

CBSE Class 12 Physics Question Paper 2024 with Solution PDF-Set 55/1/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – **Sections A, B, C, D** and **E**.
- (iii) In **Section A** – Questions no. **1 to 16** are Multiple Choice type questions. Each question carries **1** mark.
- (iv) In **Section B** – Questions no. **17 to 21** are Very Short Answer type questions. Each question carries **2** marks.
- (v) In **Section C** – Questions no. **22 to 28** are Short Answer type questions. Each question carries **3** marks.
- (vi) In **Section D** – Questions no. **29 and 30** are case study-based questions. Each question carries **4** marks.
- (vii) In **Section E** – Questions no. **31 to 33** are Long Answer type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is not allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. A thin plastic rod is bent into a circular ring of radius R . It is uniformly charged with charge density λ . The magnitude of the electric field at its centre is: [1 mark]

- (A) $\frac{\lambda}{2\epsilon_0 R}$
(B) Zero
(C) $\frac{\lambda}{4\pi\epsilon_0 R}$
(D) $\frac{\lambda}{4\epsilon_0 R}$

Correct Answer: (B) Zero

Solution:

Concept:

- Electric fields obey vector superposition: the resultant field is the vector sum of the fields of all charge elements.
- An element of charge dq at distance R produces a field of magnitude $dE = \frac{|dq|}{4\pi\epsilon_0 R^2}$, where ϵ_0 is the permittivity of free space.

Step 1: Pair opposite charge elements Choose a small element of the ring and its diametrically opposite element. Uniform linear charge density means that equal arc lengths carry equal charges. Both elements are at the same distance R from the centre, so their fields there have equal magnitudes.

Step 2: Add the field vectors The two fields point in opposite directions along the same diameter and cancel. Every element can be paired in this way, so the cancellation applies to the complete ring. Options (A), (C) and (D) are nonzero expressions; none includes the cancellation required by circular symmetry. Adding field magnitudes instead of vectors would incorrectly predict a nonzero field.

Step 3: Conclusion The net electric field at the centre is $E = 0$. Hence, the correct option is (B).

Quick Tip to Solve: At the centre of a uniformly charged complete ring, pair diametrically opposite elements before attempting any integration.

2. Ten capacitors, each of capacitance $1\ \mu\text{F}$, are connected in parallel to a source of 100 V. The total energy stored in the system is equal to: [1 mark]

- (A) 10^{-2} J
(B) 10^{-3} J
(C) $0.5 \times 10^{-3}\text{ J}$
(D) $5.0 \times 10^{-2}\text{ J}$

Correct Answer: (D) $5.0 \times 10^{-2}\text{ J}$

Solution:

Concept:

- Capacitors in parallel have the same potential difference V , and their capacitances add: $C_{\text{eq}} = \sum C_i$.
- The stored electrostatic energy is $U = \frac{1}{2}C_{\text{eq}}V^2$; use capacitance in farads and voltage in volts.

Step 1: Find the equivalent capacitance All ten capacitors are connected across the same source. Thus

$$C_{\text{eq}} = 10(1 \mu\text{F}) = 10 \times 10^{-6} \text{ F} = 10^{-5} \text{ F}.$$

The capacitance increases in parallel; the voltage across each capacitor remains 100 V.

Step 2: Calculate the stored energy Substitute the total capacitance and the common voltage:

$$U = \frac{1}{2}(10^{-5})(100)^2 = 5.0 \times 10^{-2} \text{ J}.$$

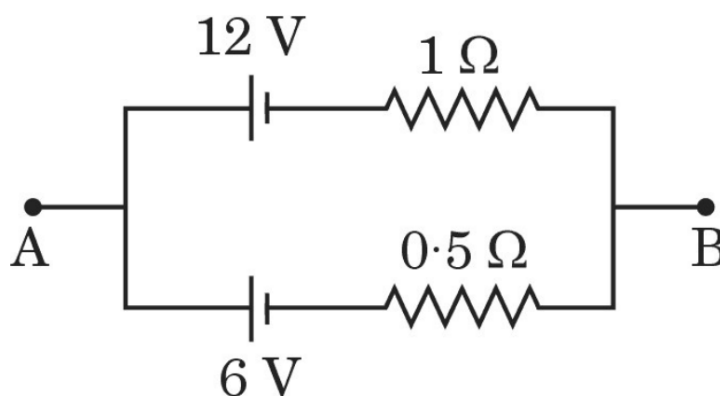
The voltage must be squared. Option (A) is smaller by a factor of five, while (B) and (C) are smaller by factors of fifty and one hundred. None follows from the parallel capacitance and energy formula.

Step 3: Conclusion The total energy stored is $5.0 \times 10^{-2} \text{ J}$. Hence, the correct option is (D).

Quick Tip to Solve: For identical parallel capacitors, multiply the energy of one capacitor by their number; all have the full source voltage.

3. Consider the circuit shown in the figure. The potential difference between points A and B is:

[1 mark]



- (A) 6 V
- (B) 8 V
- (C) 9 V
- (D) 12 V

Correct Answer: (B) 8 V

Solution:

Concept:

- The loop rule of Kirchhoff states that the algebraic sum of potential changes around a closed circuit is zero.
- A cell delivering current has terminal voltage $V = E - Ir$; a cell being charged has $V = E + Ir$, where E is emf and r is its series resistance.

Step 1: Find the circulating current Both cells have their positive terminals towards A. Their emfs oppose each other around the closed loop. The 12 V cell drives current through the 6 V cell, so

$$I = \frac{12 - 6}{1 + 0.5} = 4 \text{ A}.$$

In the upper branch current flows from B towards A, and in the lower branch it flows from A towards B.

Step 2: Calculate the terminal potential difference The stronger cell is delivering current, while the weaker cell is being charged. Either branch gives

$$V_A - V_B = 12 - (4)(1) = 8 \text{ V},$$

$$V_A - V_B = 6 + (4)(0.5) = 8 \text{ V}.$$

Options (A) and (D) ignore the resistor voltage drops. Option (C), the simple mean of the emfs, would apply only if the two series resistances were equal.

Step 3: Conclusion A is at the higher potential and $V_A - V_B = 8 \text{ V}$. Hence, the correct option is (B).

Quick Tip to Solve: For unequal cells connected positive-to-positive, first find the circulating current from the emf difference and total loop resistance.

4. A loop carrying a current I clockwise is placed in $x - y$ plane, in a uniform magnetic field directed along z -axis. The tendency of the loop will be to: [1 mark]

- (A) move along x -axis
- (B) move along y -axis
- (C) shrink
- (D) expand

Correct Answer: (C) shrink

Solution:

Concept:

- The magnetic force on a current element is $d\vec{F} = I d\vec{\ell} \times \vec{B}$, where $d\vec{\ell}$ points along the conventional current.
- A closed loop in a uniform magnetic field has zero resultant translational force, but local forces can tend to change its size.

Step 1: Determine the local force direction Take the field along the positive z -axis and view the clockwise current from the positive z side, as usual. At the rightmost part of the loop the current points along $-\hat{j}$. The force direction is therefore

$$(-\hat{j}) \times \hat{k} = -\hat{i},$$

towards the inside of the loop.

Step 2: Extend the result around the loop At the leftmost part the current is along $+\hat{j}$, giving a force along $+\hat{i}$, again inward. The same cross-product rule makes every local force point inward. Opposite forces cancel in the vector sum, so there is no translation along x or y ; this rejects (A) and (B). Option (D) would require outward local forces and would occur upon reversing either the current or the field.

Step 3: Conclusion The inward magnetic forces tend to shrink the loop. Hence, the correct option is (C).

Quick Tip to Solve: Check the force on the rightmost current element with the cross-product rule; inward or outward there identifies contraction or expansion.

5. A 10 cm long wire lies along y-axis. It carries a current of 1.0 A in positive y-direction. A magnetic field $\vec{B} = (5 \text{ mT})\hat{j} - (8 \text{ mT})\hat{k}$ exists in the region. The force on the wire is: [1 mark]

- (A) $(0.8 \text{ mN})\hat{i}$
- (B) $-(0.8 \text{ mN})\hat{i}$
- (C) $(80 \text{ mN})\hat{i}$
- (D) $-(80 \text{ mN})\hat{i}$

Correct Answer: (B) $-(0.8 \text{ mN})\hat{i}$

Solution:

Concept:

- The force on a straight current-carrying wire is $\vec{F} = I\vec{L} \times \vec{B}$, where $\vec{L}$ has the length of the wire and points along the current.
- Parallel vectors have zero cross product; for Cartesian unit vectors, $\hat{j} \times \hat{k} = \hat{i}$.

Step 1: Convert the data to SI units The current flows along the positive y-axis, so

$$\vec{L} = 0.10\hat{j} \text{ m}, \quad \vec{B} = (5 \times 10^{-3}\hat{j} - 8 \times 10^{-3}\hat{k}) \text{ T}.$$

The y-component of the field is parallel to the wire and cannot exert a magnetic force.

Step 2: Evaluate the cross product Only the negative z-component contributes:

$$\begin{aligned}\vec{F} &= (1.0)(0.10)[-8 \times 10^{-3}(\hat{j} \times \hat{k})] \text{ N} \\ &= -8.0 \times 10^{-4}\hat{i} \text{ N} = -0.8\hat{i} \text{ mN}.\end{aligned}$$

Option (A) has the wrong direction. Options (C) and (D) have a magnitude one hundred times too large, as would result from using 10 as the length in metres instead of converting centimetres.

Step 3: Conclusion The force is $\vec{F} = -(0.8 \text{ mN})\hat{i}$. Hence, the correct option is (B).

Quick Tip to Solve: Drop the field component parallel to the current, convert 10 cm to 0.10 m, then use the cyclic order of unit-vector cross products.

6. A galvanometer of resistance $G \Omega$ is converted into an ammeter of range 0 to I A. If the current through the galvanometer is 0.1% of I A, the resistance of the ammeter is: [1 mark]

- (A) $\frac{G}{999} \Omega$
(B) $\frac{G}{1000} \Omega$
(C) $\frac{G}{1001} \Omega$
(D) $\frac{G}{100.1} \Omega$

Correct Answer: (B) $\frac{G}{1000} \Omega$

Solution:

Concept:

- A galvanometer is converted into an ammeter by connecting a low shunt resistance in parallel with it.
- The galvanometer and shunt have a common voltage $V = I_g G$. The effective ammeter resistance is $R_A = V/I$, where I is the total current.

Step 1: Express the galvanometer current The given fraction is a percentage, so

$$I_g = \frac{0.1}{100} I = \frac{I}{1000}.$$

The shunt carries the remaining current $I_s = I - I_g = 999I/1000$. Therefore the voltage across the complete ammeter equals $V = I_g G = IG/1000$.

Step 2: Find the resistance of the combination Use the total current, rather than only the shunt current, to calculate the effective resistance:

$$R_A = \frac{V}{I} = \frac{G}{1000}.$$

The value $G/999$ in (A) is the shunt resistance V/I_s , not the resistance of the whole ammeter. Options (C) and (D) do not follow from the stated current fraction and the parallel connection.

Step 3: Conclusion The ammeter resistance is $\boxed{\frac{G}{1000} \Omega}$. Hence, the correct option is (B).

Quick Tip to Solve: When the galvanometer carries a fraction f of the total current, the complete ammeter resistance is fG ; the shunt resistance is different.

7. The reactance of a capacitor of capacitance C connected to an ac source of frequency ω is X . If the capacitance of the capacitor is doubled and the frequency of the source is tripled, the reactance will become:

[1 mark]

- (A) $\frac{X}{6}$
- (B) $6X$
- (C) $\frac{2}{3}X$
- (D) $\frac{3}{2}X$

Correct Answer: (A) $\frac{X}{6}$

Solution:

Concept:

- Capacitive reactance is $X_C = 1/(\omega C)$, where ω is angular frequency and C is capacitance.
- Reactance is inversely proportional to both frequency and capacitance. If ordinary frequency f is used, $X_C = 1/(2\pi fC)$, giving the same scaling ratio.

Step 1: Identify the changes in the denominator Initially $X = 1/(\omega C)$. The new capacitance is $C' = 2C$, and the new source frequency is $\omega' = 3\omega$. The product of frequency and capacitance therefore becomes six times its original value.

Step 2: Compare the new and old reactances Taking the ratio avoids any need for numerical values:

$$\frac{X'}{X} = \frac{\omega C}{(3\omega)(2C)} = \frac{1}{6}.$$

Option (B) incorrectly treats reactance as directly proportional to the product. Options (C) and (D) use a ratio of the two changes instead of multiplying their effects in the denominator. Both increases reduce the reactance, and together they reduce it by a factor of six.

Step 3: Conclusion The new reactance is $X' = X/6$. Hence, the correct option is (A).

Quick Tip to Solve: For capacitive reactance, multiply the frequency and capacitance scale factors, then divide the original reactance by that product.

8. In the four regions, I, II, III and IV, the electric fields are described as:

Region I: $E_x = E_0 \sin(kz - \omega t)$

Region II: $E_x = E_0$

Region III: $E_x = E_0 \sin kz$

Region IV: $E_x = E_0 \cos kz$

The displacement current will exist in the region:

[1 mark]

- (A) I
- (B) IV
- (C) II
- (D) III

Correct Answer: (A) I

Solution:

Concept:

- Displacement current arises from a time-varying electric flux: $I_d = \varepsilon_0 d\Phi_E/dt$, where Φ_E is electric flux.
- Locally, the displacement-current density is $\mathbf{J}_d = \varepsilon_0 \partial \mathbf{E}/\partial t$. Spatial variation alone does not produce displacement current.

Step 1: Differentiate the field in Region I At a fixed position z , the field in Region I varies with time. Its derivative is

$$\frac{\partial E_x}{\partial t} = -\omega E_0 \cos(kz - \omega t).$$

This is generally nonzero, so this region supports an oscillating displacement current. Its instantaneous value may vanish at particular phases.

Step 2: Check the remaining three regions Region II has a constant electric field. Regions III and IV vary with position z , but neither contains time t . Thus $\partial E_x/\partial t = 0$ in all three regions. Options (B), (C) and (D) therefore do not describe a region with displacement current.

Step 3: Conclusion Displacement current exists in Region I. Hence, the correct option is (A).

Quick Tip to Solve: Differentiate with respect to time at a fixed position; a sine or cosine containing only position is not enough.

9. The transition of electron that gives rise to the formation of the second spectral line of the Balmer series in the spectrum of hydrogen atom corresponds to: [1 mark]

- (A) $n_f = 2$ and $n_i = 3$
- (B) $n_f = 3$ and $n_i = 4$
- (C) $n_f = 2$ and $n_i = 4$
- (D) $n_f = 2$ and $n_i = \infty$

Correct Answer: (C) $n_f = 2$ and $n_i = 4$

Solution:

Concept:

- The Balmer series consists of hydrogen emission transitions ending at the final level $n_f = 2$.
- Successive lines correspond to initial levels $n_i = 3, 4, 5, \dots$; the first line uses the nearest higher level.

Step 1: Identify the final and initial levels Every Balmer line must end at $n_f = 2$. The first line is produced by $3 \rightarrow 2$, so the second line is produced by the next initial level, $4 \rightarrow 2$. The electron loses energy and emits a photon during this downward transition.

Step 2: Compare all four alternatives Option (A), $3 \rightarrow 2$, is the first Balmer line. Option (B), $4 \rightarrow 3$, belongs to the Paschen series because its final level is 3. Option (C) specifies $4 \rightarrow 2$. Option (D), $\infty \rightarrow 2$, is the Balmer series limit, not its second line.

Step 3: Conclusion The required transition has $n_i = 4$ and $n_f = 2$. Hence, the correct option is (C).

Quick Tip to Solve: For Balmer lines, keep the final level at 2 and count initial levels from 3: first 3, second 4.

10. Ge is doped with As. Due to doping, [1 mark]

- (A) the structure of Ge lattice is distorted.
- (B) the number of conduction electrons increases.
- (C) the number of holes increases.
- (D) the number of conduction electrons decreases.

Correct Answer: (B) the number of conduction electrons increases.

Solution:

Concept:

- Germanium is tetravalent: each atom forms four covalent bonds in the semiconductor crystal.
- Arsenic is pentavalent. As a donor impurity, it supplies an extra electron and produces an n-type semiconductor.

Step 1: Count the donor's valence electrons An arsenic atom substitutes for a germanium atom. Four of its five valence electrons participate in the surrounding covalent bonds. The fifth is weakly bound and can enter the conduction band with a small supply of thermal energy.

Step 2: Identify the carrier population that increases Donor doping therefore increases the number of conduction electrons, as stated in option (B). Holes do not become the majority carriers, so (C) is incorrect, while (D) states the opposite electron trend. In the standard dilute-doping model the host crystal structure is retained; lattice distortion in (A) is not the intended effect of doping.

Step 3: Conclusion Doping Ge with As increases the number of conduction electrons. Hence, the correct option is (B).

Quick Tip to Solve: Compare valencies: a five-valence-electron impurity in a four-valence-electron crystal donates the spare electron.

11. Two beams, A and B whose photon energies are 3.3 eV and 11.3 eV respectively, illuminate a metallic surface (work function 2.3 eV) successively. The ratio of maximum speed of electrons emitted due to beam A to that due to beam B is: [1 mark]

(A) 3

(B) 9

(C) $\frac{1}{3}$

(D) $\frac{1}{9}$

Correct Answer: (C) $\frac{1}{3}$

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = E_{\gamma} - \phi$, where E_{γ} is photon energy and ϕ is the metal's work function.
- For the same electron mass m_e , $K_{\max} = \frac{1}{2}m_e v_{\max}^2$, so the ratio of maximum speeds is the square root of the kinetic-energy ratio.

Step 1: Subtract the work function for each beam Both photon energies exceed the work function, so both beams emit photoelectrons. Their maximum kinetic energies are

$$K_A = (3.3 - 2.3) \text{ eV} = 1.0 \text{ eV},$$

$$K_B = (11.3 - 2.3) \text{ eV} = 9.0 \text{ eV}.$$

Step 2: Convert the energy ratio into the speed ratio The mass and the conversion factor from electronvolts to joules cancel in the ratio:

$$\frac{v_A}{v_B} = \sqrt{\frac{K_A}{K_B}} = \sqrt{\frac{1}{9}}.$$

Option (D) gives the energy ratio without taking its square root. Options (A) and (B) reverse the required A-to-B ratio, and (B) also confuses energy with speed.

Step 3: Conclusion The required maximum-speed ratio is $v_A/v_B = 1/3$. Hence, the correct option is (C).

Quick Tip to Solve: Subtract the same work function first, then take the square root; photon-energy ratios are not electron-speed ratios.

12. The waves associated with a moving electron and a moving proton have the same wavelength λ . It implies that they have the same: [1 mark]

- (A) momentum
- (B) angular momentum
- (C) speed
- (D) energy

Correct Answer: (A) momentum

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of linear momentum.
- For nonrelativistic particles, $v = p/m$ and $K = p^2/(2m)$; equal momenta need not imply equal speeds or kinetic energies when masses differ.

Step 1: Apply the wavelength relation to both particles The same value of λ gives

$$p_e = \frac{h}{\lambda}, \quad p_p = \frac{h}{\lambda}.$$

Thus the two particles have equal momentum magnitudes. Their directions of motion are unspecified, so equality of their momentum vectors is not implied.

Step 2: Exclude the other proposed equalities The proton and electron have different masses. At equal momentum, their speeds and kinetic energies differ, excluding (C) and (D). Angular momentum also depends on the position vector relative to a chosen origin, $\mathbf{L} = \mathbf{r} \times \mathbf{p}$; equal wavelengths alone provide no such positional information, excluding (B).

Step 3: Conclusion The particles have the same magnitude of linear momentum. Hence, the correct option is (A).

Quick Tip to Solve: Equal matter wavelengths immediately mean equal momentum magnitudes; only then use mass to compare speeds or energies.

13. Assertion (A): In photoelectric effect, the kinetic energy of the emitted photoelectrons increases with increase in the intensity of the incident light.

Reason (R): Photoelectric current depends on the wavelength of the incident light.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: As printed, Assertion (A) is false but Reason (R) can be true; this combination is absent from the options. Option (D) is the conventional intended answer if wavelength effects on photocurrent are excluded.

Solution:

Concept:

- Einstein's equation is $K_{\max} = h\nu - \phi$, where ν is frequency and ϕ is work function. At fixed frequency, increasing intensity does not increase the maximum kinetic energy.
- Photocurrent measures charge emitted and collected per unit time. It depends on photon flux, emission efficiency and collection conditions; emission also requires $hc/\lambda > \phi$.

Step 1: Evaluate the assertion Increasing intensity at a fixed frequency supplies more photons per second, but the energy per photon remains $h\nu$. It increases the number of emitted electrons under ordinary conditions, not their maximum kinetic energy. Assertion (A) is false.

Step 2: Evaluate the reason as printed The reason gives no fixed wavelength range or collection conditions. Wavelength can affect photocurrent: beyond the threshold wavelength there is no emission. Even within the emission range, for incident power P , fixed quantum efficiency η , and complete collection, the current is $I = e\eta P\lambda/(hc)$. Thus the unrestricted statement that current depends on wavelength can be true. The usual school-level contrast associates current with intensity and kinetic energy with frequency, but it does not establish universal wavelength independence.

Step 3: Check the explanatory link and the available codes A statement about the number of collected electrons cannot explain the false claim that intensity raises their kinetic energy. Options (A), (B) and (C) all require a true assertion. Option (D) requires both statements to be false. The printed options omit the physically admissible combination of a false assertion and a true reason. Under the simplified textbook convention that ignores wavelength effects on current, (D) is the intended code.

Step 4: Conclusion The assertion is false. The reason is not generally false as printed, so there is no unqualified correct option. The conventional intended answer is (D), subject to the simplified current-versus-intensity convention.

Quick Tip to Solve: Separate energy per electron from electrons per second; when judging current, check the threshold and what is held fixed.

14. Assertion (A): The mutual inductance between two coils is maximum when the coils are wound on each other.

Reason (R): The flux linkage between two coils is maximum when they are wound on each other.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- Mutual inductance is the flux linkage in one coil per unit current in the other: $M = N_2\Phi_{21}/I_1$, where $N_2\Phi_{21}$ is the linked flux due to current I_1 .
- For given coils, closer magnetic coupling increases their mutual inductance. Winding one coil over the other minimises flux leakage.

Step 1: Evaluate the assertion When the two coils are wound on each other with aligned axes, a maximum fraction of the magnetic flux produced by either coil threads the other. For the given coils and medium, this arrangement gives maximum mutual inductance. Assertion (A) is therefore true.

Step 2: Evaluate the reason Overlapping windings provide maximum shared flux for a given current because their turns enclose essentially the same flux region. Separating or misaligning the coils reduces this linkage. Reason (R) is therefore true in the stated comparison.

Step 3: Test whether the reason explains the assertion From $M = N_2\Phi_{21}/I_1$, greater linked flux at the same current directly means greater mutual inductance. The reason thus supplies the physical explanation of the assertion. Option (B) wrongly denies this link; (C) and (D) incorrectly classify one or both statements as false.

Step 4: Conclusion Both statements are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: For mutual-inductance questions, compare linked flux per unit current; overlapping coils maximise the shared flux.

15. Assertion (A): Two long parallel wires, freely suspended and connected in series to a battery, move apart.

Reason (R): Two wires carrying current in opposite directions repel each other.

[1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- Parallel wires carrying currents in the same direction attract; opposite currents repel.
- For long wires separated by distance d , the magnetic force per unit length is $F/L = \mu_0 I_1 I_2 / (2\pi d)$, where I_1, I_2 are the current magnitudes.

Step 1: Evaluate the assertion In the usual series arrangement, the two suspended parallel wires form the outgoing and return sides of the circuit: current flows down one wire and up the other. They therefore experience forces away from each other and move apart. Assertion (A) is true.

Step 2: Evaluate the reason The magnetic field of either wire acts on the current in the other. Applying the right-hand rule and $\vec{F} = I\vec{L} \times \vec{B}$ shows that the forces are repulsive when the currents are opposite. Reason (R) is true.

Step 3: Check the explanatory link The series connection gives opposite current directions along the adjacent wires, so the repulsion stated in R directly explains their separation in A. Option (B) incorrectly denies this link, while (C) and (D) incorrectly judge a true statement false.

Step 4: Conclusion Both statements are true, and R correctly explains A. Hence, the correct option is (A).

Quick Tip to Solve: Trace the current round the whole series loop: its two parallel sides carry opposite currents.

16. Assertion (A): Plane and convex mirrors cannot produce real images under any circumstance.

Reason (R): A virtual image cannot serve as an object to produce a real image.

[1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and Reason (R) is also false.

Solution:

Concept:

- A real image is formed where the outgoing rays actually meet. The familiar virtual images of plane and convex mirrors refer to real objects, not every possible incident beam.
- Converging incident rays correspond to a virtual object for a mirror. The mirror formula is $1/f = 1/v + 1/u$, with Cartesian signed focal length f , image distance v , and object distance u .

Step 1: Evaluate the assertion A plane mirror can reflect a converging beam to a real focus in front of it. A convex mirror can also form a real image for a suitable virtual object. For example, with $f = +10$ cm and $u = +5$ cm, the mirror formula gives $1/v = 1/10 - 1/5 = -1/10$, so $v = -10$ cm: a real image in front of the mirror. Thus the words “under any circumstance” make A false.

Step 2: Evaluate the reason Rays appearing to diverge from the virtual image made by one optical element can enter a converging lens. If that apparent source lies beyond the lens’s focal point, the lens brings those rays to an actual focus and forms a real image. Thus a virtual image can serve as the object for another optical element, and R is false.

Step 3: Check the explanatory link Both statements assert impossible outcomes that are physically possible with suitable incident rays or successive optical elements. Since both statements are false, R cannot explain A; options (A), (B), and (C), each of which treats A as true, are excluded.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: Test “under any circumstance” using a converging incident beam, which supplies a virtual object.

Section B (10 marks)

17. Find the temperature at which the resistance of a wire made of silver will be twice its resistance at 20°C . Take 20°C as the reference temperature and temperature coefficient of resistance of silver at $20^{\circ}\text{C} = 4.0 \times 10^{-3} \text{ K}^{-1}$. [2 marks]

Solution:

Concept:

- With reference temperature T_0 , the linear resistance law is $R_T = R_{T_0}[1 + \alpha_{T_0}(T - T_0)]$, where α_{T_0} is the temperature coefficient at that reference temperature.
- A temperature rise has the same numerical value in kelvin and degrees Celsius. The coefficient multiplies the rise $T - T_0$, not the final Celsius temperature.

Step 1: Apply the stated reference temperature Here $T_0 = 20^{\circ}\text{C}$, $\alpha_{20} = 4.0 \times 10^{-3} \text{ K}^{-1}$, and the required resistance is $R_T = 2R_{20}$. Substitute into the resistance law and cancel the nonzero reference resistance:

$$\begin{aligned} 2R_{20} &= R_{20}[1 + (4.0 \times 10^{-3})(T - 20)], \\ 2 &= 1 + (4.0 \times 10^{-3})(T - 20). \end{aligned}$$

Step 2: Calculate the rise and final temperature Doubling the resistance requires its fractional increase to equal one. Hence

$$T - 20 = \frac{2 - 1}{4.0 \times 10^{-3}} = 250^{\circ}\text{C}.$$

This is a rise of 250 K, so the final Celsius temperature is obtained by adding it to the given reference value:

$$T = 20 + 250 = 270^{\circ}\text{C}.$$

Step 3: Conclusion Using the given linear temperature law, the wire's resistance doubles at 270°C .

Quick Tip to Solve: For doubling, first find the rise $1/\alpha$; then add the stated reference temperature of 20°C .

18(a). Monochromatic light of frequency $5.0 \times 10^{14} \text{ Hz}$ passes from air into a medium of refractive index 1.5. Find the wavelength of the light (i) reflected, and (ii) refracted at the interface of the two media. [2 marks]

Solution:

Concept:

- At a stationary interface, both reflection and refraction preserve the frequency ν of monochromatic light. Wavelength obeys $\lambda = v/\nu$, where v is the speed in the relevant medium.
- Light travels approximately at $c = 3.0 \times 10^8 \text{ m s}^{-1}$ in air. In a medium of refractive index n , its speed is $v = c/n$.

Step 1: Find the reflected wavelength in air The reflected wave travels back through air. Its speed and frequency therefore remain those of the incident wave: c and $5.0 \times 10^{14} \text{ Hz}$. Thus

$$\lambda_{\text{reflected}} = \frac{c}{\nu} = \frac{3.0 \times 10^8}{5.0 \times 10^{14}} = 6.0 \times 10^{-7} \text{ m}.$$

Step 2: Find the refracted wavelength in the medium Refraction changes the propagation speed to

$$v = \frac{3.0 \times 10^8}{1.5} = 2.0 \times 10^8 \text{ m s}^{-1}.$$

The frequency stays fixed by the source, so

$$\lambda_{\text{refracted}} = \frac{v}{\nu} = \frac{2.0 \times 10^8}{5.0 \times 10^{14}} = 4.0 \times 10^{-7} \text{ m}.$$

The smaller wavelength is consistent with the smaller speed in the optically denser medium.

Step 3: Conclusion (i) Reflected wavelength: $6.0 \times 10^{-7} \text{ m} = 600 \text{ nm}$. (ii) Refracted wavelength: $4.0 \times 10^{-7} \text{ m} = 400 \text{ nm}$.

Quick Tip to Solve: Keep frequency fixed: reflection in air keeps λ , while transmission into index 1.5 divides it by 1.5.

OR

18(b). A plano-convex lens of focal length 16 cm is made of a material of refractive index 1.4. Calculate the radius of the curved surface of the lens. [2 marks]

Solution:

Concept:

- For a thin lens in air, the lensmaker's formula is $1/f = (n - 1)(1/R_1 - 1/R_2)$, where f is focal length, n is the lens refractive index, and R_1, R_2 are signed surface radii.
- A plane surface has infinite radius, so its reciprocal curvature is zero. A plano-convex lens in air has positive focal length when its refractive index exceeds one.

Step 1: Apply the plano-convex geometry Let light enter through the convex surface and leave through the plane surface. With the Cartesian sign convention, the first centre of curvature is to the right, so $R_1 = +R$; the plane second face gives $1/R_2 = 0$. The lensmaker's formula therefore reduces to

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R} - 0 \right) = \frac{n - 1}{R}.$$

Step 2: Substitute the focal length and refractive index Rearranging the reduced equation gives $R = (n - 1)f$. Since $f = 16 \text{ cm}$ and $n = 1.4$,

$$R = (1.4 - 1)(16 \text{ cm}) = 0.4 \times 16 \text{ cm} = 6.4 \text{ cm}.$$

Only one face contributes curvature. The formula for an equiconvex lens, in which both faces contribute, would incorrectly introduce an extra factor of two here.

Step 3: Conclusion The radius of the curved surface is 6.4 cm in magnitude.

Quick Tip to Solve: For a plano-convex lens in air, set the plane face's curvature to zero and use $R = (n - 1)f$.

19. An object is placed 30 cm in front of a concave mirror of radius of curvature 40 cm. Find the (i) position of the image formed and (ii) magnification of the image. [2 marks]

Solution:

Concept:

- For a spherical mirror, the focal length is half its radius of curvature: $f = R/2$. The Cartesian sign convention makes distances in front of this mirror negative.
- The mirror formula is $1/f = 1/v + 1/u$, where u and v are the signed object and image distances. Linear magnification is $M = -v/u$.

Step 1: Assign signs and find the image position The object is in front of the concave mirror, so $u = -30$ cm. Its focus is also in front, giving $f = -40/2 = -20$ cm. Substituting into the mirror formula,

$$\begin{aligned}\frac{1}{v} &= \frac{1}{f} - \frac{1}{u} = -\frac{1}{20} + \frac{1}{30} \\ &= -\frac{1}{60} \text{ cm}^{-1}, \quad v = -60 \text{ cm}.\end{aligned}$$

The negative sign places the real image on the object's side of the mirror.

Step 2: Calculate and interpret the magnification Use the signed distances, retaining the minus sign in the magnification formula:

$$M = -\frac{v}{u} = -\frac{-60}{-30} = -2.$$

The magnitude indicates an image twice the object's height; the negative sign means that it is inverted. This agrees with an object located between the focus and centre of curvature of a concave mirror.

Step 3: Conclusion (i) The image forms 60 cm in front of the mirror, with $v = -60$ cm. (ii) The magnification is $M = -2$: the image is real, inverted and twice the object's size.

Quick Tip to Solve: First halve the radius and assign a negative focal length for a concave mirror; then retain signed distances when evaluating $-v/u$.

20. Consider a neutron (mass m) of kinetic energy E and a photon of the same energy. Let λ_n and λ_p be the de Broglie wavelength of neutron and the wavelength of photon respectively. Obtain an expression for $\frac{\lambda_n}{\lambda_p}$. [2 marks]

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is momentum. For a non-relativistic neutron of mass m , kinetic energy satisfies $E = p_n^2/(2m)$.
- A photon has energy $E = p_p c = hc/\lambda_p$, where c is the speed of light. Its energy-momentum relation differs from that of a massive non-relativistic particle.

Step 1: Express the neutron wavelength in terms of its energy Applying the usual non-relativistic kinetic-energy relation for the neutron gives

$$p_n = \sqrt{2mE}, \quad \lambda_n = \frac{h}{\sqrt{2mE}}.$$

Here E denotes the neutron's kinetic energy, rather than its rest energy. Using $p_n = E/c$ would be inappropriate for this massive particle in the standard non-relativistic treatment.

Step 2: Divide by the photon wavelength The photon has the same specified energy E . Thus its wavelength is $\lambda_p = hc/E$. Taking the required ratio in the stated order,

$$\begin{aligned}\frac{\lambda_n}{\lambda_p} &= \frac{h}{\sqrt{2mE}} \frac{E}{hc} \\ &= \frac{1}{c} \sqrt{\frac{E}{2m}} = \sqrt{\frac{E}{2mc^2}}.\end{aligned}$$

Planck's constant cancels. The final ratio is dimensionless because both E and mc^2 have dimensions of energy.

Step 3: Conclusion The required ratio is $\frac{\lambda_n}{\lambda_p} = \sqrt{\frac{E}{2mc^2}}$, using the standard non-relativistic neutron relation $E = p_n^2/(2m)$.

Quick Tip to Solve: For equal energies, write the massive-particle wavelength as $h/\sqrt{2mE}$ and the photon wavelength as hc/E before taking their ratio.

21. Plot a graph showing the variation of current with voltage for the material GaAs. On the graph, mark the region where:

- (a) resistance is negative, and
- (b) Ohm's law is obeyed.

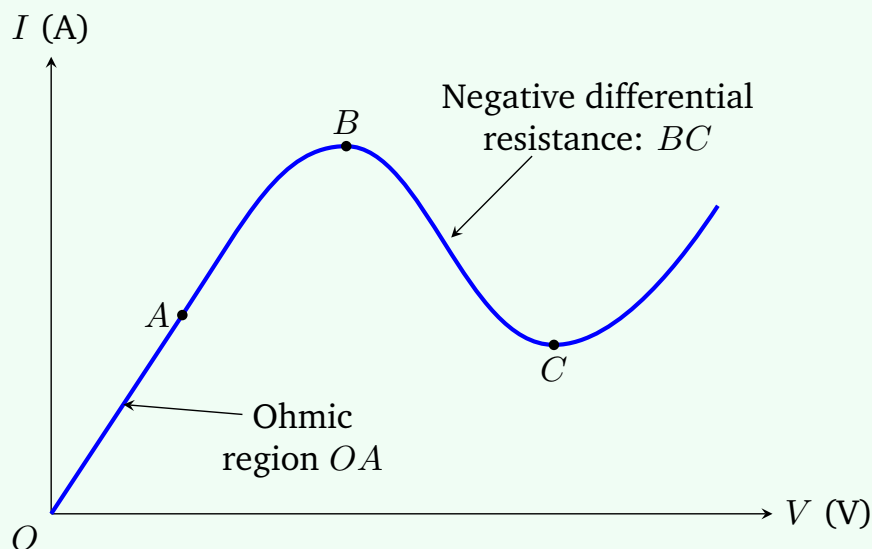
[2 marks]

Solution:

Concept:

- Ohm's law requires $I \propto V$ at fixed physical conditions, so an ohmic region is a straight line through the origin on an I -versus- V graph.
- A falling part of the characteristic has $dI/dV < 0$, hence negative differential resistance $r = dV/dI < 0$. The phrase negative resistance here refers to this differential resistance, not to the positive ratio V/I .

Step 1: Draw the characteristic and identify the ohmic part Put voltage V on the horizontal axis and current I on the vertical axis. For GaAs, the initial portion OA is a straight line through the origin. In this portion, equal increments in voltage produce equal increments in current, and the ratio V/I is constant. Therefore Ohm's law is obeyed along OA . The later rising section approaches the peak at B and is not all ohmic.



Schematic GaAs characteristic: OA is ohmic and BC has negative differential resistance.

Step 2: Identify the region of negative differential resistance After the peak B , the current decreases as the voltage increases, until the valley C is reached. Thus throughout the falling portion BC , a positive change in voltage corresponds to a negative change in current:

$$\frac{dI}{dV} < 0, \quad r = \frac{dV}{dI} < 0.$$

Beyond C , the current rises again. The falling portion, rather than the subsequent rising portion, is the requested negative-resistance region.

Step 3: Conclusion (a) The falling region BC has negative differential resistance. (b) The initial straight region OA obeys Ohm's law, as marked on the graph.

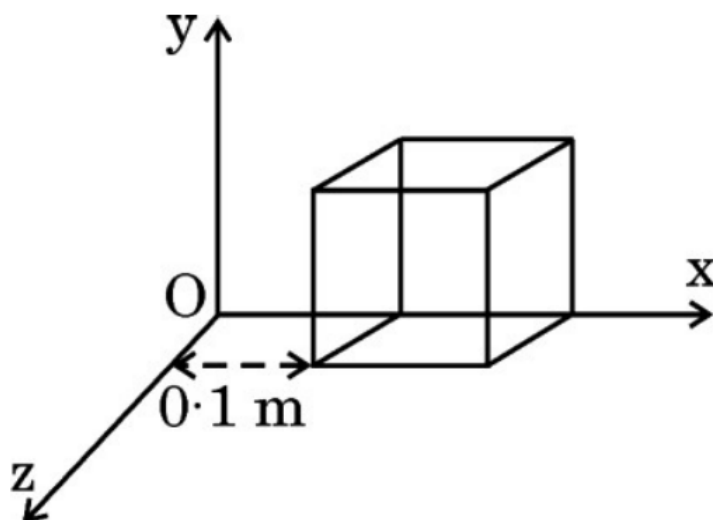
Quick Tip to Solve: On an I - V plot, identify negative differential resistance by a downward slope; identify Ohm's law by the initial straight segment through the origin.

Section C (21 marks)

22. A cube of side 0.1 m is placed, as shown in the figure, in a region where electric field $\vec{E} = 500x\hat{i}$ exists. Here x is in metres and E in NC^{-1} . Calculate:

- (a) the flux passing through the cube, and
(b) the charge within the cube.

[3 marks]



Solution:

Concept:

- Electric flux through a surface is $\Phi = \int \vec{E} \cdot d\vec{A}$, with the area vector directed outward for a closed surface. A field tangent to a face contributes zero flux through it.
- Gauss's law gives $\Phi_{\text{net}} = Q_{\text{in}}/\epsilon_0$, where Q_{in} is the net enclosed charge and $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$.

Step 1: Identify the contributing faces The cube has side $a = 0.1 \text{ m}$, so each face has area $A = a^2 = 0.01 \text{ m}^2$. The field $\vec{E} = 500x\hat{i}$ points along the positive x -direction. It is parallel to the four faces whose normals point along y or z , so their fluxes are zero. Only the two faces perpendicular to the x -axis contribute. The figure places these faces at $x_L = 0.1 \text{ m}$ and $x_R = 0.2 \text{ m}$.

Step 2: Calculate the net outward electric flux On each contributing face, x is constant, so the field is uniform over that face. The left outward normal is $-\hat{i}$, whereas the right outward normal is $+\hat{i}$. Therefore

$$\begin{aligned} E_L &= 500(0.1) = 50 \text{ N C}^{-1}, \\ E_R &= 500(0.2) = 100 \text{ N C}^{-1}, \\ \Phi_L &= -E_L A = -0.5 \text{ N m}^2\text{C}^{-1}, \\ \Phi_R &= +E_R A = +1.0 \text{ N m}^2\text{C}^{-1}. \end{aligned}$$

Adding all six faces gives

$$\Phi_{\text{net}} = \Phi_R + \Phi_L = 0.5 \text{ N m}^2\text{C}^{-1}.$$

The entering flux must be subtracted from the leaving flux, rather than added as a positive magnitude.

Step 3: Apply Gauss's law to find the charge Multiplying the net outward flux by the permittivity of free space gives

$$Q_{\text{in}} = \epsilon_0 \Phi_{\text{net}} = (8.854 \times 10^{-12})(0.5) = 4.427 \times 10^{-12} \text{ C}.$$

The enclosed charge is positive because more flux leaves the cube than enters it. As a check, the field gradient is constant, so the net flux can also be written as $500a^3 = 500(0.1)^3 = 0.5 \text{ N m}^2\text{C}^{-1}$.

Step 4: Conclusion (a) The net outward flux is $0.5 \text{ N m}^2\text{C}^{-1}$. (b) The charge enclosed by the cube is $+4.43 \times 10^{-12} \text{ C}$, approximately.

Quick Tip to Solve: For a field $E_x = kx$, subtract the fluxes of the two x -normal faces: the result is ka^3 for a cube of side a , independent of its starting x -coordinate.

23(a). Define ‘current density’. Is it a scalar or a vector? An electric field $\vec{E}$ is maintained in a metallic conductor. If n be the number of electrons (mass m , charge $-e$) per unit volume in the conductor and τ its relaxation time, show that the current density $\vec{j} = \alpha \vec{E}$, where $\alpha = \left(\frac{ne^2}{m}\right) \tau$. [3 marks]

Solution:

Concept:

- Current density is the current flowing per unit area normal to the direction of flow. For uniform flow, $j = I/A$, where I is current and A is the normal cross-sectional area.
- For charge carriers of number density n , charge q , and mean drift velocity $\vec{v}_d$, the current density is $\vec{j} = nq\vec{v}_d$.
- Relaxation time τ is the mean time between successive collisions that randomise an electron’s motion. The electric force on an electron is $-e\vec{E}$, where e is the positive magnitude of its charge.

Step 1: Define the direction and nature of current density Current density $\vec{j}$ is a vector directed along the flow of positive charge, that is, along conventional current. Its SI unit is A m^{-2} . For an area element $d\vec{A}$, the current through it is $dI = \vec{j} \cdot d\vec{A}$; thus current is a scalar even though current density is a vector. In a metal, conventional current is opposite to the electron drift velocity.

Step 2: Obtain the mean electron drift velocity An electron of mass m accelerates opposite to the applied electric field:

$$\vec{a} = \frac{-e\vec{E}}{m}.$$

The random thermal velocities have zero mean. Averaging the velocity acquired between collisions, using the relaxation time, gives

$$\vec{v}_d = \vec{a}\tau = -\frac{e\tau}{m}\vec{E}.$$

The minus sign describes electron motion opposite to the field; it must be retained when forming the current density.

Step 3: Relate drift motion to current density Since each electron carries charge $-e$,

$$\begin{aligned}\vec{j} &= n(-e)\vec{v}_d \\ &= n(-e)\left(-\frac{e\tau}{m}\vec{E}\right) \\ &= \frac{ne^2\tau}{m}\vec{E}.\end{aligned}$$

Both negative signs cancel, so the conventional current density points along $\vec{E}$. The positive proportionality factor is the electrical conductivity of the metal in this model.

Step 4: Conclusion Current density is a vector, and $\vec{j} = \alpha \vec{E}$, with $\alpha = ne^2\tau/m$, as required. The conductivity α has SI unit S m^{-1} .

Quick Tip to Solve: Keep both minus signs: electrons drift opposite to the field and carry negative charge, making current density parallel to the field.

OR

23(b). What is a Wheatstone bridge? Obtain the necessary conditions under which the Wheatstone bridge is balanced. [3 marks]

Solution:

Concept:

- A Wheatstone bridge is a network of four resistances forming a bridge, with a galvanometer across one diagonal and a cell across the other. It is used to determine an unknown resistance by a null-deflection method.
- At balance the galvanometer current is zero, so its two terminals are at equal potential. Ohm's law $V = IR$ then relates the potential drops in the four arms.

Step 1: Label the bridge and identify the balance condition Let the four junctions of the bridge be A, B, C, D in cyclic order. Put resistances P, Q, R, S in arms AB, BC, AD, DC , respectively, connect the cell across A and C , and the galvanometer between B and D . Take A at higher potential than C . When the galvanometer shows no deflection, $I_g = 0$, so $V_B = V_D$. The same current I_1 flows through P, Q , and the same current I_2 flows through R, S , because no current passes between the branches through the galvanometer.

Step 2: Equate the corresponding potential drops Since B and D are equipotential, the drops from A to these points are equal:

$$V_A - V_B = V_A - V_D \Rightarrow I_1 P = I_2 R.$$

Similarly, the drops from B and D to C are equal:

$$V_B - V_C = V_D - V_C \Rightarrow I_1 Q = I_2 S.$$

These equations follow directly from Ohm's law applied to the two series branches at the null position.

Step 3: Eliminate the branch currents Dividing the two equations eliminates I_1 and I_2 :

$$\frac{I_1 P}{I_1 Q} = \frac{I_2 R}{I_2 S} \Rightarrow \frac{P}{Q} = \frac{R}{S}.$$

Conversely, this ratio makes the two junctions divide the applied potential in the same proportion, so $V_B = V_D$ and no galvanometer current flows. Thus the resistance-ratio condition is sufficient as well as necessary for a nonzero applied voltage and finite positive arm resistances.

Step 4: Conclusion A Wheatstone bridge is balanced when $I_g = 0$, equivalently $V_B = V_D$, and its arm

resistances satisfy $\boxed{P/Q = R/S}$ or $\boxed{PS = QR}$. An unknown resistance S can then be found from $S = QR/P$.

Quick Tip to Solve: Label the two cell-to-cell branches before taking ratios: compare the upper-arm drop with the lower-arm drop in each branch.

24. A proton with kinetic energy 1.3384×10^{-14} J moving horizontally from north to south, enters a uniform magnetic field B of 2.0 mT directed eastward. Calculate:

- the speed of the proton
- the magnitude of acceleration of the proton
- the radius of the path traced by the proton

[Take (q/m) for proton = 1.0×10^8 C/kg]

[3 marks]

Solution:

Concept:

- The non-relativistic kinetic energy is $K = \frac{1}{2}m_p v^2$, where m_p is the proton mass and v is its speed.
- Magnetic force is $\vec{F} = q\vec{v} \times \vec{B}$. Here the velocity and field are perpendicular, so $F = qvB$ and $a = (q/m_p)vB$.
- The force remains perpendicular to the velocity and does no work. It supplies the centripetal force, giving $r = m_p v / (qB) = v / [(q/m_p)B]$.

Step 1: Calculate the speed from the kinetic energy For part (a), use the proton mass $m_p = 1.673 \times 10^{-27}$ kg from the paper's constants box and the printed kinetic energy:

$$\begin{aligned} v &= \sqrt{\frac{2K}{m_p}} \\ &= \sqrt{\frac{2(1.3384 \times 10^{-14})}{1.673 \times 10^{-27}}} \\ &= \sqrt{1.6000 \times 10^{13}} \text{ m s}^{-1} \\ &= 4.0 \times 10^6 \text{ m s}^{-1}. \end{aligned}$$

This speed is much smaller than the speed of light, consistent with using the classical kinetic-energy expression.

Step 2: Calculate the magnetic acceleration For part (b), southward motion is perpendicular to the eastward magnetic field. Convert $2.0 \text{ mT} = 2.0 \times 10^{-3} \text{ T}$ and use the specified $q/m_p = 1.0 \times 10^8 \text{ C kg}^{-1}$:

$$\begin{aligned} a &= \frac{q}{m_p} v B \\ &= (1.0 \times 10^8)(4.0 \times 10^6)(2.0 \times 10^{-3}) \\ &= 8.0 \times 10^{11} \text{ m s}^{-2}. \end{aligned}$$

The proton is positively charged, so the right-hand rule gives an initially upward force. Its acceleration magnitude is constant, while its direction changes as the proton turns.

Step 3: Calculate the radius of the circular path For part (c), there is no velocity component along the field, so the path is circular in a vertical north-south plane. Equating magnetic and centripetal

forces gives

$$\begin{aligned}qvB &= \frac{m_p v^2}{r}, \\r &= \frac{v}{(q/m_p)B} \\&= \frac{4.0 \times 10^6}{(1.0 \times 10^8)(2.0 \times 10^{-3})} \\&= 2.0 \times 10^1 \text{ m}.\end{aligned}$$

As a check, $v^2/r = (1.6 \times 10^{13})/20 = 8.0 \times 10^{11} \text{ m s}^{-2}$, agreeing with the magnetic acceleration.

Step 4: Conclusion (a) Speed: $4.0 \times 10^6 \text{ m s}^{-1}$. (b) Acceleration magnitude: $8.0 \times 10^{11} \text{ m s}^{-2}$. (c) Radius: 20 m .

Quick Tip to Solve: After finding the speed from kinetic energy, calculate $(q/m_p)B$ once: multiply the speed by it for acceleration and divide the speed by it for radius.

25. An inductor, a capacitor and a resistor are connected in series with an ac source $v = v_m \sin \omega t$. Derive an expression for the average power dissipated in the circuit. Also obtain the expression for the resonant frequency of the circuit. [3 marks]

Solution:

Concept:

- For a series LCR circuit, the impedance is $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$, where R , L and C are resistance, inductance and capacitance.
- The current is $i = i_m \sin(\omega t - \phi)$, with $i_m = v_m/Z$ and power factor $\cos \phi = R/Z$. The phase angle may be positive or negative.
- Over one complete cycle, $\langle \cos(2\omega t - \phi) \rangle = 0$. Root-mean-square values are $V_{\text{rms}} = v_m/\sqrt{2}$ and $I_{\text{rms}} = i_m/\sqrt{2}$.

Step 1: Express the instantaneous power Instantaneous power supplied by the source is the product of voltage and current. Using the trigonometric identity for the product of two sines,

$$\begin{aligned}p(t) &= v_m i_m \sin \omega t \sin(\omega t - \phi) \\&= \frac{v_m i_m}{2} [\cos \phi - \cos(2\omega t - \phi)].\end{aligned}$$

The first term is constant; the second represents an oscillating exchange of energy between the source and the circuit.

Step 2: Average the power over one cycle The oscillating cosine term averages to zero over a complete cycle. Hence,

$$\begin{aligned}P_{\text{av}} &= \frac{v_m i_m}{2} \cos \phi = V_{\text{rms}} I_{\text{rms}} \cos \phi \\&= I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2 R}{R^2 + (\omega L - 1/\omega C)^2}.\end{aligned}$$

Only the resistor dissipates net energy as heat. An ideal inductor and capacitor alternately store and return energy, so each consumes zero average power.

Step 3: Apply the condition for resonance In a series circuit, resonance occurs when the inductive and capacitive reactances cancel. The impedance then has its minimum value R , and the current is in

phase with the applied voltage:

$$\omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 = \frac{1}{LC}.$$

Since angular frequency and frequency obey $\omega_0 = 2\pi f_0$, this condition determines the resonant frequency.

Step 4: Conclusion The average power is $P_{av} = V_{rms} I_{rms} \cos \phi = I_{rms}^2 R$. The resonant angular frequency is $\omega_0 = 1/\sqrt{LC}$, and the resonant frequency is $f_0 = 1/(2\pi\sqrt{LC})$. At resonance, $\cos \phi = 1$.

Quick Tip to Solve: Use peak values with the factor $1/2$, or rms values without it. For resonance, set $X_L = X_C$, then divide angular frequency by 2π .

26. (a) “The wavelength of the electromagnetic wave is often correlated with the characteristic size of the system that radiates.” Give two examples to justify this statement.

(b) (i) Long distance radio broadcasts use short-wave bands. Why?

(ii) Optical and radio telescopes are built on the ground, but X-ray astronomy is possible only from satellites orbiting the Earth. Why? [3 marks]

Solution:

Concept:

- Electromagnetic radiation is produced by accelerating charges or by transitions between energy states. Its wavelength is often correlated with the characteristic dimensions of the emitting system.
- The atmosphere transmits some wavelength bands and absorbs others. Short-wave radio signals can also be returned to Earth by the ionosphere, permitting sky-wave communication.

Step 1: Give two examples of the wavelength-size correlation (a) Radio waves are produced by oscillating currents in antennas. An efficient half-wave dipole has length about $\lambda/2$; for example, a 10 m dipole can radiate waves of wavelength about 20 m. Thus radio wavelengths are associated with macroscopic antenna dimensions.

Gamma rays are emitted during transitions between nuclear energy levels. Their emitting system is the tiny atomic nucleus, and their wavelengths are extremely short compared with radio wavelengths. These examples link long wavelengths to a large radiating system and very short wavelengths to a very small one. The statement expresses a correlation; it does not require wavelength to equal source size in every process.

Step 2: Explain the use of short waves for distant broadcasts (b)(i) Short-wave radio signals of suitable frequency are returned towards Earth by the ionised layers of the upper atmosphere. This is commonly described as ionospheric reflection, arising from refraction through the ionosphere. A signal can therefore reach receivers beyond the horizon, and successive returns between the Earth and ionosphere allow very large distances to be covered. Direct line-of-sight propagation alone would not provide this range.

Step 3: Explain where the telescopes must be placed (b)(ii) The atmosphere has transmission windows in the visible region and in parts of the radio region. Radiation in these bands can reach the Earth's surface, so optical and suitable radio observations can be made with ground-based telescopes. X-rays from astronomical sources are strongly absorbed by the atmosphere before reaching the ground. X-ray telescopes must therefore be carried above the atmosphere by orbiting satellites to receive that radiation.

Step 4: Conclusion Radio antennas and atomic nuclei illustrate the wavelength-size correlation. Short-wave broadcasting exploits ionospheric return of radio waves. Ground-based optical and radio astronomy use atmospheric windows, whereas X-ray astronomy requires detectors above the absorbing atmosphere.

Quick Tip to Solve: For broadcasting, identify what returns the wave to Earth; for telescope placement, identify whether the atmosphere transmits or absorbs the radiation.

27. Write the drawbacks of Rutherford's atomic model. How did Bohr remove them? Show that different orbits in Bohr's atom are not equally spaced. [3 marks]

Solution:

Concept:

- In classical electromagnetism an accelerating charge radiates energy. An electron moving in a circular orbit has centripetal acceleration even when its speed is constant.
- Bohr postulated non-radiating stationary orbits with quantised angular momentum $m_e v r = n \hbar$, where $\hbar = h/(2\pi)$ and $n = 1, 2, 3, \dots$
- Radiation is emitted or absorbed only during a transition between stationary states, with photon energy $h\nu = |E_i - E_f|$. Coulomb attraction provides the centripetal force in a hydrogen atom.

Step 1: State Rutherford's two principal difficulties An orbiting electron is accelerated and, according to classical theory, must radiate continuously. It would lose mechanical energy, spiral towards the nucleus and collapse into it. Rutherford's model therefore could not explain the observed stability of atoms.

It also gave no rule for allowed orbits or discrete energies. During a classical inward spiral the orbital frequency would change continuously, so the model could not account for the distinct frequencies of atomic line spectra.

Step 2: Explain Bohr's resolution Bohr assumed that an electron in an allowed stationary orbit does not radiate energy, so it does not spiral into the nucleus. Only orbits satisfying $m_e v r = n \hbar$ are permitted. This restriction gives discrete energy states. A photon is emitted when an electron jumps from a higher to a lower state, with $h\nu = E_i - E_f$; the reverse transition requires absorption. Only particular energy differences occur, explaining the discrete spectral lines of hydrogen.

Step 3: Derive the allowed orbital radii For hydrogen, let $k = 1/(4\pi\epsilon_0)$, e be the magnitude of electron charge and m_e its mass. Combining Coulomb's law with the angular-momentum condition gives

$$\frac{ke^2}{r_n^2} = \frac{m_e v_n^2}{r_n}, \quad v_n = \frac{n\hbar}{m_e r_n},$$

$$ke^2 r_n = \frac{n^2 \hbar^2}{m_e}, \quad r_n = \frac{\hbar^2}{m_e k e^2} n^2 = a_0 n^2,$$

where a_0 is the Bohr radius. The separation of adjacent orbits is therefore

$$r_{n+1} - r_n = a_0 [(n+1)^2 - n^2] = a_0 (2n+1).$$

This separation grows with n : the first two gaps are $r_2 - r_1 = 3a_0$ and $r_3 - r_2 = 5a_0$. Hence successive circular orbits have increasing radial separations.

Step 4: Conclusion Rutherford's model failed to explain atomic stability and discrete line spectra.

Bohr's stationary states and transitions addressed these failures for hydrogen. Since $r_n = a_0 n^2$, the adjacent-orbit spacing $a_0(2n + 1)$ is not constant; different orbits are not equally spaced.

Quick Tip to Solve: To test equal spacing, subtract adjacent radii: use $r_{n+1} - r_n$, not just the ratio of radii. Bohr radii grow as n^2 .

28. (a) State any two properties of a nucleus.

(b) Why is the density of a nucleus much more than that of an atom?

(c) Show that the density of the nuclear matter is the same for all nuclei.

[3 marks]

Solution:

Concept:

- A nucleus contains Z protons and $A - Z$ neutrons, where Z is its atomic number and A is its mass number.
- Nuclear radius follows $R = R_0 A^{1/3}$, where R_0 is nearly constant; density is mass divided by volume.

Step 1: (a) State two nuclear properties First, the nucleus has positive charge $+Ze$, because each of its Z protons carries charge $+e$ and neutrons are uncharged. Second, its radius is approximately $R_0 A^{1/3}$, with $R_0 \simeq 1.2 \times 10^{-15}$ m. Thus its volume is proportional to the number of nucleons. These describe its charge and size, respectively.

Step 2: (b) Compare the occupied volumes Almost the entire mass of an atom is concentrated in its nucleus; the total electron mass is very small in comparison. However, the nucleus occupies a tiny fraction of the atomic volume. A typical atomic radius is of order 10^{-10} m, whereas nuclear radii are of order 10^{-15} to 10^{-14} m. For nearly the same mass, a much smaller volume gives a much greater density. In particular, $\rho_{\text{nucleus}}/\rho_{\text{atom}} \simeq (R_{\text{atom}}/R_{\text{nucleus}})^3 \gg 1$.

Step 3: (c) Eliminate the mass number from the density Taking the mass of each nucleon to be approximately m_N , the nuclear mass is $M \simeq Am_N$. Model the nucleus as a sphere and use the radius relation:

$$V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} R_0^3 A,$$
$$\rho = \frac{M}{V} = \frac{Am_N}{(4\pi/3)R_0^3 A} = \frac{3m_N}{4\pi R_0^3}.$$

The mass number cancels because both mass and volume grow in the same proportion. Using $m_N \simeq 1.67 \times 10^{-27}$ kg gives $\rho \simeq 2.3 \times 10^{17}$ kg m⁻³. Small binding-energy and nuclear-structure variations are neglected in this standard model.

Step 4: Conclusion A nucleus has charge $+Ze$ and radius $R_0 A^{1/3}$. Its very small volume explains its high density; $\rho = 3m_N/(4\pi R_0^3)$ is approximately the same for all nuclei because it is independent of A .

Quick Tip to Solve: Cube $R \propto A^{1/3}$ before calculating density: both nuclear volume and mass then contain the same factor A , which cancels.

Section D (8 marks)

Questions number 29 and 30 are case study-based questions. Read the following paragraphs and answer the questions that follow.

29. A lens is a transparent medium bounded by two surfaces, with one or both surfaces being spherical. The focal length of a lens is determined by the radii of curvature of its two surfaces and the refractive index of its medium with respect to that of the surrounding medium. The power of a lens is reciprocal of its focal length. If a number of lenses are kept in contact, the power of the combination is the algebraic sum of the powers of the individual lenses. [4 marks]

29(i). A double-convex lens, with each face having same radius of curvature R , is made of glass of refractive index n . Its power is: [1 mark]

- (A) $\frac{2(n-1)}{R}$
(B) $\frac{(2n-1)}{R}$
(C) $\frac{(n-1)}{2R}$
(D) $\frac{(2n-1)}{2R}$

Correct Answer: (A) $\frac{2(n-1)}{R}$

Solution:

Concept:

- For a thin lens in air, the lens-maker formula is $P = 1/f = (n-1)(1/R_1 - 1/R_2)$, where n is the glass refractive index.
- With light travelling from left to right, the first and second radii of a symmetric double-convex lens are $+R$ and $-R$.

Step 1: Assign the signed radii The two surfaces have equal radius magnitudes but opposite signed radii. Hence use $R_1 = R$ and $R_2 = -R$; using the same sign for both would incorrectly cancel the surface contributions.

Step 2: Substitute into the lens-maker formula

$$P = (n-1) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{2(n-1)}{R}.$$

Both curved surfaces contribute equally. Options (B) and (D) have the wrong refractive-index dependence: their power would not vanish at $n = 1$. Option (C) uses the wrong factor for two equal curved surfaces.

Step 3: Conclusion The power is $P = \frac{2(n-1)}{R}$, in dioptres when R is in metres. Hence, the correct option is (A).

Quick Tip to Solve: For a symmetric double-convex lens, use $+R, -R$: the two curvature contributions add rather than cancel.

29(ii). A double-convex lens of power P , with each face having same radius of curvature, is cut into two equal parts perpendicular to its principal axis. The power of one part of the lens will be: [1 mark]

- (A) $2P$
- (B) P
- (C) $4P$
- (D) $\frac{P}{2}$

Correct Answer: (D) $\frac{P}{2}$

Solution:

Concept:

- A thin lens in air has power $P = (n - 1)(1/R_1 - 1/R_2)$, determined by its surface curvatures.
- A cut perpendicular to the principal axis through the middle of a symmetric double-convex lens produces two equal plano-convex lenses. A plane surface has zero curvature, $1/R = 0$.

Step 1: Write the original power For the original equal-radius double-convex lens, the signed radii are $+R$ and $-R$. Thus $P = 2(n - 1)/R$. The two curved surfaces contribute equally to this power.

Step 2: Calculate the power of one half Each piece retains one original curved surface, while the other surface is the new plane cut. Therefore

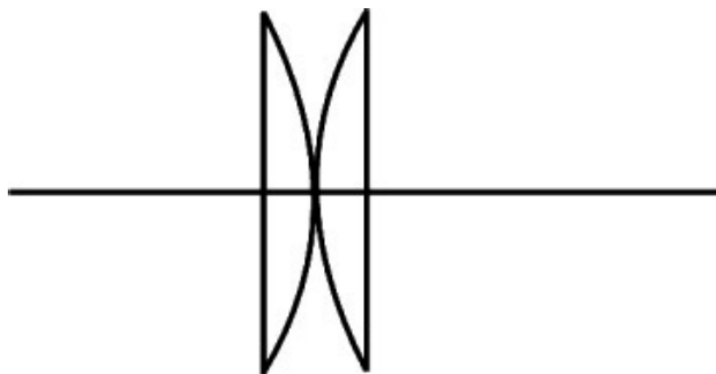
$$P_{\text{part}} = (n - 1) \left(\frac{1}{R} - 0 \right) = \frac{n - 1}{R} = \frac{P}{2}.$$

Option (B) would apply to a cut along the principal axis that preserves both curvatures. Options (A) and (C) incorrectly increase the power even though one curved-surface contribution has been removed.

Step 3: Conclusion One plano-convex part has power $P/2$, and its focal length is twice that of the original lens. Hence, the correct option is (D).

Quick Tip to Solve: Identify the cut direction first: a perpendicular central cut replaces one curved surface by a plane surface, halving this symmetric lens's power.

29(iii). The above two parts are kept in contact with each other as shown in the figure. The power of the combination will be: [1 mark]



- (A) $\frac{P}{2}$
- (B) P
- (C) $2P$
- (D) $\frac{P}{4}$

Correct Answer: (B) P

Solution:

Concept:

- Each plano-convex half obtained by the preceding perpendicular cut has power $P/2$.
- For thin lenses in contact, equivalent power is the algebraic sum $P_{\text{eq}} = P_1 + P_2$. Reversing a converging lens in air does not change the sign of its power.

Step 1: Identify the powers in the printed arrangement The figure shows the curved faces of the two plano-convex pieces facing each other, with plane faces on the outside. Each glass piece remains a converging lens of power $P/2$; facing the lenses in opposite directions does not turn either into a diverging lens.

Step 2: Add the powers of the contacting pieces Apply the thin-lens contact rule given in the passage:

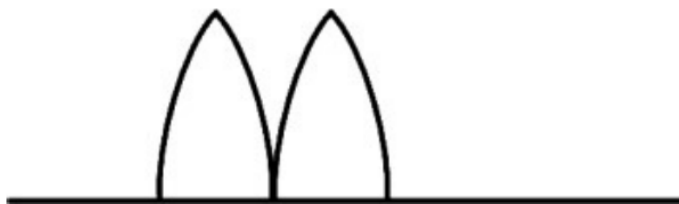
$$P_{\text{eq}} = \frac{P}{2} + \frac{P}{2} = P.$$

Option (A) counts only one piece. Option (C) wrongly assigns the original power P to each perpendicular-cut piece, and option (D) incorrectly halves the power again instead of adding the two positive contributions.

Step 3: Conclusion The combination has the original power P . Hence, the correct option is (B).

Quick Tip to Solve: Add the powers of the actual pieces: reversing a plano-convex lens does not change its positive power to a negative one.

29(iv)(a). A double-convex lens of power P , with each face having same radius of curvature, is cut along its principal axis. The two parts are arranged as shown in the figure. The power of the combination will be: [1 mark]



(A) Zero

(B) P

(C) $2P$

(D) $\frac{P}{2}$

Correct Answer: (C) $2P$

Solution:

Concept:

- Lens power depends on refractive index and the radii of the two refracting surfaces, not on the aperture area.
- Cutting a lens along its principal axis preserves the curvatures encountered by rays through either half. Thin lenses placed successively in contact have additive powers.

Step 1: Determine the power of each longitudinal half Each half retains both spherical faces with signed radii $+R$ and $-R$. The axial plane cut is not an entry or exit surface for rays through those faces. Thus

$$P_{\text{half}} = (n - 1) \left(\frac{1}{R} - \frac{1}{-R} \right) = P.$$

The smaller aperture reduces admitted light, while power is unchanged.

Step 2: Interpret the arrangement and add the powers The drawing places the halves successively along the axis, so rays pass through both. In the thin-lens contact approximation,

$$P_{\text{eq}} = P + P = 2P.$$

Both powers are positive, ruling out (A). Option (B) counts one half; (D) confuses an axial cut with a perpendicular cut.

Step 3: Conclusion The combination has power $2P$. Hence, the correct option is (C).

Quick Tip to Solve: An axial cut changes aperture, while a perpendicular cut changes a refracting surface; use the drawing to decide whether rays pass through one piece or both.

29(iv)(b). Two convex lenses of focal lengths 60 cm and 20 cm are held coaxially in contact with each other. The power of the combination is: [1 mark]

- (A) 6.6 D
 (B) 15 D
 (C) $\frac{1}{15}$ D
 (D) $\frac{1}{80}$ D

Correct Answer: (A) 6.6 D

Solution:

Concept:

- Power in dioptres is $P = 1/f$, with focal length f in metres; a convex lens in air has positive focal length and power.
- For coaxial thin lenses in contact, $P_{\text{eq}} = P_1 + P_2$. The focal lengths themselves are not added.

Step 1: Convert the focal lengths and calculate each power The given focal lengths are $f_1 = +0.60$ m and $f_2 = +0.20$ m. Therefore

$$P_1 = \frac{1}{0.60} = \frac{5}{3} \text{ D}, \quad P_2 = \frac{1}{0.20} = 5 \text{ D}.$$

The shorter-focal-length lens has the larger positive power.

Step 2: Add the powers and compare the printed choices

$$P_{\text{eq}} = \frac{5}{3} + 5 = \frac{20}{3} \text{ D} = 6.666 \dots \text{ D}.$$

The printed value 6.6 D in (A) is the intended approximate choice (the value rounds to 6.7 D at one decimal place). The equivalent focal length is 15 cm, which explains the distractor 15 but is not a power in dioptres. Option (C) takes the reciprocal of a focal length in centimetres without converting to metres. Option (D) incorrectly adds focal lengths and also omits that conversion.

Step 3: Conclusion The power is $20/3 \text{ D} \simeq 6.67 \text{ D}$, corresponding to the intended approximate printed choice 6.6 D. Hence, the correct option is (A).

Quick Tip to Solve: Convert centimetres to metres before taking reciprocals, then add powers; two positive lenses must give a power greater than either lens alone.

30. Junction Diode as a Rectifier:

The process of conversion of an ac voltage into a dc voltage is called rectification and the device which performs this conversion is called a rectifier. The characteristics of a p-n junction diode reveal that when a p-n junction diode is forward biased, it offers a low resistance and when it is reverse biased, it offers a high resistance. Hence, a p-n junction diode conducts only when it is forward biased. This property of a p-n junction diode makes it suitable for its use as a rectifier.

Thus, when an ac voltage is applied across a p-n junction, it conducts only during those alternate half cycles for which it is forward biased. A rectifier which rectifies only half cycle of an ac voltage is called a half-wave rectifier and one that rectifies both the half cycles is known as a full-wave rectifier.

[4 marks]

30(i). The root mean square value of an alternating voltage applied to a full-wave rectifier is $\frac{V_0}{\sqrt{2}}$.

Then the root mean square value of the rectified output voltage is:

[1 mark]

- (A) $\frac{V_0}{\sqrt{2}}$
- (B) $\frac{V_0^2}{\sqrt{2}}$
- (C) $\frac{2V_0}{\sqrt{2}}$
- (D) $\frac{V_0}{2\sqrt{2}}$

Correct Answer: (A) $\frac{V_0}{\sqrt{2}}$

Solution:

Concept:

- Root mean square voltage is $V_{\text{rms}} = \sqrt{\langle v^2 \rangle}$, where the average is taken over a complete period.
- An ideal full-wave rectifier reverses the negative half cycles without changing their magnitudes:
 $v_{\text{out}} = |v_{\text{in}}|$.

Step 1: Compare the squared voltages For a sinusoidal input of peak value V_0 , the output is $V_0 |\sin \omega t|$. Squaring removes the sign, so the input and output have identical squared values at every instant.

$$\langle v_{\text{out}}^2 \rangle = V_0^2 \langle \sin^2 \omega t \rangle = \frac{V_0^2}{2}.$$

Step 2: Check the alternatives Taking the positive square root leaves the RMS voltage unchanged. Option (B), as printed, contains V_0^2 and has the wrong dimensions for voltage. Option (C) incorrectly doubles the RMS value, while (D) incorrectly halves it. Rectifying both half cycles changes polarity, not the peak magnitude or the mean square.

Step 3: Conclusion The rectified RMS output voltage is $\boxed{V_0/\sqrt{2}}$. Hence, the correct option is (A).

Quick Tip to Solve: For ideal full-wave rectification, square the waveform first: reversing its negative portions cannot change its RMS value.

30(ii). In a full-wave rectifier, the current in each of the diodes flows for:

[1 mark]

- (A) Complete cycle of the input signal
- (B) Half cycle of the input signal
- (C) Less than half cycle of the input signal
- (D) Only for the positive half cycle of the input signal

Correct Answer: (B) Half cycle of the input signal

Solution:

Concept:

- In the standard two-diode full-wave rectifier, the centre-tapped secondary supplies opposite instantaneous polarities to the two diode anodes.
- A diode conducts when forward biased and blocks current when reverse biased, in the ideal diode approximation.

Step 1: Follow the two half cycles During one half cycle, the first diode is forward biased and supplies current through the load, while the second diode is reverse biased. During the next half cycle, their roles interchange. The load current retains the same direction in both intervals.

Step 2: Distinguish diode current from load current Each individual diode conducts for a duration $T/2$ in an input period T . Option (A) confuses a diode's conduction with the full-wave load output; (C) does not describe the ideal rectifier. Option (D) is wrong because one diode conducts on the positive input half cycle and the other on the negative half cycle, relative to the chosen input reference.

Step 3: Conclusion Current in each diode flows for half a cycle of the input signal. Hence, the correct option is (B).

Quick Tip to Solve: Track one diode for a complete input cycle: it conducts once and blocks once, while the two diodes share the load current alternately.

30(iii). In a full-wave rectifier:

[1 mark]

- (A) Both diodes are forward biased at the same time.
- (B) Both diodes are reverse biased at the same time.
- (C) One is forward biased and the other is reverse biased at the same time.
- (D) Both are forward biased in the first half of the cycle and reverse biased in the second half of the cycle.

Correct Answer: (C) One is forward biased and the other is reverse biased at the same time.

Solution:

Concept:

- The two ends of a centre-tapped transformer secondary have opposite instantaneous polarities relative to the centre tap.
- The standard two-diode full-wave rectifier uses alternate forward conduction so that load current has one direction during both half cycles.

Step 1: Determine the bias in either half cycle When one secondary end is positive relative to the centre tap, its diode is forward biased and conducts through the load. The other secondary end is negative, so its diode is reverse biased and blocks. When the input polarity reverses, the two diodes exchange roles.

Step 2: Compare the stated bias combinations The diodes therefore have opposite bias states during either half cycle. Options (A) and (B) wrongly give them the same bias simultaneously. Option (D) would make both conduct together and then both block, contradicting the alternate conduction needed for full-wave rectification. The instantaneous zero crossings do not alter this half-cycle description.

Step 3: Conclusion One diode is forward biased while the other is reverse biased at the same time. Hence, the correct option is (C).

Quick Tip to Solve: For a two-diode full-wave rectifier, write opposite signs at the two secondary ends; the diode connected to the positive end conducts.

30(iv)(a). An alternating voltage of frequency of 50 Hz is applied to a half-wave rectifier. Then the ripple frequency of the output will be: [1 mark]

- (A) 100 Hz
- (B) 50 Hz
- (C) 25 Hz
- (D) 150 Hz

Correct Answer: (B) 50 Hz

Solution:

Concept:

- Ripple frequency is the repetition rate of the pulses in the rectified output voltage.
- A half-wave rectifier passes only one half cycle per input period T , so its ripple frequency is $f_r = 1/T = f$, where f is the input frequency.

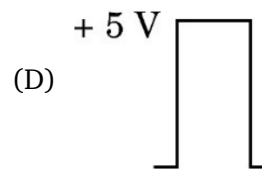
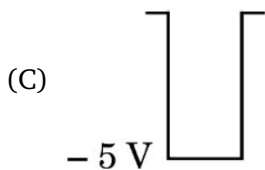
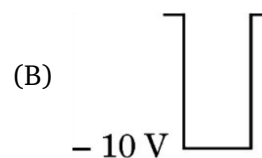
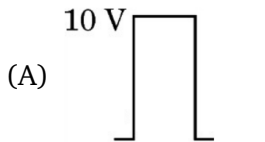
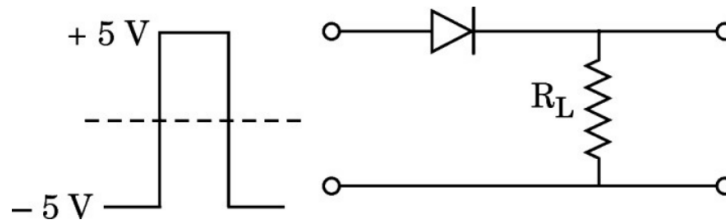
Step 1: Count the output pulses During one input half cycle the diode conducts; during the other it blocks. There is therefore one output pulse for every complete input cycle. Removing alternate half cycles does not double the interval between successive transmitted positive pulses.

Step 2: Calculate the repetition rate The input period is $T = 1/50 = 0.020$ s, and the output pattern repeats after the same interval. Thus $f_r = 1/(0.020 \text{ s})$. Option (A) would apply to full-wave rectification, which gives two pulses per input period. Option (C) incorrectly halves the frequency, and (D) would require three pulses per period.

Step 3: Conclusion The ripple frequency is 50 Hz. Hence, the correct option is (B).

Quick Tip to Solve: Count pulses in one complete input period: one gives the same ripple frequency; two give twice the input frequency.

30(iv)(b). A signal, as shown in the figure, is applied to a p-n junction diode. Identify the output across resistance R_L : [1 mark]



Correct Answer: (D)

Solution:

Concept:

- The diode in the given circuit has its anode connected to the input and its cathode connected to the upper terminal of R_L .
- An ideal forward-biased diode acts as a closed switch, whereas a reverse-biased diode acts as an open switch. The output is measured across R_L .

Step 1: Apply each input level When the input is $+5\text{ V}$, the diode is forward biased and passes the positive level to the load. When the input is -5 V , the diode is reverse biased, so load current is zero. Consequently the voltage drop across R_L is zero during that interval.

Step 2: Match the output waveform The output therefore consists of positive rectangular pulses rising from zero to $+5\text{ V}$. Option (A) incorrectly uses the input's peak-to-peak swing of 10 V as the pulse height. Options (B) and (C) show negative pulses, which this diode orientation blocks. No diode threshold voltage is specified, so the ideal diode model is used.

Step 3: Conclusion The output is the $+5\text{ V}$ positive rectangular pulse shown in option (D). Hence, the correct option is (D).

Quick Tip to Solve: Read the diode's cathode bar first, then retain only the input levels that forward bias it; the blocked intervals give zero load voltage.

Section E (15 marks)

31(a). (i) Derive an expression for potential energy of an electric dipole $\vec{p}$ in an external uniform electric field $\vec{E}$. When is the potential energy of the dipole (1) maximum, and (2) minimum?

(ii) An electric dipole consists of point charges -1.0 pC and $+1.0 \text{ pC}$ located at $(0, 0)$ and $(3 \text{ mm}, 4 \text{ mm})$ respectively in $x - y$ plane. An electric field $\vec{E} = \left(\frac{1000 \text{ V}}{\text{m}}\right) \hat{i}$ is switched on in the region. Find the torque $\vec{\tau}$ acting on the dipole. [5 marks]

Solution:

Concept:

- The dipole moment is $\vec{p} = q(\vec{r}_+ - \vec{r}_-)$, directed from negative to positive charge, where q is the magnitude of either charge.
- A uniform electric field exerts torque $\vec{\tau} = \vec{p} \times \vec{E}$. If θ is the angle between the two vectors, the torque magnitude is $pE \sin \theta$, tending to align the dipole with the field.
- The change in potential energy equals the work done by an external agent in rotating the dipole slowly against the electric torque. The conventional reference is $U = 0$ at $\theta = 90^\circ$.

Step 1: (i) Calculate the work of slow rotation Let an external agent rotate the dipole quasistatically from $\pi/2$ to θ , keeping the kinetic energy unchanged. Taking increasing θ as the positive angular direction, the electric torque component is $-pE \sin \theta$, so the balancing external torque is $+pE \sin \theta$. Thus

$$\begin{aligned} U(\theta) - U(\pi/2) &= \int_{\pi/2}^{\theta} pE \sin \theta' d\theta' \\ &= [-pE \cos \theta']_{\pi/2}^{\theta} = -pE \cos \theta. \end{aligned}$$

With the stated zero of energy, $U = -pE \cos \theta = -\vec{p} \cdot \vec{E}$. The negative sign means that alignment lowers the dipole's energy; no translational work is needed because a uniform field produces zero net force on the two charges.

Step 2: (i) Identify the extreme orientations Since $-1 \leq \cos \theta \leq 1$, the maximum potential energy is $+pE$ at $\theta = 180^\circ$, when the dipole moment is antiparallel to the field. This is unstable equilibrium: a small angular displacement allows the field to rotate the dipole farther away from this orientation. The minimum is $-pE$ at $\theta = 0^\circ$, when the vectors are parallel. This is stable equilibrium because a small displacement produces a restoring torque. The torque vanishes at both orientations, but their energies differ.

Step 3: (ii) Express the dipole moment in SI units The displacement from the negative charge at the origin to the positive charge is $(3\hat{i} + 4\hat{j}) \times 10^{-3} \text{ m}$, and $q = 1.0 \times 10^{-12} \text{ C}$. Hence

$$\vec{p} = (3\hat{i} + 4\hat{j}) \times 10^{-15} \text{ C m}.$$

Its magnitude is $5.0 \times 10^{-15} \text{ C m}$; the positive y component fixes the sign of the torque in the given positive x -directed field.

Step 4: (ii) Evaluate the vector product Use the printed field $\vec{E} = 1000\hat{i} \text{ V m}^{-1}$. The component of $\vec{p}$ parallel to $\vec{E}$ contributes no torque, while $\hat{j} \times \hat{i} = -\hat{k}$. Therefore

$$\begin{aligned} \vec{\tau} &= [(3\hat{i} + 4\hat{j}) \times 10^{-15}] \times (10^3\hat{i}) \\ &= -4.0 \times 10^{-12} \hat{k} \text{ N m}. \end{aligned}$$

The negative z direction is into the $x - y$ plane as viewed from the positive z side, corresponding to clockwise rotation toward the positive x -axis.

Step 5: Conclusion (i) $U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$; $U_{\max} = +pE$ at 180° , and $U_{\min} = -pE$ at 0° . (ii) $\vec{\tau} = -4.0 \times 10^{-12} \hat{k} \text{ N m}$.

Quick Tip to Solve: For a field along positive x , use $\tau_z = -p_y E_x$. Convert both picocoulombs and millimetres before forming the dipole moment.

OR

31(b). (i) An electric dipole (dipole moment $\vec{p} = p\hat{i}$), consisting of charges $-q$ and q , separated by distance $2a$, is placed along the x -axis, with its centre at the origin. Show that the potential V , due to this dipole, at a point x , ($x \gg a$) is equal to $\frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p} \cdot \hat{i}}{x^2}$.

(ii) Two isolated metallic spheres S_1 and S_2 of radii 1 cm and 3 cm respectively are charged such that both have the same charge density $\left(\frac{2}{\pi} \times 10^{-9}\right) \text{ C/m}^2$. They are placed far away from each other and connected by a thin wire. Calculate the new charge on sphere S_1 . [5 marks]

Solution:

Concept:

- Electric potential is a scalar and obeys superposition: a point charge Q produces $V = Q/(4\pi\epsilon_0 r)$, taking potential zero at infinity. The dipole moment has magnitude $p = 2aq$.
- A metallic sphere of radius R and surface charge density σ has total charge $Q = 4\pi R^2 \sigma$. For an isolated sphere its surface potential is $Q/(4\pi\epsilon_0 R)$.
- Connecting conductors by a wire equalises their potentials while conserving their total charge. For spheres far apart, mutual potential contributions can be neglected and the final charges are proportional to their radii.

Step 1: (i) Add the potentials of the two charges Because $\vec{p} = p\hat{i}$, place $-q$ at $x = -a$ and $+q$ at $x = +a$. At the positive axial point $x > a$, the distances from these charges are $x + a$ and $x - a$, respectively. With $k = 1/(4\pi\epsilon_0)$, scalar addition gives

$$\begin{aligned} V(x) &= \frac{kq}{x-a} - \frac{kq}{x+a} \\ &= kq \frac{(x+a) - (x-a)}{(x-a)(x+a)} \\ &= k \frac{2aq}{x^2 - a^2} = k \frac{p}{x^2 - a^2}. \end{aligned}$$

The charge signs are included explicitly; adding the magnitudes of the two potentials would incorrectly give the monopole-like contribution that cancels for a neutral dipole.

Step 2: (i) Use the distant-point approximation For $x \gg a$, the ratio a^2/x^2 is negligible, so $x^2 - a^2 \simeq x^2$. Since $\vec{p} \cdot \hat{i} = p$,

$$V(x) \simeq \frac{1}{4\pi\epsilon_0} \frac{p}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{i}}{x^2}.$$

This is the required expression, valid to the dipole approximation for the stated positive axial point. Its $1/x^2$ dependence follows from the cancellation of the leading $1/x$ contributions of equal and opposite

charges.

Step 3: (ii) Find the initial charges and their sum Convert the radii to $R_1 = 0.01$ m and $R_2 = 0.03$ m. The common initial density is $\sigma = (2/\pi) \times 10^{-9}$ C m⁻². Thus

$$\begin{aligned}Q_1 &= 4\pi(0.01)^2 \frac{2}{\pi} \times 10^{-9} = 8.0 \times 10^{-13} \text{ C}, \\Q_2 &= 4\pi(0.03)^2 \frac{2}{\pi} \times 10^{-9} = 7.2 \times 10^{-12} \text{ C}, \\Q_{\text{total}} &= Q_1 + Q_2 = 8.0 \times 10^{-12} \text{ C}.\end{aligned}$$

Initially the larger sphere has the higher potential because $V = \sigma R/\epsilon_0$ for equal surface charge density. Positive charge therefore transfers from S_2 to S_1 .

Step 4: (ii) Apply equal potential after connection Let the final charges be Q'_1 and Q'_2 . Ignoring the negligible charge stored by the thin connecting wire, electrostatic equilibrium requires

$$\frac{Q'_1}{R_1} = \frac{Q'_2}{R_2}, \quad \frac{Q'_1}{Q'_2} = \frac{1}{3}, \quad Q'_1 + Q'_2 = 8.0 \times 10^{-12} \text{ C}.$$

Substitute $Q'_2 = 3Q'_1$, giving $4Q'_1 = 8.0 \times 10^{-12}$ C. Hence $Q'_1 = 2.0 \times 10^{-12}$ C. As a check, $Q'_2 = 6.0 \times 10^{-12}$ C preserves the total and yields the same surface potential. Equal final potential does not require equal charge or equal surface charge density.

Step 5: Conclusion (i) For $x \gg a$, $V \simeq \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{i}}{x^2}$. (ii) The new charge on sphere S_1 is $+2.0 \times 10^{-12}$ C = +2.0 pC.

Quick Tip to Solve: Equal initial surface density gives charges in the ratio $R_1^2 : R_2^2 = 1 : 9$; connection changes the ratio to $R_1 : R_2 = 1 : 3$ while preserving their sum.

32(a). (i) A resistor and a capacitor are connected in series to an ac source $v = v_m \sin \omega t$. Derive an expression for the impedance of the circuit.

(ii) When does an inductor act as a conductor in a circuit? Give reason for it.

(iii) An electric lamp is designed to operate at 110 V dc and 11 A current. If the lamp is operated on 220 V, 50 Hz ac source with a coil in series, then find the inductance of the coil. [5 marks]

Solution:

Concept:

- In a series circuit, the same current flows through every component. Resistor voltage is in phase with current, whereas capacitor voltage lags current by 90° .
- Capacitive reactance is $X_C = 1/(\omega C)$, and inductive reactance is $X_L = \omega L = 2\pi fL$, where C is capacitance, L is inductance and f is frequency.
- Impedance Z relates voltage and current amplitudes, $v_m = i_m Z$, and also their rms values, $V = IZ$. Perpendicular voltage phasors add by the Pythagorean theorem.

Step 1: (i) Express the resistor and capacitor voltages Let the current in the series RC circuit be $i = i_m \sin(\omega t + \phi)$. The phase ϕ is to be determined relative to the given source $v = v_m \sin \omega t$. Ohm's law gives the resistor voltage. Using $i = dq/dt$ and $v_C = q/C$, the steady-state capacitor voltage

follows by integration:

$$\begin{aligned}v_R &= i_m R \sin(\omega t + \phi), \\q &= -\frac{i_m}{\omega} \cos(\omega t + \phi), \\v_C &= -\frac{i_m}{\omega C} \cos(\omega t + \phi).\end{aligned}$$

Thus the voltage amplitudes are $V_{Rm} = i_m R$ and $V_{Cm} = i_m X_C$. Their phase difference is 90° , so these amplitudes cannot be added arithmetically.

Step 2: (i) Combine the perpendicular voltage components Kirchhoff's voltage law gives $v = v_R + v_C$. Taking current as the phasor reference, the resistor voltage lies along the current and the capacitor voltage lies perpendicular to it, lagging by 90° . Therefore,

$$\begin{aligned}v_m^2 &= V_{Rm}^2 + V_{Cm}^2, \\v_m &= i_m \sqrt{R^2 + X_C^2}, \\Z = \frac{v_m}{i_m} &= \sqrt{R^2 + \frac{1}{\omega^2 C^2}}.\end{aligned}$$

The current leads the source voltage by ϕ , where $\tan \phi = X_C/R = 1/(\omega CR)$. Thus, for the specified source, the current is $i = (v_m/Z) \sin(\omega t + \phi)$. This also checks that the derived impedance corresponds to a capacitive circuit.

Step 3: (ii) Identify when the inductor behaves as a conductor An ideal inductor acts as a conductor when a steady direct current flows through it, after the switching transient has died away. The current is then constant, so its self-induced emf is

$$\varepsilon_L = -L \frac{di}{dt} = 0.$$

Equivalently, steady dc has $f = 0$, giving $X_L = 2\pi fL = 0$. It therefore offers no inductive opposition to steady dc. A real coil still has its winding resistance, but this is distinct from inductive reactance.

Step 4: (iii) Calculate the lamp resistance and required impedance Assume the series coil is ideal, with negligible resistance, and the lamp operates at its rated filament resistance. Its dc rating gives

$$R = \frac{110}{11} = 10 \, \Omega.$$

To run the lamp normally, its ac rms current must be 11 A, so its rms voltage remains $IR = 110$ V. The stated ac supply voltage is an rms value. The required total impedance is therefore

$$Z = \frac{V}{I} = \frac{220}{11} = 20 \, \Omega.$$

Step 5: (iii) Determine the coil reactance and inductance The lamp is resistive and the coil is inductive.

Their voltages are perpendicular phasors, so use the series RL relation $Z^2 = R^2 + X_L^2$:

$$\begin{aligned} X_L &= \sqrt{Z^2 - R^2} \\ &= \sqrt{20^2 - 10^2} = 10\sqrt{3} \, \Omega, \\ L &= \frac{X_L}{2\pi f} \\ &= \frac{10\sqrt{3}}{2\pi \times 50} \, \text{H} \\ &\simeq 5.51 \times 10^{-2} \, \text{H}. \end{aligned}$$

As a check, the coil voltage is $IX_L = 110\sqrt{3} \, \text{V}$; its vector sum with the lamp's 110 V is 220 V, as required.

Step 6: Conclusion (i) The RC impedance is $Z = \sqrt{R^2 + 1/(\omega^2 C^2)}$. (ii) An ideal inductor acts as a conductor for steady dc because $di/dt = 0$ and its induced emf vanishes. (iii) The required coil inductance is $L \simeq 0.0551 \, \text{H} = 55.1 \, \text{mH}$.

Quick Tip to Solve: For a lamp with a series choke, first find the hot resistance from its dc rating, then use $X_L = \sqrt{(V/I)^2 - R^2}$. Do not subtract the lamp voltage directly from the supply voltage: these phasors are perpendicular.

OR

32(b). (i) Draw a labelled diagram of a step-up transformer and describe its working principle. Explain any three causes for energy losses in a real transformer.

(ii) A step-up transformer converts a low voltage into high voltage. Does it violate the principle of conservation of energy? Explain.

(iii) A step-up transformer has 200 and 3000 turns in its primary and secondary coils respectively. The input voltage given to the primary coil is 90 V. Calculate:

(1) The output voltage across the secondary coil

(2) The current in the primary coil if the current in the secondary coil is 2.0 A.

[5 marks]

Solution:

Concept:

- A transformer works by mutual induction: alternating current in its primary winding produces a changing magnetic flux Φ in a common iron core, inducing emf in the secondary winding.
- For an ideal transformer, $V_s/V_p = N_s/N_p$, where N_p, N_s are the primary and secondary turns and V_p, V_s are their rms voltages. A step-up transformer has $N_s > N_p$.
- Neglecting losses, input power equals output power: $V_p I_p = V_s I_s$. Thus $I_p/I_s = N_s/N_p$; voltage increases at the expense of current.

Step 1: (i) Draw the transformer and explain mutual induction Wind two insulated coils on a common laminated soft-iron core. Connect the primary, having fewer turns, to an ac supply and connect the secondary, having more turns, to the load, as shown. The two windings are electrically insulated from each other and from the core.

The primary alternating current sets up a changing magnetic flux in the core. The core provides a

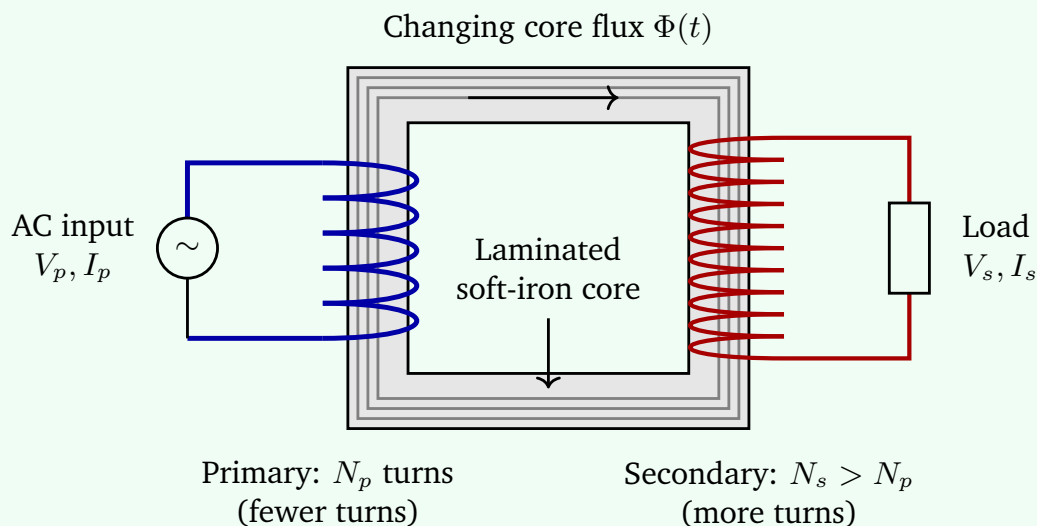
low-reluctance path so that nearly the same flux links both windings. Faraday's law gives

$$\varepsilon_p = -N_p \frac{d\Phi}{dt}, \quad \varepsilon_s = -N_s \frac{d\Phi}{dt}.$$

Taking the ratio of emf magnitudes, and neglecting winding voltage drops,

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} > 1.$$

The secondary therefore supplies a higher alternating voltage. A steady dc supply cannot sustain the changing flux required for transformer action.



Step-up transformer: insulated windings share a laminated soft-iron core; $N_s > N_p$. The turns shown are schematic.

Step 2: (i) Explain three energy losses in a real transformer **Copper loss:** The primary and secondary wires have nonzero resistances R_p and R_s . Their currents cause Joule heating at the rate $I_p^2 R_p + I_s^2 R_s$. Thick, low-resistance copper wire reduces this loss.

Eddy-current loss: The changing core flux induces circulating currents within the conducting iron. These currents heat the core. Thin laminations insulated from one another interrupt the current paths and increase their resistance, reducing this loss.

Hysteresis loss: The ac magnetic field repeatedly magnetises and demagnetises the core. Energy is dissipated during every cycle, proportional to the area of its hysteresis loop. Soft iron with a narrow hysteresis loop reduces this loss.

Step 3: (ii) Apply conservation of energy Stepping up voltage does not violate conservation of energy. In an ideal transformer, the current decreases in the same ratio as the voltage increases:

$$V_p I_p = V_s I_s, \quad I_s = I_p \frac{V_p}{V_s}.$$

Hence an increase in voltage does not create extra power. In a real transformer, output electrical power is smaller than input electrical power; the difference is converted mainly into heat through the losses described above. Total energy remains conserved.

Step 4: (iii)(1) Calculate the secondary voltage The given turns are $N_p = 200$ and $N_s = 3000$, and $V_p = 90$ V. With no loss or efficiency data supplied, use the ideal-transformer approximation. The

turns ratio is

$$k = \frac{N_s}{N_p} = \frac{3000}{200} = 15.$$

Therefore the rms output voltage is

$$V_s = kV_p = 15 \times 90 = 1350 \text{ V}.$$

Step 5: (iii)(2) Calculate the primary current Using the specified secondary current $I_s = 2.0 \text{ A}$ and equality of ideal input and output powers,

$$I_p = \frac{V_s I_s}{V_p} = \frac{1350 \times 2.0}{90} = 30 \text{ A}.$$

As a check, both powers equal 2700 W : $90 \times 30 = 1350 \times 2.0$. The primary carries the larger current because it operates at the lower voltage.

Step 6: Conclusion The step-up transformer works by mutual induction with $N_s > N_p$. Copper heating, eddy currents and hysteresis cause energy losses. Voltage amplification obeys conservation of energy because the current decreases. Under the ideal-transformer approximation, $V_s = 1350 \text{ V}$ and $I_p = 30 \text{ A}$.

Quick Tip to Solve: First calculate $N_s/N_p = 15$: multiply the primary voltage by 15, and multiply the secondary current by 15 to find the primary current. Check that both sides give the same power.

33(a). (i) A ray of light passes through a triangular prism. Show graphically, how the angle of deviation varies with the angle of incidence? Hence define the angle of minimum deviation.

(ii) A ray of light is incident normally on a refracting face of a prism of prism angle A and suffers a deviation of angle δ . Prove that the refractive index n of the material of the prism is given by
$$n = \frac{\sin(A + \delta)}{\sin A}.$$

(iii) The refractive index of the material of a prism is $\sqrt{2}$. If the refracting angle of the prism is 60° , find the:

(1) Angle of minimum deviation, and

(2) Angle of incidence.

[5 marks]

Solution:

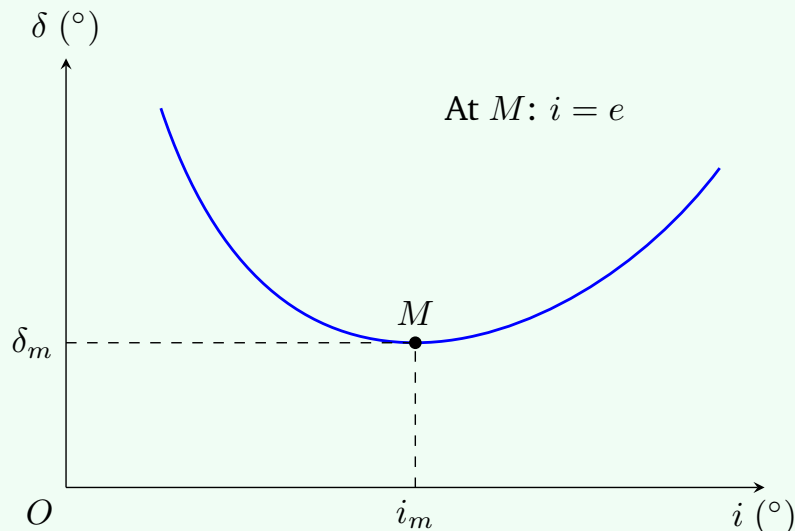
Concept:

- For a prism in air, the internal refraction angles obey $r_1 + r_2 = A$, and the deviation is $\delta = i + e - A$. Here A is the prism angle, while i and e are the incidence and emergence angles measured from the respective normals.
- Snell's law gives $\sin i = n \sin r_1$ at entry and $n \sin r_2 = \sin e$ at exit, where n is the refractive index of the prism relative to air.
- At minimum deviation, the path is symmetric: $i = e$, $r_1 = r_2 = A/2$, and $n = \sin[(A + \delta_m)/2] / \sin(A/2)$, where δ_m is the minimum deviation.

Step 1: (i) Plot the deviation and identify its minimum For a fixed prism and wavelength, increase the angle of incidence within the range for which a ray emerges from the second face. The angle of deviation first decreases, reaches a least value, and then increases. The schematic graph below shows

this smooth minimum at M ; the vertical axis is deviation and the horizontal axis is incidence.

The **angle of minimum deviation**, δ_m , is the smallest angle through which the prism deviates the ray for that wavelength. At this position the incident and emergent rays make equal angles with their respective face normals, so $i = e = i_m$. The internal ray then has equal refraction angles at the two faces.



Schematic variation of deviation with incidence for a fixed prism and wavelength; M is the minimum-deviation position.

Step 2: (ii) Use normal incidence at the first face Normal incidence means $i = 0$. Snell's law at the first face therefore gives $r_1 = 0$: the ray enters without bending. The prism-angle relation now yields

$$r_2 = A - r_1 = A.$$

Write the total deviation as the sum of deviations at the faces, or use $\delta = i + e - A$. With $i = 0$,

$$\delta = e - A, \quad e = A + \delta.$$

At the second face light travels from the prism into air. Applying Snell's law there gives

$$n \sin r_2 = \sin e \quad \Rightarrow \quad n \sin A = \sin(A + \delta).$$

Thus the required relation is $n = \sin(A + \delta) / \sin A$. This applies to the stated emerging ray; the exit incidence must permit refraction into air.

Step 3: (iii)(1) Calculate the minimum deviation Here $n = \sqrt{2}$ and $A = 60^\circ$. At minimum deviation, $r_1 = r_2 = A/2 = 30^\circ$. Substituting in the minimum-deviation formula,

$$\begin{aligned} \sqrt{2} &= \frac{\sin[(60^\circ + \delta_m)/2]}{\sin 30^\circ}, \\ \sin\left(\frac{60^\circ + \delta_m}{2}\right) &= \frac{\sqrt{2}}{2} = \sin 45^\circ. \end{aligned}$$

The angle $(A + \delta_m)/2$ is the external incidence angle, which lies between 0° and 90° ; hence the physical inverse-sine value is 45° . Therefore

$$\delta_m = 2(45^\circ) - 60^\circ = 30^\circ.$$

Step 4: (iii)(2) Find the corresponding incidence angle The requested angle of incidence is the one corresponding to minimum deviation. By symmetry $i_m = e_m$, so

$$i_m = \frac{A + \delta_m}{2} = \frac{60^\circ + 30^\circ}{2} = 45^\circ.$$

As a check, Snell's law at entry gives $\sin i_m = \sqrt{2} \sin 30^\circ = 1/\sqrt{2}$, in agreement with this angle. The normally incident ray in part (ii) and the minimum-deviation ray in part (iii) are different configurations.

Step 5: Conclusion The deviation-incidence curve has a minimum δ_m at $i = e$. For normal incidence, $n = \frac{\sin(A + \delta)}{\sin A}$. For the prism with $n = \sqrt{2}$ and $A = 60^\circ$, $\delta_m = 30^\circ$ and the corresponding $i_m = 45^\circ$.

Quick Tip to Solve: First identify the configuration: normal incidence sets $r_1 = 0$, whereas minimum deviation sets $r_1 = r_2 = A/2$. Use the matching condition before applying Snell's law.

OR

33(b). (i) State Huygens' principle. A plane wave is incident at an angle i on a reflecting surface. Construct the corresponding reflected wavefront. Using this diagram, prove that the angle of reflection is equal to the angle of incidence.

(ii) What are the coherent sources of light? Can two independent sodium lamps act like coherent sources? Explain.

(iii) A beam of light consisting of a known wavelength 520 nm and an unknown wavelength λ , used in Young's double slit experiment produces two interference patterns such that the fourth bright fringe of unknown wavelength coincides with the fifth bright fringe of known wavelength. Find the value of λ .

[5 marks]

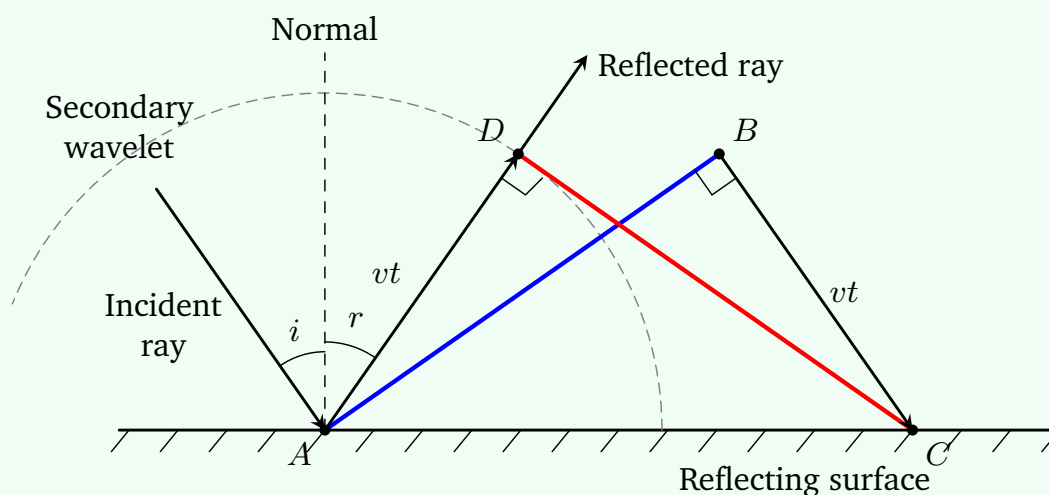
Solution:

Concept:

- Huygens' principle constructs a later wavefront as the forward envelope of secondary wavelets emitted by points on the earlier wavefront. Rays are normal to wavefronts.
- For reflection in the same medium, incident and reflected disturbances have the same speed v , so both travel a distance vt in time t .
- Coherent sources emit waves of the same frequency with a constant phase difference. In Young's experiment the position of the m th bright fringe is $y_m = m\lambda D/d$, where D is screen distance and d is slit separation.

Step 1: (i) State Huygens' principle Every point on a wavefront acts as a source of secondary wavelets that spread in all directions with the speed of light in the medium. The common tangent surface, or forward envelope, of these wavelets after a given time gives the new wavefront.

Step 2: Construct the reflected wavefront Let AB be the incident plane wavefront, with A reaching the reflecting surface first. In time t , the disturbance at B travels along the incident ray to C , so $BC = vt$. During the same time, the secondary reflected wavelet from A has radius $AD = vt$. Draw the tangent CD from C to this wavelet. The plane through CD , perpendicular to the plane of the diagram, is the reflected wavefront; in the diagram it is shown by the line CD . The reflected ray AD is perpendicular to it. The dashed arc represents the secondary wavelet in the incident medium.



Huygens' construction: incident wavefront AB , reflected wavefront CD , and equal distances $BC = AD = vt$.

Step 3: Prove the law of reflection from the construction Since rays are normal to wavefronts, $AB \perp BC$, and the radius to the tangent point gives $AD \perp DC$. The right triangles ABC and CDA have the same hypotenuse AC and equal sides $BC = AD = vt$. They are congruent by the right-angle-hypotenuse-side rule. Equivalently,

$$\sin \angle BAC = \frac{BC}{AC} = \frac{AD}{AC} = \sin \angle DCA.$$

The angle between two lines equals the angle between their corresponding perpendiculars. Thus the angle between the incident wavefront AB and the surface AC is the incidence angle i ; the angle between the reflected wavefront DC and the surface CA is the reflection angle r . Both are measured between a ray and the surface normal in the usual definition. The two acute angles of the congruent triangles therefore obey $i = r$. The construction also places the incident ray, reflected ray and normal in the same plane.

Step 4: (ii) Explain coherence and independent sodium lamps Coherent light sources have the same frequency and maintain a constant phase difference. Two independent sodium lamps cannot act as coherent sources for sustained interference. Even if they emit light of the same nominal wavelength, the atomic emissions in the two lamps occur independently and their relative phase changes randomly with time. Their interference term therefore averages out over the observation time, so no stable fringe pattern is obtained. Coherent beams are normally produced by splitting light from one common source.

Step 5: (iii) Equate the positions of the coincident bright fringes The fourth bright fringe of the unknown wavelength and the fifth bright fringe of the known wavelength lie at the same distance from the common central maximum. Applying the bright-fringe formula to the same slit separation and screen distance gives

$$\begin{aligned} \frac{4\lambda D}{d} &= \frac{5(520 \text{ nm})D}{d}, \\ 4\lambda &= 2600 \text{ nm}, \\ \lambda &= 650 \text{ nm}. \end{aligned}$$

The smaller fringe order belongs to the longer wavelength, consistent with the larger fringe spacing for that wavelength.

Step 6: Conclusion Huygens' construction gives $i = r$. Independent sodium lamps are not coherent

because they do not maintain a constant phase difference. The unknown wavelength is 650 nm .

Quick Tip to Solve: In the reflection construction set $BC = AD = vt$; for coincident bright fringes set the two products $m\lambda$ equal before substituting the orders.