

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. The number of solutions of the differential equation $\frac{dy}{dx} - y = 1$, given that $y(0) = 1$, is:

- (A) 0 (B) 1
(C) 2 (D) infinitely many

2. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true?

- (A) $\vec{a} \cdot \vec{b} \geq |\vec{a}| |\vec{b}|$ (B) $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}|$
(C) $\vec{a} \cdot \vec{b} \leq |\vec{a}| |\vec{b}|$ (D) $\vec{a} \cdot \vec{b} < |\vec{a}| |\vec{b}|$

3. The coordinates of the foot of the perpendicular drawn from the point $(0, 1, 2)$ on the x -axis are given by:

- (A) $(1, 0, 0)$ (B) $(2, 0, 0)$
(C) $(\sqrt{5}, 0, 0)$ (D) $(0, 0, 0)$

4. The common region determined by all the constraints of a linear programming problem is called:

- (A) an unbounded region (B) an optimal region
(C) a bounded region (D) a feasible region

5. Let E be an event of a sample space S of an experiment, then $P(S | E) =$

- (A) $P(S \cap E)$ (B) $P(E)$
(C) 1 (D) 0

6. The number of all scalar matrices of order 3, with each entry $-1, 0$ or 1 , is:

- (A) 1 (B) 3
(C) 2 (D) 3^9

7. $\frac{d}{dx} [\cos(\log x + e^x)]$ at $x = 1$ is:

- (A) $-\sin e$ (B) $\sin e$
(C) $-(1 + e) \sin e$ (D) $(1 + e) \sin e$

8. The degree of the differential equation $(y'')^2 + (y')^3 = x \sin(y')$ is:

- (A) 1 (B) 2
(C) 3 (D) not defined

9. The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is:

- (A) $2\hat{j}$ (B) $\hat{j}$
(C) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$

10. Direction ratios of a vector parallel to the line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are:

- (A) 2, -1, 6 (B) 2, 1, 6
(C) 2, 1, 3 (D) 2, -1, 3

11. If $F(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is:

- (A) 1 (B) 2
(C) 0 (D) -2

12. If a line makes an angle of 30° with the positive direction of x -axis, 120° with the positive direction of y -axis, then the angle which it makes with the positive direction of z -axis is:

- (A) 90° (B) 120°
(C) 60° (D) 0°

13. If the sum of all the elements of a 3×3 scalar matrix is 9, then the product of all its elements is:

- (A) 0 (B) 9
(C) 27 (D) 729

14. Which of the following statements is not true about equivalence classes A_i ($i = 1, 2, \dots, n$) formed by an equivalence relation R defined on a set A ?

- (A) $\bigcup_i A_i = A$ (B) $A_i \cap A_j \neq \emptyset, i \neq j$
(C) $x \in A_i$ and $x \in A_j \Rightarrow A_i = A_j$ (D) All elements of A_i are related to each other, for all i

15. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = k abc$, then the value of k is:

- (A) 0 (B) 1
(C) 2 (D) 4

16. The number of points of discontinuity of

$$f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$$

- (A) 0 (B) 1
(C) 2 (D) infinite

17. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is:

- (A) strictly decreasing on $\mathbb{R}$ (B) strictly increasing on $\mathbb{R}$
(C) neither strictly increasing nor strictly decreasing on $\mathbb{R}$ (D) strictly decreasing on $(-\infty, 0)$

18. If $\int_0^2 2e^{2x} dx = \int_0^a e^x dx$, then the value of 'a' is:

- (A) 1 (B) 2
(C) 4 (D) $1/2$

19. **Assertion (A):** For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.

Reason (R): For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

Select the correct answer from the codes (A), (B), (C) and (D):

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** For any symmetric matrix A , $B'AB$ is a skew-symmetric matrix.

Reason (R): A square matrix P is skew-symmetric if $P' = -P$.

Select the correct answer from the codes (A), (B), (C) and (D):

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

Section B

21. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ ($x \neq 0$) respectively, find the value of $(M - m)$.

22. Evaluate: $\int \frac{x^2}{x^9 + a^6} dx$.

23. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

24. (a) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

OR

(b) Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.

25. (a) Check the differentiability of $f(x) = |\cos x|$ at $x = \frac{\pi}{2}$.

OR

(b) If $y = A \sin 2x + B \cos 2x$ and $\frac{d^2 y}{dx^2} - ky = 0$, find the value of k .

Section C

26. (a) Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$; $y = 2$, when $x = 1$.

OR

(b) Find the general solution of the differential equation: $y dx = (x + 2y^2) dy$.

27. If vectors $\vec{a}$, $\vec{b}$ and $2\vec{a} + 3\vec{b}$ are unit vectors, then find the angle between $\vec{a}$ and $\vec{b}$.

28. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X .

29. Find: $\int x^2 \sin^{-1}(x^{3/2}) dx$.

30. (a) If $x^{30}y^{20} = (x + y)^{50}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

OR

(b) Find $\frac{dy}{dx}$, if $5^x + 5^y = 5^{x+y}$.

31. (a) Evaluate: $\int_{-2}^2 \sqrt{\frac{2-x}{2+x}} dx$.

OR

(b) Find: $\int \frac{1}{x[(\log x)^2 - 3 \log x - 4]} dx$.

32. Find the value of p for which the lines $\vec{r} = \lambda\hat{i} + (2\lambda + 1)\hat{j} + (3\lambda + 2)\hat{k}$ and $\vec{r} = \hat{i} - 3\mu\hat{j} + (\mu p + 7)\hat{k}$ are perpendicular to each other and also intersect. Also, find the point of intersection of the given lines.

33. (a) If $A = \begin{pmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$, then find $(AB)^{-1}$. Also, find $|(AB)^{-1}|$.

OR

(b) Given $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{pmatrix}$, find A^{-1} . Use it to solve the following system of equations: $x + y + z = 1$, $2x + 3y + 2z = 2$, $x + y + 2z = 4$.

34. If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x -axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

35. (a) Show that a function $f : R \rightarrow R$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \rightarrow A$ becomes an onto function.

OR

(b) A relation R is defined on $N \times N$ (where N is the set of natural numbers) as: $(a, b) R (c, d) \iff a - c = b - d$. Show that R is an equivalence relation.

36. The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

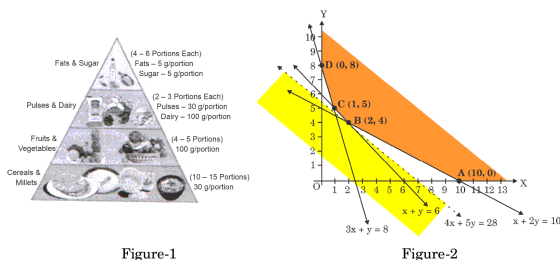


Figure-1

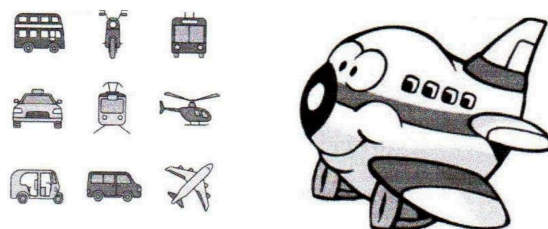
Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg) which are available at the rate of Rs. 16/kg and Rs. 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

On the basis of the above information, answer the following questions:

- Identify and write all the constraints which determine the given feasible region in Figure-2.
- If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region.

37. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.



Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

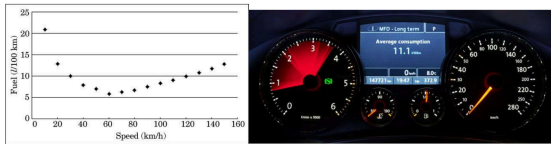
On the basis of the above information, answer the following questions:

- Find the probability that the airplane will not crash.
- Find $P(A | E_1) + P(A | E_2)$.
- (a) Find $P(A)$.

OR

- (b) Find $P(E_2 | A)$.

38. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.



The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

On the basis of the above information, answer the following questions:

(i) Find F , when $V = 40$ km/h.

(ii) Find $\frac{dF}{dV}$.

(iii) (a) Find the speed V for which fuel consumption F is minimum.

OR

(iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$.

- End of Question Paper -