

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. If $y = \cos^{-1}(e^x)$, then $\frac{dy}{dx}$ is:

- (A) $\frac{1}{\sqrt{e^{-2x} + 1}}$ (B) $-\frac{1}{\sqrt{e^{-2x} + 1}}$
 (C) $\frac{1}{\sqrt{e^{-2x} - 1}}$ (D) $-\frac{1}{\sqrt{e^{-2x} - 1}}$

2. The degree and order of the differential equation $y'' + \log(y') = x^5$ respectively are:

- (A) not defined, 5 (B) not defined, 2
 (C) 5, not defined (D) 2, 2

3. The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is:

- (A) $2\hat{j}$ (B) $\hat{j}$
 (C) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$

4. Direction ratios of a vector parallel to the line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are:

- (A) 2, -1, 6 (B) 2, 1, 6
 (C) 2, 1, 3 (D) 2, -1, 3

5. If for the matrix $A = \begin{pmatrix} \tan x & 1 \\ -1 & \tan x \end{pmatrix}$, $A + A' = 2\sqrt{3}I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is:

- (A) 0 (B) $\frac{\pi}{4}$
 (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

6. If a line makes an angle of 30° with the positive direction of x -axis, 120° with the positive direction of y -axis, then the angle which it makes with the positive direction of z -axis is:

- (A) 90° (B) 120°
 (C) 60° (D) 0°

7. If the sum of all the elements of a 3×3 scalar matrix is 9, then the product of all its elements is:

- (A) 0 (B) 9
 (C) 27 (D) 729

8. Let $f : R_+ \rightarrow [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is:

- (A) one-one (B) onto
 (C) bijective (D) neither one-one nor onto

9. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = k abc$, then the value of k is:

- (A) 0 (B) 1
 (C) 2 (D) 4

10. The number of points of discontinuity of

$$f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$$

- (A) 0 (B) 1
 (C) 2 (D) infinite

11. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is:

- (A) strictly decreasing on R (B) strictly increasing on R
 (C) neither strictly increasing nor strictly decreasing on R (D) strictly decreasing on $(-\infty, 0)$

12. Anti-derivative of $\sqrt{1 + \sin 2x}$, $x \in \left(0, \frac{\pi}{2}\right)$ is:

- (A) $\cos x + \sin x$ (B) $-\cos x + \sin x$
(C) $\cos x - \sin x$ (D) $-\cos x - \sin x$

13. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is:

- (A) $\cos x - \sin\left(\frac{y}{x}\right)$ (B) $\frac{y}{x}$
(C) $\frac{x^2 + y^2}{xy}$ (D) $\cos^2\left(\frac{x}{y}\right)$

14. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true?

- (A) $\vec{a} \cdot \vec{b} \geq |\vec{a}| |\vec{b}|$ (B) $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}|$
(C) $\vec{a} \cdot \vec{b} \leq |\vec{a}| |\vec{b}|$ (D) $\vec{a} \cdot \vec{b} < |\vec{a}| |\vec{b}|$

15. The coordinates of the foot of the perpendicular drawn from the point $(0, 1, 2)$ on the x -axis are given by:

- (A) $(1, 0, 0)$ (B) $(2, 0, 0)$
(C) $(\sqrt{5}, 0, 0)$ (D) $(0, 0, 0)$

16. The common region determined by all the constraints of a linear programming problem is called:

- (A) an unbounded region (B) an optimal region
(C) a bounded region (D) a feasible region

17. Let E be an event of a sample space S of an experiment, then $P(S | E) =$

- (A) $P(S \cap E)$ (B) $P(E)$
(C) 1 (D) 0

18. If $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = i - 3j$, then which of the following is false?

- (A) $a_{11} < 0$ (B) $a_{12} + a_{21} = -6$
(C) $a_{13} > a_{31}$ (D) $a_{31} = 0$

19. **Assertion (A):** For any symmetric matrix A , $B'AB$ is a skew-symmetric matrix.

Reason (R): A square matrix P is skew-symmetric if $P' = -P$.

Select the correct answer from the codes (A), (B), (C) and (D):

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is a scalar quantity.

Reason (R): Dot product of two vectors is a scalar quantity.

Select the correct answer from the codes (A), (B), (C) and (D):

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Section B

21. Determine whether the function $f(x) = x^2 - 6x + 3$ is increasing or decreasing in $[4, 6]$.

22. (a) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left[\tan\left(-\frac{\pi}{2}\right)\right]$.

OR

(b) Find the domain of $f(x) = \cos^{-1}(1 - x^2)$. Also, find its range.

23. (a) If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.

OR

(b) If $y = \operatorname{cosec}(\cot^{-1} x)$, then prove that $\sqrt{1+x^2} \frac{dy}{dx} - x = 0$.

24. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ ($x \neq 0$) respectively, find the value of $(M - m)$.

25. Find: $\int \frac{e^{4x} - 1}{e^{4x} + 1} dx$.

Section C

26. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X .

27. Find: $\int x^2 \log(x^2 - 1) dx$.

28. (a) If $y = (\log x)^2$, prove that $x^2 y'' + xy' = 2$.

OR

(b) If $y = \sin(\tan^{-1} e^x)$, then find $\frac{dy}{dx}$ at $x = 0$.

29. (a) Evaluate: $\int_{-2}^2 \sqrt{\frac{2-x}{2+x}} dx$.

OR

(b) Find: $\int \frac{1}{x[(\log x)^2 - 3 \log x - 4]} dx$.

30. (a) Find the particular solution of the differential equation given by $2xy + y^2 -$

$$2x^2 \frac{dy}{dx} = 0; y = 2, \text{ when } x = 1.$$

OR

(b) Find the general solution of the differential equation: $y dx = (x + 2y^2) dy$.

31. The position vectors of vertices of $\triangle ABC$ are $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$ and $C(3\hat{i} - 4\hat{j} - 4\hat{k})$. Find all the angles of $\triangle ABC$.

32. If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x -axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

33. (a) Show that a function $f : R \rightarrow R$ defined as $f(x) = x^2 + x + 1$ is neither one-one nor onto. Also, find all the values of x for which $f(x) = 3$.

OR

(b) A relation R is defined on $N \times N$ (where N is the set of natural numbers) as $(a, b) R (c, d) \iff \frac{a}{b} = \frac{c}{d}$. Show that R is an equivalence relation.

34. The vertices of $\triangle ABC$ are $A(1, 1, 0)$, $B(1, 2, 1)$ and $C(-2, 2, -1)$. Find the equations of the medians through A and B . Use the equations so obtained to find the coordinates of the centroid.

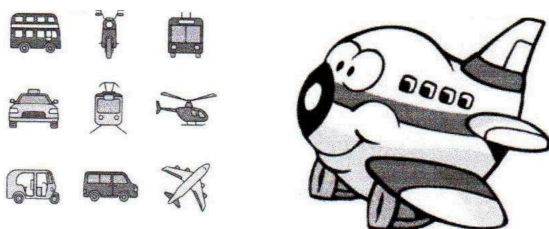
35. (a) Solve the following system of equations, using matrices: $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$, $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$, where $x, y, z \neq 0$.

OR

(b) If $A = \begin{pmatrix} 1 & \cot x \\ -\cot x & 1 \end{pmatrix}$, show that $A' A^{-1} = \begin{pmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{pmatrix}$.

Section E

36. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.



Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

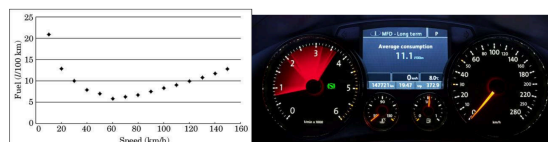
On the basis of the above information, answer the following questions:

- (i) Find the probability that the airplane will not crash.
- (ii) Find $P(A | E_1) + P(A | E_2)$.
- (iii) (a) Find $P(A)$.

OR

- (iii) (b) Find $P(E_2 | A)$.

37. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.



The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints

$$\text{is given as } F = \frac{V^2}{500} - \frac{V}{4} + 14.$$

On the basis of the above information, answer the following questions:

- (i) Find F , when $V = 40$ km/h.
- (ii) Find $\frac{dF}{dV}$.
- (iii) (a) Find the speed V for which fuel consumption F is minimum.

OR

- (iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$.

38. The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

- End of Question Paper -

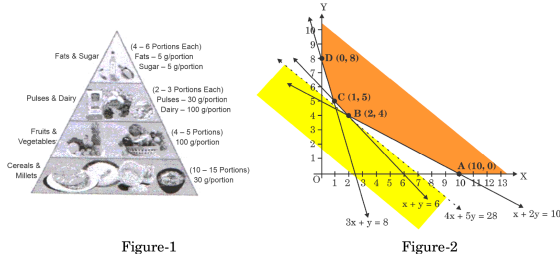


Figure-1

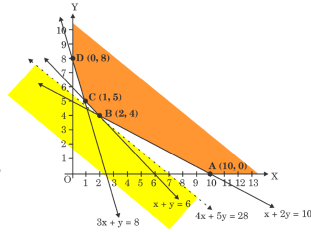


Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg) which are available at the rate of Rs. 16/kg and Rs. 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

On the basis of the above information, answer the following questions:

- Identify and write all the constraints which determine the given feasible region in Figure-2.
- If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region.