

CBSE Class 12 Physics Question Paper 2024 with Solution PDF-Set 55/3/2



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – **Sections A, B, C, D** and **E**.
- (iii) In **Section A** – Questions no. **1 to 16** are Multiple Choice type questions. Each question carries **1** mark.
- (iv) In **Section B** – Questions no. **17 to 21** are Very Short Answer type questions. Each question carries **2** marks.
- (v) In **Section C** – Questions no. **22 to 28** are Short Answer type questions. Each question carries **3** marks.
- (vi) In **Section D** – Questions no. **29 and 30** are case study based questions. Each question carries **4** marks.
- (vii) In **Section E** – Questions no. **31 to 33** are Long Answer type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is not allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. An isolated conductor, with a cavity, has a net charge $+Q$. A point charge $+q$ is inside the cavity. The charges on the cavity wall and the outer surface are respectively: [1 mark]

- (A) 0 and Q
- (B) $-q$ and $Q - q$
- (C) $-q$ and $Q + q$
- (D) 0 and $Q - q$

Correct Answer: (C) $-q$ and $Q + q$

Solution:

Concept:

- In electrostatic equilibrium the electric field inside the material of a conductor is zero.
- Gauss's law gives $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enclosed}}/\epsilon_0$; the total charge belonging to the isolated conductor remains $+Q$.

Step 1: Find the charge induced on the cavity wall Choose a Gaussian surface entirely inside the conducting material, enclosing the cavity. Its electric flux is zero, so the point charge and the cavity-wall charge sum to zero:

$$q + Q_{\text{wall}} = 0 \Rightarrow Q_{\text{wall}} = -q.$$

The point charge lies in the cavity and is not part of the conductor's own net charge.

Step 2: Apply conservation of the conductor's charge The charges on the inner and outer surfaces together equal Q :

$$-q + Q_{\text{outer}} = Q \Rightarrow Q_{\text{outer}} = Q + q.$$

Options (A) and (D) incorrectly leave the cavity wall uncharged. Option (B) has the correct inner charge but violates the specified net charge of the conductor.

Step 3: Conclusion The cavity wall carries $-q$ and the outer surface carries $Q + q$. Hence, the correct option is (C).

Quick Tip to Solve: First cancel the enclosed cavity charge using Gauss's law; then use the conductor's own net charge to find the outer charge.

2. A proton is taken from point P_1 to point P_2 , both located in an electric field. The potentials at points P_1 and P_2 are -5 V and $+5\text{ V}$ respectively. Assuming that kinetic energies of the proton at points P_1 and P_2 are zero, the work done on the proton is: [1 mark]

- (A) $-1.6 \times 10^{-18}\text{ J}$
- (B) $1.6 \times 10^{-18}\text{ J}$
- (C) Zero
- (D) $0.8 \times 10^{-18}\text{ J}$

Correct Answer: (B) $1.6 \times 10^{-18}\text{ J}$

Solution:

Concept:

- The change in electric potential energy is $\Delta U = q(V_2 - V_1)$, where a proton has charge $q = +1.6 \times 10^{-19}\text{ C}$.
- The external work needed to transfer the charge obeys $W_{\text{ext}} = \Delta K + \Delta U$. With zero initial and final kinetic energies, it equals ΔU .

Step 1: Calculate the potential increase Subtract the initial potential with its negative sign retained:

$$\Delta V = +5 - (-5) = 10\text{ V}.$$

A positive charge moved to a higher potential gains electric potential energy, so the required external work is positive.

Step 2: Evaluate the work needed to transfer the proton Taking the requested work as the work done by the external agent,

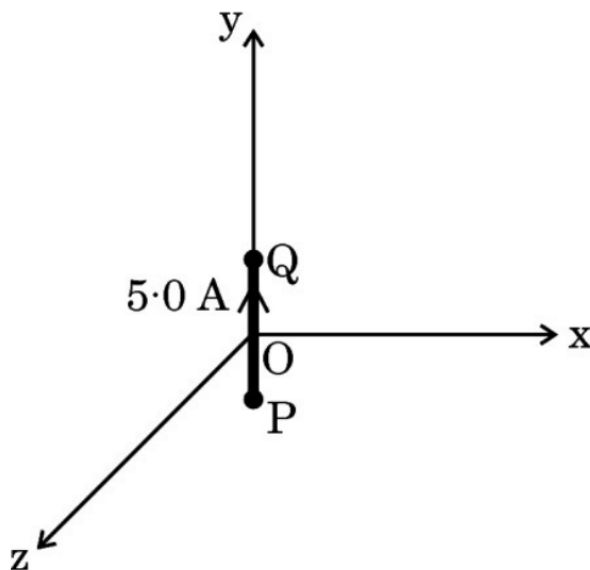
$$W_{\text{ext}} = (1.6 \times 10^{-19})(10) = 1.6 \times 10^{-18}\text{ J}.$$

Option (A) is the work done by the electric field, which has the opposite sign. Option (C) is the total work of all forces, equal to $\Delta K = 0$, rather than the external transfer work. Option (D) incorrectly uses a potential change of only 5 V .

Step 3: Conclusion The external work required is $+1.6 \times 10^{-18}\text{ J}$. Hence, the correct option is (B).

Quick Tip to Solve: For transfer at unchanged kinetic energy use $q(V_2 - V_1)$, and distinguish external work from electric-field work.

3. A 2.0 cm segment of wire, carrying 5.0 A current in positive y-direction lies along y-axis, as shown in the figure. The magnetic field at a point (3 m, 4 m, 0) due to this segment (part of a circuit) is: [1 mark]



- (A) $(0.12 \text{ nT}) \hat{j}$
- (B) $-(0.10 \text{ nT}) \hat{j}$
- (C) $-(0.24 \text{ nT}) \hat{k}$
- (D) $(0.24 \text{ nT}) \hat{k}$

Correct Answer: (C) $-(0.24 \text{ nT}) \hat{k}$

Solution:

Concept:

- For a short current segment, Biot–Savart law gives $\vec{B} = \frac{\mu_0 I}{4\pi} \frac{\Delta \vec{l} \times \vec{r}}{r^3}$, where $\vec{r}$ points from the segment to the observation point.
- The field direction follows the cross product: $\hat{j} \times \hat{i} = -\hat{k}$. Also, $1 \text{ nT} = 10^{-9} \text{ T}$.

Step 1: Determine the distance and the cross product The segment length is $2.0 \text{ cm} = 0.020 \text{ m}$, much smaller than the distance to the point. Treating it as a short element at the origin,

$$\vec{r} = 3\hat{i} + 4\hat{j}, \quad r = \sqrt{3^2 + 4^2} = 5 \text{ m}.$$

Since current flows along positive y ,

$$\Delta \vec{l} \times \vec{r} = 0.020\hat{j} \times (3\hat{i} + 4\hat{j}) = -0.060\hat{k} \text{ m}^2.$$

Step 2: Substitute into Biot–Savart law Using $\mu_0/(4\pi) = 10^{-7} \text{ T m A}^{-1}$,

$$\vec{B} = 10^{-7} \frac{5.0(-0.060)}{5^3} \hat{k} = -2.4 \times 10^{-10} \hat{k} \text{ T}.$$

Options (A) and (B) have the wrong direction: the field must be perpendicular to the xy -plane. Option (D) has the correct magnitude but reverses the cross-product direction.

Step 3: Conclusion The magnetic field is $-(0.24 \text{ nT})\hat{k}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $\Delta \vec{l} \times \vec{r}$ first to fix the sign; here only the x -component of $\vec{r}$ contributes.

4. Two thin long parallel wires separated by a distance ' a ' carry current I in opposite directions. The wires will: [1 mark]

- (A) Repel each other with a force $\frac{\mu_0 I^2}{2\pi a^2}$ per unit length.
(B) Attract each other with a force $\frac{\mu_0 I^2}{2\pi a^2}$ per unit length.
(C) Attract each other with a force $\frac{\mu_0 I^2}{2\pi a}$ per unit length.
(D) Repel each other with a force $\frac{\mu_0 I^2}{2\pi a}$ per unit length.

Correct Answer: (D) Repel each other with a force $\frac{\mu_0 I^2}{2\pi a}$ per unit length.

Solution:

Concept:

- A long straight wire carrying current I produces magnetic field $B = \mu_0 I / (2\pi a)$ at perpendicular distance a .
- A second parallel wire has magnetic force per unit length $F/L = IB$. Parallel currents in opposite directions repel.

Step 1: Calculate the force per unit length Each wire lies in the magnetic field of the other. That field is perpendicular to the current in the second wire, so the magnetic force has magnitude

$$\frac{F}{L} = I \left(\frac{\mu_0 I}{2\pi a} \right) = \frac{\mu_0 I^2}{2\pi a}.$$

The force per unit length therefore varies inversely with the separation, not with its square. This eliminates options (A) and (B).

Step 2: Determine whether the force attracts or repels Apply the right-hand rule for the field of one wire and then $\vec{F} = I\vec{L} \times \vec{B}$ to the other. For opposite current directions, the force on each wire points away from its neighbour. Thus option (C) has the right magnitude but the wrong force direction.

Step 3: Conclusion The wires repel with force $\mu_0 I^2 / (2\pi a)$ per unit length. Hence, the correct option is (D).

Quick Tip to Solve: Check direction and distance dependence separately: opposite parallel currents repel, and F/L varies as $1/a$.

5. A current carrying circular loop of magnetic moment $\vec{M}$ is suspended in a vertical plane in an external magnetic field $\vec{B}$ such that its plane is normal to $\vec{B}$. The work done in rotating this loop by 45° about an axis perpendicular to $\vec{B}$ is closest to: [1 mark]

- (A) $-0.3 MB$
- (B) $0.3 MB$
- (C) $-1.7 MB$
- (D) $1.7 MB$

Correct Answer: (B) $0.3 MB$

Solution:

Concept:

- A current loop has magnetic moment normal to its plane, with direction fixed by the right-hand rule for current.
- Its potential energy is $U = -MB \cos \theta$, where θ is the angle between $\vec{M}$ and $\vec{B}$. External work in a slow rotation is ΔU .

Step 1: Identify the initial and final angles For the usual initially stable orientation of the suspended loop, its magnetic moment is parallel to the field. Thus $\theta_i = 0^\circ$, since the plane of the loop is normal to $\vec{B}$. Rotating through 45° about an axis perpendicular to the field gives $\theta_f = 45^\circ$.

Step 2: Calculate the increase in potential energy The external agent works against the restoring magnetic torque:

$$W = U_f - U_i = MB(\cos 0^\circ - \cos 45^\circ) = MB \left(1 - \frac{1}{\sqrt{2}}\right).$$

The numerical factor is 0.2929, closest to 0.3. Negative work in (A) has the opposite sign for this rotation from stable equilibrium; values in (C) and (D) do not describe a 45° displacement from alignment. The unstated alternative initial anti-parallel orientation would instead give $-0.2929MB$; the standard answer assumes initial stable alignment.

Step 3: Conclusion For an initially stable loop, the required external work is approximately $0.3MB$. Hence, the correct option is (B).

Quick Tip to Solve: Use the angle of the magnetic moment, not the plane: a plane normal to the field gives an aligned moment in stable equilibrium.

6. The mutual inductance of two coils C_1 and C_2 is 20 mH. In coil C_1 , the current changes from 4 A to zero in 0.2 s. If the resistance of coil C_2 is $4\ \Omega$, then the charge that flows through it per second will be: [1 mark]

- (A) 4.0 C
- (B) 1.5 C
- (C) 0.05 C
- (D) 0.1 C

Correct Answer: (D) 0.1 C

Solution:

Concept:

- Mutual induction produces emf of average magnitude $|\mathcal{E}_2| = M|\Delta I_1|/\Delta t$, where M is the mutual inductance.
- With secondary resistance R , average induced current is $I_2 = |\mathcal{E}_2|/R$. Current is charge flow per second, $I_2 = \Delta Q/\Delta t$.

Step 1: Calculate the average induced emf Convert $M = 20\text{ mH} = 0.020\text{ H}$. The magnitude of the primary current change is 4 A over 0.2 s, giving

$$|\mathcal{E}_2| = 0.020 \frac{4}{0.2} = 0.40\text{ V}.$$

Lenz's law sets the current direction; only the magnitude of charge flow is asked here.

Step 2: Distinguish the rate of flow from the total charge The charge flowing per second, averaged over the change interval, is

$$\frac{\Delta Q}{\Delta t} = \frac{0.40}{4} = 0.10\text{ A} = 0.10\text{ C s}^{-1}.$$

The total charge during the actual 0.2 s interval is $0.10 \times 0.2 = 0.020\text{ C}$. The question asks for the per-second rate, so the printed choice 0.1 C is understood as charge per second. None of (A), (B) or (C) gives the calculated rate.

Step 3: Conclusion The average charge-flow rate is 0.10 C s^{-1} , corresponding to the printed 0.1 C per second. Hence, the correct option is (D).

Quick Tip to Solve: For charge per second use $M|\Delta I|/(R\Delta t)$; for total charge over the interval omit Δt from the denominator.

7. Consider a solenoid of length l and area of cross-section A with fixed number of turns. The self-inductance of the solenoid will increase if: [1 mark]

- (A) both l and A are increased
- (B) l is decreased and A is increased
- (C) l is increased and A is decreased
- (D) both l and A are decreased

Correct Answer: (B) l is decreased and A is increased

Solution:

Concept:

- A long solenoid with N turns, length l , cross-sectional area A and core permeability μ has self-inductance $L = \mu N^2 A/l$.
- With the number of turns and core material fixed, $L \propto A/l$: increasing area or decreasing length raises the inductance.

Step 1: Apply the two changes in option (B) Increasing A increases the numerator in $L = \mu N^2 A/l$, while decreasing l decreases its denominator. Both changes therefore act in the same direction and necessarily increase L . The fixed quantity is the total number of turns N , not the turns per unit length N/l .

Step 2: Test the remaining options In (A), both numerator and denominator increase, so the change in A/l depends on their relative fractional increases; an increase in L is not assured. In (C), the numerator decreases and denominator increases, so L must decrease. In (D), both decrease and the result again depends on their relative changes. Only (B) guarantees an increase without additional numerical information.

Step 3: Conclusion The self-inductance increases when l is decreased and A is increased. Hence, the correct option is (B).

Quick Tip to Solve: For fixed total turns, compare A/l , not Al ; choose changes that increase the numerator and decrease the denominator.

8. Electromagnetic waves with frequency 1.0×10^{18} Hz are known as: [1 mark]

- (A) Infrared rays
- (B) Ultraviolet rays
- (C) X-rays
- (D) Gamma rays

Correct Answer: (C) X-rays

Solution:

Concept:

- The electromagnetic spectrum is arranged by frequency or wavelength; the two are related by $c = \nu\lambda$, where c is the speed of light.
- X-rays have wavelengths of approximately 10^{-8} to 10^{-13} m; infrared and ultraviolet radiation have longer wavelengths.

Step 1: Convert the frequency to wavelength With $\nu = 1.0 \times 10^{18}$ Hz, use the speed of electromagnetic

waves in vacuum:

$$\lambda = \frac{3.0 \times 10^8}{1.0 \times 10^{18}} = 3.0 \times 10^{-10} \text{ m} = 0.30 \text{ nm}.$$

Step 2: Locate this wavelength in the spectrum A wavelength of 0.30 nm lies in the X-ray region in the standard school spectrum. Infrared and ultraviolet have lower frequencies, so (A) and (B) do not fit. Gamma rays normally occupy the higher-frequency region in this classification, so (D) is not the intended band.

Step 3: Conclusion The radiation is classified as X-rays. Hence, the correct option is (C).

Quick Tip to Solve: Convert the frequency to wavelength: a fraction of a nanometre points to the X-ray region.

9. A proton and an alpha particle having equal velocities approach a target nucleus. They come momentarily to rest and then reverse their directions. The ratio of the distance of closest approach of the proton to that of the alpha particle will be: [1 mark]

- (A) $\frac{1}{2}$
- (B) 2
- (C) $\frac{1}{4}$
- (D) 4

Correct Answer: (B) 2

Solution:

Concept:

- In a head-on approach to a heavy positive nucleus, initial kinetic energy becomes electrostatic potential energy at the closest distance r .
- Energy conservation gives $\frac{1}{2}mv^2 = kZeq/r$, hence $r = 2kZeq/(mv^2)$, where Ze is the target charge and q is the projectile charge.

Step 1: Compare charge and mass factors The proton has charge e and mass m_p . The alpha particle has charge $2e$ and mass approximately $4m_p$. Since their speeds and the target nucleus are the same, the distance scales as q/m , not simply as charge.

Step 2: Form the requested ratio Cancel the common target and speed factors:

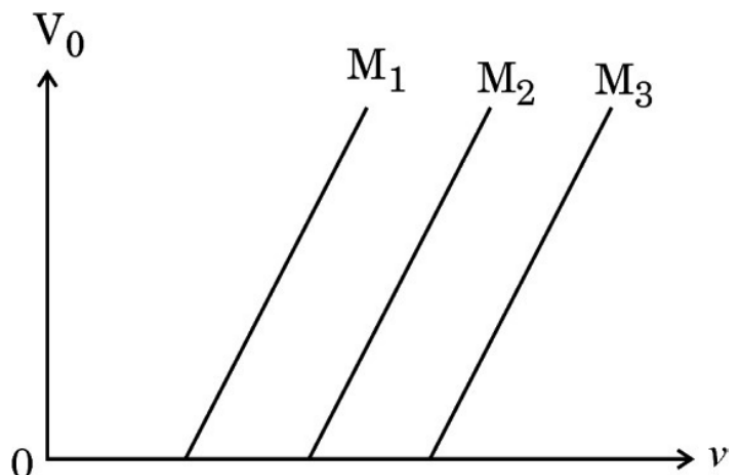
$$\frac{r_p}{r_\alpha} = \frac{e/m_p}{2e/(4m_p)} = 2.$$

The alpha particle has four times the kinetic energy but only twice the charge and can approach more closely. Option (A) reverses the requested ratio; (C) and (D) incorrectly use only the mass ratio.

Step 3: Conclusion The ratio of closest distances, proton to alpha particle, is 2 : 1. Hence, the correct option is (B).

Quick Tip to Solve: For equal speeds use closest distance proportional to charge divided by mass; equal kinetic energies would give a different ratio.

10. The figure shows the variation of stopping potential V_0 with frequency ν of incident radiation, for three materials M_1 , M_2 and M_3 with work functions ϕ_1 , ϕ_2 and ϕ_3 respectively. Then: [1 mark]



- (A) $\phi_1 > \phi_2 > \phi_3$
- (B) $\phi_2 > \phi_3 > \phi_1$
- (C) $\phi_3 > \phi_2 > \phi_1$
- (D) $\phi_2 > \phi_1 > \phi_3$

Correct Answer: (C) $\phi_3 > \phi_2 > \phi_1$

Solution:

Concept:

- Einstein's photoelectric equation is $eV_0 = h\nu - \phi$, where V_0 is stopping potential and ϕ is the work function.
- At the horizontal intercept, $V_0 = 0$, so the threshold frequency is $\nu_0 = \phi/h$. A larger threshold frequency means a larger work function.

Step 1: Read the threshold frequencies The horizontal intercepts of the plotted lines increase from M_1 to M_2 to M_3 . Thus the graph directly gives $\nu_{01} < \nu_{02} < \nu_{03}$. The equal slopes are h/e and do not imply equal work functions.

Step 2: Convert the intercept ordering to work functions Multiplication by the same positive Planck constant preserves the ordering:

$$h\nu_{01} < h\nu_{02} < h\nu_{03} \Rightarrow \phi_1 < \phi_2 < \phi_3.$$

Option (A) reverses the ordering. Options (B) and (D) wrongly put ϕ_2 first, although M_3 has the greatest threshold frequency.

Step 3: Conclusion The work functions satisfy $\phi_3 > \phi_2 > \phi_1$. Hence, the correct option is (C).

Quick Tip to Solve: Read where each stopping-potential line meets the frequency axis: farther right means a larger work function.

11. An electron makes a transition from $n = 2$ level to $n = 1$ level in the Bohr model of a hydrogen atom. Its period of revolution: [1 mark]

- (A) increases by 87.5%
- (B) decreases by 87.5%
- (C) increases by 43.75%
- (D) decreases by 43.75%

Correct Answer: (B) decreases by 87.5%

Solution:

Concept:

- In the Bohr model of hydrogen, the orbital radius varies as $r_n \propto n^2$, while electron speed varies as $v_n \propto 1/n$.
- The revolution period is $T_n = 2\pi r_n/v_n$, so $T_n \propto n^3$. Percentage change is measured relative to the initial period.

Step 1: Compare the initial and final periods The initial orbit has $n = 2$ and the final orbit has $n = 1$. Applying the cubic dependence gives

$$\frac{T_1}{T_2} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

Thus the final period is smaller than the original period, which eliminates the increase options (A) and (C).

Step 2: Calculate the percentage decrease Divide the reduction by the initial period, not the final one:

$$\frac{T_2 - T_1}{T_2} \times 100\% = \left(1 - \frac{1}{8}\right) 100\% = 87.5\%.$$

The 43.75% decrease in (D) is inconsistent with the cubic period ratio; it is not the fraction lost in this transition.

Step 3: Conclusion The electron's period of revolution decreases by 87.5%. Hence, the correct option is (B).

Quick Tip to Solve: Remember the Bohr scalings: radius goes as the square of the orbit number, speed as its inverse, and period as its cube.

12. Si is doped with a pentavalent element. The energy required to set the additional electron free is about:

[1 mark]

- (A) 0.01 eV
- (B) 0.05 eV
- (C) 0.72 eV
- (D) 1.1 eV

Correct Answer: (B) 0.05 eV

Solution:

Concept:

- A pentavalent impurity in silicon forms four covalent bonds and leaves one additional, weakly bound electron; it therefore acts as a donor.
- The donor level lies slightly below the conduction band. The energy needed to release the extra electron is the small donor ionisation energy, approximately 0.05 eV for silicon.

Step 1: Identify the transition that frees the electron The fifth electron does not need to be raised from the valence band across the entire silicon band gap. It moves from the donor level into the nearby conduction band and then becomes available for conduction.

Step 2: Distinguish donor energy from band-gap energy Silicon requires about 0.05 eV to free a donor electron. The value 0.01 eV is the familiar donor-energy scale for germanium. The values 0.72 eV and 1.1 eV are of the order of germanium and silicon band gaps, respectively, and are too large for this shallow donor transition.

Step 3: Conclusion The required energy is approximately 0.05 eV. Hence, the correct option is (B).

Quick Tip to Solve: For a doped semiconductor, first check whether the question frees a donor electron or promotes a valence electron across the full band gap.

13. Assertion (A): In a semiconductor, the electrons in the conduction band have lesser energy than those in the valence band.

Reason (R): Donor energy level is just above the valence band in a semiconductor.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and Reason (R) is also false.

Solution:

Concept:

- The conduction band lies above the valence band in energy, separated by the forbidden energy gap of the semiconductor.
- Donor states lie just below the conduction band, whereas acceptor states lie just above the valence band.

Step 1: Evaluate the assertion An electron must gain energy to move from the valence band to the conduction band. Therefore conduction-band electrons have higher, not lesser, energy than valence-band electrons. The assertion is false.

Step 2: Evaluate the reason A donor easily supplies an electron to the conduction band because its level is just below that band. The location just above the valence band belongs to an acceptor level. The reason confuses the two types of impurity levels and is false.

Step 3: Check the explanation and printed codes The false reason cannot explain the false assertion. The printed code (D) specifically represents both statements being false. Codes (A) and (B) require both statements true, while (C) requires the assertion true; none matches this evaluation.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: Place donors near the conduction band and acceptors near the valence band before judging an energy-level statement.

14. Assertion (A): Photoelectric effect demonstrates the particle nature of light.

Reason (R): Photoelectric current is proportional to frequency of incident radiation.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- In the photon model, light transfers energy in discrete packets $h\nu$. Photoemission requires $h\nu \geq \phi$, where ϕ is the metal's work function.
- Photoelectric current measures the number of collected electrons per second. At a fixed frequency above threshold and fixed collection conditions, current is proportional to incident intensity.

Step 1: Evaluate the assertion The threshold frequency and the relation $K_{\max} = h\nu - \phi$ are explained by one photon transferring its energy to one electron. The photoelectric effect therefore demonstrates the particle nature of light, so the assertion is true.

Step 2: Evaluate the reason Increasing frequency chiefly increases the maximum kinetic energy of emitted electrons and the stopping potential. It does not make photoelectric current proportional to frequency. The number of emitted and collected electrons depends on the photon flux and collection conditions; the stated reason is false.

Step 3: Decide whether the reason explains the assertion A false current-frequency claim cannot explain the particle character demonstrated by photoemission. Codes (A) and (B) both require a true

reason, and code (D) requires a false assertion. The printed code for a true assertion with a false reason is (C).

Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Keep the two photoelectric controls separate: frequency controls maximum electron energy, while intensity controls current at fixed frequency.

15. Assertion (A): A proton and an electron enter a uniform magnetic field $\vec{B}$ with the same momentum $\vec{p}$ such that $\vec{p}$ is perpendicular to $\vec{B}$. They describe circular paths of the same radius.

Reason (R): In a magnetic field, orbital radius r is equal to $\frac{p}{qB}$. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- A magnetic field perpendicular to the velocity provides the centripetal force: $|q|vB = mv^2/r$.
- The radius is $r = p/(|q|B)$, where p is momentum magnitude and $|q|$ is charge magnitude.

Step 1: Evaluate the assertion A proton and an electron have equal charge magnitudes e . Their momentum magnitudes and the magnetic field strength are also equal. Thus $r_p = p/(eB) = r_e$, so Assertion (A) is true. Opposite signs of charge reverse the sense of curvature, not its radius.

Step 2: Evaluate the reason Dividing $|q|vB = mv^2/r$ by v gives $r = mv/(|q|B) = p/(|q|B)$. In the printed radius formula, q denotes the magnitude of the charge. Reason (R) is therefore true.

Step 3: Test the explanatory link Substituting the same p , B , and charge magnitude into the reason gives equal radii directly. Unequal particle masses do not alter this result at equal momentum. Consequently (B) incorrectly denies the explanation, while (C) and (D) incorrectly reject true statements.

Step 4: Conclusion Both A and R are true, and R correctly explains A. Hence, the correct option is (A).

Quick Tip to Solve: At equal momentum compare charge magnitudes in $r = p/(|q|B)$; the charge sign controls the bending direction only.

16. Assertion (A): A convex lens, when immersed in a liquid, disappears.

Reason (R): The refractive indices of material of the lens and the liquid are equal.

[1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- Refraction depends on the relative refractive index $n_{\text{rel}} = n_{\text{lens}}/n_{\text{liquid}}$, not the lens index alone.
- For a thin lens in a liquid, $1/f = (n_{\text{rel}} - 1)(1/R_1 - 1/R_2)$, where R_1, R_2 are the signed surface radii.

Step 1: Evaluate the assertion In the stated situation of a lens disappearing in its surrounding liquid, the lens and liquid are optically indistinguishable: rays do not deviate at the lens boundaries. Assertion (A) describes this index-matched situation and is true; disappearance is not a property of every arbitrary liquid.

Step 2: Evaluate the reason The given equality $n_{\text{lens}} = n_{\text{liquid}}$ makes the relative refractive index unity. Snell's law then gives equal angles of incidence and refraction at each surface. Thus Reason (R) is true for the stated situation.

Step 3: Test the explanatory link With $n_{\text{rel}} = 1$, the lens power becomes zero and its focal length becomes infinite. The absence of optical contrast at an ideal transparent, index-matched boundary explains the disappearance. Option (B) wrongly denies this link, and (C) and (D) wrongly declare a statement false.

Step 4: Conclusion Both A and R are true, and equality of refractive indices explains the disappearance. Hence, the correct option is (A).

Quick Tip to Solve: For a lens immersed in a medium, use the relative index; equal indices mean zero lens power and no refraction at the boundary.

Section B (10 marks)

17(a). What is meant by 'relaxation time' of free electrons in a conductor? Show that the resistance of a conductor can be expressed by $R = \frac{ml}{ne^2\tau A}$, where symbols have their usual meanings. [2 marks]

Solution:

Concept:

- Relaxation time τ is the average time interval between two successive collisions of a conduction electron with the lattice ions.
- The drift-speed magnitude is $v_d = eE\tau/m$, and current is $I = neAv_d$. Here n is the number of free electrons per unit volume, e their charge magnitude and m their mass.

Step 1: Relate drift speed to the potential difference Let the uniform conductor have length l , cross-sectional area A , and a potential difference V between its ends. The electric field magnitude is $E = V/l$. An electron's acceleration magnitude between collisions is eE/m , giving the average drift speed

$$v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml}.$$

Electrons drift opposite to the field; conventional current is along the field, so the current relation below uses positive magnitudes.

Step 2: Obtain current and resistance In unit time, the charge crossing any cross-section is $neAv_d$. Therefore

$$I = neA \left(\frac{eV\tau}{ml} \right) = \frac{ne^2 A \tau}{ml} V.$$

Using the definition $R = V/I$, divide the applied potential difference by this current. The factor V cancels because the conductor is assumed ohmic at fixed temperature.

Step 3: Conclusion Relaxation time is the mean interval between successive electron-lattice collisions, and the conductor's resistance is

$$R = \frac{ml}{ne^2 \tau A}.$$

Quick Tip to Solve: Start with $I = neAv_d$, substitute $v_d = eV\tau/(ml)$, then invert I/V to obtain resistance.

OR

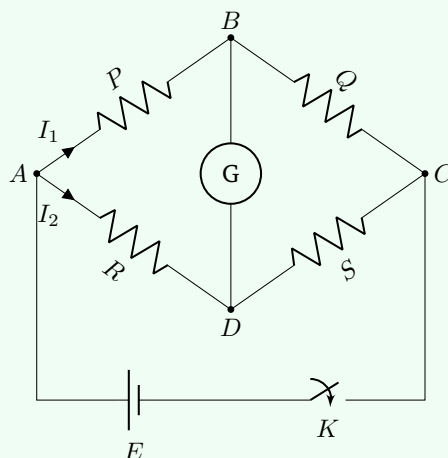
17(b). Draw the circuit diagram of a Wheatstone bridge. Obtain the condition when no current flows through the galvanometer in it. [2 marks]

Solution:

Concept:

- A Wheatstone bridge has four resistances in its arms, a galvanometer across one diagonal and a cell across the other diagonal.
- At balance the galvanometer current $I_G = 0$, so its two terminals have equal potentials. Ohm's law then relates the voltage drops in the two branches.

Step 1: Draw and identify the bridge circuit Let the arms AB, BC, AD, DC have resistances P, Q, R, S , respectively. Connect the galvanometer G between B and D , and a cell with key K between A and C . With the cell positive terminal at A , let branch currents be I_1 along ABC and I_2 along ADC .



Wheatstone bridge: galvanometer across BD, cell and key across AC.

Step 2: Equate the potential drops at balance When no current flows through G , $V_B = V_D$. Since the galvanometer draws no current, P, Q carry the same current I_1 , and R, S carry the same current I_2 . Comparing potential drops from A to the equal-potential junctions gives

$$V_A - V_B = V_A - V_D \Rightarrow I_1 P = I_2 R.$$

Comparing the drops from those junctions to C gives

$$V_B - V_C = V_D - V_C \Rightarrow I_1 Q = I_2 S.$$

Divide the first current equation by the second to eliminate the unknown currents.

Step 3: Conclusion The Wheatstone bridge is balanced, with no galvanometer current, when

$$\boxed{\frac{P}{Q} = \frac{R}{S}} \quad \text{or} \quad PS = QR.$$

Quick Tip to Solve: Label the four arms before forming ratios: compare the two resistances along one cell-to-cell branch with those along the other.

18. The magnifying power of an astronomical telescope is 24. In normal adjustment, distance between its two lenses is 150 cm. Find the focal length of the objective lens. [2 marks]

Solution:

Concept:

- For an astronomical telescope in normal adjustment, the final image is at infinity and the lens separation is $L = f_o + f_e$, where f_o and f_e are the positive focal lengths of the objective and eyepiece.
- Its angular magnifying power has magnitude $|M| = f_o/f_e$. The signed magnification is negative because the image is inverted.

Step 1: Use the magnification ratio The given magnifying power 24 is its magnitude. Hence

$$\frac{f_o}{f_e} = 24 \Rightarrow f_o = 24f_e.$$

The objective therefore has a focal length twenty-four times that of the eyepiece. Both lenses are converging, so their focal lengths enter the normal-adjustment separation as a sum.

Step 2: Apply the lens separation Substituting the focal-length ratio into the given separation 150 cm,

$$\begin{aligned}f_o + f_e &= 150 \text{ cm}, \\24f_e + f_e &= 150 \text{ cm}, \\f_e &= \frac{150}{25} \text{ cm} = 6 \text{ cm}, \\f_o &= 24 \times 6 \text{ cm} = 144 \text{ cm}.\end{aligned}$$

As a check, these values add to 150 cm and their ratio is $144/6 = 24$, satisfying both conditions independently.

Step 3: Conclusion The focal length of the objective lens is $f_o = 144 \text{ cm} = 1.44 \text{ m}$.

Quick Tip to Solve: In normal adjustment use $f_o = |M|L/(|M| + 1)$; here the objective takes 24/25 of the total lens separation.

19. Write two points of difference between interference and diffraction of light.

[2 marks]

Solution:

Concept:

- Interference and diffraction both arise from the superposition of coherent light waves, producing intensity maxima and minima.
- In Young's double-slit interference, two coherent sources are involved; in single-slit diffraction, contributions from different portions of the same wavefront interfere.

Step 1: Compare the origin of the two patterns Interference is the redistribution of intensity due to superposition of light from two coherent sources, such as the two slits in Young's experiment. Diffraction is the spreading of light at an aperture or obstacle, with the observed pattern produced by superposition of secondary wavelets from different parts of the same incident wavefront.

Step 2: Compare the widths of the fringes In the usual Young's double-slit arrangement, the bright and dark interference fringes have equal width, given by $\beta = \lambda D/d$, where d is the slit separation and D is the screen distance. In single-slit diffraction, the central bright maximum is approximately twice as wide as a secondary bright maximum. Thus the diffraction pattern does not have a sequence of identical-width bright fringes.

Step 3: Conclusion The two differences are: interference combines waves from two coherent sources and has equal-width fringes in Young's experiment; single-slit diffraction combines waves from different parts of one wavefront and has a central maximum twice as wide as the secondary maxima.

Quick Tip to Solve: Compare the standard double-slit and single-slit patterns: identify the contributing sources first, then compare the central fringe width with the others.

20. Light of wavelength 500 nm is incident on caesium metal (work function 2.14 eV) and photoemission of electrons occurs. Calculate the (i) kinetic energy (in eV) of the fastest electrons and (ii) stopping potential for this situation. (Take $hc = 1240 \text{ eV}\cdot\text{nm}$) [2 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = hc/\lambda - \phi$, where ϕ is the work function and hc/λ is the energy of one incident photon.
- The stopping potential magnitude V_0 satisfies $eV_0 = K_{\max}$, where e is the magnitude of the electron charge. An energy in electron-volts has the same numerical value as its stopping potential in volts.

Step 1: Calculate the incident photon energy Use the given value of hc directly with wavelength in nanometres, so that the energy is obtained in electron-volts:

$$E_{\gamma} = \frac{1240 \text{ eV} \cdot \text{nm}}{500 \text{ nm}} = 2.48 \text{ eV}.$$

This exceeds the work function 2.14 eV, consistent with the stated photoemission.

Step 2: Find the maximum kinetic energy and stopping potential The fastest electron uses energy equal to the work function to escape and retains the remaining photon energy as kinetic energy:

$$K_{\max} = 2.48 - 2.14 = 0.34 \text{ eV}.$$

A retarding potential must remove this entire kinetic energy to stop even the fastest electron. Since $1 \text{ eV} = e \times 1 \text{ V}$,

$$V_0 = \frac{K_{\max}}{e} = 0.34 \text{ V}.$$

Step 3: Conclusion (i) The maximum kinetic energy is 0.34 eV . (ii) The stopping potential has magnitude 0.34 V , with the collector negative relative to the emitting surface.

Quick Tip to Solve: With $hc = 1240 \text{ eV} \cdot \text{nm}$, divide by wavelength in nm and subtract the work function; the resulting number also gives the stopping potential in volts.

21. Suppose a pure Si crystal has $5 \times 10^{28} \text{ atoms m}^{-3}$. It is doped by 1 ppm concentration of boron. Calculate the concentration of holes and electrons, given that $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$. Is the doped crystal n -type or p -type? [2 marks]

Solution:

Concept:

- Boron is a trivalent acceptor impurity in silicon: each ionised acceptor introduces one hole. For complete ionisation and dominant acceptor doping, the hole density is $p \simeq N_A$.
- At thermal equilibrium, the mass-action law gives $np = n_i^2$, where n and p are electron and hole concentrations and n_i is the intrinsic carrier concentration at the same temperature.

Step 1: Convert the doping fraction into a hole concentration A concentration of one part per million means one boron atom per 10^6 silicon atoms. Therefore the acceptor density is

$$N_A = (5 \times 10^{28})(10^{-6}) = 5 \times 10^{22} \text{ m}^{-3}.$$

Under the usual complete-ionisation assumption, each acceptor supplies a hole. Since $N_A \gg n_i$, intrinsic carriers make a negligible correction and $p \simeq 5 \times 10^{22} \text{ m}^{-3}$.

Step 2: Apply the mass-action law and identify the semiconductor Use the increased hole concentration to find the reduced minority electron concentration:

$$n = \frac{n_i^2}{p} = \frac{(1.5 \times 10^{16})^2}{5 \times 10^{22}} = 4.5 \times 10^9 \text{ m}^{-3}.$$

Boron accepts an electron in completing its bonding, leaving a mobile hole. Holes greatly outnumber electrons, so they are the majority charge carriers and the semiconductor is p -type.

Step 3: Conclusion The hole concentration is $p \simeq 5 \times 10^{22} \text{ m}^{-3}$, the electron concentration is $n \simeq 4.5 \times 10^9 \text{ m}^{-3}$, and the doped silicon is p -type.

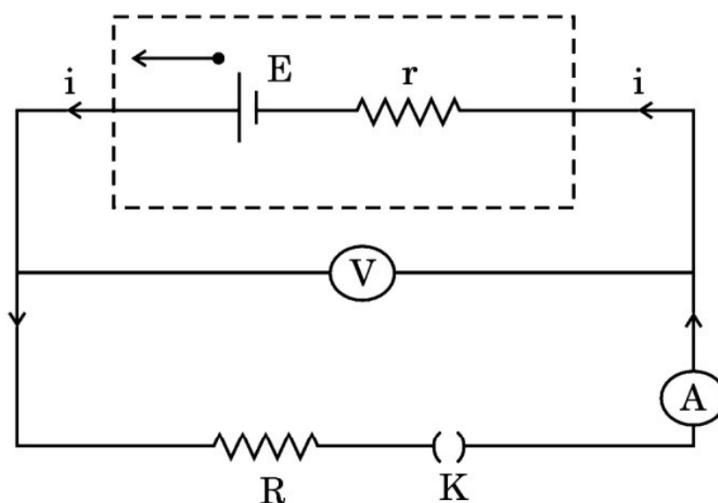
Quick Tip to Solve: Convert ppm to 10^{-6} first. For boron-doped silicon, use the dopant density for holes and obtain electrons from n_i^2/p .

Section C (21 marks)

A battery of unknown emf E and internal resistance r is connected in a circuit as shown in the figure. When the key (K) is open, the voltmeter reads 10.0 V and ammeter reads 0 A . In the closed circuit, the voltmeter reads 6.0 V and ammeter reads 2.0 A . Calculate:

22(a). emf of the battery,

[3 marks total]



Solution:

Concept:

- The emf E is the energy supplied by the battery per unit charge and equals its terminal potential difference when no current is drawn.
- For a discharging battery, $V = E - Ir$, where V is terminal voltage, I is current and r is internal resistance.

Step 1: Select the open-circuit reading When key K is open, the ammeter reads 0 A . Thus there is no voltage drop Ir across the battery's internal resistance. The voltmeter is connected across the battery terminals, so its open-circuit reading directly measures the emf.

Step 2: Apply the terminal-voltage relation Substitute the open-circuit values into the battery equation:

$$V_{\text{open}} = E - 0 \times r = E.$$

The printed open-circuit voltmeter reading is 10.0 V. The smaller reading of 6.0 V is obtained only when current flows and some voltage is lost internally.

Step 3: Conclusion The emf of the battery is $E = 10.0 \text{ V}$.

Quick Tip to Solve: Use the voltmeter reading with the key open for emf; the closed-circuit reading is reduced by the internal voltage drop.

22(b). internal resistance of the battery (r), and

Solution:

Concept:

- When a battery supplies current I , its terminal voltage is $V = E - Ir$, where E is the emf and r is its internal resistance.
- The lost voltage inside the battery is $E - V = Ir$. The emf is given by the open-circuit terminal voltage because the current is then zero.

Step 1: Find the internal voltage drop The open-circuit reading gives $E = 10.0 \text{ V}$. On closing the key, the battery supplies $I = 2.0 \text{ A}$ and the terminal voltage falls to $V = 6.0 \text{ V}$. The difference is the potential drop across the internal resistance:

$$Ir = E - V = 10.0 - 6.0 = 4.0 \text{ V}.$$

Step 2: Divide the lost voltage by the current Rearrange the battery equation and use the closed-circuit current:

$$r = \frac{E - V}{I} = \frac{4.0 \text{ V}}{2.0 \text{ A}} = 2.0 \Omega.$$

This is positive, as expected for an internal resistance that reduces the terminal voltage while the battery delivers current.

Step 3: Conclusion The internal resistance of the battery is $r = 2.0 \Omega$.

Quick Tip to Solve: Subtract the loaded voltage from the open-circuit voltage before dividing by current; this isolates the internal resistance.

22(c). external resistance (R).

Solution:

Concept:

- Ohm's law for the external resistor is $V_R = IR$, where V_R is the potential difference across resistance R and I is its current.
- With the key closed and an ideal ammeter, the battery's terminal voltage appears across the external resistance. The emf includes the additional internal voltage drop and must not be used as V_R .

Step 1: Identify the voltage across the external resistor In the closed circuit, the voltmeter reads

6.0 V across the battery terminals and hence across the external branch. The key and ammeter have negligible resistance, so the entire terminal voltage appears across R . The ammeter reading gives the resistor current as 2.0 A.

Step 2: Apply Ohm's law to the external branch Divide the loaded terminal voltage by the current through the resistor:

$$R = \frac{V_R}{I} = \frac{6.0 \text{ V}}{2.0 \text{ A}} = 3.0 \Omega.$$

Using 10.0 V instead would give the total resistance $R + r$, because that is the battery emf rather than its loaded terminal voltage.

Step 3: Conclusion The external resistance is $R = 3.0 \Omega$.

Quick Tip to Solve: For the external resistor, divide the closed-circuit voltmeter reading by the simultaneous ammeter reading.

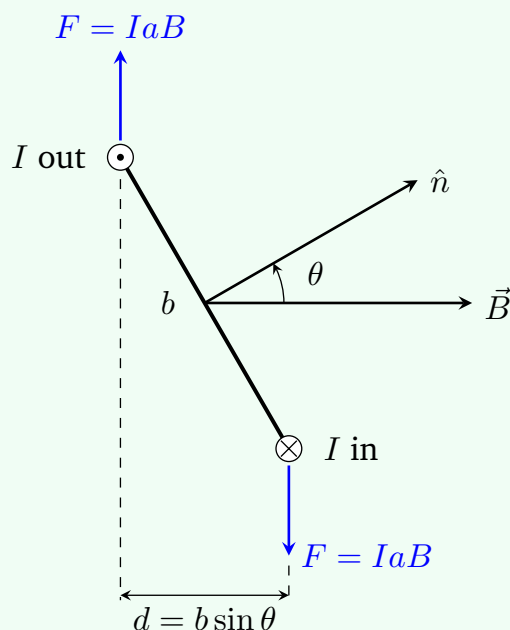
23. A rectangular loop of area $\vec{A}$, carrying current I , is placed in a uniform magnetic field $\vec{B}$. With the help of a suitable diagram, derive an expression, in vector form, for the torque acting on the loop. [3 marks]

Solution:

Concept:

- A straight side carrying current I in a uniform field experiences $\vec{F} = I\vec{\ell} \times \vec{B}$, where $\vec{\ell}$ points along the current.
- The area vector is $\vec{A} = A\hat{n}$, with $\hat{n}$ given by the right-hand rule for the loop current. The magnetic moment is $\vec{m} = I\vec{A}$.
- Two equal, opposite, parallel forces form a couple of torque magnitude Fd , where d is the perpendicular distance between their lines of action.

Step 1: Choose the loop orientation Let the rectangular loop have sides a and b , so its area is $A = ab$. Let θ be the angle between the area normal $\hat{n}$ and the uniform magnetic field. Choose the two sides of length a perpendicular to the field. The diagram shows the loop edge-on: these sides run perpendicular to the page and are separated by the side of length b . A dot denotes current towards us and a cross denotes current away from us.



Edge-on view: the two sides of length a are perpendicular to the page; their force separation is $b \sin \theta$.

Step 2: Find the force couple and its lever arm On each length- a side, the current is perpendicular to $\vec{B}$, hence the force magnitude is $F = IaB$. The currents are opposite, so these forces are equal and opposite. Their lines of action are separated by $d = b \sin \theta$, since the loop plane makes angle $90^\circ - \theta$ with the field. The forces on the other two sides are equal, opposite and collinear; they contribute no net torque. Thus the resultant force on the loop is zero, but the first pair produces a turning effect:

$$\tau = Fd = (IaB)(b \sin \theta) = IAB \sin \theta.$$

Step 3: Determine the vector direction Applying $\vec{F} = I\vec{\ell} \times \vec{B}$ to the two active sides shows that the couple turns the loop so that its area normal approaches the field direction. Its torque is perpendicular to both $\vec{A}$ and $\vec{B}$, in the right-hand-rule direction of $\vec{A} \times \vec{B}$. In the orientation drawn, this is into the page. Therefore the vector expression combines this direction with the magnitude just derived.

Step 4: Conclusion The torque on a single rectangular current loop is

$$\boxed{\vec{\tau} = I\vec{A} \times \vec{B} = \vec{m} \times \vec{B}}, \quad \tau = IAB \sin \theta.$$

Here θ is the angle between the area vector and the magnetic field; torque is measured in N m.

Quick Tip to Solve: Measure the angle from the area normal, not from the loop plane. The torque is maximum when the field lies in the plane of the loop.

24. Distinguish between reactance and impedance of an ac circuit. Show that an ideal inductor in an ac circuit does not dissipate any power. [3 marks]

Solution:

Concept:

- Reactance is the opposition to alternating current due to inductance or capacitance: $X_L = \omega L$ and $X_C = 1/(\omega C)$, where ω is angular frequency.

- Impedance combines resistance and net reactance. For a series RLC circuit its magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2} = V_{\text{rms}}/I_{\text{rms}}$. Both reactance and impedance have the unit ohm.
- Mean ac power is $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}} \cos \phi$, where ϕ is the phase difference between voltage and current.

Step 1: Distinguish reactance from impedance Reactance describes opposition arising from an inductor or capacitor storing and returning energy. An ideal reactive element does not convert electrical energy into heat. Inductive reactance increases with frequency, whereas capacitive reactance decreases with frequency. In a series circuit the signed net reactance is $X = X_L - X_C$. Impedance is the total opposition to ac, accounting for both resistance and reactance, together with their phase relation. Resistance produces irreversible energy dissipation, whereas reactance causes a phase shift. Thus impedance is not the arithmetic sum $R + X$; its magnitude is $\sqrt{R^2 + X^2}$. In a purely resistive circuit $Z = R$, and in a pure inductor $Z = X_L$.

Step 2: Obtain the phase relation for an ideal inductor An ideal inductor has zero resistance. Apply a sinusoidal voltage $v = V_m \sin \omega t$. Since $v = L di/dt$, integration gives the sinusoidal steady-state current

$$i = -\frac{V_m}{\omega L} \cos \omega t = I_m \sin \left(\omega t - \frac{\pi}{2} \right), \quad I_m = \frac{V_m}{\omega L}.$$

Therefore current lags voltage by $\pi/2$, so the power factor is $\cos \phi = 0$.

Step 3: Average the power over a complete cycle The instantaneous power is not always zero:

$$p = vi = -V_m I_m \sin \omega t \cos \omega t = -\frac{V_m I_m}{2} \sin 2\omega t.$$

Its positive and negative portions cancel over a complete period. Equivalently,

$$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \frac{\pi}{2} = 0.$$

Energy is stored in the magnetic field as $U = \frac{1}{2} Li^2$ while current magnitude increases and is returned to the source while it decreases. No energy is irreversibly dissipated.

Step 4: Conclusion Reactance represents inductive or capacitive opposition; impedance represents the combined effect of resistance and reactance. An ideal inductor has a phase difference of 90° and zero average power consumption: $P_{\text{av}} = 0$.

Quick Tip to Solve: For an ideal inductor use a 90° phase lag, then apply $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi$. Zero average power does not mean zero instantaneous power.

25. The electric field in an electromagnetic wave in vacuum is given by:

$$\vec{E} = (6.3 \text{ N/C})[\cos((1.5 \text{ rad/m})y + (4.5 \times 10^8 \text{ rad/s})t)]\hat{i}$$

(a) Find the wavelength and frequency of the wave.

(b) What is the amplitude of the magnetic field of the wave?

(c) Write an expression for the magnetic field of this wave.

[3 marks]

Solution:

Concept:

- For a harmonic wave the wave number is $k = 2\pi/\lambda$ and angular frequency is $\omega = 2\pi f$, where λ is wavelength and f is frequency.

- In vacuum, $E_0 = cB_0$ and $c = \omega/k$. The electric and magnetic fields oscillate in phase and are perpendicular to each other and to the propagation direction.
- The propagation direction is that of $\vec{E} \times \vec{B}$. A phase of the form $ky + \omega t$ travels along the negative y -axis.

Step 1: Read the wave parameters and solve part (a) Comparing the given expression with $\vec{E} = E_0 \cos(ky + \omega t)\hat{i}$, identify

$$E_0 = 6.3 \text{ N C}^{-1}, \quad k = 1.5 \text{ rad m}^{-1}, \quad \omega = 4.5 \times 10^8 \text{ rad s}^{-1}.$$

The coefficient of position is wave number, not wavelength; the coefficient of time is angular frequency, not ordinary frequency. Therefore

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.5} \text{ m} \simeq 4.19 \text{ m},$$

$$f = \frac{\omega}{2\pi} = \frac{4.5 \times 10^8}{2\pi} \text{ Hz} \simeq 7.16 \times 10^7 \text{ Hz}.$$

As a check, $\omega/k = 3.0 \times 10^8 \text{ m s}^{-1}$, the speed of light in vacuum.

Step 2: Calculate the magnetic amplitude for part (b) Use the vacuum relation between the field amplitudes:

$$B_0 = \frac{E_0}{c} = \frac{6.3}{3.0 \times 10^8} \text{ T} = 2.1 \times 10^{-8} \text{ T}.$$

This is a positive amplitude; the direction of the magnetic-field vector must be determined separately from the wave's direction of travel.

Step 3: Determine the magnetic vector for part (c) Follow a point of fixed phase: $ky + \omega t = \text{constant}$ gives $dy/dt = -\omega/k$. Thus the wave travels along $-\hat{j}$. Since the electric field is along $\hat{i}$, the magnetic field must have its positive amplitude along $+\hat{k}$, because $\hat{i} \times \hat{k} = -\hat{j}$. Choosing $-\hat{k}$ would incorrectly reverse the propagation direction. Both fields have the same cosine phase, so the magnetic field uses the given phase without a phase shift.

Step 4: Conclusion (a) $\lambda \simeq 4.19 \text{ m}$ and $f \simeq 7.16 \times 10^7 \text{ Hz}$. (b) $B_0 = 2.1 \times 10^{-8} \text{ T}$. (c) The magnetic field is

$$\vec{B} = (2.1 \times 10^{-8} \text{ T}) \cos[(1.5 \text{ rad/m})y + (4.5 \times 10^8 \text{ rad/s})t] \hat{k}.$$

Quick Tip to Solve: First read the propagation direction from the sign in the phase; then choose the magnetic direction so that $\vec{E} \times \vec{B}$ points that way.

26. Explain the origin of the spectral lines of hydrogen atom using Bohr's theory. Draw the energy level diagram of hydrogen atom showing its various spectral series. [3 marks]

Solution:

Concept:

- Bohr's stationary orbits have discrete energies $E_n = -13.6/n^2 \text{ eV}$, where $n = 1, 2, 3, \dots$ is the principal quantum number. An electron in a stationary orbit does not radiate energy.
- A transition from an initial level n_i to a lower final level n_f emits a photon of energy $h\nu = E_{n_i} - E_{n_f}$, where h is Planck's constant and ν is the photon frequency.

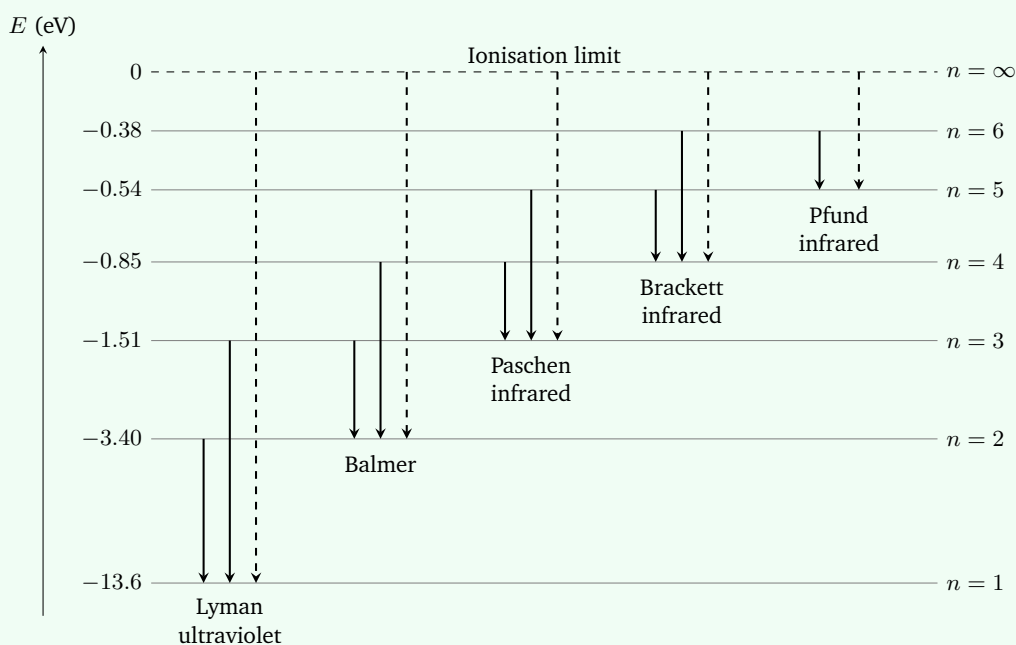
Step 1: Explain the origin of discrete lines When hydrogen is excited, its electron absorbs energy and moves from a lower stationary level to a higher one. On returning to a lower level it emits the energy difference as a photon. Since only specified energy levels are permitted, only specified differences and therefore discrete frequencies occur. These produce separate spectral lines instead of a continuous spectrum. Upward transitions absorb photons of the same corresponding frequencies.

Step 2: Obtain the wavelength relation For an emission transition, $n_i > n_f$. Substituting Bohr's energy expression gives

$$\frac{hc}{\lambda} = 13.6 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \text{ eV}, \quad \frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right),$$

where c is the speed of light, λ is the wavelength, and R_H is the hydrogen Rydberg constant. Transitions ending at the same lower level form a spectral series. Increasing n_i makes successive lines approach the series limit at $n_i \rightarrow \infty$.

Step 3: Identify the series on the energy level diagram The Lyman series ends at $n_f = 1$ and lies in the ultraviolet region. The Balmer series ends at $n_f = 2$ and includes the visible hydrogen lines. Paschen, Brackett and Pfund series end at $n_f = 3, 4, 5$, respectively, and lie in the infrared region. The downward arrows below show representative emissions; dashed arrows indicate series limits. The energy levels approach $E = 0$ as $n \rightarrow \infty$.



Hydrogen energy levels and representative spectral series (vertical spacing schematic).

Step 4: Conclusion Hydrogen's line spectrum arises from transitions between quantised Bohr energy levels. Each fixed final level defines a series: Lyman (1), Balmer (2), Paschen (3), Brackett (4) and Pfund (5).

Quick Tip to Solve: Identify a hydrogen spectral series from the final level, not the starting level: final levels 1 to 5 give Lyman, Balmer, Paschen, Brackett and Pfund in order.

27. (a) Define atomic mass unit (u).

(b) Calculate the energy required to separate a deuteron into its constituent parts (a proton and a neutron). Given:

$$m(D) = 2.014102 \text{ u}$$

$$m_H = 1.007825 \text{ u}$$

$$m_n = 1.008665 \text{ u.}$$

[3 marks]

Solution:

Concept:

- One atomic mass unit, 1 u, is one twelfth of the mass of a free, neutral carbon-12 atom at rest in its ground state; $1 \text{ u} \simeq 1.6605 \times 10^{-27} \text{ kg}$.
- Nuclear binding energy is the energy required to separate a nucleus completely into its constituent nucleons. It equals $B = \Delta mc^2$, where Δm is the mass defect and c is the speed of light; $1 \text{ u } c^2 \simeq 931.5 \text{ MeV}$.

Step 1: (a) State the atomic mass unit definition The carbon-12 standard defines a common unit for atomic and nuclear masses:

$$1 \text{ u} = \frac{1}{12} m(\text{one neutral carbon-12 atom}).$$

Thus one carbon-12 atom has mass exactly 12 u. The unit is a mass unit; multiplying a mass defect expressed in this unit by 931.5 MeV/u gives its energy equivalent.

Step 2: (b) Determine the mass defect consistently A deuteron contains one proton and one neutron. The supplied $m_H = 1.007825 \text{ u}$ is the mass of a neutral hydrogen atom, while $m(D) = 2.014102 \text{ u}$ is the supplied deuterium atomic mass. Each atom contains one electron, so the electron rest masses cancel on subtraction. Atomic electron binding energies are negligible at the precision required here. Hence

$$\begin{aligned}\Delta m &= m_H + m_n - m(D) \\ &= (1.007825 + 1.008665 - 2.014102) \text{ u} \\ &= 0.002388 \text{ u.}\end{aligned}$$

The positive defect means that the bound system has less rest energy than the separated constituents; energy must be supplied to separate them.

Step 3: Convert the mass defect into separation energy Using mass-energy equivalence,

$$\begin{aligned}E_{\text{required}} &= \Delta mc^2 \\ &= 0.002388 \times 931.5 \text{ MeV} \\ &= 2.224422 \text{ MeV.}\end{aligned}$$

With $1 \text{ MeV} = 1.602 \times 10^{-13} \text{ J}$, this corresponds to approximately $3.56 \times 10^{-13} \text{ J}$ per deuteron. This is the total binding energy of the deuteron; no division by its two nucleons is needed for the requested separation energy.

Step 4: Conclusion (a) 1 u is one twelfth of the mass of a neutral carbon-12 atom at rest in its ground state. (b) The required energy is $2.224 \text{ MeV} \simeq 3.56 \times 10^{-13} \text{ J}$ per deuteron.

Quick Tip to Solve: With atomic masses, subtract deuterium from hydrogen plus neutron: the two atomic electron masses cancel. Multiply the mass defect in u by 931.5 to obtain the separation energy in MeV.

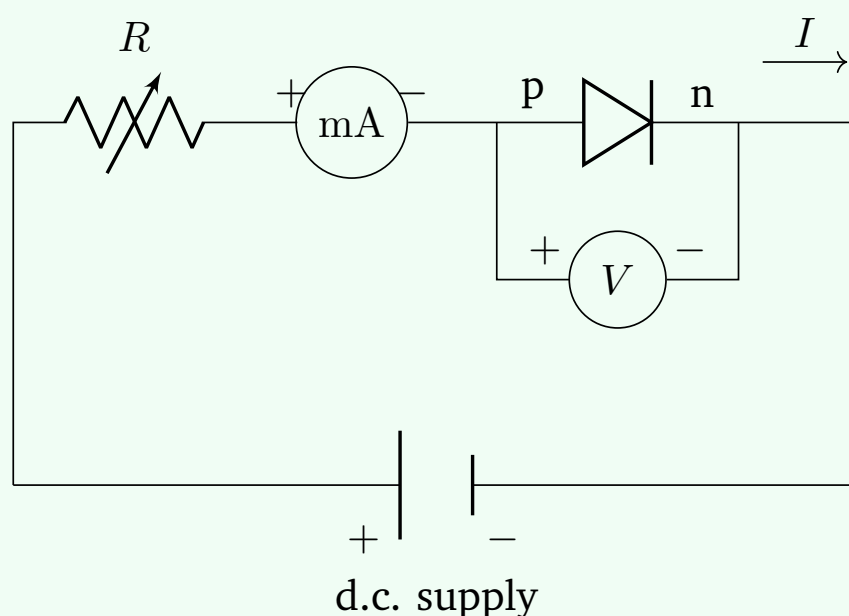
28(a). Draw the circuit diagrams for obtaining the $V - I$ characteristics of a $p-n$ junction diode. Explain briefly the salient features of the $V - I$ characteristics in (i) forward biasing, and (ii) reverse biasing. [3 marks]

Solution:

Concept:

- A forward bias connects the p-side to positive and the n-side to negative; it reduces the junction barrier and narrows the depletion layer.
- A reverse bias connects the p-side to negative and the n-side to positive; it raises the barrier and widens the depletion layer. Measure diode voltage in parallel and current in series.

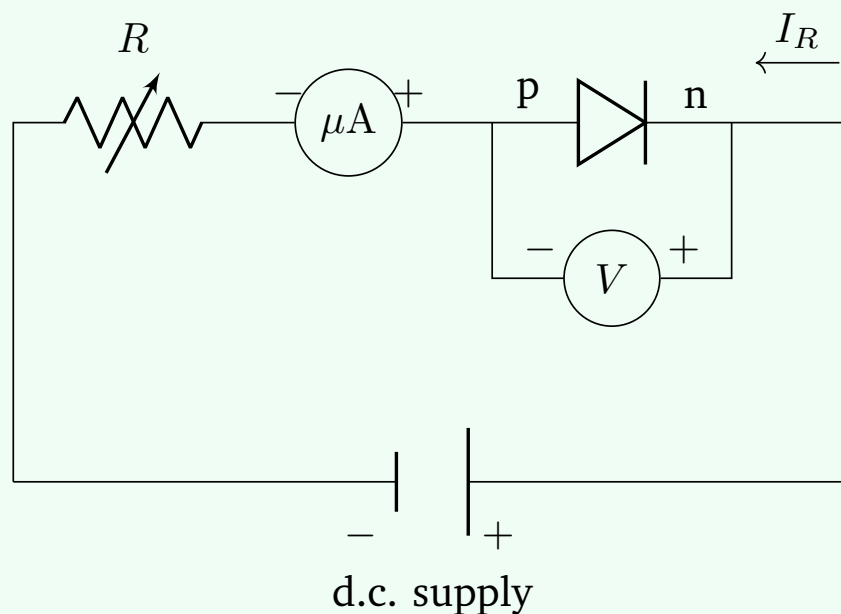
Step 1: Set up the forward-bias circuit Connect the diode, a protective variable resistance R and a milliammeter in series with a d.c. supply. Connect the voltmeter across the diode, with its positive terminal on the p-side. Vary R to obtain successive pairs of diode voltage V and current I , and plot I against V . The supply polarity and meter polarities are shown below.



Forward-bias circuit; the longer battery plate is positive.

Step 2: Explain the forward characteristic Initially the forward current is very small because the junction barrier still opposes majority-carrier motion. Around the knee (cut-in) voltage, the current starts increasing rapidly for a small increase in applied voltage. Typical knee voltages are about 0.7 V for silicon and 0.3 V for germanium. Above the knee the forward dynamic resistance dV/dI is low. The characteristic is nonlinear, so the diode does not obey Ohm's law over its full operating range.

Step 3: Set up and explain reverse bias Reverse the supply polarity and use a microammeter, since the reverse current is much smaller. Connect the voltmeter positive terminal to the n-side. Before breakdown, a small minority-carrier current flows from n to p; its magnitude is nearly independent of reverse voltage at a fixed temperature. This is the reverse saturation current. At the breakdown voltage its magnitude rises abruptly with little further change in voltage. The series resistance limits current and protects the diode; an ordinary diode should not be driven into uncontrolled breakdown.



Reverse-bias circuit; the microammeter measures current from n to p.

Step 4: Conclusion Forward bias gives a rapid current increase beyond the knee; reverse bias gives a small, nearly constant current until breakdown. Use a milliammeter for forward bias and a microammeter for reverse bias, with a voltmeter across the junction in both circuits.

Quick Tip to Solve: First mark the diode's p and n sides: p to supply positive means forward bias. Put the current meter in series and the voltmeter across only the diode.

OR

28(b). On the basis of energy band diagrams, distinguish between (i) an insulator, (ii) a semiconductor, and (iii) a conductor.

[3 marks]

Solution:

Concept:

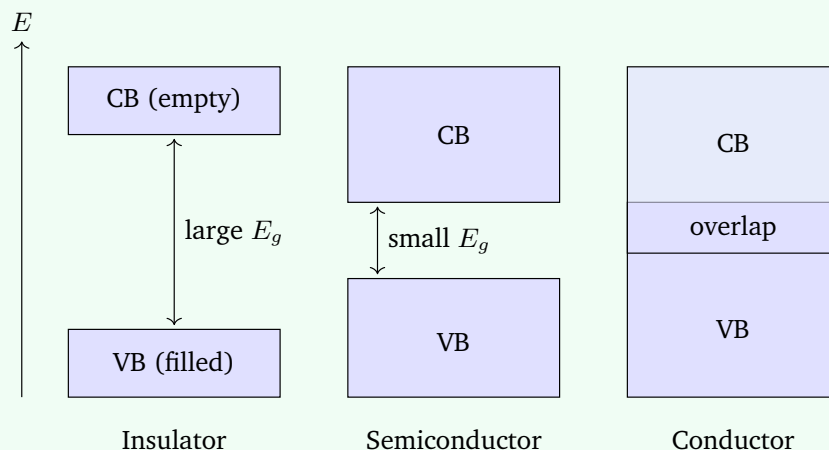
- The valence band contains the electrons participating in bonding; the conduction band provides states in which electrons can move through the solid. A completely filled band cannot carry a net current.
- The forbidden energy gap E_g is the separation between the valence-band maximum and conduction-band minimum.

Step 1: Identify the large gap in an insulator The valence band is full and the conduction band is empty at low temperature. A large forbidden gap, typically greater than about 3 eV, separates them. At ordinary temperatures thermal energy cannot excite an appreciable number of electrons across this gap. Consequently there are almost no mobile carriers and the electrical conductivity is extremely small.

Step 2: Identify the small gap in a semiconductor At 0 K, an intrinsic semiconductor has a filled valence band and an empty conduction band. Its forbidden gap is small, of order 1 eV: about 1.1 eV in silicon and 0.7 eV in germanium. At ordinary temperatures some electrons are thermally excited into the conduction band, leaving holes in the valence band. Both electrons and holes contribute to

current.

Step 3: Identify available states in a conductor In a conductor a band is partially filled, or the valence and conduction bands overlap. Thus electrons have nearby empty states available without needing to cross a forbidden gap. An applied electric field can redistribute electrons into these states, producing a large current.



Schematic band arrangements (not to an energy scale); VB is valence band and CB is conduction band.

Step 4: Conclusion An insulator has a large forbidden gap, a semiconductor has a small forbidden gap, and a conductor has overlapping bands or a partially filled band.

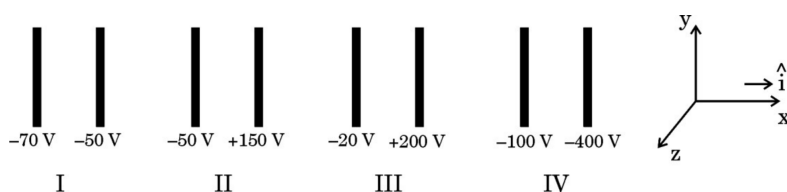
Quick Tip to Solve: Read a band diagram by checking available empty states first: a large gap suppresses carriers, a small gap permits thermal excitation, and overlap permits easy conduction.

Section D (8 marks)

Case Study Based Questions

Questions number 29 and 30 are case study based questions. Read the following paragraphs and answer the questions that follow.

29. The figure shows four pairs of parallel identical conducting plates, separated by the same distance 2.0 cm and arranged perpendicular to the x-axis. The electric potential of each plate is mentioned. The electric field between a pair of plates is uniform and normal to the plates. [4 marks]



29(i). For which pair of the plates is the electric field $\vec{E}$ along $\hat{i}$?

[1 mark]

- (A) I
- (B) II
- (C) III
- (D) IV

Correct Answer: (D) IV

Solution:

Concept:

- The electric field points in the direction of decreasing potential: $E_x = -dV/dx$.
- For uniform field between plates separated by d , $E_x = -(V_R - V_L)/d$, where V_L and V_R are the left and right plate potentials.

Step 1: Compare the change towards positive x The positive x-direction is from left to right. For pairs I, II and III, the right potential exceeds the left potential: their changes are respectively +20 V, +200 V and +220 V. Their fields therefore point along $-\hat{i}$, eliminating (A), (B) and (C).

Step 2: Check pair IV For pair IV the potential decreases from -100 V to -400 V towards the right. With $d = 0.020$ m,

$$E_x = -\frac{-400 - (-100)}{0.020} = +1.5 \times 10^4 \text{ V m}^{-1}.$$

The positive sign fixes the direction along the positive x-axis.

Step 3: Conclusion Only pair IV has its electric field along $\hat{i}$. Hence, the correct option is (D).

Quick Tip to Solve: Compare signed potentials, not their magnitudes: electric field points from the higher potential to the lower potential even when both are negative.

29(ii). An electron is released midway between the plates of pair IV. It will:

[1 mark]

- (A) move along $\hat{i}$ at constant speed
- (B) move along $-\hat{i}$ at constant speed
- (C) accelerate along $\hat{i}$
- (D) accelerate along $-\hat{i}$

Correct Answer: (D) accelerate along $-\hat{i}$

Solution:

Concept:

- The electric field points from higher to lower potential; for uniform field $E_x = -(V_R - V_L)/d$.
- Electric force is $\vec{F} = q\vec{E}$. For an electron $q = -e$, so $\vec{a} = -e\vec{E}/m_e$, opposite to the electric field.

Step 1: Determine the field direction In pair IV, potential decreases to the right from -100 V to -400 V . Hence the field points along $+\hat{i}$. Its magnitude is $300/0.020 = 1.5 \times 10^4\text{ V m}^{-1}$, so it is nonzero everywhere between the plates, including the midpoint.

Step 2: Apply the negative electron charge The electron's force and acceleration are opposite to this field, along $-\hat{i}$, towards the left plate. Released from rest in a uniform nonzero field, it gains speed under constant acceleration. Thus both constant-speed choices (A) and (B) are incorrect; choice (C) has the direction appropriate to a positive charge.

Step 3: Conclusion The electron accelerates along $-\hat{i}$. Hence, the correct option is (D).

Quick Tip to Solve: Find the field from decreasing potential first, then reverse that direction once for an electron; uniform field implies constant acceleration, not constant speed.

29(iii). Let V_0 be the potential at the left plate of any set, taken to be at $x = 0\text{ m}$. Then the potential V at any point ($0 \leq x \leq 2\text{ cm}$) between the plates of that set can be expressed as: (Here α is a constant, positive or negative.)

[1 mark]

- (A) $V = V_0 + \alpha x$
- (B) $V = V_0 + \alpha x^2$
- (C) $V = V_0 + \alpha x^{1/2}$
- (D) $V = V_0 + \alpha x^{3/2}$

Correct Answer: (A) $V = V_0 + \alpha x$

Solution:

Concept:

- Along the x-axis, the electric field is the negative potential gradient: $E_x = -dV/dx$.
- A uniform electric field has constant E_x , so potential varies linearly with distance. The given boundary condition is $V(0) = V_0$.

Step 1: Integrate the constant potential gradient Since E_x is independent of position between each pair of plates,

$$V(x) - V_0 = - \int_0^x E_x dx' = -E_x x.$$

Thus the coefficient of x is the constant $\alpha = -E_x$. It may have either sign depending on the plate potentials and has units of V m^{-1} if x is in metres.

Step 2: Compare all four expressions A term proportional to x gives a constant derivative and hence a uniform field. Terms in x^2 , $x^{1/2}$ and $x^{3/2}$ have derivatives proportional to x , $x^{-1/2}$ and $x^{1/2}$, respectively. Their fields depend on position, so options (B), (C) and (D) contradict the given uniform field.

Step 3: Conclusion $V = V_0 + \alpha x$, where $\alpha = -E_x$ is constant. Hence, the correct option is (A).

Quick Tip to Solve: Differentiate each proposed potential once: only a linear function of position produces the constant gradient required by a uniform electric field.

29(iv)(a). Let E_1, E_2, E_3 and E_4 be the magnitudes of the electric field between the pairs of plates I, II, III and IV respectively. Then: [1 mark]

- (A) $E_1 > E_2 > E_3 > E_4$
- (B) $E_3 > E_4 > E_1 > E_2$
- (C) $E_4 > E_3 > E_2 > E_1$
- (D) $E_2 > E_3 > E_4 > E_1$

Correct Answer: (C) $E_4 > E_3 > E_2 > E_1$

Solution:

Concept:

- For parallel plates with uniform field, the field magnitude is $E = |V_R - V_L|/d$, where d is their separation.
- All four separations are equal, so comparing field magnitudes is equivalent to comparing absolute potential differences.

Step 1: Calculate the potential differences Take the difference between the two potentials of each pair, retaining its magnitude:

$$\begin{aligned} |\Delta V_1| &= |-50 - (-70)| = 20 \text{ V}, \\ |\Delta V_2| &= |150 - (-50)| = 200 \text{ V}, \\ |\Delta V_3| &= |200 - (-20)| = 220 \text{ V}, \\ |\Delta V_4| &= |-400 - (-100)| = 300 \text{ V}. \end{aligned}$$

Step 2: Compare the field magnitudes Using $d = 0.020 \text{ m}$, the magnitudes are 1000, 10000, 11000, and 15000 V m^{-1} , respectively. The negative plate potentials of pair IV do not make its field magnitude negative. Options (A), (B), and (D) each contradict the ordering of these differences.

Step 3: Conclusion $E_4 > E_3 > E_2 > E_1$. Hence, the correct option is (C).

Quick Tip to Solve: For equal plate separations, rank absolute potential differences; the individual plate potentials do not determine the field strength.

29(iv)(b). An electron is projected from the right plate of set I directly towards its left plate. It just comes to rest at the plate. The speed with which it was projected is about: (Take $e/m = 1.76 \times 10^{11} \text{ C/kg}$) [1 mark]

(A) $1.3 \times 10^5 \text{ m/s}$

(B) $2.6 \times 10^6 \text{ m/s}$

(C) $6.5 \times 10^5 \text{ m/s}$

(D) $5.2 \times 10^7 \text{ m/s}$

Correct Answer: (B) $2.6 \times 10^6 \text{ m/s}$

Solution:

Concept:

- Electrostatic energy is conserved: $K_i + qV_i = K_f + qV_f$, where $q = -e$ for an electron.
- For a particle just stopping at its destination, $K_f = 0$; its initial kinetic energy equals its increase in electric potential energy.

Step 1: Find the energy needed to reach the left plate The electron starts at $V_i = -50 \text{ V}$ and stops at $V_f = -70 \text{ V}$. Thus $V_f - V_i = -20 \text{ V}$, and

$$\frac{1}{2}mv^2 = (-e)(-20 \text{ V}) = 20e \text{ V}.$$

The positive energy increase is consistent with the electron slowing as it moves left, opposite to the force on it.

Step 2: Calculate the projection speed Using the given charge-to-mass ratio,

$$v = \sqrt{2 \frac{e}{m} (20 \text{ V})} = \sqrt{2(1.76 \times 10^{11})(20)} = 2.65 \times 10^6 \text{ m s}^{-1}.$$

The other listed speeds do not supply the required kinetic energy; omitting the square root or using a potential instead of its difference gives an incorrect result.

Step 3: Conclusion The required speed is approximately $2.6 \times 10^6 \text{ m s}^{-1}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the positive stopping energy $e|\Delta V|$, then take the square root in $v = \sqrt{2(e/m)|\Delta V|}$.

30. Diffraction and interference are closely related phenomena that occur together. Diffraction is the phenomenon of bending of light around the edges of the obstacle, while interference is the combination of waves that results in a new wave pattern. In order to get interference, there must be at least two waves that are diffracting. So while diffraction can occur without interference, interference cannot occur without diffraction.

Two slits of width $2\text{ }\mu\text{m}$ each in an opaque material are separated by a distance of $6\text{ }\mu\text{m}$. Monochromatic light of wavelength 450 nm is incident normally on the slits. One finds a combined interference and diffraction pattern on the screen. [4 marks]

30(i). The number of peaks of the interference fringes formed within the central peak of the envelope of the diffraction pattern will be: [1 mark]

- (A) 2
- (B) 3
- (C) 4
- (D) 6

Correct Answer: (D) 6

Solution:

Concept:

- The central diffraction maximum has width $W = 2\lambda D/a$, where a is slit width and D is screen distance, in the small-angle approximation.
- Interference fringe width is $\beta = \lambda D/d$, where d is slit separation. The usual CBSE fringe-count estimate is $N = W/\beta$.

Step 1: Compare the envelope and fringe widths Dividing the central-envelope width by the interference-fringe spacing gives

$$N \simeq \frac{2\lambda D/a}{\lambda D/d} = \frac{2d}{a}.$$

Both λ and D cancel. Thus the count obtained by this conventional width-ratio method depends only on the slit separation relative to the individual slit width.

Step 2: Apply the convention used by the options With $d = 6\text{ }\mu\text{m}$ and $a = 2\text{ }\mu\text{m}$,

$$N \simeq \frac{2 \times 6}{2} = 6.$$

The other listed numbers do not follow this ratio. This is the conventional estimate intended by the options, not a rigorous count of individual local intensity maxima: counting nominal bright orders alone gives $m = 0, \pm 1, \pm 2$, because the boundary orders $m = \pm 3$ are suppressed by diffraction minima.

Step 3: Conclusion Using the usual central-envelope width to fringe-width convention, the answer is six. Hence, the correct option is (D).

Quick Tip to Solve: For the standard width-ratio question use $N \simeq 2d/a$; wavelength changes both widths equally and therefore cancels.

30(ii). The number of peaks of the interference formed if the slit width is doubled while keeping the distance between the slits same will be: [1 mark]

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (C) 3

Solution:

Concept:

- When the slit width is doubled, a becomes $4\ \mu\text{m}$, while the centre separation remains $d = 6\ \mu\text{m}$.
- Bright interference orders within the central diffraction envelope obey $|m| < d/a$, with integral order m .

Step 1: Apply the central-envelope condition The first diffraction minima now satisfy $a \sin \theta = \pm \lambda$, while the interference maxima satisfy $d \sin \theta = m\lambda$. Dividing these conditions gives

$$|m| < \frac{6}{4} = 1.5.$$

Increasing slit width narrows the central envelope; it does not alter the interference fringe spacing because d remains fixed.

Step 2: List the surviving bright orders The permitted integer orders are $m = -1, 0, +1$, giving three bright peaks. The next orders $m = \pm 2$ lie outside the central envelope. Option (A) omits both side peaks, option (B) omits the central peak, and option (D) overcounts the allowed orders.

Step 3: Conclusion Three interference peaks lie within the central diffraction envelope. Hence, the correct option is (C).

Quick Tip to Solve: After changing slit width, recompute d/a and list the allowed integer orders, including the central order $m = 0$.

30(iii)(a). If instead of $450\ \text{nm}$ light, another light of wavelength $680\ \text{nm}$ is used, number of peaks of the interference formed in the central peak of the envelope of the diffraction pattern will be: [1 mark]

- (A) 2
- (B) 4
- (C) 6
- (D) 9

Correct Answer: (C) 6

Solution:

Concept:

- The central diffraction maximum has width $W = 2\lambda D/a$, where a is slit width and D is screen distance, in the small-angle approximation.
- Interference fringe width is $\beta = \lambda D/d$, where d is slit separation. The usual CBSE fringe-count estimate is $N = W/\beta$.

Step 1: Compare the envelope and fringe widths Dividing the central-envelope width by the

interference-fringe spacing gives

$$N \simeq \frac{2\lambda D/a}{\lambda D/d} = \frac{2d}{a}.$$

Both λ and D cancel. Thus the count obtained by this conventional width-ratio method depends only on the slit separation relative to the individual slit width.

Step 2: Apply the convention used by the options With $d = 6 \mu\text{m}$ and $a = 2 \mu\text{m}$,

$$N \simeq \frac{2 \times 6}{2} = 6.$$

Changing the wavelength from 450 nm to 680 nm widens the diffraction envelope and the fringes in the same ratio, leaving this result unchanged. The other listed numbers do not follow this ratio. This is the conventional estimate intended by the options, not a rigorous count of individual local intensity maxima: counting nominal bright orders alone gives $m = 0, \pm 1, \pm 2$, because the boundary orders $m = \pm 3$ are suppressed by diffraction minima.

Step 3: Conclusion Using the usual central-envelope width to fringe-width convention, the answer is six. Hence, the correct option is (C).

Quick Tip to Solve: For the standard width-ratio question use $N \simeq 2d/a$; wavelength changes both widths equally and therefore cancels.

OR

30(iii)(b). Consider the diffraction of light by a single slit described in this case study. The first minimum falls at an angle θ equal to: [1 mark]

- (A) $\sin^{-1}(0.12)$
- (B) $\sin^{-1}(0.225)$
- (C) $\sin^{-1}(0.32)$
- (D) $\sin^{-1}(0.45)$

Correct Answer: (B) $\sin^{-1}(0.225)$

Solution:

Concept:

- For a single slit of width a , diffraction minima occur at $a \sin \theta = n\lambda$, for nonzero integral n .
- The first minimum corresponds to $n = 1$. Its location depends on slit width, not on the separation of the two slits.

Step 1: Select the width and wavelength The single slit described in the original passage has width $a = 2 \mu\text{m} = 2 \times 10^{-6} \text{ m}$, and the incident wavelength is $\lambda = 450 \text{ nm} = 450 \times 10^{-9} \text{ m}$. This alternative does not adopt the changed wavelength of the preceding OR alternative.

Step 2: Evaluate the first-minimum condition Substitute into $a \sin \theta = \lambda$:

$$\sin \theta = \frac{450 \times 10^{-9}}{2 \times 10^{-6}} = 0.225.$$

Hence the inverse sine must have argument 0.225. The other arguments do not equal λ/a ; using the slit separation would instead give an interference condition.

Step 3: Conclusion $\theta = \sin^{-1}(0.225)$. Hence, the correct option is (B).

Quick Tip to Solve: For a diffraction minimum use the individual slit width a , and reset to the passage values when starting a separate OR alternative.

30(iv). The number of bright fringes formed due to interference on 1 m of screen placed at $\frac{4}{3}$ m away from the slits is: [1 mark]

- (A) 2
- (B) 3
- (C) 6
- (D) 10

Correct Answer: (D) 10

Solution:

Concept:

- In the usual small-angle interference approximation, adjacent bright fringes are separated by $\beta = \lambda D/d$, with screen distance D and slit separation d .
- The standard number of fringes spanning a screen length L is $N = L/\beta$; this uses the fringe-spacing convention for the stated screen length.

Step 1: Calculate the fringe width The official paper gives $D = 4/3$ m. With $\lambda = 450 \times 10^{-9}$ m and $d = 6 \times 10^{-6}$ m,

$$\beta = \frac{(450 \times 10^{-9})(4/3)}{6 \times 10^{-6}} = 0.10 \text{ m.}$$

The diffraction-envelope width is not the quantity requested here.

Step 2: Divide the screen length by the spacing For $L = 1$ m,

$$N = \frac{1}{0.10} = 10.$$

This is the conventional interference-fringe count over the specified length; the question does not specify an endpoint-inclusive count of individual maxima. Options (A), (B), and (C) do not match the ratio of screen length to fringe spacing.

Step 3: Conclusion The screen length spans ten interference fringes. Hence, the correct option is (D).

Quick Tip to Solve: Convert nanometres and micrometres to the same unit before computing β , then divide the screen length by that spacing.

Section E (15 marks)

31(a). (i) Obtain the expression for the capacitance of a parallel plate capacitor with a dielectric medium between its plates.

(ii) A charge of $6 \mu\text{C}$ is given to a hollow metallic sphere of radius 0.2 m . Find the potential at (i) the surface and (ii) the centre of the sphere. [5 marks]

Solution:

Concept:

- Capacitance is $C = Q/V$, where Q is the magnitude of charge on either plate and V is the potential difference between them.
- A dielectric of relative permittivity K has permittivity $\varepsilon = K\varepsilon_0$. Polarisation reduces the field due to a fixed free charge to $E = \sigma/(K\varepsilon_0)$.
- In electrostatic equilibrium a conductor is equipotential. For an isolated charged spherical conductor, with potential zero at infinity, $V = Q/(4\pi\varepsilon_0 R)$ at its surface and everywhere inside its empty cavity.

Step 1: (i) Specify the parallel-plate arrangement Consider two large parallel conducting plates, each of area A , separated by a distance d . They carry equal and opposite free charges $+Q$ and $-Q$, giving free surface-charge densities $+\sigma$ and $-\sigma$, where $\sigma = Q/A$. The separation is small compared with the plate dimensions, so edge effects can be neglected and the field between the plates is uniform. Let a homogeneous linear dielectric of relative permittivity K completely fill the space between them.

Step 2: (i) Find the field and potential difference Without the dielectric, the fields of the two oppositely charged plates add between them, giving $E_0 = \sigma/\varepsilon_0$. The dielectric becomes polarised: its bound surface charges produce a field opposing the original field. For the same free charge on the plates, the resulting field is

$$E = \frac{E_0}{K} = \frac{\sigma}{K\varepsilon_0} = \frac{Q}{K\varepsilon_0 A}.$$

Since this field is uniform, the magnitude of the potential difference across the separation is

$$V = Ed = \frac{Qd}{K\varepsilon_0 A}.$$

Thus a given free charge produces a smaller voltage when the dielectric is present.

Step 3: (i) Obtain the capacitance Substitute the potential difference into the definition of capacitance:

$$C = \frac{Q}{V} = \frac{Q}{Qd/(K\varepsilon_0 A)} = \frac{K\varepsilon_0 A}{d}.$$

If $C_0 = \varepsilon_0 A/d$ denotes the capacitance with vacuum between the plates, then $C = KC_0$. Capacitance depends on the geometry and dielectric, rather than on the particular charge used in the derivation.

Step 4: (ii) Calculate the surface potential For the hollow metallic sphere, $Q = 6 \times 10^{-6} \text{ C}$ and $R = 0.2 \text{ m}$. Its charge resides on the outer surface, and the exterior potential is that of a point charge

at its centre. Taking potential zero at infinity gives

$$\begin{aligned} V_{\text{surface}} &= \frac{1}{4\pi\epsilon_0} \frac{Q}{R} \\ &= \frac{(9 \times 10^9)(6 \times 10^{-6})}{0.2} \\ &= 2.7 \times 10^5 \text{ V.} \end{aligned}$$

The positive sign follows from the positive charge given to the sphere.

Step 5: (ii) Determine the centre potential There is no charge inside the cavity. Spherical symmetry and Gauss's law give zero electric field for $r < R$. Since potential difference is the negative line integral of electric field,

$$V_{\text{centre}} - V_{\text{surface}} = - \int_{\text{surface}}^{\text{centre}} \vec{E} \cdot d\vec{l} = 0.$$

Therefore the centre has the same potential as the surface. Zero interior electric field means a constant potential, not necessarily zero potential; the value is fixed by the charged surface and the chosen reference at infinity.

Step 6: Conclusion The capacitance with dielectric filling is $C = K\epsilon_0 A/d$. For the charged sphere,

$$V_{\text{surface}} = V_{\text{centre}} = +2.7 \times 10^5 \text{ V, relative to infinity.}$$

Quick Tip to Solve: For a fully filled capacitor multiply the vacuum capacitance by K . Inside a charged hollow conductor, keep the surface potential constant; never substitute $r = 0$ into the exterior formula.

OR

31(b). (i) A charge $+Q$ is placed on a thin conducting spherical shell of radius R . Use Gauss's theorem to derive an expression for the electric field at a point lying (i) inside and (ii) outside the shell.

(ii) Show that the electric field for same charge density (σ) is twice in case of a conducting plate or surface than in a nonconducting sheet. [5 marks]

Solution:

Concept:

- Gauss's law states $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the net charge enclosed by a closed Gaussian surface.
- For an isolated, uniformly charged spherical shell, symmetry makes the field radial and its magnitude constant on every concentric spherical Gaussian surface.
- The field inside a conductor in electrostatic equilibrium is zero. A Gaussian pillbox across its surface therefore has electric flux only through the cap outside the conductor.

Step 1: (i) Choose a spherical Gaussian surface Let the conducting spherical shell have radius R and total charge $+Q$. In the absence of external fields or charges inside the cavity, its charge distributes uniformly over the outer surface. Consider a concentric Gaussian sphere of radius r . By spherical symmetry, $\vec{E}$ is radial and has the same magnitude at every point of this sphere. The area vector is also radial, so

$$\oint \vec{E} \cdot d\vec{A} = E \oint dA = E(4\pi r^2).$$

We determine the enclosed charge separately for the two regions.

Step 2: (i) Derive the field inside the shell When $r < R$, the Gaussian surface lies inside the empty shell and encloses no charge. All the charge is on the shell at radius R . Gauss's law then gives

$$E(4\pi r^2) = \frac{0}{\epsilon_0}, \quad E = 0 \quad (0 < r < R).$$

The field at the centre is also zero by symmetry. Here zero enclosed charge implies zero field because spherical symmetry has already established that the radial field magnitude is constant on the Gaussian sphere; zero net flux alone would not establish this for an arbitrary charge arrangement.

Step 3: (i) Derive the field outside the shell When $r > R$, the Gaussian surface encloses the full charge $+Q$. Thus

$$E(4\pi r^2) = \frac{Q}{\epsilon_0}, \quad \vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}.$$

The field points radially outwards because the charge is positive. The exterior field is identical to that of a point charge $+Q$ placed at the centre. Immediately outside the shell it approaches $Q/(4\pi\epsilon_0 R^2)$, whereas immediately inside it is zero.

Step 4: (ii) Find the field just outside a conducting surface Let σ denote the charge per unit area on the particular conducting surface being considered. Choose a very short cylindrical Gaussian pillbox with cap area A , one cap inside the conductor and one just outside. The interior field is zero, while the exterior field is perpendicular to the surface. The curved surface contributes no flux in the limit of vanishing pillbox height. Hence

$$E_{\text{conductor}} A = \frac{\sigma A}{\epsilon_0}, \quad E_{\text{conductor}} = \frac{\sigma}{\epsilon_0}.$$

For positive σ , the field points normally outwards. This expression refers to the local charge density on that surface, not the combined charge per projected area on both faces of a plate.

Step 5: (ii) Compare with a nonconducting sheet For a large, uniformly charged nonconducting sheet with the same surface charge density σ , neglect edge effects. Reflection symmetry gives equal field magnitudes on both sides, directed away from a positive sheet. A pillbox straddling the sheet therefore has outward flux through both caps, each of area A :

$$2E_{\text{sheet}} A = \frac{\sigma A}{\epsilon_0}, \quad E_{\text{sheet}} = \frac{\sigma}{2\epsilon_0}.$$

Taking the ratio at equal surface charge density gives

$$\frac{E_{\text{conductor}}}{E_{\text{sheet}}} = \frac{\sigma/\epsilon_0}{\sigma/(2\epsilon_0)} = 2.$$

The factor of two comes from one cap carrying flux for the conductor, compared with two caps for the nonconducting sheet.

Step 6: Conclusion The shell produces $\vec{E} = 0 \ (r < R)$ and $\vec{E} = Q\hat{r}/(4\pi\epsilon_0 r^2) \ (r > R)$. At equal surface charge density, $E_{\text{conductor}} = \sigma/\epsilon_0 = 2E_{\text{sheet}}$, where $E_{\text{sheet}} = \sigma/(2\epsilon_0)$.

Quick Tip to Solve: Count the pillbox caps with nonzero flux: one for a conducting surface, two for a symmetric nonconducting sheet. Use σ for the charge density on the specific surface being compared.

32(a). (i) (1) What is meant by current sensitivity of a galvanometer? Mention the factors on which it depends.

(2) A galvanometer of resistance G is converted into a voltmeter of range $(0 - V)$ by using a resistance R . Find the resistance, in terms of R and G , required to convert it into a voltmeter of range $\left(0 - \frac{V}{2}\right)$.

(ii) The magnetic flux through a coil of resistance $5\ \Omega$ increases with time as: $\phi = (2.0t^3 + 5.0t^2 + 6.0t)$ mWb. Find the magnitude of induced current through the coil at $t = 2$ s. [5 marks]

Solution:

Concept:

- In a moving-coil galvanometer with a radial magnetic field, the magnetic torque is $NIAB$ and the restoring torque is $k\theta$. Here N is the number of turns, A the area of each turn, B the field and k the torsional constant.
- A voltmeter is made by adding a high resistance in series with the galvanometer. Its range is $V = I_g(G + R)$, where I_g is the full-scale galvanometer current.
- Faraday's law gives the induced emf $\varepsilon = -d\phi/dt$. The magnitude of the current in a coil of resistance r is $|I| = |\varepsilon|/r$.

Step 1: (i)(1) Define and derive current sensitivity Current sensitivity is the angular deflection produced per unit current through the galvanometer. At equilibrium the magnetic torque is balanced by the restoring torque:

$$NIAB = k\theta, \quad S_I = \frac{\theta}{I} = \frac{NAB}{k}.$$

Thus sensitivity increases with the number of coil turns N , magnetic field strength B and area A of the coil, and decreases with increasing torsional constant k of the suspension. It is measured in rad A^{-1} . The radial field ensures that the magnetic torque remains proportional to current throughout the working deflection range.

Step 2: (i)(2) Keep the full-scale current unchanged Let R' be the new resistance connected in series. The galvanometer itself is unchanged, so both its resistance G and full-scale current I_g are unchanged. For the two ranges,

$$V = I_g(G + R),$$
$$\frac{V}{2} = I_g(G + R').$$

Dividing the second equation by the first gives

$$G + R' = \frac{G + R}{2}, \quad R' = \frac{R - G}{2}.$$

It is the total voltmeter resistance that is halved, so merely replacing R by $R/2$ would be incorrect. A nonnegative added series resistance requires $R \geq G$; when $R = G$, no added resistance is needed for the lower range.

Step 3: (ii) Differentiate the flux in SI units Convert milliwebers to webers before differentiating. With t in seconds, the given flux is

$$\phi = (2.0t^3 + 5.0t^2 + 6.0t) \times 10^{-3} \text{ Wb}.$$

The instantaneous rate of change is

$$\frac{d\phi}{dt} = (6.0t^2 + 10.0t + 6.0) \times 10^{-3} \text{ V}.$$

At $t = 2 \text{ s}$,

$$|\varepsilon| = (24 + 20 + 6) \times 10^{-3} = 0.050 \text{ V}.$$

The minus sign in Faraday's law specifies the opposing sense of the induced emf; the question asks only for its resulting current magnitude.

Step 4: (ii) Use the coil resistance For $r = 5 \Omega$, Ohm's law gives

$$|I| = \frac{|\varepsilon|}{r} = \frac{0.050}{5} = 0.010 \text{ A}.$$

The flux stated is the flux through the coil, so no extra, unspecified turn factor is introduced. The induced current acts to oppose the increase of flux, in accordance with Lenz's law.

Step 5: Conclusion $S_I = \theta/I = NAB/k$, directly proportional to N , A , B and inversely proportional to k . The required series resistance is $R' = (R - G)/2$. At $t = 2 \text{ s}$, the induced current magnitude is $0.010 \text{ A} = 10 \text{ mA}$.

Quick Tip to Solve: For a changed voltmeter range, scale the total resistance $G + R$, then subtract G . Differentiate the flux polynomial before substituting time, and convert mWb to Wb.

OR

32(b). (i) A rectangular coil of N turns and area of cross-section A is rotated at a steady angular speed ω in a uniform magnetic field. Obtain an expression for the emf induced in the coil at any instant of time.

(ii) Two coplanar and concentric circular loops L_1 and L_2 are placed coaxially with their centres coinciding. The radii of L_1 and L_2 are 1 cm and 100 cm respectively. Calculate the mutual inductance of the loops. (Take $\pi^2 = 10$)

[5 marks]

Solution:

Concept:

- Magnetic flux through one turn is $\phi = BA \cos \theta$, where θ is the angle between the field and the normal to the coil. For N turns the flux linkage is $N\phi$.
- Faraday's law is $\varepsilon = -d(N\phi)/dt$; the negative sign expresses Lenz's law. Uniform rotation makes the flux linkage vary sinusoidally.
- Mutual inductance is the flux linkage of one circuit per unit current in the other: $M = \Phi_{21}/I_1 = \Phi_{12}/I_2$. The field at the centre of a circular single-turn loop of radius r is $B = \mu_0 I/(2r)$.

Step 1: (i) Express the rotating coil's flux linkage Let the uniform field have magnitude B . Take the usual generator arrangement: the rotation axis lies in the plane of the coil and is perpendicular to the magnetic field. Choose $t = 0$ when the area normal is parallel to the field. At angular speed ω , the angle between the normal and field is $\theta = \omega t$. Therefore,

$$\phi(t) = BA \cos \omega t, \quad N\phi(t) = NBA \cos \omega t.$$

The factor N is necessary because each turn links the same flux. The area A is the area enclosed by each turn, not the area of the wire's cross-section.

Step 2: (i) Differentiate to obtain the instantaneous emf Applying Faraday's law gives

$$\begin{aligned}\varepsilon(t) &= -\frac{d}{dt}(NBA \cos \omega t) \\ &= NBA\omega \sin \omega t.\end{aligned}$$

The peak emf is $\varepsilon_0 = NBA\omega$, so $\varepsilon = \varepsilon_0 \sin \omega t$. Its polarity reverses every half-turn. At maximum flux the instantaneous flux change is zero, whereas at zero flux the flux changes fastest and the emf magnitude is maximum. If the initial angle is θ_0 , the general phase-dependent expression is $\varepsilon = NBA\omega \sin(\omega t + \theta_0)$.

Step 3: (ii) Calculate the field of the larger loop Write the radii in SI units:

$$r_1 = 1 \text{ cm} = 10^{-2} \text{ m}, \quad r_2 = 100 \text{ cm} = 1 \text{ m}.$$

Let a current I_2 flow in the larger loop L_2 . Its field at the common centre is

$$B_2 = \frac{\mu_0 I_2}{2r_2}.$$

Since $r_1/r_2 = 0.01$, the field varies negligibly across the much smaller loop. We therefore use the centre field as approximately uniform over its area. Because the loops are coplanar, this field is normal to the area of L_1 .

Step 4: (ii) Find the linked flux per unit current The smaller loop is a single turn, so its flux linkage is

$$\Phi_{12} \simeq B_2 \pi r_1^2 = \frac{\mu_0 I_2 \pi r_1^2}{2r_2}.$$

Divide by I_2 and substitute $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$:

$$\begin{aligned}M &\simeq \frac{\mu_0 \pi r_1^2}{2r_2} \\ &= \frac{(4\pi \times 10^{-7})\pi(10^{-2})^2}{2(1)} \\ &= 2\pi^2 \times 10^{-11} \text{ H} \\ &= 2.0 \times 10^{-10} \text{ H}, \quad (\pi^2 = 10).\end{aligned}$$

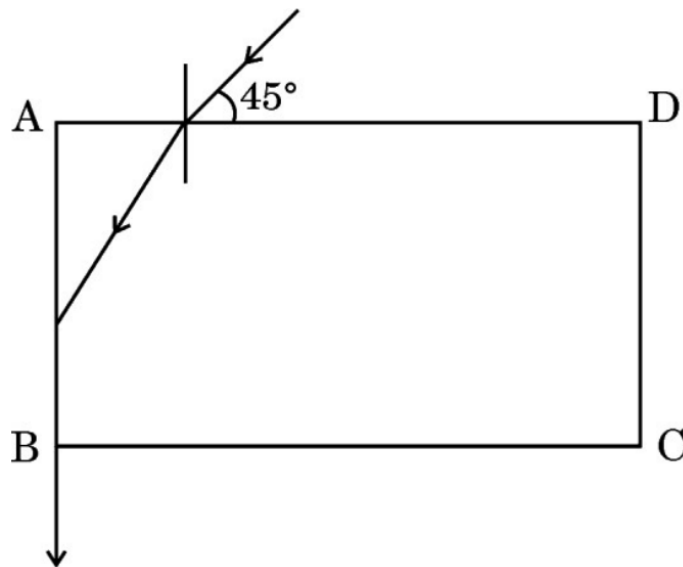
By reciprocity this is also the mutual inductance obtained when the smaller loop carries the source current. Choosing the larger loop as the source makes the flux calculation simple and justified.

Step 5: Conclusion The induced emf is $\varepsilon(t) = NBA\omega \sin \omega t$, with peak value $NBA\omega$, for an initially field-parallel area normal. The mutual inductance of the two loops is approximately $2.0 \times 10^{-10} \text{ H}$, or 0.20 nH .

Quick Tip to Solve: Differentiate the total flux linkage $NBA \cos \omega t$. For widely unequal concentric loops, take current in the larger loop and multiply its centre field by the smaller loop's area.

33(a). (i) Trace the path of a ray of light showing refraction through a triangular prism and hence obtain an expression for angle of deviation (δ) in terms of A , i and e , where symbols have their usual meanings. Draw a graph showing the variation of angle of deviation with the angle of incidence.

(ii) In the figure, a ray of light is incident on a transparent liquid contained in a thin glass box at an angle of 45° with its one face. The emergent ray passes along the face AB . Find the refractive index of the liquid. [5 marks]

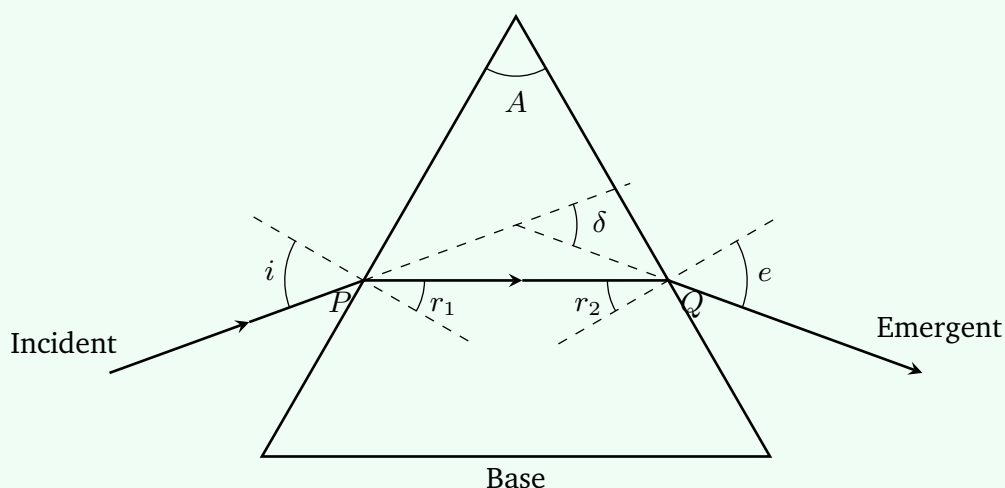


Solution:

Concept:

- For a prism of apex angle A , the internal refraction angles satisfy $r_1 + r_2 = A$. All incidence, refraction and emergence angles are measured from the corresponding face normals.
- The deviation is the total change in the ray's direction at the two refracting faces. At minimum deviation the path is symmetric: $i = e$ and $r_1 = r_2 = A/2$.
- Snell's law at an air-liquid interface is $\sin i = \mu \sin r$, where μ is the liquid's refractive index relative to air. At the critical angle C , $\mu \sin C = 1$.

Step 1: (i) Trace refraction through the prism Let a ray enter the first face at P with incidence angle i , refract at angle r_1 , reach the second face at Q with internal incidence angle r_2 , and emerge into air at angle e . It bends towards the normal on entering glass and away from the normal on leaving it. The overall deviation is towards the prism's base. The dashed lines through P and Q are the face normals; the other dashed segments extend the incident and emergent directions to show δ .



Refraction through a prism; the symmetric path is shown for clarity.

Step 2: Derive the deviation formula The two face normals and the refracted ray form the usual interior geometry. The angle between the relevant normal segments is $180^\circ - A$; the triangle angle sum therefore gives

$$r_1 + r_2 + (180^\circ - A) = 180^\circ, \quad r_1 + r_2 = A.$$

The changes in direction at the first and second faces are $i - r_1$ and $e - r_2$, respectively. Adding them,

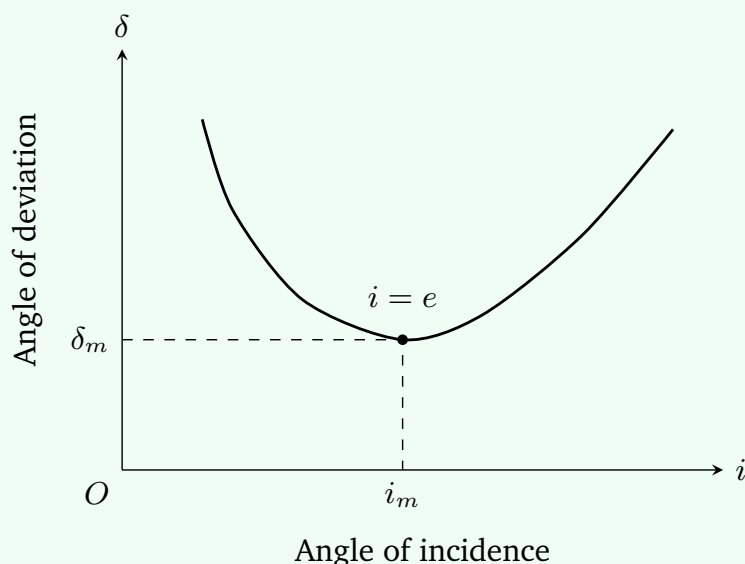
$$\begin{aligned} \delta &= (i - r_1) + (e - r_2) \\ &= i + e - (r_1 + r_2) = i + e - A. \end{aligned}$$

Thus the prism angle must be subtracted from the sum of the external angles; it is not added to that sum.

Step 3: Sketch the deviation-incidence graph For a fixed prism and colour of light, over the range in which a ray emerges, δ first decreases as i increases, reaches a minimum δ_m , and then increases. The minimum occurs for the symmetric path, where $i = e = i_m$. Hence

$$\delta_m = 2i_m - A, \quad r_1 = r_2 = A/2.$$

The graph below is schematic; its minimum is labelled, and no numerical angles are implied. Reversing a ray interchanges i and e while keeping its deviation unchanged.



Schematic variation of deviation with incidence for a fixed prism.

Step 4: (ii) Relate the angles at the liquid surfaces The incident ray makes 45° with the horizontal face AD , so its angle to that face's vertical normal is also 45° . Let the refracted angle in the liquid, measured from the vertical normal, be r . Refraction into the liquid gives

$$\sin 45^\circ = \mu \sin r, \quad \mu \sin r = \frac{1}{\sqrt{2}}.$$

The normal to the vertical face AB is horizontal. The ray's incidence there is therefore $90^\circ - r$. Since the emergent ray travels along AB , its emergence angle is 90° , so the internal incidence equals the critical angle C . Thus

$$C = 90^\circ - r, \quad \mu \sin C = 1 \Rightarrow \mu \cos r = 1.$$

The thin parallel glass walls introduce no net change in the Snell relation between air and liquid; the intermediate glass index cancels at their parallel interfaces.

Step 5: Eliminate the unknown refraction angle Squaring the two equations and adding eliminates r , because $\sin^2 r + \cos^2 r = 1$:

$$\begin{aligned} \mu^2 \sin^2 r + \mu^2 \cos^2 r &= \frac{1}{2} + 1, \\ \mu^2 &= \frac{3}{2}, \\ \mu &= \sqrt{\frac{3}{2}} \simeq 1.225. \end{aligned}$$

The positive root is appropriate for the refractive index of this transparent liquid. As a consistency check, $\sin r = 1/\sqrt{3}$ gives $r \simeq 35.3^\circ$ and $C \simeq 54.7^\circ$, whose sum is 90° .

Step 6: Conclusion (i) The angle of deviation is $\delta = i + e - A$; its graph has a minimum δ_m at $i = e$.
(ii) The refractive index of the liquid is $\mu = \sqrt{3/2} \simeq 1.225$, a dimensionless quantity.

Quick Tip to Solve: For perpendicular entry and exit faces, the internal angles are complementary. Combine $\mu \sin r = \sin i$ with $\mu \cos r = 1$ for grazing emergence, then square and add.

33(b). (i) The displacement of two light waves, each of amplitude ' a ' and frequency ω , emanating from two coherent sources of light, are given by $y_1 = a \cos \omega t$ and $y_2 = a \cos(\omega t + \phi)$. ϕ is the phase difference between the two waves. These light waves superpose at a point. Obtain the expression for the resultant intensity at that point.

(ii) In Young's double slit experiment, find the ratio of intensities at two points on a screen when waves emanating from two slits reaching these points have path differences (i) $\frac{\lambda}{6}$ and (ii) $\frac{\lambda}{12}$. [5 marks]

Solution:

Concept:

- The superposition principle gives the resultant displacement as the algebraic sum of the individual wave displacements. Coherent sources maintain a constant phase difference, so a stable interference pattern is obtained.
- In the same medium, intensity is proportional to the square of the wave amplitude: $I = KA^2$, where K is the common proportionality constant. Write the intensity due to either slit alone as $I_0 = Ka^2$.
- A path difference Δ produces phase difference $\phi = 2\pi\Delta/\lambda$, where λ is the wavelength. For equal slit intensities, $I = 4I_0 \cos^2(\phi/2)$.

Step 1: (i) Add the two displacements At the given point, apply the superposition principle to the two coherent waves:

$$y = y_1 + y_2 = a \cos \omega t + a \cos(\omega t + \phi).$$

Use the identity $\cos u + \cos v = 2 \cos[(u - v)/2] \cos[(u + v)/2]$. Since cosine is even, this gives

$$y = 2a \cos \frac{\phi}{2} \cos \left(\omega t + \frac{\phi}{2} \right).$$

Thus the resultant is a harmonic wave of the same angular frequency, with amplitude magnitude $A = 2a |\cos(\phi/2)|$. The sign of the cosine factor affects the phase convention, not the intensity.

Step 2: (i) Obtain the resultant intensity Squaring the resultant amplitude and using $I_0 = Ka^2$, the intensity becomes

$$\begin{aligned} I &= KA^2 = 4Ka^2 \cos^2 \frac{\phi}{2} \\ &= 4I_0 \cos^2 \frac{\phi}{2} = 2I_0(1 + \cos \phi). \end{aligned}$$

The term $2I_0 \cos \phi$ is the interference contribution in addition to the individual intensities $I_0 + I_0$. Therefore intensities cannot simply be added for coherent waves. As checks, the formula gives $I_{\max} = 4I_0$ at $\phi = 0$ and $I_{\min} = 0$ at $\phi = \pi$, consistent with constructive and destructive interference respectively.

Step 3: (ii) Calculate the intensity for the first path difference For the equally illuminated slits of the standard Young's double slit experiment, use the same single-slit intensity I_0 at the two nearby screen points. At the first point, $\Delta_1 = \lambda/6$, so

$$\phi_1 = \frac{2\pi \lambda}{\lambda} \frac{\lambda}{6} = \frac{\pi}{3}.$$

Hence, denoting its resultant intensity by I_P ,

$$I_P = 4I_0 \cos^2 \frac{\pi}{6} = 4I_0 \frac{3}{4} = 3I_0.$$

Step 4: (ii) Calculate the second intensity and compare The second printed path difference is $\Delta_2 = \lambda/12$. Its phase difference is

$$\phi_2 = \frac{2\pi}{\lambda} \frac{\lambda}{12} = \frac{\pi}{6}.$$

Using the equivalent form $I = 2I_0(1 + \cos \phi)$ avoids squaring $\cos 15^\circ$:

$$\begin{aligned} I_Q &= 2I_0 \left(1 + \cos \frac{\pi}{6} \right) \\ &= 2I_0 \left(1 + \frac{\sqrt{3}}{2} \right) = (2 + \sqrt{3})I_0. \end{aligned}$$

The common factor I_0 cancels when the intensities are compared in the order of the stated path differences. The second point is brighter because its smaller phase difference lies closer to the constructive-interference condition.

Step 5: Conclusion (i) The resultant intensity is $I = 4I_0 \cos^2(\phi/2) = 2I_0(1 + \cos \phi)$, where I_0 is the intensity due to either wave alone. (ii) For path differences $\lambda/6$ and $\lambda/12$, respectively,

$$I_P : I_Q = 3 : (2 + \sqrt{3}).$$

Quick Tip to Solve: First convert each path difference to phase using $2\pi\Delta/\lambda$. For $\Delta = \lambda/12$, use $2I_0(1 + \cos \phi)$ to evaluate $\cos 30^\circ$ directly instead of $\cos^2 15^\circ$.