

CBSE Class 12 Physics Question Paper 2024 with Solution PDF-Set 55/2/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. All questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – **Sections A, B, C, D** and **E**.
- (iii) In **Section A** – Questions no. **1 to 16** are Multiple Choice type questions. Each question carries **1** mark.
- (iv) In **Section B** – Questions no. **17 to 21** are Very Short Answer type questions. Each question carries **2** marks.
- (v) In **Section C** – Questions no. **22 to 28** are Short Answer type questions. Each question carries **3** marks.
- (vi) In **Section D** – Questions no. **29 and 30** are case study based questions. Each question carries **4** marks.
- (vii) In **Section E** – Questions no. **31 to 33** are Long Answer type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is not allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. Two charged particles P and Q, having the same charge but different masses m_P and m_Q , start from rest and travel equal distances in a uniform electric field $\vec{E}$ in time t_P and t_Q respectively. Neglecting the effect of gravity, the ratio $\frac{t_P}{t_Q}$ is: [1 mark]

- (A) $\frac{m_P}{m_Q}$
(B) $\frac{m_Q}{m_P}$
(C) $\sqrt{\frac{m_P}{m_Q}}$
(D) $\sqrt{\frac{m_Q}{m_P}}$

Correct Answer: (C) $\sqrt{\frac{m_P}{m_Q}}$

Solution:

Concept:

- A charge of magnitude q in a uniform electric field of magnitude E experiences a constant force $F = qE$, giving acceleration $a = qE/m$.
- Starting from rest, the distance travelled under constant acceleration is $s = \frac{1}{2}at^2$, where t is the travel time.

Step 1: Express time in terms of mass The two particles have equal charge and experience the same field, so the force magnitudes are equal. For either particle,

$$s = \frac{qE}{2m}t^2, \quad t = \sqrt{\frac{2ms}{qE}}.$$

Thus the heavier particle accelerates less and takes longer to cover the same distance.

Step 2: Compare the two times The quantities s , q and E cancel when the time expressions are divided. Therefore the time ratio depends on the square root of the mass ratio. Options (A) and (B) omit this square root; option (D) reverses the ratio and would incorrectly make the heavier particle faster.

Step 3: Conclusion $\frac{t_P}{t_Q} = \sqrt{\frac{m_P}{m_Q}}$. Hence, the correct option is (C).

Quick Tip to Solve: For equal distances from rest under equal forces, use $t \propto \sqrt{m}$; the heavier particle must take longer.

2. Electrons drift with speed v_d in a conductor with potential difference V across its ends. If V is reduced to $\left(\frac{V}{2}\right)$, their drift speed will become: [1 mark]

- (A) $\frac{v_d}{2}$
- (B) v_d
- (C) $2v_d$
- (D) $4v_d$

Correct Answer: (A) $\frac{v_d}{2}$

Solution:

Concept:

- The magnitude of electron drift velocity is $v_d = eE\tau/m_e$, where e is the electronic charge magnitude, m_e the electron mass and τ the mean relaxation time.
- In a uniform conductor of length L , the electric field magnitude is $E = V/L$. For the same conductor at unchanged temperature, $v_d \propto V$.

Step 1: Relate drift speed to the potential difference Keeping the conductor and its physical conditions fixed gives

$$v_d = \frac{e\tau}{m_e L} V.$$

The coefficient multiplying V remains constant, so halving the applied potential difference halves the driving electric field.

Step 2: Apply the voltage change If the new voltage is $V' = V/2$, then

$$\frac{v'_d}{v_d} = \frac{V'}{V} = \frac{1}{2}.$$

An unchanged speed, option (B), ignores the reduced field. Options (C) and (D) predict an increase even though the electric force decreases, so neither follows the drift-speed relation.

Step 3: Conclusion The new drift speed is $v'_d = v_d/2$. Hence, the correct option is (A).

Quick Tip to Solve: For the same conductor at fixed temperature, multiply the drift speed by the same factor as the applied voltage.

3. A wire of length 4.4 m is bent round in the shape of a circular loop and carries a current of 1.0 A. The magnetic moment of the loop will be: [1 mark]

- (A) 0.7 Am^2
- (B) 1.54 Am^2
- (C) 2.10 Am^2
- (D) 3.5 Am^2

Correct Answer: (B) 1.54 Am^2

Solution:

Concept:

- The magnetic dipole moment magnitude of a single current loop is $\mu = IA$, where I is its current and A is its enclosed area.

- A wire bent into one circular loop has length $\ell = 2\pi r$ and encloses area $A = \pi r^2$, where r is the loop radius.

Step 1: Obtain the radius from the wire length The given 4.4 m is the circumference, not the radius. Taking $\pi = 22/7$,

$$r = \frac{\ell}{2\pi} = \frac{4.4}{2(22/7)} = 0.70 \text{ m}.$$

Step 2: Calculate area and multiply by current The enclosed area and magnetic moment are

$$A = \frac{22}{7}(0.70)^2 = 1.54 \text{ m}^2,$$

$$\mu = (1.0)(1.54) = 1.54 \text{ A m}^2.$$

Option (A) uses the numerical radius instead of the area. Options (C) and (D) also disagree with $I\pi r^2$; there is no extra turn factor because the wire forms a single loop.

Step 3: Conclusion The magnetic moment is 1.54 A m^2 . Hence, the correct option is (B).

Quick Tip to Solve: When the wire length is supplied for one circular loop, use $\mu = I\ell^2/(4\pi)$ to avoid confusing circumference and radius.

4. A circular coil of radius 10 cm is placed in a magnetic field $\vec{B} = (1.0\hat{i} + 0.5\hat{j}) \text{ mT}$ such that the outward unit vector normal to the surface of the coil is $(0.6\hat{i} + 0.8\hat{j})$. The magnetic flux linked with the coil is: [1 mark]

- (A) $0.314 \mu\text{Wb}$
- (B) $3.14 \mu\text{Wb}$
- (C) $31.4 \mu\text{Wb}$
- (D) $1.256 \mu\text{Wb}$

Correct Answer: (C) $31.4 \mu\text{Wb}$

Solution:

Concept:

- Magnetic flux through a plane surface in a uniform field is $\Phi = \vec{B} \cdot \vec{A} = A(\vec{B} \cdot \hat{n})$, where $\hat{n}$ is the chosen unit normal.
- A circle of radius r has area $A = \pi r^2$. Use $1 \text{ mT} = 10^{-3} \text{ T}$ and $1 \mu\text{Wb} = 10^{-6} \text{ Wb}$.

Step 1: Project the field onto the normal Multiply corresponding Cartesian components and add:

$$\vec{B} \cdot \hat{n} = (1.0 \times 0.6 + 0.5 \times 0.8) \text{ mT} = 1.0 \times 10^{-3} \text{ T}.$$

Both contributions are positive for the specified outward normal. The full field magnitude cannot replace its normal component.

Step 2: Multiply by the circular area Convert the radius first: $r = 10 \text{ cm} = 0.10 \text{ m}$. Then

$$\Phi = (1.0 \times 10^{-3})\pi(0.10)^2 = 3.14 \times 10^{-5} \text{ Wb}.$$

The micro-weber conversion multiplies this number by 10^6 . Options (A) and (B) are too small by factors of 100 and 10 respectively; option (D) also disagrees with the dot product times area.

Step 3: Conclusion The flux is $\Phi = 31.4 \mu\text{Wb}$. Hence, the correct option is (C).

Quick Tip to Solve: Calculate $B_x n_x + B_y n_y$ first, and convert the radius to metres before squaring it for the area.

5. Which of the following quantity/quantities remains same in primary and secondary coils of an ideal transformer?

Current, Voltage, Power, Magnetic flux

[1 mark]

- (A) Current only
- (B) Voltage only
- (C) Power only
- (D) Magnetic flux and Power both

Correct Answer: (D) Magnetic flux and Power both

Solution:

Concept:

- An ideal transformer has no energy losses, so its input and output powers are equal: $P_p = P_s$, with p and s denoting primary and secondary.
- Perfect magnetic coupling means the same core magnetic flux $\Phi(t)$ passes through each turn of both coils. Voltage depends on turn number through Faraday's law.

Step 1: Compare voltage and current If the turn numbers are N_p and N_s , an ideal transformer obeys

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}, \quad \frac{I_s}{I_p} = \frac{N_p}{N_s}.$$

Thus voltage and current generally change when the turn numbers differ. Options (A) and (B) cannot describe a general ideal transformer.

Step 2: Identify the shared quantities Since there is no leakage flux, both coils are threaded by the same core flux. Their flux linkages $N_p\Phi$ and $N_s\Phi$ need not be equal. With no copper, core or other losses, the power transferred to the secondary equals the input power. Option (C) omits the unchanged magnetic flux and is therefore incomplete.

Step 3: Conclusion Magnetic flux and power both remain the same in the primary and secondary of an ideal transformer. Hence, the correct option is (D).

Quick Tip to Solve: Separate flux Φ from flux linkage $N\Phi$: ideal coils share the former even when their turn numbers differ.

6. A resistor and an ideal inductor are connected in series to a $100\sqrt{2}$ V, 50 Hz ac source. When a voltmeter is connected across the resistor or the inductor, it shows the same reading. The reading of the voltmeter is: [1 mark]

- (A) $100\sqrt{2}$ V
- (B) 100 V
- (C) $50\sqrt{2}$ V
- (D) 50 V

Correct Answer: (B) 100 V

Solution:

Concept:

- An ac voltmeter measures root-mean-square (rms) voltage. An ac source voltage stated without a peak-value qualification is its rms value.
- In a series resistor–ideal-inductor circuit, V_R is in phase with the current and V_L leads it by 90° . Therefore $V^2 = V_R^2 + V_L^2$, where V is the source rms voltage.

Step 1: Use the equal voltmeter readings Let the reading across either component be U . Since $V_R = V_L = U$ and the two component voltages are perpendicular phasors,

$$V = \sqrt{U^2 + U^2} = \sqrt{2} U.$$

They cannot be added as ordinary in-phase voltages.

Step 2: Substitute the source voltage With $V = 100\sqrt{2}$ V,

$$U = \frac{V}{\sqrt{2}} = \frac{100\sqrt{2}}{\sqrt{2}} = 100 \text{ V}.$$

Option (A) equates a component voltage to the resultant. Option (C) incorrectly divides the source by two; option (D) is also inconsistent with the phasor sum. The stated frequency is not needed once the two readings are known to be equal.

Step 3: Conclusion The voltmeter reads 100 V across either component. Hence, the correct option is (B).

Quick Tip to Solve: Equal resistor and inductor rms voltages each equal the supply rms voltage divided by $\sqrt{2}$, since they are in quadrature.

7. Electromagnetic waves with wavelength 10 nm are called:

[1 mark]

- (A) Infrared waves
- (B) Ultraviolet rays
- (C) Gamma rays
- (D) X-rays

Correct Answer: (B) Ultraviolet rays

Solution:

Concept:

- Electromagnetic radiation is classified by wavelength λ or frequency ν , related in vacuum by $c = \lambda\nu$, where c is the speed of light.
- In the usual school-level classification, ultraviolet radiation extends from about 400 nm down to about 1 nm, beyond the violet end of visible light.

Step 1: Convert the given wavelength Since $1 \text{ nm} = 10^{-9} \text{ m}$, the wavelength is

$$\lambda = 10 \text{ nm} = 10^{-8} \text{ m}.$$

Its corresponding frequency is $\nu = c/\lambda = 3 \times 10^{16} \text{ Hz}$, which is higher than visible-light frequencies.

Step 2: Locate the radiation in the spectrum The value 10 nm lies in the ultraviolet range used for this classification. Infrared has wavelengths longer than visible red light, so option (A) fails. Gamma rays have far shorter wavelengths, ruling out (C). X-rays are assigned the shorter-wavelength region, approximately below 1 nm in this school-level convention, so (D) is not the intended classification here.

Step 3: Conclusion Electromagnetic waves of wavelength 10 nm are ultraviolet rays. Hence, the correct option is (B).

Quick Tip to Solve: Convert nanometres before comparing ranges: $10 \text{ nm} = 10^{-8} \text{ m}$, well below visible wavelengths but within the ultraviolet range.

8. The work function for a photosensitive surface is 3.315 eV. The cut-off wavelength for photoemission of electrons from this surface is:

[1 mark]

- (A) 150 nm
- (B) 200 nm
- (C) 375 nm
- (D) 500 nm

Correct Answer: (C) 375 nm

Solution:

Concept:

- The work function ϕ is the minimum energy needed to eject an electron from the surface.
- At the cut-off wavelength λ_0 , the emitted electron has zero maximum kinetic energy, so $hc/\lambda_0 = \phi$, where h is Planck's constant and c is the speed of light.

Step 1: Convert the work function to joules Since $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$,

$$\phi = 3.315 \times 1.6 \times 10^{-19} = 5.304 \times 10^{-19} \text{ J}.$$

Step 2: Apply the threshold condition Using the supplied constants,

$$\lambda_0 = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5.304 \times 10^{-19}} = 3.75 \times 10^{-7} \text{ m} = 375 \text{ nm}.$$

The shorter wavelengths 150 nm and 200 nm can cause emission but are not the cut-off wavelength. At 500 nm, the photon energy is too small to overcome the work function.

Step 3: Conclusion The cut-off wavelength is 375 nm. Hence, the correct option is (C).

Quick Tip to Solve: For a work function in eV, use $\lambda_0(\text{nm}) \approx 1240/\phi(\text{eV})$; the cut-off is the longest wavelength that can eject electrons.

9. Energy levels A, B and C of an atom correspond to increasing values of energy i.e. $E_A < E_B < E_C$. Let λ_1 , λ_2 and λ_3 be the wavelengths of radiation corresponding to the transitions C to B, B to A and C to A, respectively. The correct relation between λ_1 , λ_2 and λ_3 is: [1 mark]

- (A) $\lambda_1^2 + \lambda_2^2 = \lambda_3^2$
(B) $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_3}$
(C) $\lambda_1 + \lambda_2 + \lambda_3 = 0$
(D) $\lambda_1 + \lambda_2 = \lambda_3$

Correct Answer: (B) $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_3}$

Solution:

Concept:

- In an atomic transition, the emitted photon energy equals the energy difference between the initial and final levels.
- Photon energy is $E = hc/\lambda$, where h is Planck's constant, c is the speed of light and λ is the wavelength.

Step 1: Express the three transition energies Using the transitions assigned in the question,

$$E_C - E_B = \frac{hc}{\lambda_1}, \quad E_B - E_A = \frac{hc}{\lambda_2}, \quad E_C - E_A = \frac{hc}{\lambda_3}.$$

Every energy difference is positive because $E_A < E_B < E_C$.

Step 2: Add the successive energy gaps The gap from C to A equals the sum of the gaps from C to B and B to A:

$$\frac{hc}{\lambda_3} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}.$$

Cancel the common factor hc . Energies, and thus reciprocal wavelengths, add; wavelengths and their squares do not add. Option (C) is also impossible because all wavelengths are positive. Since the C-to-A gap is largest, λ_3 is the shortest wavelength, contrary to (A) and (D).

Step 3: Conclusion $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_3}$. Hence, the correct option is (B).

Quick Tip to Solve: For a direct transition spanning two successive gaps, add photon frequencies or reciprocal wavelengths, never the wavelengths themselves.

10. An alpha particle approaches a gold nucleus in Geiger-Marsden experiment with kinetic energy K . It momentarily stops at a distance d from the nucleus and reverses its direction. Then d is proportional to: [1 mark]

- (A) $\frac{1}{\sqrt{K}}$
- (B) $\sqrt{K}$
- (C) $\frac{1}{K}$
- (D) K

Correct Answer: (C) $\frac{1}{K}$

Solution:

Concept:

- In a head-on approach to a heavy nucleus, electrostatic repulsion converts the alpha particle's kinetic energy into electric potential energy.
- The potential energy of charges q_1 and q_2 separated by r is $U = kq_1q_2/r$, where $k = 1/(4\pi\epsilon_0)$ and the zero of potential energy is at infinity.

Step 1: Apply conservation of energy at the turning point The alpha particle has charge $+2e$, and the gold nucleus has charge $+Ze$, with $Z = 79$. At the closest distance d , the alpha particle is momentarily at rest. Taking the heavy nucleus as stationary, its incident kinetic energy is therefore

$$K = \frac{k(2e)(Ze)}{d} = \frac{2kZe^2}{d}.$$

Step 2: Isolate the distance and compare the options Rearranging gives $d = 2kZe^2/K$. For the same alpha particle and gold nucleus, the numerator is constant. Doubling K halves d , so the dependence is neither $1/\sqrt{K}$ nor a positive power of K . Options (B) and (D) incorrectly predict that a more energetic particle turns back farther away.

Step 3: Conclusion The distance of closest approach obeys $d \propto 1/K$. Hence, the correct option is (C).

Quick Tip to Solve: At a head-on turning point, write $K = U$ first; because Coulomb potential energy varies as $1/r$, the closest distance varies as $1/K$.

11. An n-type semiconducting Si is obtained by doping intrinsic Si with:

[1 mark]

- (A) Al
- (B) B
- (C) P
- (D) In

Correct Answer: (C) P

Solution:

Concept:

- Silicon is tetravalent: each Si atom contributes four valence electrons to covalent bonds with neighbouring atoms.
- A pentavalent donor impurity supplies an extra, weakly bound electron and produces an n-type semiconductor, in which electrons are the majority carriers.

Step 1: Identify the donor among the listed elements Phosphorus (P) has five valence electrons. When it replaces a silicon atom in the lattice, four electrons participate in covalent bonds. The fifth electron can be released easily into the conduction band, increasing the population of mobile electrons.

Step 2: Eliminate the acceptor impurities Aluminium (Al), boron (B) and indium (In) are trivalent. Each has only three valence electrons and leaves an electron deficiency, or hole, in the bonding arrangement. They act as acceptors and produce p-type silicon, so options (A), (B) and (D) do not give the required majority carriers. The n-type crystal remains electrically neutral overall; the term refers to its majority mobile carriers.

Step 3: Conclusion Intrinsic silicon must be doped with phosphorus (P). Hence, the correct option is (C).

Quick Tip to Solve: Compare the dopant valency with silicon's four: five supplies an extra electron (n-type), whereas three creates a hole (p-type).

12. When a p-n junction diode is subjected to reverse biasing:

[1 mark]

- (A) the barrier height decreases and the depletion region widens.
- (B) the barrier height increases and the depletion region widens.
- (C) the barrier height decreases and the depletion region shrinks.
- (D) the barrier height increases and the depletion region shrinks.

Correct Answer: (B) the barrier height increases and the depletion region widens.

Solution:

Concept:

- The depletion region of a p-n junction contains uncovered, fixed ionised donors and acceptors; their electric field creates the junction's potential barrier.
- Reverse bias connects the p-side to the negative terminal and the n-side to the positive terminal, reinforcing the built-in electric field.

Step 1: Determine how reverse bias moves majority carriers The negative terminal attracts holes away from the junction on the p-side, while the positive terminal attracts electrons away from the junction on the n-side. More fixed ions are left uncovered near the junction, so the depletion region

becomes wider.

Step 2: Determine the change in barrier height The applied reverse voltage adds to the built-in potential barrier. If V_0 is the unbiased barrier potential and V_R is the magnitude of the reverse voltage, the barrier becomes $V_0 + V_R$. Thus both the barrier height and depletion width increase. Option (A) gives the wrong barrier change, (D) gives the wrong width change, and (C) describes the changes associated with forward bias.

Step 3: Conclusion Under reverse bias, the barrier height increases and the depletion region widens. Hence, the correct option is (B).

Quick Tip to Solve: Reverse bias pulls majority carriers away from the junction: remember that both the barrier and the depletion width increase together.

13. Assertion (A): Photoelectric current increases with an increase in intensity of incident radiation, for a given frequency of incident radiation and the accelerating potential.

Reason (R): Increase in the intensity of incident radiation results in an increase in the number of photoelectrons emitted per second and hence an increase in the photocurrent. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- For incident frequency ν above the threshold, each photon has energy $h\nu$, where h is Planck's constant.
- At fixed frequency, higher intensity means more photons incident per unit area per second. Photocurrent is $I = e\dot{N}$, where $\dot{N}$ is the number of collected electrons per second and e is the elementary charge.

Step 1: Evaluate the assertion In the photoemission regime, keeping the frequency and accelerating potential fixed while increasing the light intensity increases the number of photoelectrons available for collection. The photoelectric current therefore increases. Assertion (A) is true.

Step 2: Evaluate the reason At fixed frequency, increasing intensity increases the incident photon rate without increasing each photon's energy. More photons eject more electrons per second from the same photosensitive surface. At the fixed accelerating potential, the collected electron rate and hence the photocurrent increase. Reason (R) is true.

Step 3: Test the explanatory link The reason gives the direct cause of the asserted current increase: a greater emission rate produces a greater collected charge per second. It does not claim that intensity increases the maximum electron kinetic energy, which depends on frequency. Thus (B) misses the causal link, while (C) and (D) incorrectly reject true statements.

Step 4: Conclusion Both Assertion (A) and Reason (R) are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: Separate photon number from photon energy: at fixed frequency, intensity controls the photoelectron rate and current, not the maximum kinetic energy.

14. Assertion (A): Lenz's law is a consequence of the law of conservation of energy.

Reason (R): There is no power loss in an ideal inductor.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- Lenz's law states that the induced current opposes the change in magnetic flux producing it, consistent with $\varepsilon = -d\Phi_B/dt$ for a single turn.
- An ideal inductor has zero resistance and stores magnetic energy $U = \frac{1}{2}LI^2$, where L is inductance and I is current. Energy stored is distinct from energy dissipated.

Step 1: Evaluate the assertion If the induced current assisted the flux change, it would reinforce the cause of induction and could generate electrical energy without the corresponding external work. The opposing direction ensures that the energy supplied by external work accounts for the electrical energy produced or stored. Lenz's law therefore follows from conservation of energy, and Assertion (A) is true.

Step 2: Evaluate the reason An ideal inductor has no resistance, so its Joule heating loss is $I^2R = 0$. It may absorb power while its magnetic energy increases and return that energy later; neither process constitutes dissipative power loss. Thus Reason (R) is also true.

Step 3: Check whether the reason explains the assertion The absence of loss in one ideal circuit element does not explain why induced current must oppose a flux change. Lenz's law applies even to resistive circuits, where electrical energy can become heat. Therefore the statements are both true but the reason is not the explanation. Option (A) assigns an invalid explanatory link, and (C) and (D) incorrectly declare a statement false.

Step 4: Conclusion Both the assertion and the reason are true, but the reason does not explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: For an ideal inductor, distinguish zero energy loss from zero instantaneous power; then test the explanatory link separately from the two truth values.

15. Assertion (A): An electron and a proton enter with the same momentum $\vec{p}$ in a magnetic field $\vec{B}$ such that $\vec{p} \perp \vec{B}$. Then both describe a circular path of the same radius.

Reason (R): The radius of the circular path described by the charged particle (charge q , mass m) moving in the magnetic field $\vec{B}$ is given by $r = \frac{mv}{qB}$. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- A particle moving perpendicular to a uniform magnetic field experiences a magnetic force of magnitude $F = |q|vB$, which provides the centripetal force.
- Its orbit radius is $r = mv/(|q|B) = p/(|q|B)$, where p is momentum and $|q|$ is the charge magnitude.

Step 1: Evaluate the assertion An electron and a proton have equal charge magnitudes e . With equal momentum magnitudes in the same field, their radii are $r_e = p/(eB)$ and $r_p = p/(eB)$. Their different masses do not change this result because the specified quantity is momentum, not speed. Thus Assertion (A) is true.

Step 2: Evaluate the reason Equating magnetic and centripetal forces gives

$$|q|vB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{|q|B}.$$

In the printed radius formula, q denotes the magnitude of the particle's charge. With this usual convention, Reason (R) is true; a geometrical radius is positive.

Step 3: Check the explanatory link Substituting $mv = p$ into the reason gives equal radii immediately for equal p , equal $|q|$ and equal B . Opposite charge signs reverse the sense of rotation, not the radius. Therefore the reason explains the assertion, excluding (B); neither statement is false, excluding (C) and (D).

Step 4: Conclusion Both statements are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: For equal momenta, compare $p/(|q|B)$; use the charge sign only to determine the direction of bending.

16. Assertion (A): The magnifying power of a compound microscope is negative.

Reason (R): The final image formed is erect with respect to the object.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- A compound microscope's objective forms a real, magnified and inverted intermediate image of the object.
- The eyepiece acts as a magnifying glass. It preserves the intermediate image's orientation, so the final image is inverted relative to the original object.

Step 1: Evaluate the assertion The signed magnifying power is negative because the final image is inverted relative to the object. For normal adjustment, the standard approximate expression is $M = -LD/(f_o f_e)$, where L is the tube length, D the least distance of distinct vision, and f_o, f_e the positive focal lengths. Thus Assertion (A) is true.

Step 2: Evaluate the reason The objective first reverses the object's orientation. The eyepiece forms a virtual image that is erect relative to this intermediate image, but remains inverted relative to the original object. Reason (R), which calls the final image erect with respect to the object, is therefore false.

Step 3: Check the explanatory link A false statement about orientation cannot explain the negative sign. Options (A) and (B) incorrectly require the reason to be true, while (D) incorrectly calls the assertion false. The negative sign records inversion; it does not mean that the image is diminished.

Step 4: Conclusion The assertion is true and the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Track orientation through both lenses: the eyepiece preserves the inversion already produced by the objective.

Section B (10 marks)

17. Define resistivity of a conductor. How does the resistivity of a conductor depend upon the following:

(a) Number density of free electrons in the conductor (n)

(b) Their relaxation time (τ)

[2 marks]

Solution:

Concept:

- Resistivity ρ is a material property defined by $\rho = RA/\ell$, where R is resistance, A is cross-sectional area and ℓ is length of a uniform conductor.
- In the free-electron model, $\sigma = ne^2\tau/m$ and $\rho = 1/\sigma$. Here n is free-electron number density, e the

electron charge magnitude, m its mass and τ the mean relaxation time.

Step 1: Define resistivity Resistivity is numerically equal to the resistance of a conductor of unit length and unit cross-sectional area at a specified temperature. Its SI unit follows from RA/ℓ and is $\Omega \text{ m}$. Unlike resistance, resistivity does not depend on the conductor's length or cross-sectional area.

Step 2: Obtain the microscopic expression Under an applied electric field of magnitude E , the drift-speed magnitude is $v_d = eE\tau/m$. The current density is therefore

$$J = nev_d = \frac{ne^2\tau}{m}E.$$

Using $J = \sigma E$ and $\rho = 1/\sigma$ gives $\rho = m/(ne^2\tau)$.

Step 3: State both dependences (a) With τ fixed, $\rho \propto 1/n$: more free electrons carry more current for the same field, reducing resistivity. (b) With n fixed, $\rho \propto 1/\tau$: a longer time between collisions allows greater drift speed, again reducing resistivity.

Step 4: Conclusion $\rho = RA/\ell = m/(ne^2\tau)$, measured in $\Omega \text{ m}$. It varies inversely with both free-electron number density and relaxation time, with the other quantity held fixed.

Quick Tip to Solve: Start with $J = nev_d$ and insert $v_d = eE\tau/m$; the coefficient of E is conductivity, whose reciprocal is resistivity.

18(a). Two waves, each of amplitude 'a' and frequency ' ω ' emanating from two coherent sources of light superpose at a point. If the phase difference between the two waves is ϕ , obtain an expression for the resultant intensity at that point. [2 marks]

Solution:

Concept:

- Coherent waves have the same frequency and a fixed phase difference, so their superposition produces a steady interference intensity.
- Intensity is proportional to the square of amplitude. Write $I_0 = Ka^2$ for the intensity due to either wave alone, where K is the common proportionality constant.

Step 1: Represent the two waves Choose the phase origin at the observation point so that the disturbances are $y_1 = a \sin \omega t$ and $y_2 = a \sin(\omega t + \phi)$. Here ω is the common angular frequency and ϕ is the phase difference. The waves are taken to have the same polarization, as in the usual interference arrangement.

Step 2: Add their disturbances By the principle of superposition and the identity for the sum of two sines,

$$\begin{aligned} y &= a \sin \omega t + a \sin(\omega t + \phi) \\ &= 2a \cos \frac{\phi}{2} \sin \left(\omega t + \frac{\phi}{2} \right). \end{aligned}$$

Thus the resultant amplitude magnitude is $A = 2a |\cos(\phi/2)|$. The phase factor determines the amplitude rather than changing the frequency.

Step 3: Convert amplitude into intensity Squaring the resultant amplitude gives

$$I = KA^2 = 4Ka^2 \cos^2 \frac{\phi}{2} = 2I_0(1 + \cos \phi).$$

For $\phi = 0$, the amplitude doubles and the intensity is $4I_0$. For $\phi = \pi$, the two equal disturbances cancel and the intensity is zero, confirming the interference limits.

Step 4: Conclusion The resultant intensity is $I = 4I_0 \cos^2(\phi/2) = 2I_0(1 + \cos \phi)$, where I_0 is the intensity of either wave separately. Equivalently, $I \propto 4a^2 \cos^2(\phi/2)$.

Quick Tip to Solve: Add the wave amplitudes before squaring; directly adding the two individual intensities misses the interference term.

OR

18(b). What is the effect on the interference pattern in Young's double-slit experiment when (i) the source slit is moved closer to the plane of the slits, and (ii) the separation between the two slits is increased? Justify your answers. [2 marks]

Solution:

Concept:

- In Young's experiment the fringe width is $\beta = \lambda D/d$, where λ is wavelength, D is the distance from double slits to screen, and d is slit separation.
- A source slit of finite width s , at distance L from the double slits, has angular width approximately s/L . Distinct fringes require this angular width to be sufficiently small, approximately $s/L \ll \lambda/d$.

Step 1: Move the source slit closer (i) For the usual source slit of finite width, decreasing L increases its angular width s/L . Different points of this source produce displaced fringe patterns. Their superposition reduces the contrast between bright and dark bands, so the fringes become less distinct and may disappear if the source is too close. The fringe spacing itself is unchanged because λ , D and d remain fixed. For an ideal point source moved along the symmetry axis, this loss of visibility does not occur.

Step 2: Increase the double-slit separation (ii) With wavelength and screen distance unchanged, increasing d decreases the fringe width:

$$\beta' = \frac{\lambda D}{d'}, \quad \frac{\beta'}{\beta} = \frac{d}{d'} < 1 \quad (d' > d).$$

Thus adjacent bright fringes, and likewise adjacent dark fringes, come closer together. For example, doubling the slit separation halves the fringe width, so more fringes fit into the same screen interval.

Step 3: Conclusion (i) Moving a finite-width source slit closer reduces fringe visibility, while the fringe width remains unchanged. (ii) Increasing double-slit separation reduces the fringe width in inverse proportion to that separation.

Quick Tip to Solve: Use $\lambda D/d$ for fringe spacing and the source's angular width s/L for visibility; these describe different features of the pattern.

19. A convex lens ($n = 1.52$) has a focal length of 15.0 cm in air. Find its focal length when it is immersed in liquid of refractive index 1.65. What will be the nature of the lens? [2 marks]

Solution:

Concept:

- For a thin lens of refractive index n_l in a medium of index n_m , the lens maker's formula is $1/f = (n_l/n_m - 1)(1/R_1 - 1/R_2)$, where the radii have their Cartesian signs.
- Immersion changes the relative refractive index, but not the lens shape. A positive focal length denotes convergence and a negative focal length denotes divergence.

Step 1: Eliminate the unchanged curvature factor Let $K = 1/R_1 - 1/R_2$. Taking the refractive index of air as unity, the two lens maker relations are

$$\frac{1}{f_{\text{air}}} = (1.52 - 1)K, \quad \frac{1}{f_{\text{liquid}}} = \left(\frac{1.52}{1.65} - 1\right)K.$$

Their ratio gives

$$f_{\text{liquid}} = f_{\text{air}} \frac{1.52 - 1}{1.52/1.65 - 1}.$$

Step 2: Substitute and interpret the sign Use $f_{\text{air}} = +15.0$ cm, since the convex lens converges in air. Then

$$f_{\text{liquid}} = 15.0 \frac{0.52 \times 1.65}{1.52 - 1.65} = -99.0 \text{ cm}.$$

The liquid has a larger refractive index than the lens, so the relative index is less than unity and the optical power reverses sign. The geometrical shape remains convex.

Step 3: Conclusion The focal length in the liquid is -99.0 cm. The lens behaves as a **diverging lens** in this liquid.

Quick Tip to Solve: Compare the liquid and lens indices first: if the liquid index is greater, a convex lens has negative power.

20. The carbon isotope $^{12}_6\text{C}$ has a nuclear mass of 12.000000 u. Calculate the binding energy of its nucleus.

Given $m_p = 1.007825$ u; $m_n = 1.008665$ u. [2 marks]

Solution:

Concept:

- Binding energy is the energy required to separate a nucleus into its constituent nucleons. It equals $E_b = \Delta mc^2$, where Δm is the mass defect.
- For atomic number Z and mass number A , the supplied-mass calculation uses $\Delta m = Zm_p + (A - Z)m_n - M$. The conversion is $1 \text{ u } c^2 = 931.5 \text{ MeV}$.

Step 1: Count the nucleons and calculate their total mass Carbon-12 has $Z = 6$ protons and $A - Z = 12 - 6 = 6$ neutrons. Using the numerical masses provided in the question, their total mass before binding is

$$6m_p + 6m_n = 6(1.007825 + 1.008665) = 12.098940 \text{ u}.$$

Binding reduces the mass below this sum, so the mass defect must be positive.

Step 2: Convert the mass defect into energy Subtract the supplied carbon mass:

$$\Delta m = 12.098940 - 12.000000 = 0.098940 \text{ u.}$$

Therefore,

$$E_b = 0.098940 \times 931.5 = 92.16261 \text{ MeV.}$$

This is the total binding energy of the nucleus, not the binding energy per nucleon. The printed data use the conventional atomic-mass numerical values: 1.007825 u is the hydrogen-atom mass and 12.000000 u is the carbon-12 atomic mass. Their six electron masses cancel in this subtraction, giving the intended nuclear binding energy to the stated precision.

Step 3: Conclusion Using the supplied data, the binding energy of the carbon-12 nucleus is 92.16 MeV, approximately $1.48 \times 10^{-11} \text{ J}$.

Quick Tip to Solve: Carbon-12 contains six protons and six neutrons; multiply the mass defect by 931.5 for the total energy in MeV.

21. How does the energy gap of an intrinsic semiconductor effectively change when doped with a (a) trivalent impurity, and (b) pentavalent impurity? Justify your answer in each case. [2 marks]

Solution:

Concept:

- In an intrinsic semiconductor, an electron must acquire the band-gap energy $E_g = E_C - E_V$ to move from the valence-band edge E_V to the conduction-band edge E_C .
- Ordinary impurity doping introduces allowed energy levels within the forbidden gap. It reduces the energy needed to produce mobile carriers; the host's valence-to-conduction band gap itself remains approximately unchanged for dilute doping.

Step 1: Explain the effect of a trivalent impurity A trivalent atom has one fewer valence electron than the host silicon or germanium atom. It introduces an acceptor level E_A just above the valence band. A valence electron can occupy this level by absorbing the small energy $E_A - E_V$, leaving a mobile hole in the valence band. Thus the effective energy required to create the majority carriers is much smaller than E_g ; the material becomes a p-type semiconductor.

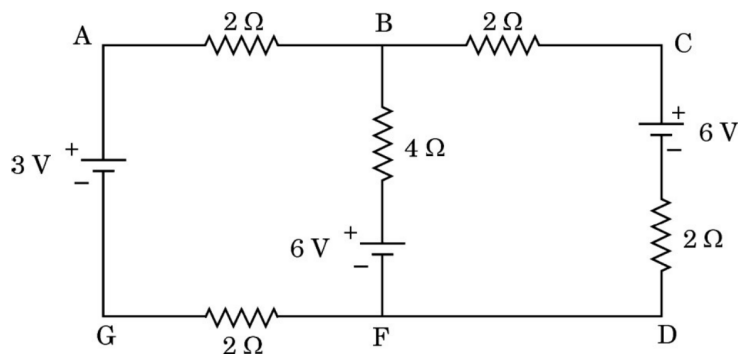
Step 2: Explain the effect of a pentavalent impurity A pentavalent atom supplies an extra, weakly bound electron and introduces a donor level E_D just below the conduction band. The electron needs only the small energy $E_C - E_D$ to enter the conduction band, where it can conduct. Thus the effective carrier-excitation energy is again much smaller than E_g ; the material becomes an n-type semiconductor. In both cases, the reduced energy refers to an impurity-to-band transition, rather than a collapse of the original forbidden gap.

Step 3: Conclusion The effective excitation gap decreases in both cases: (a) a trivalent impurity creates an acceptor level just above the valence band; (b) a pentavalent impurity creates a donor level just below the conduction band. These nearby levels make mobile holes or electrons available with much less energy.

Quick Tip to Solve: Remember acceptors above the valence band and donors below the conduction band; identify the short energy jump in each case.

Section C (21 marks)

22. The figure shows a circuit with three ideal batteries. Find the magnitude and direction of currents in the branches AG, BF and CD. [3 marks]



Solution:

Concept:

- Kirchhoff's junction rule expresses charge conservation: the algebraic sum of currents leaving any junction is zero.
- Kirchhoff's loop rule states that the sum of potential changes around a loop is zero. Crossing a cell from its positive terminal to its negative terminal produces a potential drop equal to its emf.
- Along a branch with resistance R and an opposing emf E , a trial current from the higher labelled node to the reference node satisfies $V = IR + E$. A negative result reverses the trial direction.

Step 1: Identify the three paths between the junctions Points F and D are directly joined by an ideal wire and have the same potential. Set $V_F = V_D = 0$ and let $V_B = V$. There are three paths from B to F: B–A–G–F, B–F, and B–C–D–F. Their resistances are respectively $2 + 2 = 4\Omega$, 4Ω , and $2 + 2 = 4\Omega$. Each battery has its positive terminal towards the upper part of the diagram.

Step 2: Express each branch current using its voltage drop Temporarily take all three currents from B towards F. Denote them by I_L , I_M , and I_R for the left, middle, and right paths. In every path this direction crosses the battery from positive to negative. Hence Kirchhoff's voltage rule gives

$$\begin{aligned} V &= 4I_L + 3, & I_L &= \frac{V - 3}{4}, \\ V &= 4I_M + 6, & I_M &= \frac{V - 6}{4}, \\ V &= 4I_R + 6, & I_R &= \frac{V - 6}{4}. \end{aligned}$$

Here voltages are in volts and currents in amperes; ideal batteries contribute no internal resistance.

Step 3: Apply the junction rule and determine the directions No external current enters B, so the signed currents satisfy

$$\frac{V - 3}{4} + \frac{V - 6}{4} + \frac{V - 6}{4} = 0 \Rightarrow 3V - 15 = 0.$$

Thus $V = 5\text{ V}$, giving

$$I_L = +0.50\text{ A}, \quad I_M = -0.25\text{ A}, \quad I_R = -0.25\text{ A}.$$

The left current follows the trial path, while the middle and right currents run opposite to their trial

paths. The two incoming currents at B sum to 0.50 A, balancing its outgoing current.

Step 4: Conclusion The requested currents are $I_{AG} = 0.50 \text{ A}, A \rightarrow G$, $I_{BF} = 0.25 \text{ A}, F \rightarrow B$, and $I_{CD} = 0.25 \text{ A}, D \rightarrow C$.

Quick Tip to Solve: Treat B and the common F-D wire as the two nodes: all three complete paths have resistance 4Ω , which simplifies the junction equation.

23. (a) On what factors does the speed of an electromagnetic wave in a medium depend?

(b) How is an electromagnetic wave produced?

(c) Sketch a schematic diagram depicting the electric and magnetic fields for an electromagnetic wave propagating along z-axis. [3 marks]

Solution:

Concept:

- In a uniform medium, the electromagnetic wave speed is $v = 1/\sqrt{\mu\epsilon}$, where μ is the permeability and ϵ is the permittivity of the medium.
- Accelerated electric charges radiate electromagnetic waves. An oscillating charge produces time-varying electric and magnetic fields.
- An electromagnetic wave is transverse: $\vec{E} \perp \vec{B}$, and both are perpendicular to propagation, whose direction is $\vec{E} \times \vec{B}$.

Step 1: (a) Identify the properties controlling the speed The speed depends on the electric and magnetic properties of the medium, represented by its permittivity and permeability. In terms of their relative values,

$$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r\epsilon_r}}, \quad c = \frac{1}{\sqrt{\mu_0\epsilon_0}}.$$

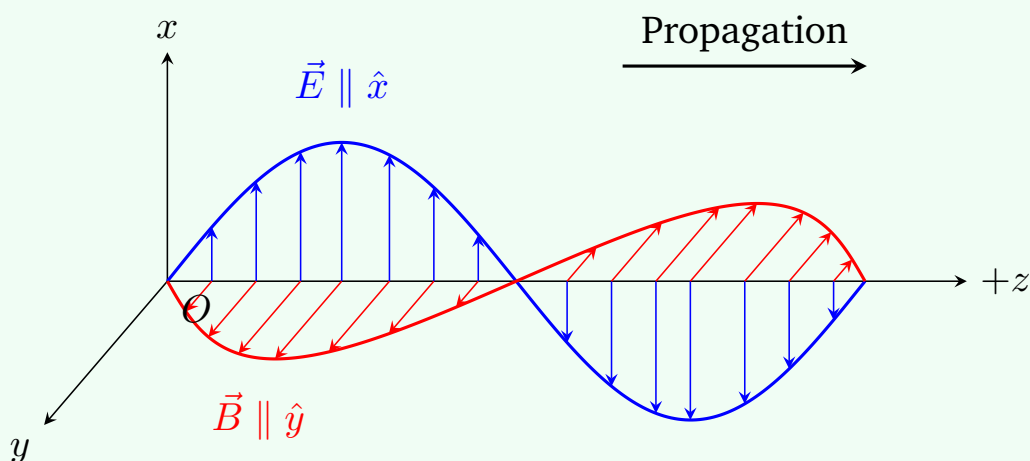
Thus, a greater product $\mu\epsilon$ gives a smaller wave speed. For a non-magnetic medium, $\mu_r \simeq 1$, so $v \simeq c/\sqrt{\epsilon_r}$.

Step 2: (b) Explain the production of the wave When an electric charge accelerates, for example when it oscillates in an antenna, its electric field changes with time. A time-varying electric field is associated with a magnetic field, and a changing magnetic field induces an electric field. These coupled disturbances propagate away from the source as an electromagnetic wave. A charge at rest produces only a static electric field and does not radiate such a wave.

Step 3: (c) Choose perpendicular field directions and sketch them For propagation along $+z$, choose the electric field along the x -axis and the magnetic field along the y -axis. One possible plane wave is

$$\begin{aligned}\vec{E}(z, t) &= E_0 \sin(kz - \omega t) \hat{x}, \\ \vec{B}(z, t) &= B_0 \sin(kz - \omega t) \hat{y}.\end{aligned}$$

The fields have the same phase: their zeros and extrema occur at the same positions at a fixed time. Since $\hat{x} \times \hat{y} = \hat{z}$, energy travels along $+z$. The schematic below shows a snapshot, with field amplitudes drawn on independent scales.



Electric and magnetic fields of a wave propagating along $+z$; amplitudes use independent scales.

Step 4: Conclusion The speed is determined by μ and ε , and accelerated charges generate electromagnetic waves. For a wave travelling along $+z$, mutually perpendicular, in-phase fields can be chosen along x and y , with $\vec{E} \times \vec{B}$ along $+z$.

Quick Tip to Solve: To check the sketch, apply the right-hand rule to $\vec{E} \times \vec{B}$: it must point along the stated propagation axis, and both waves must cross zero together.

24. A 100-turn coil of radius 1.6 cm and resistance $5.0 \, \Omega$ is co-axial with a solenoid of 250 turns/cm and radius 1.8 cm. The solenoid current drops from 1.5 A to zero in 25 ms. Calculate the current induced in the coil in this duration. (Take $\pi^2 = 10$) [3 marks]

Solution:

Concept:

- The field inside a long solenoid is $B = \mu_0 n I_s$, where n is its turns per unit length and I_s is the solenoid current.
- Faraday's law gives the magnitude of the average induced emf as $|\mathcal{E}| = N |\Delta\Phi| / \Delta t$. Here N is the number of turns in the receiving coil and $\Phi = BA$ is flux per turn.
- The induced current is $I_c = |\mathcal{E}| / R$. Lenz's law determines its direction: its magnetic field opposes the change in the original flux.

Step 1: Convert the data and select the linked area The coil radius is smaller than the solenoid radius, so the field threads the entire coil area. We therefore use the coil radius, not the solenoid radius:

$$\begin{aligned} N &= 100, & R &= 5.0 \, \Omega, \\ r_c &= 1.6 \times 10^{-2} \, \text{m}, & A &= \pi r_c^2 = 2.56\pi \times 10^{-4} \, \text{m}^2, \\ n &= 250 \times 100 = 2.50 \times 10^4 \, \text{m}^{-1}, & \Delta t &= 25 \times 10^{-3} \, \text{s}. \end{aligned}$$

The coils are coaxial, so the solenoid field is perpendicular to the receiving coil's plane and $\Phi = BA$.

Step 2: Find the rate of change of current and induced emf The magnitude of the average rate of fall of the solenoid current is

$$\frac{|\Delta I_s|}{\Delta t} = \frac{1.5 - 0}{25 \times 10^{-3}} = 60 \, \text{A s}^{-1}.$$

Since $B = \mu_0 n I_s$, the flux linkage is $N\Phi = N\mu_0 n A I_s$. Thus,

$$\begin{aligned} |\mathcal{E}| &= N\mu_0 n A \frac{|\Delta I_s|}{\Delta t} \\ &= 100(4\pi \times 10^{-7})(2.50 \times 10^4)(2.56\pi \times 10^{-4})(60) \\ &= 0.01536\pi^2 \text{ V} = 0.1536 \text{ V}, \end{aligned}$$

using the given $\pi^2 = 10$.

Step 3: Apply Ohm's law and determine the sense of induction The resistance limits the induced current to

$$I_c = \frac{0.1536}{5.0} = 0.03072 \text{ A}.$$

As the original flux is decreasing, the coil's induced field supports the original solenoid field. Hence the induced current circulates in the same sense as the solenoid current when both are viewed from the same end. The calculated value is the average over the stated interval; it is constant if the current falls uniformly, in the usual negligible-self-inductance approximation.

Step 4: Conclusion The induced current is $3.072 \times 10^{-2} \text{ A}$, or approximately 31 mA, directed so as to maintain the decreasing solenoid flux.

Quick Tip to Solve: Use the smaller coil's area for the linked flux, convert turns per centimetre to turns per metre, and keep both factors of π until substituting $\pi^2 = 10$.

25(a). Two long, straight, parallel conductors carry steady currents in opposite directions. Explain the nature of the force of interaction between them. Obtain an expression for the magnitude of the force between the two conductors. Hence define one ampere. [3 marks]

Solution:

Concept:

- A long straight conductor carrying current I produces a magnetic field $B = \mu_0 I / (2\pi d)$ at perpendicular distance d in vacuum.
- A straight length l carrying current I in a magnetic field experiences force $\vec{F} = I\vec{l} \times \vec{B}$. For a perpendicular field, $F = IlB$.
- Parallel currents in the same direction attract, whereas parallel currents in opposite directions repel. The two conductors experience equal and opposite forces.

Step 1: Determine the direction of the interaction Let conductor 1 be to the left of conductor 2, separated by distance d . Take current I_1 upward in conductor 1 and current I_2 downward in conductor 2. By the right-hand thumb rule, the magnetic field due to conductor 1 at conductor 2 points into the page. Applying $\vec{F}_2 = I_2 \vec{l} \times \vec{B}_1$, the downward current in this field experiences a force to the right, away from conductor 1. Similarly, conductor 1 experiences a force to the left. The interaction is therefore repulsive.

Step 2: Derive the magnitude of the force Neglecting end effects, the field of conductor 1 at conductor 2 is uniform along a chosen length l :

$$B_1 = \frac{\mu_0 I_1}{2\pi d}.$$

The current direction is perpendicular to this field, so the force magnitude on that length of conductor

2 is

$$F_{21} = I_2 l B_1 \sin 90^\circ = \frac{\mu_0 I_1 I_2 l}{2\pi d}.$$

Interchanging the conductors gives the same force magnitude on an equal length of conductor 1. Thus the force per unit length is

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}.$$

The formula is stated for vacuum, with air giving essentially the same result; I_1 and I_2 here denote positive current magnitudes.

Step 3: Use the force law to define one ampere In the conventional force-based definition used for this derivation, set $I_1 = I_2 = 1 \text{ A}$, $d = 1 \text{ m}$, and $\mu_0 = 4\pi \times 10^{-7} \text{ N A}^{-2}$. Then

$$\frac{F}{l} = \frac{(4\pi \times 10^{-7})(1)(1)}{2\pi(1)} = 2 \times 10^{-7} \text{ N m}^{-1}.$$

One ampere is the constant current which, when maintained in each of two straight parallel conductors of infinite length and negligible circular cross-section, placed one metre apart in vacuum, produces a force of $2 \times 10^{-7} \text{ N}$ per metre of length between them. For the opposite directions specified here, that force is repulsive.

Step 4: Conclusion Oppositely directed parallel currents repel with magnitude $F = \mu_0 I_1 I_2 l / (2\pi d)$ on a length l . The conventional one-ampere definition corresponds to $2 \times 10^{-7} \text{ N m}^{-1}$ between the specified conductors one metre apart in vacuum.

Quick Tip to Solve: Find the field of the first wire before applying IlB to the second. For the ampere definition, state vacuum, one metre separation, infinite length and negligible cross-section.

OR

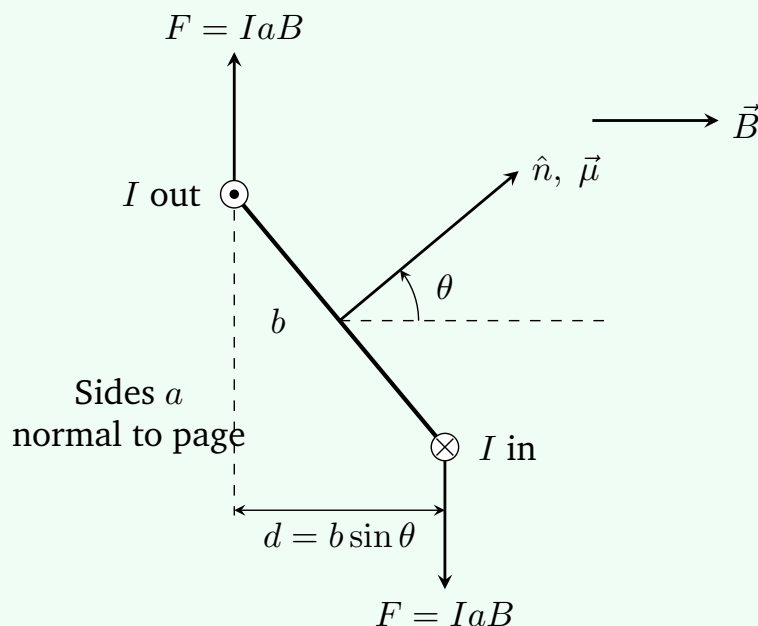
25(b). Obtain an expression for the torque $\vec{\tau}$ acting on a current carrying loop in a uniform magnetic field $\vec{B}$. Draw the necessary diagram. [3 marks]

Solution:

Concept:

- A straight conductor of length vector $\vec{l}$, carrying current I , experiences $\vec{F} = I\vec{l} \times \vec{B}$ in a uniform magnetic field.
- A current loop has magnetic moment $\vec{\mu} = IA\hat{n}$, where A is its area and $\hat{n}$ is the normal given by the right-hand rule. A couple has torque equal to either force times their perpendicular separation.

Step 1: Choose a rectangular loop and define its orientation Consider a single-turn rectangular loop with sides a and b , so its area is $A = ab$. Let the normal $\hat{n}$ make angle θ with the uniform field $\vec{B}$. Choose the two sides of length a perpendicular to the field. The diagram is an edge-on view looking along these sides; the dot and cross show their opposite current directions. The sloping line represents the plane of the loop in this view.



Edge-on view: the sides of length a carry current out of and into the page; the force separation is $b \sin \theta$.

Step 2: Identify the couple and calculate its torque Each side of length a experiences a force of magnitude $F = IaB$. These forces are equal, opposite and parallel, so their resultant is zero, but their lines of action are separated. The other two sides experience equal and opposite forces along a common line, and therefore produce no couple. From the edge-on geometry, the perpendicular distance between the first pair of forces is $d = b \sin \theta$. Hence,

$$\begin{aligned}\tau &= Fd = (IaB)(b \sin \theta) \\ &= IAB \sin \theta = \mu B \sin \theta.\end{aligned}$$

This is the magnitude of the torque; the direction follows the right-hand rule for $\vec{\mu} \times \vec{B}$. The torque tends to turn the magnetic moment towards the field, rather than translating the loop.

Step 3: Write the vector expression and generalise With the area vector along the current-defined normal, the vector relation is

$$\vec{\tau} = IA\hat{n} \times \vec{B} = \vec{\mu} \times \vec{B}.$$

The same result holds for a planar loop of any shape. For a coil of N identical turns, $\vec{\mu} = NIA\hat{n}$. The torque vanishes when the normal is parallel or antiparallel to the field and is maximum when the normal is perpendicular to it.

Step 4: Conclusion The torque on a current loop is $\vec{\tau} = \vec{\mu} \times \vec{B}$, with magnitude $\tau = IAB \sin \theta$ for one turn, or $NIAB \sin \theta$ for N turns. Here θ is the angle between the loop's normal and the field.

Quick Tip to Solve: Measure the angle from the normal, not the plane of the loop; then use $\tau = IAB \sin \theta$ without changing sine to cosine.

26. Using Bohr's postulates, derive the expression for the radius of the n th orbit of an electron in a hydrogen atom. Also find the numerical value of Bohr's radius a_0 . [3 marks]

Solution:

Concept:

- In Bohr's model, an electron moves in a permitted stationary circular orbit without radiating energy; the electrostatic attraction of the proton supplies the centripetal force.
- The angular momentum is quantised: $m_e v_n r_n = n\hbar = nh/(2\pi)$, where $n = 1, 2, \dots$, m_e is the electron mass and h is Planck's constant. The nucleus is treated as fixed in this model.

Step 1: Equate Coulomb attraction to centripetal force The hydrogen nucleus has charge $+e$, and the electron has charge $-e$. At orbit radius r_n , the attractive electric force acts radially inward. Thus,

$$\frac{m_e v_n^2}{r_n} = \frac{e^2}{4\pi\epsilon_0 r_n^2}, \quad m_e v_n^2 r_n = \frac{e^2}{4\pi\epsilon_0}.$$

This mechanical condition alone allows continuous radii; Bohr's angular-momentum postulate selects the allowed values.

Step 2: Apply angular-momentum quantisation From $m_e v_n r_n = nh/(2\pi)$, eliminate the speed using

$$v_n = \frac{nh}{2\pi m_e r_n}.$$

Substituting this into the force equation gives

$$\begin{aligned} m_e \left(\frac{nh}{2\pi m_e r_n} \right)^2 r_n &= \frac{e^2}{4\pi\epsilon_0}, \\ \frac{n^2 h^2}{4\pi^2 m_e r_n} &= \frac{e^2}{4\pi\epsilon_0}, \\ r_n &= \frac{\epsilon_0 n^2 h^2}{\pi m_e e^2} = n^2 a_0. \end{aligned}$$

Thus the allowed radii increase as the square of the principal quantum number, rather than linearly with it.

Step 3: Evaluate the first-orbit radius For $n = 1$, Bohr's radius is $a_0 = \epsilon_0 h^2 / (\pi m_e e^2)$. Using $h = 6.63 \times 10^{-34}$ J s, $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C and $\epsilon_0 = 8.854 \times 10^{-12}$ F m $^{-1}$,

$$a_0 = \frac{(8.854 \times 10^{-12})(6.63 \times 10^{-34})^2}{\pi(9.1 \times 10^{-31})(1.6 \times 10^{-19})^2} \simeq 5.3 \times 10^{-11} \text{ m}.$$

Step 4: Conclusion The radius of the n th orbit is $r_n = \frac{\epsilon_0 h^2}{\pi m_e e^2} n^2 = a_0 n^2$. The Bohr radius is

$$a_0 \simeq 5.3 \times 10^{-11} \text{ m} = 0.053 \text{ nm}.$$

Quick Tip to Solve: First eliminate v_n using quantised angular momentum; check that your final hydrogen radius scales as n^2 and reduces to a_0 at $n = 1$.

27. de Broglie wavelength λ as a function of $\frac{1}{\sqrt{K}}$ for two particles of masses m_1 and m_2 are shown in the figure. Here, K is the energy of the moving particles.

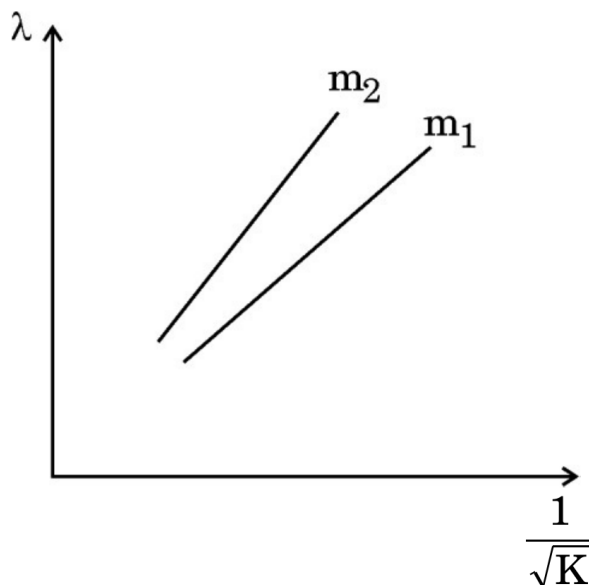
(a) What does the slope of a line represent?

(b) Which of the two particles is heavier?

(c) Is this graph also valid for a photon?

Justify your answer in each case.

[3 marks]



Solution:

Concept:

- The de Broglie relation is $\lambda = h/p$, where h is Planck's constant and p is the magnitude of momentum. For a non-relativistic material particle of mass m , its kinetic energy is $K = p^2/(2m)$.
- A graph of $y = sx$ has slope s . For a photon, the energy-momentum relation is $E = pc$, so $\lambda = hc/E$; the massive-particle expression for kinetic energy cannot be applied to it.

Step 1: (a) Express the wavelength in terms of the horizontal variable From $K = p^2/(2m)$, we have $p = \sqrt{2mK}$. Substitute this into the de Broglie relation:

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2m}} \left(\frac{1}{\sqrt{K}} \right).$$

Therefore, with $x = 1/\sqrt{K}$, the slope of the plotted straight line represents $s = h/\sqrt{2m}$. It is inversely proportional to the square root of the mass, not to the mass itself. The relation assumes the particles move non-relativistically.

Step 2: (b) Compare the two slopes The line labelled m_2 is steeper than the line labelled m_1 , so $s_2 > s_1$. Since $m = h^2/(2s^2)$, the smaller slope corresponds to the larger mass. Quantitatively,

$$\frac{s_2}{s_1} = \sqrt{\frac{m_1}{m_2}} > 1 \Rightarrow m_1 > m_2.$$

Thus particle 1 is heavier. This comparison uses the inclinations of the lines, which is the information given by the printed graph.

Step 3: (c) Test the dependence for a photon A photon has zero rest mass and obeys $E = pc$. Calling

its energy K , as the horizontal-axis notation requires, gives

$$\lambda = \frac{hc}{K} = hc \left(\frac{1}{\sqrt{K}} \right)^2 = hc x^2.$$

The plot of λ against $1/\sqrt{K}$ is therefore a parabola in the positive quadrant, rather than either straight line shown. Its slope varies with x . Substituting zero mass into $h/\sqrt{2mK}$ is invalid because that expression was obtained from non-relativistic massive-particle mechanics.

Step 4: Conclusion (a) The slope is $\boxed{h/\sqrt{2m}}$. (b) $\boxed{m_1 > m_2}$, so particle 1 is heavier. (c) The straight-line graph is not valid for a photon: $\lambda = hc(1/\sqrt{K})^2$ gives a parabolic dependence.

Quick Tip to Solve: For this graph, mass varies as $1/(\text{slope})^2$: choose the less steep line as the heavier particle, and use $E = pc$ separately for photons.

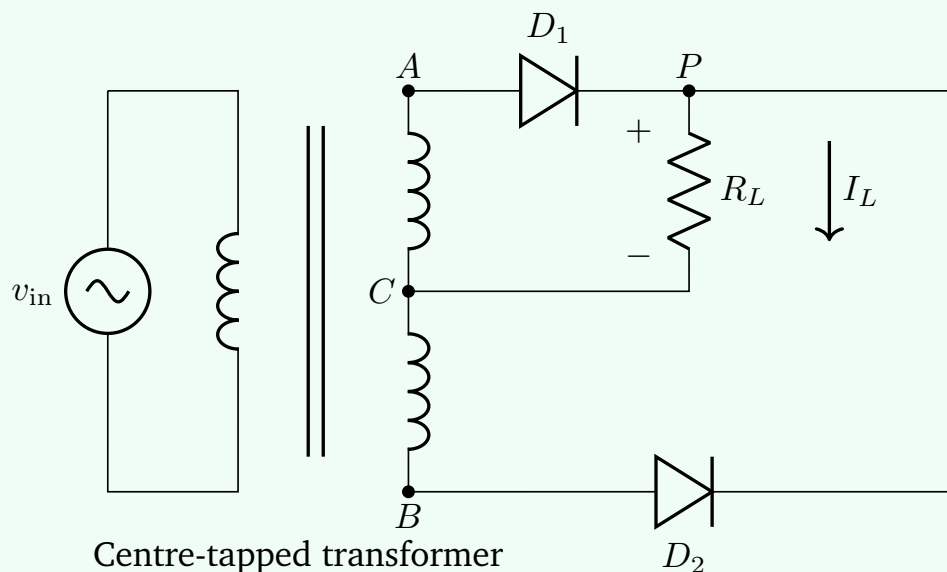
28. With the help of a circuit diagram, explain the working of a p-n junction diode as a full wave rectifier. Draw its input and output waveforms. [3 marks]

Solution:

Concept:

- A p-n junction diode conducts when forward biased and blocks when reverse biased. Rectification uses this one-way conduction to obtain a unidirectional load current from an alternating input.
- A centre-tapped transformer provides equal secondary voltages of opposite phase relative to its centre tap C . Two diodes conduct alternately, using both half-cycles of the input.

Step 1: Connect a full-wave rectifier Connect the anodes of D_1 and D_2 to the secondary ends A and B , respectively. Join their cathodes at P . Connect the load resistance R_L between P and the centre tap C . The alternating supply is applied to the transformer's primary. The circuit below labels the load polarity and its conventional current direction.



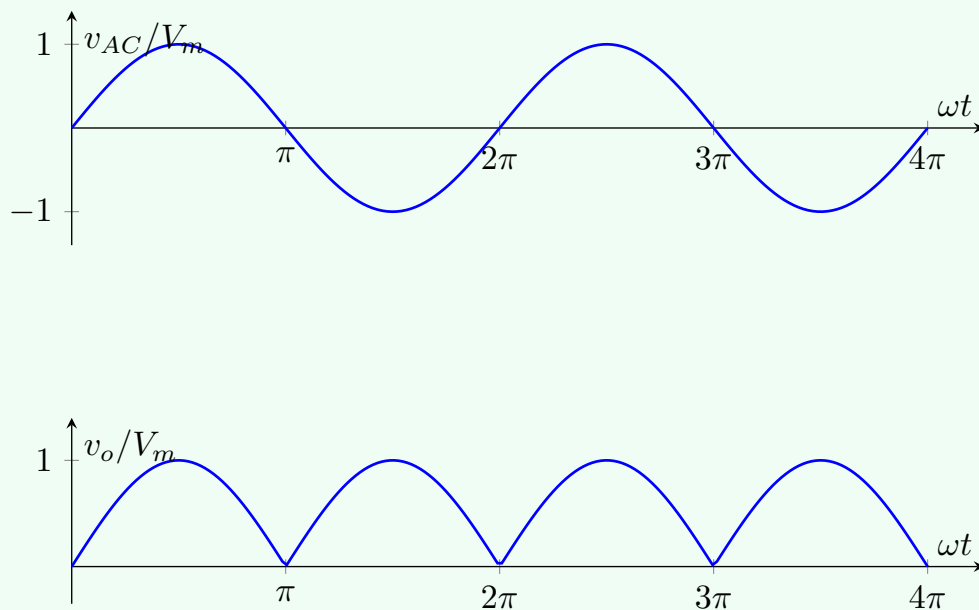
Centre-tapped full-wave rectifier; joined cathodes make P positive.

Step 2: Follow the positive half-cycle When A is positive relative to C , B is negative relative to C . Thus D_1 is forward biased and conducts, while D_2 is reverse biased. Current follows $A \rightarrow D_1 \rightarrow P \rightarrow$

$R_L \rightarrow C$, returning through the upper secondary winding. The load current flows from P towards C .

Step 3: Follow the negative half-cycle The secondary polarities reverse: B is now positive and A negative relative to C . Diode D_2 conducts and D_1 blocks. Current follows $B \rightarrow D_2 \rightarrow P \rightarrow R_L \rightarrow C$, returning through the lower secondary winding. The direction through the load is again from P towards C .

Step 4: Compare the input and output waveforms For ideal diodes and a sinusoidal half-secondary voltage $v_{AC} = V_m \sin \omega t$, the load voltage is $v_o = V_m |\sin \omega t|$. Each negative half-cycle is converted to a positive load pulse. The output is pulsating direct voltage, with two pulses per input period T ; its ripple frequency is $2f$, where $f = 1/T$ is the input frequency. The waveform below assumes negligible diode voltage drops.



Sinusoidal input and full-wave rectified output, shown over two input periods.

Step 5: Conclusion The two diodes conduct on alternate half-cycles, but maintain the same load-current direction. Both halves of the alternating input produce a positive output across R_L , giving full-wave rectification and an output ripple frequency twice the input frequency.

Quick Tip to Solve: Trace each conducting path from the positive secondary end to the centre tap: both paths must traverse the load in the same direction.

Section D (8 marks)

Case Study Based Questions

Questions number 29 and 30 are case study based questions. Read the following paragraphs and answer the questions that follow.

29. When the terminals of a cell are connected to a conductor of resistance R , an electric current flows through the circuit. The electrolyte of the cell also offers some resistance in the path of the current, like the conductor. This resistance offered by the electrolyte is called internal resistance of the cell (r). It depends upon the nature of the electrolyte, the area of the electrodes immersed in the electrolyte and the temperature. Due to internal resistance, a part of the energy supplied by the cell is wasted in the form of heat.

When no current is drawn from the cell, the potential difference between the two electrodes is known as emf of the cell (ε). With a current drawn from the cell, the potential difference between the two electrodes is termed as terminal potential difference (V). [4 marks]

29(i). Choose the incorrect statement: [1 mark]

- (A) The potential difference (V) between the two terminals of a cell in a closed circuit is always less than its emf (ε), during discharge of the cell.
- (B) The internal resistance of a cell decreases with the decrease in temperature of the electrolyte.
- (C) When current is drawn from the cell then $V = \varepsilon - Ir$.
- (D) The graph between potential difference between the two terminals of the cell (V) and the current (I) through it is a straight line with a negative slope.

Correct Answer: (B) The internal resistance of a cell decreases with the decrease in temperature of the electrolyte.

Solution:

Concept:

- During discharge a cell of emf ε and internal resistance r supplies current I , so its terminal voltage is $V = \varepsilon - Ir$.
- Electrolytic conduction is due to mobile ions. A rise in electrolyte temperature generally improves ionic mobility and decreases the internal resistance.

Step 1: Check the temperature statement Cooling the electrolyte reduces ionic mobility and generally increases its resistance. Statement (B) claims that the internal resistance decreases on cooling, which reverses the usual temperature dependence of an electrolyte.

Step 2: Check the remaining statements For a discharging cell with $I > 0$ and $r > 0$, the internal voltage drop Ir is positive. Consequently $V < \varepsilon$, as stated in (A), and the relation in (C) is correct. Writing $V = \varepsilon - rI$ also gives a straight-line graph of V against I , with intercept ε and negative slope $-r$, so (D) is correct.

Step 3: Conclusion The incorrect statement is that internal resistance decreases when the electrolyte is cooled. Hence, the correct option is (B).

Quick Tip to Solve: For an electrolyte, heating generally increases ionic mobility; do not apply the usual metallic-resistance temperature trend.

29(ii). Two cells of emfs 2.0 V and 6.0 V and internal resistances $0.1\ \Omega$ and $0.4\ \Omega$ respectively, are connected in parallel. The equivalent emf of the combination will be: [1 mark]

- (A) 2.0 V
- (B) 2.8 V
- (C) 6.0 V
- (D) 8.0 V

Correct Answer: (B) 2.8 V

Solution:

Concept:

- For two cells connected in parallel with like terminals together, their equivalent emf is the common open-circuit terminal voltage.
- With emfs $\varepsilon_1, \varepsilon_2$ and internal resistances r_1, r_2 , it is $\varepsilon_{eq} = (\varepsilon_1/r_1 + \varepsilon_2/r_2)/(1/r_1 + 1/r_2)$.

Step 1: Identify the resistance-weighted emf In a parallel combination unequal emfs are not added. At open circuit, the currents in the two branches sum to zero, so $(\varepsilon_1 - V)/r_1 + (\varepsilon_2 - V)/r_2 = 0$. Solving for the common voltage gives the equivalent-emf formula.

Step 2: Substitute and compare the options Using the supplied internal resistances,

$$\varepsilon_{eq} = \frac{2.0/0.1 + 6.0/0.4}{1/0.1 + 1/0.4} = \frac{20 + 15}{10 + 2.5} = 2.8\text{ V}.$$

This value lies between the two cell emfs and is closer to the 2.0 V cell because its internal resistance is smaller. Options (A) and (C) ignore the other cell's contribution, while (D) incorrectly adds the emfs as in a series-aiding combination.

Step 3: Conclusion The equivalent emf is 2.8 V. Hence, the correct option is (B).

Quick Tip to Solve: Parallel unequal cells give a conductance-weighted average emf, closer to the emf of the cell with smaller internal resistance.

29(iii). Dipped in the solution, the electrode exchanges charges with the electrolyte. The positive electrode develops a potential V_+ ($V_+ > 0$), and the negative electrode develops a potential $-(V_-)$ ($V_- \geq 0$), relative to the electrolyte adjacent to it. When no current is drawn from the cell then: [1 mark]

- (A) $\varepsilon = V_+ + V_- > 0$
- (B) $\varepsilon = V_+ - V_- > 0$
- (C) $\varepsilon = V_+ + V_- < 0$
- (D) $\varepsilon = V_+ + V_- = 0$

Correct Answer: (A) $\varepsilon = V_+ + V_- > 0$

Solution:

Concept:

- When no current is drawn, the emf of a cell equals the potential of its positive terminal minus the potential of its negative terminal.
- The magnitude V_+ is positive and V_- is nonnegative; the actual negative-electrode potential is $-V_-$.

Step 1: Retain the sign of each electrode potential Taking the electrolyte as the reference in this open-circuit cell, the positive electrode has potential $+V_+$, whereas the negative electrode has potential $-V_-$. Subtracting terminal potentials must therefore retain this explicitly printed minus sign.

Step 2: Find the open-circuit potential difference The potential rise from the negative terminal to the positive terminal is

$$\varepsilon = (+V_+) - (-V_-) = V_+ + V_- > 0.$$

Option (B) wrongly subtracts the magnitude V_- , instead of subtracting the signed potential $-V_-$. Since $V_+ > 0$ and $V_- \geq 0$, their sum is positive and cannot be negative as in (C) or zero as in (D).

Step 3: Conclusion The cell emf is $\varepsilon = V_+ + V_- > 0$. Hence, the correct option is (A).

Quick Tip to Solve: Write each electrode's signed potential in brackets before subtracting; subtracting a negative potential produces a sum of magnitudes.

29(iv)(a). Five identical cells, each of emf 2 V and internal resistance 0.1Ω are connected in parallel. This combination in turn is connected to an external resistor of 9.98Ω . The current flowing through the resistor is: [1 mark]

(A) 0.05 A

(B) 0.1 A

(C) 0.15 A

(D) 0.2 A

Correct Answer: (D) 0.2 A

Solution:

Concept:

- For n identical cells connected in parallel with like terminals joined, the equivalent emf remains ε , while the equivalent internal resistance becomes r/n .
- The external load current is $I = \varepsilon / (R + r/n)$, where R is the load resistance and r is one cell's internal resistance.

Step 1: Replace the parallel cells by one equivalent cell All five cells have the same emf, so their parallel combination has emf 2 V. Their five identical internal resistances are in parallel:

$$r_{\text{eq}} = \frac{0.1}{5} = 0.02 \Omega.$$

Adding the five emfs would incorrectly treat the cells as a series combination.

Step 2: Calculate the total load current The equivalent internal resistance is in series with the external resistor. Hence,

$$I = \frac{2}{9.98 + 0.02} = \frac{2}{10.00} = 0.2 \text{ A}.$$

Options (A), (B) and (C) do not satisfy this loop equation with the printed values. The question asks

for the current through the external resistor, not the current supplied by each individual cell; each cell supplies one-fifth of the load current.

Step 3: Conclusion The current through the external resistor is 0.2 A. Hence, the correct option is (D).

Quick Tip to Solve: Parallel identical cells keep the same emf; divide only their internal resistance by five before adding the external resistance.

OR

29(iv)(b). Potential difference across a cell in the open circuit is 6 V. It becomes 4 V when a current of 2 A is drawn from it. The internal resistance of the cell is: [1 mark]

- (A) 1.0 Ω
- (B) 1.5 Ω
- (C) 2.0 Ω
- (D) 2.5 Ω

Correct Answer: (A) 1.0 Ω

Solution:

Concept:

- In an open circuit no current flows, so there is no internal voltage drop and the measured terminal potential difference equals the cell emf ε .
- During discharge the terminal voltage is $V = \varepsilon - Ir$, where r is internal resistance and I is the current delivered by the cell.

Step 1: Identify the emf and lost voltage The open-circuit measurement gives $\varepsilon = 6$ V. When the current is 2 A, the terminal voltage falls to 4 V. Thus the voltage lost inside the cell is $\varepsilon - V = 6 - 4 = 2$ V.

Step 2: Apply the discharge equation Rearranging $V = \varepsilon - Ir$ gives

$$r = \frac{\varepsilon - V}{I} = \frac{6 - 4}{2} = 1.0 \Omega.$$

The larger resistances in (B), (C) and (D) would cause internal drops of 3, 4 and 5 V, respectively, instead of the observed 2 V. In particular, $V/I = 2 \Omega$ describes the external load resistance, not the internal resistance.

Step 3: Conclusion The internal resistance of the cell is 1.0 Ω . Hence, the correct option is (A).

Quick Tip to Solve: Divide the fall from open-circuit voltage to loaded voltage by the current; dividing the loaded voltage itself gives the external resistance.

30. When a ray of light propagates from a denser medium to a rarer medium, it bends away from the normal. When the incident angle is increased, the refracted ray deviates more from the normal. For a particular angle of incidence in the denser medium, the refracted ray just grazes the interface of the two surfaces. This angle of incidence is called the critical angle for the pair of media involved.

[4 marks]

30(i). For a ray incident at the critical angle, the angle of reflection is:

[1 mark]

- (A) 0°
- (B) $< 90^\circ$
- (C) $> 90^\circ$
- (D) 90°

Correct Answer: (B) $< 90^\circ$

Solution:

Concept:

- The law of reflection gives $r_{\text{refl}} = i$, where both angles are measured from the normal.
- The critical angle C satisfies $\sin C = n_{\text{rarer}}/n_{\text{denser}}$; for distinct transparent media, $0 < C < 90^\circ$.

Step 1: Apply the law of reflection The incident angle is $i = C$. Reflection still obeys its usual law at this incidence, so the reflected ray makes an angle $r_{\text{refl}} = C$ with the normal in the denser medium.

Step 2: Distinguish reflection from refraction The ray that grazes the interface is the refracted ray, whose angle is 90° . This does not make the angle of reflection 90° . The critical angle is positive and acute, excluding 0° , 90° , and angles greater than 90° .

Step 3: Conclusion The angle of reflection equals the critical angle and is $< 90^\circ$. Hence, the correct option is (B).

Quick Tip to Solve: At critical incidence, assign 90° to refraction; reflection always retains the incident angle.

30(ii). A ray of light of wavelength 600 nm is incident in water $\left(n = \frac{4}{3}\right)$ on the water-air interface at an angle less than the critical angle. The wavelength associated with the refracted ray is:

[1 mark]

- (A) 400 nm
- (B) 450 nm
- (C) 600 nm
- (D) 800 nm

Correct Answer: (D) 800 nm

Solution:

Concept:

- Frequency f remains unchanged when light crosses a stationary interface, while speed and wavelength may change: $v = f\lambda$.
- The refractive index is $n = c/v$. Thus wavelengths in two media satisfy $\lambda_2/\lambda_1 = n_1/n_2$.

Step 1: Identify the wavelength in water The given 600 nm is the wavelength of the incident light in water. On entering air, its speed rises from $c/(4/3)$ to approximately c , while its frequency stays fixed. Its wavelength must therefore increase by a factor of $4/3$.

Step 2: Calculate the refracted wavelength Taking the refractive index of air as unity,

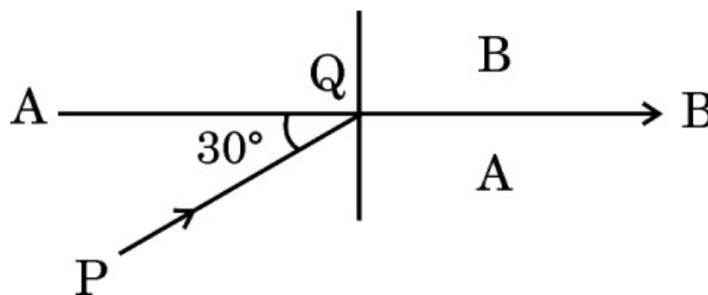
$$\lambda_{\text{air}} = \frac{n_{\text{water}}}{n_{\text{air}}} \lambda_{\text{water}} = \frac{4}{3} \times 600 = 800 \text{ nm}.$$

The values 400 nm and 450 nm incorrectly decrease the wavelength; 600 nm incorrectly keeps wavelength unchanged. It is frequency, rather than wavelength, that is conserved across the interface.

Step 3: Conclusion The refracted wavelength is 800 nm. Hence, the correct option is (D).

Quick Tip to Solve: In water-to-air refraction, multiply the wavelength in water by its refractive index; frequency stays constant.

30(iii)(a). The interface AB between the two media A and B is shown in the figure. In the denser medium A, the incident ray PQ makes an angle of 30° with the horizontal. The refracted ray is parallel to the interface. The refractive index of medium B w.r.t. medium A is: [1 mark]



- (A) $\frac{\sqrt{3}}{2}$
- (B) $\frac{\sqrt{5}}{2}$
- (C) $\frac{4}{\sqrt{3}}$
- (D) $\frac{2}{\sqrt{3}}$

Correct Answer: (A) $\frac{\sqrt{3}}{2}$

Solution:

Concept:

- In Snell's law, $n_A \sin i = n_B \sin r$, the angles are measured from the normal to the interface.
- The refractive index of B with respect to A is $n_{BA} = n_B/n_A$; it is less than unity when B is optically rarer than A.

Step 1: Convert the given angle to incidence The ray makes 30° with the horizontal interface. Since the normal is perpendicular to that interface, the incidence angle is $i = 90^\circ - 30^\circ = 60^\circ$. The refracted ray travels along the interface, so $r = 90^\circ$.

Step 2: Apply Snell's law to the two media Divide Snell's law by $n_A \sin r$ to obtain

$$n_{BA} = \frac{n_B}{n_A} = \frac{\sin 60^\circ}{\sin 90^\circ} = \frac{\sqrt{3}}{2}.$$

Each of the other three printed values exceeds unity, which contradicts the fact that B is rarer than A.

In particular, $2/\sqrt{3}$ is the reversed relative index, that of A with respect to B.

Step 3: Conclusion The refractive index of B with respect to A is $\sqrt{3}/2$. Hence, the correct option is (A).

Quick Tip to Solve: First subtract an angle measured from the surface from 90° ; then use the requested order of the relative index.

OR

30(iii)(b). Two media A and B are separated by a plane boundary. The speed of light in medium A and B is $2 \times 10^8 \text{ ms}^{-1}$ and $2.5 \times 10^8 \text{ ms}^{-1}$ respectively. The critical angle for a ray of light going from medium A to medium B is: [1 mark]

- (A) $\sin^{-1} \frac{1}{2}$
(B) $\sin^{-1} \frac{4}{5}$
(C) $\sin^{-1} \frac{3}{5}$
(D) $\sin^{-1} \frac{2}{5}$

Correct Answer: (B) $\sin^{-1} \frac{4}{5}$

Solution:

Concept:

- Optical refractive index is inversely proportional to light speed: $n = c/v$, where c is the speed in vacuum.
- At the critical angle C for travel from denser A to rarer B, Snell's law gives $n_A \sin C = n_B \sin 90^\circ$.

Step 1: Identify the denser medium The speeds are $v_A = 2 \times 10^8 \text{ m s}^{-1}$ and $v_B = 2.5 \times 10^8 \text{ m s}^{-1}$. Light is slower in A, so $n_A > n_B$, and a critical angle exists for the stated direction from A to B.

Step 2: Calculate the sine of the critical angle Using $n = c/v$,

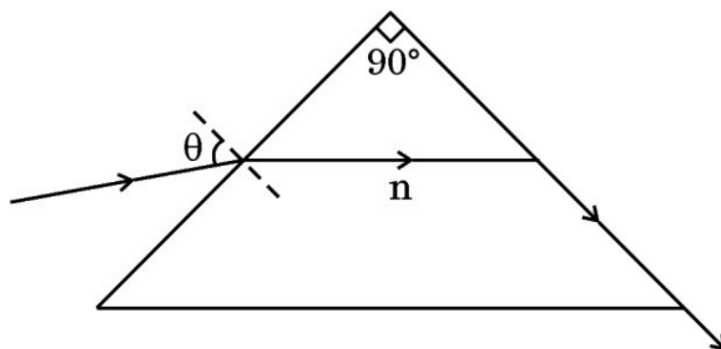
$$\sin C = \frac{n_B}{n_A} = \frac{c/v_B}{c/v_A} = \frac{v_A}{v_B} = \frac{2 \times 10^8}{2.5 \times 10^8} = \frac{4}{5}.$$

The printed ratios $1/2$, $3/5$, and $2/5$ do not equal the given speed ratio. Reversing the speed ratio would give a sine greater than one, signalling an incorrect ordering.

Step 3: Conclusion The critical angle is $C = \sin^{-1}(4/5)$, approximately 53.1° . Hence, the correct option is (B).

Quick Tip to Solve: For critical-angle questions using speeds, divide the smaller speed in the denser medium by the larger speed.

30(iv). The figure shows the path of a light ray through a triangular prism. In this phenomenon, the angle θ is given by: [1 mark]



- (A) $\sin^{-1} \sqrt{n^2 - 1}$
 (B) $\sin^{-1}(n^2 - 1)$
 (C) $\sin^{-1} \sqrt{\frac{1}{n^2 - 1}}$
 (D) $\sin^{-1} \frac{1}{(n^2 - 1)}$

Correct Answer: (A) $\sin^{-1} \sqrt{n^2 - 1}$

Solution:

Concept:

- For a prism of angle A , its internal refraction angles satisfy $r_1 + r_2 = A$; both are measured from the corresponding face normals.
- At a prism-air boundary with grazing emergence, the internal angle is critical: $\sin C = 1/n$, where n is the prism's refractive index relative to air.
- At the first face, Snell's law gives $\sin \theta = n \sin r_1$, since the marked θ is the angle between the incident ray and the normal.

Step 1: Use grazing emergence and prism geometry At the second face the emerging ray runs along the face, so $r_2 = C$. The prism angle shown is $A = 90^\circ$; therefore $r_1 = 90^\circ - C$ and $\sin r_1 = \cos C$.

Step 2: Apply refraction at the first face Substitute this relation into Snell's law at entry:

$$\begin{aligned} \sin \theta &= n \cos C = n \sqrt{1 - \sin^2 C} \\ &= n \sqrt{1 - \frac{1}{n^2}} = \sqrt{n^2 - 1}. \end{aligned}$$

The square root follows from $\cos^2 C = 1 - \sin^2 C$; dropping it produces option (B). Taking its reciprocal produces (C), and both taking the reciprocal and dropping the square root produces (D), neither following Snell's law. The shown path requires a real incident angle, hence $1 < n \leq \sqrt{2}$.

Step 3: Conclusion The angle is $\theta = \sin^{-1} \sqrt{n^2 - 1}$. Hence, the correct option is (A).

Quick Tip to Solve: For a right-angled prism with grazing exit, use $r_1 + C = 90^\circ$ before applying Snell's law at entry.

Section E (15 marks)

31(a). (i) Obtain an expression for the electric potential due to a small dipole of dipole moment $\vec{p}$, at a point $\vec{r}$ from its centre, for much larger distances compared to the size of the dipole.

(ii) Three point charges q , $2q$ and nq are placed at the vertices of an equilateral triangle. If the potential energy of the system is zero, find the value of n . [5 marks]

Solution:

Concept:

- Electric potential is a scalar. With zero potential at infinity, a point charge Q produces $V = Q/(4\pi\epsilon_0 r)$, so potentials add algebraically.
- For charges $-Q$ and $+Q$ separated by $2a$, the dipole moment is $\vec{p} = 2Qa \hat{p}$, directed from the negative charge to the positive charge.
- The electrostatic potential energy of distinct point charges is $U = \sum_{i < j} q_i q_j / (4\pi\epsilon_0 r_{ij})$; each distinct pair is counted once.

Step 1: (i) Write the exact dipole potential Put the dipole centre at the origin and let the observation point have position vector $\vec{r}$, magnitude r . Let θ be the angle between $\vec{p}$ and $\vec{r}$. The charges $+Q$ and $-Q$ are at $+a\hat{p}$ and $-a\hat{p}$, respectively. Their distances from the observation point are

$$r_+ = \sqrt{r^2 + a^2 - 2ar \cos \theta}, \quad r_- = \sqrt{r^2 + a^2 + 2ar \cos \theta}.$$

Adding their potentials with their signs gives

$$V(\vec{r}) = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_+} - \frac{1}{r_-} \right).$$

Step 2: (i) Expand for distances much larger than the dipole size Since $r \gg a$, use $(1+x)^{-1/2} \simeq 1-x/2$, retaining first-order terms in a/r . Thus

$$\begin{aligned} \frac{1}{r_+} &= \frac{1}{r} \left(1 + \frac{a^2}{r^2} - \frac{2a \cos \theta}{r} \right)^{-1/2} \simeq \frac{1}{r} \left(1 + \frac{a \cos \theta}{r} \right), \\ \frac{1}{r_-} &\simeq \frac{1}{r} \left(1 - \frac{a \cos \theta}{r} \right). \end{aligned}$$

Subtracting cancels the equal $1/r$ terms because the dipole has zero net charge:

$$V(\vec{r}) \simeq \frac{2Qa \cos \theta}{4\pi\epsilon_0 r^2} = \frac{p \cos \theta}{4\pi\epsilon_0 r^2} = \frac{\vec{p} \cdot \vec{r}}{4\pi\epsilon_0 r^3}.$$

This potential falls as $1/r^2$; it is zero on the equatorial plane, positive on the positive-charge side and negative on the opposite side.

Step 3: (ii) Add the three pair energies Let the equilateral triangle have side l , with $q \neq 0$. All three pair separations are l . The pairs are $(q, 2q)$, (q, nq) and $(2q, nq)$, giving

$$\begin{aligned} U &= \frac{1}{4\pi\epsilon_0 l} [q(2q) + q(nq) + (2q)(nq)] \\ &= \frac{q^2}{4\pi\epsilon_0 l} (2 + 3n). \end{aligned}$$

The charges are assembled from infinity; no self-energy terms are included. For the prescribed zero energy, the nonzero prefactor cancels:

$$2 + 3n = 0 \implies n = -\frac{2}{3}.$$

The third charge must therefore have the opposite sign to q , so its two negative interaction energies balance the positive energy of the first pair.

Step 4: Conclusion (i) For $r \gg a$, $V(\vec{r}) = \frac{\vec{p} \cdot \vec{r}}{4\pi\epsilon_0 r^3} = \frac{p \cos \theta}{4\pi\epsilon_0 r^2}$, to the dipole approximation. (ii) The required dimensionless value is $n = -2/3$.

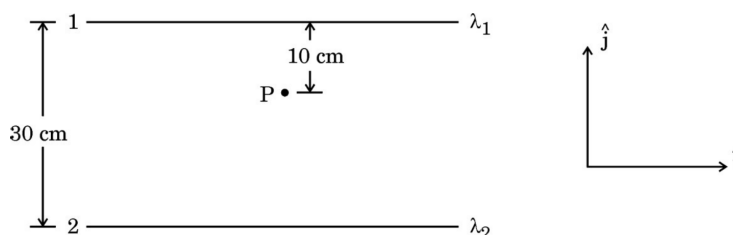
Quick Tip to Solve: For a distant dipole, subtract the two reciprocal distances before substituting $p = 2Qa$. For three charges, list exactly three pairs; equal separations let you factor out $1/(4\pi\epsilon_0 l)$.

OR

31(b). (i) State Gauss's Law in electrostatics. Apply this to obtain the electric field $\vec{E}$ at a point near a uniformly charged infinite plane sheet.

(ii) Two long straight wires 1 and 2 are kept as shown in the figure. The linear charge density of the two wires are $\lambda_1 = 10 \mu\text{C}/\text{m}$ and $\lambda_2 = -20 \mu\text{C}/\text{m}$. Find the net force $\vec{F}$ experienced by an electron held at point P.

[5 marks]



Solution:

Concept:

- Gauss's law relates the total outward electric flux through a closed surface to its enclosed charge: $\oint_S \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$.
- The symmetry of a uniformly charged infinite sheet makes the electric field perpendicular to the sheet and equal in magnitude on both sides.
- An infinitely long uniformly charged wire produces a radial field of magnitude $E = |\lambda|/(2\pi\epsilon_0 r)$, where r is the perpendicular distance and λ is the linear charge density.
- Fields add as vectors. The force on an electron is $\vec{F} = -e\vec{E}$, opposite to the net field, with $e = 1.6 \times 10^{-19} \text{ C}$.

Step 1: (i) State Gauss's law and choose the Gaussian surface Gauss's law states that the total electric flux through any closed surface equals $1/\epsilon_0$ times the net charge enclosed by that surface:

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}.$$

Here $d\vec{A}$ points outward. For an infinite plane sheet of uniform surface charge density σ , translational symmetry rules out dependence on position along the sheet, while symmetry rules out a tangential

field. Choose a cylindrical pillbox that straddles the sheet, with flat faces parallel to the sheet and each of area A . The two faces are equally distant from the sheet.

Step 2: (i) Evaluate flux and obtain the sheet field First take $\sigma > 0$. The field points normally away from the sheet. On both flat faces, the field and outward area vector are parallel, so each contributes EA . On the curved surface they are perpendicular, so its flux is zero. The enclosed charge is σA . Hence

$$2EA = \frac{\sigma A}{\epsilon_0} \implies E = \frac{\sigma}{2\epsilon_0}.$$

More generally, on either side let $\hat{n}$ point normally from the sheet into that side. Then

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n}, \quad |\vec{E}| = \frac{|\sigma|}{2\epsilon_0}.$$

Thus the field is independent of distance: away from a positive sheet and toward a negative sheet. The factor of two arises because flux leaves through both flat faces.

Step 3: (ii) Read the perpendicular distances and field directions The wires are separated by 30 cm, and P is 10 cm below wire 1. Therefore

$$r_1 = 0.10 \text{ m}, \quad r_2 = (30 - 10) \text{ cm} = 0.20 \text{ m}.$$

Wire 1 is positive, so its field at P points away from it, vertically downward. Wire 2 is negative, so its field at P points toward it, also downward. With the printed upward $\hat{j}$ direction, both fields have negative $\hat{j}$ components. There is no $\hat{i}$ component.

Step 4: (ii) Add the wire fields and find the electron force Treat the long wires as effectively infinite. The wire-field formula follows by applying Gauss's law to a coaxial cylinder: $E(2\pi rL) = |\lambda|L/\epsilon_0$. Using $1/(4\pi\epsilon_0) = 9 \times 10^9 \text{ N m}^2\text{C}^{-2}$, the separate fields are

$$\begin{aligned} \vec{E}_1 &= -2(9 \times 10^9) \frac{10 \times 10^{-6}}{0.10} \hat{j} = -1.8 \times 10^6 \hat{j} \text{ N C}^{-1}, \\ \vec{E}_2 &= -2(9 \times 10^9) \frac{20 \times 10^{-6}}{0.20} \hat{j} = -1.8 \times 10^6 \hat{j} \text{ N C}^{-1}. \end{aligned}$$

Their magnitudes are equal because the second wire has twice the charge-density magnitude and twice the distance. Adding, then multiplying by the negative electron charge, gives

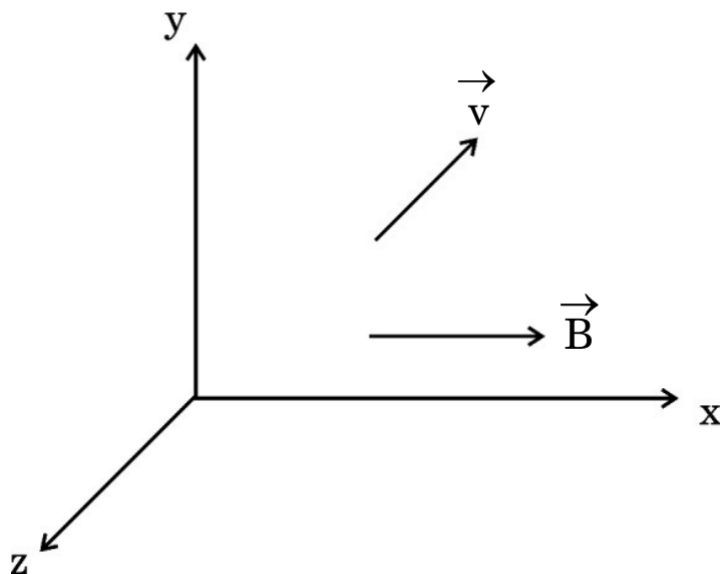
$$\begin{aligned} \vec{E}_{\text{net}} &= -3.6 \times 10^6 \hat{j} \text{ N C}^{-1}, \\ \vec{F} &= (-1.6 \times 10^{-19})(-3.6 \times 10^6) \hat{j} \text{ N} \\ &= 5.76 \times 10^{-13} \hat{j} \text{ N}. \end{aligned}$$

Step 5: Conclusion (i) The infinite-sheet field is $\vec{E} = \sigma \hat{n}/(2\epsilon_0)$, with $\hat{n}$ directed away from the sheet on the chosen side. (ii) The electron experiences $\boxed{\vec{F} = 5.76 \times 10^{-13} \hat{j} \text{ N}}$, upward, toward wire 1.

Quick Tip to Solve: Between oppositely charged wires, draw both field directions first: here both point downward. Add their magnitudes, then reverse the direction once for the electron's negative charge.

32(a). (i) A particle of mass m and charge q is moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ as shown in the figure. Show that it follows a helical path. Hence, obtain its frequency of revolution.

(ii) In a hydrogen atom, the electron moves in an orbit of radius $2 \text{ \AA}$ making 8×10^{14} revolutions per second. Find the magnetic moment associated with the orbital motion of the electron. [5 marks]



Solution:

Concept:

- The magnetic force is $\vec{F} = q\vec{v} \times \vec{B}$. It acts perpendicular to velocity and therefore changes the direction, but not the speed or kinetic energy, of the particle.
- Resolve the velocity into $v_{\parallel} = v \cos \theta$ along the field and $v_{\perp} = v \sin \theta$ perpendicular to it, where θ is the angle between $\vec{v}$ and $\vec{B}$.
- A charge completing f revolutions per second constitutes a current of magnitude $I = |q|f$. The magnetic dipole moment of a circular orbit of radius r is $\mu = I\pi r^2$.

Step 1: (i) Separate the two components of motion In the printed figure the uniform magnetic field is along the positive x -axis, and the incident velocity has both parallel and perpendicular components. The parallel component produces no magnetic force because its cross product with $\vec{B}$ is zero. Consequently the particle moves uniformly along x , with constant speed $v_{\parallel}$. The perpendicular component experiences a force of magnitude $|q|v_{\perp}B$. This force continually turns that component in the plane perpendicular to the field, while its magnitude remains constant.

Step 2: (i) Establish the circular motion and the helix The magnetic force supplies the centripetal force for the transverse circular motion. If r is the radius of this circle,

$$\frac{mv_{\perp}^2}{r} = |q|v_{\perp}B \quad \Rightarrow \quad r = \frac{mv_{\perp}}{|q|B}.$$

Thus the particle undergoes uniform circular motion in a plane perpendicular to $\vec{B}$, together with uniform translation along $\vec{B}$. The superposition is a helix whose axis is parallel to the magnetic field. Both components are nonzero in the figure, so the path is neither a straight line nor a plane circle.

Step 3: (i) Obtain the period and frequency One transverse revolution covers a circumference $2\pi r$ at

speed $v_{\perp}$. Hence

$$T = \frac{2\pi r}{v_{\perp}} = \frac{2\pi m}{|q|B},$$
$$f = \frac{1}{T} = \frac{|q|B}{2\pi m}.$$

The distance travelled along the axis in this period is the pitch,

$$p = v_{\parallel}T = \frac{2\pi m v \cos \theta}{|q|B}.$$

In the nonrelativistic treatment, the revolution frequency is independent of speed and pitch angle. The charge sign reverses the sense of rotation, but cannot make the frequency negative.

Step 4: (ii) Calculate the orbital current and moment The electron's orbit has radius $r = 2 \times 10^{-10}$ m and revolution frequency $f = 8 \times 10^{14} \text{ s}^{-1}$. Using the magnitude of the electronic charge, $e = 1.6 \times 10^{-19} \text{ C}$, gives

$$I = ef = (1.6 \times 10^{-19})(8 \times 10^{14}) = 1.28 \times 10^{-4} \text{ A}.$$

Therefore,

$$\begin{aligned}\mu &= I\pi r^2 \\ &= (1.28 \times 10^{-4})\pi(2 \times 10^{-10})^2 \\ &= 1.61 \times 10^{-23} \text{ A m}^2.\end{aligned}$$

Conventional current runs opposite to the electron's motion. By the right-hand rule its magnetic moment is therefore opposite to the electron's orbital angular momentum.

Step 5: Conclusion (i) The particle follows a helix along $\vec{B}$, with frequency $f = |q|B/(2\pi m)$, or $qB/(2\pi m)$ when q denotes the charge magnitude. (ii) Its orbital magnetic moment has magnitude $1.61 \times 10^{-23} \text{ A m}^2$, opposite to the orbital angular momentum.

Quick Tip to Solve: Use only the perpendicular speed in the circular-motion equation; the parallel speed fixes the pitch. For an orbiting electron, calculate the moment directly from $\mu = ef\pi r^2$, converting the radius to metres before squaring it.

OR

32(b). (i) What is current sensitivity of a galvanometer? Show how the current sensitivity of a galvanometer may be increased. "Increasing the current sensitivity of a galvanometer may not necessarily increase its voltage sensitivity." Explain.

(ii) A moving coil galvanometer has a resistance 15Ω and takes 20 mA to produce full scale deflection. How can this galvanometer be converted into a voltmeter of range 0 to 100 V ? [5 marks]

Solution:

Concept:

- In a moving-coil galvanometer with a radial magnetic field, the magnetic torque is $NIAB$, where N is the number of turns, I the current, A the coil area and B the field strength.
- The suspension provides a restoring torque $k\theta$, where k is the torsional constant and θ is the angular deflection. At equilibrium, $NIAB = k\theta$.
- Current sensitivity is deflection per unit current, $S_I = \theta/I$, and voltage sensitivity is deflection per unit applied voltage, $S_V = \theta/V$. A galvanometer becomes a voltmeter by adding a suitable high

resistance in series.

Step 1: (i) Define and derive current sensitivity The current sensitivity of a galvanometer is the angular deflection produced per unit current passing through its coil. A radial field keeps the magnetic torque proportional to current throughout the deflection. Balancing the magnetic torque against the restoring torque gives

$$NIAB = k\theta \Rightarrow S_I = \frac{\theta}{I} = \frac{NAB}{k}.$$

If deflection is measured in radians, the unit of current sensitivity is rad A^{-1} .

Step 2: (i) Identify ways to increase current sensitivity From $S_I = NAB/k$, keeping the other factors fixed, sensitivity increases by increasing the number of turns N , increasing the area A of each turn, or increasing the magnetic field strength B . It also increases by decreasing the torsional constant k , using a suspension that provides less restoring torque for a given angular deflection. Each change increases the deflection obtained for the same small current.

Step 3: (i) Compare voltage sensitivity with current sensitivity Let G be the resistance of the galvanometer coil. For a potential difference V across it, Ohm's law gives $I = V/G$. Therefore

$$S_V = \frac{\theta}{V} = \frac{\theta}{I} \frac{I}{V} = \frac{S_I}{G} = \frac{NAB}{kG}.$$

Voltage sensitivity depends on both current sensitivity and coil resistance. For example, adding turns of the same wire on approximately the same coil geometry increases N , but also increases the wire length and hence G . If both increase in the same proportion, S_I increases while $S_V = S_I/G$ stays approximately unchanged. Thus greater current sensitivity does not necessarily mean greater voltage sensitivity; the accompanying change in resistance must also be considered.

Step 4: (ii) Determine the series multiplier resistance Here the coil resistance is $G = 15 \Omega$, and the current for full-scale deflection is $I_g = 20 \text{ mA} = 0.020 \text{ A}$. Let the required series resistance be R_s . To obtain full-scale deflection when the voltage across the complete instrument is 100 V,

$$\begin{aligned} V &= I_g(G + R_s), \\ R_s &= \frac{V}{I_g} - G \\ &= \frac{100}{0.020} - 15 \\ &= 4985 \Omega. \end{aligned}$$

The total resistance is 5000Ω , so applying 100 V sends exactly 20 mA through the coil. The coil drops $I_g G = 0.30 \text{ V}$, and the series resistor drops the remaining 99.70 V. Connect this series combination across the two points whose potential difference is to be measured, and calibrate the deflection scale from 0 to 100 V.

Step 5: Conclusion (i) Current sensitivity is $S_I = NAB/k$; it increases with N , A , B , or a reduction in k . Since $S_V = S_I/G$, a simultaneous increase in coil resistance can prevent any rise in voltage sensitivity. (ii) Connect a 4985Ω (4.985 k Ω) resistance in series to obtain the 0–100 V voltmeter.

Quick Tip to Solve: For voltmeter conversion, first calculate the required total resistance V/I_g , then subtract the coil resistance. When comparing sensitivities, always check whether the proposed coil change also changes G .

33(a). (i) Give any two differences between the interference pattern obtained in Young's double-slit experiment and a diffraction pattern due to a single slit.

(ii) Draw an intensity distribution graph in case of a double-slit interference pattern.

(iii) In Young's double-slit experiment using monochromatic light of wavelength λ , the intensity of light at a point on the screen, where path difference is λ , is K units. Find the intensity of light at a point on the screen where the path difference is $\frac{\lambda}{6}$. [5 marks]

Solution:

Concept:

- Interference in Young's double-slit experiment is superposition of light from two coherent slits. Single-slit diffraction is superposition of secondary wavelets from different parts of the same slit.
- For identical, equally illuminated narrow slits, each giving intensity I_0 , the phase difference is $\phi = 2\pi\Delta/\lambda$ and the resultant intensity is $I = 4I_0 \cos^2(\phi/2)$, where Δ is path difference and λ is wavelength.
- In the usual ideal Young's experiment, the fringe width is $\beta = \lambda D/d$, where D is the slit-to-screen distance and d is slit separation. The plotted coordinate y is distance from the central bright fringe.

Step 1: (i) Compare the widths of the bright fringes In the ideal Young's double-slit pattern, successive bright fringes are equally spaced and all bright fringes have the same width $\beta = \lambda D/d$. The central bright fringe has the same width as the other bright fringes.

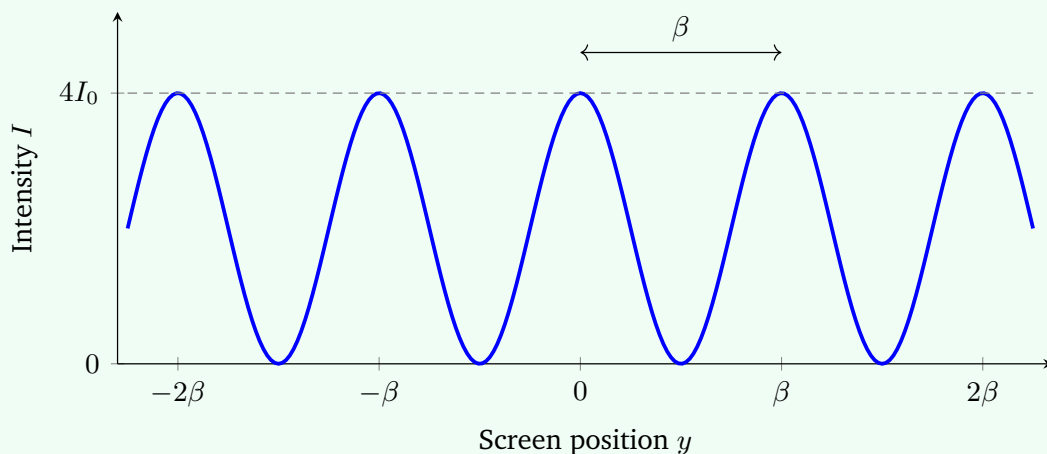
For single-slit diffraction with slit width a , the first minima occur at $y = \pm\lambda D/a$. Thus the central bright maximum has width $2\lambda D/a$, twice the width $\lambda D/a$ of a secondary bright fringe between adjacent minima. This is the first difference.

Step 2: (i) Compare the peak intensities For two identical narrow slits, the ideal interference maxima have equal intensity $4I_0$, and complete destructive interference gives zero intensity at each minimum. In a single-slit diffraction pattern, the central maximum is the brightest; the secondary maxima are weaker and decrease in intensity away from the centre. This is the second difference. The equal-height interference comparison assumes that variation of the individual slits' diffraction envelope is neglected.

Step 3: (ii) Draw and interpret the interference intensity graph Since $\Delta \simeq dy/D$, the intensity as a function of screen position is

$$I(y) = 4I_0 \cos^2\left(\frac{\pi y}{\beta}\right).$$

The graph therefore has equally high, equally spaced maxima at $y = n\beta$ and zero-intensity minima at $y = (n + \frac{1}{2})\beta$, where n is an integer. The central bright fringe is at $y = 0$. Both axes and the maximum intensity are labelled below.



Ideal double-slit interference intensity; $\beta = \lambda D/d$.

Step 4: (iii) Use the given intensity to determine the maximum At the first specified point, $\Delta = \lambda$, so the waves reach in phase:

$$\phi = \frac{2\pi\lambda}{\lambda} = 2\pi, \quad K = 4I_0 \cos^2 \pi = 4I_0.$$

Hence the given intensity K is the constructive-interference maximum. It is the combined intensity, not the intensity due to one slit alone.

Step 5: (iii) Substitute the required path difference At the second point, the printed path difference is $\Delta = \lambda/6$. The phase difference and intensity ratio are

$$\phi = \frac{2\pi\lambda}{\lambda} \frac{1}{6} = \frac{\pi}{3},$$

$$\frac{I}{K} = \cos^2\left(\frac{\phi}{2}\right) = \cos^2\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}.$$

The half-phase angle is used in the squared cosine; using the whole phase angle would give the wrong intensity.

Step 6: Conclusion The ideal interference pattern has bright fringes of equal width and equal peak intensity; single-slit diffraction has a central maximum twice as wide as the secondary fringes, with weaker secondary maxima. The labelled interference graph is shown above. At path difference $\lambda/6$, the required intensity is $I = 3K/4$ units.

Quick Tip to Solve: First identify $\Delta = \lambda$ as a maximum, then use $I/K = \cos^2(\pi\Delta/\lambda)$; for $\Delta = \lambda/6$, square $\cos(\pi/6)$.

33(b). (i) Draw a labelled ray diagram of a compound microscope showing image formation at least distance of distinct vision. Derive an expression for its magnifying power.

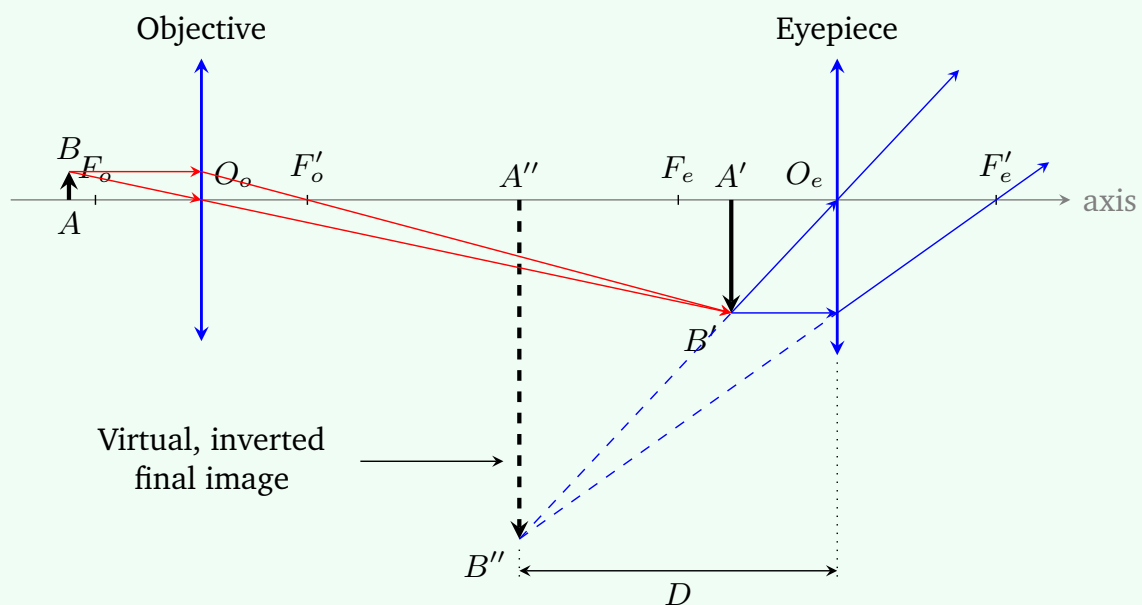
(ii) A telescope consists of two lenses of focal length 100 cm and 5 cm. Find the magnifying power when the final image is formed at infinity. [5 marks]

Solution:

Concept:

- A compound microscope combines the objective's linear magnification with the eyepiece's angular magnification: $M = m_o m_e$. The objective forms a real, inverted intermediate image; the eyepiece acts as a simple microscope.
- For a thin lens, $1/f = 1/v - 1/u$, using Cartesian signs. The least distance of distinct vision is D , conventionally 25 cm; the final image for near-point adjustment has $v_e = -D$.
- For an astronomical telescope in normal adjustment, the objective's focal-plane image lies at the eyepiece's first focus, giving signed angular magnification $M = -f_o/f_e$.

Step 1: (i) Construct the microscope image Place the small object AB just beyond the first focus F_o of the objective. The objective produces a real, inverted and enlarged image $A'B'$. Adjust the eyepiece so that $A'B'$ lies between its first focus F_e and its optical centre O_e . Rays emerging from the eyepiece diverge; their backward extensions meet at B'' . Thus $A''B''$ is virtual, enlarged and inverted relative to AB , at distance D to the left of the eyepiece (the eye is assumed close to it). The diagram shows representative construction rays for each lens; dashed lines are backward extensions.



Compound microscope adjusted for a virtual final image at the near point; schematic, not to scale.

Step 2: (i) Derive the eyepiece magnification Let the magnitude of the intermediate image height be h' and let its distance from the eyepiece be $s = |u_e|$. With $u_e = -s$ and $v_e = -D$, the eyepiece lens equation gives

$$\frac{1}{f_e} = -\frac{1}{D} + \frac{1}{s}, \quad \frac{D}{s} = 1 + \frac{D}{f_e}.$$

For small angles, the intermediate image subtends $\beta \simeq h'/s$ at the eyepiece. Viewed directly at the

near point it would subtend $\alpha' \simeq h'/D$. Therefore

$$m_e = \frac{\beta}{\alpha'} = \frac{D}{s} = 1 + \frac{D}{f_e}.$$

The extra term 1 is present because the final image is at the near point rather than at infinity.

Step 3: (i) Combine the two magnifications For objective object and image distances $u_o < 0$ and $v_o > 0$, similar triangles give $m_o = v_o/u_o$. If the original object height is h , its unaided angular size at the near point is h/D ; multiplying the objective's height ratio by the eyepiece's angular ratio gives

$$M = \frac{v_o}{u_o} \left(1 + \frac{D}{f_e} \right), \quad |M| = \frac{v_o}{|u_o|} \left(1 + \frac{D}{f_e} \right).$$

For a microscope of large magnification, $|u_o| \simeq f_o$ and $v_o \simeq L$, where L denotes the effective tube length in this approximation. Hence the usual board-level expression is $|M| \simeq (L/f_o)(1 + D/f_e)$. The signed magnification is negative because the final image is inverted.

Step 4: (ii) Calculate the telescope's magnifying power In an astronomical telescope the longer-focal-length lens is the objective, so $f_o = 100$ cm and $f_e = 5$ cm. A final image at infinity means normal adjustment. If the common focal-plane image has height h , the incident and emergent small angles have magnitudes h/f_o and h/f_e ; their ratio is therefore f_o/f_e . Substitution gives

$$M_{\text{tel}} = -\frac{f_o}{f_e} = -\frac{100}{5} = -20.$$

Angular magnification has no unit, and the minus sign describes inversion of the astronomical image.

Step 5: Conclusion (i) At the least distance of distinct vision, the microscope has magnifying power $|M| = \frac{v_o}{|u_o|} \left(1 + \frac{D}{f_e} \right) \simeq \frac{L}{f_o} \left(1 + \frac{D}{f_e} \right)$, with an inverted final image. (ii) The telescope's magnifying power is 20, or signed magnification -20 .

Quick Tip to Solve: Use $1 + D/f_e$ for a microscope eyepiece adjusted to the near point; for a telescope focused at infinity, use the objective-to-eyepiece focal-length ratio directly.