

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio $3 : 1$ and S is the mid-point of line segment PR . The position vector of S is:

- (A) $\frac{\vec{p} + 3\vec{q}}{4}$ (B) $\frac{\vec{p} + 3\vec{q}}{8}$
 (C) $\frac{5\vec{p} + 3\vec{q}}{4}$ (D) $\frac{5\vec{p} + 3\vec{q}}{8}$

2. For the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{pmatrix}$ to be invertible, the value of λ is:

- (A) 0 (B) 10
 (C) $\mathbb{R} - \{10\}$ (D) $\mathbb{R} - \{-10\}$

3. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y -axis is:

- (A) $\frac{5\pi}{6}$ (B) $\frac{3\pi}{4}$
 (C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$

4. The Cartesian equation of the line passing through the point $(1, -3, 2)$ and parallel to the line $\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is:

- (A) $\frac{x-1}{\frac{2}{z-2} - 1} = \frac{y+3}{0} = \frac{z-2}{-1}$ (B) $\frac{x+1}{\frac{1}{z+2} - 2} = \frac{y-3}{1} = \frac{z-2}{2}$
 (C) $\frac{x+1}{\frac{2}{z+2} - 1} = \frac{y-3}{0} = \frac{z-2}{-1}$ (D) $\frac{x-1}{\frac{1}{z-2} - 2} = \frac{y+3}{1} = \frac{z-2}{2}$

5. If $A = \begin{pmatrix} x & 0 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 0 \\ -1 & 1 \end{pmatrix}$, then value of x for which $A^2 = B$ is:

- (A) -2 (B) 2
 (C) 2 or -2 (D) 4

6. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is:

- (A) -60 units/sec (B) 60 units/sec
 (C) -70 units/sec (D) -140 units/sec

7. Let $f(x) = \begin{vmatrix} x^2 & \sin x \\ p & -1 \end{vmatrix}$, where p is a constant. The value of p for which $f'(0) = 1$ is:

- (A) $\mathbb{R}$ (B) 1
 (C) 0 (D) -1

8. If A and B are two events such that $P(A/B) = P(B/A) \neq 0$, then:

- (A) $A \subset B$, but $A \neq B$ (B) $A = B$
 (C) $A \cap B = \phi$ (D) $P(A) = P(B)$

9. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is:

- (A) injective but not surjective. (B) surjective but not injective.
 (C) both injective and surjective. (D) neither injective nor surjective.

10. If A is a square matrix of order 3 and $|\text{adj} \cdot A| = 8$, then the value of $|A^T|$ is:

- (A) $\sqrt{2}$ (B) $-\sqrt{2}$
 (C) 8 (D) $2\sqrt{2}$

11. If $\int_{-2}^3 x^2 dx = k \int_0^2 x^2 dx + \int_2^3 x^2 dx$, then the value of k is:

- (A) 2 (B) 1
 (C) 0 (D) $\frac{1}{2}$

12. The value of $\int_1^e \log x \, dx$ is:

- (A) 0 (B) 1
(C) e (D) $e \log e$

13. The area bounded by the curve $y = \sqrt{x}$, Y-axis and between the lines $y = 0$ and $y = 3$ is:

- (A) $2\sqrt{3}$ (B) 27
(C) 9 (D) 3

14. The order of the following differential equation $\frac{d^3 y}{dx^3} + \left(\frac{dy}{dx}\right)^2 = 4 \log\left(\frac{d^2 y}{dx^2}\right)$ is:

- (A) not defined (B) 3
(C) 4 (D) 5

15. If inverse of matrix $\begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ is the

matrix $\begin{pmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{pmatrix}$, then value of λ is:

- (A) -4 (B) 1
(C) 3 (D) 4

16. Find the matrix A^2 , where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = \text{maximum}(i, j) - \text{minimum}(i, j)$:

- (A) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
(C) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

17. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is:

- (A) $\sin x e^{\sin^2 x}$ (B) $\cos x e^{\sin^2 x}$
(C) $-2 \cos x e^{\sin^2 x}$ (D) $-2 \sin^2 x \cos x e^{\sin^2 x}$

18. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to:

- (A) 2 (B) 1
(C) 0 (D) -2

19. **Assertion (A):** Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.

Reason (R): The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** The vectors $\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$, $\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$ and $\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$ represent the sides of a right angled triangle.

Reason (R): Three non-zero vectors of which no two are collinear, form a triangle, if their resultant is zero vector or sum of any two vectors is equal to the third.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Section B

21. Simplify: $\cos^{-1} x + \cos^{-1} \left(\frac{x + \sqrt{3 - 3x^2}}{2} \right)$,
 $\frac{1}{2} \leq x \leq 1$.

22. (a) Find: $\int \cos^3 x e^{\log \sin x} dx$.

OR

(b) Find: $\int \frac{1}{5 + 4x - x^2} dx$.

23. The surface area of a cube increases at the rate of $72 \text{ cm}^2/\text{sec}$. Find the rate of change of its volume, when the edge of the cube measures 3 cm.

24. Find the vector equation of the line passing through the point $(2, 3, -5)$ and making equal angles with the co-ordinate axes.

25. Verify whether the function f defined by

$$f(x) = \begin{cases} x \sin \left(\frac{1}{x} \right), & x \neq 0 \\ 0, & x = 0 \end{cases} \text{ is continuous at } x =$$

0 or not.

Section C

26. (a) Evaluate: $\int_0^\pi \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx.$

OR

(b) Find: $\int \frac{2x+1}{(x+1)^2(x-1)} dx.$

27. If $y = (\tan^{-1} x)^2$, show that $(x^2 + 1)^2 \frac{d^2 y}{dx^2} + 2x(x^2 + 1) \frac{dy}{dx} = 2.$

28. (a) Find the particular solution of the differential equation $\frac{dy}{dx} - 2xy = 3x^2 e^{x^2}; y(0) = 5.$

OR

(b) Solve the following differential equation: $x^2 dy + y(x+y) dx = 0.$

29. (a) If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}.$

OR

(b) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}.$

30. Find the projection of vector $(\vec{b} + \vec{c})$ on vector $\vec{a}$, where $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{k}.$

31. An urn contains 3 red and 2 white marbles. Two marbles are drawn one by one with replacement from the urn. Find the probability distribution of the number of white balls. Also, find the mean of the number of white balls drawn.

32. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

33. (a) Find the co-ordinates of the foot of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also, find the perpendicular distance of the given point from the line.

OR

(b) Find the shortest distance between the lines L_1 and L_2 given below:

L_1 : the line passing through $(2, -1, 1)$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$, and

L_2 : $\vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

34. (a) If $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{pmatrix}$, then find A^{-1} and

hence solve the following system of equations:

$$x + 2y - 3z = 1$$

$$2x - 3z = 2$$

$$x + 2y = 3$$

OR

(b) Find the product of the matrices

$$\begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix} \begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix} \text{ and hence}$$

solve the following system of linear equations:

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

35. Solve the following L.P.P. graphically:

Minimise $Z = 6x + 3y$

Subject to constraints $4x + y \geq 80$; $x + 5y \geq 115$;
 $3x + 2y \leq 150$; $x, y \geq 0$.

36. A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be $1\frac{1}{2}$ cm.

On the basis of the above information, answer the following questions:

(i) Write the expression for the area of the visiting card in terms of x .

(ii) Obtain the dimensions of the card of minimum area.

37. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions:

(i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time. Find $P(E_1)$, $P(E_2)$.

(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.

(iii) Find the probability of customer paying second month's bill in time.

OR

(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

38. Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$$

On the basis of the above information, answer the following questions:

(i) Find whether the relation R is symmetric or not.

(ii) Find whether the relation R is transitive or not.

(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

(b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$, check whether the relation S is symmetric and transitive.

- End of Question Paper -