

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is:

- (A) $\sin x e^{\sin^2 x}$ (B) $\cos x e^{\sin^2 x}$
 (C) $-2 \cos x e^{\sin^2 x}$ (D) $-2 \sin^2 x \cos x e^{\sin^2 x}$

2. If A is a square matrix of order 2 and $|A| = -2$, then the value of $|5A'|$ is:

- (A) -50 (B) -10
 (C) 10 (D) 50

3. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to:

- (A) 2 (B) 1
 (C) 0 (D) -2

4. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is:

- (A) -60 units/sec (B) 60 units/sec
 (C) -70 units/sec (D) -140 units/sec

5. The product of matrix P and Q is equal to a diagonal matrix. If the order of matrix Q is 3×2 , then order of matrix P is:

- (A) 2×2 (B) 3×3
 (C) 2×3 (D) 3×2

6. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is:

- (A) injective but not surjective. (B) surjective but not injective.
 (C) both injective and surjective. (D) neither injective nor surjective.

7. If $\sin(xy) = 1$, then $\frac{dy}{dx}$ is equal to:

- (A) $\frac{x}{y}$ (B) $-\frac{x}{y}$
 (C) $\frac{y}{x}$ (D) $-\frac{y}{x}$

8. If inverse of matrix $\begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ is the

matrix $\begin{pmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{pmatrix}$, then value of λ is:

- (A) -4 (B) 1
 (C) 3 (D) 4

9. Find the matrix A^2 , where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = \text{maximum}(i, j) - \text{minimum}(i, j)$:

- (A) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
 (C) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

10. If A is a square matrix of order 3 and $|\text{adj} \cdot A| = 8$, then the value of $|A^T|$ is:

- (A) $\sqrt{2}$ (B) $-\sqrt{2}$
 (C) 8 (D) $2\sqrt{2}$

11. $\int_{\pi/4}^{\pi/2} \cot \theta \operatorname{cosec}^2 \theta d\theta$ is:

- (A) $\frac{1}{2}$ (B) $-\frac{1}{2}$
 (C) 0 (D) $-\frac{\pi}{2}$

12. The integral $\int \frac{dx}{\sqrt{9-4x^2}}$ is equal to:

- (A) $\frac{1}{6} \sin^{-1}\left(\frac{2x}{3}\right) + c$ (B) $\frac{1}{2} \sin^{-1}\left(\frac{2x}{3}\right) + c$
 (C) $\sin^{-1}\left(\frac{2x}{3}\right) + c$ (D) $3 \sin^{-1}\left(\frac{2x}{3}\right) + c$

13. The area of the region bounded by the curve $y^2 = 4x$ and $x = 1$ is:

- (A) $\frac{4}{3}$ (B) $\frac{8}{3}$
 (C) $\frac{64}{3}$ (D) $\frac{32}{3}$

14. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is:

- (A) $e^x + e^{-y} = c$ (B) $e^{-x} + e^{-y} = c$
 (C) $e^{x+y} = c$ (D) $2e^{x+y} = c$

15. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is:

- (A) $\frac{5\pi}{6}$ (B) $\frac{3\pi}{4}$
 (C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$

16. The Cartesian equation of the line passing through the point $(1, -3, 2)$ and parallel to the line $\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is:

- (A) $\frac{x-1}{\frac{2}{z-2}-1} = \frac{y+3}{0} = \frac{z-2}{2}$ (B) $\frac{x+1}{\frac{1}{z+2}-2} = \frac{y-3}{1} = \frac{z-2}{2}$
 (C) $\frac{x+1}{\frac{2}{z+2}-1} = \frac{y-3}{0} = \frac{z-2}{2}$ (D) $\frac{x-1}{\frac{1}{z-2}-2} = \frac{y+3}{1} = \frac{z-2}{2}$

17. If A and B are two events such that $P(A/B) = P(B/A) \neq 0$, then:

- (A) $A \subset B$, but $A \neq B$ (B) $A = B$
 (C) $A \cap B = \phi$ (D) $P(A) = P(B)$

18. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio $3 : 1$ and S is the mid-point of line segment PR . The position vector of S is:

- (A) $\frac{\vec{p} + 3\vec{q}}{4}$ (B) $\frac{\vec{p} + 3\vec{q}}{8}$
 (C) $\frac{5\vec{p} + 3\vec{q}}{4}$ (D) $\frac{5\vec{p} + 3\vec{q}}{8}$

19. **Assertion (A):** The vectors $\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$, $\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$ and $\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$ represent the sides of a right angled triangle.

Reason (R): Three non-zero vectors of which no two are collinear, form a triangle, if their resultant is zero vector or sum of any two vectors is equal to the third.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
 (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.

Reason (R): The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
 (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

Section B

21. If $a = \sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$ and $b = \tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$, then find the value of $a + b$.

22. (a) Find: $\int \cos^3 x e^{\log \sin x} dx$.

OR

(b) Find: $\int \frac{1}{5 + 4x - x^2} dx$.

23. Sand is pouring from a pipe at the rate of $15 \text{ cm}^3/\text{minute}$. The falling sand forms a cone on the ground such that the height of the cone is always one-third of the radius of the base. How fast is the height of the sand cone increasing at the instant when the height is 4 cm ?

24. Find the vector equation of the line passing through the point $(2, 3, -5)$ and making equal angles with the co-ordinate axes.

25. Verify whether the function f defined by

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases} \text{ is continuous at } x =$$

0 or not.

Section C

26. (a) Find the particular solution of the differential equation $\frac{dy}{dx} - 2xy = 3x^2e^{x^2}$; $y(0) = 5$.

OR

(b) Solve the following differential equation:
 $x^2 dy + y(x + y) dx = 0$.

27. Find the values of a and b so that the following function is differentiable for all values of x :

$$f(x) = \begin{cases} ax + b, & x > -1 \\ bx^2 - 3, & x \leq -1 \end{cases}$$

28. (a) Evaluate: $\int_0^\pi \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$.

OR

(b) Find: $\int \frac{2x + 1}{(x + 1)^2(x - 1)} dx$.

29. (a) If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$.

OR

(b) If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$.

30. Given $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 3$.

31. Bag I contains 3 red and 4 black balls, Bag II contains 5 red and 2 black balls. Two balls are transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. Find the probability that the drawn ball is red in colour.

32. (a) Find the co-ordinates of the foot of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also, find the perpendicular distance of the given point from the line.

OR

(b) Find the shortest distance between the lines L_1 and L_2 given below:

L_1 : the line passing through $(2, -1, 1)$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$, and

L_2 : $\vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

33. (a) If $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{pmatrix}$, then find A^{-1} and

hence solve the following system of equations:

$$x + 2y - 3z = 1$$

$$2x - 3z = 2$$

$$x + 2y = 3$$

OR

(b) Find the product of the matrices

$$\begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix} \begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix} \text{ and hence}$$

solve the following system of linear equations:

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

34. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

35. Solve the following Linear Programming problem graphically:

Maximise $Z = 300x + 600y$

Subject to $x + 2y \leq 12$

$$2x + y \leq 12$$

$$x + \frac{5}{4}y \geq 5$$

$$x \geq 0, y \geq 0.$$

36. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions:

(i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time. Find $P(E_1)$, $P(E_2)$.

(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.

(iii) Find the probability of customer paying second month's bill in time.

OR

(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

37. Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$$

On the basis of the above information, answer the following questions:

(i) Find whether the relation R is symmetric or not.

(ii) Find whether the relation R is transitive or not.

(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

(b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$, check whether the relation S is symmetric and transitive.

38. A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be $1\frac{1}{2}$ cm.

On the basis of the above information, answer the following questions:

(i) Write the expression for the area of the visiting card in terms of x .

(ii) Obtain the dimensions of the card of minimum area.

- End of Question Paper -