

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin \theta = \frac{3}{5}$. Then, $\hat{a} \cdot \hat{b}$ is equal to:

- (A) $\pm \frac{3}{5}$ (B) $\pm \frac{3}{4}$
(C) $\pm \frac{4}{5}$ (D) $\pm \frac{4}{3}$

2. The integrating factor of the differential equation $x \frac{dy}{dx} - y = x^4 - 3x$ is:

- (A) x (B) $-x$
(C) x^{-1} (D) $\log(x^{-1})$

3. If the direction cosines of a line are $\sqrt{3}k, \sqrt{3}k, \sqrt{3}k$, then the value of k is:

- (A) ± 1 (B) $\pm \sqrt{3}$
(C) ± 3 (D) $\pm \frac{1}{3}$

4. A linear programming problem deals with the optimization of a/an:

- (A) logarithmic function (B) linear function
(C) quadratic function (D) exponential function

5. If $P(A | B) = P(A' | B)$, then which of the following statements is true?

- (A) $P(A) = P(A')$ (B) $P(A) = 2P(B)$
(C) $P(A \cap B) = \frac{1}{2}P(B)$ (D) $P(A \cap B) = 2P(B)$

6. If a_{ij} and A_{ij} represent the $(i, j)^{\text{th}}$ element and its cofactor of $\begin{pmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{pmatrix}$ respectively, then the value of $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$ is:

- (A) 0 (B) -28
(C) 114 (D) -114

7. The derivative of $\sin(x^2)$ w.r.t. x , at $x = \sqrt{\pi}$, is:

- (A) 1 (B) -1
(C) $-2\sqrt{\pi}$ (D) $2\sqrt{\pi}$

8. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ respectively are:

- (A) 1, 2 (B) 2, 3
(C) 2, 1 (D) 2, 6

9. The vector with terminal point $A(2, -3, 5)$ and initial point $B(3, -4, 7)$ is:

- (A) $\hat{i} - \hat{j} + 2\hat{k}$ (B) $\hat{i} + \hat{j} + 2\hat{k}$
(C) $-\hat{i} - \hat{j} - 2\hat{k}$ (D) $-\hat{i} + \hat{j} - 2\hat{k}$

10. The distance of point $P(a, b, c)$ from y -axis is:

- (A) b (B) b^2
(C) $\sqrt{a^2 + c^2}$ (D) $a^2 + c^2$

11. The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0, x + y \geq 4$ is:

- (A) 0 (B) 1
(C) 2 (D) 3

12. If A and B are two non-zero square matrices of same order such that $(A + B)^2 = A^2 + B^2$, then:

- (A) $AB = O$ (B) $AB = -BA$
(C) $BA = O$ (D) $AB = BA$

13. A relation R defined on set $A = \{x : x \in \mathbb{Z} \text{ and } 0 \leq x \leq 10\}$ as $R = \{(x, y) : x = y\}$ is given to be an equivalence relation. The number of equivalence classes is:

- (A) 1 (B) 2
(C) 10 (D) 11

14. If a matrix has 36 elements, the number of possible orders it can have, is:

- (A) 13 (B) 3
(C) 5 (D) 9

15. The number of points, where $f(x) = [x]$, $0 < x < 3$ ($[\cdot]$ denotes greatest integer function) is not differentiable is:

- (A) 1 (B) 2
(C) 3 (D) 4

16. Let $f(x)$ be a continuous function on $[a, b]$ and differentiable on (a, b) . Then, this function $f(x)$ is strictly increasing in (a, b) if:

- (A) $f'(x) < 0, \forall x \in (a, b)$ (B) $f'(x) > 0, \forall x \in (a, b)$
(C) $f'(x) = 0, \forall x \in (a, b)$ (D) $f'(x) > 0, \forall x \in (a, b)$

17. If $\begin{pmatrix} x+y & 2 \\ 5 & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is:

- (A) 7 (B) 6
(C) 8 (D) 18

18. If $\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{6}$, then the value of a' is:

- (A) 3 (B) $2\sqrt{3}$
(C) $\frac{5}{3}$ (D) $\frac{1}{2}$

19. **Assertion (A):** A line in space cannot be drawn perpendicular to x, y and z axes simultaneously.

Reason (R): For any line making angles α, β, γ with the positive directions of x, y and z axes respectively, $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** For matrix $A =$

$$\begin{pmatrix} 1 & \cos \theta & 1 \\ -\cos \theta & 1 & \cos \theta \\ -1 & -\cos \theta & 1 \end{pmatrix}, \text{ where } \theta \in [0, 2\pi], |A| \in [2, 4].$$

Reason (R): $\cos \theta \in [-1, 1], \forall \theta \in [0, 2\pi]$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

Section B

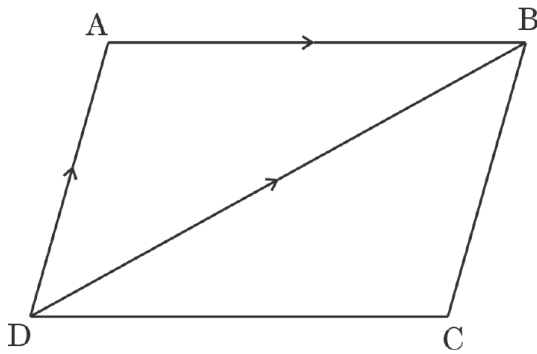
21. (a) Find: $\int x \sqrt{1+2x} \, dx$

OR

(b) Evaluate: $\int_0^{\pi^2/4} \frac{\sin \sqrt{x}}{\sqrt{x}} \, dx$

22. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a} + \vec{b}) \perp \vec{a}$ and $(2\vec{a} + \vec{b}) \perp \vec{b}$, then prove that $|\vec{b}| = \sqrt{2} |\vec{a}|$.

23. In the given figure, $ABCD$ is a parallelogram. If $\vec{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, then find $\vec{AD}$ and hence find the area of parallelogram $ABCD$.



24. (a) If $y = \sqrt{\cos x + y}$, prove that $\frac{dy}{dx} = \frac{\sin x}{1 - 2y}$.

OR

(b) Show that the function $f(x) = |x|^3$ is differentiable at all points of its domain.

25. Find the absolute maximum and minimum values of the function $f(x) = 12x^{4/3} - 6x^{1/3}$, $x \in [0, 1]$.

Section C

26. (a) Find: $\int \frac{x^2}{(x^2 + 4)(x^2 + 9)} dx$

OR

(b) Evaluate: $\int_1^3 (|x - 1| + |x - 2| + |x - 3|) dx$

27. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$.

28. Solve the following linear programming problem graphically:

Maximise $z = 5x + 4y$ subject to the constraints

$$x + 2y \geq 4$$

$$3x + y \leq 6$$

$$x + y \leq 4$$

$$x, y \geq 0$$

29. E and F are two independent events such that $P(\overline{E}) = 0.6$ and $P(E \cup F) = 0.6$. Find $P(F)$ and $P(\overline{E} \cup \overline{F})$.

30. (a) A relation R on set $A = \{1, 2, 3, 4, 5\}$ is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive.

OR

(b) A function f is defined from $\mathbb{R} \rightarrow \mathbb{R}$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function $f(x)$. Hence, check whether function $f(x)$ is one-one and onto or not.

31. (a) If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$.

OR

(b) If $y = (\tan x)^x$, then find $\frac{dy}{dx}$.

Section D

32. (a) Evaluate: $\int_0^{\pi/2} e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$

OR

(b) Find: $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

33. Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines $x = -2$ and $x = 2$.

34. Equations of sides of a parallelogram $ABCD$ are as follows:

$$\begin{aligned} AB : \frac{x+1}{1} &= \frac{y-2}{-2} = \frac{z-1}{2} \\ BC : \frac{x-1}{3} &= \frac{y+2}{-5} = \frac{z-5}{3} \\ CD : \frac{x-4}{1} &= \frac{y+7}{-2} = \frac{z-8}{2} \\ DA : \frac{x-2}{3} &= \frac{y+3}{-5} = \frac{z-4}{3} \end{aligned}$$

Find the equation of diagonal BD .

35. (a) If $A = \begin{pmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{pmatrix}$, find A^{-1} and use it to solve the following system of equations: $x - 2y = 10$, $2x - y - z = 8$, $-2y + z = 7$.

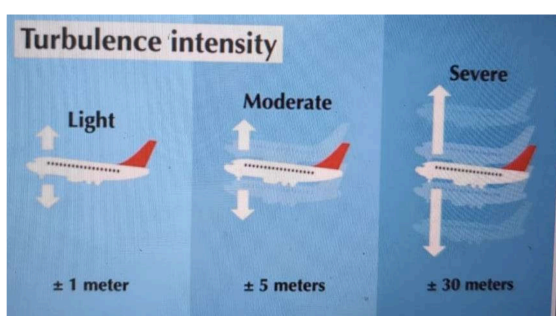
OR

(b) If $A = \begin{pmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{pmatrix}$, find the value of $(a + x) - (b + y)$.

36. Case Study - 2

According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights.

Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.



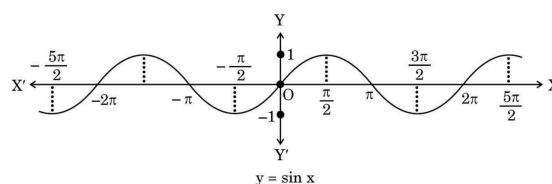
On the basis of the above information, answer the following questions:

- (i) Find the probability that an airplane reached its destination late. [2]
- (ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence. [2]

37. Case Study - 3

If a function $f : X \rightarrow Y$ defined as $f(x) = y$ is one-one and onto, then we can define a unique function $g : Y \rightarrow X$ such that $g(y) = x$, where $x \in X$ and $y = f(x)$, $y \in Y$. Function g is called the inverse of function f .

The domain of sine function is $\mathbb{R}$ and function sine: $\mathbb{R} \rightarrow \mathbb{R}$ is neither one-one nor onto. The following graph shows the sine function.



Let sine function be defined from set A to $[-1, 1]$ such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from $[-1, 1]$ to A .

On the basis of the above information, answer the following questions:

- (i) If A is the interval other than principal value branch, give an example of one such interval. [1]
- (ii) If $\sin^{-1}(x)$ is defined from $[-1, 1]$ to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$. [1]
- (iii) (a) Draw the graph of $\sin^{-1} x$ from $[-1, 1]$ to its principal value branch. [2]

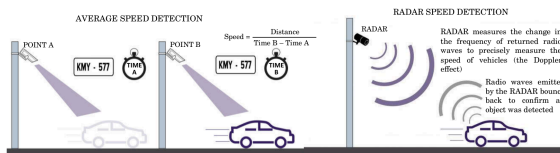
OR

- (iii) (b) Find the domain and range of $f(x) = 2 \sin^{-1}(1 - x)$. [2]

38. Case Study - 1

The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.

- End of Question Paper -



A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ .

On the basis of the above information, answer the following questions:

(i) Express θ in terms of height of the camera installed on the pole and x . [1]

(ii) Find $\frac{d\theta}{dx}$. [1]

(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. [2]

OR

(iii) (b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{1}{101}$ rad/s, then find the speed of the car. [2]