

CBSE Class 12 Mathematics 2023 - Set 65-C-1 - Compartment



Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains 38 questions. All questions are compulsory.
- (ii) This question paper is divided into five Sections A, B, C, D and E.
- (iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
- (iv) In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions carrying 2 marks each.
- (v) In Section C, Questions no. 26 to 31 are short answer (SA) type questions carrying 3 marks each.
- (vi) In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.
- (vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
- (ix) Use of calculators is not allowed.

Section A

1. If A is a square matrix of order 3 and $|A| = 6$, then the value of $|\text{adj } A|$ is:

- (A) 6 (B) 36
(C) 27 (D) 216

2. The value of $\int_0^{\pi/6} \sin 3x \, dx$ is:

- (A) $-\frac{\sqrt{3}}{2}$ (B) $-\frac{1}{3}$
(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{3}$

3. If $\vec{a}$, $\vec{b}$ and $(\vec{a} + \vec{b})$ are all unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of θ is:

- (A) $\frac{2\pi}{3}$ (B) $\frac{5\pi}{6}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

4. The projection of vector $\hat{i}$ on the vector $\hat{i} + \hat{j} + 2\hat{k}$ is:

- (A) $\frac{1}{\sqrt{6}}$ (B) $\sqrt{6}$
(C) $\frac{2}{\sqrt{6}}$ (D) $\frac{3}{\sqrt{6}}$

5. A family has 2 children and the elder child is a girl. The probability that both children are girls is:

- (A) $\frac{1}{4}$ (B) $\frac{1}{8}$
(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

6. The vector equation of a line which passes through the point $(2, -4, 5)$ and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{2} = \frac{z+8}{6}$ is:

- (A) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ (B) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$
(C) $\vec{r} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ (D) $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} - 2\hat{j} - 6\hat{k})$

7. For which value of x , are the determinants $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ equal?

- (A) ± 3 (B) -3
(C) ± 2 (D) 2

8. The value of the cofactor of the element of second row and third column in the matrix

$$\begin{pmatrix} 4 & 3 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 3 \end{pmatrix} \text{ is:}$$

- (A) 5 (B) -5
(C) -11 (D) 11

9. The difference of the order and the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$ is:

- (A) 1 (B) 2
(C) -1 (D) 0

10. If matrix $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ and $A^2 = kA$, then the value of k is:

- (A) 1 (B) -2
(C) 2 (D) -1

11. $\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$ is equal to:

- (A) $\tan x - \cot x + C$ (B) $-\cot x - \tan x + C$
(C) $\cot x + \tan x + C$ (D) $\tan x - \cot x - C$

12. The integrating factor of the differential equation $(3x^2 + y)\frac{dx}{dy} = x$ is:

- (A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$
(C) $\frac{2}{x}$ (D) $-\frac{1}{x}$

13. The point which lies in the half-plane $2x + y - 4 \leq 0$ is:

- (A) (0, 8) (B) (1, 1)
(C) (5, 5) (D) (2, 2)

14. If $(\cos x)^y = (\cos y)^x$, then $\frac{dy}{dx}$ is equal to:

- (A) $\frac{y \tan x + \log(\cos y)}{x \tan y - \log(\cos x)}$ (B) $\frac{x \tan y + \log(\cos x)}{y \tan x + \log(\cos y)}$
(C) $\frac{y \tan x - \log(\cos y)}{x \tan y - \log(\cos x)}$ (D) $\frac{y \tan x + \log(\cos y)}{x \tan y + \log(\cos x)}$

15. It is given that $X \begin{pmatrix} 3 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$. Then matrix X is:

- (A) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$
(C) $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$

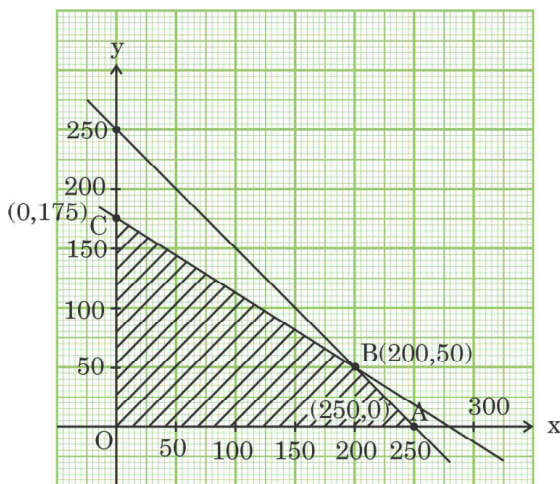
16. If ABCD is a parallelogram and AC and BD are its diagonals, then $\vec{AC} + \vec{BD}$ is:

- (A) $2\vec{DA}$ (B) $2\vec{AB}$
(C) $2\vec{BC}$ (D) $2\vec{BD}$

17. If $x = a \cos \theta + b \sin \theta$, $y = a \sin \theta - b \cos \theta$, then which one of the following is true ?

- (A) $y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$ (B) $y^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$
(C) $y^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 0$ (D) $y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - y = 0$

18. The corner points of the bounded feasible region of an LPP are $O(0, 0)$, $A(250, 0)$, $B(200, 50)$ and $C(0, 175)$. If the maximum value of the objective function $Z = 2ax + by$ occurs at the points $A(250, 0)$ and $B(200, 50)$, then the relation between a and b is:



- (A) $2a = b$ (B) $2a = 3b$
(C) $a = b$ (D) $a = 2b$

19. **Assertion (A):** The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$.

Reason (R): Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1, 1\}$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** Quadrilateral formed by vertices $A(0, 0, 0)$, $B(3, 4, 5)$, $C(8, 8, 8)$ and $D(5, 4, 3)$ is a rhombus.

Reason (R): ABCD is a rhombus if $AB = BC = CD = DA$, $AC \neq BD$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Section B

21. If three non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$.

22(a). Simplify:

$$\tan^{-1} \left(\frac{\cos x}{1 - \sin x} \right)$$

OR

22(b). Prove that the greatest integer function $f : \mathbb{R} \rightarrow \mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto.

23. Function f is defined as

$$f(x) = \begin{cases} 2x + 2, & \text{if } x < 2 \\ k, & \text{if } x = 2 \\ 3x, & \text{if } x > 2 \end{cases}$$

Find the value of k for which the function f is continuous at $x = 2$.

24. Find the intervals in which the function $f(x) = x^4 - 4x^3 + 4x^2 + 15$ is strictly increasing.

25(a). If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 7$, $|\vec{b}| = 24$, $|\vec{c}| = 25$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

OR

25(b). If a line makes angles α , β and γ with the x-axis, y-axis and z-axis respectively, then prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.

Section C

26(a). Evaluate:

$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

OR

26(b). Evaluate:

$$\int_1^3 (|x-1| + |x-2|) dx$$

27(a). Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$, given that $y = 1$ when $x = 0$.

OR

27(b). Find the particular solution of the differential equation $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$, given that $y = 0$ when $x = 1$.

28(a). Out of two bags, bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag B.

OR

28(b). Out of a group of 50 people, 20 always speak the truth. Two persons are selected at random from the group (without replacement). Find the probability distribution of the number of selected persons who always speak the truth.

29. Find:

$$\int \frac{\cos \theta}{\sqrt{3-3\sin \theta - \cos^2 \theta}} d\theta$$

30. Solve the following Linear Programming Problem graphically:

$$\text{Minimise } z = 3x + 8y$$

subject to the constraints:

$$3x + 4y \geq 8$$

$$5x + 2y \geq 11$$

$$x \geq 0, y \geq 0$$

31. Find:

$$\int \frac{2x^2 + 1}{x^2(x^2 + 4)} dx$$

32. If matrix $A = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$, find A^{-1} , and

hence solve the following system of linear equations:

$$3x + 2y + z = 2000$$

$$4x + y + 3z = 2500$$

$$x + y + z = 900$$

33(a). Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also find their point of intersection.

OR

33(b). Find the shortest distance between the pair of lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}; z = 2$.

34. Find the area of the triangle ABC bounded by the lines represented by the equations $5x - 2y - 10 = 0$, $x - y - 9 = 0$ and $3x - 4y - 6 = 0$, using the integration method.

35(a). Show that the relation S in the set of real numbers $\mathbb{R}$ defined by

$$S = \{(a, b) : a \leq b^3, a \in \mathbb{R}, b \in \mathbb{R}\}$$

is neither reflexive, nor symmetric, nor transitive.

OR

35(b). Let R be the relation defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b) : \text{both } a \text{ and } b \text{ are either odd or even}\}$. Show that R is an equivalence relation. Hence, find the elements of equivalence class $[1]$.

Section E

36(i). In a group activity class, there are 10 students whose ages are 16, 17, 15, 14, 19, 17, 16, 19, 16 and 15 years. One student is selected at random such that each has equal chance of being chosen and age of the student is recorded.



On the basis of the above information, answer the following questions:

(i) Find the probability that the age of the selected student is a composite number.

36(ii). Let X be the age of the selected student. What can be the value of X ?

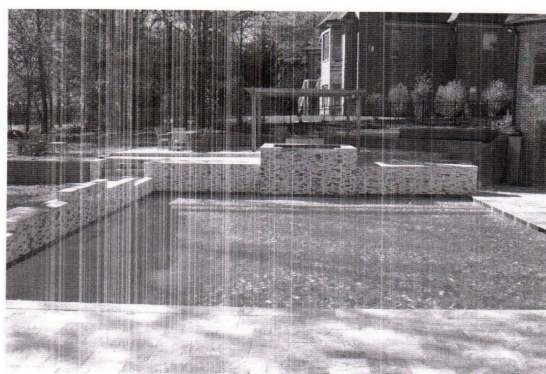
36(iii)(a). Find the probability distribution of random variable X and hence find the mean age.

OR

36(iii)(b). A student was selected at random and his age was found to be greater than 15 years. Find the probability that his age is a prime number.

37(i). A housing society wants to commission a swimming pool for its residents. For this, they have to purchase a square piece of land and dig this to such a depth that its capacity is 250 cubic metres. Cost of land is Rs 500 per square metre. The cost of digging increases with the depth and cost for the whole pool is Rs $4000(\text{depth})^2$.

Suppose the side of the square plot is x metres and depth is h metres.



On the basis of the above information, answer the following questions:

(i) Write cost $C(h)$ as a function in terms of h .

37(ii). Find critical point.

37(iii)(a). Use second derivative test to find the value of h for which cost of constructing the pool is minimum. What is the minimum cost of construction of the pool ?

OR

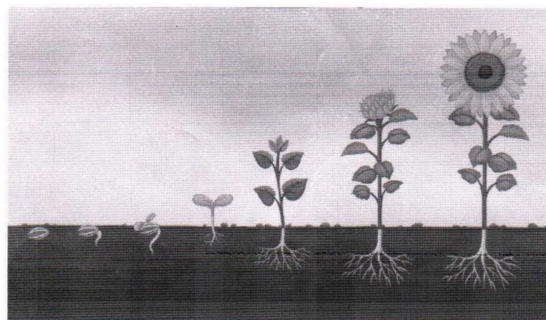
37(iii)(b). Use first derivative test to find the depth of the pool so that cost of construction is minimum. Also, find the relation between x and h for minimum cost.

38(i). In an agricultural institute, scientists do experiments with varieties of seeds to grow them in different environments to produce healthy plants and get more yield.

A scientist observed that a particular seed grew very fast after germination. He had recorded growth of the plant since germination and he said that its growth can be defined by the function

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, \quad 0 \leq x \leq 10$$

where x is the number of days the plant is exposed to sunlight.



On the basis of the above information, answer the following questions:

(i) What are the critical points of the function $f(x)$?

38(ii). Using second derivative test, find the minimum value of the function.

- End of Question Paper -