

# CBSE Class 12 Physics Question Paper 2023 with Solution PDF-Set 55/5/1

## Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

### General Instructions

- (i) This question paper contains **35** questions. **All** questions are compulsory.
- (ii) Question paper is divided into **FIVE** sections – Section **A, B, C, D** and **E**.
- (iii) **In section – A** : question number **1** to **18** are Multiple Choice (MCQ) type questions carrying **1** mark each.
- (iv) **In section – B** : question number **19** to **25** are Short Answer-1 (SA-1) type questions carrying **2** marks each.
- (v) **In section – C** : question number **26** to **30** are Short Answer-2 (SA-2) type questions carrying **3** marks each.
- (vi) **In section – D** : question number **31** to **33** are Long Answer (LA) type questions carrying **5** marks each.
- (vii) **In section – E** : question number **34** and **35** are case-based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section – **B**, 2 questions in Section – **C**, 3 questions in Section – **D** and 2 questions in Section – **E**.
- (ix) Use of calculators is NOT allowed.

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

### Section A (18 marks)

1. An electric dipole of length 2 cm is placed at an angle of  $30^\circ$  with an electric field  $2 \times 10^5$  N/C. If the dipole experiences a torque of  $8 \times 10^{-3}$  Nm, the magnitude of either charge of the dipole, is [1 mark]

- (A)  $4 \mu\text{C}$
- (B)  $7 \mu\text{C}$
- (C)  $8 \text{ mC}$
- (D)  $2 \text{ mC}$

**Correct Answer:** (A)  $4 \mu\text{C}$

#### Solution:

##### Concept:

- For charges of magnitude  $q$  separated by length  $l$ , the electric dipole moment has magnitude  $p = ql$ .
- A uniform field exerts torque  $\tau = pE \sin \theta$ , where  $\theta$  is the angle between the dipole moment and the field.

**Step 1: Convert the dipole length** The stated length is the full separation of the charges, so  $l = 2 \text{ cm} = 0.02 \text{ m}$ . Here  $\theta = 30^\circ$ , hence  $\sin \theta = 1/2$ . Do not double the given length again.

**Step 2: Substitute in the torque relation** Rearranging  $\tau = qlE \sin \theta$  gives

$$q = \frac{8 \times 10^{-3}}{0.02 \times 2 \times 10^5 \times (1/2)} = 4 \times 10^{-6} \text{ C.}$$

Option (B) does not satisfy the given torque. Options (C) and (D) are in millicoulombs and would produce much larger torques.

**Step 3: Conclusion** The magnitude of either charge is  $4 \mu\text{C}$ . Hence, the correct option is (A).

**Quick Tip to Solve:** Use the full stated dipole length in  $p = ql$ , and convert centimetres to metres before substituting.

2. Two long parallel wires kept 2 m apart carry 3A current each, in the same direction. The force per unit length on one wire due to the other is [1 mark]

- (A)  $4.5 \times 10^{-5} \text{ N m}^{-1}$ , attractive
- (B)  $4.5 \times 10^{-7} \text{ N/m}$ , repulsive
- (C)  $9 \times 10^{-7} \text{ N/m}$ , repulsive
- (D)  $9 \times 10^{-5} \text{ N/m}$ , attractive

**Correct Answer:** No printed option is correct: the force per unit length is  $9 \times 10^{-7} \text{ N m}^{-1}$ , attractive.

#### Solution:

##### Concept:

- Two long parallel wires at separation  $d$ , carrying currents  $I_1$  and  $I_2$ , exert force per unit length  $F/L = \mu_0 I_1 I_2 / (2\pi d)$ .
- Parallel currents in the same direction attract; currents in opposite directions repel.

**Step 1: Calculate the force per unit length** Use  $I_1 = I_2 = 3\text{ A}$ ,  $d = 2\text{ m}$ , and  $\mu_0 = 4\pi \times 10^{-7}\text{ N A}^{-2}$ :

$$\frac{F}{L} = \frac{4\pi \times 10^{-7} \times 3 \times 3}{2\pi \times 2} = 9 \times 10^{-7}\text{ N m}^{-1}.$$

**Step 2: Check the direction and all printed options** Since both currents flow in the same direction, the interaction is attractive. Option (C) has the correct magnitude but incorrectly says repulsive. Option (B) has both an incorrect magnitude and direction. Options (A) and (D) say attractive, but their printed magnitudes,  $4.5 \times 10^{-5}$  and  $9 \times 10^{-5}\text{ N m}^{-1}$ , are incorrect. Thus the printed alternatives contain no fully correct answer.

**Step 3: Conclusion** The force per unit length is  $9 \times 10^{-7}\text{ N m}^{-1}$ , attractive. No printed option is correct.

**Quick Tip to Solve:** Check magnitude and direction separately: same-direction currents attract, even when a distractor has the right numerical value.

**3. Which of the following has its permeability less than that of free space?**

[1 mark]

- (A) Copper
- (B) Aluminium
- (C) Copper chloride
- (D) Nickel

**Correct Answer:** (A) Copper

**Solution:**

**Concept:**

- Magnetic permeability  $\mu$  measures a material's magnetic response;  $\mu_0$  is the permeability of free space.
- For a linear material,  $\mu = \mu_0(1 + \chi_m)$ , where  $\chi_m$  is its magnetic susceptibility. Diamagnets have  $\chi_m < 0$ , so  $\mu < \mu_0$ .

**Step 1: Identify the required magnetic behaviour** Permeability less than that of free space requires a negative magnetic susceptibility. Such a material is diamagnetic: its induced magnetisation opposes the applied magnetic field.

**Step 2: Compare the materials** Copper is diamagnetic. Aluminium and copper chloride are paramagnetic, with positive susceptibility and permeability slightly greater than  $\mu_0$ . Nickel is ferromagnetic, with a strong magnetic response and permeability greater than that of free space. Thus only copper satisfies the stated inequality.

**Step 3: Conclusion** Copper has permeability less than that of free space. Hence, the correct option is (A).

**Quick Tip to Solve:** Translate  $\mu < \mu_0$  into  $\chi_m < 0$ , then identify the diamagnetic material among the choices.

4. A square shaped coil of side 10 cm, having 100 turns is placed perpendicular to a magnetic field which is increasing at 1 T/s. The induced emf in the coil is [1 mark]

- (A) 0.1 V
- (B) 0.5 V
- (C) 0.75 V
- (D) 1.0 V

**Correct Answer:** (D) 1.0 V

**Solution:**

**Concept:**

- Faraday's law gives the magnitude of the induced emf as  $|\varepsilon| = N|d\Phi_B/dt|$ , where  $N$  is the number of turns and  $\Phi_B$  is magnetic flux through one turn.
- For a coil whose plane is perpendicular to the magnetic field, its area vector is parallel to the field, so  $\Phi_B = BA$ .

**Step 1: Find the area of one turn** The square has side 10 cm = 0.10 m, hence its area is

$$A = (0.10)^2 = 0.010 \text{ m}^2.$$

The coil is stationary, so only the magnetic field changes the flux.

**Step 2: Apply Faraday's law to all turns** Since  $N = 100$  and  $dB/dt = 1 \text{ T s}^{-1}$ ,

$$|\varepsilon| = NA \frac{dB}{dt} = 100 \times 0.010 \times 1 = 1.0 \text{ V}.$$

Options (A), (B) and (C) do not equal this product. In particular, the number of turns multiplies the flux-change rate; it must not be omitted.

**Step 3: Conclusion** The magnitude of the induced emf is 1.0 V. Hence, the correct option is (D).

**Quick Tip to Solve:** Square the side after converting it to metres, then multiply the area by the turns and the rate of change of field.

5. Which one of the following electromagnetic radiation has the least wavelength? [1 mark]

- (A) Gamma rays
- (B) Microwaves
- (C) Visible light
- (D) X-rays

**Correct Answer:** (A) Gamma rays

**Solution:**

**Concept:**

- Electromagnetic waves in vacuum obey  $c = \nu\lambda$ , where  $\nu$  is frequency and  $\lambda$  is wavelength.
- In the standard electromagnetic spectrum, gamma rays occupy the highest-frequency, shortest-wavelength end.

**Step 1: Order the listed radiations** In the usual spectrum ordering, the wavelengths decrease in the

sequence

Microwaves > Visible light > X-rays > Gamma rays.

Gamma rays therefore lie beyond X-rays towards shorter wavelengths.

**Step 2: Compare with the distractors** Microwaves have long wavelengths, and visible light has shorter wavelengths than microwaves but longer wavelengths than X-rays. Although X-rays also have very short wavelengths, gamma rays are the shortest-wavelength category in the standard CBSE spectrum ordering. Thus options (B), (C) and (D) do not describe its shortest-wavelength end.

**Step 3: Conclusion** Gamma rays have the least wavelength in the standard spectrum ordering. Hence, the correct option is (A).

**Quick Tip to Solve:** For least wavelength, look for the highest-frequency end of the electromagnetic spectrum: gamma rays.

**6. In a Young's double-slit experiment, the screen is moved away from the plane of the slits. What will be its effect on the following?**

(i) Angular separation of the fringes.

(ii) Fringe-width.

[1 mark]

(A) Both (i) and (ii) remain constant.

(B) (i) remains constant, but (ii) decreases.

(C) (i) remains constant, but (ii) increases.

(D) Both (i) and (ii) increase.

**Correct Answer:** (C) (i) remains constant, but (ii) increases.

**Solution:**

**Concept:**

- In Young's double-slit experiment, the linear fringe width is  $\beta = \lambda D/d$ , where  $D$  is the screen distance and  $d$  is the slit separation.
- In the small-angle approximation, the angular separation is  $\Delta\theta = \beta/D = \lambda/d$ , independent of  $D$ .

**Step 1: Check the angular separation** Moving the screen leaves the wavelength  $\lambda$  and slit separation  $d$  unchanged. Since  $\Delta\theta = \lambda/d$ , the angular separation of consecutive fringes remains constant. Increasing the distance does not change their angular directions from the slits.

**Step 2: Check the linear fringe width** Moving the screen away increases  $D$ . The relation  $\beta = \lambda D/d$  shows that the fringe width increases in direct proportion to  $D$ . Options (A) and (B) incorrectly keep this width fixed or decrease it, while option (D) incorrectly predicts an increase in angular separation.

**Step 3: Conclusion** (i) Angular separation remains constant; (ii) fringe width increases. Hence, the correct option is (C).

**Quick Tip to Solve:** Divide the fringe width by screen distance to obtain angular separation; the factor  $D$  cancels.

7. The energy of a photon of wavelength  $\lambda$  is:

[1 mark]

- (A)  $hc\lambda$
- (B)  $hc/\lambda$
- (C)  $\lambda/hc$
- (D)  $\lambda h/c$

**Correct Answer:** (B)  $hc/\lambda$

**Solution:**

**Concept:**

- A photon of frequency  $\nu$  carries energy  $E = h\nu$ , where  $h$  is Planck's constant.
- For light in vacuum,  $c = \nu\lambda$ , where  $c$  is the speed of light and  $\lambda$  is its wavelength.

**Step 1: Express frequency using wavelength** Rearrange the wave relation to obtain  $\nu = c/\lambda$ . Thus a larger wavelength corresponds to a smaller frequency and therefore to a smaller photon energy.

**Step 2: Substitute into Planck's relation** Combining the two relations gives

$$E = h\nu = h\frac{c}{\lambda} = \frac{hc}{\lambda}.$$

Option (A) wrongly makes energy proportional to wavelength. Option (C) is the reciprocal of the required expression, while option (D) also has the wrong wavelength dependence. The units of  $hc/\lambda$  are  $\text{J s} \times \text{m s}^{-1}/\text{m} = \text{J}$ , as required.

**Step 3: Conclusion** The photon energy is  $E = hc/\lambda$ . Hence, the correct option is (B).

**Quick Tip to Solve:** Start from  $E = h\nu$  and replace  $\nu$  by  $c/\lambda$ ; shorter wavelengths mean higher photon energies.

8. The ratio of the nuclear densities of two nuclei having mass numbers 64 and 125 is

[1 mark]

- (A)  $\frac{64}{125}$
- (B)  $\frac{4}{5}$
- (C)  $\frac{5}{4}$
- (D) 1

**Correct Answer:** (D) 1

**Solution:**

**Concept:**

- The nuclear radius follows  $R = R_0 A^{1/3}$ , where  $A$  is mass number and  $R_0$  is a constant.
- Nuclear mass is approximately  $Am_N$ , where  $m_N$  is the mean nucleon mass, and density is mass divided by volume.

**Step 1: Express the nuclear density** Treating a nucleus as a sphere, its volume is  $V = (4\pi/3)R^3 = (4\pi/3)R_0^3 A$ . Therefore

$$\rho = \frac{Am_N}{(4\pi/3)R_0^3 A} = \frac{3m_N}{4\pi R_0^3}.$$

The factor  $A$  cancels because both mass and volume are proportional to it.

**Step 2: Compare the two nuclei** Nuclear density is consequently independent of mass number in this model, so the nuclei with  $A = 64$  and  $A = 125$  have equal densities. Option (A) compares masses without accounting for volume; options (B) and (C) correspond to ratios of radii or their reciprocal, not densities.

**Step 3: Conclusion** The nuclear-density ratio is  $\rho_{64}/\rho_{125} = 1$ . Hence, the correct option is (D).

**Quick Tip to Solve:** Cube  $R \propto A^{1/3}$  to get  $V \propto A$ ; mass and volume scale together, leaving nuclear density constant.

## 9. During the formation of a p-n junction:

[1 mark]

- (A) diffusion current keeps increasing.
- (B) drift current remains constant.
- (C) both the diffusion current and drift current remain constant.
- (D) diffusion current remains almost constant but drift current increases till both currents become equal.

**Correct Answer:** (D) diffusion current remains almost constant but drift current increases till both currents become equal.

### Solution:

#### Concept:

- Diffusion transfers majority carriers across the junction because of the carrier concentration gradient.
- The depletion-region electric field drives a drift current opposite to diffusion; at equilibrium their magnitudes are equal.

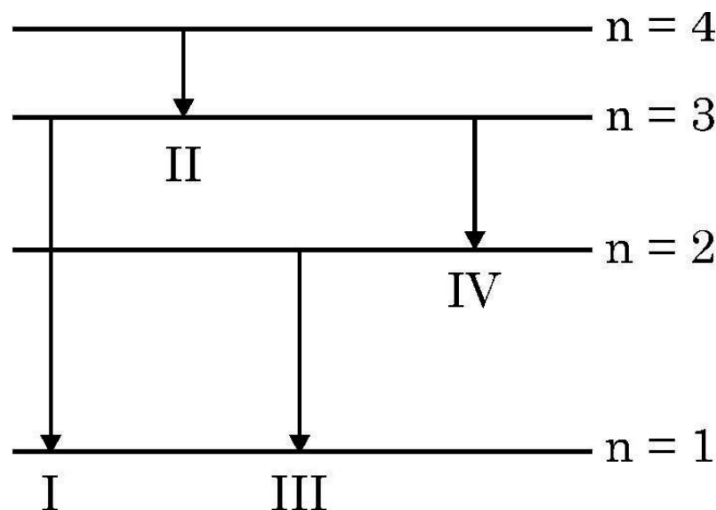
**Step 1: Track the formation of the barrier** Initially, electrons diffuse from the n-side to the p-side and holes diffuse in the opposite direction. The uncovered ionised donors and acceptors create a depletion layer and a built-in electric field. As this field develops, the opposing drift current increases.

**Step 2: Compare the stated current trends** In the approximate description used by the options, diffusion current remains almost constant while drift current grows until it balances diffusion. A continuously increasing diffusion current would not describe equilibration, so (A) fails. Drift cannot remain constant while the barrier is forming, ruling out (B) and (C). The net equilibrium current is zero, although the two opposing carrier currents are nonzero.

**Step 3: Conclusion** Drift current increases until it balances the approximately constant diffusion current in the stated model. Hence, the correct option is (D).

**Quick Tip to Solve:** During junction formation, follow the growing barrier field; equilibrium requires equal and opposite drift and diffusion currents.

10. The diagram shows four energy level of an electron in Bohr model of hydrogen atom. Identify the transition in which the emitted photon will have the highest energy. [1 mark]



- (A) I
- (B) II
- (C) III
- (D) IV

**Correct Answer:** (A) I

**Solution:**

**Concept:**

- Hydrogen energy levels are  $E_n = -13.6/n^2$  eV, where  $n$  is the principal quantum number.
- A downward transition emits a photon of energy  $E_\gamma = E_i - E_f = 13.6(1/n_f^2 - 1/n_i^2)$  eV.

**Step 1: Read the four arrows** In the printed diagram, I is  $3 \rightarrow 1$ , II is  $4 \rightarrow 3$ , III is  $2 \rightarrow 1$ , and IV is  $3 \rightarrow 2$ . The arrowheads indicate the final levels; all four arrows represent emission.

**Step 2: Compare the actual energy differences** Apply the energy-level formula rather than judging the drawing's scale:

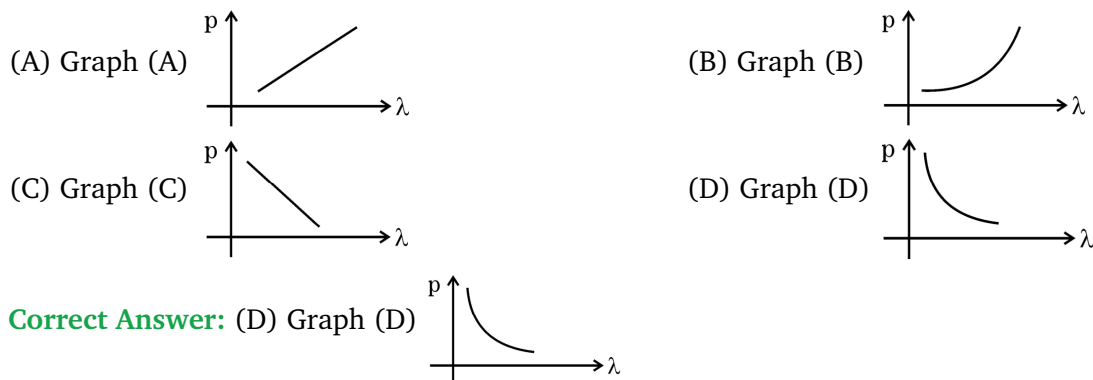
$$\begin{aligned} E_I &= 13.6(1 - 1/9) = 12.09 \text{ eV}, \\ E_{II} &= 13.6(1/9 - 1/16) = 0.66 \text{ eV}, \\ E_{III} &= 13.6(1 - 1/4) = 10.20 \text{ eV}, \\ E_{IV} &= 13.6(1/4 - 1/9) = 1.89 \text{ eV}. \end{aligned}$$

Transition III also ends at the ground state, but starts from a lower initial energy than I. Transitions II and IV have much smaller gaps between excited levels.

**Step 3: Conclusion** Transition I emits the highest-energy photon, approximately 12.09 eV. Hence, the correct option is (A).

**Quick Tip to Solve:** First select arrows ending at the lowest level, then choose the one starting at the highest initial level.

11. Which of the following graphs correctly represents the variation of a particle momentum with its associated de-Broglie wavelength? [1 mark]



### Solution:

#### Concept:

- The de Broglie relation is  $\lambda = h/p$ , where  $h$  is Planck's constant and  $p$  is momentum magnitude.
- Thus  $p = h/\lambda$ : momentum and wavelength are inversely proportional, and their product is constant.

**Step 1: Identify the shape and limiting behaviour** With  $p$  on the vertical axis and  $\lambda$  on the horizontal axis, increasing wavelength decreases momentum. The curve is a rectangular hyperbola in the first quadrant. It approaches the axes but never meets either axis for finite positive values:  $p \rightarrow 0$  only as  $\lambda \rightarrow \infty$ .

**Step 2: Test all four graphs** Graphs (A) and (B) show increasing momentum as wavelength increases, contradicting inverse proportionality. Graph (C) decreases along a straight line, which would make momentum zero at a finite wavelength and does not keep  $p\lambda$  constant. Graph (D) decreases rapidly at first and then flattens, as required by  $p = h/\lambda$ .

**Step 3: Conclusion** Graph (D) correctly represents the inverse relation between momentum and de Broglie wavelength. Hence, the correct option is (D).

**Quick Tip to Solve:** For a de Broglie graph, check that doubling wavelength halves momentum; a descending straight line cannot satisfy this rule everywhere.

12. The capacitors, each of  $4\ \mu F$  are to be connected in such a way that the effective capacitance of the combination is  $6\ \mu F$ . This can be achieved by connecting [1 mark]

- (A) All three in parallel
- (B) All three in series
- (C) Two of them connected in series and the combination in parallel to the third.
- (D) Two of them connected in parallel and the combination in series to the third.

**Correct Answer:** (C) Two of them connected in series and the combination in parallel to the third.

### Solution:

#### Concept:

- For capacitors in parallel, capacitances add:  $C_p = C_1 + C_2$ .
- For capacitors in series, reciprocals add:  $1/C_s = 1/C_1 + 1/C_2$ . Two equal capacitors of capacitance  $C$  in series give  $C/2$ .

**Step 1: Combine a series pair with the third capacitor** Each capacitor is  $4\ \mu\text{F}$ . Connecting two in series gives

$$C_s = \frac{4 \times 4}{4 + 4} = 2\ \mu\text{F}.$$

This pair in parallel with the remaining capacitor gives  $C_{\text{eq}} = 2 + 4 = 6\ \mu\text{F}$ , the required value.

**Step 2: Check the other arrangements** All three in parallel give  $12\ \mu\text{F}$ , while all three in series give  $4/3\ \mu\text{F}$ , ruling out (A) and (B). For (D), the parallel pair is  $8\ \mu\text{F}$ ; connecting it in series with  $4\ \mu\text{F}$  gives  $8 \times 4 / (8 + 4) = 8/3\ \mu\text{F}$ , not the required value.

**Step 3: Conclusion** Connect two capacitors in series, then connect this pair in parallel with the third. Hence, the correct option is (C).

**Quick Tip to Solve:** With three identical capacitors, a series pair in parallel with the third gives  $C + C/2 = 3C/2$ .

**13. Which of the following statements about a series LCR circuit connected to an ac source is correct?**

[1 mark]

- (A) If the frequency of the source is increased, the impedance of the circuit first decreases and then increases.
- (B) If the net reactance  $(X_L - X_C)$  of circuit becomes equal to its resistance, then the current leads the voltage by  $45^\circ$ .
- (C) At resonance, the voltage drop across the inductor is more than that across the capacitor.
- (D) At resonance, the voltage drop across the capacitor is more than that across the inductor.

**Correct Answer:** (A) If the frequency of the source is increased, the impedance of the circuit first decreases and then increases.

**Solution:**

**Concept:**

- A series LCR circuit has impedance  $Z = \sqrt{R^2 + (X_L - X_C)^2}$ , with  $X_L = \omega L$ ,  $X_C = 1/(\omega C)$ , and  $\omega = 2\pi f$ .
- At resonance  $\omega_0 = 1/\sqrt{LC}$ , the reactances cancel and impedance is minimum,  $Z = R$ .

**Step 1: Follow impedance through resonance** As frequency is swept upward from low values, capacitive reactance initially dominates. Its excess over inductive reactance decreases, so impedance decreases. At resonance it reaches  $R$ . Above resonance, inductive reactance dominates and its excess increases, so impedance increases. Thus (A) describes the full frequency dependence across resonance.

**Step 2: Check phase and resonant voltages** If  $X_L - X_C = R > 0$ , the circuit is inductive. Since  $\tan \phi = (X_L - X_C)/R = 1$ , current lags voltage by  $45^\circ$ , rather than leading it; (B) is false. At resonance the same current flows through both components and  $X_L = X_C$ , so the voltage-drop magnitudes  $V_L = IX_L$  and  $V_C = IX_C$  are equal. Both (C) and (D) are false.

**Step 3: Conclusion** When frequency is increased through resonance, impedance first decreases and then increases. Hence, the correct option is (A).

**Quick Tip to Solve:** Sketch the impedance minimum at resonance, and remember that positive net reactance means current lags voltage.

14. According to Huygens principle, the amplitude of secondary wavelets is

[1 mark]

- (A) equal in both the forward and the backward directions.
- (B) maximum in the forward direction and zero in the backward direction.
- (C) large in the forward direction and small in the backward direction.
- (D) small in the forward direction and large in the backward direction.

**Correct Answer:** (B) maximum in the forward direction and zero in the backward direction.

**Solution:**

**Concept:**

- In Huygens' construction, each point of a wavefront acts as a source of secondary wavelets, whose forward envelope forms the next wavefront.
- In the usual directional-amplitude prescription for these wavelets, amplitude is greatest in the forward direction and vanishes in the backward direction.

**Step 1: Identify forward and backward directions** Forward means along the propagation direction, normal to the original wavefront. The secondary wavelets contribute most strongly in this direction. Their contribution in the exactly opposite direction is zero, so the construction does not produce a backward-travelling wavefront.

**Step 2: Compare the proposed amplitudes** Option (A) would give equal forward and backward contributions, contradicting the directional prescription. Option (C) makes the backward amplitude small but nonzero, whereas it is zero in the exactly backward direction. Option (D) reverses the actual directional dependence. Only (B) specifies both the forward maximum and the backward zero correctly.

**Step 3: Conclusion** The amplitude is maximum in the forward direction and zero in the backward direction. Hence, the correct option is (B).

**Quick Tip to Solve:** For the standard Huygens wavelet question, pair the forward envelope with maximum amplitude and the exactly backward direction with zero amplitude.

15. The radius of the  $n^{\text{th}}$  orbit in Bohr model of hydrogen atom is proportional to

[1 mark]

- (A)  $n^2$
- (B)  $1/n^2$
- (C)  $n$
- (D)  $1/n$

**Correct Answer:** (A)  $n^2$

**Solution:**

**Concept:**

- In Bohr's model, Coulomb attraction supplies centripetal force:  $m_e v^2 / r = k e^2 / r^2$ , where  $k = 1 / (4\pi\epsilon_0)$ .
- Angular momentum is quantised:  $m_e v r = n\hbar$ , where  $\hbar = h / (2\pi)$  and  $n$  is the orbit number.

**Step 1: Eliminate the orbital speed** From angular-momentum quantisation,  $v = n\hbar / (m_e r)$ . Substitut-

ing this into the force equation gives

$$\frac{n^2 \hbar^2}{m_e r^3} = \frac{ke^2}{r^2}.$$

Multiplying by  $r^3$  yields  $n^2 \hbar^2 / m_e = ke^2 r$ .

**Step 2: Identify the orbit-number dependence** Rearranging gives

$$r_n = \frac{\hbar^2}{m_e ke^2} n^2 = a_0 n^2,$$

where  $a_0$  is the first Bohr radius, a constant for hydrogen. Thus the dependence is quadratic, not linear as in (C), and not decreasing with  $n$  as in (B) or (D).

**Step 3: Conclusion** The radius of the  $n$ th orbit is proportional to  $n^2$ . Hence, the correct option is (A).

**Quick Tip to Solve:** Keep Bohr scalings distinct: hydrogen orbit radius grows as  $n^2$ , whereas the magnitude of its energy falls as  $1/n^2$ .

**16. Assertion (A):** The resistance of an intrinsic semiconductor decreases with increase in its temperature.

**Reason (R):** The number of conduction electrons as well as hole increase in an intrinsic semiconductor with rise in its temperature. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and (R) is the correct explanation of (A).  
(B) Both Assertion (A) and Reason (R) are true and (R) is NOT the correct explanation of (A).  
(C) Assertion (A) is true and Reason (R) is false.  
(D) Assertion (A) is false and Reason (R) is also false.

**Correct Answer:** (A) Both Assertion (A) and Reason (R) are true and (R) is the correct explanation of (A).

**Solution:**

**Concept:**

- An intrinsic semiconductor has equal electron and hole concentrations:  $n = p = n_i$ . Thermal excitation generates electron-hole pairs.
- Its conductivity is  $\sigma = en_i(\mu_e + \mu_h)$ , where  $\mu_e$  and  $\mu_h$  are carrier mobilities. For fixed dimensions, resistance is inversely proportional to conductivity.

**Step 1: Evaluate the assertion** An intrinsic semiconductor has a negative temperature coefficient of resistance over the usual temperature range considered here. Raising its temperature increases conductivity and decreases resistance. Therefore Assertion (A) is true.

**Step 2: Evaluate the reason** Thermal energy excites more electrons from the valence band into the conduction band, leaving holes behind. Each excitation produces one conduction electron and one hole, so both carrier populations increase together. Therefore Reason (R) is true.

**Step 3: Test the explanatory connection** The strong increase in intrinsic carrier concentration outweighs the reduction in carrier mobility caused by increased scattering. Consequently  $\sigma$  rises and

resistance falls. Thus the increase in electron and hole populations directly explains the assertion. Option (B) incorrectly denies this connection, while (C) and (D) incorrectly reject true statements.

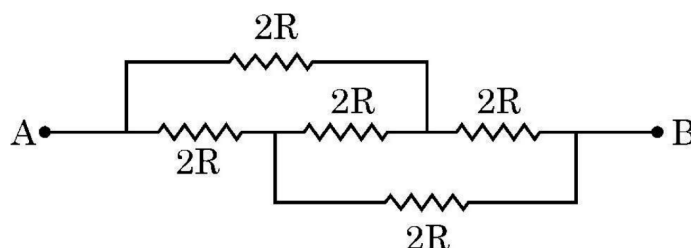
**Step 4: Conclusion** Both Assertion (A) and Reason (R) are true, and R correctly explains A. Hence, the correct option is (A).

**Quick Tip to Solve:** For an intrinsic semiconductor heated at fixed dimensions, track the increase in electron-hole pairs before judging the change in resistance.

**17. Assertion (A):** The equivalent resistance between points A and B in the given network is  $2R$ .

**Reason (R):** All the resistors are connected in parallel.

[1 mark]



(A) Both Assertion (A) and Reason (R) are true and (R) is the correct explanation of (A).

(B) Both Assertion (A) and Reason (R) are true and (R) is NOT the correct explanation of (A).

(C) Assertion (A) is true and Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

**Correct Answer:** (C) Assertion (A) is true and Reason (R) is false.

### Solution:

#### Concept:

- A balanced Wheatstone bridge has equal resistance ratios in its two arms, so no current flows through its bridge resistor.
- Series resistances add; for two equal resistances  $X$  in parallel, the equivalent resistance is  $X/2$ .

**Step 1: Evaluate the assertion** Call the two internal junctions on the horizontal line  $C$  and  $D$ , from left to right. The arms  $A - C - B$  and  $A - D - B$  each contain two resistors of  $2R$ . Their ratios are equal,  $2R/(2R) = 2R/(2R)$ , so  $C$  and  $D$  are equipotential and the middle resistor carries no current. Hence

$$R_{AB} = (4R) \parallel (4R) = \frac{4R \times 4R}{4R + 4R} = 2R.$$

The assertion is true.

**Step 2: Evaluate the reason** All five resistors do not connect across the same two nodes, so they are not all in parallel. The reason is false; the central resistor is the bridge arm.

**Step 3: Check the explanation and options** The result follows from bridge balance followed by a series-parallel reduction. The proposed parallel connection cannot explain it. Options (A) and (B)

wrongly treat the reason as true, while (D) wrongly treats the assertion as false.

**Step 4: Conclusion** The assertion is true and the reason is false. Hence, the correct option is (C).

**Quick Tip to Solve:** Redraw the two internal junctions as a bridge: equal arm ratios make the middle resistor inactive.

**18. Assertion (A):** The deflecting torque acting on a current carrying loop is zero when its plane is perpendicular to the direction of magnetic field.

**Reason (R):** The deflecting torque acting on a loop of magnetic moment  $\vec{m}$  in a magnetic field  $\vec{B}$  is given by the dot product of  $\vec{m}$  and  $\vec{B}$ . [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and (R) is the correct explanation of (A).  
(B) Both Assertion (A) and Reason (R) are true and (R) is NOT the correct explanation of (A).  
(C) Assertion (A) is true and Reason (R) is false.  
(D) Assertion (A) is false and Reason (R) is also false.

**Correct Answer:** (C) Assertion (A) is true and Reason (R) is false.

**Solution:**

**Concept:**

- The magnetic moment  $\vec{m} = NI\vec{A}$  of a current loop is normal to its plane, with direction given by the right-hand rule.
- Magnetic torque is  $\vec{\tau} = \vec{m} \times \vec{B}$ , with magnitude  $mB \sin \theta$ , where  $\theta$  is the angle between the normal and the field.

**Step 1: Evaluate the assertion** When the plane is perpendicular to the magnetic field, its normal is parallel or antiparallel to the field. Thus  $\theta = 0^\circ$  or  $180^\circ$ , giving  $\tau = mB \sin \theta = 0$ . The assertion is true.

**Step 2: Evaluate the reason** Torque is the cross product, not the dot product, of magnetic moment and magnetic field. The dot product is a scalar and instead appears in the potential energy  $U = -\vec{m} \cdot \vec{B}$ . Therefore the reason is false.

**Step 3: Check the explanatory relationship** The zero torque follows from the sine of the angle between  $\vec{m}$  and  $\vec{B}$ ; the incorrect dot-product statement cannot explain it. Options (A) and (B) require a true reason, while (D) requires a false assertion, so none applies.

**Step 4: Conclusion** The assertion is true and the reason is false. Hence, the correct option is (C).

**Quick Tip to Solve:** Measure the torque angle from the loop's normal, not its plane; then use the cross product.

### Section B (14 marks)

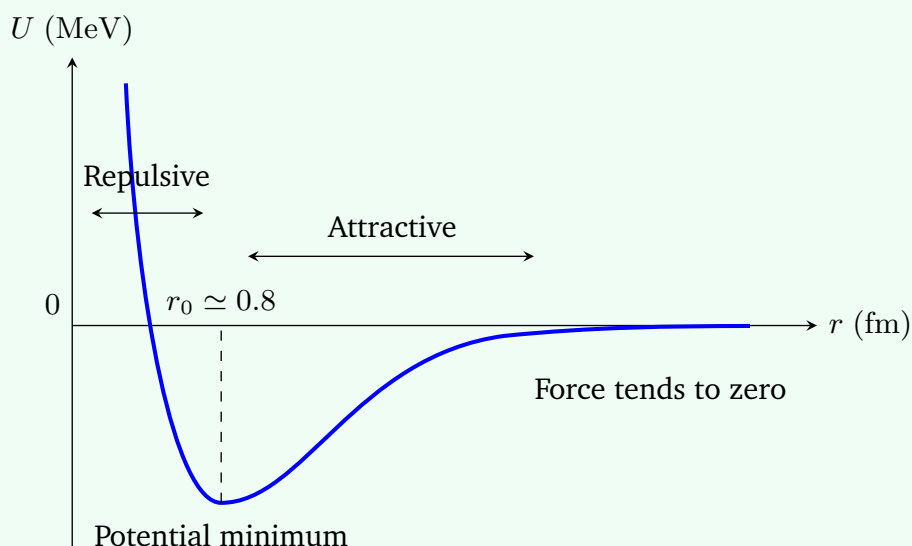
19. Draw a graph showing the variation of potential energy of a pair of nucleons as a function of their separation. Indicate the region in which the nuclear force is (a) attractive and (b) repulsive. [2 marks]

#### Solution:

##### Concept:

- Nuclear potential energy  $U(r)$  is taken as zero for widely separated nucleons; it has a negative well and rises steeply at very small separation  $r$ .
- The radial nuclear force is  $F_r = -dU/dr$ . Positive  $F_r$  pushes the nucleons apart (repulsion), while negative  $F_r$  pulls them together (attraction).

**Step 1: Draw the potential-energy curve** Put separation  $r$  on the horizontal axis and potential energy  $U$  on the vertical axis. The curve rises steeply to positive values as separation tends towards zero, falls through zero into a negative minimum at  $r = r_0$  (about 0.8 fm), and then rises towards zero from below. Nuclear interaction becomes negligible beyond a few femtometres. The sketch is qualitative;  $1 \text{ fm} = 10^{-15} \text{ m}$ .



*Qualitative nucleon potential: repulsion left of the minimum and attraction to its right.*

**Step 2: Identify force regions from the slope** To the left of the minimum,  $dU/dr < 0$ , so  $F_r > 0$ : the force is repulsive. To the right, while the curve rises towards zero,  $dU/dr > 0$ , so  $F_r < 0$ : the force is attractive. At the minimum  $dU/dr = 0$ , and hence the net force is zero. At large separations the curve becomes nearly flat and the nuclear force vanishes.

**Step 3: Conclusion** The nuclear force is repulsive for  $r < r_0$ , attractive for  $r > r_0$  within its short range, and negligible beyond a few femtometres. The boundary is the potential minimum, not the crossing of  $U = 0$ .

**Quick Tip to Solve:** Read the slope of the potential graph and reverse its sign to find the force; negative potential alone does not prove attraction.

**20(a).** How will the De Broglie wavelength associated with an electron be affected when the (i) velocity of the electron decreases? and (ii) accelerating potential is increased? Justify your answer.

[2 marks]

**Solution:**

**Concept:**

- De Broglie's relation is  $\lambda = h/p$ , where  $h$  is Planck's constant and  $p$  is the magnitude of the electron momentum.
- In the non-relativistic regime,  $p = mv$ . An electron accelerated from rest through a potential difference  $V$  gains kinetic energy  $eV = p^2/(2m)$ , where  $e$  is the magnitude of its charge.

**Step 1: Relate wavelength to electron velocity** Substituting the momentum in the de Broglie relation gives

$$\lambda = \frac{h}{mv}, \quad \frac{\lambda_2}{\lambda_1} = \frac{v_1}{v_2}.$$

Thus a decrease in speed reduces momentum and increases the associated wavelength. For example, halving the non-relativistic speed doubles the wavelength. It is the magnitude of velocity that matters in this relation.

**Step 2: Relate wavelength to accelerating potential** Equating the gain in kinetic energy to the electric work gives

$$eV = \frac{p^2}{2m} \Rightarrow p = \sqrt{2meV}, \quad \lambda = \frac{h}{\sqrt{2meV}}.$$

Consequently  $\lambda \propto V^{-1/2}$ . Increasing the accelerating potential increases momentum and decreases wavelength; multiplying the potential by four halves the wavelength under these assumptions.

**Step 3: Conclusion** (i) The wavelength increases when the electron speed decreases. (ii) It decreases when the accelerating potential increases, with  $\lambda \propto 1/\sqrt{V}$  in the non-relativistic regime.

**Quick Tip to Solve:** First write  $\lambda = h/p$ : speed changes give an inverse relation, but potential changes give an inverse square-root relation.

OR

**20(b).** How would the stopping potential for a given photosensitive surface change if (i) the frequency of the incident radiation were increased? and (ii) the intensity of incident radiation were decreased? Justify your answer.

[2 marks]

**Solution:**

**Concept:**

- Einstein's photoelectric equation is  $K_{\max} = h\nu - \phi$ , where  $\nu$  is incident frequency and  $\phi$  is the work function of the given surface.
- The stopping potential magnitude  $V_s$  satisfies  $eV_s = K_{\max}$ , where  $e$  is the magnitude of the electron charge. Intensity at fixed frequency controls photon flux, not the energy  $h\nu$  of each photon.

**Step 1: Increase the incident frequency** For the same photosensitive surface, the work function stays constant. Hence

$$V_s = \frac{h\nu - \phi}{e}, \quad \Delta V_s = \frac{h}{e} \Delta \nu.$$

Above the threshold frequency, increasing frequency gives each photon more energy, so the most energetic emitted electrons have greater kinetic energy and require a larger opposing potential to stop them.

**Step 2: Decrease the incident intensity** At unchanged frequency above threshold, reducing intensity decreases the number of incident photons per second. Fewer photoelectrons are emitted and the photocurrent decreases. However, each photon still supplies the same energy  $h\nu$ , so  $K_{\max}$  and the stopping potential remain unchanged, provided emission is still present.

**Step 3: Conclusion** (i) Increasing frequency increases the magnitude of the stopping potential. (ii) Decreasing intensity at fixed frequency leaves that magnitude unchanged; it reduces the photocurrent.

**Quick Tip to Solve:** Use  $eV_s = h\nu - \phi$ : frequency appears in the equation, while intensity changes the number of photoelectrons instead.

**21. Identify the electromagnetic wave whose wavelengths range is from about**

(a)  $10^{-12}$  m to about  $10^{-8}$  m.

(b)  $10^{-3}$  m to about  $10^{-1}$  m.

Write one use of each.

[2 marks]

**Solution:**

**Concept:**

- Electromagnetic waves are classified by wavelength  $\lambda$ ; their frequency is  $\nu = c/\lambda$ , where  $c$  is the speed of light in vacuum.
- X-rays have very short wavelengths and can penetrate soft tissue. Microwaves have wavelengths of order millimetres to centimetres and are useful in radar.

**Step 1: (a) Identify the shorter wavelength band** The range  $10^{-12}$  m to  $10^{-8}$  m is the conventional X-ray band. One use is medical radiography to detect bone fractures: X-rays pass more readily through soft tissue than through bone, producing contrast in the recorded image.

**Step 2: (b) Identify the longer wavelength band** The range  $10^{-3}$  m to  $10^{-1}$  m, or about 1 mm to 10 cm, is the microwave band. One use is radar, in which a transmitted microwave signal reflects from a distant object; the echo can be used to determine its distance.

**Step 3: Conclusion** (a) X-rays: detecting bone fractures by medical radiography. (b) Microwaves: radar detection and ranging.

**Quick Tip to Solve:** Convert the second range to millimetres and centimetres to recognise microwaves immediately.

**22. Depict the orientation of an electric dipole in (a) stable and (b) unstable equilibrium in an external uniform electric field. Write the potential energy of the dipole in each case.**

[2 marks]

**Solution:**

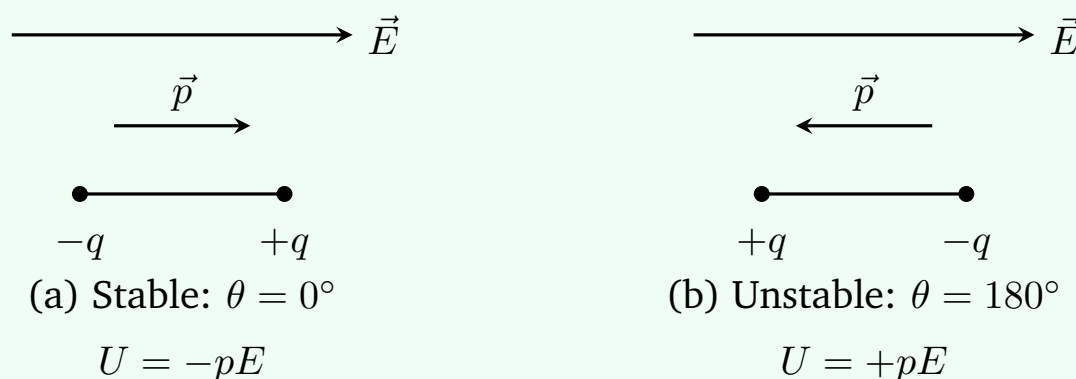
**Concept:**

- The electric dipole moment  $\vec{p}$  points from  $-q$  to  $+q$ . In a uniform field  $\vec{E}$ , its torque is  $\vec{\tau} = \vec{p} \times \vec{E}$ .

- With zero potential energy at  $90^\circ$ , the potential energy is  $U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$ , where  $\theta$  is the angle between  $\vec{p}$  and  $\vec{E}$ .

**Step 1: (a) Stable equilibrium** The dipole is parallel to the external field, so  $\theta = 0^\circ$ . The torque vanishes and a small angular displacement produces a restoring torque toward this orientation. Substituting in the potential-energy expression gives  $U = -pE \cos 0^\circ = -pE$ , the minimum possible energy.

**Step 2: (b) Unstable equilibrium** The dipole is antiparallel to the external field, so  $\theta = 180^\circ$ . Although the torque is initially zero, a small angular displacement produces a torque that turns it farther away from this orientation. Its energy is  $U = -pE \cos 180^\circ = +pE$ , the maximum possible energy.



*Dipole orientations in stable and unstable equilibrium.*

**Step 3: Conclusion** Stable:  $\vec{p} \parallel \vec{E}$ ,  $U = -pE$ . Unstable:  $\vec{p}$  opposite to  $\vec{E}$ ,  $U = +pE$ .

**Quick Tip to Solve:** Draw the dipole arrow from negative to positive charge; alignment with the field gives minimum energy.

**23(a).** Write the expression for the Lorentz force on a particle of charge  $q$  moving with a velocity  $\vec{v}$  in a magnetic field  $\vec{B}$ . When is the magnitude of this force maximum? Show that no work is done by this force on the particle during its motion from a point  $\vec{r}_1$  to point  $\vec{r}_2$ . [2 marks]

**Solution:**

**Concept:**

- In a magnetic field alone, the Lorentz force is  $\vec{F} = q(\vec{v} \times \vec{B})$ , where  $q$  is the signed charge,  $\vec{v}$  its velocity and  $\vec{B}$  the magnetic field.
- The force magnitude is  $F = |q|vB \sin \theta$ , where  $\theta$  is the angle between velocity and field. Work depends on the component of force along displacement.

**Step 1: State the force and its maximum value** The magnetic force is  $\vec{F} = q(\vec{v} \times \vec{B})$ . For fixed  $|q|$ , speed  $v$  and field magnitude  $B$ , its magnitude is greatest when  $\sin \theta = 1$ , that is, when  $\vec{v} \perp \vec{B}$ . Thus the maximum magnitude is  $F_{\max} = |q|vB$ . The sign of the charge reverses the direction, not the magnitude.

**Step 2: Evaluate the work along the trajectory** During a time  $dt$ , the infinitesimal displacement is

$d\vec{r} = \vec{v} dt$ . Since a cross product is perpendicular to each of its factors,

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = q \int_{t_1}^{t_2} (\vec{v} \times \vec{B}) \cdot \vec{v} dt = 0.$$

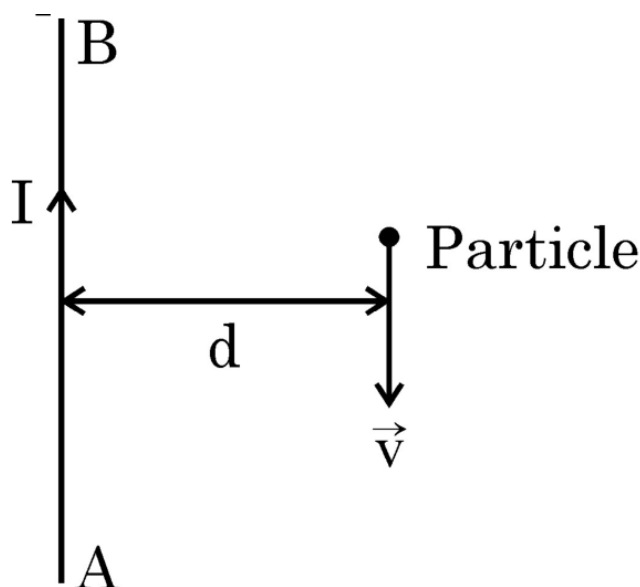
This holds at every point of the actual path, even though the direction of velocity can change. A magnetic force alone changes the direction of motion but cannot change the kinetic energy.

**Step 3: Conclusion**  $\vec{F} = q(\vec{v} \times \vec{B})$ ; its maximum magnitude  $|q|vB$  occurs for perpendicular velocity and field. The work done from  $\vec{r}_1$  to  $\vec{r}_2$  is zero.

**Quick Tip to Solve:** Use  $d\vec{r} = \vec{v} dt$  to prove zero work directly; the magnetic force is always perpendicular to velocity.

OR

**23(b).** A long straight wire AB carries a current  $I$ . A particle (mass  $m$  and charge  $q$ ) moves with a velocity  $\vec{v}$ , parallel to the wire, at a distance  $d$  from it as shown in the figure. Obtain the expression for the force experienced by the particle and mention its directions. [2 marks]



**Solution:**

**Concept:**

- At perpendicular distance  $d$  from a long straight wire carrying current  $I$ , the magnetic field magnitude is  $B = \mu_0 I / (2\pi d)$ , where  $\mu_0$  is the permeability of free space.
- A moving charge experiences  $\vec{F} = q(\vec{v} \times \vec{B})$ . Use the right-hand rule for the wire's field first, then for the cross product; a negative charge reverses the resulting force.

**Step 1: Find the magnetic field at the particle** The diagram shows current from A to B, vertically upward, and the particle to the right of the wire. By the right-hand thumb rule, the field at the particle is into the page, with magnitude  $B = \mu_0 I / (2\pi d)$ . The particle's velocity is downward in the page, so  $\vec{v} \perp \vec{B}$ .

**Step 2: Calculate the force and determine its direction** Take  $+\hat{x}$  to the right,  $+\hat{y}$  upward and  $+\hat{z}$  out

of the page. Then  $\vec{v} = -v\hat{y}$  and  $\vec{B} = -B\hat{z}$ , giving

$$\vec{F} = q(-v\hat{y}) \times (-B\hat{z}) = \frac{\mu_0 I q v}{2\pi d} \hat{x}.$$

Thus a positive charge experiences a force to the right, away from the wire. For a negative charge the direction is leftward, toward the wire. The mass  $m$  does not enter the magnetic force; it would be needed to calculate the resulting acceleration.

**Step 3: Conclusion** The force magnitude is  $F = \mu_0 I |q| v / (2\pi d)$ . It is away from the wire for  $q > 0$  and toward the wire for  $q < 0$ , for the current and velocity shown.

**Quick Tip to Solve:** With upward current, the field on the right is into the page; downward velocity gives an outward force for a positive charge.

**24. The potential difference applied across a given conductor is doubled. How will this affect (i) the mobility of electrons and (ii) the current density in the conductor? Justify your answers.** [2 marks]

**Solution:**

**Concept:**

- Electron mobility is the magnitude of drift velocity per unit electric field:  $\mu = v_d / E = e\tau / m$ , where  $e$  is the magnitude of electronic charge,  $m$  is electron mass and  $\tau$  is relaxation time.
- For a uniform conductor of length  $L$ ,  $E = V / L$  and current density has magnitude  $J = nev_d = ne\mu E$ , where  $n$  is the free-electron number density.

**Step 1: Examine the mobility** For the given conductor, take its temperature and physical dimensions as unchanged, as in the usual ohmic model. Doubling the potential difference does not change  $e$ ,  $m$ , or the relaxation time  $\tau$ . Therefore the ratio of drift speed to electric field remains fixed:

$$\mu' = \frac{e\tau}{m} = \mu.$$

Although the electrons acquire twice the drift speed, this does not mean twice the mobility, because the electric field has also doubled.

**Step 2: Calculate the new current density** The same length  $L$  is now subjected to potential difference  $2V$ , so

$$E' = \frac{2V}{L} = 2E.$$

Since  $n$  and  $\mu$  remain unchanged,

$$J' = ne\mu E' = ne\mu(2E) = 2J.$$

This also agrees with Ohm's law: constant resistance gives twice the current, and the same cross-sectional area gives twice the current density.

**Step 3: Conclusion** (i) Electron mobility remains unchanged. (ii) Current density doubles, assuming unchanged temperature and conductor dimensions.

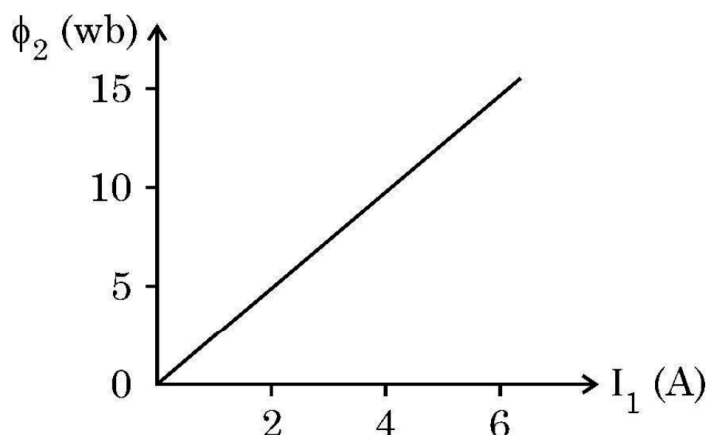
**Quick Tip to Solve:** Distinguish mobility  $v_d / E$  from drift speed: doubling the field doubles  $v_d$ , while their ratio stays constant.

25. Two coils  $C_1$  and  $C_2$  are placed close to each other. The magnetic flux  $\phi_2$  linked with the coil  $C_2$  varies with the current  $I_1$  flowing in coil  $C_1$ , as shown in the figure. Find

(i) the mutual inductance of the arrangement, and

(ii) the rate of change of current  $\left(\frac{dI_1}{dt}\right)$  that will induce an emf of 100 V in coil  $C_2$ .

[2 marks]



### Solution:

#### Concept:

- Mutual inductance  $M$  is the flux linkage in the secondary coil per unit current in the primary:  $\phi_2 = MI_1$ . Hence the slope of the given flux-linkage versus current graph is  $M$ .
- Faraday's law for the secondary coil gives  $\varepsilon_2 = -M dI_1/dt$ . Its magnitude is  $|\varepsilon_2| = M|dI_1/dt|$ ; the minus sign expresses Lenz's law.

**Step 1: Read the slope of the graph** The straight line passes through the origin and the plotted values  $I_1 = 6$  A,  $\phi_2 = 15$  Wb (equivalently 4 A, 10 Wb). Thus the flux linkage increases uniformly with current. The mutual inductance is

$$M = \frac{\Delta\phi_2}{\Delta I_1} = \frac{15 - 0}{6 - 0} \text{ Wb A}^{-1} = 2.5 \text{ H}.$$

Here the quantity described as flux linked with the coil is the linkage represented by the graph; no additional turns factor is supplied or needed.

**Step 2: Find the required current-change rate** To obtain an induced emf of magnitude 100 V, substitute the mutual inductance into Faraday's law:

$$\left|\frac{dI_1}{dt}\right| = \frac{|\varepsilon_2|}{M} = \frac{100}{2.5} = 40 \text{ A s}^{-1}.$$

An increasing or decreasing current at this rate produces the same emf magnitude, with opposite polarities. The question gives the magnitude of the emf and therefore determines the magnitude of the rate.

**Step 3: Conclusion** (i)  $M = 2.5$  H. (ii) The required rate of change of primary current has magnitude  $40 \text{ A s}^{-1}$ .

**Quick Tip to Solve:** First take the slope of the flux-current graph to obtain  $M$ ; then divide the required emf by this slope.

## Section C (15 marks)

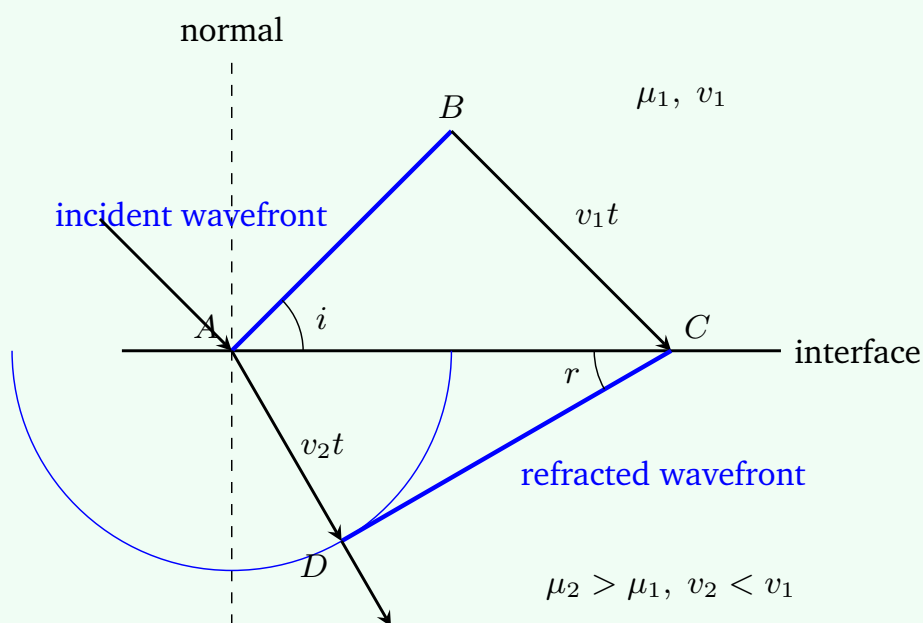
**26(a).** A plane wave-front propagating in a medium of refractive index ' $\mu_1$ ' is incident on a plane surface making an angle of incidence ( $i$ ). It enters into a medium of refractive index  $\mu_2$  ( $\mu_2 > \mu_1$ ). Use Huygens's construction of secondary wavelets to trace the refracted wave-front. Hence verify Snell's law of refraction. [3 marks]

### Solution:

#### Concept:

- Huygens' principle: every point on a wavefront emits secondary wavelets. Their common forward envelope gives the wavefront at a later time; rays are normal to wavefronts.
- Light speeds in the two media are  $v_1 = c/\mu_1$  and  $v_2 = c/\mu_2$ . Since  $\mu_2 > \mu_1$ , the wavelets advance more slowly in the second medium.

**Step 1: Construct the incident advance and refracted wavelet** Let  $AB$  be the incident plane wavefront and  $AC$  the plane interface. Point  $A$  reaches the interface first. During the time  $t$  in which the disturbance at  $B$  reaches  $C$ , it covers  $BC = v_1 t$  along the incident ray. Meanwhile, the secondary wavelet from  $A$  advances a distance  $AD = v_2 t$  into medium 2. Draw this wavelet with centre  $A$  and radius  $v_2 t$ . From  $C$ , draw tangent  $CD$  to it. The tangent is the common forward envelope and is the refracted plane wavefront. Since a radius is perpendicular to a tangent,  $AD \perp CD$ ;  $AD$  gives the refracted-ray direction.



*Huygens construction for refraction; the rays are perpendicular to their wavefronts.*

**Step 2: Relate the construction to the angles** The incident ray  $BC$  is perpendicular to  $AB$ , while the normal to the interface is perpendicular to  $AC$ . Therefore the angle between  $AB$  and  $AC$  equals the angle of incidence  $i$ . Similarly, the acute angle between  $CD$  and  $CA$  equals the angle of refraction  $r$ . From right triangles  $ABC$  and  $ADC$ , which share hypotenuse  $AC$ ,

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}, \quad \sin r = \frac{AD}{AC} = \frac{v_2 t}{AC}.$$

**Step 3: Derive Snell's law** Dividing the two expressions eliminates both the construction time and the

common length:

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{c/\mu_1}{c/\mu_2} = \frac{\mu_2}{\mu_1}.$$

Thus the denser medium has the smaller angle to the normal. The construction lies in the plane of incidence, so the incident ray, refracted ray and normal are coplanar.

**Step 4: Conclusion** The tangent  $CD$  is the refracted wavefront and  $\mu_1 \sin i = \mu_2 \sin r$ , verifying Snell's law. Since  $\mu_2 > \mu_1$ ,  $r < i$  for oblique incidence: the ray bends towards the normal.

**Quick Tip to Solve:** Use the same time  $t$  and common hypotenuse  $AC$  in both right triangles; the sine ratio then becomes the speed ratio immediately.

OR

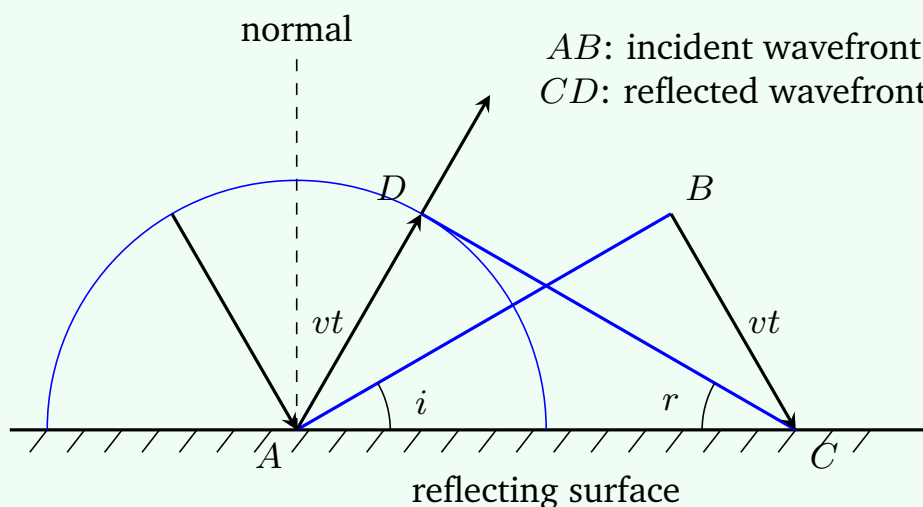
**26(b).** Using Huygen's construction, show how a plane wave is reflected from a surface. Hence verify the law of reflection. [3 marks]

**Solution:**

**Concept:**

- By Huygens' principle, each point reached by a wavefront produces secondary wavelets. Their common forward tangent gives the new plane wavefront.
- Incident and reflected waves travel in the same medium at speed  $v$ . Rays are perpendicular to their corresponding wavefronts, and incidence and reflection angles are measured from the surface normal.

**Step 1: Construct the reflected plane wavefront** Let  $AC$  be the reflecting surface and  $AB$  an incident plane wavefront. Point  $A$  strikes the surface first. While the disturbance at  $B$  travels to  $C$  in a time  $t$ , it covers  $BC = vt$  along the incident ray. During the same time, a secondary wavelet centred at  $A$  travels back into the original medium through distance  $AD = vt$ . Draw this wavelet and the tangent  $CD$  from  $C$  to it. This tangent is the reflected plane wavefront. The radius  $AD$  is perpendicular to  $CD$  and represents the direction of the reflected ray.



*Huygens construction for reflection, with  $BC = AD = vt$ .*

**Step 2: Identify the angles in the two right triangles** Since  $BC \perp AB$ , triangle  $ABC$  is right-angled

at  $B$ . Since  $AD \perp CD$ , triangle  $ADC$  is right-angled at  $D$ . Both have the same hypotenuse  $AC$ . The angle between the incident wavefront  $AB$  and the surface  $AC$  equals incidence angle  $i$ , because their respective perpendiculars are the incident ray and normal. Similarly, the angle between reflected wavefront  $CD$  and the surface  $CA$  equals reflection angle  $r$ . Therefore

$$\sin i = \frac{BC}{AC}, \quad \sin r = \frac{AD}{AC}.$$

**Step 3: Verify the law of reflection** Reflection occurs in the same medium, so the two distances travelled during the common interval are equal:  $BC = AD = vt$ . Hence

$$\sin i = \frac{vt}{AC} = \sin r.$$

For the physical incidence and reflection angles between  $0^\circ$  and  $90^\circ$ , this implies  $i = r$ . The construction is in the plane of incidence; the incident ray, reflected ray and normal to the surface all lie in this same plane.

**Step 4: Conclusion** The tangent  $CD$  is the reflected plane wavefront. Huygens' construction gives  $i = r$  and coplanarity of the incident ray, reflected ray and normal, verifying the laws of reflection.

**Quick Tip to Solve:** For reflection, both wavefront advances use the same speed: set  $BC = AD = vt$  and compare their ratios to the common hypotenuse  $AC$ .

**27.** An alternating voltage of 220 V is applied across a device X. A current of 0.22 A flows in the circuit and it lags behind the applied voltage in phase by  $\pi/2$  radian. When the same voltage is applied across another device Y, the current in the circuit remains the same and it is in phase with the applied voltage.

(i) Name the devices X and Y and,

(ii) Calculate the current flowing in the circuit when the same voltage is applied across the series combination of X and Y.

[3 marks]

**Solution:**

**Concept:**

- In an ideal inductor the current lags the applied voltage by  $\pi/2$ , whereas in a resistor the current and voltage are in phase.
- Inductive reactance is  $X_L = V/I$ . A series resistor-inductor circuit has impedance  $Z = \sqrt{R^2 + X_L^2}$ , because its resistive and inductive voltage drops are perpendicular phasors.

**Step 1: Identify the two devices** For X the current lags the voltage by exactly  $\pi/2$ , identifying X as a pure inductor (a coil of negligible resistance). A capacitor would make the current lead instead. For Y the current is in phase with the voltage, so Y is a resistor. The identical current magnitudes for the separate devices do not imply identical phase behaviour.

**Step 2: Calculate the resistance and reactance separately** Treat the quoted alternating voltage and current as rms values. Each device alone carries 0.22 A under 220 V, hence

$$X_L = \frac{220}{0.22} = 1000 \Omega, \quad R = \frac{220}{0.22} = 1000 \Omega.$$

The same supply is subsequently used, so its frequency and therefore the inductive reactance remain

unchanged.

**Step 3: Combine the impedances and apply the ac form of Ohm's law** In series, the current through both devices is the same. The resistor voltage is in phase with this current and the inductor voltage leads it by  $90^\circ$ . Therefore their magnitudes cannot be added directly:

$$Z = \sqrt{1000^2 + 1000^2} = 1000\sqrt{2} \Omega,$$
$$I = \frac{V}{Z} = \frac{220}{1000\sqrt{2}} = \frac{0.22}{\sqrt{2}} \text{ A}.$$

**Step 4: Conclusion** (i) X is a pure inductor and Y is a resistor. (ii) The rms current in their series combination is  $I \approx 0.156 \text{ A}$ .

**Quick Tip to Solve:** Equal separate currents here give  $R = X_L$ ; therefore the series current is the original current divided by  $\sqrt{2}$ , not by 2.

**28. State the basic principle behind the working of an ac generator. Briefly describe its working and obtain the expression for the instantaneous value of emf induced.** [3 marks]

**Solution:**

**Concept:**

- Faraday's law states that a changing magnetic flux linkage induces emf:  $\varepsilon = -d(N\Phi)/dt$ . The minus sign gives the direction prescribed by Lenz's law.
- For a coil of  $N$  turns and area  $A$  rotating in a uniform field  $B$ , the flux linkage is  $N\Phi = NBA \cos \theta$ , where  $\theta$  is the angle between the field and the coil's area normal.

**Step 1: State the principle and describe the arrangement** An ac generator works on electromagnetic induction: rotation of a coil changes the magnetic flux linked with it and induces an emf. A rectangular conducting coil is rotated mechanically between the poles of a magnet about an axis perpendicular to the magnetic field. Its two ends connect to two separate slip rings that rotate with the coil. Stationary carbon brushes touch the rings and connect them to the external circuit.

**Step 2: Explain the production of alternating emf** As the coil rotates, its orientation relative to the magnetic field changes continuously. The changing flux induces emf, which drives current through a closed external circuit. After every half revolution the direction of motion of each active side reverses relative to the field, so the induced emf reverses polarity. The slip rings maintain each coil end's connection to its own brush. The generator thus converts mechanical energy into electrical energy and supplies alternating emf.

**Step 3: Derive the instantaneous induced emf** Let the coil rotate uniformly with angular speed  $\omega$ , and choose  $t = 0$  when its area normal is parallel to the magnetic field. Then  $\theta = \omega t$ , so

$$N\Phi(t) = NBA \cos \omega t,$$
$$\varepsilon(t) = -\frac{d}{dt}(NBA \cos \omega t)$$
$$= NBA\omega \sin \omega t.$$

Thus emf is zero when the flux is maximum, and its magnitude is maximum when the coil's plane is parallel to the field and flux passes through zero. The angular frequency of the emf equals the rotation angular speed.

**Step 4: Conclusion** The ac generator uses electromagnetic induction. Its instantaneous emf is  $\varepsilon(t) = \varepsilon_0 \sin \omega t$ , where  $\varepsilon_0 = NBA\omega$  is the peak emf for the stated initial orientation.

**Quick Tip to Solve:** Define the angle using the coil's normal, write flux as a cosine, and differentiate; maximum flux gives zero emf, not maximum emf.

**29. (a)** Briefly describe how the current sensitivity of a moving coil galvanometer can be increased.  
**(b)** A galvanometer shows full scale deflection for current  $I_g$ . A resistance  $R_1$  is required to convert it into a voltmeter of range  $(0 - V)$  and a resistance  $R_2$  to convert it into a voltmeter of range  $(0 - 2V)$ . Find the resistance of the galvanometer. [3 marks]

**Solution:**

**Concept:**

- In a radial magnetic field the magnetic torque on a galvanometer coil is  $NIAB$ . At equilibrium it balances the restoring torque  $C\theta$ , where  $C$  is the torsional constant of the suspension.
- Current sensitivity is the deflection per unit current:  $S_I = \theta/I = NAB/C$ . A galvanometer of resistance  $G$ , connected in series with resistance  $R$ , has full-scale voltage  $V_{fs} = I_g(G + R)$ .

**Step 1: Obtain the current sensitivity** Let the coil have  $N$  turns and area  $A$ , in a radial field of magnitude  $B$ . At steady deflection, the magnetic and restoring torques are equal:

$$NIAB = C\theta \quad \Rightarrow \quad \frac{\theta}{I} = \frac{NAB}{C}.$$

Therefore current sensitivity can be increased by increasing the number of turns, increasing the coil area, strengthening the magnetic field, or decreasing the suspension's torsional constant. A soft iron core helps produce a strong radial field; a suspension with a smaller restoring torque for a given twist increases deflection for the same current. These changes concern current sensitivity, rather than voltage sensitivity.

**Step 2: Write the two full-scale voltage equations** The extra resistance must be connected in series so that the galvanometer carries only its full-scale current  $I_g$  at the selected maximum voltage. The same galvanometer has the same  $I_g$  and resistance  $G$  in both conversions:

$$V = I_g(G + R_1), \quad 2V = I_g(G + R_2).$$

The added resistances are  $R_1$  and  $R_2$ ; neither equals the total voltmeter resistance by itself.

**Step 3: Eliminate the full-scale voltage and current** Divide the second range equation by the first, using their ratio of 2:

$$\begin{aligned} 2 &= \frac{G + R_2}{G + R_1}, \\ 2G + 2R_1 &= G + R_2, \\ G &= R_2 - 2R_1. \end{aligned}$$

The result has units of resistance. Since a real galvanometer has positive resistance, the stated

conversion resistances must satisfy  $R_2 > 2R_1$ .

**Step 4: Conclusion** (a) Increase  $N$ ,  $A$  or  $B$ , or decrease  $C$ , to increase current sensitivity  $NAB/C$ .  
(b) The galvanometer resistance is  $G = R_2 - 2R_1$ .

**Quick Tip to Solve:** When a voltmeter range doubles, its total series resistance doubles; include the galvanometer's own resistance before taking the ratio.

**30(a). (i) Differentiate between 'distance of closest approach' and 'impact parameter'.**

**(ii) Determine the distance of closest approach when an alpha particle of kinetic energy 3.95 MeV approaches a nucleus of  $Z = 79$ , stops and reverses its directions.** [3 marks]

**Solution:**

**Concept:**

- In Rutherford scattering the impact parameter  $b$  is the perpendicular distance from the nucleus to the initial straight-line direction of the incoming particle, far from the nucleus.
- The distance of closest approach is the minimum particle-nucleus separation along the actual trajectory. For a head-on approach, conservation of energy gives  $K = k(2e)(Ze)/r_{\min}$ , taking potential energy as zero at infinity and neglecting nuclear recoil.

**Step 1: Distinguish the two distances** The impact parameter describes the initial aiming offset of the incoming alpha particle: it is measured perpendicular to the incident direction before appreciable deflection occurs. The closest approach is the smallest actual separation between the alpha particle and the nucleus during its deflected motion. Thus the former characterises the incoming trajectory, whereas the latter describes its nearest point to the nucleus and depends on the repulsive interaction and incident kinetic energy. In a head-on collision,  $b = 0$  but the closest approach is nonzero because electrostatic repulsion stops the alpha particle before it reaches the nucleus.

**Step 2: Apply energy conservation at the turning point** The alpha particle stops and reverses direction, so this is a head-on encounter. Its charge is  $+2e$ , and the nuclear charge is  $+Ze = +79e$ . At the turning point the initial kinetic energy is converted completely into electrostatic potential energy:

$$K = \frac{2kZe^2}{r_{\min}} \Rightarrow r_{\min} = \frac{2kZe^2}{K}.$$

Here  $k = 1/(4\pi\epsilon_0)$ . The conversion of all kinetic energy applies to the stated head-on stopping point; an oblique encounter retains tangential motion.

**Step 3: Convert the energy and substitute** Use  $e = 1.6 \times 10^{-19}$  C and  $k = 9 \times 10^9$  N m<sup>2</sup>C<sup>-2</sup>:

$$\begin{aligned} K &= 3.95 \times 10^6 \times 1.6 \times 10^{-19} \\ &= 6.32 \times 10^{-13} \text{ J}, \\ r_{\min} &= \frac{2(9 \times 10^9)(79)(1.6 \times 10^{-19})^2}{6.32 \times 10^{-13}} \\ &= 5.76 \times 10^{-14} \text{ m}. \end{aligned}$$

**Step 4: Conclusion** The impact parameter is the initial perpendicular offset, while the distance of closest approach is the minimum actual separation. For the given head-on alpha particle,  $b = 0$  and  $r_{\min} = 5.76 \times 10^{-14} \text{ m} = 57.6 \text{ fm}$ .

**Quick Tip to Solve:** Use charge  $2e$  for the alpha particle and convert MeV to joules before substitution; zero impact parameter does not mean zero closest distance.

OR

**30(b). (i) State three postulates of Bohr's theory of hydrogen atom.**

**(ii) Find the angular momentum of an electron revolving in the second orbit in Bohr's hydrogen atom.** [3 marks]

**Solution:**

**Concept:**

- Bohr's model permits only certain stationary orbits of an electron around the hydrogen nucleus, with a definite energy associated with each orbit.
- The orbital angular momentum is quantised:  $L = m_e v r = n h / (2\pi) = n \hbar$ , where  $m_e$  is the electron mass,  $v$  its speed,  $r$  the orbital radius and  $n = 1, 2, 3, \dots$
- Radiation of frequency  $\nu$  is emitted or absorbed only in a transition between permitted energy levels; the photon energy is  $h\nu = |E_i - E_f|$ .

**Step 1: State the three postulates for part (i)** First, an electron revolves around the nucleus in certain permitted circular stationary orbits without radiating energy. Each such orbit has a fixed energy; the electrostatic attraction supplies the centripetal force.

Second, only those orbits are allowed for which the electron's angular momentum is an integral multiple of  $h/(2\pi)$ :

$$m_e v r = \frac{n h}{2\pi}, \quad n = 1, 2, 3, \dots$$

Third, radiation is emitted when an electron jumps from a higher energy orbit to a lower energy orbit, and is absorbed for the reverse transition. Its frequency is fixed by the difference in the two orbit energies:

$$h\nu = E_{\text{higher}} - E_{\text{lower}}.$$

Thus the electron does not radiate continuously while remaining in a stationary orbit.

**Step 2: Apply angular momentum quantisation to part (ii)** The second orbit corresponds to  $n = 2$ , not to a second value of electron spin. Hence use orbital angular momentum and substitute Planck's constant  $h = 6.63 \times 10^{-34} \text{ J s}$ :

$$\begin{aligned} L_2 &= \frac{2h}{2\pi} = \frac{h}{\pi}, \\ &= \frac{6.63 \times 10^{-34}}{3.1416} \text{ J s}. \end{aligned}$$

The unit J s is equivalent to  $\text{kg m}^2 \text{ s}^{-1}$ , the SI unit of angular momentum.

**Step 3: Conclusion** Bohr's three postulates are stationary non-radiating orbits, quantised orbital angular momentum, and radiation during transitions with  $h\nu = |\Delta E|$ . In the second orbit,

$$L_2 = 2\hbar = \frac{h}{\pi} \simeq 2.11 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}.$$

**Quick Tip to Solve:** For the angular momentum in the  $n$ th Bohr orbit, use  $L_n = n\hbar$  directly; the orbital radius and speed need not be calculated separately.

### Section D (15 marks)

**31(a).** (i) Explain how free electrons in a metal at constant temperature attain an average velocity under the action of an electric field. Hence obtain an expression for it.

(ii) Consider two conducting wires A and B of the same diameter but made of different materials joined in series across a battery. The number density of electrons in A is 1.5 times that in B. Find the ratio of drift velocity of electrons in wire A to that in wire B. [5 marks]

**Solution:**

**Concept:**

- Drift velocity  $\vec{v}_d$  is the average velocity acquired by free electrons in a conductor under an electric field, superposed on their random thermal motion.
- For an electron of charge  $-e$  and mass  $m_e$ ,  $\vec{F} = -e\vec{E}$  and  $\vec{a} = -e\vec{E}/m_e$ . The relaxation time  $\tau$  is the average time between successive collisions.
- The magnitude of the steady current is  $I = neAv_d$ , where  $n$  is the number of free electrons per unit volume,  $A$  is the wire's cross-sectional area and  $v_d$  is drift speed.

**Step 1: Describe motion without an applied field in part (i)** Free electrons in a metal move rapidly and randomly due to thermal motion and undergo frequent collisions with the lattice. At constant temperature, there is no preferred direction in the absence of an electric field. The average of their random velocity vectors is therefore zero, so thermal motion alone gives no net current.

**Step 2: Average the velocity gained between collisions** When a uniform electric field  $\vec{E}$  is applied, each electron accelerates opposite to the field because its charge is negative. If  $\vec{u}_i$  is its random velocity just after its last collision and  $t_i$  is the time since that collision, its velocity immediately before observation is

$$\vec{v}_i = \vec{u}_i - \frac{e\vec{E}}{m_e}t_i.$$

Collisions randomise the electron's motion, preventing indefinite acceleration in one direction. In the steady state, averaging over the large number of electrons gives  $\langle \vec{u}_i \rangle = 0$  and  $\langle t_i \rangle = \tau$ . Thus

$$\begin{aligned} \vec{v}_d = \langle \vec{v}_i \rangle &= \langle \vec{u}_i \rangle - \frac{e\vec{E}}{m_e} \langle t_i \rangle \\ &= -\frac{e\tau}{m_e} \vec{E}. \end{aligned}$$

The drift speed is  $v_d = eE\tau/m_e$ . A steady average velocity is attained because the directional velocity gained from the field between collisions is repeatedly randomised. At constant temperature the material's relaxation time is taken as constant in the usual ohmic regime.

**Step 3: Relate drift speed to the series current in part (ii)** In a time interval  $\Delta t$ , electrons drifting through a cross section occupy a cylinder of length  $v_d \Delta t$  and volume  $Av_d \Delta t$ . This volume contains  $nAv_d \Delta t$  free electrons, so the magnitude of charge crossing is

$$\Delta Q = neAv_d \Delta t, \quad I = \frac{\Delta Q}{\Delta t} = neAv_d.$$

The two wires are joined in series, so their steady currents are equal by conservation of charge. They have the same diameter and therefore the same cross-sectional area. The electron charge is also the same in both materials.

**Step 4: Substitute the electron densities** Write the equal-current condition for wires A and B and cancel the common area and electron charge:

$$n_A e A v_{dA} = n_B e A v_{dB},$$

$$\frac{v_{dA}}{v_{dB}} = \frac{n_B}{n_A} = \frac{n_B}{1.5n_B} = \frac{2}{3}.$$

The wire containing more free electrons per unit volume needs a smaller drift speed to carry the same current. Different materials may have different relaxation times and electric fields; these differences do not alter the equal-current relation used here.

**Step 5: Conclusion** The average electron velocity is  $\vec{v}_d = -(e\tau/m_e)\vec{E}$ , opposite to the electric field, with magnitude  $eE\tau/m_e$ . For the series wires of equal diameter,  $v_{dA} : v_{dB} = 2 : 3$ .

**Quick Tip to Solve:** In series wires first set the currents equal. Then use  $v_d \propto 1/(nA)$ ; equal diameters cancel the areas, leaving the inverse ratio of electron densities.

OR

**31(b).** (i) A cell of emf ( $E$ ) and internal resistance ( $r$ ) is connected across a variable load resistance ( $R$ ). Draw plots showing the variation of terminal voltage  $V$  with (i)  $R$  and (ii) the current ( $I$ ) in the load.

(ii) Three cells, each of emf  $E$  but internal resistances  $2r$ ,  $3r$  and  $6r$  are connected in parallel across a resistor  $R$ . Obtain expressions for (i) current flowing in the circuit, and (ii) the terminal potential difference across the equivalent cell. [5 marks]

**Solution:**

**Concept:**

- A cell of emf  $E$  and internal resistance  $r$ , supplying current  $I$ , has terminal voltage  $V = E - Ir$ . Across an external resistance  $R$ , Ohm's law gives  $V = IR$ .
- For cells joined in parallel with like terminals together, all branches have the same terminal voltage. Kirchhoff's junction rule gives the load current as the sum of the branch currents.
- Cells of equal emf  $E$  in parallel have equivalent emf  $E$ ; their internal resistances combine in

parallel, increasing the available current without adding the emfs.

**Step 1: (i) Express terminal voltage in terms of load resistance** Applying the loop rule to the cell and the load gives  $E = I(R + r)$ . Thus

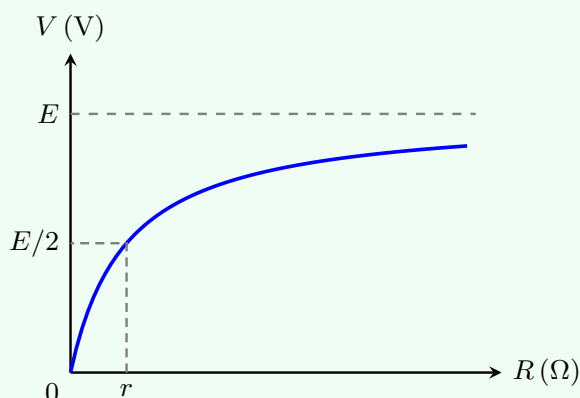
$$I = \frac{E}{R + r}, \quad V = IR = \frac{ER}{R + r}.$$

For a short circuit,  $R = 0$  and  $V = 0$ . As  $R$  increases, the load current and the internal voltage drop decrease, so the terminal voltage rises. At  $R = r$ , it is  $E/2$ . The rise becomes progressively slower, and  $V$  approaches  $E$  as  $R \rightarrow \infty$ . Therefore the  $V$ -against- $R$  graph is an increasing, concave-down curve through the origin, with horizontal asymptote  $V = E$ .

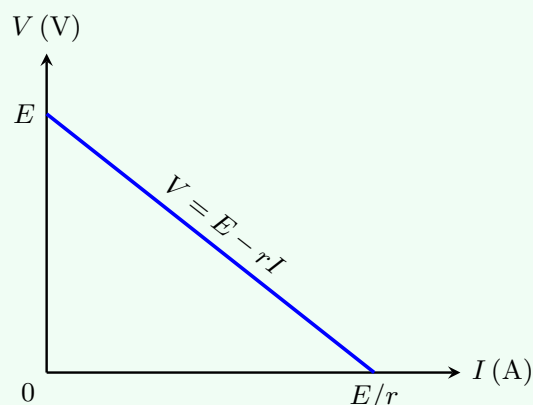
**Step 2: (i) Express terminal voltage in terms of load current** Rearranging the loop rule gives

$$V = E - rI.$$

This is a straight line of negative slope  $-r$ . At open circuit,  $I = 0$  and  $V = E$ ; at short circuit,  $V = 0$  and  $I = E/r$ . For a passive load, the relevant part of the graph lies between these intercepts. These limiting cases also agree with the resistance plot.



Variation with  $R$



Variation with  $I$

*Terminal voltage for a cell with internal resistance  $r > 0$ : variation with load resistance and load current.*

**Step 3: (ii) Find the equivalent cell** Assume the usual parallel arrangement with all positive terminals joined together and all negative terminals joined together. Since the three emfs are equal, their equivalent emf is  $E_{\text{eq}} = E$ . Denote the equivalent internal resistance by  $r_{\text{eq}}$ . Then

$$\frac{1}{r_{\text{eq}}} = \frac{1}{2r} + \frac{1}{3r} + \frac{1}{6r} = \frac{3 + 2 + 1}{6r} = \frac{1}{r}.$$

Thus the combination behaves as a single cell of emf  $E$  and internal resistance  $r$ . Equivalently, if  $V$  is the common terminal voltage, the three branch currents are  $(E - V)/(2r)$ ,  $(E - V)/(3r)$  and  $(E - V)/(6r)$ ; their sum is  $I = (E - V)/r$ .

**Step 4: (ii) Apply the loop rule to the equivalent circuit** The external resistance  $R$  is in series with

the equivalent internal resistance. Hence

$$E = I(R + r_{\text{eq}}), \quad I = \frac{E}{R + r}.$$

The potential difference across the equivalent cell is the voltage across the load, rather than its emf alone. Using either Ohm's law or the internal drop,

$$V = IR = E - Ir = E - \frac{Er}{R + r} = \frac{ER}{R + r}.$$

The load current is the sum of the currents supplied by all three cells; it is not the current through any one branch.

**Step 5: Conclusion** In (i),  $V = ER/(R + r)$  rises from zero towards  $E$ , while  $V = E - rI$  is a straight line joining  $(0, E)$  to  $(E/r, 0)$ . In (ii),  $E_{\text{eq}} = E$ ,  $r_{\text{eq}} = r$ , the load current is  $I = E/(R + r)$ , and the terminal potential difference is  $V = ER/(R + r)$ .

**Quick Tip to Solve:** Check both plots using open circuit ( $I = 0, V = E$ ) and short circuit ( $R = 0, V = 0$ ). For equal-emf cells in parallel, keep the emf unchanged and combine only their internal resistances in parallel.

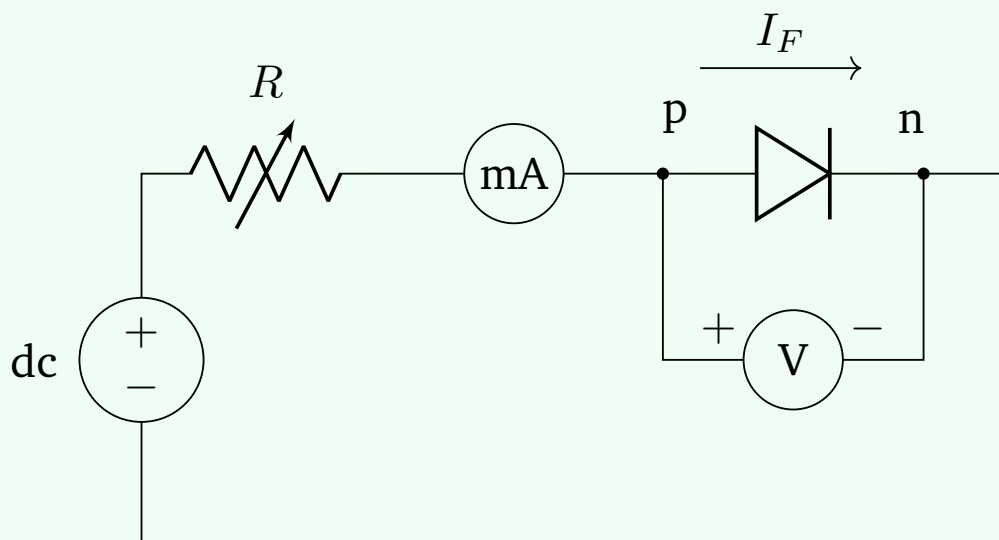
**32(a).** Draw the circuit arrangement for studying V-I characteristics of a p-n junction diode in (i) forward biasing and (ii) reverse biasing. Draw the typical V-I characteristics of a silicon diode. Describe briefly the following terms: (i) minority carrier injection in forward biasing and (ii) breakdown voltage in reverse biasing. [5 marks]

**Solution:**

**Concept:**

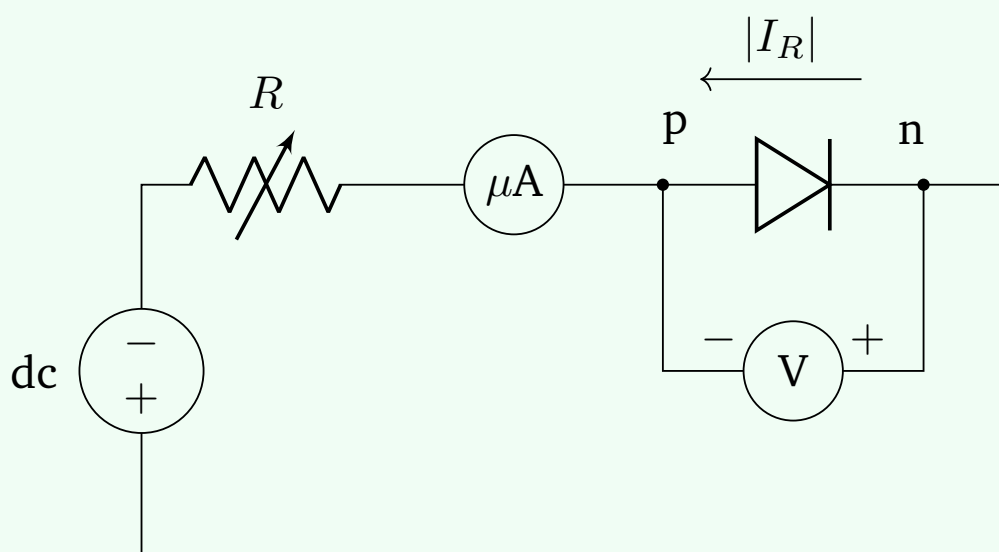
- Forward bias connects the p-side to the positive terminal and the n-side to the negative terminal of a dc supply; it lowers the junction potential barrier and narrows the depletion layer.
- Reverse bias connects the p-side to the negative terminal and the n-side to the positive terminal; it raises the barrier and widens the depletion layer. A small minority-carrier current flows before breakdown.
- To obtain a diode characteristic, measure the current through the diode with a series ammeter and the potential difference across it with a parallel voltmeter, varying the applied dc voltage.

**Step 1: Set up the forward-bias circuit** Connect the diode's p-side (anode) towards the positive supply terminal and its n-side (cathode, marked by the bar) towards the negative terminal. Put a rheostat  $R$  and a milliammeter in series with the diode. Connect a voltmeter across the diode, with its positive terminal at the p-side. Starting at zero applied voltage, vary the rheostat and record corresponding diode voltage and current readings. The rheostat limits the current and protects the diode when its resistance becomes small.



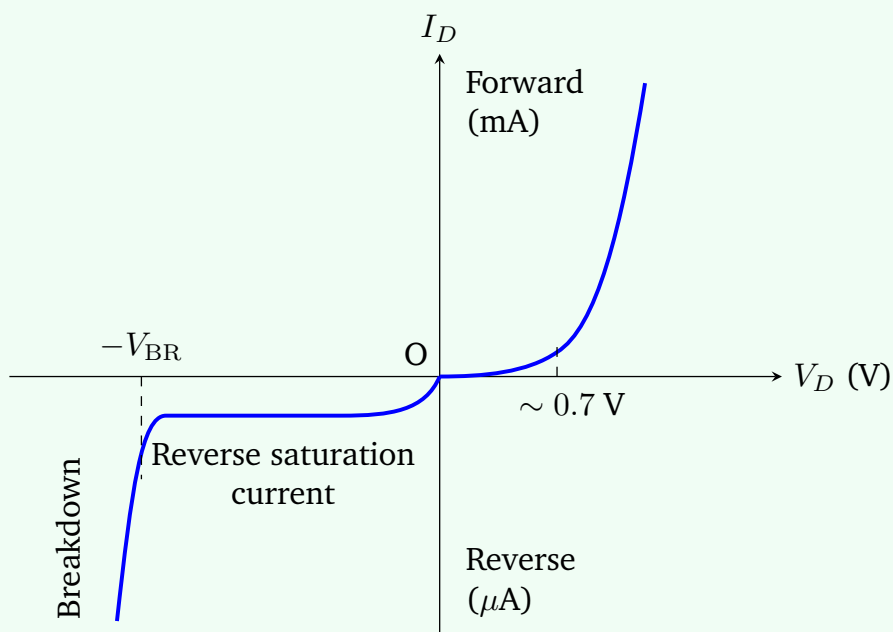
*Forward bias: series milliammeter and rheostat; voltmeter across the diode.*

**Step 2: Reverse the supply to study reverse bias** Keep the diode orientation fixed but reverse the supply connections, so that the n-side is positive and the p-side is negative. Use a microammeter in series because the reverse current is much smaller than the normal forward current. Reverse the voltmeter leads so that its positive terminal is at the n-side. Vary the applied voltage and record the reverse voltage and current; plot these with negative signs on the combined characteristic. Use a suitable current-limiting resistance and do not exceed the diode's safe ratings.



*Reverse bias: supply and voltmeter polarities reversed; microammeter in series.*

**Step 3: Draw and explain the silicon diode characteristic** Take diode voltage  $V_D = V_p - V_n$  on the horizontal axis and conventional current from p to n as positive on the vertical axis. In forward bias the current is initially small. Near the knee, approximately 0.7 V for silicon, it starts increasing rapidly for a small further increase in voltage. This is an approximate turn-on voltage, not an exact threshold below which the current is identically zero. In reverse bias a very small, nearly constant reverse saturation current flows until the reverse voltage reaches the breakdown value. The sketch uses different current scales for the two branches: milliamperes forward and microamperes reverse.



*Qualitative silicon characteristic; forward and reverse current scales differ.*

**Step 4: Explain minority carrier injection** The reduced barrier in forward bias allows majority carriers to diffuse across the junction: electrons move from the n-region into the p-region, and holes move from the p-region into the n-region. After crossing, the electrons are minority carriers in the p-region and the holes are minority carriers in the n-region. Their concentrations near the junction therefore rise above their equilibrium values. This introduction of excess minority carriers is called minority carrier injection. The injected carriers diffuse further into the neutral regions and recombine with majority carriers; the source continuously replenishes the carriers.

**Step 5: Define reverse breakdown voltage** The breakdown voltage  $V_{BR}$  is the magnitude of the reverse bias at which the reverse current increases sharply with only a small further change in voltage. The strong electric field can produce Zener breakdown in a heavily doped junction or avalanche multiplication in a more lightly doped junction. On the signed characteristic the breakdown point lies at  $V_D = -V_{BR}$ . Current must be limited externally to avoid excessive heating; breakdown does not by itself imply that the junction has been destroyed.

**Step 6: Conclusion** The forward circuit uses p-positive bias and a milliammeter; the reverse circuit uses n-positive bias and a microammeter. Silicon has an approximate forward knee at  $0.7 V$ . Forward bias injects excess minority carriers across the junction, while reverse breakdown at  $V_{BR}$  produces a sharp rise in reverse current.

**Quick Tip to Solve:** Mark the p and n ends first, then set the supply polarity. Use mA for forward current and microamperes for reverse current, and label the reverse breakdown point on the negative-voltage axis.

**32(b).** Name two important processes involved in the formation of a p-n junction diode. With the help of a circuit diagram, explain the working of junction diode as a full wave rectifier. Draw its input and output waveforms. State the characteristic property of a junction diode that makes it suitable for rectification. [5 marks]

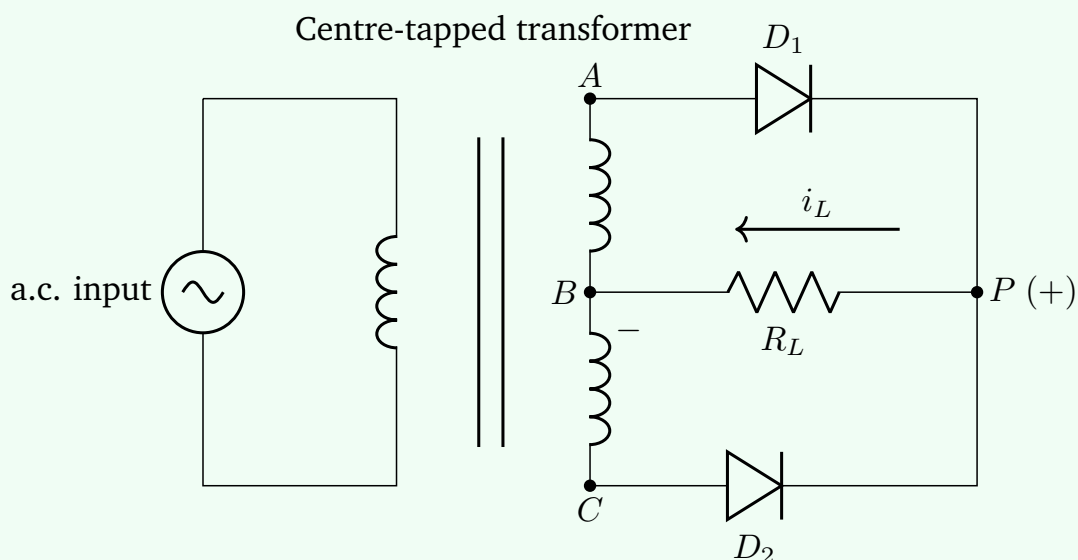
**Solution:**

**Concept:**

- A p-n junction forms through diffusion of majority carriers and drift of charge carriers in the electric field of the depletion region.
- A diode offers low resistance when forward biased and very high resistance when reverse biased, so its conduction is strongly directional.
- A centre-tapped full-wave rectifier uses two diodes on alternate half-cycles. For ideal diodes, if one half-secondary voltage is  $v_{AB} = V_m \sin \omega t$ , the load voltage is  $v_o = V_m |\sin \omega t|$ .

**Step 1: Name and explain the junction-formation processes** The two important processes are **diffusion and drift**. Because of the concentration difference, electrons diffuse from the n-side into the p-side and holes diffuse from the p-side into the n-side. Recombination near the junction leaves fixed positive donor ions on the n-side and fixed negative acceptor ions on the p-side. This produces a depletion region and an electric field directed from the n-side to the p-side. The field drives a drift current opposite to the diffusion current. At equilibrium the two currents balance, giving zero net current through the unbiased junction.

**Step 2: Arrange the full-wave rectifier circuit** Connect the ends  $A$  and  $C$  of a centre-tapped transformer secondary to the anodes of diodes  $D_1$  and  $D_2$ , respectively. Join their cathodes at  $P$ , and connect the load resistance  $R_L$  between  $P$  and the centre tap  $B$ . An a.c. supply is connected to the primary. The two equal secondary halves have opposite instantaneous voltages relative to  $B$ :  $v_{CB} = -v_{AB}$ . The load polarity shown is positive at  $P$ .



*Centre-tapped full-wave rectifier; the joined cathodes at  $P$  form the positive output terminal.*

**Step 3: Explain conduction in the first half-cycle** When  $A$  is positive relative to  $B$ ,  $C$  is negative relative to  $B$ . Thus  $D_1$  is forward biased and conducts, whereas  $D_2$  is reverse biased and blocks. The conventional current path is  $A \rightarrow D_1 \rightarrow P \rightarrow R_L \rightarrow B$ , completing the circuit through the upper half

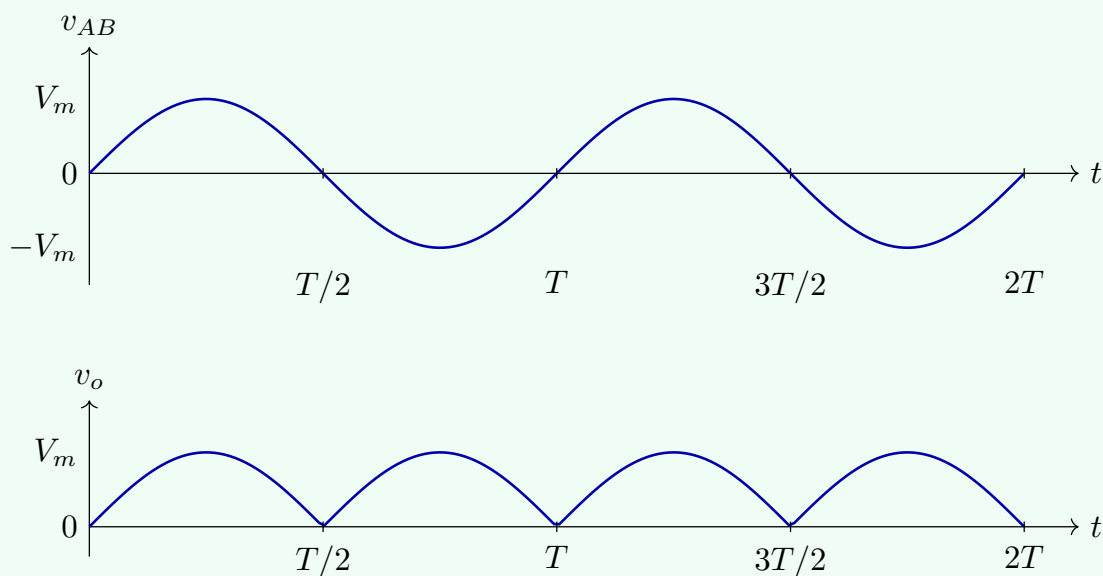
of the secondary. The current through the load flows from  $P$  towards  $B$ .

**Step 4: Explain conduction in the next half-cycle** When the supply reverses,  $C$  becomes positive and  $A$  becomes negative relative to  $B$ . Now  $D_2$  conducts and  $D_1$  blocks. Current follows  $C \rightarrow D_2 \rightarrow P \rightarrow R_L \rightarrow B$ , returning through the lower half of the secondary. The load current again flows from  $P$  towards  $B$ . Both input half-cycles therefore contribute to an output of the same polarity; the output is pulsating d.c., not a constant voltage.

**Step 5: Draw and interpret the input and output waveforms** The input waveform shown is the sinusoidal voltage  $v_{AB}$  of one half of the secondary; the primary input has the same sinusoidal shape and frequency. Neglecting diode voltage drops, the negative input half-cycles are inverted at the load:

$$v_o = V_m |\sin \omega t|, \quad i_L = \frac{v_o}{R_L}.$$

If the input period is  $T$  and its frequency is  $f = 1/T$ , the output pulses repeat every  $T/2$ , so the ripple frequency is  $2f$ . In a practical silicon-diode circuit, the forward diode drop slightly reduces the output peaks. Rectification is possible because the diode conducts readily under forward bias and offers very high resistance under reverse bias.



*Sinusoidal input and ideal full-wave output;  $T$  is the input period.*

**Step 6: Conclusion** Junction formation involves **diffusion and drift**. In the full-wave rectifier,  $D_1$  and  $D_2$  conduct on alternate half-cycles while the load current always flows from  $P$  to  $B$ . The output is unidirectional pulsating d.c. with frequency  $2f$ , enabled by the diode's low forward and high reverse resistance.

**Quick Tip to Solve:** Trace the load current in both half-cycles: both paths must enter the same end of the load, even though different diodes conduct.

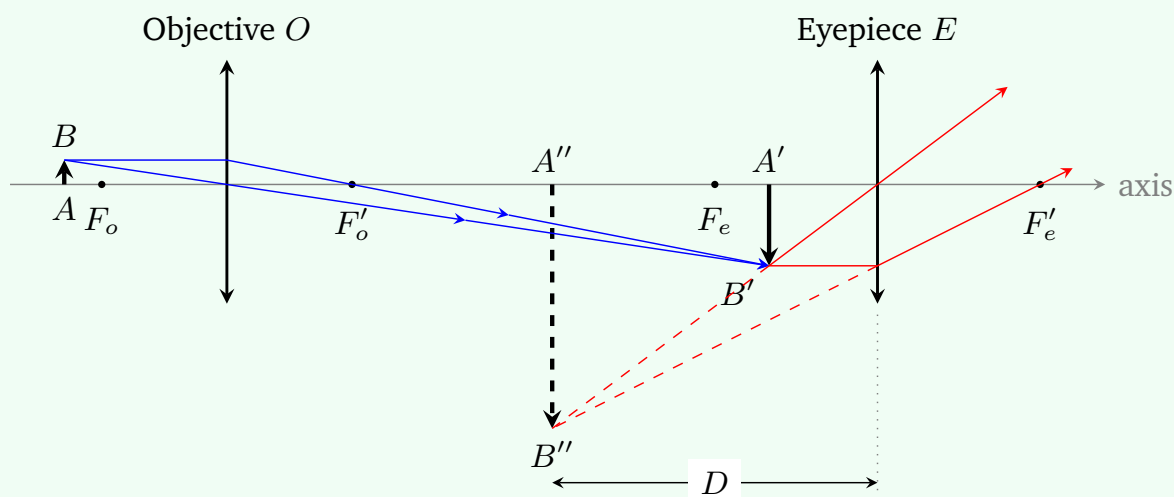
**33(a)(i).** Draw a ray diagram to show the working of a compound microscope. Obtain the expression for the total magnification for the final image to be formed at the near point. [3 marks]

**Solution:**

**Concept:**

- A compound microscope uses a short-focal-length objective to form a real enlarged intermediate image, which the eyepiece views as a simple microscope.
- The total angular magnification is  $M = m_o M_e$ , where  $m_o = v_o/u_o$  is the signed linear magnification of the objective and  $M_e$  is the eyepiece's angular magnification relative to unaided viewing at  $D = 25$  cm.
- For either thin lens,  $1/f = 1/v - 1/u$ , with Cartesian signs and light travelling from left to right. A virtual final image at the near point has  $v_e = -D$ .

**Step 1: Trace the image formation** Place the object  $AB$  just beyond the first focal point of the objective  $O$ . The objective forms a real, inverted and enlarged image  $A'B'$ . Adjust the eyepiece  $E$  so that  $A'B'$  lies between its first focus  $F_e$  and the eyepiece. The rays leaving the eyepiece diverge; their backward extensions meet at  $B''$ , giving the virtual enlarged image  $A''B''$  at the near point. The final image remains inverted relative to  $AB$ . In the schematic, solid arrows show light propagation and dashed lines show backward extensions.



*Compound microscope with final image at the near point (schematic, not to scale).*

**Step 2: Derive the eyepiece magnification at the near point** Write  $u_e = -s$ , where  $s$  is the positive distance of the intermediate image from the eyepiece. With the eye close to the eyepiece and  $v_e = -D$ , its lens equation gives

$$\frac{1}{f_e} = -\frac{1}{D} + \frac{1}{s}, \quad \frac{D}{s} = 1 + \frac{D}{f_e}.$$

If the intermediate image has height magnitude  $h'$ , it subtends the small angle  $\beta \simeq h'/s$ . Viewed directly at the near point, the same height subtends  $\alpha_0 \simeq h'/D$ . Hence

$$M_e = \frac{\beta}{\alpha_0} = \frac{D}{s} = 1 + \frac{D}{f_e}.$$

**Step 3: Combine the objective and eyepiece factors** The objective's image is inverted, so  $m_o =$

$v_o/u_o < 0$ . Multiplying the increase in image height by the eyepiece's angular factor gives

$$M = \frac{v_o}{u_o} \left( 1 + \frac{D}{f_e} \right), \quad |M| = \frac{v_o}{|u_o|} \left( 1 + \frac{D}{f_e} \right).$$

In the usual microscope approximation,  $|u_o| \simeq f_o$  and  $v_o \simeq L$ , where  $L$  is the effective tube length. Thus the familiar approximate magnifying power is

$$|M| \simeq \frac{L}{f_o} \left( 1 + \frac{D}{f_e} \right).$$

The exact objective image distance should be calculated when an actual object distance is supplied.

**Step 4: Conclusion** For a final image at the near point, the signed total magnification is  $M = (v_o/u_o)(1 + D/f_e)$ . Its magnitude is approximately  $|M| = (L/f_o)(1 + D/f_e)$ . The final image is virtual, enlarged and inverted.

**Quick Tip to Solve:** For near-point viewing, multiply the objective magnification by  $1 + D/f_e$ ; the extra 1 distinguishes this result from relaxed-eye viewing at infinity.

**33(a)(ii).** In a compound microscope an object is placed at a distance of 1.5 cm from the objective of focal length 1.25 cm. If the eye-piece has a focal length of 5 cm and the final image is formed at the near point, find the magnifying power of the microscope. [2 marks]

**Solution:**

**Concept:**

- A compound microscope uses a short-focal-length objective to form a real enlarged intermediate image, which the eyepiece views as a simple microscope.
- The total angular magnification is  $M = m_o M_e$ , where  $m_o = v_o/u_o$  is the signed linear magnification of the objective and  $M_e$  is the eyepiece's angular magnification relative to unaided viewing at  $D = 25$  cm.
- For either thin lens,  $1/f = 1/v - 1/u$ , with Cartesian signs and light travelling from left to right. A virtual final image at the near point has  $v_e = -D$ .

**Step 1: Calculate the objective image distance** Here  $u_o = -1.5$  cm and  $f_o = +1.25$  cm. Therefore

$$\begin{aligned} \frac{1}{v_o} &= \frac{1}{f_o} + \frac{1}{u_o} = \frac{1}{1.25} - \frac{1}{1.5} \\ &= \frac{4}{5} - \frac{2}{3} = \frac{2}{15} \text{ cm}^{-1}, \\ v_o &= 7.5 \text{ cm}, \quad m_o = \frac{7.5}{-1.5} = -5. \end{aligned}$$

The positive image distance confirms that the objective produces a real image on its opposite side.

**Step 2: Calculate the total magnifying power** For  $f_e = 5$  cm and the final image at the normal near point,

$$M_e = 1 + \frac{25}{5} = 6, \quad M = (-5)(6) = -30.$$

The negative sign denotes inversion. Magnifying power quoted as a positive magnitude is therefore 30, a dimensionless ratio of angular sizes.

**Step 3: Conclusion** The microscope has magnifying power  $\boxed{30}$  (signed angular magnification  $M = -30$ ); its final image is inverted relative to the object.

**Quick Tip to Solve:** When the objective object distance is given, find its image distance from the lens formula before multiplying by  $1 + D/f_e$ ; do not replace it by an unspecified tube length.

OR

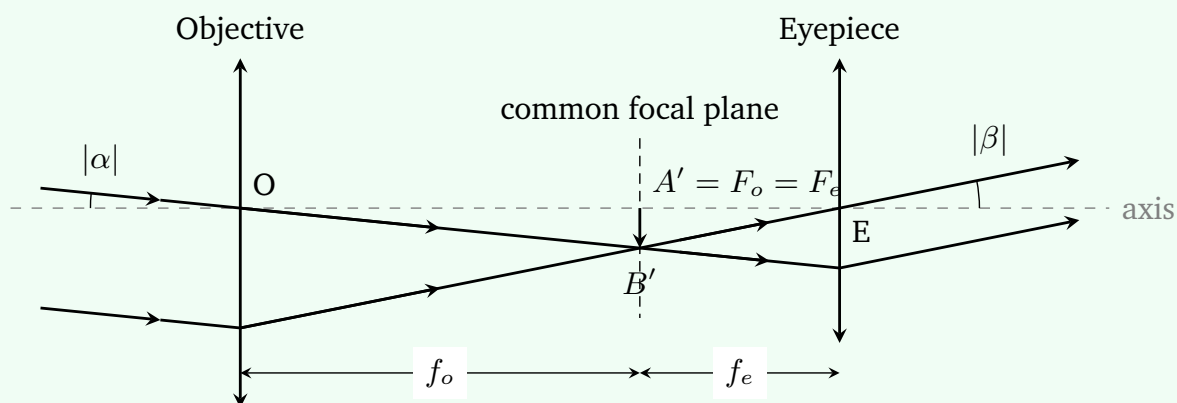
**33(b)(i).** Draw a ray diagram for the formation of image of an object by an astronomical telescope, in normal adjustment. Obtain the expression for its magnifying power. [5 marks total]

**Solution:**

**Concept:**

- An astronomical telescope has a converging objective of long focal length  $f_o$  and a converging eyepiece of shorter focal length  $f_e$ .
- In normal adjustment, the objective's real image lies in the eyepiece's first focal plane. The final image is at infinity and the separation is  $L = f_o + f_e$ .
- Angular magnifying power is  $M = \beta/\alpha$ , where  $\alpha$  is the angle subtended by the distant object at the unaided eye and  $\beta$  is the angle subtended by the final image.

**Step 1: Trace the rays in normal adjustment** Parallel rays from a distant off-axis object point reach objective O, forming the real inverted image  $A'B'$  at its second focal plane. This plane coincides with the eyepiece's first focal plane:  $F_o = F_e = A'$ . Rays from  $B'$  leave eyepiece E parallel, producing a final inverted image at infinity for the relaxed eye.



*Astronomical telescope in normal adjustment; common focal plane at  $A'B'$ , with incident and emergent parallel beams. Schematic, not to scale.*

**Step 2: Derive the angular magnifying power** Let  $h = A'B'$  be the positive magnitude of the intermediate image height. The objective triangle and the eyepiece triangle give the angular magnitudes

$$\tan |\alpha| = \frac{h}{f_o}, \quad \tan |\beta| = \frac{h}{f_e}.$$

For paraxial rays,  $\tan \theta \simeq \theta$ . Dividing eliminates the intermediate image height:

$$|M| = \frac{|\beta|}{|\alpha|} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}.$$

The final image is inverted relative to the distant object. Consequently, the signed angular magnification is  $M = -f_o/f_e$ ; the minus sign describes inversion, not a negative focal length.

**Step 3: Conclusion** In normal adjustment, the signed angular magnification is  $M = -f_o/f_e$ , and the magnifying power is  $|M| = f_o/f_e$ . The final image is inverted and at infinity.

**Quick Tip to Solve:** The common focal plane makes the emergent rays parallel; divide the eyepiece angular size  $h/f_e$  by the objective angular size  $h/f_o$ .

**33(b)(ii).** The magnifying power of an astronomical telescope in normal adjustment is 2.9 and the objective and the eyepiece are separated by a distance of 150 cm. Find the focal lengths of the two lenses.

**Solution:**

**Concept:**

- An astronomical telescope has a converging objective of long focal length  $f_o$  and a converging eyepiece of shorter focal length  $f_e$ .
- In normal adjustment, the objective's real image lies in the eyepiece's first focal plane. The final image is at infinity and the separation is  $L = f_o + f_e$ .
- Angular magnifying power is  $M = \beta/\alpha$ , where  $\alpha$  is the angle subtended by the distant object at the unaided eye and  $\beta$  is the angle subtended by the final image.

**Step 1: Form equations for the focal lengths** The quoted magnifying power 2.9 is its magnitude. Both lenses are converging, so both focal lengths are positive. Normal adjustment supplies the two independent relations

$$\frac{f_o}{f_e} = 2.9, \quad f_o + f_e = 150 \text{ cm}.$$

Substitute  $f_o = 2.9f_e$  into the length equation to obtain  $3.9f_e = 150 \text{ cm}$ .

**Step 2: Evaluate both focal lengths and check** Solving for the eyepiece and then the objective gives

$$f_e = \frac{150}{3.9} \text{ cm} \simeq 38.46 \text{ cm},$$
$$f_o = 2.9 \left( \frac{150}{3.9} \right) \text{ cm} \simeq 111.54 \text{ cm}.$$

The rounded focal lengths add to 150.00 cm, and their ratio is approximately 2.9. The objective has the longer focal length, as required for angular enlargement.

**Step 3: Conclusion** The required focal lengths are  $f_o \simeq 111.54 \text{ cm}$  for the objective and  $f_e \simeq 38.46 \text{ cm}$  for the eyepiece.

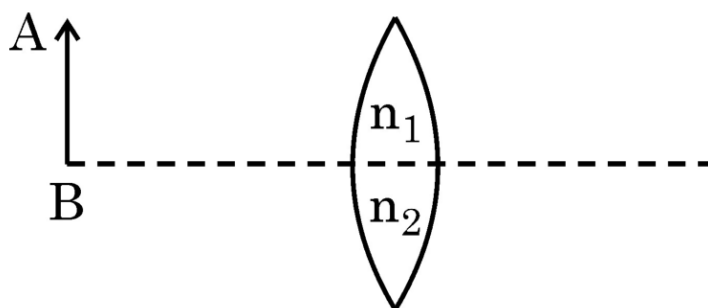
**Quick Tip to Solve:** Use the ratio and sum together:  $f_e = L/(|M| + 1)$  and  $f_o = |M|L/(|M| + 1)$ .

### Section E (8 marks)

**Note :** Questions number 34 and 35 are Case Study based questions. Read the following paragraph and answer the questions.

**34.** A lens is a transparent optical medium bounded by two surfaces; at least one of which should be spherical. Considering image formation by a single spherical surface successively at the two surfaces of a lens, lens maker's formula is obtained. It is useful to design lenses of desired focal length using surfaces of suitable radii of curvature. This formula helps us obtain a relation between  $u$ ,  $v$  and  $f$  for a lens. Lenses form images of objects and they are used in a number of optical devices, for example microscopes and telescopes. [4 marks]

**34(i).** An object AB is kept in front of a composite convex lens, as shown in figure. Will the lens produce one image? If not, explain. [1 mark]



#### Solution:

##### Concept:

- Every illuminated part of a lens receives rays from the whole object and can form a complete image.
- For fixed surface curvatures in air, lens power is  $1/f = (n - 1)(1/R_1 - 1/R_2)$ ; it depends on the refractive index  $n$ .

**Step 1: Compare the two lens portions** The upper and lower portions have different refractive indices,  $n_1$  and  $n_2$ . Although their bounding surfaces have the same curvature, their powers, and hence their focal lengths  $f_1$  and  $f_2$ , differ.

**Step 2: Apply the imaging condition separately** Both portions have the same object distance  $u$ , but each obeys  $1/v_j = 1/f_j + 1/u$ . Different focal lengths therefore give different image distances. Each portion forms the whole image of AB; it does not form only the corresponding half of the object.

**Step 3: Conclusion** No. For the two different refractive indices shown, the composite lens forms two complete images at different positions.

**Quick Tip to Solve:** Compare refractive indices before treating a composite lens as a single focal-length lens.

**34(ii).** A real image of an object formed by a convex lens is observed on a screen. If the screen is removed, will the image still be formed? Explain. [1 mark]

#### Solution:

##### Concept:

- A real image forms where rays refracted by the lens actually converge.

- A screen intercepts and scatters light at the image plane, making the image visible on its surface; it does not cause the convergence.

**Step 1: Identify what determines the image position** The convex lens refracts rays from each object point so that they meet at its corresponding image point. The object distance and the lens focal length determine this position through the lens formula, independently of whether a screen is present.

**Step 2: Remove the screen without changing the lens** Removing the screen changes neither the incident rays nor their refraction. The rays still meet at the same image points and then diverge. An eye placed suitably beyond the image can receive these rays and see the real image; only the image displayed on the screen disappears.

**Step 3: Conclusion** Yes. The real image still forms at the same position because convergence of the refracted rays is independent of the screen.

**Quick Tip to Solve:** Separate image formation by convergence from image display by scattering at a screen.

**34(iii).** A double convex lens is made of glass of refractive index 1.55 with both faces of the same radius of curvature. Find the radius of curvature required if focal length is 20 cm. [2 marks]

**Solution:**

**Concept:**

- For a thin lens in air, lens maker's formula is  $1/f = (n - 1)(1/R_1 - 1/R_2)$ , where  $n$  is the refractive index of its material and  $R_1, R_2$  are signed surface radii.
- With light travelling from left to right, an equiconvex lens has  $R_1 = +R$  and  $R_2 = -R$ , where  $R$  is the common positive radius magnitude.

**Step 1: Insert the signs of the two curvatures** The centre of curvature of the first face lies to its right, while that of the second face lies to its left. Equal curvature magnitudes therefore do not mean equal signed radii. Substituting their signs gives

$$\frac{1}{f} = (n - 1) \left( \frac{1}{R} - \frac{1}{-R} \right) = \frac{2(n - 1)}{R}.$$

**Step 2: Calculate the required radius** Rearrange to  $R = 2(n - 1)f$ . The given converging lens has  $f = +20$  cm and  $n = 1.55$ , so

$$R = 2(1.55 - 1)(20 \text{ cm}) = 22 \text{ cm}.$$

Keeping the focal length in centimetres gives the radius in centimetres too. As a check,  $2(0.55)/22 = 1/20$ , reproducing the specified reciprocal focal length.

**Step 3: Conclusion** Each face requires radius of curvature of magnitude 22 cm; the signed radii are  $R_1 = +22$  cm and  $R_2 = -22$  cm.

**Quick Tip to Solve:** For an equiconvex lens in air, use  $R = 2(n - 1)f$ , with opposite signs for the two signed surface radii.

**34(iii).** Two convex lenses A and B of focal lengths 15 cm and 10 cm respectively are placed coaxially 'd' distance apart. A point object is kept at a distance of 30 cm in front of lens A. Find the value of 'd' so that the rays emerging from lens B are parallel to its principal axis. [2 marks]

**Solution:**

**Concept:**

- For a thin lens,  $1/f = 1/v - 1/u$ , where distances use the Cartesian sign convention and the positive direction is the direction of incident light.
- A convex lens produces parallel emergent rays when the incident rays diverge from its first principal focus, so its object distance must be  $u = -f$ .

**Step 1: Locate the image formed by lens A** Here  $f_A = +15$  cm and  $u_A = -30$  cm. Applying the lens formula,

$$\frac{1}{v_A} = \frac{1}{15} - \frac{1}{30} = \frac{1}{30} \text{ cm}^{-1}.$$

Thus A forms a real image 30 cm to its right. This also follows because the object is at twice its focal length.

**Step 2: Place lens B relative to the intermediate image** This image acts as the object for B. For parallel emergence, it must be at B's front focus, 10 cm to the left of B. If B is  $d$  to the right of A, its object distance is  $u_B = 30 - d$ . Hence,

$$30 - d = -10 \text{ cm}.$$

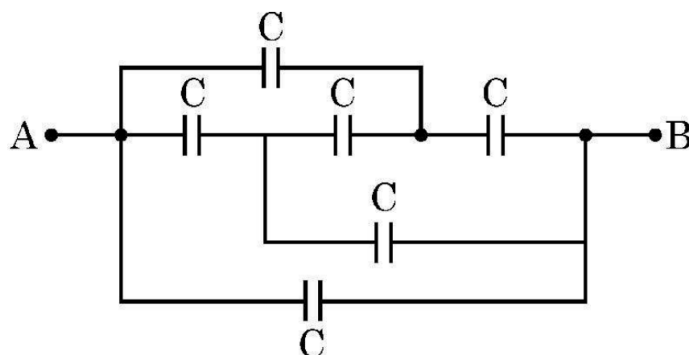
B must therefore lie beyond the image formed by A, so the rays first converge and then diverge before entering B.

**Step 3: Conclusion** The required lens separation is  $d = 40$  cm.

**Quick Tip to Solve:** First locate the image of A, then place B one focal length beyond that image to obtain parallel emergent rays.

**35.** A capacitor is a system of two conductors separated by an insulator. The two conductors have equal and opposite charges with a potential difference between them. The capacitance of a capacitor depends on the geometrical configuration (shape, size and separation) of the system and also on the nature of the insulator separating the two conductors. They are used to store charges. Like resistors, capacitors can be arranged in series or parallel or a combination of both to obtain desired value of capacitance. [4 marks]

**35(i).** Find the equivalent capacitance between points A and B in the given diagram. [1 mark]



**Solution:**

**Concept:**

- A balanced capacitor bridge has equal potentials at its two internal junctions, so the capacitor joining those junctions carries no charge.
- Two equal capacitors  $C$  in series have capacitance  $C/2$ ; parallel branch capacitances add.

**Step 1: Identify the balanced bridge** Call the two internal junctions on the horizontal line X and Y. Each is connected to A through one capacitor  $C$  and to B through another capacitor  $C$ . By symmetry,  $V_X = V_Y$ . The middle capacitor between X and Y therefore has zero potential difference and may be omitted when finding the equivalent capacitance.

**Step 2: Combine the remaining branches** The paths A–X–B and A–Y–B each contain two capacitors  $C$  in series. Each path contributes  $C/2$ . The bottom capacitor  $C$  is directly across A and B and is in parallel with these paths:

$$C_{\text{eq}} = \frac{C}{2} + \frac{C}{2} + C.$$

**Step 3: Conclusion** The equivalent capacitance is  $C_{\text{eq}} = 2C$ .

**Quick Tip to Solve:** Check the two internal junction potentials before combining capacitors; the balanced middle branch can be removed.

**35(ii).** A dielectric slab is inserted between the plates of a parallel plate capacitor. The electric field between the plates decreases. Explain. [1 mark]

**Solution:**

**Concept:**

- An applied electric field polarises a dielectric, inducing bound charges on its opposite surfaces.
- The field  $E_p$  of these bound charges opposes the original field  $E_0$ , reducing the net field for fixed free charge on the plates.

**Step 1: Explain dielectric polarisation** When the slab enters the capacitor, its molecules acquire or align electric dipole moments. The slab face near the positive plate develops negative bound charge, while the opposite face develops positive bound charge.

**Step 2: Add the fields with their directions** These bound charges produce a field opposite to the field of the capacitor plates. Consequently, the resultant field is  $E = E_0 - E_p$ . The stated decrease corresponds to unchanged free charge on the plates, as for an isolated charged capacitor. If a battery instead held the voltage fixed, the uniform field of a fully filled capacitor would remain  $V/d$ .

**Step 3: Conclusion** Dielectric polarisation produces an opposing electric field, thereby reducing the net field between the plates at fixed free charge.

**Quick Tip to Solve:** Draw the induced negative charge next to the positive plate; its opposing field immediately explains the reduction.

**35(iii).** A capacitor A of capacitance  $C$ , having charge  $Q$  is connected across another uncharged capacitor B of capacitance  $2C$ . Find an expression for (a) the potential difference across the combination and (b) the charge lost by capacitor A. [2 marks]

**Solution:**

**Concept:**

- Capacitors connected across each other are in parallel and attain the same final potential difference  $V_f$ .
- In an isolated charge-sharing combination, total charge on the connected positive-side plates is conserved. The charge on a capacitor is  $Q_i = C_i V_f$ .

**Step 1: Find the common potential difference** Initially, capacitor A carries charge  $Q$ , while B is uncharged. The parallel equivalent capacitance is  $C + 2C = 3C$ . No external source supplies charge, so conservation of charge gives

$$Q = CV_f + 2CV_f = 3CV_f.$$

Therefore the potential difference across the combination is  $V_f = Q/(3C)$ . It is one-third of the initial potential difference  $Q/C$  across A.

**Step 2: Calculate the charge transferred from A** After sharing, A retains

$$Q_{A,f} = CV_f = C \frac{Q}{3C} = \frac{Q}{3}.$$

The charge lost by A is its initial charge minus its final charge:

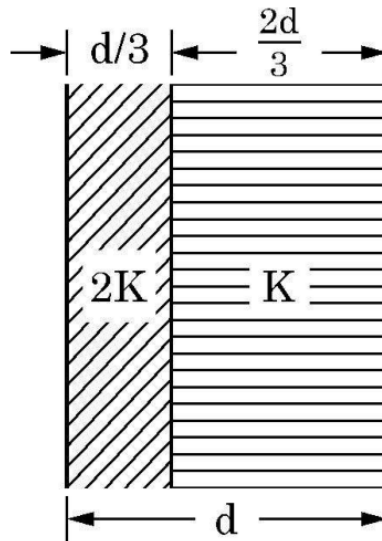
$$\Delta Q_A = Q - Q_{A,f} = Q - \frac{Q}{3} = \frac{2Q}{3}.$$

B gains precisely this charge, since  $Q_{B,f} = 2CV_f = 2Q/3$ , which checks conservation.

**Step 3: Conclusion** (a) The common potential difference is  $Q/(3C)$ . (b) Capacitor A loses charge  $2Q/3$ .

**Quick Tip to Solve:** In parallel charge sharing, first divide total initial charge by total capacitance, then use that common voltage for each capacitor.

**35(iii).** Two slabs of dielectric constants  $2K$  and  $K$  fill the space between the plates of a parallel plate capacitor of plate area  $A$  and plate separation  $d$  as shown in figure. Find an expression for capacitance of the system. [2 marks]



### Solution:

#### Concept:

- A dielectric layer of relative permittivity  $K_i$ , area  $A$ , and thickness  $t_i$  has equivalent capacitance  $C_i = K_i \epsilon_0 A / t_i$ , where  $\epsilon_0$  is vacuum permittivity.
- Layers arranged along the plate separation act in series: their potential drops add for the same free plate charge, giving  $1/C_{eq} = 1/C_1 + 1/C_2$ .

**Step 1: Read the layer thicknesses from the figure** Both layers cover the full plate area  $A$ . The layer of dielectric constant  $2K$  has thickness  $d/3$ , and the layer of dielectric constant  $K$  has thickness  $2d/3$ . Thus their equivalent capacitances are

$$C_1 = \frac{2K\epsilon_0 A}{d/3} = \frac{6K\epsilon_0 A}{d}, \quad C_2 = \frac{K\epsilon_0 A}{2d/3} = \frac{3K\epsilon_0 A}{2d}.$$

The division is across the thickness, so these are series layers rather than parallel areas.

**Step 2: Add the reciprocal capacitances** Using the series rule,

$$\begin{aligned} \frac{1}{C_{eq}} &= \frac{d}{6K\epsilon_0 A} + \frac{2d}{3K\epsilon_0 A} \\ &= \frac{5d}{6K\epsilon_0 A}. \end{aligned}$$

Equivalently, the effective dielectric constant is  $6K/5$ , which lies between  $K$  and  $2K$ , as expected.

**Step 3: Conclusion** The capacitance of the system is  $C_{eq} = \frac{6K\epsilon_0 A}{5d}$ .

**Quick Tip to Solve:** For slabs stacked across the gap, use  $C = \epsilon_0 A / \sum(t_i/K_i)$ ; never add their capacitances directly.