ll R$ ) are arranged coaxially. Obtain the expression for their mutual inductance. [3 marks total]

**Solution:**

**Concept:**

- Mutual inductance is the flux linked with one loop per unit current in the other:  $M = \Phi/I$ , for a single-turn receiving loop.
- A circular loop of radius  $R$  carrying current  $I$  produces a field  $B = \mu_0 I/(2R)$  at its centre, normal to its plane. Here  $\mu_0$  is the permeability of free space.

**Step 1: Find the field over the smaller loop** Pass a current  $I$  through the larger loop. Since the loops are concentric and coplanar, the magnetic field at the centre is perpendicular to the area of the smaller loop. Since  $r \ll R$ , the field changes negligibly over this small area, so we may use the central value throughout it:

$$B \simeq \frac{\mu_0 I}{2R}.$$

**Step 2: Calculate the flux linkage and divide by the current** The smaller loop has area  $A = \pi r^2$ ; its area normal is parallel to the field. Thus its magnetic flux, in magnitude, is

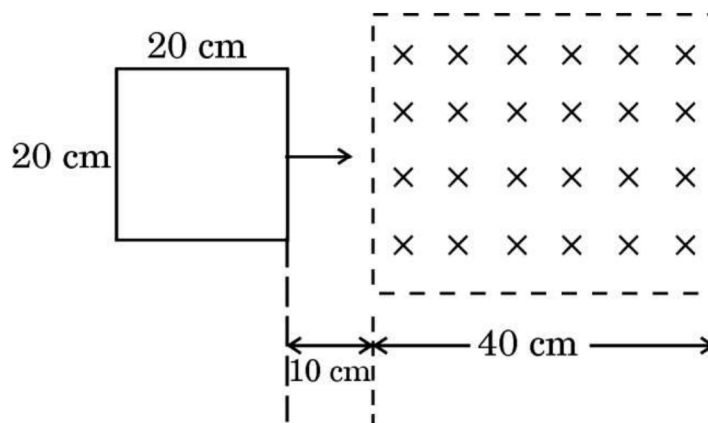
$$\Phi \simeq BA = \frac{\mu_0 I \pi r^2}{2R}.$$

There is one turn in each loop, so no additional turn factor is needed. Dividing this flux by the current in the larger loop gives the mutual inductance. Reciprocity means the same mutual inductance applies if the smaller loop carries the current instead.

**Step 3: Conclusion** The mutual inductance is  $M \simeq \frac{\mu_0 \pi r^2}{2R}$ , in henry, valid for  $r \ll R$ . This is its magnitude; the sign of the flux linkage depends on the chosen loop orientations.

**Quick Tip to Solve:** Put the trial current in the larger loop so its nearly uniform central field can be multiplied directly by the smaller loop's area.

**30(b).** A square loop of side 20 cm starts moving at  $t = 0$  with a velocity of 5 cm/s towards a region of uniform magnetic field as shown in the figure. Specify the time interval(s) during which induced emf is produced in the loop.



### Solution:

#### Concept:

- Faraday's law gives  $\varepsilon = -d\Phi/dt$ . In a uniform stationary magnetic field,  $\Phi = BA_{\text{in}}$ , where  $A_{\text{in}}$  is the area of the loop inside the field.
- A translating loop develops induced emf only while its overlap area with the magnetic region changes. For constant speed, the time to cover a distance  $d$  is  $t = d/v$ .

**Step 1: Find when the loop enters the field** From the figure, the initial gap between the leading edge of the loop and the near boundary is 10 cm, the loop side is 20 cm, the field region is 40 cm wide, and  $v = 5 \text{ cm s}^{-1}$ . The leading edge reaches the field at  $t_1 = 10/5 = 2 \text{ s}$ . The trailing edge reaches the same boundary at  $t_2 = (10 + 20)/5 = 6 \text{ s}$ . Between these times, the area inside the field increases, so the flux changes.

**Step 2: Find when the loop leaves the field** The leading edge reaches the far boundary after travelling the initial gap plus the field width:  $t_3 = (10 + 40)/5 = 10 \text{ s}$ . The trailing edge leaves the far boundary at  $t_4 = (10 + 40 + 20)/5 = 14 \text{ s}$ . Between these times, the overlap area decreases. From 6 to 10 s, the whole loop is inside the uniform field and its flux is constant. Before 2 s and after 14 s, the loop is outside the field. Hence these other intervals give zero emf. Entry and exit emfs have opposite signs because the flux changes in opposite senses.

**Step 3: Conclusion** Induced emf is produced during entry from **2 s to 6 s** and during exit from **10 s to 14 s**. At the ideal sharp boundaries, the emf switches between zero and its nonzero value.

**Quick Tip to Solve:** Track the leading and trailing edges across both field boundaries; motion entirely within a uniform field gives constant flux and zero emf.

## Section D (15 marks)

**31(a).** (i) Draw a ray diagram to show how the final image is formed at infinity in an astronomical refracting telescope. Obtain an expression for its magnifying power.

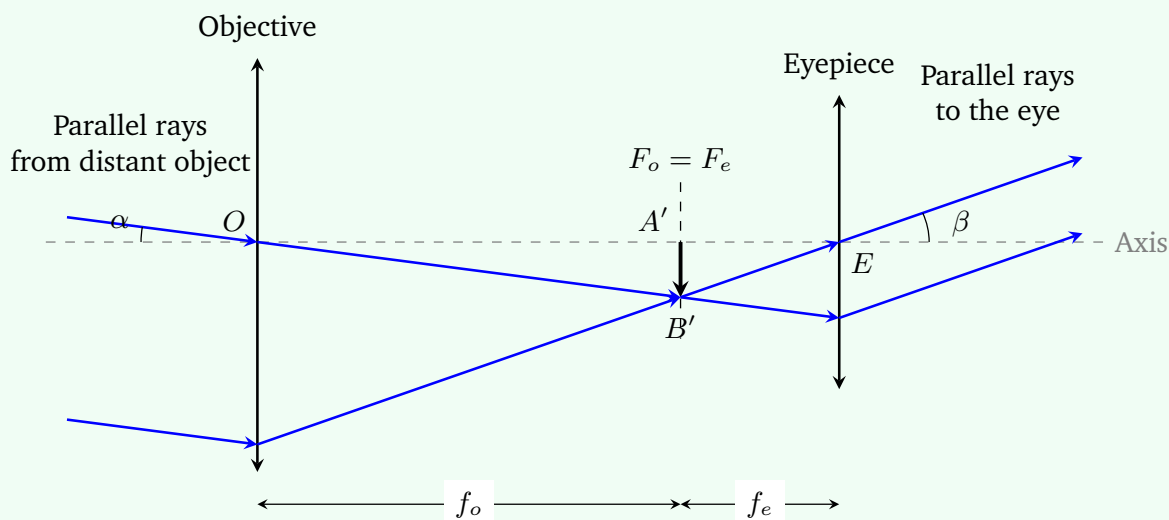
(ii) Two thin lenses  $L_1$  and  $L_2$ ,  $L_1$  being a convex lens of focal length 24 cm and  $L_2$  a concave lens of focal length 18 cm are placed coaxially at a separation of 45 cm. A 1 cm tall object is placed in front of the lens  $L_1$  at a distance of 36 cm. Find the location and height of the image formed by the combination. [5 marks]

### Solution:

#### Concept:

- In normal adjustment of an astronomical telescope, the real image made by the objective lies in the first focal plane of the eyepiece, so the final image is at infinity.
- Angular magnification is  $M = \beta/\alpha$ , where  $\alpha$  is the angle subtended by the distant object at the objective and  $\beta$  is the angle subtended by its final image at the eye.
- For each thin lens,  $1/v - 1/u = 1/f$  and  $m = v/u$ . Distances are measured from that lens, positive along the incident light; the total linear magnification is  $m_1 m_2$ .

**Step 1: (i) Set the telescope in normal adjustment** The objective has a large aperture and a long focal length  $f_o$ . It receives nearly parallel rays from a distant object and forms a real, inverted image  $A'B'$  at its second focal plane. Place the eyepiece, of shorter focal length  $f_e$ , so that its first focal plane coincides with this image plane. Rays from each point of  $A'B'$  then emerge parallel through the eyepiece: the observer sees the final image at infinity without accommodating the eye. The lens separation is  $f_o + f_e$ .



*Telescope in normal adjustment:  $A'B'$  lies at the common focal plane, and the emergent rays are parallel. Angles are exaggerated.*

**Step 2: Derive the angular magnification** Let the magnitude of the intermediate image height be  $h$ . Using the small-angle approximation and the two triangles in the ray diagram,

$$|\alpha| \simeq \frac{h}{f_o}, \quad |\beta| \simeq \frac{h}{f_e}.$$

The final image remains inverted relative to the distant object. Thus the angular magnification has a

negative sign:

$$M = -\frac{|\beta|}{|\alpha|} = -\frac{h/f_e}{h/f_o} = -\frac{f_o}{f_e}.$$

Its magnitude increases when the objective focal length is increased or the eyepiece focal length is decreased. The eyepiece magnifies the angular size of the objective's image; it does not erect that image.

**Step 3: (ii) Find the image produced by the convex lens** Take light as travelling from left to right, with  $L_1$  at  $x = 0$ . For the first lens,  $u_1 = -36$  cm and  $f_1 = +24$  cm. Hence

$$\frac{1}{v_1} = \frac{1}{24} - \frac{1}{36} = \frac{1}{72},$$
$$v_1 = +72 \text{ cm}, \quad m_1 = \frac{72}{-36} = -2.$$

Without the second lens, the first image would form at  $x = 72$  cm, with height  $-2$  cm. Its negative height indicates inversion relative to the original object.

**Step 4: Use the first image as a virtual object for the concave lens** The concave lens is at  $x = 45$  cm, before the converging rays reach the first image. They are therefore incident on  $L_2$  as a converging beam aimed at a point  $72 - 45 = 27$  cm to its right. This is a virtual object for  $L_2$ , so  $u_2 = +27$  cm, not negative. With  $f_2 = -18$  cm,

$$\frac{1}{v_2} = -\frac{1}{18} + \frac{1}{27} = -\frac{1}{54},$$
$$v_2 = -54 \text{ cm}, \quad m_2 = \frac{-54}{27} = -2.$$

The outgoing rays diverge and their backward extensions meet on the left of  $L_2$ . The image coordinate relative to  $L_1$  is  $x = 45 - 54 = -9$  cm.

**Step 5: Calculate the final height and orientation** Magnifications multiply because the image height from the first lens is the object height for the second lens:

$$m = m_1 m_2 = (-2)(-2) = +4, \quad h_{\text{final}} = m h_{\text{object}} = 4(1) = +4 \text{ cm}.$$

The positive total magnification means the final image is erect relative to the original object. Its negative image distance from the second lens identifies it as virtual.

**Step 6: Conclusion** (i) For normal adjustment,  $M = -f_o/f_e$  and the telescope length is  $f_o + f_e$ . (ii) The final image is virtual and erect, 54 cm to the left of  $L_2$ , equivalently 9 cm to the left of  $L_1$ , and has height 4 cm.

**Quick Tip to Solve:** If the second lens intercepts rays before their first image forms, that image is a virtual object for it: use positive  $u_2$ , then multiply the two signed magnifications.

**31(b).** (i) Explain the working principle of an optical fibre with the help of a diagram. Mention one use of a light pipe.

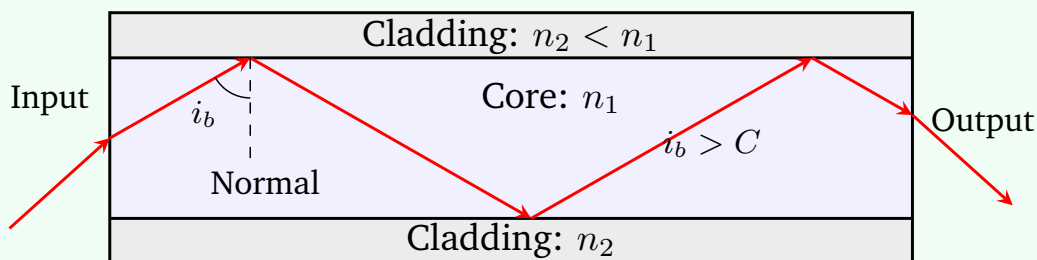
(ii) A ray of light is incident at an angle of  $60^\circ$  on one face of a prism with the prism angle  $A = 60^\circ$ . The ray passes symmetrically through the prism. Find the angle of minimum deviation ( $\delta_m$ ) and refractive index of the material of the prism. If the prism is immersed in water, how will  $\delta_m$  be affected? Justify your answer. [5 marks]

### Solution:

#### Concept:

- Total internal reflection occurs when light travels from a denser medium to a rarer medium with incidence angle greater than the critical angle. For core and cladding indices  $n_1 > n_2$ ,  $\sin C = n_2/n_1$ .
- For a prism,  $r_1 + r_2 = A$  and  $\delta = i + e - A$ , where  $A$  is the prism angle,  $i, e$  are the external angles and  $r_1, r_2$  are the internal refraction angles.
- At minimum deviation the ray passes symmetrically:  $i = e$ ,  $r_1 = r_2 = A/2$ , and the refractive index relative to the surrounding medium is  $\mu_{\text{rel}} = \sin[(A + \delta_m)/2] / \sin(A/2)$ .

**Step 1: (i) Explain the guiding of light in a fibre** An optical fibre has a transparent core of refractive index  $n_1$ , surrounded by cladding of a slightly lower refractive index  $n_2$ . Light admitted into the core at a suitable angle strikes the core-cladding boundary with incidence angle  $i_b > C$ , measured from the normal. Since it approaches a rarer medium from a denser one, it undergoes total internal reflection. Repeated total internal reflections at successive points guide the light along the core, with little loss in an ideal fibre. The cladding maintains the required refractive-index difference. A fibre can guide light around gentle bends provided the incidence angle at each reflecting boundary remains above the critical angle.



*Light guided through the higher-index core by repeated total internal reflection.*

**Step 2: (i) State a use of a light pipe** A bundle of optical fibres acts as a light pipe in a medical endoscope. It carries light into an internal organ for illumination; an imaging fibre bundle can carry the reflected image back for viewing. This makes otherwise inaccessible regions visible without a large surgical opening.

**Step 3: (ii) Use the symmetric path to find minimum deviation** The given symmetric path is the minimum-deviation path. Hence the emergence angle equals the incidence angle, and the two internal refraction angles are equal. With  $i = 60^\circ$  and  $A = 60^\circ$ ,

$$r_1 = r_2 = \frac{A}{2} = 30^\circ, \quad e = i = 60^\circ.$$

Substitution in the prism-deviation relation gives

$$\delta_m = i + e - A = 60^\circ + 60^\circ - 60^\circ = 60^\circ.$$

**Step 4: (ii) Calculate the prism's refractive index** Taking the surrounding air to have refractive index unity, apply Snell's law at the first face:

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} \simeq 1.73.$$

This agrees with the minimum-deviation formula using  $A = 60^\circ$  and  $\delta_m = 60^\circ$ . Refractive index is a ratio and has no unit.

**Step 5: (ii) Explain the effect of immersion in water** In water, the refractive index of the prism relative to its surroundings becomes  $\mu_{\text{rel}} = \mu/\mu_w$ . Since water has  $\mu_w > 1$ , this relative index is smaller than the prism's index relative to air. Thus the contrast between the media and the bending at the faces both decrease. For the same prism angle, the new minimum deviation satisfies

$$\delta_{m,w} = 2 \sin^{-1} \left[ \frac{\mu}{\mu_w} \sin \frac{A}{2} \right] - A.$$

The argument of the inverse sine decreases on immersion, so the minimum deviation decreases. This comparison concerns the minimum obtained after adjusting the prism to its new symmetric path; the original incidence angle need not remain the minimum-deviation angle in water.

**Step 6: Conclusion** An optical fibre guides light by repeated total internal reflection and is used as a light pipe in endoscopy. For the prism in air,  $\delta_m = 60^\circ$  and  $\mu = \sqrt{3} \simeq 1.73$ . Its minimum deviation decreases on immersion in water because its relative refractive index decreases.

**Quick Tip to Solve:** For a symmetric prism path, write  $e = i$  and  $r = A/2$  first. On changing the surrounding medium, replace the prism index by its relative index before comparing minimum deviations.

**32(a). (i)** A germanium crystal is doped with antimony. With the help of energy-band diagram, explain how the conductivity of the doped crystal is affected.

**(ii)** Briefly explain the two processes involved in the formation of a p-n junction.

**(iii)** What will the effect of (1) forward biasing, and (2) reverse biasing be on the width of depletion layer in a p-n junction diode? [5 marks]

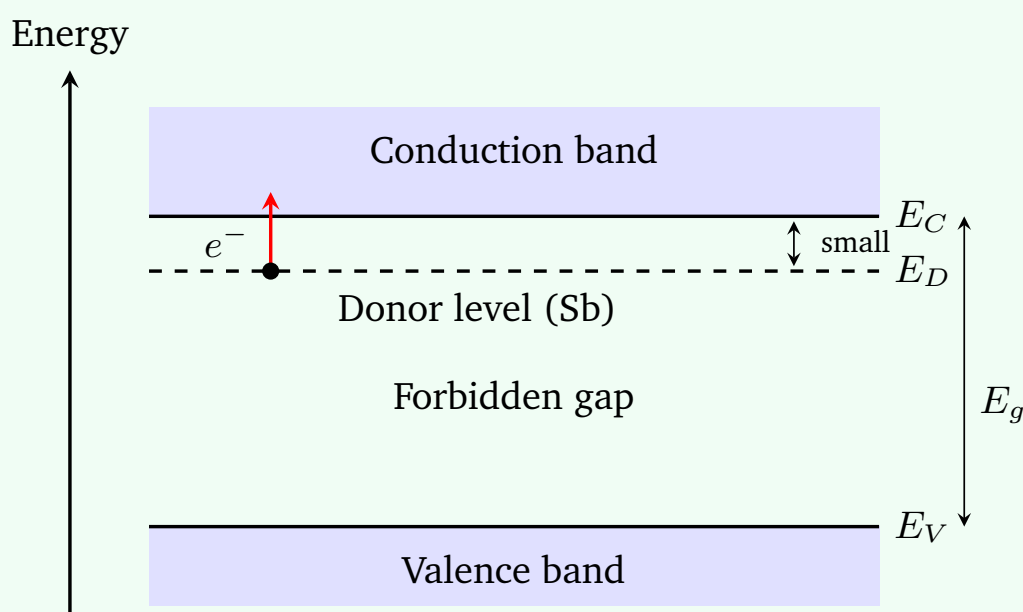
**Solution:**

**Concept:**

- A pentavalent donor added to tetravalent germanium produces an n-type semiconductor. Its donor energy level  $E_D$  lies just below the conduction-band edge  $E_C$ .
- Semiconductor conductivity is  $\sigma = e(n\mu_e + p\mu_h)$ , where  $n, p$  are the electron and hole number densities,  $\mu_e, \mu_h$  are their mobilities, and  $e$  is the magnitude of electronic charge.
- Diffusion results from a carrier concentration gradient; drift results from an electric field. At equilibrium their opposing currents in a p-n junction balance.
- The depletion layer contains fixed ionised donors and acceptors and very few mobile carriers. Its width responds to the potential barrier across the junction.

**Step 1: (i) Explain the donor action of antimony** Germanium has four valence electrons, while antimony has five. A substitutional antimony atom forms four covalent bonds with neighbouring germanium atoms. Its fifth electron is weakly bound, so only a small thermal energy is required to free it. The antimony atom then remains as a fixed positive donor ion, while the released electron can carry current through the crystal.

**Step 2: (i) Interpret the energy-band diagram** Antimony introduces donor levels just below the conduction band, as shown in the schematic diagram. The energy  $E_C - E_D$  needed to lift a donor electron into the conduction band is much smaller than the band gap  $E_g = E_C - E_V$ . Thus many donor electrons enter the conduction band at ordinary temperatures without needing excitation across the full forbidden gap. The electron concentration increases greatly and electrons become the majority carriers; holes are minority carriers. For ordinary donor doping, the increased carrier population raises  $\sigma$ , so the crystal becomes more conducting and its resistivity decreases. The doped crystal remains electrically neutral overall because mobile electrons are balanced by the positive donor ions.



*Schematic energy bands of antimony-doped germanium (not to scale).*

**Step 3: (ii) Describe diffusion and depletion-layer formation** On forming a p-n junction, the large concentration gradients cause electrons to diffuse from the n-side to the p-side and holes from the p-side to the n-side. Near the junction, electrons and holes recombine. This removes mobile carriers and exposes fixed positive donor ions on the n-side and fixed negative acceptor ions on the p-side. The resulting region depleted of mobile carriers is the depletion layer. Its space charge establishes an electric field directed from the n-side towards the p-side and a potential barrier that opposes further majority-carrier diffusion.

**Step 4: (ii) Explain drift and junction equilibrium** The junction field drives minority electrons from the p-side to the n-side, opposite to the field, and minority holes from the n-side to the p-side, along the field. This field-driven motion is drift. The associated drift current is opposite to the conventional diffusion current. As the barrier builds, diffusion is reduced until, at thermal equilibrium without external bias, the drift and diffusion currents are equal in magnitude. The net current is then zero even though microscopic carrier motion continues.

**Step 5: (iii) Compare forward and reverse bias** In forward bias, the p-side is connected to the positive terminal and the n-side to the negative terminal. The applied voltage opposes the junction barrier

and pushes majority carriers towards the junction. The barrier is reduced and the depletion-layer width decreases. In reverse bias, the p-side is connected to the negative terminal and the n-side to the positive terminal. Majority carriers are drawn away from the junction, exposing more fixed ions. The barrier increases and the depletion-layer width increases.

**Step 6: Conclusion** (i) Antimony makes germanium an n-type semiconductor and increases its conductivity by supplying easily excited donor electrons. (ii) Diffusion forms the depletion region and barrier; drift opposes diffusion, with equal opposing currents at equilibrium. (iii) Forward bias narrows the depletion layer, whereas reverse bias widens it.

**Quick Tip to Solve:** Place a donor level just below the conduction band. For bias effects, track majority carriers: driving them towards the junction narrows the depletion layer; pulling them away widens it.

OR

**32(b).** (i) With the help of a circuit diagram, briefly explain the working of a full-wave rectifier using p-n junction diodes.

(ii) Draw V-I characteristics of a p-n junction diode. Explain how these characteristics make a diode suitable for rectification.

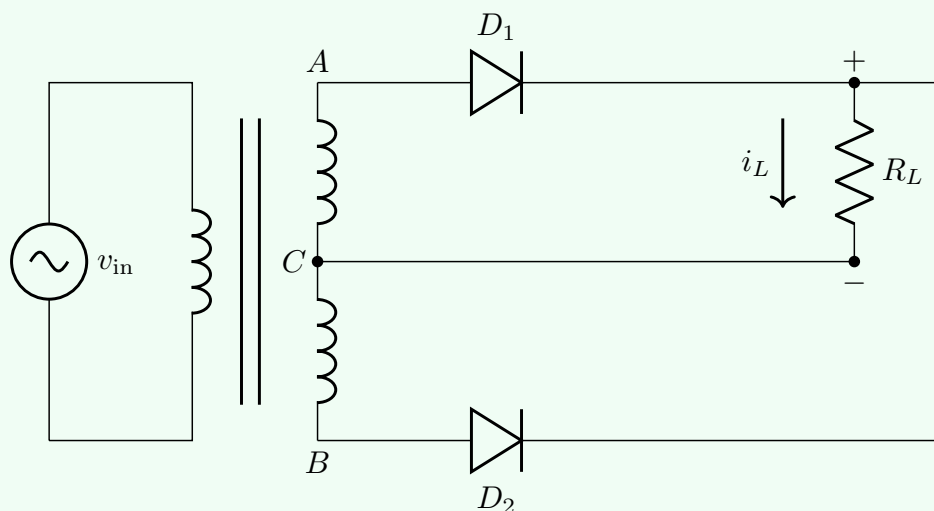
(iii) Carbon and silicon have the same lattice structure. Then why is carbon an insulator but silicon a semiconductor? [5 marks]

**Solution:**

**Concept:**

- A p-n junction diode conducts appreciably in forward bias but carries only a very small reverse saturation current below breakdown. This asymmetry enables rectification.
- A centre-tapped transformer supplies two equal secondary voltages, opposite in phase relative to the centre tap. Two diodes can therefore supply load current in the same direction on alternate half-cycles.
- The forbidden energy gap  $E_g$  separates the valence and conduction bands. Thermal excitation across a smaller gap produces more mobile charge carriers at a given temperature.

**Step 1: (i) Connect the full-wave rectifier** Connect the ends  $A$  and  $B$  of a centre-tapped transformer secondary to the anodes of diodes  $D_1$  and  $D_2$ . Join their cathodes to the positive output terminal. Connect the load resistance  $R_L$  between this common cathode terminal and the centre tap  $C$ , which is the return terminal. Apply the alternating input to the transformer primary. The circuit below shows the load-current direction for both half-cycles.



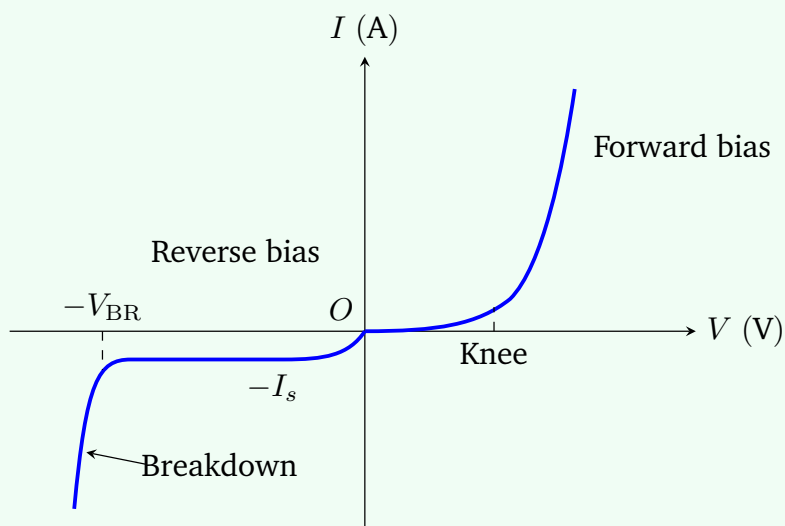
*Centre-tapped full-wave rectifier; the common cathodes form the positive output.*

**Step 2: (i) Trace current during both half-cycles** When  $A$  is positive relative to  $C$ ,  $B$  is negative. Diode  $D_1$  is forward biased and  $D_2$  is reverse biased. Conventional current follows  $A \rightarrow D_1 \rightarrow R_L \rightarrow C$ , returning through the upper half of the secondary. During the next half-cycle,  $B$  is positive relative to  $C$ , while  $A$  is negative. Now  $D_2$  conducts and  $D_1$  blocks. Current follows  $B \rightarrow D_2 \rightarrow R_L \rightarrow C$ , returning through the lower secondary half. Thus both halves of the input are used, and current through  $R_L$  always flows from its upper terminal to  $C$ . Neglecting diode drops, if the peak voltage of either secondary half is  $V_m$ ,

$$v_o = V_m |\sin \omega t|, \quad i_L = \frac{V_m}{R_L} |\sin \omega t|.$$

The output is unidirectional but pulsating; it is not constant DC. Its ripple frequency is twice the input frequency.

**Step 3: (ii) Interpret the diode characteristic** Take diode voltage  $V = V_p - V_n$  and forward conventional current as positive. In forward bias the current is initially small, then increases rapidly beyond the knee voltage as the potential barrier is reduced. The forward resistance in the conducting region is low. In reverse bias the depletion layer widens and only a very small reverse saturation current  $I_s$ , due to minority carriers, flows until breakdown. Hence the reverse resistance below breakdown is very high. The graph is schematic; its reverse-current scale is enlarged for visibility. In an alternating circuit this unequal conduction allows a diode to pass the forward-biased half-cycle and suppress the reverse-biased half-cycle. In the full-wave circuit the two diodes alternate this role, giving the same load polarity throughout. Ordinary rectifiers must operate below their reverse-breakdown voltage.



*Schematic diode V-I curve; reverse current is exaggerated and axes are not to scale.*

**Step 4: (iii) Compare the energy gaps of carbon and silicon** Here carbon means diamond, which has the same crystal structure as silicon. Equal lattice structure does not imply equal energy-band separation. Diamond has stronger covalent bonding and a large forbidden gap of about 5.4 eV, whereas silicon has a gap of about 1.1 eV. At ordinary temperature thermal energy cannot excite an appreciable number of electrons across diamond's large gap, so its conduction band remains essentially empty and it behaves as an insulator. Silicon's smaller gap permits a useful population of thermally generated conduction electrons and valence-band holes, so it behaves as a semiconductor. This comparison concerns diamond, not the graphite allotrope of carbon.

**Step 5: Conclusion** The two diodes conduct on alternate half-cycles and produce a pulsating output of fixed polarity across  $R_L$ . The diode's low forward resistance and high reverse resistance make rectification possible. Diamond is insulating because its energy gap is much larger than silicon's, despite their similar lattice structure.

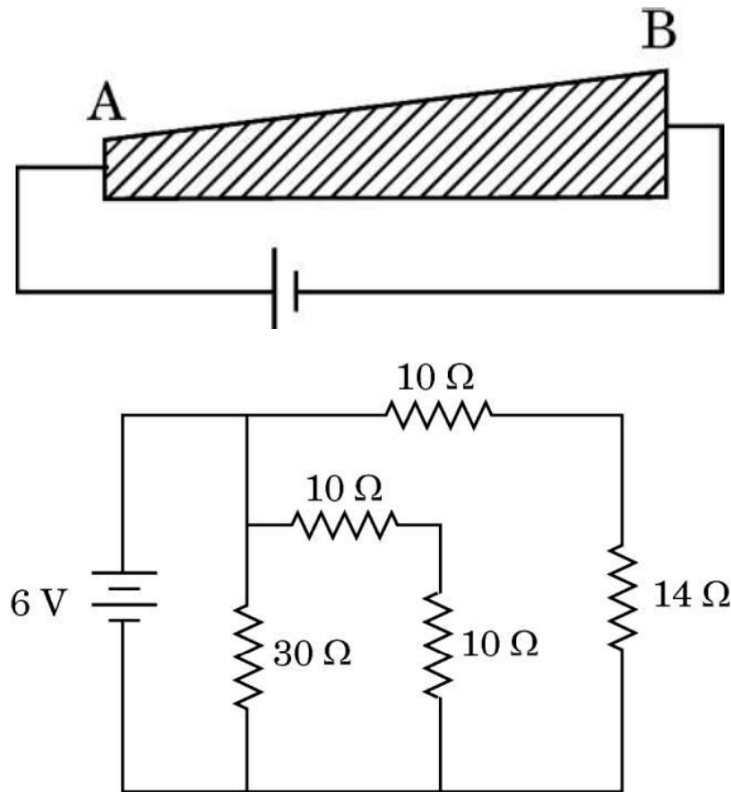
**Quick Tip to Solve:** Trace each conducting path from a secondary end back to the centre tap: both paths must cross the load in the same direction. For carbon versus silicon, compare energy gaps rather than lattice structure alone.

**33(a).** (i) Define mobility of electrons. Give its SI units.

(ii) A steady current flows through a wire AB, as shown in the figure. What happens to the electric field and the drift velocity along the wire? Justify your answer.

(iii) Consider the circuit shown in the figure. Find the effective resistance of the circuit and the current drawn from the battery.

[5 marks]



### Solution:

#### Concept:

- Electron mobility is the drift-speed magnitude per unit electric-field magnitude:  $\mu = v_d/E = e\tau/m_e$ , where  $e$  is the magnitude of electron charge,  $\tau$  is relaxation time and  $m_e$  is electron mass.
- In a steady current,  $I = neAv_d$  and  $E = \rho J = \rho I/A$ , where  $n$  is free-electron number density,  $A$  is local cross-sectional area,  $J$  is current density and  $\rho$  is resistivity.
- Resistances in series add, while parallel branches share the same voltage and obey  $1/R_{eq} = \sum 1/R_k$ . Ohm's law gives the battery current as  $I = V/R_{eq}$ .

**Step 1: (i) Define electron mobility and its unit** Mobility of electrons is the magnitude of the average drift velocity acquired by free electrons in a conductor per unit applied electric field. It measures how readily the electrons drift in response to the field. Thus

$$\mu = \frac{v_d}{E}, \quad [\mu] = \frac{\text{m s}^{-1}}{\text{V m}^{-1}} = \text{m}^2 \text{V}^{-1} \text{s}^{-1}.$$

Mobility is taken as positive; because electrons are negatively charged, their drift-velocity vector is opposite to the electric field:  $\vec{v}_d = -\mu \vec{E}$ .

**Step 2: (ii) Apply current continuity to the widening wire** The cross-sectional area of the wire increases from A towards B. In steady state, charge cannot continually accumulate at any section, so the same current passes through every cross-section. For a uniform material at constant temperature,

$n$  and  $\rho$  are constant. Therefore

$$v_d = \frac{I}{neA}, \quad E = \frac{\rho I}{A}.$$

Both the drift speed and the magnitude of electric field are inversely proportional to the local cross-sectional area. Hence both decrease from the narrow end A to the wider end B. The battery polarity shown drives conventional current and the electric field from A to B; electrons drift from B to A. The statement about decrease describes the magnitudes at successive locations from A to B, not the direction of electron motion.

**Step 3: (iii) Identify the three parallel branches** The upper left junction and the common bottom wire are the two battery terminals. Between them, the left branch contains  $30\ \Omega$ . The middle branch contains two  $10\ \Omega$  resistors in series, since their connecting junction has no other path. The outer right branch similarly contains  $10\ \Omega$  and  $14\ \Omega$  in series. Thus

$$R_1 = 30\ \Omega, \quad R_2 = 10 + 10 = 20\ \Omega, \quad R_3 = 10 + 14 = 24\ \Omega.$$

These three branches share both endpoints, so they are in parallel, even though the drawing places some components close together.

**Step 4: (iii) Combine the branches and calculate the current** Adding their conductances gives

$$\frac{1}{R_{\text{eq}}} = \frac{1}{30} + \frac{1}{20} + \frac{1}{24} = \frac{4 + 6 + 5}{120} = \frac{1}{8}\ \Omega^{-1}.$$

The equivalent resistance is therefore  $8\ \Omega$ . With the ideal  $6\ \text{V}$  battery shown,

$$I = \frac{6}{8} = 0.75\ \text{A}.$$

As a check, the separate branch currents are  $6/30 = 0.20\ \text{A}$ ,  $6/20 = 0.30\ \text{A}$  and  $6/24 = 0.25\ \text{A}$ ; their sum is  $0.75\ \text{A}$ .

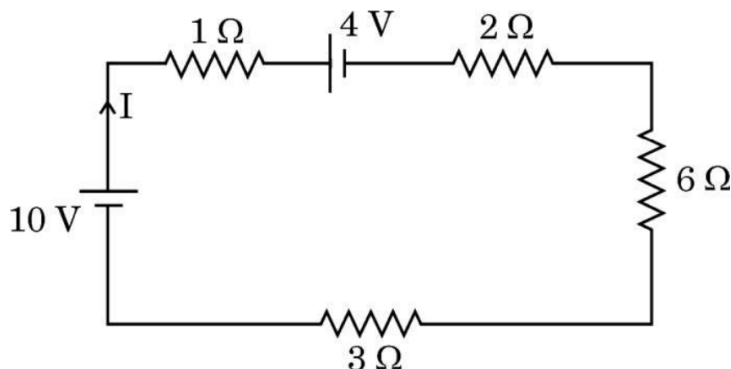
**Step 5: Conclusion** (i)  $\mu = v_d/E$ , with SI unit  $\text{m}^2\ \text{V}^{-1}\ \text{s}^{-1}$ . (ii) Both electric-field magnitude and electron drift speed decrease from A to B. (iii) The effective resistance is  $8\ \Omega$ , and the current drawn is  $0.75\ \text{A}$ .

**Quick Tip to Solve:** Mark the two battery nodes first: this reveals the  $30\ \Omega$ ,  $20\ \Omega$  and  $24\ \Omega$  parallel branches. For the tapered wire, keep current constant and use the local area.

**33(b).** (i) Define electrical conductivity of a wire. Give its SI unit.

(ii) High current is to be drawn safely from (1) a low-voltage battery, and (2) a high-voltage battery. What can you say about the internal resistance of the two batteries?

(iii) Calculate the total energy supplied by the batteries to the circuit shown in the figure, in one minute. [5 marks]



### Solution:

#### Concept:

- Conductivity  $\sigma$  is the reciprocal of resistivity  $\rho$ ; the local form of Ohm's law is  $\vec{J} = \sigma \vec{E}$ , where  $\vec{J}$  is current density and  $\vec{E}$  is electric field.
- A battery of emf  $\mathcal{E}$ , internal resistance  $r$  and external load resistance  $R$  delivers  $I = \mathcal{E}/(R + r)$ . Its internal voltage drop is  $Ir$ .
- Kirchhoff's loop rule adds emfs algebraically. A cell supplies electrical energy when current leaves its positive terminal and absorbs energy when current enters that terminal. The signed energy contribution over time  $t$  is  $W = \pm \mathcal{E}It$ .

**Step 1: (i) Define electrical conductivity and its SI unit** Electrical conductivity expresses how readily a material conducts electric current. Quantitatively, it is the current density produced per unit applied electric field, or the reciprocal of resistivity:

$$\sigma = \frac{J}{E} = \frac{1}{\rho}.$$

For a uniform wire of length  $l$ , area  $A$  and resistance  $R$ ,  $\rho = RA/l$ , so  $\sigma = l/(RA)$ . Its SI unit is  $\Omega^{-1} \text{ m}^{-1}$ , equivalently siemens per metre  $\text{S m}^{-1}$ . Conductivity is a material property; it is distinct from conductance  $1/R$ , which also depends on the wire's dimensions.

**Step 2: (ii) Compare the required internal resistances** The load current satisfies  $I = \mathcal{E}/(R + r)$ . A low-voltage battery must have very small internal resistance to deliver a high current: otherwise much of its small emf is lost as  $Ir$ , and the current is limited severely. A high-voltage battery should have comparatively large internal resistance to limit the current to a safe value and avoid an excessive current if the external resistance becomes small. In particular, the short-circuit current is  $I_{\text{sc}} = \mathcal{E}/r$ , so a large emf with negligible  $r$  can drive a dangerously large current. Thus the expected comparison is low internal resistance for the low-voltage battery and comparatively high internal resistance for the high-voltage battery; no numerical values can be assigned without a specified load and safe-current limit.

**Step 3: (iii) Determine the net emf and loop current** Follow the indicated clockwise current. The

10 V battery is crossed from its negative to its positive terminal and provides a potential rise. The 4 V battery is crossed from its positive to its negative terminal and opposes it. All four resistors are in a single series loop, giving

$$R_{\text{total}} = 1 + 2 + 6 + 3 = 12 \Omega.$$

Kirchhoff's loop rule therefore gives

$$10 - 4 - 12I = 0, \quad I = \frac{10 - 4}{12} = 0.50 \text{ A}.$$

The positive result confirms the indicated direction. The emfs must be subtracted, since the cells are connected in opposition.

**Step 4: (iii) Calculate the net energy supplied in one minute** In  $t = 60 \text{ s}$ , the charge passing through each cell is  $Q = It = 0.50 \times 60 = 30 \text{ C}$ . The 10 V battery supplies

$$W_{10} = 10Q = 10 \times 30 = 300 \text{ J}.$$

Current enters the positive terminal of the 4 V battery, so it absorbs  $W_4 = 4Q = 120 \text{ J}$ , rather than delivering this energy to the resistors. The total net energy supplied by the battery combination is

$$W_{\text{net}} = 300 - 120 = 180 \text{ J}.$$

This agrees with the energy dissipated in all the resistors:

$$I^2 R_{\text{total}} t = (0.50)^2 \times 12 \times 60 = 180 \text{ J}.$$

The energy balance separates the energy delivered by the driving cell from the part absorbed by the opposing cell.

**Step 5: Conclusion** (i)  $\sigma = 1/\rho = J/E$ , with SI unit  $\text{S m}^{-1}$ . (ii) The low-voltage battery should have low internal resistance; the high-voltage battery should have comparatively high internal resistance for current limitation. (iii) The circuit current is 0.50 A, and the net energy supplied by the batteries in one minute is 180 J.

**Quick Tip to Solve:** Trace the current through each cell's long plate before adding emfs: here the 4 V cell absorbs energy, so use  $(10 - 4)It$ , not  $(10 + 4)It$ .

### Section E (8 marks)

**34(a).** The lens maker's formula is useful to design lenses of desired focal lengths. The focal length also depends on the refractive index of the material of the lens and the surrounding medium. The refractive index depends on the wavelength of the light used. The power of a lens is related to its focal length.

Answer the following questions based on the above:

(a) How will the power of a lens be affected with an increase of wavelength of light?

[1 mark]

**Solution:**

**Concept:**

- For a thin lens in air, its power is  $P = (\mu - 1)(1/R_1 - 1/R_2)$ , where  $\mu$  is its refractive index and  $R_1, R_2$  are the signed radii.
- In normal dispersion, the refractive index of glass decreases as the wavelength of light increases.

**Step 1: Keep the lens geometry fixed** Changing the light does not change either radius of curvature. Thus the curvature factor in the lens maker's formula stays constant, while  $\mu - 1$  becomes smaller for a longer wavelength.

**Step 2: Relate refractive index to optical strength** Hence  $|P|$  decreases and  $|f| = 1/|P|$  increases. For a convex lens, the positive power decreases; for a concave lens, the negative power becomes less negative, with its magnitude decreasing.

**Step 3: Conclusion** The power (in magnitude) decreases with increasing wavelength; the lens becomes optically weaker.

**Quick Tip to Solve:** Remember that red light bends less than violet light, so a glass lens has a longer focal length in red light.

**34(b).** The radius of curvature of two surfaces of a convex lens is  $R$  each. For what value of  $\mu$  of its material will its focal length become equal to  $R$ ?

[1 mark]

**Solution:**

**Concept:**

- For a thin lens in air,  $1/f = (\mu - 1)(1/R_1 - 1/R_2)$ , where  $f$  is the focal length and  $\mu$  is the refractive index of its material.
- For a symmetric biconvex lens with light travelling from left to right,  $R_1 = +R$  and  $R_2 = -R$  in the Cartesian sign convention.

**Step 1: Insert the signed radii** Although both radii have magnitude  $R$ , their centres of curvature lie on opposite sides. Therefore,

$$\frac{1}{f} = (\mu - 1) \left( \frac{1}{R} - \frac{1}{-R} \right) = \frac{2(\mu - 1)}{R}.$$

**Step 2: Apply the required focal length** Set  $f = R$  and multiply both sides by the nonzero radius:

$$\frac{1}{R} = \frac{2(\mu - 1)}{R} \Rightarrow 1 = 2(\mu - 1).$$

This determines the dimensionless refractive index directly, independent of the size of the lens.

**Step 3: Conclusion** The required refractive index is  $\mu = 1.5$ .

**Quick Tip to Solve:** For an equiconvex lens in air use  $f = R/[2(\mu - 1)]$ ; the two signed radii must have opposite signs.

**34(c).** The focal length of a concave lens of  $\mu = 1.5$  is 20 cm in air. It is completely immersed in water of  $\mu = \frac{4}{3}$ . Calculate its focal length in water. [2 marks]

**Solution:**

**Concept:**

- The lens maker's formula in a medium is  $1/f = (\mu_l/\mu_m - 1)K$ , where  $\mu_l$  and  $\mu_m$  are the lens and medium indices, and  $K = 1/R_1 - 1/R_2$ .
- Immersion changes the relative refractive index but not the curvature factor  $K$ . A diverging concave lens has a negative focal length.

**Step 1: Express the power in each medium** The given focal-length magnitude in air is 20 cm, so its signed focal length is  $f_a = -20$  cm. In air, taking  $\mu_a = 1$ , and in water respectively,

$$\frac{1}{f_a} = (1.5 - 1)K = \frac{1}{2}K, \quad \frac{1}{f_w} = \left(\frac{1.5}{4/3} - 1\right)K = \frac{1}{8}K.$$

The relative refractive index in water is  $9/8$ , which is still greater than unity, so the lens remains diverging.

**Step 2: Eliminate the unchanged curvature factor** Dividing the two equations gives a ratio of inverse focal lengths:

$$\frac{1/f_w}{1/f_a} = \frac{1/8}{1/2} = \frac{1}{4} \Rightarrow f_w = 4f_a.$$

Thus the optical power is reduced to one fourth of its air value. The magnitude of the focal length must increase fourfold, rather than decrease, because power and focal length are reciprocals.

**Step 3: Conclusion** The focal length in water is  $f_w = -80$  cm, of magnitude 80 cm. The negative sign indicates a diverging lens.

**Quick Tip to Solve:** For immersion, compare  $(\mu_l/\mu_m - 1)$  factors first and retain the negative sign of a concave lens.

OR

**34(c).** An object is placed in front of a lens which forms its erect image of magnification 3. The power of the lens is 5 D. Calculate the distance of the object and the image from the lens. [2 marks]

**Solution:**

**Concept:**

- Lens power  $P = 1/f$ , with  $f$  measured in metres and  $P$  in dioptres. Positive power corresponds to a converging lens.

- In the Cartesian sign convention,  $1/f = 1/v - 1/u$  and the signed linear magnification is  $m = v/u$ . An erect image has positive magnification.

**Step 1: Find the focal length and the signed magnification** The given power is  $+5\text{ D}$ , so

$$f = \frac{1}{5} \text{ m} = +20 \text{ cm}.$$

Because the image is erect, take  $m = +3$ , not  $-3$ . Thus  $v/u = 3$ , or  $v = 3u$ . The object in front of the lens has  $u < 0$ , so this erect image also has  $v < 0$  and is virtual.

**Step 2: Substitute the magnification relation in the lens formula** Working consistently in centimetres,

$$\begin{aligned} \frac{1}{20} &= \frac{1}{3u} - \frac{1}{u} = -\frac{2}{3u}, \\ 3u &= -40 \text{ cm}, \quad u = -\frac{40}{3} \text{ cm}, \\ v &= 3u = -40 \text{ cm}. \end{aligned}$$

The object is within the focal length, since  $|u| = 13.33 \text{ cm} < 20 \text{ cm}$ , consistent with the erect, magnified virtual image formed by a convex lens.

**Step 3: Conclusion** The object is  $13.33 \text{ cm}$  in front of the lens and the image is  $40 \text{ cm}$  from it on the same side. Their signed distances are  $u = -13.33 \text{ cm}$  and  $v = -40 \text{ cm}$ .

**Quick Tip to Solve:** For an erect lens image, use positive magnification: here  $v = 3u$ , so both distances have the same sign.

**35(a).** Electrostatics deals with the study of forces, fields and potentials arising from static charges. Force and electric field, due to a point charge is basically determined by Coulomb's law. For symmetric charge configurations, Gauss's law, which is also based on Coulomb's law, helps us to find the electric field. A charge/a system of charges like a dipole experience a force/torque in an electric field. Work is required to be done to provide a specific orientation to a dipole with respect to an electric field.

Answer the following questions based on the above:

(a) Consider a uniformly charged thin conducting shell of radius  $R$ . Plot a graph showing the variation of  $|\vec{E}|$  with distance  $r$  from the centre, for points  $0 \leq r \leq 3R$ . [1 mark]

**Solution:**

**Concept:**

- A spherical Gaussian surface gives  $E(4\pi r^2) = |Q_{\text{enc}}|/\epsilon_0$ , where  $Q_{\text{enc}}$  is the enclosed charge.
- The charge  $Q$  of a thin conducting spherical shell lies on its surface. Its interior field is zero; outside, it acts as a point charge at its centre.

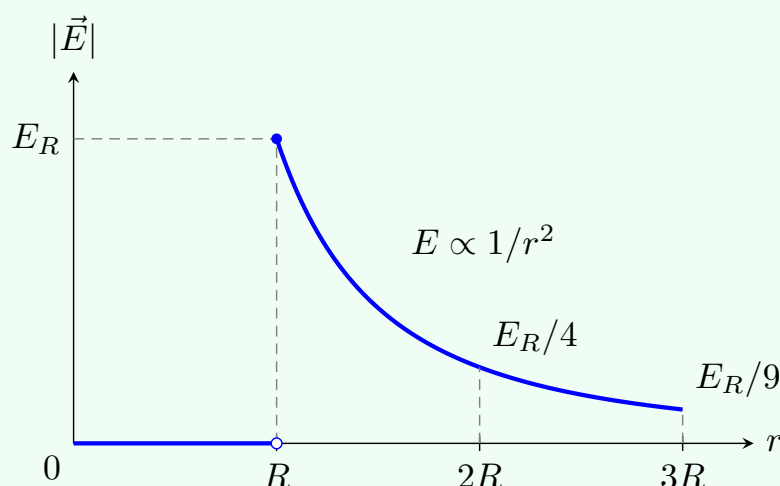
**Step 1: Write the field in the two regions** For  $0 \leq r < R$ , no charge is enclosed, so  $E = 0$ . For  $r > R$ , the enclosed charge is  $Q$ , hence

$$E = \frac{|Q|}{4\pi\epsilon_0 r^2}.$$

Define  $E_R = |Q|/(4\pi\epsilon_0 R^2)$ , the field immediately outside the shell.

**Step 2: Plot the zero interior field and inverse-square exterior field** The graph lies along the horizontal axis up to  $R$ . At the surface the field jumps from zero to  $E_R$ , then decreases as  $1/r^2$ , reaching  $E_R/4$  at

$2R$  and  $E_R/9$  at  $3R$ . The dashed vertical line indicates the discontinuity, not a continuous field inside the shell.

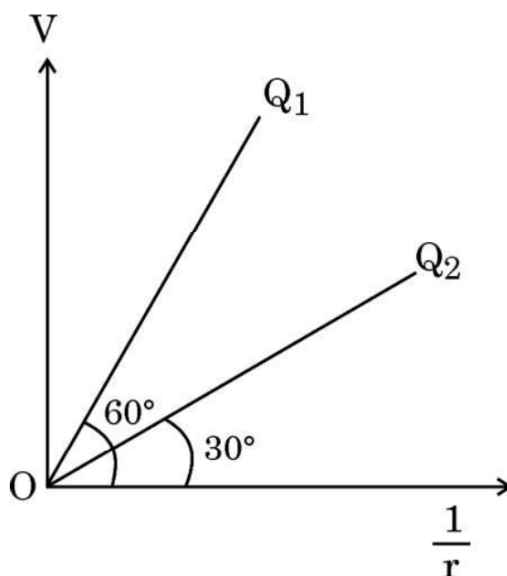


Field magnitude for a thin charged conducting shell;  $E_R = |Q|/(4\pi\epsilon_0 R^2)$ .

**Step 3: Conclusion** The required graph has zero field inside and an inverse-square decrease outside, with a discontinuity at  $r = R$ .

**Quick Tip to Solve:** A conducting shell has zero field throughout its interior; do not draw the rising straight line used for a uniformly charged solid insulator.

**35(b).** The figure shows the variation of potential  $V$  with  $\frac{1}{r}$  for two point charges  $Q_1$  and  $Q_2$ , where  $V$  is the potential at a distance  $r$  due to a point charge. Find  $\frac{Q_1}{Q_2}$ . [1 mark]



**Solution:**

**Concept:**

- Taking potential zero at infinity, a point charge  $Q$  produces  $V = Q/(4\pi\epsilon_0 r)$  at distance  $r$ .
- In a graph of  $V$  against  $1/r$ , the slope is  $Q/(4\pi\epsilon_0)$ . Hence the ratio of slopes equals the ratio of

charges.

**Step 1: Read the angles from the horizontal axis** The line for  $Q_1$  makes  $60^\circ$  with the  $1/r$  axis, whereas the line for  $Q_2$  makes  $30^\circ$ . Both slopes are positive, so both charges are positive. The steeper line must represent the larger charge.

**Step 2: Divide the slopes in the printed order** Since both lines use the same coordinate axes, their scale factor cancels when taking the ratio:

$$\frac{Q_1}{Q_2} = \frac{\tan 60^\circ}{\tan 30^\circ} = \frac{\sqrt{3}}{1/\sqrt{3}} = 3.$$

The angles themselves cannot be divided; it is their tangents that give the slopes.

**Step 3: Conclusion** The requested ratio is  $Q_1/Q_2 = 3$ , or  $Q_1 : Q_2 = 3 : 1$ .

**Quick Tip to Solve:** On a  $V$  versus  $1/r$  plot, charge is proportional to slope; check the order of the requested ratio before dividing.

**35(c).** An electric dipole of dipole moment of  $6 \times 10^{-7}$  C-m is kept in a uniform electric field of  $10^4$  N/C such that the dipole moment and the electric field are parallel. Calculate the potential energy of the dipole. [2 marks]

**Solution:**

**Concept:**

- The potential energy of a dipole in a uniform electric field is  $U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$ , where  $p$  is its dipole moment,  $E$  is the field magnitude and  $\theta$  is their mutual angle.
- The usual reference is  $U = 0$  when the dipole is perpendicular to the field. Parallel alignment has the minimum potential energy and is the stable equilibrium orientation.

**Step 1: Identify the orientation and convert no units** The given values are already in SI units:  $p = 6 \times 10^{-7}$  C m and  $E = 10^4$  N/C. Since the dipole moment is parallel to the field, the angle is  $\theta = 0^\circ$ , so  $\cos \theta = 1$ . The angle is measured between the dipole moment vector and the field, not between the dipole axis and an arbitrary reference direction.

**Step 2: Substitute into the energy expression** Using the dipole potential energy formula gives

$$U = -(6 \times 10^{-7})(10^4) \cos 0^\circ \text{ J} = -6 \times 10^{-3} \text{ J}.$$

Here (C m)(N/C) = N m = J. The negative sign follows from the chosen perpendicular reference; it must not be dropped simply because energy is a scalar.

**Step 3: Conclusion** The potential energy is  $U = -6 \times 10^{-3} \text{ J}$ , or  $-6 \text{ mJ}$ .

**Quick Tip to Solve:** For parallel alignment use  $U = -pE$ ; for antiparallel alignment use  $U = +pE$ .

**35(c).** An electric dipole of dipole moment  $\vec{p}$  is initially kept in a uniform electric field  $\vec{E}$  such that  $\vec{p}$  is perpendicular to  $\vec{E}$ . Find the amount of work done in rotating the dipole to a position at which  $\vec{p}$  becomes antiparallel to  $\vec{E}$ . [2 marks]

**Solution:**

**Concept:**

- A dipole has potential energy  $U(\theta) = -pE \cos \theta$  in a uniform electric field, where  $\theta$  is the angle between  $\vec{p}$  and  $\vec{E}$ , and  $p, E$  are their magnitudes.
- For a slow rotation with no change in kinetic energy, the work done by an external agent equals the increase in potential energy:  $W_{\text{ext}} = U_f - U_i$ . Work done by the electric field has the opposite sign.

**Step 1: Evaluate the initial and final potential energies** Initially the dipole is perpendicular to the field, so  $\theta_i = 90^\circ$ . In the required final position it is antiparallel, so  $\theta_f = 180^\circ$ . Therefore,

$$U_i = -pE \cos 90^\circ = 0,$$

$$U_f = -pE \cos 180^\circ = +pE.$$

The final orientation has higher energy because the field tends to align the dipole parallel to itself.

**Step 2: Find the work required for rotation** The external agent must rotate the dipole against the restoring torque of the electric field. Its work is

$$W_{\text{ext}} = \Delta U = pE - 0 = pE.$$

Equivalently, integrating the opposing torque gives  $W_{\text{ext}} = \int_{\pi/2}^{\pi} pE \sin \theta d\theta = pE$ . The field itself does negative work over this rotation,  $W_{\text{field}} = -pE$ ; this distinguishes the work supplied to rotate the dipole from the field's work.

**Step 3: Conclusion** The work required to rotate the dipole from perpendicular to antiparallel alignment is  $W_{\text{ext}} = pE$ , positive and measured in joules for SI values of  $p$  and  $E$ .

**Quick Tip to Solve:** Write the two angles first, then use  $W_{\text{ext}} = pE(\cos \theta_i - \cos \theta_f)$  to preserve the sign.