

CBSE Class 12 Physics Question Paper 2024 with Solution PDF-Set 55/1/3



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – **Sections A, B, C, D** and **E**.
- (iii) In **Section A** – Questions no. **1** to **16** are Multiple Choice type questions. Each question carries **1** mark.
- (iv) In **Section B** – Questions no. **17** to **21** are Very Short Answer type questions. Each question carries **2** marks.
- (v) In **Section C** – Questions no. **22** to **28** are Short Answer type questions. Each question carries **3** marks.
- (vi) In **Section D** – Questions no. **29** and **30** are case study-based questions. Each question carries **4** marks.
- (vii) In **Section E** – Questions no. **31** to **33** are Long Answer type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is not allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. A thin plastic rod is bent into a circular ring of radius R . It is uniformly charged with charge density λ . The magnitude of the electric field at its centre is: [1 mark]

- (A) $\frac{\lambda}{2\epsilon_0 R}$
(B) Zero
(C) $\frac{\lambda}{4\pi\epsilon_0 R}$
(D) $\frac{\lambda}{4\epsilon_0 R}$

Correct Answer: (B) Zero

Solution:

Concept:

- Electric field is a vector; the net field is the vector sum of the fields due to all charge elements.
- A small ring element of length dl carries charge $dq = \lambda dl$, producing a central field of magnitude $dE = dq/(4\pi\epsilon_0 R^2)$.

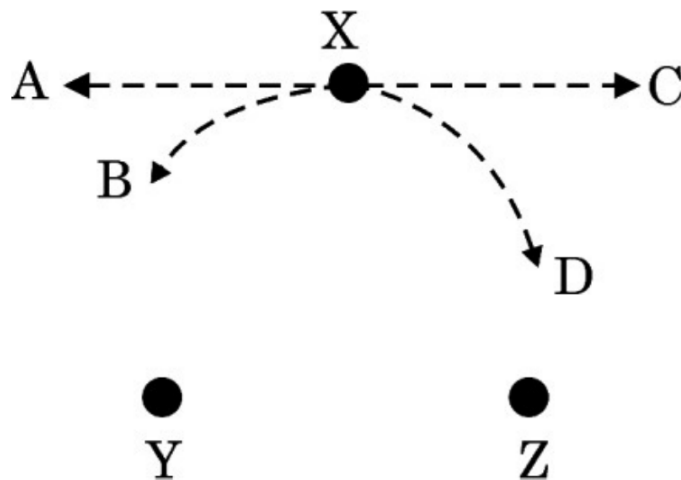
Step 1: Pair opposite charge elements Choose any small element on the uniformly charged ring. Its diametrically opposite element has the same charge and is also a distance R from the centre. Their electric fields at the centre have equal magnitudes and opposite directions, so their vector sum vanishes.

Step 2: Apply cancellation to the entire ring Every element has an opposite partner, so all such pairs cancel. The nonzero expressions in (A), (C) and (D) cannot describe the central field of a complete, uniformly charged ring: they ignore this vector cancellation. A nonzero total charge does not imply a nonzero field at every point.

Step 3: Conclusion The magnitude of the electric field at the centre is $E = 0$. Hence, the correct option is (B).

Quick Tip to Solve: For a uniformly charged complete ring, pair diametrically opposite elements before attempting any integration.

2. Three small charged spheres X , Y and Z carrying charges $+q$, $-q$ and $+q$ respectively are placed equidistant from each other, as shown in the figure. The spheres Y and Z are held in place. Initially X is also held in place, but is otherwise free to move. When X is released, the path followed by it will be: [1 mark]



- (A) A
- (B) B
- (C) C
- (D) D

Correct Answer: (B) B

Solution:

Concept:

- Coulomb force acts along the joining line: unlike charges attract and like charges repel, with magnitude $F = kq^2/r^2$, where $k = 1/(4\pi\epsilon_0)$.
- A particle released from rest initially accelerates along the resultant force; its subsequent path bends as the force direction changes.

Step 1: Resolve the initial forces on X The negative charge at Y attracts X downwards and to the left. The positive charge at Z repels X upwards and to the left. Since the initial distances and charge magnitudes are equal, the vertical components cancel and the leftward components add. Thus X initially moves left, ruling out paths C and D.

Step 2: Follow the changing resultant After a small leftward displacement, X is closer to Y and farther from Z . The attraction towards Y becomes stronger, while the repulsion from Z becomes weaker. A downward resultant component therefore develops, bending the motion towards Y . Path A incorrectly assumes that the initial cancellation of vertical forces persists throughout the motion.

Step 3: Conclusion The particle follows the leftward path curving down towards Y , labelled B. Hence, the correct option is (B).

Quick Tip to Solve: Determine both the initial resultant and how the two distances change; the initial force alone does not fix the whole path.

3. In a uniform straight wire, conduction electrons move along $+x$ direction. Let $\vec{E}$ and $\vec{j}$ be the electric field and current density in the wire, respectively. Then: [1 mark]

- (A) $\vec{E}$ and $\vec{j}$ both are along $+x$ direction.
 (B) $\vec{E}$ and $\vec{j}$ both are along $-x$ direction.
 (C) $\vec{E}$ is along $+x$ direction, but $\vec{j}$ is along $-x$ direction.
 (D) $\vec{E}$ is along $-x$ direction, but $\vec{j}$ is along $+x$ direction.

Correct Answer: (B) $\vec{E}$ and $\vec{j}$ both are along $-x$ direction.

Solution:

Concept:

- For conduction electrons, $\vec{j} = -ne\vec{v}_d$, where n is the number density, $e > 0$ is the elementary charge and $\vec{v}_d$ is their drift velocity.
- In an ordinary conducting wire, $\vec{j} = \sigma\vec{E}$, where the conductivity σ is positive; the current density and electric field point in the same direction.

Step 1: Find the conventional current direction Electrons carry negative charge. When their drift is along $+x$, conventional current, defined as the flow of positive charge, is along $-x$. Thus $\vec{j}$ points along $-x$, excluding options (A) and (D).

Step 2: Determine the electric field direction The force on an electron is $\vec{F} = -e\vec{E}$, so its steady drift is opposite to the electric field. An electron drift along $+x$ therefore requires $\vec{E}$ along $-x$. This also agrees with $\vec{j} = \sigma\vec{E}$. Option (C) incorrectly makes the field parallel to electron drift.

Step 3: Conclusion Both $\vec{E}$ and $\vec{j}$ are along the $-x$ direction. Hence, the correct option is (B).

Quick Tip to Solve: Reverse the electron drift direction to obtain conventional current; the electric field then has the same direction as that current.

4. Two charged particles, P and Q , each having charge q but of masses m_1 and m_2 , are accelerated through the same potential difference V . They enter a region of magnetic field $\vec{B}$ ($\perp \vec{v}$) and describe the circular paths of radii a and b respectively. Then $\left(\frac{m_1}{m_2}\right)$ is equal to: [1 mark]

- (A) $\frac{a}{b}$
 (B) $\frac{b}{a}$
 (C) $\left(\frac{a}{b}\right)^2$
 (D) $\left(\frac{b}{a}\right)^2$

Correct Answer: (C) $\left(\frac{a}{b}\right)^2$

Solution:

Concept:

- Acceleration from rest through a potential difference of magnitude V gives kinetic energy $\frac{1}{2}mv^2 = |q|V$, where m is particle mass and $|q|$ its charge magnitude.
- For velocity perpendicular to a uniform magnetic field, $|q|vB = mv^2/r$, giving the circular radius $r = mv/(|q|B)$.

Step 1: Eliminate the particle speed The particles gain the same kinetic energy, but their speeds need not be equal because their masses differ. Combining the energy equation with the radius formula gives

$$r^2 = \frac{m^2 v^2}{q^2 B^2} = \frac{2mV}{|q|B^2}.$$

Step 2: Compare the two radii The quantities $|q|$, V and B are common, so mass is proportional to the square of radius:

$$\frac{m_1}{m_2} = \frac{a^2}{b^2}.$$

Option (A) assumes equal speeds instead of equal accelerating potentials. Options (B) and (D) incorrectly predict that the heavier particle has the smaller orbit under these conditions.

Step 3: Conclusion $\frac{m_1}{m_2} = \left(\frac{a}{b}\right)^2$. Hence, the correct option is (C).

Quick Tip to Solve: For equal charge, accelerating voltage and magnetic field, use $r \propto \sqrt{m}$, not $r \propto m$.

5. A galvanometer of resistance $G \Omega$ is converted into an ammeter of range 0 to I A. If the current through the galvanometer is 0.1% of I A, the resistance of the ammeter is: [1 mark]

- (A) $\frac{G}{999} \Omega$
 (B) $\frac{G}{1000} \Omega$
 (C) $\frac{G}{1001} \Omega$
 (D) $\frac{G}{100.1} \Omega$

Correct Answer: (B) $\frac{G}{1000} \Omega$

Solution:

Concept:

- An ammeter is made by connecting a low resistance shunt S in parallel with a galvanometer of resistance G .
- Both parallel branches have the same potential difference: $V_g = I_g G$. The effective ammeter resistance is $R_A = V_g / I$, where I is total current.

Step 1: Express the galvanometer current The galvanometer carries only 0.1% of the total full-scale current, so

$$I_g = \frac{0.1}{100} I = \frac{I}{1000}, \quad I_s = I - I_g = \frac{999I}{1000}.$$

Here I_s is the current through the shunt, which carries nearly all the input current.

Step 2: Find the resistance of the whole ammeter The terminal voltage is $V_g = I_g G = IG/1000$. Therefore

$$R_A = \frac{V_g}{I} = \frac{G}{1000}.$$

The shunt alone has resistance $S = I_g G / I_s = G/999$, so option (A) confuses the shunt with the parallel combination. Options (C) and (D) do not give the stated fraction $I_g / I = 1/1000$.

Step 3: Conclusion The resistance of the ammeter is $\frac{G}{1000} \Omega$. Hence, the correct option is (B).

Quick Tip to Solve: When asked for the ammeter resistance, use $R_A = (I_g/I)G$; $G/999$ here is only the shunt resistance.

6. A 10 cm long wire lies along y-axis. It carries a current of 1.0 A in positive y-direction. A magnetic field $\vec{B} = (5 \text{ mT})\hat{j} - (8 \text{ mT})\hat{k}$ exists in the region. The force on the wire is: [1 mark]

- (A) $(0.8 \text{ mN})\hat{i}$
- (B) $-(0.8 \text{ mN})\hat{i}$
- (C) $(80 \text{ mN})\hat{i}$
- (D) $-(80 \text{ mN})\hat{i}$

Correct Answer: (B) $-(0.8 \text{ mN})\hat{i}$

Solution:

Concept:

- The magnetic force on a straight current-carrying wire is $\vec{F} = I\vec{L} \times \vec{B}$, where $\vec{L}$ points along the conventional current and has magnitude equal to the wire length.
- The unit-vector products are $\hat{j} \times \hat{j} = 0$ and $\hat{j} \times \hat{k} = \hat{i}$; a field component parallel to the wire exerts no force.

Step 1: Convert the length and magnetic field to SI units The current is along $+y$, so $\vec{L} = 0.10\hat{j} \text{ m}$. The field is

$$\vec{B} = (5 \times 10^{-3}\hat{j} - 8 \times 10^{-3}\hat{k}) \text{ T}.$$

Only the negative z -component contributes, since the y -component is parallel to the wire.

Step 2: Evaluate the cross product With $I = 1.0 \text{ A}$,

$$\begin{aligned}\vec{F} &= (1.0)(0.10)(-8 \times 10^{-3})(\hat{j} \times \hat{k}) \text{ N} \\ &= -8.0 \times 10^{-4}\hat{i} \text{ N}.\end{aligned}$$

The negative sign excludes (A) and (C). Since $1 \text{ mN} = 10^{-3} \text{ N}$, the magnitude is 0.8 mN , not the 80 mN in (D).

Step 3: Conclusion $\vec{F} = -(0.8 \text{ mN})\hat{i}$, directed along $-x$. Hence, the correct option is (B).

Quick Tip to Solve: Discard the field component parallel to the wire, convert centimetres to metres, and retain the minus sign in the perpendicular component.

7. The primary and secondary coils of a transformer have 500 turns and 5000 turns respectively. The primary coil is connected to an ac source of 220 V – 50 Hz. The output across the secondary coil is: [1 mark]

- (A) 220 V – 50 Hz
- (B) 1100 V – 50 Hz
- (C) 2200 V – 5 Hz
- (D) 2200 V – 50 Hz

Correct Answer: (D) 2200 V – 50 Hz

Solution:

Concept:

- For an ideal transformer, the voltage ratio equals the turns ratio: $V_s/V_p = N_s/N_p$, where subscripts p and s denote primary and secondary coils.
- Both coils link the same alternating magnetic flux. A transformer changes the voltage amplitude but preserves the frequency of the supply.

Step 1: Calculate the secondary voltage The turns ratio is $N_s/N_p = 5000/500 = 10$, so this is a step-up transformer. Applying the voltage relation gives

$$V_s = V_p \frac{N_s}{N_p} = 220 \times \frac{5000}{500} = 2200 \text{ V.}$$

Thus (A) and (B) give incorrect voltage values for the stated turns ratio.

Step 2: Determine the secondary frequency The primary supply produces flux oscillations at 50 Hz. The induced secondary emf follows the same oscillations, so its frequency remains 50 Hz. Option (C) wrongly divides the frequency by the turns ratio; the voltage ratio does not act on frequency.

Step 3: Conclusion The secondary output is 2200 V at 50 Hz. Hence, the correct option is (D).

Quick Tip to Solve: Multiply the primary voltage by the secondary-to-primary turns ratio, but copy the supply frequency unchanged.

8. The first scientist who produced and observed electromagnetic waves of wavelengths in the range 25 mm – 5 mm was: [1 mark]

- (A) J.C. Maxwell
- (B) H.R. Hertz
- (C) J.C. Bose
- (D) G. Marconi

Correct Answer: (C) J.C. Bose

Solution:

Concept:

- Maxwell predicted electromagnetic waves theoretically; their production and detection required experiments.
- The wavelength range identifies the experiment: J.C. Bose produced and observed millimetre electromagnetic waves in the range 25 mm to 5 mm.

Step 1: Identify the specified wavelength range The question specifies very short radio waves, with wavelengths measured in millimetres. This range is associated with the pioneering experiments of J.C. Bose, rather than with the first general theoretical prediction of electromagnetic waves.

Step 2: Distinguish the other scientists' contributions Maxwell developed the electromagnetic theory, so (A) does not describe an experimental production of these waves. Hertz experimentally demonstrated electromagnetic waves, but the particular millimetre range quoted here is associated with Bose, so (B) is unsuitable. Marconi developed wireless communication using radio waves, so (D) also does not match the stated achievement.

Step 3: Conclusion J.C. Bose first produced and observed electromagnetic waves in the stated wavelength range. Hence, the correct option is (C).

Quick Tip to Solve: Match the wavelength clue: 25 mm to 5 mm points to Bose's millimetre-wave experiments.

9. The waves associated with a moving electron and a moving proton have the same wavelength λ . It implies that they have the same: [1 mark]

- (A) momentum
- (B) angular momentum
- (C) speed
- (D) energy

Correct Answer: (A) momentum

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of linear momentum.
- For a non-relativistic particle of mass m , speed $v = p/m$ and kinetic energy $K = p^2/(2m)$.

Step 1: Apply the de Broglie relation Both particles have the same wavelength, so their momentum magnitudes satisfy

$$p_e = \frac{h}{\lambda} = p_p.$$

The electron and proton need not travel in the same direction; the equality concerns the magnitudes of their momenta.

Step 2: Check the other proposed equalities Their masses differ greatly, so equal momentum does not give equal speed or equal kinetic energy: $v = p/m$ and $K = p^2/(2m)$. Thus (C) and (D) do not follow. Angular momentum depends additionally on the position relative to a chosen origin, $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, so the wavelength alone cannot imply (B).

Step 3: Conclusion The two particles have equal magnitudes of linear momentum. Hence, the correct option is (A).

Quick Tip to Solve: For equal de Broglie wavelengths, equate momentum first; do not equate speeds for particles of different masses.

10. Two beams, A and B whose photon energies are 3.3 eV and 11.3 eV respectively, illuminate a metallic surface (work function 2.3 eV) successively. The ratio of maximum speed of electrons emitted due to beam A to that due to beam B is: [1 mark]

- (A) 3
- (B) 9
- (C) $\frac{1}{3}$
- (D) $\frac{1}{9}$

Correct Answer: (C) $\frac{1}{3}$

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = E_{\gamma} - \phi$, where E_{γ} is photon energy and ϕ is the metal's work function.
- Since $K_{\max} = \frac{1}{2}m_e v_{\max}^2$, the maximum speed of an emitted electron is proportional to $\sqrt{K_{\max}}$.

Step 1: Subtract the work function for each beam Both photon energies exceed $\phi = 2.3$ eV, so both beams eject electrons. Their maximum kinetic energies are

$$K_A = (3.3 - 2.3) \text{ eV} = 1.0 \text{ eV},$$

$$K_B = (11.3 - 2.3) \text{ eV} = 9.0 \text{ eV}.$$

Step 2: Convert the energy ratio into a speed ratio The electron mass and the conversion from electron volts to joules cancel in the ratio:

$$\frac{v_A}{v_B} = \sqrt{\frac{K_A}{K_B}} = \sqrt{\frac{1}{9}} = \frac{1}{3}.$$

Option (D) is the kinetic-energy ratio, not the speed ratio. Options (A) and (B) are greater than one, although beam A leaves less energy for the emitted electron.

Step 3: Conclusion The ratio of maximum speeds is $v_A : v_B = 1 : 3$. Hence, the correct option is (C).

Quick Tip to Solve: Subtract the work function before taking the square root of the energy ratio; the photon-energy ratio is not sufficient.

11. The transition of electron that gives rise to the formation of the second spectral line of the Balmer series in the spectrum of hydrogen atom corresponds to: [1 mark]

- (A) $n_f = 2$ and $n_i = 3$
- (B) $n_f = 3$ and $n_i = 4$
- (C) $n_f = 2$ and $n_i = 4$
- (D) $n_f = 2$ and $n_i = \infty$

Correct Answer: (C) $n_f = 2$ and $n_i = 4$

Solution:

Concept:

- Every Balmer-series emission line in hydrogen arises from a transition to the final principal quantum number $n_f = 2$.
- Successive lines correspond to $n_i = 3, 4, 5, \dots$, where n_i is the initial principal quantum number, with $n_i > n_f$.

Step 1: Fix the final energy level The name Balmer fixes the final level at $n_f = 2$. A transition ending at $n_f = 3$, as in (B), belongs to the Paschen series and cannot produce a Balmer line.

Step 2: Count the transitions from the lowest upper level The first Balmer line is the transition $3 \rightarrow 2$, which is option (A). The next possible initial level is $n_i = 4$, so the second line is $4 \rightarrow 2$. Option (D), $\infty \rightarrow 2$, denotes the Balmer series limit rather than its second discrete line.

Step 3: Conclusion The second Balmer line corresponds to $n_i = 4$ and $n_f = 2$. Hence, the correct option is (C).

Quick Tip to Solve: For the k -th Balmer line, keep $n_f = 2$ and set $n_i = 2 + k$; the second line therefore starts at 4.

12. Ge is doped with As. Due to doping, [1 mark]

- (A) the structure of Ge lattice is distorted.
- (B) the number of conduction electrons increases.
- (C) the number of holes increases.
- (D) the number of conduction electrons decreases.

Correct Answer: (B) the number of conduction electrons increases.

Solution:

Concept:

- Germanium is tetravalent: each Ge atom contributes four valence electrons to covalent bonds in its crystal lattice.
- Arsenic is pentavalent. Substitutional As doping supplies donor electrons and produces an n-type semiconductor.

Step 1: Count the bonds formed by an arsenic impurity When an As atom replaces a Ge atom, four of its five valence electrons form covalent bonds with neighbouring Ge atoms. The fifth electron is weakly bound and is readily released into the conduction band at ordinary temperatures.

Step 2: Identify the carrier population that increases Each ionised donor supplies an extra conduction electron, so electrons become the majority carriers. Holes remain minority carriers; adding As does

not create an extra hole for every donated electron. Thus (C) is incorrect and (D) reverses the donor effect. In the standard dilute-doping model the host crystal structure is retained, so lattice distortion in (A) is not the intended effect.

Step 3: Conclusion Doping Ge with As increases the number of conduction electrons. Hence, the correct option is (B).

Quick Tip to Solve: Compare valence electrons: five from As versus four from Ge leaves one donor electron and identifies n-type doping.

13. Assertion (A): Two long parallel wires, freely suspended and connected in series to a battery, move apart.

Reason (R): Two wires carrying current in opposite directions repel each other.

[1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- Two parallel current-carrying wires attract for currents in the same direction and repel for currents in opposite directions.
- The force per unit length is $F/L = \mu_0 I_1 I_2 / (2\pi d)$, where d is the separation and I_1, I_2 are the current magnitudes.

Step 1: Evaluate the assertion In the usual series arrangement of two freely suspended parallel wires, current travels down one wire and returns up the other to complete the circuit. The two parallel portions therefore carry equal currents in opposite directions. They repel and move apart, so Assertion (A) is true.

Step 2: Evaluate the reason Each wire creates a magnetic field at the other wire. Applying the right-hand rule for the field and the force law $\mathbf{F} = I\mathbf{L} \times \mathbf{B}$ shows that opposite parallel currents experience forces directed away from each other. Reason (R) is therefore true.

Step 3: Check whether the reason explains the assertion The series connection gives the opposite current directions in the suspended parallel portions, and the repulsion stated in R explains their separation. Thus the explanation is direct, ruling out (B). Since both statements are true, (C) and (D) are also excluded.

Step 4: Conclusion Both A and R are true, and R correctly explains A. Hence, the correct option is (A).

Quick Tip to Solve: Trace the complete series loop before judging the force: the current goes along one suspended wire and returns along the other.

14. Assertion (A): Plane and convex mirrors cannot produce real images under any circumstance.

Reason (R): A virtual image cannot serve as an object to produce a real image.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and Reason (R) is also false.

Solution:

Concept:

- A real image forms where the emerging rays actually converge. Converging incident rays can act as a virtual object for a mirror.
- The mirror formula is $1/v + 1/u = 1/f$. In the Cartesian convention, a virtual object behind the mirror has $u > 0$, while a real image in front has $v < 0$.

Step 1: Evaluate the assertion using a virtual object Plane and convex mirrors form virtual images for ordinary real objects, but the words “under any circumstance” are too strong. For a plane mirror, $1/f = 0$, so $v = -u < 0$ for a virtual object with $u > 0$. A convex mirror has $f > 0$; if $0 < u < f$, then

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} < 0.$$

Thus either mirror can form a real image from appropriately converging incident rays, and Assertion (A) is false.

Step 2: Evaluate the reason for successive optical elements An image formed by one optical element may serve as the object for another. Even a virtual image can supply an effective object position for a subsequent converging lens, which can form a real image when that object is beyond its focal point. Therefore the claimed impossibility in Reason (R) is false.

Step 3: Match the two truth values to the printed codes Both the assertion and the reason make absolute claims that the examples disprove. There is consequently no true explanatory relation to assess. The printed code for both statements being false is (D), whereas (A), (B) and (C) all require a true assertion.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: When a mirror statement says “under any circumstance”, test converging incident rays and a virtual object before accepting it.

15. Assertion (A): The mutual inductance between two coils is maximum when the coils are wound on each other.

Reason (R): The flux linkage between two coils is maximum when they are wound on each other.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- Mutual inductance M measures magnetic coupling: $N_2\Phi_{21} = MI_1$, where $N_2\Phi_{21}$ is the flux linkage of the second coil due to current I_1 in the first.
- For given coils and medium, closer coupling reduces leakage flux and increases the fraction of flux shared by the coils.

Step 1: Evaluate the assertion When the coils are wound on each other, their magnetic fields occupy the same region and the coupling is greatest. Thus their mutual inductance is maximum for this arrangement. Assertion (A) is true.

Step 2: Evaluate the reason With the coils closely superposed, the maximum possible magnetic flux produced by one coil links the other. This is the maximum-flux-linkage arrangement for the given coils, so Reason (R) is true.

Step 3: Connect the two statements At a fixed current I_1 , the relation $M = N_2\Phi_{21}/I_1$ shows directly that maximum flux linkage means maximum mutual inductance. Therefore R explains A, excluding (B). Since both statements are true, (C) and (D) are also excluded.

Step 4: Conclusion Both A and R are true, and R correctly explains A. Hence, the correct option is (A).

Quick Tip to Solve: To link coil geometry to mutual inductance, hold the source current fixed and compare the flux linking the other coil.

16. Assertion (A): In photoelectric effect, the kinetic energy of the emitted photoelectrons increases with increase in the intensity of the incident light.

Reason (R): Photoelectric current depends on the wavelength of the incident light.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and Reason (R) is also false.

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = hc/\lambda - \phi$, where ϕ is the metal's work function. Photon energy determines the maximum electron kinetic energy.
- In the usual school-level comparison with light above threshold and fixed collection conditions, intensity controls the photoelectric current, while frequency controls $K_{\max}$.

Step 1: Evaluate the assertion At a fixed frequency, increasing intensity increases the number of incident photons per second; it does not increase the energy of an individual photon. The work function is unchanged, so $K_{\max} = h\nu - \phi$ remains unchanged. Assertion (A) is false.

Step 2: Evaluate the reason in the stated textbook context Photoelectric current measures the rate of collection of emitted electrons. In the standard above-threshold comparison, this current is associated with the intensity of illumination, whereas wavelength is associated with photon energy and stopping potential. Thus the intended statement R is false. The threshold condition must still hold: light of wavelength greater than the threshold wavelength cannot cause emission.

Step 3: Choose the assertion–reason code Since A is false, options (A), (B) and (C), which all describe A as true, are excluded. Within the conventional interpretation of R used here, both statements are false; R therefore cannot explain A.

Step 4: Conclusion On the standard above-threshold textbook interpretation, A and R are both false. Hence, the correct option is (D).

Quick Tip to Solve: Separate photon energy from photon number: frequency sets the maximum kinetic energy, while increased intensity at fixed frequency raises the photocurrent.

Section B (10 marks)

17. A uniform wire of length L and area of cross-section A has resistance R . The wire is uniformly stretched so that its length increases by 25%. Calculate the percentage increase in the resistance of the wire. [2 marks]

Solution:

Concept:

- The resistance of a uniform wire is $R = \rho L/A$, where ρ is resistivity, L is length and A is cross-sectional area.
- Uniform stretching conserves the wire's volume: $AL = A'L'$. Assume the material and temperature remain unchanged, so ρ is constant.

Step 1: Find the stretched dimensions The new length is the original length plus one quarter of it:

$$L' = L + 0.25L = 1.25L = \frac{5}{4}L.$$

Stretching also makes the wire thinner. Conservation of volume gives

$$A' = \frac{AL}{L'} = \frac{4}{5}A.$$

It would be incorrect to retain the original area while increasing the length.

Step 2: Compare the resistances Divide the new resistance by the original resistance so that the common resistivity cancels:

$$\frac{R'}{R} = \frac{\rho L'/A'}{\rho L/A} = \frac{L'}{L} \frac{A}{A'} = \frac{5}{4} \times \frac{5}{4} = \frac{25}{16}.$$

Hence the fractional increase, measured relative to the original resistance, is

$$\frac{R' - R}{R} = \frac{25}{16} - 1 = \frac{9}{16}.$$

Step 3: Conclusion The percentage increase is $\frac{9}{16} \times 100\% = 56.25\%$. The new resistance is $1.5625R$.

Quick Tip to Solve: For a stretched wire of constant volume and resistivity, use $R'/R = (L'/L)^2$; subtract one before converting the ratio to a percentage increase.

18. An object is placed 30 cm in front of a concave mirror of radius of curvature 40 cm. Find the (i) position of the image formed and (ii) magnification of the image. [2 marks]

Solution:

Concept:

- The mirror formula is $1/f = 1/v + 1/u$, with distances measured from the pole using the Cartesian sign convention. A concave mirror facing a real object has $f < 0$ and $u < 0$.
- The focal length is half the radius of curvature, $f = R/2$, with its sign included. Linear magnification is $m = h_i/h_o = -v/u$.

Step 1: Find the image position Take the direction of incident light as positive. The object and centre of curvature are in front of the mirror, so

$$u = -30 \text{ cm}, \quad R = -40 \text{ cm}, \quad f = -20 \text{ cm}.$$

Substitute these signed distances into the mirror formula:

$$\begin{aligned} \frac{1}{v} &= \frac{1}{f} - \frac{1}{u} = -\frac{1}{20} + \frac{1}{30} \\ &= -\frac{1}{60} \text{ cm}^{-1}. \end{aligned}$$

The negative image distance places the image on the same side of the mirror as the object; the reflected rays actually meet there.

Step 2: Find and interpret the magnification Using the image distance obtained above,

$$m = -\frac{v}{u} = -\frac{-60}{-30} = -2.$$

The negative sign indicates inversion, and the magnitude 2 means the image is twice the object's height. This agrees with an object between the focus and centre of curvature of a concave mirror: its real image forms beyond the centre.

Step 3: Conclusion (i) $v = -60 \text{ cm}$: the real image forms 60 cm in front of the mirror. (ii) $m = -2$: the image is inverted and twice the object's size.

Quick Tip to Solve: Write $u = -30 \text{ cm}$ and $f = -20 \text{ cm}$ before substitution; retain the minus sign in $m = -v/u$ to identify inversion.

19. Consider a neutron (mass m) of kinetic energy E and a photon of the same energy. Let λ_n and λ_p be the de Broglie wavelength of neutron and the wavelength of photon respectively. Obtain an expression for $\frac{\lambda_n}{\lambda_p}$. [2 marks]

Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where h is Planck's constant and p is momentum.
- A non-relativistic neutron has kinetic energy $E = p_n^2/(2m)$. A photon obeys $E = p_p c = hc/\lambda_p$, where c is the speed of light.

Step 1: Express the neutron wavelength Use the neutron's kinetic energy to obtain its momentum. In

the non-relativistic approximation used for this problem,

$$E = \frac{p_n^2}{2m} \Rightarrow p_n = \sqrt{2mE}.$$

Its de Broglie wavelength is therefore

$$\lambda_n = \frac{h}{\sqrt{2mE}}.$$

The energy given for the neutron is kinetic energy, rather than its rest energy mc^2 .

Step 2: Express the photon wavelength and divide A photon has zero rest mass, so the neutron's kinetic-energy formula cannot be applied to it. Instead, for photon energy equal to the neutron's kinetic energy,

$$\lambda_p = \frac{hc}{E}.$$

Divide the two wavelengths:

$$\begin{aligned} \frac{\lambda_n}{\lambda_p} &= \frac{h}{\sqrt{2mE}} \frac{E}{hc} \\ &= \frac{1}{c} \sqrt{\frac{E}{2m}} = \sqrt{\frac{E}{2mc^2}}. \end{aligned}$$

Planck's constant cancels. The final ratio is dimensionless because both E and mc^2 have units of energy.

Step 3: Conclusion The required ratio is $\frac{\lambda_n}{\lambda_p} = \sqrt{\frac{E}{2mc^2}}$, using the non-relativistic neutron relation $E \ll mc^2$.

Quick Tip to Solve: For equal energies, write the two momenta separately: $p_n = \sqrt{2mE}$ and $p_p = E/c$. Then the wavelength ratio is p_p/p_n .

20(a). Monochromatic light of frequency 5.0×10^{14} Hz passes from air into a medium of refractive index 1.5. Find the wavelength of the light (i) reflected, and (ii) refracted at the interface of the two media. [2 marks]

Solution:

Concept:

- At a stationary interface, the frequency of light remains unchanged during reflection and refraction; it is fixed by the source.
- The wave relation is $v = \nu\lambda$, where v is speed, ν frequency and λ wavelength. In a medium of refractive index n , $v = c/n$.

Step 1: Calculate the wavelength of reflected light The reflected wave travels back into air, so its speed is approximately $c = 3.0 \times 10^8 \text{ m s}^{-1}$. Its frequency is the incident frequency, $\nu = 5.0 \times 10^{14} \text{ Hz}$. Therefore,

$$\lambda_{\text{reflected}} = \frac{c}{\nu} = \frac{3.0 \times 10^8}{5.0 \times 10^{14}} = 6.0 \times 10^{-7} \text{ m}.$$

Reflection changes the direction of propagation, but does not change the wavelength in the same medium.

Step 2: Calculate the wavelength of refracted light In the second medium the speed decreases to

$v = c/n = 2.0 \times 10^8 \text{ m s}^{-1}$. Since the frequency stays the same, the wavelength decreases in the same ratio as the speed:

$$\lambda_{\text{refracted}} = \frac{c}{n\nu} = \frac{6.0 \times 10^{-7}}{1.5} = 4.0 \times 10^{-7} \text{ m}.$$

Step 3: Conclusion (i) Reflected light: $\lambda = 6.0 \times 10^{-7} \text{ m} = 600 \text{ nm}$. (ii) Refracted light: $\lambda = 4.0 \times 10^{-7} \text{ m} = 400 \text{ nm}$.

Quick Tip to Solve: First find the wavelength in air using c/ν ; divide it by n only for the light transmitted into the medium.

OR

20(b). A plano-convex lens of focal length 16 cm is made of a material of refractive index 1.4. Calculate the radius of the curved surface of the lens. [2 marks]

Solution:

Concept:

- For a thin lens in air, the lensmaker's formula is $\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$, where f is focal length, n is refractive index and R_1, R_2 are signed surface radii.
- A plane surface has infinite radius of curvature and therefore zero reciprocal curvature. A plano-convex lens in air is converging, so its focal length is positive.

Step 1: Apply the surface sign convention Take light to travel from left to right, entering the convex surface first. The centre of curvature of this first surface lies to its right, so $R_1 = +R$, where R is the required positive radius. The second surface is plane, giving $R_2 = \infty$. Thus the lensmaker's formula becomes

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R} - 0 \right) = \frac{n - 1}{R}.$$

Step 2: Substitute the focal length and refractive index Rearranging gives $R = (n - 1)f$. Using $f = 16 \text{ cm}$ and $n = 1.4$,

$$R = (1.4 - 1) \times 16 \text{ cm} = 0.4 \times 16 \text{ cm} = 6.4 \text{ cm}.$$

The refractive index is dimensionless, so the calculated radius has the same length unit as the focal length. Reversing the lens changes the signed surface assignments but leaves its focal length and radius magnitude unchanged.

Step 3: Conclusion The radius of the curved surface is 6.4 cm, or 0.064 m.

Quick Tip to Solve: For a plano-convex thin lens in air, set the plane surface's reciprocal radius to zero to obtain $R = (n - 1)f$.

21. Differentiate between ‘diffusion current’ and ‘drift current’. Explain their role in the formation of p-n junction. [2 marks]

Solution:

Concept:

- Diffusion is carrier transport from a region of higher carrier concentration to one of lower concentration. It can occur without an applied electric field.
- Drift is carrier transport caused by an electric field. Holes drift along the field and electrons opposite to it; conventional current is opposite to electron motion.

Step 1: Distinguish the two currents Diffusion current is driven by a concentration gradient, whereas drift current is driven by an electric field. In an unbiased p-n junction, diffusion is mainly due to majority carriers: electrons move from n to p and holes from p to n. Both contributions give conventional diffusion current from p to n. The junction field sweeps thermally generated minority carriers across the junction, producing drift current from n to p, opposite to the diffusion current.

Step 2: Explain depletion-layer formation and equilibrium When the p and n regions are joined, majority carriers diffuse across the boundary and recombine. They leave fixed positive donor ions on the n side and fixed negative acceptor ions on the p side. This region is depleted of mobile carriers and is called the depletion layer. Its electric field points from n to p and creates a potential barrier that opposes further majority-carrier diffusion. The same field causes the opposing drift current. At thermal equilibrium, the magnitudes of drift and diffusion currents are equal:

$$I_{\text{drift}} + I_{\text{diffusion}} = 0$$

when their opposite directions are included in the signs. Thus zero net current does not mean that the individual carrier processes cease.

Step 3: Conclusion Diffusion, driven by the concentration gradient, creates the depletion layer and barrier; field-driven drift opposes it. In an unbiased junction at equilibrium, the two currents balance and the net current is zero.

Quick Tip to Solve: Track conventional current separately from electron motion: junction diffusion current is p to n, while drift current is n to p.

Section C (21 marks)

22. An air-filled parallel plate capacitor with plate separation 1 mm has a capacitance of 20 pF. It is charged to 4.0 μC. Calculate the amount of work done to pull its plates to a separation of 5 mm. Assume the charge on the plates remains the same. [3 marks]

Solution:

Concept:

- For an air-filled parallel-plate capacitor, $C = \epsilon_0 A/d$, where A is plate area and d is plate separation. For fixed plate area, capacitance is inversely proportional to separation.
- At constant charge Q , electrostatic energy is $U = Q^2/(2C)$. The external work in slowly separating the plates equals the increase in stored energy when no charge flows to or from a source.

Step 1: Find the capacitance at the larger separation The initial and final separations are $d_i = 1 \text{ mm}$

and $d_f = 5$ mm. The plate area remains unchanged, so

$$C_f = C_i \frac{d_i}{d_f} = 20 \text{ pF} \times \frac{1}{5} = 4 \text{ pF}.$$

Increasing the separation by a factor of five therefore reduces the capacitance by the same factor. The specified fixed charge is $Q = 4.0 \times 10^{-6}$ C.

Step 2: Calculate the initial and final stored energies Convert the capacitances to farads: $C_i = 20 \times 10^{-12}$ F and $C_f = 4 \times 10^{-12}$ F. Since the charge is fixed rather than the potential difference, use $Q^2/(2C)$:

$$U_i = \frac{(4.0 \times 10^{-6})^2}{2(20 \times 10^{-12})} = 0.40 \text{ J},$$

$$U_f = \frac{(4.0 \times 10^{-6})^2}{2(4 \times 10^{-12})} = 2.0 \text{ J}.$$

The final stored energy is five times the initial energy, as expected because at fixed charge U is proportional to d .

Step 3: Relate the energy change to external work The oppositely charged plates attract each other. Pulling them apart therefore requires positive external work against the electric force. With no energy exchange with a battery and no change in kinetic energy, this work is stored as additional electrostatic energy:

$$W_{\text{ext}} = U_f - U_i = 2.0 - 0.40 = 1.6 \text{ J}.$$

The work done by the electric force itself has the opposite sign; the question asks for the work supplied to pull the plates apart.

Step 4: Conclusion The external work required to increase the plate separation to 5 mm at constant charge is $\boxed{+1.6 \text{ J}}$.

Quick Tip to Solve: With charge fixed, $U \propto d$. A fivefold separation makes the energy fivefold, so the required work is $4U_i$.

23(a). Define current density. Is it a scalar or a vector? An electric field $\vec{E}$ is maintained in a metallic conductor. If n be the number of electrons (mass m , charge $-e$) per unit volume in the conductor and τ its relaxation time, show that the current density $\vec{j} = \alpha \vec{E}$, where $\alpha = \left(\frac{ne^2}{m}\right) \tau$. [3 marks]

Solution:

Concept:

- Current density is the electric current crossing unit area normal to the flow. For uniform flow, $j = I/A$, where I is current and A is the perpendicular cross-sectional area.
- For charge carriers of number density n , charge q and mean drift velocity $\vec{v}_d$, $\vec{j} = nq\vec{v}_d$. Electrons have charge $q = -e$, where e is positive.
- The relaxation time τ is the mean time between successive collisions. The field produces electron acceleration $\vec{a} = -e\vec{E}/m$.

Step 1: Define the direction and nature of current density Current density is a vector directed along the motion of positive charge, hence along conventional current. For electrons it is opposite to their drift velocity. More generally, the current across a small oriented surface is $dI = \vec{j} \cdot d\vec{A}$. Its SI unit is

A m^{-2} ; the scalar current through a surface does not make current density a scalar.

Step 2: Determine the electron drift velocity Without an applied field, random thermal electron velocities average to zero. Between collisions the applied field changes each electron's velocity with acceleration $-e\vec{E}/m$. Averaging the time since the last collision gives the relaxation time, so

$$\vec{v}_d = \vec{a}\tau = -\frac{e\tau}{m}\vec{E}.$$

The negative sign indicates that the mean electron drift is opposite to the electric field.

Step 3: Relate the drift to current density During time dt , the magnitude of charge crossing a perpendicular area A is $neAv_d dt$. Therefore $j = nev_d$. Including directions and the negative electron charge gives

$$\begin{aligned}\vec{j} &= -ne\vec{v}_d \\ &= -ne\left(-\frac{e\tau}{m}\vec{E}\right) = \frac{ne^2\tau}{m}\vec{E}.\end{aligned}$$

Thus the two negative signs cancel and conventional current density is parallel to the applied field.

Step 4: Conclusion Current density is a vector. The required relation is $\boxed{\vec{j} = \alpha\vec{E}}$, with $\boxed{\alpha = ne^2\tau/m}$. The coefficient α is the electrical conductivity, measured in S m^{-1} .

Quick Tip to Solve: Keep both electron signs: use $\vec{v}_d = -e\tau\vec{E}/m$ and $\vec{j} = -ne\vec{v}_d$; their cancellation gives positive conductivity.

OR

23(b). What is a Wheatstone bridge? Obtain the necessary conditions under which the Wheatstone bridge is balanced. [3 marks]

Solution:

Concept:

- A Wheatstone bridge is an arrangement of four resistances in a quadrilateral, with a galvanometer across one diagonal and a source of emf across the other. It compares resistances and can determine an unknown resistance.
- At balance, no current passes through the galvanometer, so its two terminals have equal potential. Ohm's law then relates the potential drops in the two branches.

Step 1: Label the arms and the branch currents Label the four vertices A, B, C, D in cyclic order. Let the arm resistances be $AB = P$, $BC = Q$, $AD = R$ and $DC = S$. Connect the source across A and C , with A at higher potential, and connect the galvanometer between B and D . This specifies two source branches: $A \rightarrow B \rightarrow C$ and $A \rightarrow D \rightarrow C$. At balance the galvanometer current $I_g = 0$. Therefore the same current I_1 flows through P, Q , while the same current I_2 flows through R, S .

Step 2: Equate the potential drops to the middle junctions The null galvanometer reading requires $V_B = V_D$. Hence the potential drops from A to these junctions are equal:

$$V_A - V_B = V_A - V_D \Rightarrow I_1P = I_2R.$$

Similarly, the potential drops from the junctions to C are equal:

$$V_B - V_C = V_D - V_C \Rightarrow I_1 Q = I_2 S.$$

Step 3: Eliminate the currents and obtain the balance condition Dividing the first drop equation by the second eliminates both branch currents:

$$\frac{I_1 P}{I_1 Q} = \frac{I_2 R}{I_2 S} \Rightarrow \frac{P}{Q} = \frac{R}{S}.$$

Conversely, this resistance ratio gives equal voltage-divider fractions $Q/(P+Q) = S/(R+S)$. Thus B and D have equal potentials and connecting the galvanometer produces no current, confirming balance.

Step 4: Conclusion The Wheatstone bridge is balanced when $I_g = 0$, equivalently $V_B = V_D$. For the stated arm labels, its resistance condition is $P/Q = R/S$, or $PS = QR$. An unknown arm S can then be found as $S = QR/P$.

Quick Tip to Solve: Label the four arms before quoting the ratio: equate drops from the same supply terminal to the two galvanometer junctions, then divide.

24. A circular coil with cross-sectional area 0.2 cm^2 carries a current of 4 A . It is kept in a uniform magnetic field of magnitude 0.5 T normal to the plane of the coil. Calculate:

(a) the net force on the coil.

(b) the torque on the coil.

(c) the average force on each electron in the coil due to the magnetic field. The free electron density in the material of the coil is 10^{28} m^{-3} .

[3 marks]

Solution:

Concept:

- A current element experiences $d\vec{F} = I d\vec{\ell} \times \vec{B}$. The resultant force on a closed current loop in a uniform magnetic field is zero.
- A plane coil has magnetic moment $\vec{\mu} = NIA_{\text{loop}}\hat{n}$ and torque $\vec{\tau} = \vec{\mu} \times \vec{B}$, where A_{loop} is the area enclosed by a turn and $\hat{n}$ is its normal.
- For conducting wire of cross-sectional area A , the current magnitude is $I = neAv_d$. The average magnetic force associated with electron drift has magnitude $ev_d B \sin \theta$.

Step 1: Calculate the net force on the coil Since the field is uniform, it can be taken outside the closed-loop integral:

$$\vec{F}_{\text{net}} = I \oint d\vec{\ell} \times \vec{B} = I \left(\oint d\vec{\ell} \right) \times \vec{B} = \vec{0}.$$

The vector sum of directed displacements around a closed loop is zero. Individual coil elements experience magnetic forces, but opposite elements experience equal and opposite forces, so the resultant is 0 N .

Step 2: Calculate the torque on the coil The field is normal to the plane of the coil. Its magnetic moment, also normal to the coil, is therefore parallel or antiparallel to the field. The angle between $\vec{\mu}$

and $\vec{B}$ is 0° or 180° , giving

$$\tau = NIA_{\text{loop}}B \sin \theta = 0 \text{ N m}.$$

No value of the enclosed loop area or number of turns is needed, because the sine factor vanishes.

Step 3: Find the drift speed from the wire cross-section The given cross-sectional area is the area of the conducting wire, used in $I = neAv_d$. Convert it to SI units:

$$A = 0.2 \text{ cm}^2 = 0.2 \times 10^{-4} \text{ m}^2 = 2.0 \times 10^{-5} \text{ m}^2.$$

Using $e = 1.6 \times 10^{-19} \text{ C}$,

$$v_d = \frac{4}{(10^{28})(1.6 \times 10^{-19})(2.0 \times 10^{-5})} = 1.25 \times 10^{-4} \text{ m s}^{-1}.$$

Step 4: Calculate the average magnetic force per electron Electron drift is tangential to the wire, hence in the plane of the coil and perpendicular to $\vec{B}$. Random thermal velocity averages to zero, so the average Lorentz force at a point is $-e\vec{v}_d \times \vec{B}$. Its magnitude is

$$\begin{aligned} F_e &= ev_d B = \frac{IB}{nA} \\ &= (1.6 \times 10^{-19})(1.25 \times 10^{-4})(0.5) \\ &= 1.0 \times 10^{-23} \text{ N}. \end{aligned}$$

This local force is radial; its direction depends on the current and field directions. Forces around the coil cancel vectorially, consistently with part (a).

Step 5: Conclusion (a) Net force: 0 N . (b) Torque: 0 N m . (c) Magnitude of the average magnetic force on each drifting electron: $1.0 \times 10^{-23} \text{ N}$.

Quick Tip to Solve: For part (c), combine $I = neAv_d$ with $F_e = ev_d B$ to get $F_e = IB/(nA)$; use wire cross-section, not enclosed loop area.

25. (a) Draw the graphs showing the variation of the following with the frequency of ac source in a circuit:

(i) Resistance

(ii) Capacitive reactance

(iii) Inductive reactance

(b) Can the voltage drop across the inductor or the capacitor in a series LCR circuit be greater than the applied voltage of the ac source? Justify your answer. [3 marks]

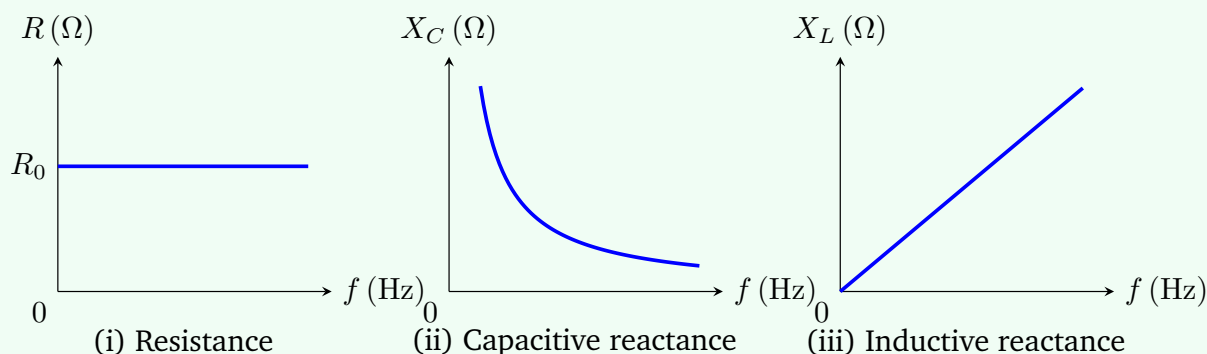
Solution:

Concept:

- For ideal circuit elements at fixed temperature, resistance R is independent of frequency f , while $X_C = 1/(2\pi fC)$ and $X_L = 2\pi fL$. Here C is capacitance and L is inductance.
- In a series LCR circuit, the same rms current I flows through all elements. Voltages add as phasors: $V = \sqrt{V_R^2 + (V_L - V_C)^2}$, with $V_R = IR$, $V_L = IX_L$ and $V_C = IX_C$.

Step 1: (a) Plot resistance and reactance against frequency Resistance is constant, so its graph is a

horizontal straight line. Capacitive reactance decreases inversely with frequency, giving a rectangular hyperbola in the first quadrant: $X_C \rightarrow \infty$ as $f \rightarrow 0^+$, and $X_C \rightarrow 0$ as $f \rightarrow \infty$. Inductive reactance increases directly with frequency, so it is a straight line through the origin with slope $2\pi L$. The sketches below use positive, arbitrary scales; both reactances and resistance are measured in ohms.



Frequency dependence of ideal resistance, capacitive reactance and inductive reactance.

Step 2: (b) Compare individual voltages with the source voltage Yes, the voltage across an inductor or a capacitor can exceed the applied voltage. The inductor voltage leads the current by 90° , while the capacitor voltage lags it by 90° . Thus these two voltages oppose each other in the phasor sum; their magnitudes are not added arithmetically. At resonance, $\omega_0 L = 1/(\omega_0 C)$, so $V_L = V_C$ and the applied voltage is only $V = IR$. Consequently,

$$\frac{V_L}{V} = \frac{V_C}{V} = \frac{\omega_0 L}{R} = Q.$$

If the quality factor $Q > 1$, each reactive voltage exceeds V , even though the two reactive phasors cancel. This is consistent with Kirchhoff's voltage law, which applies to the instantaneous voltages or their phasor sum.

Step 3: Conclusion R is constant, $X_C \propto 1/f$, and $X_L \propto f$, as shown. Yes, either reactive voltage may exceed the source voltage; at resonance both do so when $Q > 1$, owing to their opposite phases.

Quick Tip to Solve: Sketch R horizontal, X_C falling hyperbolically and X_L rising linearly. For series LCR voltages, subtract the reactive phasors before combining with V_R .

26. (a) State any two properties of a nucleus.

(b) Why is the density of a nucleus much more than that of an atom?

(c) Show that the density of the nuclear matter is the same for all nuclei.

[3 marks]

Solution:

Concept:

- A nucleus contains Z protons and $A - Z$ neutrons, where Z is its atomic number and A its mass number. Its charge is $+Ze$, and its mass is approximately Am_N , with m_N the mean nucleon mass.
- The empirical nuclear radius relation is $R = R_0 A^{1/3}$, where $R_0 \simeq 1.2 \times 10^{-15}$ m. Mean mass density is mass divided by volume, with $V = 4\pi R^3/3$ for a spherical nucleus.

Step 1: (a) State two nuclear properties First, the nucleus is positively charged: its Z protons give it total charge $+Ze$, since neutrons carry no net charge. Second, its radius is approximately $R_0 A^{1/3}$; hence its volume increases in direct proportion to the number of nucleons A . These describe its

electrical charge and its size dependence respectively.

Step 2: (b) Compare nuclear and atomic volumes Almost the entire mass of an atom is concentrated in its nucleus because protons and neutrons are much heavier than electrons. Yet the nuclear radius is of order 10^{-15} to 10^{-14} m, far smaller than the atomic radius of order 10^{-10} m. Thus almost the same mass occupies a much smaller volume in the nucleus. Approximately,

$$\frac{\rho_{\text{nucleus}}}{\rho_{\text{atom}}} \simeq \left(\frac{r_{\text{atom}}}{R}\right)^3 \gg 1.$$

This large volume ratio accounts for the much greater nuclear density; most atomic volume is outside the nucleus.

Step 3: (c) Derive the density of nuclear matter Substitute $M \simeq Am_N$ and $R = R_0 A^{1/3}$ in the density formula:

$$\begin{aligned}\rho &= \frac{M}{(4/3)\pi R^3} \simeq \frac{Am_N}{(4/3)\pi R_0^3 A} \\ &= \frac{3m_N}{4\pi R_0^3}.\end{aligned}$$

The factor A cancels. Since m_N and R_0 are approximately universal, the density is independent of mass number. Using $m_N \simeq 1.67 \times 10^{-27}$ kg gives $\rho \simeq 2.3 \times 10^{17}$ kg m $^{-3}$. Small mass-defect and nuclear-structure effects are neglected in this standard model.

Step 4: Conclusion A nucleus has positive charge $+Ze$ and radius $R_0 A^{1/3}$. Its very small volume contains almost all atomic mass, making it extremely dense. Its mean density $3m_N/(4\pi R_0^3)$ is approximately the same for all nuclei.

Quick Tip to Solve: Cube $R = R_0 A^{1/3}$ before substituting into density: both nuclear mass and volume contain exactly one factor of A , which cancels.

27. State the three postulates of Bohr's theory of hydrogen atom.

A hydrogen atom de-excites from level n to level $(n - 1)$. Show that, according to Bohr's theory, the frequency of radiation emitted $\nu \cong \frac{\alpha}{n^3}$, for large values of n , where α is a constant. This result exactly agrees with that obtained from classical physics — one of the successes of Bohr's theory. [3 marks]

Solution:

Concept:

- Bohr's model permits discrete stationary electron orbits and angular momenta $m_e v r = n\hbar$, where $n = 1, 2, \dots$, m_e is electron mass and $\hbar = h/(2\pi)$.
- Hydrogen energy levels are $E_n = -K/n^2$, where $K = 13.6$ eV. Emission from a higher to a lower level obeys $h\nu = E_i - E_f$, with h Planck's constant.

Step 1: State the three postulates **1. Stationary orbits:** An electron revolves around the nucleus in certain permitted circular orbits without radiating energy. Each stationary orbit has a definite energy, despite the electron's acceleration.

2. Quantised angular momentum: Only those orbits are allowed for which the electron's orbital angular momentum is an integral multiple of $h/(2\pi)$:

$$m_e v r = \frac{nh}{2\pi}, \quad n = 1, 2, 3, \dots$$

3. Radiation during transitions: Radiation is emitted or absorbed only when the electron changes between stationary states. Its frequency is fixed by $h\nu = |E_i - E_f|$; a downward transition emits a photon and an upward transition absorbs one.

Step 2: Calculate the frequency for adjacent levels For emission from level n to $n - 1$, take $n \geq 2$. The final state is more negative in energy, so the photon energy is positive:

$$\begin{aligned} h\nu &= E_n - E_{n-1} \\ &= K \left[\frac{1}{(n-1)^2} - \frac{1}{n^2} \right] \\ &= K \frac{2n-1}{n^2(n-1)^2}. \end{aligned}$$

This is the exact adjacent-level result within the model; no large- n approximation has yet been made.

Step 3: Apply the large quantum number limit If $n \gg 1$, then $2n - 1 \simeq 2n$ and $(n - 1)^2 \simeq n^2$. Therefore,

$$\nu \simeq \frac{K}{h} \frac{2n}{n^4} = \frac{2K/h}{n^3} = \frac{\alpha}{n^3}, \quad \alpha = \frac{2K}{h}.$$

The numerator is independent of n . With K expressed in joules, $\alpha \simeq 6.58 \times 10^{15} \text{ s}^{-1}$. The same leading expression follows for the electron's classical orbital frequency: from $m_e v_n r_n = n\hbar$ and kinetic energy $m_e v_n^2/2 = K/n^2$,

$$f_{\text{orbit}} = \frac{v_n}{2\pi r_n} = \frac{m_e v_n^2}{nh} = \frac{2K}{hn^3}.$$

Hence the adjacent-transition radiation frequency approaches the classical orbital frequency at large n , illustrating the correspondence principle.

Step 4: Conclusion Bohr postulated stationary nonradiating orbits, angular momentum $m_e v r = n\hbar$, and photon emission or absorption with $h\nu = |\Delta E|$. For adjacent high levels, $\nu \simeq \alpha/n^3$, where $\alpha = 2K/h$, in agreement with the classical orbital frequency.

Quick Tip to Solve: First subtract the negative energy levels exactly, then use $n - 1 \simeq n$ only for large n . The numerator scales as n and the denominator as n^4 , giving n^{-3} .

28. (a) “The wavelength of the electromagnetic wave is often correlated with the characteristic size of the system that radiates.” Give two examples to justify this statement.

(b) (i) Long distance radio broadcasts use short-wave bands. Why?

(ii) Optical and radio telescopes are built on the ground, but X-ray astronomy is possible only from satellites orbiting the Earth. Why?

[3 marks]

Solution:

Concept:

- The wavelength λ and frequency ν of electromagnetic radiation are related by $c = \nu\lambda$, where c is its speed in vacuum.
- The characteristic size of a radiating system is often correlated with its radiation wavelength; this is a qualitative correlation, not an exact universal equality.
- Atmospheric transmission depends on wavelength: the ionosphere returns suitable radio waves, whereas the atmosphere absorbs cosmic X-rays.

Step 1: (a) Compare two radiating systems Radio waves are emitted by oscillating charges in antennas. Efficient transmitting antennas have lengths comparable with the emitted wavelength; for example, a half-wave dipole has length approximately $\lambda/2$. Thus metre-scale antennas can radiate metre-scale radio waves.

Gamma rays originate in transitions of atomic nuclei. Nuclei are extremely small systems, and the gamma radiation they emit has very short wavelengths. Thus a large antenna is associated with long-wavelength radiation, whereas a tiny nucleus is associated with short-wavelength radiation. These examples illustrate the stated correlation without asserting exact equality of source size and wavelength.

Step 2: (b)(i) Explain short-wave broadcasting Short-wave radio signals of suitable frequency are returned towards Earth by the ionosphere. The wave can therefore reach a receiver far beyond the direct line of sight of the transmitter. Successive returns between the ionosphere and Earth's surface permit communication over very long distances. This sky-wave propagation is why short-wave bands are useful for distant broadcasts.

Step 3: (b)(ii) Compare the atmospheric windows Earth's atmosphere transmits visible light and useful radio-frequency bands sufficiently well for astronomical signals in these bands to reach ground-based telescopes. In contrast, the atmosphere absorbs incoming X-rays before they reach the ground. X-ray detectors must therefore be placed above the absorbing atmosphere, for example on satellites orbiting Earth, to observe astronomical X-ray sources.

Step 4: Conclusion Antennas and nuclei illustrate the correlation between source size and wavelength. Short waves support long-distance sky-wave communication, while atmospheric transmission allows ground-based optical and radio astronomy but requires space-based X-ray astronomy.

Quick Tip to Solve: For each band, ask whether the atmosphere transmits it, returns it to Earth, or absorbs it; these three behaviours explain the applications here.

Section D (8 marks)

Questions number 29 and 30 are case study-based questions. Read the following paragraphs and answer the questions that follow.

29. A lens is a transparent medium bounded by two surfaces, with one or both surfaces being spherical. The focal length of a lens is determined by the radii of curvature of its two surfaces and the refractive index of its medium with respect to that of the surrounding medium. The power of a lens is reciprocal of its focal length. If a number of lenses are kept in contact, the power of the combination is the algebraic sum of the powers of the individual lenses. [4 marks]

29(i). A double-convex lens, with each face having same radius of curvature R , is made of glass of refractive index n . Its power is: [1 mark]

- (A) $\frac{2(n-1)}{R}$
(B) $\frac{(2n-1)}{R}$
(C) $\frac{(n-1)}{2R}$
(D) $\frac{(2n-1)}{2R}$

Correct Answer: (A) $\frac{2(n-1)}{R}$

Solution:

Concept:

- The power of a thin lens in air is $P = 1/f = (n-1)(1/R_1 - 1/R_2)$, where n is the refractive index of glass and R_1, R_2 are signed radii.
- For light travelling from left to right through a symmetric double-convex lens, $R_1 = +R$ and $R_2 = -R$.

Step 1: Apply the signs of the two radii The centre of curvature of the first face lies to its right, whereas that of the second face lies to its left. Equal magnitudes therefore do not mean equal signed radii. Substituting in the lens-maker formula gives

$$P = (n-1) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{2(n-1)}{R}.$$

Step 2: Check the printed choices Both curved surfaces contribute equally, so their powers add. Option (C) incorrectly divides the single-surface contribution by two. Options (B) and (D), containing $2n-1$, would predict a nonzero power even at $n=1$, when there is no refractive-index contrast and no lens action. They are therefore inconsistent.

Step 3: Conclusion The power is $P = 2(n-1)/R$, in dioptres when R is in metres. Hence, the correct option is (A).

Quick Tip to Solve: For a double-convex lens, put opposite signs on the two radii before subtracting their reciprocals.

29(ii). A double-convex lens of power P , with each face having same radius of curvature, is cut into two equal parts perpendicular to its principal axis.

The power of one part of the lens will be:

[1 mark]

(A) $2P$

(B) P

(C) $4P$

(D) $\frac{P}{2}$

Correct Answer: (D) $\frac{P}{2}$

Solution:

Concept:

- The power of the original symmetric double-convex lens in air is $P = 2(n - 1)/R$, where R is the magnitude of either radius.
- Cutting perpendicular to the principal axis through the centre gives two plano-convex lenses; a plane face has infinite radius and zero reciprocal curvature.

Step 1: Identify the surfaces of each part Each equal part retains one of the original curved faces, while the cut creates a plane face. The glass refractive index remains n , and the radius of the remaining curved face is still R . The lens-maker formula therefore gives the positive power of either part as

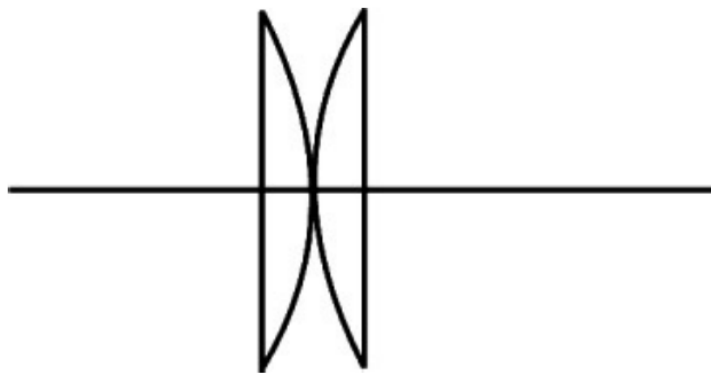
$$P_{\text{part}} = (n - 1) \left(\frac{1}{R} - \frac{1}{\infty} \right) = \frac{n - 1}{R}.$$

Step 2: Compare with the original lens The original lens had two equal contributions from its curved faces; each part now has only one. Thus its focal length doubles and its power halves. Option (B) would apply to a cut along the principal axis, which retains both curvatures. Options (A) and (C) incorrectly increase the power despite removing one curved face.

Step 3: Conclusion Each part has power $\boxed{P/2}$. Hence, the correct option is (D).

Quick Tip to Solve: A cut perpendicular to the axis makes a plane face: remove that face's curvature contribution before calculating power.

29(iii). The above two parts are kept in contact with each other as shown in the figure. The power of the combination will be: [1 mark]



- (A) $\frac{P}{2}$
(B) P
(C) $2P$
(D) $\frac{P}{4}$

Correct Answer: (B) P

Solution:

Concept:

- For thin lenses placed in contact, the equivalent power is the algebraic sum $P_{eq} = P_1 + P_2$.
- Each plano-convex half from a perpendicular cut has positive power $P/2$; turning it around does not change the sign of its power in air.

Step 1: Identify the powers in the arrangement The figure places the curved sides of the two plano-convex pieces towards one another. Each piece still has one plane face and one convex face, so it remains a converging lens of power $P/2$. Reversing a lens exchanges its faces but does not convert a converging lens into a diverging lens.

Step 2: Add the two powers Using the thin-lens contact approximation stated in the passage,

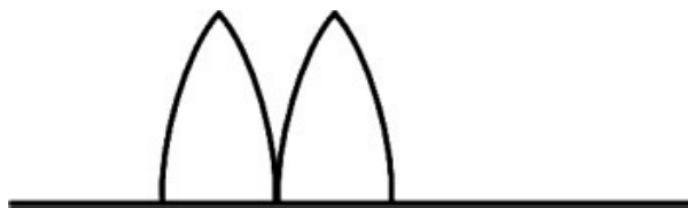
$$P_{eq} = \frac{P}{2} + \frac{P}{2} = P.$$

Option (A) counts only one piece. Option (C) wrongly assigns the original power P to each plano-convex half. Option (D) incorrectly halves a half-power again; contact powers add rather than multiply or divide.

Step 3: Conclusion The combination has power P . Hence, the correct option is (B).

Quick Tip to Solve: Turning a convex lens around leaves its power positive; add the two half-powers for this contact arrangement.

29(iv)(a). A double-convex lens of power P , with each face having same radius of curvature, is cut along its principal axis. The two parts are arranged as shown in the figure. The power of the combination will be: [1 mark]



(A) Zero

(B) P

(C) $2P$

(D) $\frac{P}{2}$

Correct Answer: (C) $2P$

Solution:

Concept:

- Lens power depends on refractive index and the curvatures of the faces traversed by light: $P = (n - 1)(1/R_1 - 1/R_2)$.
- Cutting a lens along its principal axis reduces its aperture but does not change its focal length or power. Thin lenses in contact have additive powers.

Step 1: Find the power of each half lens The cut is along the principal axis, so a ray passing through either half still meets the same two curved surfaces as before. The values of n , R_1 , and R_2 are unchanged. Each half therefore retains power P , although the smaller aperture allows less light to pass. It is not a plano-convex half formed by a perpendicular cut.

Step 2: Use the printed arrangement The two half lenses are placed one after the other in the light path, as shown. Under the thin-lens contact approximation, their positive powers add:

$$P_{\text{eq}} = P + P = 2P.$$

The power cannot be zero because neither piece is diverging. Option (B) gives only one piece's power; option (D) confuses an axial cut with the perpendicular cut in part (ii).

Step 3: Conclusion The power of the combination is $2P$. Hence, the correct option is (C).

Quick Tip to Solve: First distinguish the cut: along the axis changes aperture only, so each half keeps the original power before contact powers are added.

29(iv)(b). Two convex lenses of focal lengths 60 cm and 20 cm are held coaxially in contact with each other. The power of the combination is: [1 mark]

- (A) 6.6 D
 (B) 15 D
 (C) $\frac{1}{15}$ D
 (D) $\frac{1}{80}$ D

Correct Answer: (A) 6.6 D

Solution:

Concept:

- Lens power in dioptres is $P = 1/f$ when focal length f is expressed in metres; a convex lens has positive power.
- The powers of thin coaxial lenses in contact add: $P_{\text{eq}} = P_1 + P_2$.

Step 1: Convert the focal lengths to metres The given positive focal lengths are $f_1 = 60 \text{ cm} = 0.60 \text{ m}$ and $f_2 = 20 \text{ cm} = 0.20 \text{ m}$. Thus

$$P_1 = \frac{1}{0.60} = \frac{5}{3} \text{ D}, \quad P_2 = \frac{1}{0.20} = 5 \text{ D}.$$

Step 2: Add the powers and compare the options

$$P_{\text{eq}} = \frac{5}{3} + 5 = \frac{20}{3} \text{ D} = 6.666 \dots \text{ D}.$$

The printed 6.6 D is the intended approximate choice; the value correctly rounded to one decimal place would be 6.7 D. The equivalent focal length is 15 cm, which is not a power of 15 D. The choice $1/15 \text{ D}$ omits conversion from centimetres to metres, and $1/80 \text{ D}$ incorrectly adds focal lengths and also uses centimetres.

Step 3: Conclusion The calculated power is $\frac{20}{3} \text{ D} \approx 6.67 \text{ D}$, corresponding to the printed approximate value 6.6 D. Hence, the correct option is (A).

Quick Tip to Solve: Convert centimetres to metres before taking reciprocals; for lenses in contact add powers, not focal lengths.

30. Junction Diode as a Rectifier:

The process of conversion of an ac voltage into a dc voltage is called rectification and the device which performs this conversion is called a rectifier. The characteristics of a p-n junction diode reveal that when a p-n junction diode is forward biased, it offers a low resistance and when it is reverse biased, it offers a high resistance. Hence, a p-n junction diode conducts only when it is forward biased. This property of a p-n junction diode makes it suitable for its use as a rectifier.

Thus, when an ac voltage is applied across a p-n junction, it conducts only during those alternate half cycles for which it is forward biased. A rectifier which rectifies only half cycle of an ac voltage is called a half-wave rectifier and one that rectifies both the half cycles is known as a full-wave rectifier.

[4 marks]

30(i). The root mean square value of an alternating voltage applied to a full-wave rectifier is $\frac{V_0}{\sqrt{2}}$.

Then the root mean square value of the rectified output voltage is:

[1 mark]

- (A) $\frac{V_0}{\sqrt{2}}$
- (B) $\frac{V_0^2}{\sqrt{2}}$
- (C) $\frac{2V_0}{\sqrt{2}}$
- (D) $\frac{V_0}{2\sqrt{2}}$

Correct Answer: (A) $\frac{V_0}{\sqrt{2}}$

Solution:

Concept:

- The root mean square voltage is $V_{\text{rms}} = \sqrt{\langle v^2 \rangle}$, where the average is taken over a complete period.
- An ideal full-wave rectifier reverses the negative half cycles without changing their magnitudes:
 $v_{\text{out}} = |v_{\text{in}}|$.

Step 1: Compare the squared voltages Let the sinusoidal input be $v_{\text{in}} = V_0 \sin \omega t$, with peak voltage V_0 . Squaring removes the sign, so rectification leaves the squared waveform unchanged:

$$v_{\text{out}}^2 = V_0^2 \sin^2 \omega t, \quad \langle v_{\text{out}}^2 \rangle = \frac{V_0^2}{2}.$$

Step 2: Evaluate the RMS value and compare the options Taking the positive square root gives the same RMS value as the input. Option (B), as printed with V_0^2 , has the dimensions of voltage squared and cannot be an RMS voltage. Option (C) incorrectly doubles the amplitude, whereas (D) incorrectly halves the RMS value. Full-wave rectification changes polarity, not the magnitude of either half cycle.

Step 3: Conclusion The rectified output has $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$. Hence, the correct option is (A).

Quick Tip to Solve: For full-wave rectification, square the waveform first: reversing a sign cannot change its RMS value.

30(ii). In a full-wave rectifier, the current in each of the diodes flows for:

[1 mark]

- (A) Complete cycle of the input signal
- (B) Half cycle of the input signal
- (C) Less than half cycle of the input signal
- (D) Only for the positive half cycle of the input signal

Correct Answer: (B) Half cycle of the input signal

Solution:

Concept:

- An ideal p-n junction diode conducts when forward biased and blocks current when reverse biased.
- In the centre-tapped full-wave rectifier, the two diodes conduct on alternate half cycles, while load current has the same direction in both halves.

Step 1: Follow conduction through one input cycle During the first half cycle, one end of the transformer secondary is positive relative to the centre tap. Its diode is forward biased and conducts; the other diode is reverse biased. In the next half cycle the secondary polarities reverse, so the second diode conducts and the first blocks.

Step 2: Distinguish each diode from the complete rectifier Each diode therefore carries current for a duration $T/2$ in every input period T . Option (A) confuses the load's conduction over both halves with conduction in a single diode. For ideal diodes the conducting interval is a whole half cycle, so (C) is incorrect. Option (D) wrongly assigns the same positive half cycle of the input signal to both diodes; their conducting halves alternate.

Step 3: Conclusion Current in each diode flows for half a cycle of the input signal. Hence, the correct option is (B).

Quick Tip to Solve: Track one diode at a time: each conducts for one half cycle, although the load receives current in both.

30(iii). In a full-wave rectifier:

[1 mark]

- (A) Both diodes are forward biased at the same time.
- (B) Both diodes are reverse biased at the same time.
- (C) One is forward biased and the other is reverse biased at the same time.
- (D) Both are forward biased in the first half of the cycle and reverse biased in the second half of the cycle.

Correct Answer: (C) One is forward biased and the other is reverse biased at the same time.

Solution:

Concept:

- Forward bias allows appreciable current through a p-n junction diode; reverse bias blocks it in the ideal rectifier model.
- The two ends of a centre-tapped secondary have opposite instantaneous polarities relative to the centre tap, causing the two rectifier diodes to conduct alternately.

Step 1: Examine the first half cycle When the upper secondary end is positive relative to the centre tap, its diode is forward biased. The other end is simultaneously negative, so the second diode is reverse biased. The conducting diode supplies current to the load while the other does not conduct.

Step 2: Reverse the input polarity In the next half cycle both secondary voltages reverse: the previously conducting diode becomes reverse biased and the other becomes forward biased. Thus the two diodes always have opposite bias states during either half cycle. Options (A) and (B) incorrectly give them the same bias; (D) would make both diodes conduct together and then both block, contrary to full-wave operation. The instants of zero applied voltage are merely switching instants.

Step 3: Conclusion One diode is forward biased while the other is reverse biased. Hence, the correct option is (C).

Quick Tip to Solve: In a two-diode full-wave rectifier, label the secondary ends with opposite signs before deciding the diode biases.

30(iv)(a). An alternating voltage of frequency of 50 Hz is applied to a half-wave rectifier. Then the ripple frequency of the output will be: [1 mark]

- (A) 100 Hz
- (B) 50 Hz
- (C) 25 Hz
- (D) 150 Hz

Correct Answer: (B) 50 Hz

Solution:

Concept:

- Ripple is the periodic variation remaining in a rectifier's unfiltered output. Its frequency is the repetition frequency of the output pulses.
- A half-wave rectifier passes one half cycle and blocks the other, producing one output pulse per input period: $f_{\text{ripple}} = f_{\text{in}}$.

Step 1: Find the interval between successive pulses The input frequency is $f_{\text{in}} = 50$ Hz, so the input period is

$$T = \frac{1}{f_{\text{in}}} = \frac{1}{50} \text{ s} = 0.020 \text{ s}.$$

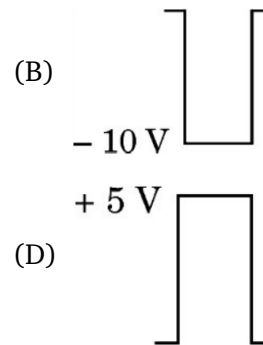
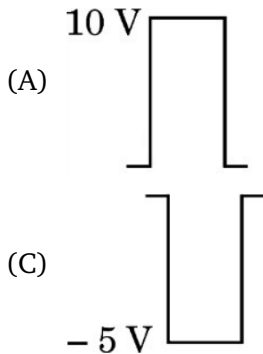
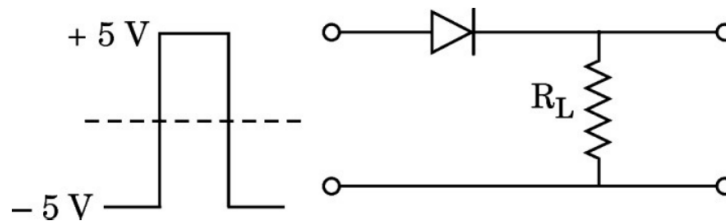
After a transmitted half cycle, the next half cycle is blocked. The next transmitted pulse begins one complete input period after the previous pulse.

Step 2: Calculate the ripple frequency The output repetition period is therefore $T_{\text{out}} = T$, giving $f_{\text{ripple}} = 1/T_{\text{out}}$. Option (A), 100 Hz, applies to a full-wave rectifier with two pulses per input cycle. Option (C) would require one pulse every two cycles, and (D) would require three pulses per cycle; neither is produced by this half-wave rectifier.

Step 3: Conclusion The ripple frequency is 50 Hz. Hence, the correct option is (B).

Quick Tip to Solve: Count output pulses per input cycle: one gives f for half-wave rectification; two give $2f$ for full-wave rectification.

30(iv)(b). A signal, as shown in the figure, is applied to a p-n junction diode. Identify the output across resistance R_L : [1 mark]



Correct Answer: (D)

Solution:

Concept:

- An ideal diode conducts from anode to cathode when forward biased and acts as an open circuit when reverse biased. The vertical bar in its symbol identifies the cathode.
- With the shown series diode and load resistance R_L , the load voltage follows the input during conduction and is zero when the diode blocks.

Step 1: Apply the positive input level The diode's anode is on the input side and its cathode is on the load side. When the input is $+5\text{ V}$ relative to the lower wire, the diode is forward biased. Neglecting the diode's forward voltage drop, the upper end of R_L is at $+5\text{ V}$, giving a positive rectangular output pulse of the same height as the input.

Step 2: Apply the negative input level and compare the waveforms At an input of -5 V , the diode is reverse biased and load current is zero. Hence $V_{\text{out}} = IR_L = 0$. The output switches between zero and $+5\text{ V}$. Option (A) wrongly doubles the amplitude to 10 V ; (B) and (C) show negative output pulses, which this diode orientation blocks. The input's peak-to-peak value of 10 V is not the output pulse height.

Step 3: Conclusion The output is the positive rectangular pulse of height $+5\text{ V}$, with zero voltage during the negative input level. Hence, the correct option is (D).

Quick Tip to Solve: Locate the cathode bar, then test each input level separately; a blocked series diode makes the load voltage zero.

Section E (15 marks)

31(a). (i) A resistor and a capacitor are connected in series to an ac source $v = v_m \sin \omega t$. Derive an expression for the impedance of the circuit.

(ii) When does an inductor act as a conductor in a circuit? Give reason for it.

(iii) An electric lamp is designed to operate at 110 V dc and 11 A current. If the lamp is operated on 220 V, 50 Hz ac source with a coil in series, then find the inductance of the coil. [5 marks]

Solution:

Concept:

- In a series circuit the same current flows through every element. The resistor voltage is in phase with the current, while the capacitor voltage lags it by $\pi/2$. Capacitive reactance is $X_C = 1/(\omega C)$, where C is capacitance and ω is angular frequency.
- Impedance is the ratio of voltage amplitude to current amplitude, $Z = V_m/I_m$, and also equals $V_{\text{rms}}/I_{\text{rms}}$. Voltages with different phases must be added as phasors.
- The self-induced emf of an inductor is $\varepsilon_L = -L di/dt$; its inductive reactance is $X_L = \omega L = 2\pi fL$, where L is inductance and f is frequency.

Step 1: (i) Express the resistor and capacitor voltages Let the steady-state current be $i = I_m \sin(\omega t + \phi)$, with the phase ϕ to be found. The resistor voltage is

$$v_R = I_m R \sin(\omega t + \phi).$$

For the capacitor, $i = dq/dt$, so integration gives the alternating charge $q = -(I_m/\omega) \cos(\omega t + \phi)$. Therefore

$$v_C = \frac{q}{C} = -I_m X_C \cos(\omega t + \phi).$$

Thus the voltage amplitudes $V_{Rm} = I_m R$ and $V_{Cm} = I_m X_C$ are mutually perpendicular phasors.

Step 2: (i) Combine the voltages to obtain impedance Kirchhoff's voltage law gives $v = v_R + v_C$. Since the two voltage phasors are perpendicular, the square of the resultant amplitude is the sum of their squared amplitudes:

$$v_m^2 = V_{Rm}^2 + V_{Cm}^2 = I_m^2 (R^2 + X_C^2),$$
$$Z = \frac{v_m}{I_m} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}.$$

Relative to the current phasor, the resultant voltage lies below the current axis. Hence the current leads the applied voltage by ϕ , where $\tan \phi = X_C/R = 1/(\omega CR)$. This is consistent with the applied voltage $v = v_m \sin \omega t$.

Step 3: (ii) Identify when an inductor acts as a conductor An ideal inductor acts as an ordinary conducting connection for steady direct current, after the switching transient has ended. The current is then constant, so $di/dt = 0$, and the induced opposing emf is zero. Equivalently, for dc $f = 0$, giving $X_L = 2\pi fL = 0$. A real coil still has the resistance of its wire; the statement means that its inductance offers no opposition to a constant current.

Step 4: (iii) Find the lamp resistance and required total impedance Treat the lamp as a resistor at its rated operating temperature and the series coil as having negligible resistance. From the dc rating,

the lamp resistance is

$$R = \frac{110}{11} = 10 \Omega.$$

For the lamp to operate normally on the ac supply, the rms current must be 11 A. The stated ac voltage 220 V is its rms value. Thus the required magnitude of the series RL impedance is

$$Z = \frac{220}{11} = 20 \Omega.$$

Step 5: (iii) Calculate the coil inductance The resistor and inductor voltage phasors are at right angles, so $Z^2 = R^2 + X_L^2$. This gives

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{20^2 - 10^2} = 10\sqrt{3} \Omega,$$

$$L = \frac{X_L}{2\pi f} = \frac{10\sqrt{3}}{2\pi(50)} \text{ H} \approx 5.51 \times 10^{-2} \text{ H}.$$

As a check, the lamp voltage remains $IR = 110 \text{ V}$, while the coil voltage is $IX_L = 110\sqrt{3} \text{ V}$. Their phasor resultant is $\sqrt{110^2 + (110\sqrt{3})^2} = 220 \text{ V}$, as required.

Step 6: Conclusion (i) The series RC impedance is $Z = \sqrt{R^2 + 1/(\omega^2 C^2)}$. (ii) An ideal inductor behaves as a conductor for steady dc because $di/dt = 0$ and hence its induced emf vanishes. (iii) The required coil inductance is $L \approx 0.0551 \text{ H} = 55.1 \text{ mH}$, assuming negligible coil resistance.

Quick Tip to Solve: First use the lamp's dc rating to find its hot resistance. For the ac circuit, combine the lamp and coil voltages using squares; subtracting 110 V directly from 220 V ignores their phase difference.

OR

31(b). (i) Draw a labelled diagram of a step-up transformer and describe its working principle. Explain any three causes for energy losses in a real transformer.

(ii) A step-up transformer converts a low voltage into high voltage. Does it violate the principle of conservation of energy? Explain.

(iii) A step-up transformer has 200 and 3000 turns in its primary and secondary coils respectively. The input voltage given to the primary coil is 90 V. Calculate:

(1) The output voltage across the secondary coil

(2) The current in the primary coil if the current in the secondary coil is 2.0 A.

[5 marks]

Solution:

Concept:

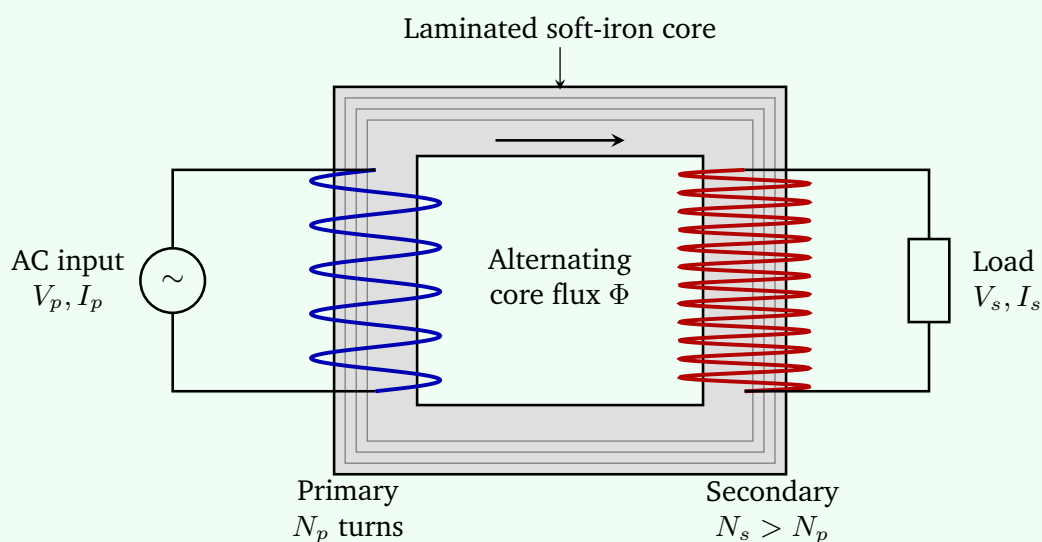
- A transformer works on mutual induction: a changing current in the primary coil produces a changing magnetic flux through the secondary coil and hence induces an emf in it.
- For an ideal transformer, $V_s/V_p = N_s/N_p$, where N_p, N_s are the primary and secondary turns and V_p, V_s are their rms voltages.
- Energy conservation gives $V_p I_p = V_s I_s$ for the ideal transformer and resistive load assumed here. Thus voltage and current transform in inverse ratios.

Step 1: (i) Draw and explain the step-up transformer The primary and secondary are insulated coils wound on a common laminated soft-iron core. The secondary has more turns than the primary,

$N_s > N_p$, as shown schematically below. An alternating voltage applied to the primary drives an alternating current. Its changing magnetic flux is guided by the core so that approximately the same flux Φ links each turn of both coils. By Faraday's law,

$$e_p = -N_p \frac{d\Phi}{dt}, \quad e_s = -N_s \frac{d\Phi}{dt}.$$

Neglecting winding resistance and flux leakage, the ratio of the rms terminal voltages is therefore $V_s/V_p = N_s/N_p > 1$. The secondary supplies the load at a higher voltage. The two windings are electrically insulated from one another; energy is transferred through the changing magnetic field. A steady direct current cannot sustain this process because its magnetic flux is constant after switching transients.



Step-up transformer: insulated windings on a common laminated core, with more secondary turns (schematic).

Step 2: (i) Explain three real energy losses **Copper loss:** Both windings have resistance, so current heats the wires. The loss is approximately $I_p^2 R_p + I_s^2 R_s$; low-resistance copper windings reduce it.

Eddy-current loss: The changing core flux induces circulating currents within the iron. These currents produce Joule heating. Thin core laminations insulated from one another increase the resistance of these current paths and reduce the loss.

Hysteresis loss: The core is repeatedly magnetised and demagnetised during each ac cycle. Energy is dissipated as heat in reversing its magnetisation. A soft magnetic material with a narrow hysteresis loop reduces this loss.

Step 3: (ii) Apply conservation of energy Increasing the voltage does not create energy. In an ideal transformer the increase in secondary voltage is accompanied by a corresponding decrease in secondary current:

$$V_p I_p = V_s I_s, \quad \frac{I_s}{I_p} = \frac{N_p}{N_s}.$$

Thus the output power equals the input power. In a real transformer, some input energy is converted into heat, so the useful output power is smaller than the input power; output energy plus losses equals input energy. Neither the ideal nor the real transformer violates energy conservation.

Step 4: (iii)(1) Calculate the secondary voltage Use the ideal-transformer approximation because no

efficiency or winding resistance is specified. The turns ratio is

$$\frac{N_s}{N_p} = \frac{3000}{200} = 15.$$

The rms voltage is stepped up by this factor:

$$V_s = V_p \frac{N_s}{N_p} = 90 \times 15 = 1350 \text{ V}.$$

Step 5: (iii)(2) Calculate the primary current The primary current must be greater than the secondary current in this step-up transformer. Equating the input and output powers gives

$$I_p = \frac{V_s I_s}{V_p} = \frac{1350 \times 2.0}{90} = 30 \text{ A}.$$

As a check, $90 \times 30 = 2700 \text{ W}$ at the input and $1350 \times 2.0 = 2700 \text{ W}$ at the output. The equal powers confirm the ideal approximation is internally consistent.

Step 6: Conclusion The step-up transformer uses mutual induction with $N_s > N_p$. Copper heating, eddy currents and hysteresis cause real energy losses. Energy is conserved because a higher output voltage is accompanied by a lower current. Assuming an ideal transformer, $V_s = 1350 \text{ V}$ and $I_p = 30 \text{ A}$.

Quick Tip to Solve: First find the turns ratio: here it is 15. Multiply the primary voltage by 15, but divide the primary current by 15 to get the secondary current.

32(a). (i) Derive an expression for potential energy of an electric dipole $\vec{p}$ in an external uniform electric field $\vec{E}$. When is the potential energy of the dipole (1) maximum, and (2) minimum?

(ii) An electric dipole consists of point charges -1.0 pC and $+1.0 \text{ pC}$ located at $(0, 0)$ and $(3 \text{ mm}, 4 \text{ mm})$ respectively in $x - y$ plane. An electric field $\vec{E} = \left(\frac{1000 \text{ V}}{\text{m}}\right) \hat{i}$ is switched on in the region. Find the torque $\vec{\tau}$ acting on the dipole.

[5 marks]

Solution:

Concept:

- The dipole moment is $\vec{p} = q(\vec{r}_+ - \vec{r}_-)$, directed from the negative charge to the positive charge; q denotes the positive charge magnitude.
- A uniform field exerts torque $\vec{\tau} = \vec{p} \times \vec{E}$. Its magnitude is $pE \sin \theta$, where θ is the angle between the dipole and the field.
- The increase of electrostatic potential energy equals the work done by an external agent in rotating the dipole slowly against the electric torque. Choose $U = 0$ at $\theta = \pi/2$.

Step 1: (i) Evaluate the work needed for a slow rotation The electric torque tends to reduce θ , bringing the dipole into alignment with the field. To increase the angle slowly, the external agent applies an equal and opposite torque. For a small angular displacement $d\theta$, the external work is

$$dW_{\text{ext}} = pE \sin \theta d\theta = dU.$$

Starting from the perpendicular orientation and integrating up to the final angle gives

$$\begin{aligned} U(\theta) - U(\pi/2) &= \int_{\pi/2}^{\theta} pE \sin \theta' d\theta' \\ &= [-pE \cos \theta']_{\pi/2}^{\theta} = -pE \cos \theta. \end{aligned}$$

With the stated zero of energy this is $U = -pE \cos \theta = -\vec{p} \cdot \vec{E}$. An alternative choice of zero would add only a constant and would not change the torque or equilibrium orientations.

Step 2: (i) Identify the maximum and minimum energy orientations Since $-1 \leq \cos \theta \leq 1$, the energy lies between $-pE$ and $+pE$. At $\theta = \pi$, the dipole is antiparallel to the field and its energy is maximum, $U_{\max} = +pE$. This is unstable equilibrium: a small angular disturbance lets the field turn the dipole further away from the antiparallel orientation. At $\theta = 0$, it is parallel to the field and its energy is minimum, $U_{\min} = -pE$. This is stable equilibrium because the torque opposes a small displacement from alignment.

Step 3: (ii) Express the dipole moment in SI units The displacement from the negative charge at the origin to the positive charge is

$$\vec{r}_+ - \vec{r}_- = (3\hat{i} + 4\hat{j}) \times 10^{-3} \text{ m}.$$

Using $q = 1.0 \text{ pC} = 1.0 \times 10^{-12} \text{ C}$, we obtain

$$\vec{p} = (3\hat{i} + 4\hat{j}) \times 10^{-15} \text{ C m}.$$

The dipole's component parallel to the field contributes no torque; the positive y-component determines its sense of rotation.

Step 4: (ii) Take the vector product with the applied field The printed field is $\vec{E} = 1000\hat{i} \text{ V m}^{-1}$. Thus, using $\hat{i} \times \hat{i} = 0$ and $\hat{j} \times \hat{i} = -\hat{k}$,

$$\begin{aligned} \vec{\tau} &= [(3\hat{i} + 4\hat{j}) \times 10^{-15}] \times (10^3\hat{i}) \\ &= -4.0 \times 10^{-12} \hat{k} \text{ N m}. \end{aligned}$$

The negative z-direction is into the x-y plane for the usual right-handed axes. It represents clockwise rotation as viewed from the positive z-axis, consistent with the field turning the dipole toward the positive x-axis.

Step 5: Conclusion (i) $U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$, with zero at 90° . It is maximum ($+pE$) at 180° and minimum ($-pE$) at 0° . (ii) $\boxed{\vec{\tau} = -4.0 \times 10^{-12} \hat{k} \text{ N m}}$.

Quick Tip to Solve: When the field points along positive x, use $\tau_z = -p_y E_x$; convert both pC and mm to SI before multiplying.

32(b). (i) An electric dipole (dipole moment $\vec{p} = p\hat{i}$), consisting of charges $-q$ and q , separated by distance $2a$, is placed along the x-axis, with its centre at the origin. Show that the potential V , due to this dipole, at a point x , ($x \gg a$) is equal to $\frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p} \cdot \hat{i}}{x^2}$.

(ii) Two isolated metallic spheres S_1 and S_2 of radii 1 cm and 3 cm respectively are charged such that both have the same charge density $\left(\frac{2}{\pi} \times 10^{-9}\right) \text{ C/m}^2$. They are placed far away from each other and connected by a thin wire. Calculate the new charge on sphere S_1 . [5 marks]

Solution:

Concept:

- Electric potential is a scalar and obeys superposition: the potential of a point charge Q at distance r is kQ/r , where $k = 1/(4\pi\epsilon_0)$.
- For charges separated by $2a$, the dipole moment magnitude is $p = 2qa$; it points from $-q$ to $+q$.
- A spherical conductor of radius R , carrying surface charge density σ , has charge $Q = 4\pi R^2\sigma$. For widely separated spheres, its potential is approximately kQ/R .
- Connecting conductors redistributes charge until their potentials are equal. Total charge is conserved; the thin connecting wire is assumed to carry negligible charge.

Step 1: (i) Add the potentials of the two charges Since $\vec{p} = p\hat{i}$, the negative charge is at $x = -a$ and the positive charge at $x = +a$. At the stated distant point on the positive x-axis, $x > a$, their respective distances are $x + a$ and $x - a$. Retaining the signs of the charges, scalar addition gives

$$\begin{aligned} V(x) &= k \left(\frac{q}{x-a} - \frac{q}{x+a} \right) \\ &= kq \frac{(x+a) - (x-a)}{(x-a)(x+a)} \\ &= \frac{k(2qa)}{x^2 - a^2} = \frac{kp}{x^2 - a^2}. \end{aligned}$$

This is the exact potential on this part of the axis, taking potential zero at infinity. Adding potential magnitudes without the negative charge's sign would incorrectly produce a monopole-like expression.

Step 2: (i) Apply the far-field approximation For $x \gg a$, the ratio a^2/x^2 is negligible, so $x^2 - a^2 \simeq x^2$. Hence

$$V(x) \simeq \frac{kp}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{i}}{x^2}.$$

We used $\vec{p} \cdot \hat{i} = p$. Thus the requested expression follows in the stated distant-point limit. The dipole potential falls as the inverse square of the axial distance, because the leading inverse-distance contributions of the equal and opposite charges cancel.

Step 3: (ii) Calculate the charge initially present on each sphere Convert the radii: $R_1 = 10^{-2} \text{ m}$ and $R_2 = 3 \times 10^{-2} \text{ m}$. With the common surface density $\sigma = (2/\pi) \times 10^{-9} \text{ C m}^{-2}$,

$$\begin{aligned} Q_1 &= 4\pi(10^{-2})^2 \frac{2}{\pi} \times 10^{-9} = 8.0 \times 10^{-13} \text{ C}, \\ Q_2 &= 4\pi(3 \times 10^{-2})^2 \frac{2}{\pi} \times 10^{-9} = 7.2 \times 10^{-12} \text{ C}, \\ Q_{\text{total}} &= Q_1 + Q_2 = 8.0 \times 10^{-12} \text{ C}. \end{aligned}$$

Equal initial surface charge densities give initial charges in the ratio $R_1^2 : R_2^2 = 1 : 9$; this is not the ratio after connection.

Step 4: (ii) Use equal final potentials and charge conservation Denote the final charges by Q'_1 and Q'_2 . Since the spheres are far apart, their mutual influence is neglected in the standard isolated-sphere approximation. The wire makes their final potentials equal:

$$\frac{kQ'_1}{R_1} = \frac{kQ'_2}{R_2} \Rightarrow Q'_1 : Q'_2 = R_1 : R_2 = 1 : 3.$$

Combining this ratio with conserved total charge gives

$$\begin{aligned} Q'_1 &= \frac{R_1}{R_1 + R_2} Q_{\text{total}} \\ &= \frac{1}{1 + 3} (8.0 \times 10^{-12}) = 2.0 \times 10^{-12} \text{ C}. \end{aligned}$$

As a check, $Q'_2 = 6.0 \times 10^{-12} \text{ C}$; their sum is unchanged. Initially $V_2/V_1 = R_2/R_1 = 3$, so charge flows to the smaller sphere and increases its positive charge, as this result predicts.

Step 5: Conclusion (i) For $x \gg a$, $V \simeq \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{i}}{x^2}$. (ii) The new charge on the sphere S_1 asked for in the printed paper is $Q'_1 = +2.0 \times 10^{-12} \text{ C} = +2.0 \text{ pC}$.

Quick Tip to Solve: Equal surface charge density means initial charges scale as R^2 , whereas a wire makes final charges scale as R ; conserve the total before applying the final ratio.

33(a). (i) A ray of light passes through a triangular prism. Show graphically, how the angle of deviation varies with the angle of incidence? Hence define the angle of minimum deviation.

(ii) A ray of light is incident normally on a refracting face of a prism of prism angle A and suffers a deviation of angle δ . Prove that the refractive index n of the material of the prism is given by

$$n = \frac{\sin(A + \delta)}{\sin A}.$$

(iii) The refractive index of the material of a prism is $\sqrt{2}$. If the refracting angle of the prism is 60° , find the:

(1) Angle of minimum deviation, and

(2) Angle of incidence.

[5 marks]

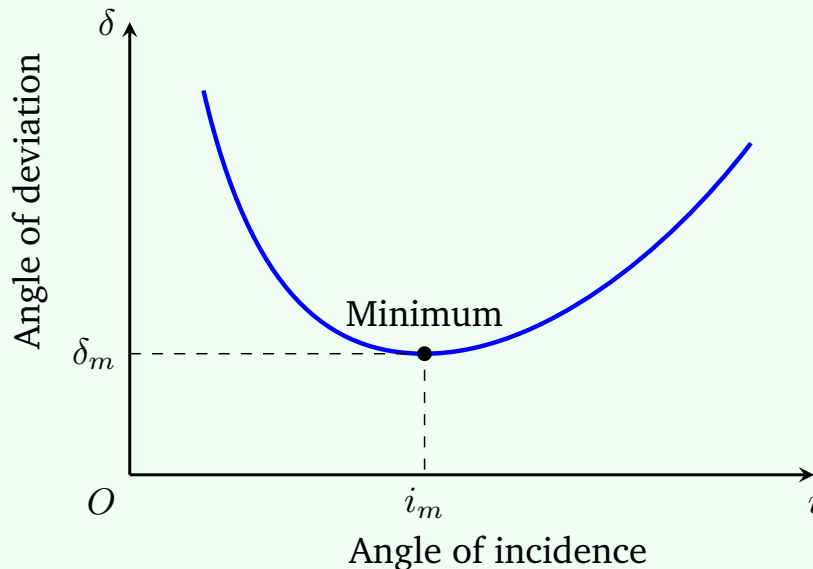
Solution:

Concept:

- For a prism in air, $A = r_1 + r_2$ and $\delta = i + e - A$, where A is the prism angle, i and e are the incidence and emergence angles, and r_1, r_2 are the internal refraction angles. All ray angles are measured from the corresponding normals.
- Snell's law at entry is $\sin i = n \sin r_1$, and at emergence it is $n \sin r_2 = \sin e$, where n is the refractive index of the prism relative to air.
- At minimum deviation δ_m , the path through the prism is symmetric: $i = e$ and $r_1 = r_2 = A/2$. Thus $n = \sin[(A + \delta_m)/2] / \sin(A/2)$.

Step 1: (i) Plot deviation against incidence For a fixed prism and a fixed wavelength, increase the angle of incidence over the range in which the ray emerges from the second face. Initially the

deviation decreases, reaches its least value, and then increases. The graph therefore has a rounded minimum; it is not a straight line. The dashed horizontal line through the lowest point gives δ_m , and the dashed vertical line gives the corresponding incidence i_m . The angle of minimum deviation is the least angle between the direction of the incident ray produced forward and the emergent ray, obtained by varying the angle of incidence for the given prism and wavelength. At this position, reversibility gives a symmetric ray path with $i = e = i_m$.



Qualitative deviation-incidence curve for a prism; the lowest point defines δ_m .

Step 2: (ii) Apply normal incidence at the first face Normal incidence means $i = 0$, so Snell's law gives $r_1 = 0$: the ray enters without bending. Using the prism-angle relation and then the deviation relation,

$$r_2 = A - r_1 = A,$$

$$\delta = 0 + e - A,$$

$$e = A + \delta.$$

The ray bends away from the normal at the second face as it travels from glass to air. Applying Snell's law there gives

$$n \sin r_2 = \sin e \quad \Rightarrow \quad n \sin A = \sin(A + \delta).$$

Dividing by $\sin A$ proves the required relation $n = \sin(A + \delta) / \sin A$. This derivation uses the stated emergent ray; it applies when the second face permits emergence rather than total internal reflection.

Step 3: (iii)(1) Calculate the minimum deviation The numerical prism has $n = \sqrt{2}$ and $A = 60^\circ$. At minimum deviation the internal angles are equal, so $r_1 = r_2 = 30^\circ$. Substituting in the prism formula,

$$\begin{aligned} \sqrt{2} &= \frac{\sin[(60^\circ + \delta_m)/2]}{\sin 30^\circ}, \\ \sin\left(\frac{60^\circ + \delta_m}{2}\right) &= \frac{1}{\sqrt{2}}, \\ \frac{60^\circ + \delta_m}{2} &= 45^\circ, \\ \delta_m &= 30^\circ. \end{aligned}$$

The physical incidence and emergence angles lie between 0° and 90° , so the relevant inverse-sine value is 45° .

Step 4: (iii)(2) Find the corresponding angle of incidence The angle requested here is the incidence at minimum deviation, rather than the normal-incidence condition of part (ii). Since $i_m = e_m$, the deviation relation becomes $\delta_m = 2i_m - A$. Hence

$$i_m = \frac{A + \delta_m}{2} = \frac{60^\circ + 30^\circ}{2} = 45^\circ.$$

As a check, Snell's law gives $\sin i_m = \sqrt{2} \sin 30^\circ = 1/\sqrt{2}$, consistent with the same angle.

Step 5: Conclusion (i) The deviation first decreases and then increases; its least value is δ_m . (ii) For normal incidence, $n = \frac{\sin(A + \delta)}{\sin A}$. (iii) The numerical prism has $\delta_m = 30^\circ$ and the corresponding $i_m = 45^\circ$.

Quick Tip to Solve: Keep the two conditions separate: normal incidence gives $r_1 = 0$ and $r_2 = A$; minimum deviation gives $r_1 = r_2 = A/2$ and $i = e$.

OR

33(b). (i) **State Huygens' principle.** A plane wave is incident at an angle i on a reflecting surface. Construct the corresponding reflected wavefront. Using this diagram, prove that the angle of reflection is equal to the angle of incidence.

(ii) What are the coherent sources of light? Can two independent sodium lamps act like coherent sources? Explain.

(iii) A beam of light consisting of a known wavelength 520 nm and an unknown wavelength λ , used in Young's double slit experiment produces two interference patterns such that the fourth bright fringe of unknown wavelength coincides with the fifth bright fringe of known wavelength. Find the value of λ . [5 marks]

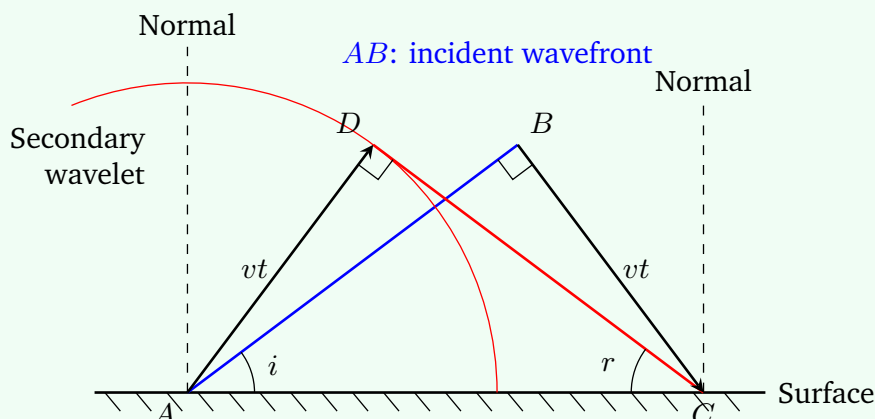
Solution:

Concept:

- Huygens' principle: every point on a wavefront acts as a source of secondary wavelets, spreading with the speed of light in that medium. Their forward envelope at a later instant gives the new wavefront.
- Rays are perpendicular to wavefronts. Reflection occurs in the same medium, so incident and reflected disturbances travel with the same speed v .
- Coherent sources have the same frequency and a constant phase difference. In Young's experiment the position of the n th bright fringe is $y_n = n\lambda D/d$, where D is screen distance and d is slit separation.

Step 1: (i) Construct the reflected wavefront Let AB be a plane incident wavefront and AC the reflecting surface. Point A reaches the surface first. In the time t taken by the disturbance at B to travel along the incident ray to C , the secondary wavelet centred at A acquires radius vt . Thus $BC = vt$. Draw this wavelet and the tangent CD from C , touching it at D . Since the disturbance

reflected from A has travelled $AD = vt$, CD is the reflected wavefront, the common envelope of secondary wavelets emitted by the surface during this interval. The incident ray BC is perpendicular to AB , and the reflected ray AD is perpendicular to CD .



Huygens' reflection construction: AB incident wavefront, CD reflected wavefront, and $BC = AD = vt$. Angles i, r shown between wavefronts and surface equal the corresponding ray-normal angles.

Step 2: (i) Prove the law of reflection The angle between the incident wavefront AB and surface AC equals the angle of incidence i : both are angles between pairs of mutually perpendicular lines. Similarly, the angle between reflected wavefront CD and surface CA equals the angle of reflection r . In right triangles ABC and ADC , AC is the common hypotenuse, and $BC = AD = vt$. Therefore,

$$\sin i = \frac{BC}{AC} = \frac{vt}{AC}, \quad \sin r = \frac{AD}{AC} = \frac{vt}{AC}.$$

Both angles lie between 0° and 90° , so equality of these sines proves $i = r$. The construction also places the incident ray, reflected ray and surface normal in the same plane.

Step 3: (ii) Identify coherent sources and assess the lamps Two light sources are coherent if they emit light of the same frequency with a phase difference that remains constant in time. Two independent sodium lamps do not act as coherent sources, even when the same sodium spectral line is selected from each. Their atoms emit independently, and their phases change randomly and independently. Consequently the relative phase is not constant, so any interference term averages out over ordinary observation times and no stable fringe pattern is obtained. Coherent sources are normally obtained by splitting light from one source, as in Young's double slit arrangement.

Step 4: (iii) Use the coincident bright fringes The fourth bright fringe of the unknown wavelength and fifth bright fringe of 520 nm have the same displacement from the central maximum. The central bright fringe has order zero, so the relevant orders are four and five. With the same D and d for both patterns,

$$\begin{aligned} \frac{4\lambda D}{d} &= \frac{5(520 \text{ nm})D}{d}, \\ 4\lambda &= 2600 \text{ nm}, \\ \lambda &= \frac{2600}{4} \text{ nm} = 650 \text{ nm}. \end{aligned}$$

This larger wavelength has the smaller fringe order at the common position, consistent with fringe width increasing with wavelength.

Step 5: Conclusion Huygens' construction gives the reflected wavefront CD and proves $i = r$. Independent sodium lamps are not coherent because their relative phase fluctuates randomly. The unknown wavelength is 650 nm .

Quick Tip to Solve: In the reflection construction, compare the equal distances $BC = AD = vt$. For coincident bright fringes, cancel D/d immediately and equate the products of order and wavelength.