

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is:

- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$
(C) $\frac{5\pi}{6}$ (D) $\frac{11\pi}{6}$

2. The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represent the sides of:

- (A) an equilateral triangle (B) an obtuse-angled triangle
(C) an isosceles triangle (D) a right-angled triangle

3. Let $\vec{a}$ be any vector such that $|\vec{a}| = a$. The value of $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ is:

- (A) a^2 (B) $2a^2$
(C) $3a^2$ (D) 0

4. If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ and $A^2 + 7I = kA$, then the value of k is:

- (A) 1 (B) 2
(C) 5 (D) 7

5. Let $A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}$ and $B = \frac{1}{3} \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & \lambda \end{pmatrix}$. If $AB = I$, then the value of λ is:

- (A) $-\frac{9}{4}$ (B) -2
(C) $-\frac{3}{2}$ (D) 0

6. Derivative of x^2 with respect to x^3 , is:

- (A) $\frac{2}{3x}$ (B) $\frac{3x}{2}$
(C) $\frac{2x}{3}$ (D) $6x^5$

7. The function $f(x) = |x| + |x - 2|$ is:

- (A) continuous, but not differentiable at $x = 0$ and $x = 2$. (B) differentiable but not continuous at $x = 0$ and $x = 2$.
(C) continuous but not differentiable at $x = 0$ only. (D) neither continuous nor differentiable at $x = 0$ and $x = 2$.

8. The value of $\int_0^\pi \tan^2\left(\frac{\theta}{3}\right) d\theta$ is:

- (A) $\pi + \sqrt{3}$ (B) $3\sqrt{3} - \pi$
(C) $\sqrt{3} - \pi$ (D) $\pi - \sqrt{3}$

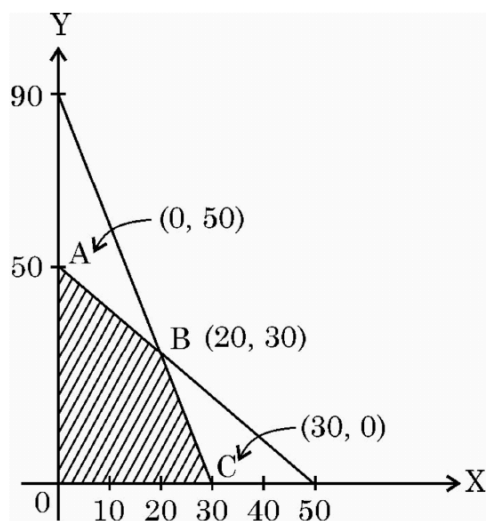
9. The integrating factor of the differential equation $\frac{dy}{dx} + \frac{2}{x}y = 0$, $x \neq 0$ is:

- (A) $\frac{2}{x}$ (B) x^2
(C) $e^{2/x}$ (D) $e^{\log(2x)}$

10. The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to:

- (A) $-\frac{1}{2}$ (B) $\frac{1}{2}$
(C) 2 (D) 3

11. The maximum value of $Z = 4x + y$ for a L.P.P. whose feasible region is given below is:



- (A) 50 (B) 110
(C) 120 (D) 170

12. The probability distribution of a random variable X is:

X	0	1	2	3	4
$P(X)$	0.1	k	$2k$	k	0.1

where k is some unknown constant.

The probability that the random variable X takes the value 2 is:

- (A) $\frac{1}{5}$ (B) $\frac{2}{5}$
(C) $\frac{4}{5}$ (D) 1

13. The function $f(x) = kx - \sin x$ is strictly increasing for:

- (A) $k > 1$ (B) $k < 1$
(C) $k > -1$ (D) $k < -1$

14. The Cartesian equation of a line passing through the point with position vector $\vec{a} = \hat{i} - \hat{j}$ and parallel to the line $\vec{r} = \hat{i} + \hat{k} + \mu(2\hat{i} - \hat{j})$, is:

- (A) $\frac{x-2}{1} = \frac{y+1}{0} = \frac{z}{1}$ (B) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$
(C) $\frac{x+1}{2} = \frac{y+1}{1} = \frac{z}{0}$ (D) $\frac{x-1}{2} = \frac{y}{-1} = \frac{z-1}{0}$

15. If $\begin{pmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{pmatrix}$ is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is:

- (A) 0 (B) 5
(C) 10 (D) 25

16. Given that $A^{-1} = \frac{1}{7} \begin{pmatrix} 2 & 1 \\ -3 & 2 \end{pmatrix}$, matrix A is:

- (A) $7 \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ (B) $\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$
(C) $\frac{1}{7} \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ (D) $\frac{1}{49} \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$

17. If $A = \begin{pmatrix} 2 & 1 \\ -4 & -2 \end{pmatrix}$, then the value of $I - A + A^2 - A^3 + \dots$ is:

- (A) $\begin{pmatrix} -1 & -1 \\ 4 & 3 \end{pmatrix}$ (B) $\begin{pmatrix} 3 & 1 \\ -4 & -1 \end{pmatrix}$
 (C) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

18. The integrating factor of the differential equation $(x + 2y^2)\frac{dy}{dx} = y$ ($y > 0$) is:

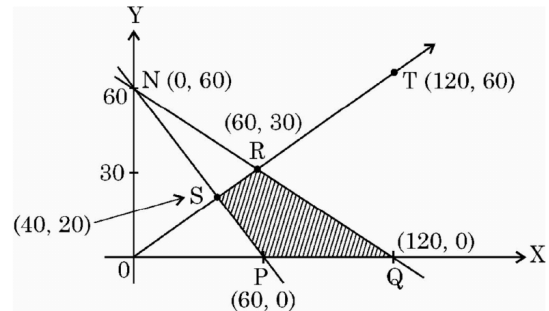
- (A) $\frac{1}{x}$ (B) x
 (C) y (D) $\frac{1}{y}$

19. **Assertion (A):** The relation $R = \{(x, y) : (x + y) \text{ is a prime number and } x, y \in \mathbb{N}\}$ is not a reflexive relation.

Reason (R): The number ' $2n$ ' is composite for all natural numbers n .

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
 (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
 (C) Assertion (A) is true, but Reason (R) is false.
 (D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** The corner points of the bounded feasible region of a L.P.P. are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.



Reason (R): The optimal solution of a LPP having bounded feasible region must occur at corner points.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
 (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
 (C) Assertion (A) is true, but Reason (R) is false.
 (D) Assertion (A) is false, but Reason (R) is true.

Section B

21. (a) If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$.

OR

(b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$.

22. The volume of a cube is increasing at the rate of $6 \text{ cm}^3/\text{s}$. How fast is the surface area of cube increasing, when the length of an edge is 8 cm ?

23. Show that the function f given by $f(x) = \sin x + \cos x$, is strictly decreasing in the interval $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$.

24. (a) Express $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$, in the simplest form.

OR

(b) Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

25. Find: $\int \frac{2x}{(x^2 + 1)(x^2 - 4)} dx$.

Section C

26. Find $\frac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1} \sqrt{x}$ is given.

27. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = y \cot 2x$, given that $y\left(\frac{\pi}{4}\right) = 2$.

OR

(b) Find the particular solution of the differential equation $(xe^{y/x} + y) dx = x dy$, given that $y = 1$ when $x = 1$.

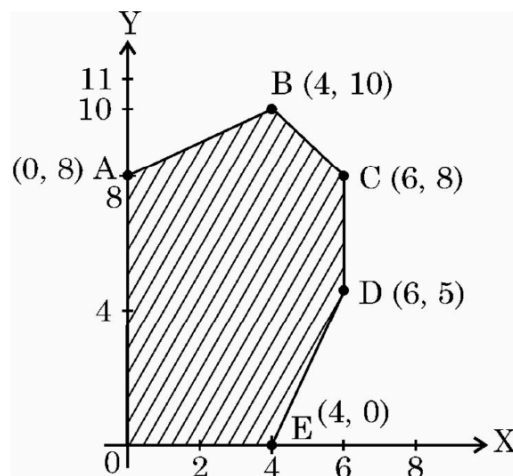
28. Find: $\int \sec^3 \theta d\theta$

29. (a) A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.

OR

(b) A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.

30. The corner points of the feasible region determined by the system of linear constraints are as shown in the following figure:



(i) If $Z = 3x - 4y$ be the objective function, then find the maximum value of Z .

(ii) If $Z = px + qy$ where $p, q > 0$ be the objective function. Find the condition on p and q so that maximum value of Z occurs at $B(4, 10)$ and $C(6, 8)$.

31. (a) Evaluate: $\int_0^{\pi/4} \frac{x dx}{1 + \cos 2x + \sin 2x}$

OR

(b) Find: $\int e^x \left[\frac{1}{(1+x^2)^{3/2}} + \frac{x}{\sqrt{1+x^2}} \right] dx$

Section D

32. (a) Let $A = \mathbb{R} - \{5\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f : A \rightarrow B$, defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one-one and onto.

OR

(b) Check whether the relation S in the set of real numbers $\mathbb{R}$ defined by $S = \{(a, b) : \text{where } a - b + \sqrt{2} \text{ is an irrational number}\}$ is reflexive, symmetric or transitive.

33. (a) Find the distance between the line $\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1}$ and another line parallel to it passing through the point $(4, 0, -5)$.

OR

(b) If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point $(3, -4, 7)$.

34. Find A^{-1} , if $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & 0 & 1 \end{pmatrix}$. Hence, solve the following system of equations:

$$x + 2y + z = 5$$

$$2x + 3y = 1$$

$$x - y + z = 8$$

35. (a) Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X -axis and the ordinates $x = -2$ and $x = 2$, using integration.

OR

(b) Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X -axis.

36. Case Study - 3: Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.

36(i). What is the probability that at least one of them is selected?

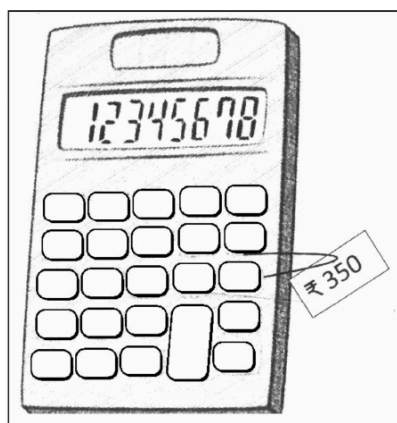
36(ii). Find $P(G | H)$ where G is the event of Jaspreet's selection and H denotes the event that Rohit is not selected.

36(iii). (a) Find the probability that exactly one of them is selected.

OR

(b) Find the probability that exactly two of them are selected.

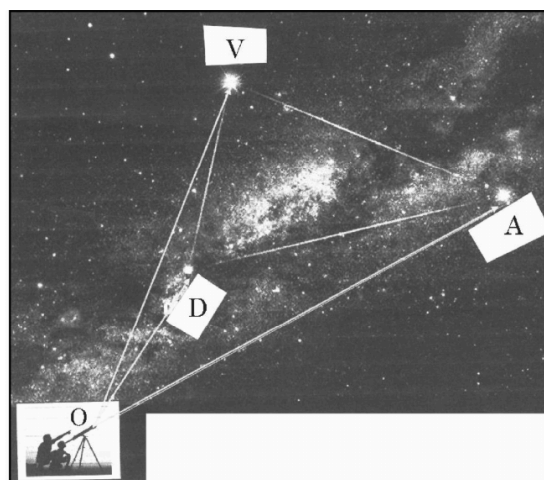
37. Case Study - 1: A store has been selling calculators at Rs.350 each. A market survey indicates that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function $p = 450 - \frac{1}{2}x$.



37(i). Determine the number of units (x) that should be sold to maximise the revenue $R(x) = x p(x)$. Also, verify the result.

37(ii). What rebate in price of calculator should the store give to maximise the revenue?

38. Case Study - 2: An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at $O(0, 0, 0)$ and the three stars have their locations at the points D , A and V having position vectors $2\hat{i} + 3\hat{j} + 4\hat{k}$, $7\hat{i} + 5\hat{j} + 8\hat{k}$ and $-3\hat{i} + 7\hat{j} + 11\hat{k}$ respectively.



38(i). How far is the star V from star A ?

38(ii). Find a unit vector in the direction of $\vec{DA}$.

38(iii). (a) Find the measure of $\angle VDA$.

OR

(b) What is the projection of vector $\vec{DV}$ on vector $\vec{DA}$?

- End of Question Paper -