

CBSE Class 12 Physics Question Paper 2024 with Solution PDF-Set 55/5/2



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are compulsory.
- (ii) Question paper is divided into **FIVE** sections – Section **A, B, C, D** and **E**.
- (iii) In **Section A** : Question number **1** to **16** are Multiple Choice (MCQ) type questions carrying **1** mark each.
- (iv) In **Section B** : Question number **17** to **21** are Very Short Answer (VSA) type questions carrying **2** marks each.
- (v) In **Section C** : Question number **22** to **28** are Short Answer (SA) type questions carrying **3** marks each.
- (vi) In **Section D** : Question number **29 & 30** are Long Answer (LA) type questions carrying **4** marks each.
- (vii) In **Section E** : Question number **31** to **33** are Case-Based questions carrying **5** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in **1** question in Section–**B**, **1** question in Section–**C**, **2** questions in Section–**D** and **3** questions in Section–**E**.
- (ix) Use of calculators is NOT allowed.

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. An ammeter and a voltmeter are connected in series to a battery. Their readings are noted as 'A' and 'V' respectively. If a resistor is connected in parallel with the voltmeter, then: [1 mark]

- (A) A will increase, V will decrease.
- (B) A will decrease, V will increase.
- (C) Both A and V will decrease.
- (D) Both A and V will increase.

Correct Answer: (A) A will increase, V will decrease.

Solution:

Concept:

- A voltmeter measures the potential difference across its own resistance R_V ; an ammeter measures the total series current.
- Adding a parallel resistance reduces the equivalent resistance of that branch. With emf E and other series resistance r_s , current is $I = E/(r_s + R)$.

Step 1: Compare the circuit currents Initially the voltmeter branch has resistance R_V . After a resistor R_x is connected across it, the branch resistance is $R_p = R_V R_x / (R_V + R_x) < R_V$. Thus total circuit resistance falls, so the current through the series ammeter increases.

Step 2: Compare the voltmeter readings The voltage across the branch is $V = ER/(r_s + R)$, where r_s includes the finite ammeter resistance and any battery internal resistance. This expression increases with R , so replacing R_V by the smaller R_p decreases the voltmeter reading. Equivalently, the increased current causes a larger voltage drop in the other series resistance. Options (B) and (C) wrongly decrease the total current, while (D) wrongly increases the branch voltage.

Step 3: Conclusion The ammeter reading increases and the voltmeter reading decreases. Hence, the correct option is (A).

Quick Tip to Solve: Distinguish total current from branch voltage: a shunt lowers branch resistance, raising the total current while lowering its voltage.

2. An ac voltage is applied across an ideal inductor. The current in it:

[1 mark]

- (A) leads the voltage by $\left(\frac{1}{4}\right)$ cycle.
- (B) lags the voltage by $\left(\frac{1}{4}\right)$ cycle.
- (C) leads the voltage by $\left(\frac{1}{2}\right)$ cycle.
- (D) lags the voltage by $\left(\frac{1}{2}\right)$ cycle.

Correct Answer: (B) lags the voltage by $\left(\frac{1}{4}\right)$ cycle.

Solution:

Concept:

- For an ideal inductor of inductance L , the applied voltage is $v = L di/dt$.
- A phase difference of $\pi/2$ radians corresponds to one quarter of a complete cycle.

Step 1: Obtain the current from the applied voltage Let the applied sinusoidal voltage be $v = V_m \sin \omega t$. Integrating $di/dt = (V_m/L) \sin \omega t$ for the steady alternating current gives

$$i = -\frac{V_m}{\omega L} \cos \omega t = \frac{V_m}{\omega L} \sin \left(\omega t - \frac{\pi}{2} \right).$$

The current waveform therefore reaches any corresponding phase later than the voltage.

Step 2: Identify the phase relation The negative phase shift shows a lag of $\pi/2$, which is $(\pi/2)/(2\pi) = 1/4$ cycle. Option (A) reverses lead and lag; options (C) and (D) state a half-cycle phase difference, which would mean opposite-phase waveforms rather than the inductive relation.

Step 3: Conclusion The current lags the voltage by one quarter cycle. Hence, the correct option is (B).

Quick Tip to Solve: For an inductor remember that voltage leads current by 90° ; divide by 360° to express the lag in cycles.

3. An iron needle is kept near a strong bar magnet. It will experience:

[1 mark]

- (A) a force of attraction and no torque.
- (B) a force of attraction and a torque.
- (C) a torque and no force.
- (D) neither a force nor a torque.

Correct Answer: (B) a force of attraction and a torque.

Solution:

Concept:

- Iron is ferromagnetic: a nearby strong magnet induces magnetisation in it.
- A magnetic dipole in a non-uniform field experiences a force; an inclined dipole experiences an aligning torque $\tau = \mathbf{m} \times \mathbf{B}$.

Step 1: Determine the translational effect The field near a bar magnet is non-uniform and becomes

stronger near its poles. The iron needle becomes magnetised and is pulled towards the stronger-field region. The attraction at the nearer end is greater than the opposing effect at its farther end, producing a net attractive force.

Step 2: Determine the rotational effect The elongated needle also tends to align its magnetic axis with the local field. For a general initial orientation this produces a torque as well as attraction. The torque becomes zero once alignment is reached, but the question asks for the usual effects on the needle. Option (A) omits the aligning effect, and (C) and (D) omit the attractive force in the non-uniform field.

Step 3: Conclusion The iron needle experiences attraction and, in general, an aligning torque. Hence, the correct option is (B).

Quick Tip to Solve: Separate the two effects: field non-uniformity gives attraction, while an inclined magnetic axis gives an aligning torque.

4. A galvanometer shows full scale deflection for a current I_g . If a shunt of resistance S_1 is connected to the galvanometer, it gets converted into an ammeter of range $(0-I)$. When resistance of the shunt is made S_2 , its range becomes $(0-2I)$. Then $\frac{S_1}{S_2}$ is: [1 mark]

- (A) $\frac{I + I_g}{I - I_g}$
 (B) $\frac{I - I_g}{I + I_g}$
 (C) $\frac{2I - I_g}{I - I_g}$
 (D) $\frac{I - I_g}{2I - I_g}$

Correct Answer: (C) $\frac{2I - I_g}{I - I_g}$

Solution:

Concept:

- A shunt is connected parallel to a galvanometer, so their potential differences are equal.
- If galvanometer resistance is G and full-scale current is I_g , a total current I_T requires $I_g G = (I_T - I_g)S$.

Step 1: Write the two full-scale conditions At total current I , only I_g passes through the galvanometer, and the remaining $I - I_g$ passes through shunt S_1 . When the range becomes $2I$, the galvanometer still carries I_g . Thus

$$S_1 = \frac{I_g G}{I - I_g}, \quad S_2 = \frac{I_g G}{2I - I_g}.$$

Step 2: Divide the shunt resistances Division cancels the common factor $I_g G$:

$$\frac{S_1}{S_2} = \frac{2I - I_g}{I - I_g}.$$

The numerator contains $2I - I_g$, ruling out (A) and (B). Option (D) is the reciprocal; it is also less than one, whereas the larger current range requires a smaller shunt, so $S_1 > S_2$.

Step 3: Conclusion $\frac{S_1}{S_2} = \frac{2I - I_g}{I - I_g}$. Hence, the correct option is (C).

Quick Tip to Solve: Keep the galvanometer current fixed at I_g ; subtract it from each total range before taking the inverse ratio.

5. A coil of area of cross-section 0.5 m^2 is placed in a magnetic field acting normally to its plane. The field varies as $B = 0.5t^2 + 2t$, where B is in tesla and t in seconds. The emf induced in the coil at $t = 1 \text{ s}$ is: [1 mark]

- (A) 0.5 V
- (B) 1.0 V
- (C) 1.5 V
- (D) 3.0 V

Correct Answer: (C) 1.5 V

Solution:

Concept:

- Faraday's law gives the magnitude of induced emf as $|\varepsilon| = |d\Phi/dt|$.
- For a field normal to a coil of fixed area A , $\Phi = BA$, so $|\varepsilon| = A|dB/dt|$ for the stated coil.

Step 1: Differentiate the field The field varies as $B = 0.5t^2 + 2t$. Differentiating with respect to time rather than merely substituting into B gives

$$\frac{dB}{dt} = t + 2.$$

At $t = 1 \text{ s}$, the rate of change is 3 T s^{-1} .

Step 2: Calculate the induced emf magnitude The field is normal to the plane, so there is no additional angular factor. Using $A = 0.5 \text{ m}^2$,

$$|\varepsilon| = 0.5 \times 3 = 1.5 \text{ V}.$$

The value 3.0 V in (D) omits the area factor; (A) and (B) do not use the full derivative at the specified time. The positive options refer to magnitude; the Lenz-law sign specifies an orientation-dependent polarity.

Step 3: Conclusion The induced emf has magnitude 1.5 V. Hence, the correct option is (C).

Quick Tip to Solve: Differentiate $B(t)$ first, evaluate at the stated time, and only then multiply by the coil area.

6. A pure Si crystal having 5×10^{28} atoms m^{-3} is doped with 1 ppm concentration of antimony. If the concentration of holes in the doped crystal is found to be $4.5 \times 10^9 \text{ m}^{-3}$, the concentration (in m^{-3}) of intrinsic charge carriers in Si crystal is about: [1 mark]

- (A) 1.2×10^{15}
- (B) 1.5×10^{16}
- (C) 3.0×10^{15}
- (D) 2.0×10^{16}

Correct Answer: (B) 1.5×10^{16}

Solution:

Concept:

- Antimony is a pentavalent donor in silicon; each ionised donor supplies approximately one free electron.
- At thermal equilibrium, the mass-action law is $np = n_i^2$, where n , p and n_i are electron, hole and intrinsic carrier concentrations.

Step 1: Find the donor electron concentration One part per million corresponds to a fraction 10^{-6} . Thus the donor concentration, approximately equal to the majority electron concentration, is

$$n \simeq N_D = (5 \times 10^{28})(10^{-6}) = 5 \times 10^{22} \text{ m}^{-3}.$$

The given hole concentration is negligible compared with this donor concentration.

Step 2: Use the mass-action law Using $p = 4.5 \times 10^9 \text{ m}^{-3}$,

$$n_i = \sqrt{np} = \sqrt{(5 \times 10^{22})(4.5 \times 10^9)} = \sqrt{2.25 \times 10^{32}} = 1.5 \times 10^{16} \text{ m}^{-3}.$$

Squaring this result reproduces the required product np . The smaller concentrations in (A) and (C) give products too small, while (D) gives 4×10^{32} , too large.

Step 3: Conclusion The intrinsic carrier concentration is approximately $1.5 \times 10^{16} \text{ m}^{-3}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert ppm to 10^{-6} , then use $n_i = \sqrt{np}$; rewrite the power of ten as an even exponent before taking the root.

7. The potential energy between two nucleons inside a nucleus is minimum at a distance of about:

[1 mark]

- (A) 0.8 fm
- (B) 1.6 fm
- (C) 2.0 fm
- (D) 2.8 fm

Correct Answer: (A) 0.8 fm

Solution:

Concept:

- The nuclear interaction has a strongly repulsive core at very small nucleon separations and an attractive region at larger separations.
- The minimum of the nucleon potential-energy curve occurs near the equilibrium separation, where the force $F = -dU/dr$ changes sign.

Step 1: Locate the bottom of the potential well For the standard nucleon interaction potential used in the Class 12 description, the potential energy has its minimum at a separation of approximately 0.8 fm. At smaller separations the repulsive core makes the potential rise steeply; beyond the minimum the attractive part approaches zero as separation grows.

Step 2: Distinguish minimum position from interaction range The separation giving the most negative potential energy is not the same as the distance over which nuclear attraction is appreciable. Values such as 1.6 fm, 2.0 fm and 2.8 fm lie farther along the attractive tail and do not represent the bottom of the standard potential well. Here $1 \text{ fm} = 10^{-15} \text{ m}$.

Step 3: Conclusion The potential energy is minimum at about 0.8 fm. Hence, the correct option is (A).

Quick Tip to Solve: Identify the bottom of the nuclear potential well near 0.8 fm, rather than confusing it with the few-femtometre force range.

8. In a Young's double-slit experiment in air, the fringe width is found to be 0.44 mm. If the entire setup is immersed in water $\left(n = \frac{4}{3}\right)$, the fringe width will be:

[1 mark]

- (A) 0.88 mm
- (B) 0.59 mm
- (C) 0.33 mm
- (D) 0.44 mm

Correct Answer: (C) 0.33 mm

Solution:

Concept:

- Young's fringe width is $\beta = \lambda D/d$, where D is the slit-to-screen distance and d is the slit separation.
- In a medium of refractive index n , wavelength becomes λ/n ; the frequency and the apparatus dimensions remain unchanged.

Step 1: Compare the fringe widths Immersion changes only the wavelength in the fringe-width

formula. Therefore

$$\frac{\beta_{\text{water}}}{\beta_{\text{air}}} = \frac{1}{n}.$$

Since water has $n > 1$, the fringe pattern contracts rather than expands.

Step 2: Substitute and check the choices

$$\beta_{\text{water}} = \frac{0.44}{4/3} = 0.44 \times \frac{3}{4} = 0.33 \text{ mm}.$$

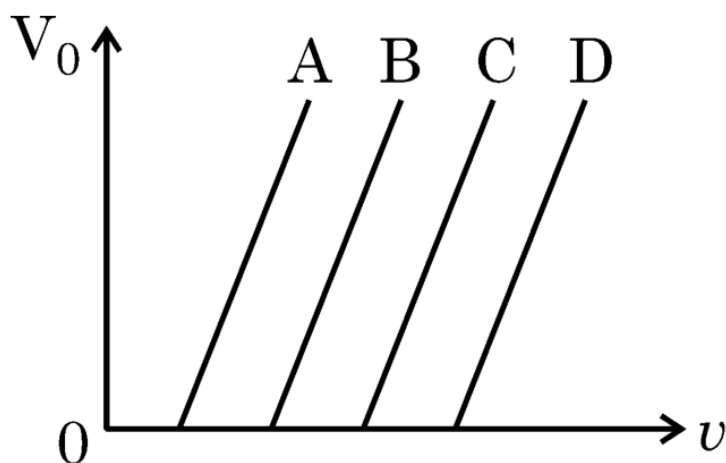
Both 0.88 mm and 0.59 mm imply an increase, while 0.44 mm incorrectly assumes that the wavelength stays unchanged.

Step 3: Conclusion The fringe width in water is 0.33 mm. Hence, the correct option is (C).

Quick Tip to Solve: For complete immersion with fixed slit geometry, divide the air fringe width by the refractive index.

9. The variation of the stopping potential (V_0) with the frequency (ν) of the incident radiation for four metals A, B, C and D is shown in the figure. For the same frequency of incident radiation producing photo-electrons in all metals, the kinetic energy of photo-electrons will be maximum for metal

[1 mark]



- (A) A
- (B) B
- (C) C
- (D) D

Correct Answer: (A) A

Solution:

Concept:

- Einstein's photoelectric equation gives $K_{\text{max}} = h\nu - \phi = eV_0$, where ϕ is the metal's work function and V_0 its stopping potential.
- The frequency-axis intercept is the threshold frequency $\nu_0 = \phi/h$. All four lines have the same slope h/e .

Step 1: Read the threshold frequencies The graph places the frequency-axis intercepts in the order

$\nu_{0A} < \nu_{0B} < \nu_{0C} < \nu_{0D}$. Thus metal A has the smallest work function, because multiplying each threshold frequency by the same positive constant h preserves this order.

Step 2: Compare the energies at one frequency For a common incident frequency above all four thresholds, the photon energy $h\nu$ is the same. The smallest work function leaves the largest maximum kinetic energy. Equivalently, a vertical line at this frequency meets line A at the greatest V_0 . Metals B, C and D have successively larger work functions and hence lower maximum kinetic energies.

Step 3: Conclusion Photoelectrons have the greatest maximum kinetic energy for metal A. Hence, the correct option is (A).

Quick Tip to Solve: On parallel stopping-potential graphs, the leftmost threshold means the greatest photoelectron energy at a fixed frequency.

10. The energy of an electron in the ground state of hydrogen atom is -13.6 eV. The kinetic and potential energy of the electron in the first excited state will be: [1 mark]

- (A) -13.6 eV, 27.2 eV
- (B) -6.8 eV, 13.6 eV
- (C) 3.4 eV, -6.8 eV
- (D) 6.8 eV, -3.4 eV

Correct Answer: (C) 3.4 eV, -6.8 eV

Solution:

Concept:

- The hydrogen energy levels obey $E_n = -13.6/n^2$ eV, with the first excited state corresponding to $n = 2$.
- For a bound electron in a Coulomb orbit, $U = -2K$ and $E = K + U = -K$, so $K = -E$ and $U = 2E$.

Step 1: Find the first excited-state energy The ground state is $n = 1$, so the next state is $n = 2$, not $n = 1$. Its total energy is

$$E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV.}$$

Step 2: Separate kinetic and potential energies

$$K = -E_2 = 3.4 \text{ eV,} \quad U = 2E_2 = -6.8 \text{ eV.}$$

Their sum is -3.4 eV, as required. Options (A) and (B) incorrectly give negative kinetic energy. Option (D) violates the Coulomb relation $U = -2K$ and would give positive total energy.

Step 3: Conclusion The kinetic and potential energies are respectively 3.4 eV and -6.8 eV. Hence, the correct option is (C).

Quick Tip to Solve: For any hydrogen orbit, write $K = -E$ and $U = 2E$ after selecting the correct principal quantum number.

11. The electromagnetic waves used to purify water are:

[1 mark]

- (A) Infrared rays
- (B) Ultraviolet rays
- (C) X-rays
- (D) Gamma rays

Correct Answer: (B) Ultraviolet rays

Solution:

Concept:

- Ultraviolet radiation has a germicidal action and is used to disinfect water by inactivating disease-causing microorganisms.
- Different regions of the electromagnetic spectrum have characteristic applications; infrared is mainly associated with heating, while X-rays are commonly used for imaging.

Step 1: Identify the required action Water purification in this question refers to disinfection. Ultraviolet radiation damages the genetic material of microorganisms, preventing their reproduction and making it useful for water-treatment systems. This action does not imply that irradiation removes every dissolved impurity.

Step 2: Distinguish the other radiation types Infrared rays primarily provide heating. X-rays are associated with medical imaging, and gamma rays with applications such as radiotherapy and sterilisation of medical equipment. Although other energetic radiation can also damage cells, ultraviolet rays are the standard water-purification application asked here.

Step 3: Conclusion Ultraviolet rays are used to purify water by disinfection. Hence, the correct option is (B).

Quick Tip to Solve: Associate ultraviolet lamps with germicidal water treatment; do not confuse this application with infrared heating.

12. The focal lengths of the objective and the eyepiece of a compound microscope are 1 cm and 2 cm respectively. If the tube length of the microscope is 10 cm, the magnification obtained by the microscope for most suitable viewing by relaxed eye is:

[1 mark]

- (A) 250
- (B) 200
- (C) 150
- (D) 125

Correct Answer: (D) 125

Solution:

Concept:

- For relaxed-eye viewing, a compound microscope is in normal adjustment: the final image is at infinity.
- The standard microscope approximation gives magnifying power $|M| = (L/f_o)(D/f_e)$, where L is the tube length, f_o , f_e are the objective and eyepiece focal lengths, and $D = 25$ cm.

Step 1: Select the relaxed-eye formula The eyepiece views the objective's real enlarged image at

its focal plane. Its angular magnification is therefore D/f_e ; the extra factor $1 + D/f_e$ applies to an image at the near point and is inappropriate here. The objective's approximate linear magnification magnitude is L/f_o .

Step 2: Calculate the magnifying power Using centimetres for all lengths,

$$|M| = \frac{10}{1} \times \frac{25}{2} = 10 \times 12.5 = 125.$$

The alternatives 250, 200, and 150 do not follow from the normal-adjustment formula with the supplied focal lengths. The image is inverted, but the choices give the magnitude of magnification.

Step 3: Conclusion The magnification for a relaxed eye is 125. Hence, the correct option is (D).

Quick Tip to Solve: The words 'relaxed eye' mean image at infinity: use D/f_e for the eyepiece, with $D = 25$ cm.

13. Assertion (A): An alpha particle is moving towards a gold nucleus. The impact parameter is maximum for the scattering angle of 180° .

Reason (R): The impact parameter in an alpha particle scattering experiment does not depend on the atomic number of the target nucleus. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).
 (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
 (C) Assertion (A) is true but Reason (R) is false.
 (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (D) Both Assertion (A) and Reason (R) are false.

Solution:

Concept:

- The impact parameter b is the perpendicular distance from the target nucleus to the alpha particle's initial straight-line path.
- Rutherford scattering gives $b = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{K} \cot(\theta/2)$, where Z is the target atomic number, K the incident alpha-particle kinetic energy and θ the scattering angle.

Step 1: Test the assertion At $\theta = 180^\circ$, $\cot(\theta/2) = \cot 90^\circ = 0$, so $b = 0$. This is a head-on approach and gives the minimum impact parameter, not the maximum. The assertion is false.

Step 2: Test the reason For fixed incident kinetic energy and scattering angle, the Rutherford relation shows that b is proportional to Z . Thus the impact parameter associated with a specified deflection depends on the target's atomic number. The reason is false in the scattering relation being tested.

Step 3: Check the explanatory connection The reversal at 180° occurs for a head-on trajectory, while changing the nuclear charge changes the deflection of a given incident trajectory. The reason therefore neither states the correct dependence nor explains the assertion. Options (A), (B), and (C) all require a true assertion, so each is excluded.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: Remember that 180° scattering is head-on: its impact parameter is zero. Compare atomic-number dependence at fixed energy and angle.

14. Assertion (A): In a Young's double-slit experiment, interference pattern is not observed when two coherent sources are infinitely close to each other.

Reason (R): The fringe width is proportional to the separation between the two sources. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (C) Assertion (A) is true but Reason (R) is false.

Solution:

Concept:

- The fringe width in Young's experiment is $\beta = \lambda D/d$, with wavelength λ , screen distance D , and source separation d .
- Distinct interference fringes require the phase difference, and hence the path difference, to vary appreciably across the observation screen.

Step 1: Test the assertion When the coherent sources become infinitely close, $d \rightarrow 0$, and $\beta \rightarrow \infty$. The path difference $\Delta \approx dy/D$ tends to zero at each finite screen coordinate y . Thus distinct alternating bright and dark fringes cannot be observed on a finite screen. The assertion is true; it does not mean that superposition of the waves ceases.

Step 2: Test the reason With λ and D fixed, $\beta \propto 1/d$. Fringe width is inversely proportional to the separation of the sources, whereas the reason states direct proportionality. The reason is false.

Step 3: Check whether the reason explains the assertion It is the increase of fringe width without limit, caused by decreasing d , that accounts for the loss of a visible fringe pattern. The given reason has the dependence reversed and cannot explain the assertion. Options (A) and (B) require a true reason; option (D) requires a false assertion.

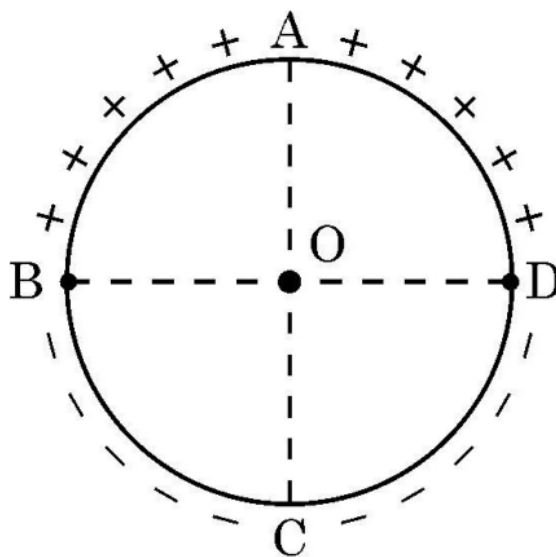
Step 4: Conclusion The assertion is true but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Start with $\beta = \lambda D/d$: reducing the slit separation broadens the fringes, and $d \rightarrow 0$ sends their spacing to infinity.

15. Assertion (A): Equal amount of positive and negative charges are distributed uniformly on two halves of a thin circular ring as shown in figure. The resultant electric field at the centre O of the ring is along OC.

Reason (R): It is so because the net potential at O is not zero.

[1 mark]



- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).
 (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
 (C) Assertion (A) is true but Reason (R) is false.
 (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (C) Assertion (A) is true but Reason (R) is false.

Solution:

Concept:

- Electric fields add vectorially. At O, the field due to positive charge points away from the charge, whereas that due to negative charge points towards it.
- Potential adds algebraically: $V = k \int dq/r$, where $k = 1/(4\pi\epsilon_0)$ and r is the distance from the charge element.

Step 1: Evaluate the assertion using symmetry For the positively charged upper semicircle, horizontal components from symmetric charge elements cancel, while vertical components point downward along OC. For the negatively charged lower semicircle, horizontal components also cancel and vertical components again point along OC. Both halves therefore produce a nonzero resultant field along OC. Assertion (A) is true.

Step 2: Evaluate the reason by adding potentials Let the ring radius be r , with charges $+Q$ and $-Q$ on its two halves. Every charge element is at the same distance from O, so

$$V_O = \frac{kQ}{r} + \frac{k(-Q)}{r} = 0.$$

The statement that the net potential is not zero is false.

Step 3: Check whether the reason explains the assertion A zero potential at one point does not imply a zero electric field there; field depends on the spatial variation of potential. The asserted direction

follows from vector addition and symmetry, not from the false reason. Options (A) and (B) require a true reason, and option (D) incorrectly calls the assertion false.

Step 4: Conclusion The assertion is true but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: At the centre of a ring, add signed charges for potential, but add directed field components for electric field.

16. Assertion (A): The energy of a charged particle moving in a magnetic field does not change.

Reason (R): It is because the work done by the magnetic force on the charge moving in a magnetic field is zero. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).

Solution:

Concept:

- The magnetic Lorentz force is $\vec{F}_B = q(\vec{v} \times \vec{B})$, perpendicular to the instantaneous velocity $\vec{v}$.
- The work-energy theorem gives $dK/dt = \vec{F} \cdot \vec{v}$, where K is the kinetic energy of the particle.

Step 1: Evaluate the assertion A magnetic force can change the direction of motion, but it cannot change the speed of a charged particle. Thus the kinetic energy, $K = mv^2/2$, remains constant under the magnetic force alone. In the stated magnetic-field situation, the assertion is true.

Step 2: Evaluate the reason The instantaneous power delivered by the magnetic force is

$$P_B = \vec{F}_B \cdot \vec{v} = q(\vec{v} \times \vec{B}) \cdot \vec{v} = 0.$$

A cross product is perpendicular to $\vec{v}$, so its dot product with $\vec{v}$ vanishes. Consequently, the magnetic force does zero work throughout the motion. The reason is true.

Step 3: Connect the reason with the assertion Since the magnetic work is zero, the work-energy theorem gives $\Delta K = W_B = 0$. This directly explains the constant energy asserted. Option (B) misses this causal connection, while options (C) and (D) wrongly declare one or both statements false.

Step 4: Conclusion Both statements are true, and the reason correctly explains the assertion. Hence, the correct option is (A).

Quick Tip to Solve: Test energy changes using $\vec{F} \cdot \vec{v}$: a magnetic force gives zero even when the trajectory curves.

Section B (10 marks)

17(a). Four point charges of $1\ \mu\text{C}$, $-2\ \mu\text{C}$, $1\ \mu\text{C}$ and $-2\ \mu\text{C}$ are placed at the corners A, B, C and D respectively, of a square of side 30 cm. Find the net force acting on a charge of $4\ \mu\text{C}$ placed at the centre of the square. [2 marks]

Solution:

Concept:

- Coulomb's law gives the force magnitude $F = k|qQ|/r^2$, where r is the charge separation and $k = 9 \times 10^9\ \text{N m}^2\text{C}^{-2}$.
- By superposition, the net force is the vector sum of the four individual forces. Equal charges at opposite vertices of a square produce equal and opposite forces at its centre.

Step 1: Find the common distance from the centre Let O be the square's centre and $a = 30\ \text{cm} = 0.30\ \text{m}$. Since each diagonal has length $a\sqrt{2}$,

$$OA = OB = OC = OD = \frac{a}{\sqrt{2}}, \quad r^2 = \frac{a^2}{2} = 0.045\ \text{m}^2.$$

The test charge is $Q = 4 \times 10^{-6}\ \text{C}$. Vertices A and C are opposite, as are B and D.

Step 2: Add the forces from each opposite pair The equal positive charges at A and C repel the test charge along opposite directions on diagonal AC. Each force has magnitude

$$F_A = F_C = \frac{(9 \times 10^9)(1 \times 10^{-6})(4 \times 10^{-6})}{0.045} = 0.80\ \text{N}.$$

Hence $\vec{F}_A + \vec{F}_C = \vec{0}$. The equal negative charges at B and D attract the test charge towards B and D respectively. Each force has magnitude 1.60 N, but their directions are opposite on diagonal BD, giving $\vec{F}_B + \vec{F}_D = \vec{0}$. The difference between the magnitudes of the positive and negative corner charges does not matter: cancellation occurs within each opposite pair.

Step 3: Conclusion The net force on the central charge is $\vec{F}_{\text{net}} = \vec{0}$, with magnitude 0 N.

Quick Tip to Solve: Pair opposite corners first: identical charges on each diagonal cancel at the centre without resolving components.

OR

17(b). Three point charges, 1 pC each, are kept at the vertices of an equilateral triangle of side 10 cm. Find the net electric field at the centroid of triangle. [2 marks]

Solution:

Concept:

- The field of a point charge has magnitude $E = k|q|/r^2$, directed away from a positive charge.
- In an equilateral triangle the centroid is equidistant from the vertices. The three vertex-to-centroid directions are separated by 120° , so equal field vectors along them cancel.

Step 1: Calculate the magnitude of each field The printed charge is $q = 1\ \text{pC} = 10^{-12}\ \text{C}$, and the side is $a = 10\ \text{cm} = 0.10\ \text{m}$. The centroid divides each median in the ratio 2 : 1, so its distance from a

vertex is

$$r = \frac{2}{3} \left(\frac{\sqrt{3}a}{2} \right) = \frac{a}{\sqrt{3}}.$$

Therefore each charge produces the same field magnitude at the centroid:

$$E_0 = \frac{kq}{r^2} = \frac{3kq}{a^2} = \frac{3(9 \times 10^9)(10^{-12})}{(0.10)^2} = 2.7 \text{ N C}^{-1}.$$

Step 2: Add the three field vectors Choose the positive x -axis along one field vector. The other two then make angles 120° and 240° with it. The components add as

$$E_x = E_0(1 + \cos 120^\circ + \cos 240^\circ) = E_0(1 - \frac{1}{2} - \frac{1}{2}) = 0,$$

$$E_y = E_0(\sin 120^\circ + \sin 240^\circ) = E_0(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}) = 0.$$

The individual fields are nonzero, but their vector sum vanishes by the threefold symmetry.

Step 3: Conclusion The net electric field at the centroid is $\vec{E}_{\text{net}} = \vec{0}$, with magnitude $\boxed{0 \text{ N C}^{-1}}$.

Quick Tip to Solve: Three equal charges at the vertices of an equilateral triangle give zero field at its centroid because equal vectors at 120° cancel.

18. Derive an expression for magnetic force $\vec{F}$ acting on a straight conductor of length L carrying current I in an external magnetic field $\vec{B}$. Is it valid when the conductor is in zig-zag form? Justify.

[2 marks]

Solution:

Concept:

- A moving charge experiences magnetic force $\vec{f} = q(\vec{v}_d \times \vec{B})$, where $\vec{v}_d$ is its drift velocity.
- Current density is $\vec{J} = nq\vec{v}_d$, where n is carrier number density. For a wire of cross-sectional area A , $\vec{J} = (I/A)\hat{\ell}$, with $\hat{\ell}$ along conventional current.

Step 1: Sum the carrier forces in the straight wire Take a straight conductor of length L and cross-sectional area A wholly in a uniform external magnetic field. It contains $N = nAL$ mobile charge carriers. Their combined magnetic force, transmitted to the conductor, is

$$\begin{aligned}\vec{F} &= nALq(\vec{v}_d \times \vec{B}) \\ &= AL(\vec{J} \times \vec{B}) = IL(\hat{\ell} \times \vec{B}).\end{aligned}$$

Defining $\vec{L} = L\hat{\ell}$ along conventional current gives $\vec{F} = I(\vec{L} \times \vec{B})$. Its magnitude is $F = ILB \sin \theta$, where θ is the angle between the current and field; the cross product gives its direction.

Step 2: Add the forces on the zig-zag segments For a bent wire in the same uniform field, apply the formula separately to each straight segment, with $\vec{\ell}_j$ directed along current:

$$\vec{F}_{\text{net}} = \sum_j I(\vec{\ell}_j \times \vec{B}) = I \left[\left(\sum_j \vec{\ell}_j \right) \times \vec{B} \right].$$

Successive displacement vectors telescope, so $\sum_j \vec{\ell}_j = \vec{r}_{\text{end}} - \vec{r}_{\text{start}}$. Thus the vector formula remains

valid for a zig-zag wire if its length vector is the displacement between its endpoints along current, rather than the total wire length. For a nonuniform field one must instead use $I \int d\vec{\ell} \times \vec{B}(\vec{r})$.

Step 3: Conclusion For a straight wire, $\vec{F} = I(\vec{L} \times \vec{B})$. For a zig-zag wire in a uniform field, the same form applies with $\vec{L} = \vec{r}_{\text{end}} - \vec{r}_{\text{start}}$; the actual bent-wire length cannot replace this endpoint displacement.

Quick Tip to Solve: In a uniform magnetic field, join a bent wire's endpoints with an arrow along current to obtain its effective length vector.

19. The radius of curvature of a convex mirror is 30 cm. It forms an image of an object which is half the size of the object. Find the separation between the object and the image. [2 marks]

Solution:

Concept:

- In the Cartesian sign convention, a convex mirror has positive focal length $f = R/2$; a real object in front has $u < 0$, and its virtual image behind has $v > 0$.
- The mirror formula is $1/f = 1/v + 1/u$, and the transverse magnification is $m = -v/u$, where u and v are measured from the pole.

Step 1: Use the image magnification A convex mirror forms an erect, diminished image of a real object, so half the object's size means $m = +1/2$, not $-1/2$. The given radius gives

$$f = +\frac{30}{2} = +15 \text{ cm}, \quad -\frac{v}{u} = \frac{1}{2} \Rightarrow u = -2v.$$

Step 2: Locate the object and image Substitute the relation between the distances in the mirror formula:

$$\frac{1}{15} = \frac{1}{v} + \frac{1}{-2v} = \frac{1}{2v} \Rightarrow v = +7.5 \text{ cm}, \quad u = -15 \text{ cm}.$$

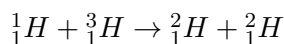
Thus the object lies in front of the mirror and the image lies behind it. Their separation is the difference of their signed coordinates, or equivalently the sum of the two positive distances from the pole:

$$d = |v - u| = |7.5 - (-15)| = 22.5 \text{ cm}.$$

Step 3: Conclusion The separation between the object and its virtual image is $\boxed{22.5 \text{ cm}}$.

Quick Tip to Solve: For a convex mirror, take the erect image's magnification as positive and add the object and image distances because they lie on opposite sides of the mirror.

20. Calculate the energy released/absorbed (in MeV) in the nuclear reaction:



Given: $m({}_1^1\text{H}) = 1.007825\text{ u}$, $m({}_1^2\text{H}) = 2.014102\text{ u}$, $m({}_1^3\text{H}) = 3.016049\text{ u}$

[2 marks]

Solution:

Concept:

- The reaction energy is $Q = (m_{\text{initial}} - m_{\text{final}})c^2$. A positive Q means energy is released; a negative Q means energy must be supplied.
- Use $1\text{ u}c^2 = 931.5\text{ MeV}$. Atomic masses may be used here because the two hydrogen atoms on either side contain the same total number of electrons, so electron rest masses cancel.

Step 1: Add the masses on each side There is one hydrogen-1 atom and one hydrogen-3 atom initially, while the products contain two hydrogen-2 atoms. Hence

$$m_{\text{initial}} = 1.007825 + 3.016049 = 4.023874\text{ u},$$

$$m_{\text{final}} = 2(2.014102) = 4.028204\text{ u}.$$

The final rest mass is larger, so the reaction cannot release energy.

Step 2: Convert the mass change into reaction energy Keep the initial-minus-final order so that the sign identifies release or absorption:

$$\Delta m = 4.023874 - 4.028204 = -0.004330\text{ u},$$

$$Q = (-0.004330)(931.5)\text{ MeV}$$

$$= -4.033395\text{ MeV} \simeq -4.03\text{ MeV}.$$

The increase in rest energy must be supplied from the available initial energy; the magnitude of the negative reaction energy is therefore the energy absorbed.

Step 3: Conclusion The reaction has $Q = -4.03\text{ MeV}$; therefore approximately 4.03 MeV of energy is absorbed.

Quick Tip to Solve: Add all reactant and product masses before multiplying by 931.5; a heavier product side immediately signals energy absorption.

21. A proton of energy 1.6 MeV approaches a gold nucleus ($Z = 79$). Find the distance of its closest approach.

[2 marks]

Solution:

Concept:

- For a head-on approach to a heavy nucleus, the proton's initial kinetic energy is completely converted to electrostatic potential energy at the turning point. The gold nucleus is treated as stationary.
- Taking the potential energy at infinity as zero, $K = kZe^2/r_{\text{min}}$, where $k = 1/(4\pi\epsilon_0)$, $Z = 79$, e is the proton charge and r_{min} is the separation of their centres at closest approach.

Step 1: Convert the incident energy to joules The proton has charge $+e$, whereas the gold nucleus

has charge $+79e$; their repulsive interaction slows the incident proton. Its energy in SI units is

$$K = 1.6 \times 10^6 \times 1.6 \times 10^{-19} = 2.56 \times 10^{-13} \text{ J.}$$

The given energy is kinetic energy far from the nucleus, where the electrostatic potential energy is negligible.

Step 2: Apply energy conservation at the turning point At the closest point in a head-on encounter the proton is momentarily at rest. Therefore

$$K = \frac{k(79e)e}{r_{\min}}, \quad r_{\min} = \frac{(9 \times 10^9)(79)(1.6 \times 10^{-19})^2}{2.56 \times 10^{-13}}.$$

The numerator is $1.82016 \times 10^{-26} \text{ J m}$, so

$$r_{\min} = 7.11 \times 10^{-14} \text{ m.}$$

This is a centre-to-centre separation; using $2e$ for the incident charge would incorrectly treat the proton as an alpha particle.

Step 3: Conclusion The distance of closest approach is $r_{\min} \simeq 7.1 \times 10^{-14} \text{ m}$, or approximately 71 fm.

Quick Tip to Solve: For a proton approaching a nucleus, use $r_{\min} = kZe^2/K$; only an alpha particle requires the extra factor of 2 in the numerator.

Section C (21 marks)

22. A photosensitive surface of work function 2.1 eV is irradiated by radiation of wavelength 150 nm. Calculate (i) the threshold wavelength, (ii) energy (in eV) of an incident photon, and (iii) maximum kinetic energy of emitted photoelectron. [3 marks]

Solution:

Concept:

- The work function ϕ is the minimum energy needed to eject an electron. At the threshold wavelength λ_0 , $hc/\lambda_0 = \phi$, where h is Planck's constant and c is the speed of light.
- Each incident photon has energy $E = hc/\lambda$. Einstein's photoelectric equation gives $K_{\max} = E - \phi$, provided $E \geq \phi$.

Step 1: Calculate the threshold wavelength Convert the work function to SI units before using $h = 6.63 \times 10^{-34} \text{ J s}$ and $c = 3 \times 10^8 \text{ m s}^{-1}$:

$$\phi = 2.1(1.6 \times 10^{-19}) = 3.36 \times 10^{-19} \text{ J.}$$

For part (i), the threshold corresponds to electrons just escaping with zero kinetic energy:

$$\lambda_0 = \frac{hc}{\phi} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{3.36 \times 10^{-19}} = 5.92 \times 10^{-7} \text{ m} \simeq 592 \text{ nm.}$$

This is the longest wavelength capable of producing photoelectric emission from this surface.

Step 2: Calculate the incident photon energy For part (ii), use the actual incident wavelength

$$\lambda = 150 \text{ nm} = 150 \times 10^{-9} \text{ m:}$$

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{150 \times 10^{-9}} = 1.326 \times 10^{-18} \text{ J.}$$

Converting to electronvolts gives

$$E = \frac{1.326 \times 10^{-18}}{1.6 \times 10^{-19}} \text{ eV} = 8.2875 \text{ eV} \simeq 8.29 \text{ eV.}$$

Since $150 \text{ nm} < \lambda_0$, this energy exceeds the work function and emission occurs.

Step 3: Find the maximum photoelectron kinetic energy For part (iii), one photon transfers its energy to one electron. After the work function is supplied, the remaining energy is the maximum possible kinetic energy:

$$K_{\max} = 8.2875 - 2.1 = 6.1875 \text{ eV} \simeq 6.19 \text{ eV.}$$

Equivalently, $K_{\max} = (6.1875)(1.6 \times 10^{-19}) = 9.90 \times 10^{-19} \text{ J}$. Electrons that lose energy inside the material can emerge with smaller kinetic energies.

Step 4: Conclusion (i) $\lambda_0 \simeq 592 \text{ nm}$; (ii) $E \simeq 8.29 \text{ eV}$; (iii) $K_{\max} \simeq 6.19 \text{ eV}$, or $9.90 \times 10^{-19} \text{ J}$.

Quick Tip to Solve: Use the work function to find the threshold wavelength, but use the incident wavelength to find photon energy; subtract the work function only after putting both energies in the same units.

23(a). (i) **State Lenz's Law.** In a closed circuit, the induced current opposes the change in magnetic flux that produced it as per the law of conservation of energy. Justify.

(ii) A metal rod of length 2 m is rotated with a frequency 60 rev/s about an axis passing through its centre and perpendicular to its length. A uniform magnetic field of 2 T perpendicular to its plane of rotation is switched-on in the region. Calculate the e.m.f. induced between the centre and the end of the rod. [3 marks]

Solution:

Concept:

- Lenz's law gives the direction of the induced current: its magnetic effect opposes the change in magnetic flux that produces it. Thus $\mathcal{E} = -d\Phi_B/dt$ for a single-turn circuit.
- A rotating radial conductor has local speed $v = \omega r$. In a perpendicular uniform field B , the magnitude of the motional emf of a small radial element is $d\mathcal{E} = B\omega r dr$.

Step 1: State the law and connect it with energy conservation (i) The induced current flows in such a direction that its magnetic field opposes the change in magnetic flux responsible for induction. For example, when a magnet approaches a closed conducting coil, the induced current makes the nearer face repel the approaching magnet. An external agent must therefore do work against this opposing force. That mechanical work supplies the electrical energy, which may appear as Joule heat in the circuit.

If the induced current instead aided the change, the approaching magnet would be accelerated while current and heat were also being produced, without an external energy supply. The growing flux change would reinforce the current further, contradicting conservation of energy. Lenz's law prevents such creation of energy; it opposes the change in flux, not necessarily the existing flux itself.

Step 2: Identify the rotating length and angular speed (ii) The rotation axis passes through the centre,

so the distance from the centre to either end is half the full rod length:

$$a = \frac{2}{2} = 1 \text{ m}, \quad \omega = 2\pi f = 2\pi(60) = 120\pi \text{ rad s}^{-1}.$$

The speed rises from zero at the centre to ωa at the end. Consequently, the full rod length cannot be substituted as the radius of rotation.

Step 3: Integrate the motional emf from centre to end The magnetic force on charges in the rotating rod separates charge along its length. Integrating the small contributions over one half of the rod gives

$$\begin{aligned} |\mathcal{E}| &= \int_0^a B\omega r \, dr = \frac{1}{2} B\omega a^2 \\ &= \frac{1}{2} (2)(120\pi)(1)^2 \text{ V} = 120\pi \text{ V}. \end{aligned}$$

This is the centre-to-end emf after the uniform field has been established; the question supplies no switching time from which to calculate a transient emf.

Step 4: Conclusion Lenz's law ensures that the external work against the induced opposition supplies the electrical energy. The induced emf magnitude between the centre and either end is $120\pi \text{ V} \approx 377 \text{ V}$. Its polarity depends on the directions of rotation and magnetic field, which are not specified.

Quick Tip to Solve: For rotation about the centre, use half the rod length in $\mathcal{E} = \frac{1}{2} B\omega a^2$; using the full length makes the centre-to-end result four times too large.

OR

23(b). (i) State and explain Ampere's circuital law.

(ii) Two long straight parallel wires separated by 20 cm, carry 5 A and 10 A current respectively, in the same direction. Find the magnitude and direction of the net magnetic field at a point midway between them.

[3 marks]

Solution:

Concept:

- Ampere's circuital law for steady currents states that $\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$, where I_{enc} is the algebraic current through a surface bounded by the closed path C .
- The magnetic field at perpendicular distance r from a long straight wire carrying current I has magnitude $B = \mu_0 I / (2\pi r)$. Its direction follows the right-hand thumb rule.

Step 1: State and explain Ampere's circuital law (i) The line integral of the magnetic field around any closed path equals μ_0 times the net current enclosed by that path:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}.$$

Here $d\vec{l}$ is a directed element tangent to the chosen path and μ_0 is the permeability of free space. Each contribution is the tangential component of $\vec{B}$ multiplied by the path-element length. Current signs are chosen using the right-hand rule: curled fingers follow the traversal direction and the thumb gives the positive surface normal. Currents crossing the surface in opposite directions enter with opposite signs.

For a circular path around one long straight wire, symmetry makes B constant and tangent everywhere; hence $B(2\pi r) = \mu_0 I$, yielding the stated straight-wire field formula.

Step 2: Calculate both fields at the midpoint (ii) The midpoint is equally distant from the wires:

$$r = \frac{20 \text{ cm}}{2} = 0.10 \text{ m}.$$

Taking $I_1 = 5 \text{ A}$, $I_2 = 10 \text{ A}$, and $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$,

$$B_1 = \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 0.10} = 1.0 \times 10^{-5} \text{ T},$$

$$B_2 = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.10} = 2.0 \times 10^{-5} \text{ T}.$$

Step 3: Combine the fields with their directions Since the parallel currents have the same direction, their magnetic fields at a point between the wires point in opposite directions. Thus their magnitudes must be subtracted:

$$B_{\text{net}} = B_2 - B_1 = (2.0 - 1.0) \times 10^{-5} \text{ T}.$$

The net field follows the field of the 10 A wire. To make the direction explicit, place the 5 A wire on the left and the 10 A wire on the right with both currents upward in the page. At the midpoint the first field is into the page, and the second is out of the page, so the resultant is out of the page for this stated orientation.

Step 4: Conclusion Ampere's law is $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$. The midpoint field is $1.0 \times 10^{-5} \text{ T} = 10 \mu\text{T}$, perpendicular to the plane containing the wires and in the direction of the field due to the 10 A wire.

Quick Tip to Solve: At the midpoint of same-direction parallel currents, subtract the field magnitudes and use the larger current's right-hand-rule direction.

24. (i) Define 'temperature coefficient of resistance' of a metal.

(ii) Show the variation of resistivity of copper with rise in temperature.

(iii) The resistance of a wire is 10Ω at 27°C . Find its resistance at -73°C . The temperature coefficient of resistance of the material of the wire is $1.70 \times 10^{-4} ^\circ\text{C}^{-1}$. [3 marks]

Solution:

Concept:

- The temperature coefficient of resistance at a reference temperature T_0 is the fractional change in resistance per unit rise in temperature: $\alpha = (1/R_0)(dR/dT)_{T_0}$. Its unit is K^{-1} or $^\circ\text{C}^{-1}$.
- Over a range where the coefficient can be treated as constant, $R = R_0[1 + \alpha(T - T_0)]$. A metal such as copper has a positive temperature coefficient; its resistivity also increases approximately linearly over an ordinary temperature range.

Step 1: Define the temperature coefficient (i) The temperature coefficient of resistance of a metal is the increase in its resistance per unit original resistance per degree rise in temperature, referred to a specified reference temperature. If R_0 is its resistance at T_0 , then for a small temperature change ΔT ,

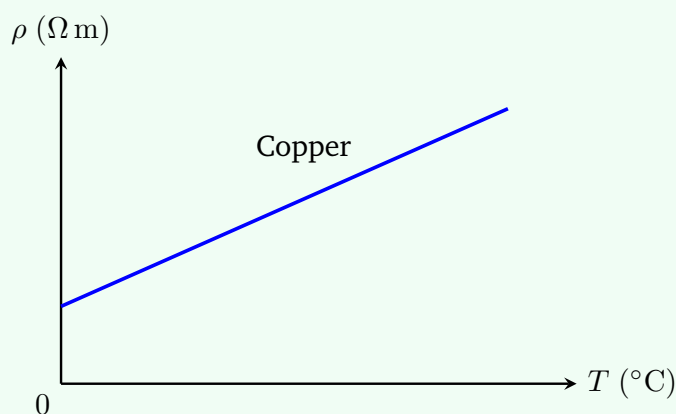
$$\alpha \simeq \frac{R - R_0}{R_0 \Delta T}.$$

A positive coefficient means that heating increases resistance and cooling decreases it. The numerical size of a temperature interval is the same in kelvin and degrees Celsius.

Step 2: Show the variation of copper resistivity (ii) As copper is heated, stronger lattice vibrations increase the scattering of conduction electrons. Its resistivity ρ therefore rises with temperature. In the usual approximately linear range,

$$\rho(T) = \rho(T_0)[1 + \alpha(T - T_0)].$$

The graph of ρ against T consequently has a positive slope. The sketch below represents this ordinary temperature range; the straight-line approximation is not intended as an extrapolation all the way to absolute zero.



Qualitative rise of copper resistivity with temperature in its approximately linear range.

Step 3: Use the actual fall in temperature (iii) Use the given initial resistance as the reference: $R_0 = 10 \, \Omega$ at $T_0 = 27^\circ\text{C}$. The final temperature is -73°C , so

$$\Delta T = T - T_0 = -73 - 27 = -100^\circ\text{C}.$$

The negative temperature change must be retained because the wire is being cooled. Taking the supplied coefficient as constant for the stated calculation,

$$\begin{aligned} R &= 10[1 + (1.70 \times 10^{-4})(-100)] \, \Omega \\ &= 10(1 - 0.0170) \, \Omega \\ &= 10(0.9830) \, \Omega. \end{aligned}$$

The fractional decrease is 0.0170, or 1.70%; this provides a check that the resistance should be slightly below its initial value.

Step 4: Conclusion The temperature coefficient measures the fractional resistance change per degree rise at a reference temperature. Copper's resistivity increases approximately linearly with temperature in the ordinary range, as shown. The wire's resistance at -73°C is $9.83 \, \Omega$.

Quick Tip to Solve: Calculate $\Delta T = T_{\text{final}} - T_{\text{initial}}$ first: cooling from 27°C to -73°C is a change of -100°C , not -73°C .

25. Name the part of the electromagnetic spectrum which are:

- (i) stopped by face mask worn by welders.
- (ii) used in detectors in Earth satellites.
- (iii) used in 'short-wave band' in communication.

Also write the order of wavelengths, in each case.

[3 marks]

Solution:

Concept:

- Electromagnetic bands are identified by their wavelength λ , and their uses depend on the interaction of the radiation with matter.
- Wavelength and frequency are related by $c = \nu\lambda$, where c is the speed of light and ν is frequency. Band boundaries are approximate; an order of wavelength is a representative power of ten in metres.

Step 1: Identify the radiation blocked by a welder's mask (i) A welding arc emits intense **ultraviolet radiation**. This can damage the eyes and exposed skin, so the protective face mask absorbs it. Ultraviolet wavelengths are approximately 10^{-9} m to 4×10^{-7} m; a representative order is 10^{-8} m. The protection is associated with the short-wavelength ultraviolet component of the arc.

Step 2: Identify the radiation sensed by Earth satellites (ii) **Infrared radiation** is used in detectors aboard Earth satellites. Objects on Earth emit thermal radiation, and infrared detectors can sense this emission for observations such as cloud and surface-temperature mapping. Infrared extends roughly from 7×10^{-7} m to 10^{-3} m; a representative thermal-infrared wavelength is of order 10^{-5} m.

Step 3: Identify the short-wave communication band (iii) The short-wave band is part of the **radio-wave** region. Its wavelengths are approximately 10 m to 100 m, corresponding to frequencies of about 30 MHz to 3 MHz. Thus the wavelength scale is tens of metres, or 10^1 to 10^2 m. These waves are useful for long-distance communication by ionospheric propagation.

Step 4: Conclusion (i) Ultraviolet: representative order 10^{-8} m. (ii) Infrared: representative order 10^{-5} m. (iii) Radio waves: short-wave wavelengths 10–100 m.

Quick Tip to Solve: Link welding to ultraviolet, thermal satellite sensing to infrared, and the short-wave communication band to radio waves; write every wavelength in metres.

26. (a) Explain the characteristics of a p-n junction diode that makes it suitable for its use as a rectifier.

(b) With the help of a circuit diagram, explain the working of a full wave rectifier.

[3 marks]

Solution:

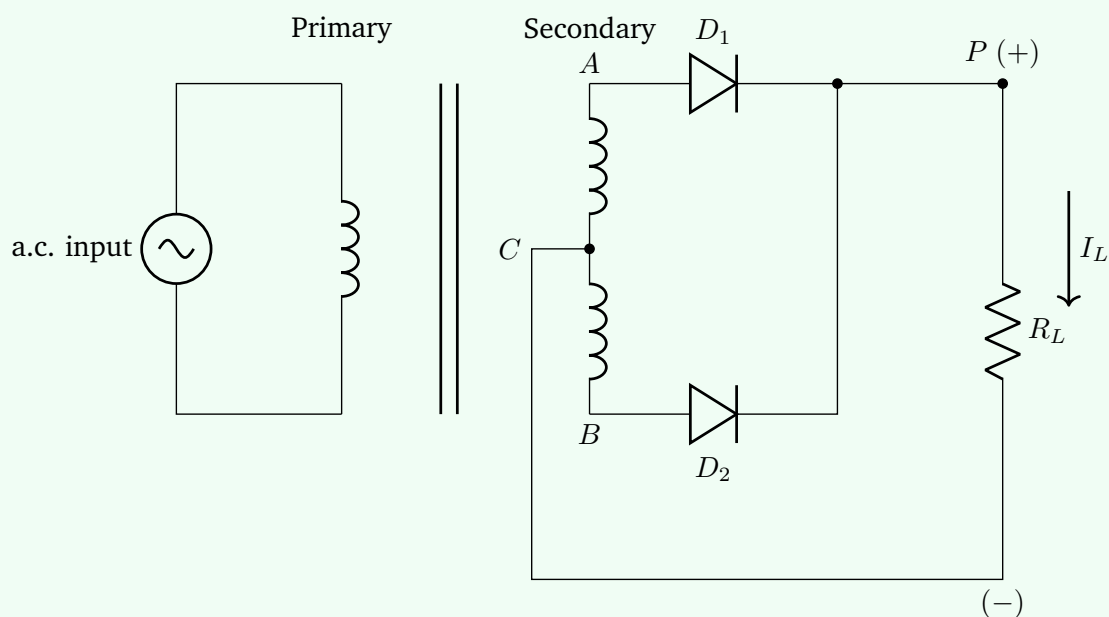
Concept:

- A p-n junction diode conducts appreciably in forward bias but has only a very small reverse saturation current in reverse bias, below breakdown. It therefore acts as a unidirectional switch.
- Rectification converts alternating current into a unidirectional current. A centre-tapped full-wave rectifier uses two diodes that conduct on alternate half-cycles and drive current through the load in the same direction.

Step 1: Explain the rectifying characteristic (a) Forward bias reduces the junction potential barrier and permits a large current once the applied voltage exceeds the effective turn-on voltage. The diode

then offers low resistance. Reverse bias increases the barrier and gives a very high resistance, with only a small minority-carrier leakage current. This strong asymmetry allows one polarity of current to pass while blocking the opposite polarity, which makes the diode suitable for rectification. Operation must remain below the reverse-breakdown voltage.

Step 2: Connect the centre-tapped rectifier (b) Connect the ends A and B of a centre-tapped transformer secondary to the anodes of D_1 and D_2 . Join their cathodes at the positive output terminal P . Connect the load R_L between P and the centre tap C . The two half-secondary voltages, measured relative to C , are equal in magnitude and opposite in phase. The diagram shows the complete a.c. input and load-return connections.



Centre-tapped full-wave rectifier; current through the load is downward in both half-cycles.

Step 3: Follow the first half-cycle When A is positive and B is negative relative to C , D_1 is forward biased and D_2 is reverse biased. Conventional current follows $A \rightarrow D_1 \rightarrow P \rightarrow R_L \rightarrow C$, returning through the upper half of the secondary. Thus current through the load flows from P towards C .

Step 4: Follow the next half-cycle When the secondary polarity reverses, B is positive relative to C . Now D_2 conducts and D_1 blocks. The current path is $B \rightarrow D_2 \rightarrow P \rightarrow R_L \rightarrow C$, returning through the lower half-secondary. The load current still flows from P towards C . Both input half-cycles therefore contribute positive output pulses; for input frequency f , the output pulse frequency is $2f$.

Step 5: Conclusion A diode's low forward resistance and high reverse resistance provide rectification. The two diodes conduct alternately in the full-wave circuit, producing a unidirectional, pulsating output across R_L during both half-cycles.

Quick Tip to Solve: Trace each half-cycle from its positive secondary end through the conducting diode and load back to the centre tap; both paths must cross the load in the same direction.

27. Explain the following, giving reasons:

(a) A doped semiconductor is electrically neutral.

(b) In a p-n junction under equilibrium, there is no net current.

(c) In a diode, the reverse current is practically not dependent on the applied voltage.

[3 marks]

Solution:

Concept:

- Doping changes the number and type of mobile charge carriers, but adding neutral impurity atoms does not add a net electric charge to the crystal.
- Across a p-n junction, diffusion due to a carrier concentration gradient competes with drift due to the electric field of the depletion region.
- Below breakdown and at fixed temperature, reverse current is controlled mainly by the supply of thermally generated minority carriers, giving an approximately constant reverse saturation current.

Step 1: Account for the charge after doping (a) In an n-type semiconductor, each donor that releases a mobile electron becomes an immobile positive ion. The electron's negative charge is balanced by the donor ion's positive charge. In a p-type semiconductor, an acceptor captures an electron and becomes an immobile negative ion, while a mobile hole of equal positive charge is produced. Thus in either case the total positive and negative charges remain equal. The terms n-type and p-type identify the majority carriers; they do not describe the net charge of the sample.

Step 2: Balance diffusion and drift at equilibrium (b) Electrons tend to diffuse from the n side to the p side, while holes diffuse from the p side to the n side, due to the concentration differences. Uncovered ions create a depletion region with an electric field directed from the n side to the p side. This field produces a drift current opposite to the diffusion current. At thermal equilibrium these two currents have equal magnitudes, so with a consistent sign convention,

$$I_{\text{net}} = I_{\text{diffusion}} + I_{\text{drift}} = 0.$$

Individual carrier motions still occur; it is their opposing contributions to current that cancel.

Step 3: Explain reverse-current saturation (c) Reverse bias increases the junction barrier and suppresses majority-carrier diffusion. The remaining current is mainly due to thermally generated minority carriers that the junction field sweeps across the depletion region. At a fixed temperature their supply is approximately fixed. Increasing the reverse voltage collects these available carriers but does not greatly increase their generation, so the current remains close to the reverse saturation value I_s . This explanation applies before reverse breakdown; at breakdown the reverse current rises sharply and is no longer nearly voltage independent.

Step 4: Conclusion Doped semiconductors remain neutral because mobile-carrier charge is balanced by impurity-ion charge. Equilibrium junction current vanishes because drift and diffusion cancel. Reverse current is almost voltage independent below breakdown because it is limited chiefly by the temperature-dependent minority-carrier supply.

Quick Tip to Solve: Separate three ideas: overall charge balance after doping, current balance at equilibrium, and minority-carrier supply under reverse bias. State the below-breakdown condition for reverse saturation.

28. An electron moving with a velocity $\vec{v} = (1.0 \times 10^7 \text{ m/s})\hat{i} + (0.5 \times 10^7 \text{ m/s})\hat{j}$ enters a region of uniform magnetic field $\vec{B} = (0.5 \text{ mT})\hat{j}$. Find the radius of the circular path described by it. While rotating, does the electron trace a linear path too? If so, calculate the linear distance covered by it during the period of one revolution. [3 marks]

Solution:

Concept:

- The magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ acts only on the velocity component perpendicular to $\vec{B}$; it does no work and leaves the parallel component unchanged.
- For mass m , charge magnitude e , perpendicular speed $v_{\perp}$ and field magnitude B , the circular radius is $r = mv_{\perp}/(eB)$, the period is $T = 2\pi m/(eB)$, and the helix pitch is $p = v_{\parallel}T$.

Step 1: Resolve the velocity along and across the field Since the magnetic field points along $+\hat{j}$, the $\hat{i}$ component produces circular motion in the plane perpendicular to the field, while the $\hat{j}$ component produces uniform forward motion. Thus

$$v_{\perp} = 1.0 \times 10^7 \text{ m s}^{-1}, \quad v_{\parallel} = 0.5 \times 10^7 \text{ m s}^{-1}, \quad B = 5.0 \times 10^{-4} \text{ T}.$$

The total speed must not be substituted for $v_{\perp}$: the magnetic force cannot bend the parallel component.

Step 2: Calculate the radius from the centripetal force Equating magnetic force to the centripetal force gives $ev_{\perp}B = mv_{\perp}^2/r$. Using $m = 9.1 \times 10^{-31} \text{ kg}$ and $e = 1.6 \times 10^{-19} \text{ C}$,

$$r = \frac{mv_{\perp}}{eB} = \frac{(9.1 \times 10^{-31})(1.0 \times 10^7)}{(1.6 \times 10^{-19})(5.0 \times 10^{-4})} = 0.11375 \text{ m}.$$

The negative charge reverses the sense of rotation but does not change this positive radius.

Step 3: Find the forward distance in one revolution There is no magnetic force along the field, so the electron retains its velocity $v_{\parallel}$ in the $+\hat{j}$ direction. Combining this translation with circular motion gives a helix, rather than a planar circle. Its period and pitch are

$$T = \frac{2\pi m}{eB} = \frac{2\pi(9.1 \times 10^{-31})}{(1.6 \times 10^{-19})(5.0 \times 10^{-4})} = 7.15 \times 10^{-8} \text{ s},$$

$$p = v_{\parallel}T = (0.5 \times 10^7)(7.15 \times 10^{-8}) = 0.357 \text{ m}.$$

This is the axial distance covered during one complete turn, not the length of the curved helical track.

Step 4: Conclusion The radius is $r \approx 0.114 \text{ m}$ (11.4 cm). Yes, the electron simultaneously moves uniformly along $+\hat{j}$, covering 0.357 m (35.7 cm) per revolution; its resultant path is helical.

Quick Tip to Solve: First pick the velocity component parallel to the field axis: use the other component for the radius and the parallel component for the pitch.

Section D (8 marks)

29. A prism is an optical medium bounded by three refracting plane surfaces. A ray of light suffers successive refractions on passing through its two surfaces and deviates by a certain angle from its original path. The refractive index of the material of the prism is given by $\mu = \sin \left(\frac{A + \delta_m}{2} \right) / \sin \frac{A}{2}$.

If the angle of incidence on the second surface is greater than an angle called critical angle, the ray will not be refracted from the second surface and is totally internally reflected. [4 marks]

29(i). The critical angle for glass is θ_1 and that for water is θ_2 . The critical angle for glass-water surface would be (given ${}_a\mu_g = 1.5$, ${}_a\mu_w = 1.33$): [1 mark]

- (A) less than θ_2
- (B) between θ_1 and θ_2
- (C) greater than θ_2
- (D) less than θ_1

Correct Answer: (C) greater than θ_2

Solution:

Concept:

- For light travelling from refractive index n_1 to a smaller index n_2 , the critical angle C satisfies $\sin C = n_2/n_1$.
- The inverse sine function increases between zero and one, so critical angles can be compared by comparing their sines.

Step 1: Write the critical-angle ratios for air The quoted critical angles for glass and water refer to their respective interfaces with air. Taking the refractive index of air as one,

$$\sin \theta_1 = \frac{1}{1.5} = 0.667, \quad \sin \theta_2 = \frac{1}{1.33} = 0.752.$$

Therefore $\theta_1 < \theta_2$, approximately $41.8^\circ < 48.8^\circ$.

Step 2: Compare the glass-to-water critical angle Glass is the denser medium, so the critical angle C_{gw} applies to light incident from glass into water:

$$\sin C_{gw} = \frac{1.33}{1.5} = 0.887 > \sin \theta_2.$$

Hence $C_{gw} \approx 62.5^\circ$. The proposed values below θ_2 , between the two air-interface angles, or below θ_1 are all excluded. Replacing air by water makes total internal reflection harder to achieve and raises the critical angle.

Step 3: Conclusion The glass-water critical angle is greater than θ_2 . Hence, the correct option is (C).

Quick Tip to Solve: Write the index of the outgoing medium over that of the incident medium; for glass to water the ratio is $1.33/1.5$, not $1/1.5$.

29(ii). When a ray of light of wavelength λ and frequency ν is refracted into a denser medium:

[1 mark]

- (A) λ and ν both increase.
- (B) λ increases but ν is unchanged.
- (C) λ decreases but ν is unchanged.
- (D) λ and ν both decrease.

Correct Answer: (C) λ decreases but ν is unchanged.

Solution:

Concept:

- The speed of light in a medium of refractive index n is $v = c/n$, where c is its speed in vacuum.
- Wave speed, frequency and wavelength obey $v = \nu\lambda$. Frequency is fixed by the source and remains continuous across a stationary interface.

Step 1: Track the frequency through refraction On entering an optically denser medium the light continues to oscillate at the source frequency. The frequency is therefore still ν . A change in frequency at the boundary would disrupt continuity of the oscillations, so options (A) and (D), which change frequency, cannot describe ordinary refraction.

Step 2: Use the lower speed to determine the wavelength If the incident and refracted media have indices n_1 and n_2 , with $n_2 > n_1$, their speeds satisfy $v_2 < v_1$. At unchanged frequency,

$$\frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1} = \frac{n_1}{n_2} < 1.$$

The wavelength must shorten. Option (B) incorrectly predicts a longer wavelength despite the reduced wave speed.

Step 3: Conclusion The wavelength λ decreases, while the frequency ν is unchanged. Hence, the correct option is (C).

Quick Tip to Solve: Keep frequency fixed at a refracting boundary; then wavelength changes in the same direction as speed.

29(iii)(a). The critical angle for a ray of light passing from glass to water is minimum for:

[1 mark]

- (A) red colour
- (B) blue colour
- (C) yellow colour
- (D) violet colour

Correct Answer: (D) violet colour

Solution:

Concept:

- For glass-to-water incidence, the critical angle C obeys $\sin C = n_w/n_g = 1/\mu_{gw}$, where $\mu_{gw} = n_g/n_w$ is the refractive index of glass relative to water.
- In the usual normal-dispersion behaviour of glass and water assumed here, the relative refractive index is largest for violet light; a larger relative index gives a smaller critical angle.

Step 1: Compare the relative indices for the colours Violet has the shortest wavelength among the listed colours and the greatest glass-to-water relative refractive index in the normal-dispersion model. Red has the longest wavelength and the smallest relative index; blue and yellow lie between them.

Step 2: Apply the inverse relation to the critical angle Since $\sin C = 1/\mu_{gw}$, the largest μ_{gw} makes $\sin C$, and hence C , smallest. Thus violet has the smallest critical angle. Red gives the largest angle rather than the smallest, and neither blue nor yellow gives the extremum. The relevant index is the ratio of the two media's indices, because the ray exits into water, not air.

Step 3: Conclusion The critical angle is minimum for violet colour. Hence, the correct option is (D).

Quick Tip to Solve: For a critical-angle colour question, compare the relative refractive indices: greatest relative index means smallest critical angle.

OR

29(iii)(b). Three beams of red, yellow and violet colours are passed through a prism, one by one under the same condition. When the prism is in the position of minimum deviation, the angles of refraction from the second surface are r_R , r_Y and r_V respectively. Then: [1 mark]

(A) $r_V < r_Y < r_R$

(B) $r_Y < r_R < r_V$

(C) $r_R < r_Y < r_V$

(D) $r_R = r_Y = r_V$

Correct Answer: (C) $r_R < r_Y < r_V$

Solution:

Concept:

- At minimum deviation, the two internal angles in a prism of apex angle A are equal to $A/2$. The external angle of emergence e satisfies $\sin e = \mu \sin(A/2)$, with air outside.
- Normal dispersion gives $\mu_R < \mu_Y < \mu_V$ for red, yellow and violet light in glass.

Step 1: Identify the angles named in the question The printed phrase “angles of refraction from the second surface” denotes the outgoing refracted angles, measured in air from the second face’s normal. These are emergence angles, denoted here by the question’s r_R, r_Y, r_V . They are distinct from the internal incidence angle $A/2$, which is the same at minimum deviation for each colour.

Step 2: Compare the outgoing angles using Snell’s law Applying Snell’s law at the second surface for each colour in its minimum-deviation position,

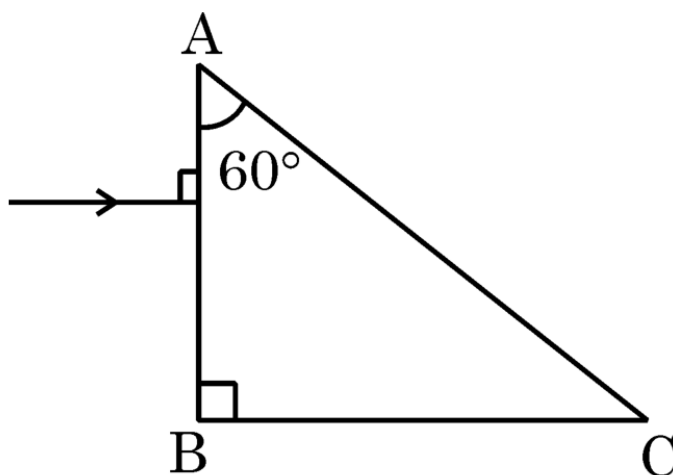
$$\sin r_R = \mu_R \sin \frac{A}{2}, \quad \sin r_Y = \mu_Y \sin \frac{A}{2}, \quad \sin r_V = \mu_V \sin \frac{A}{2}.$$

Because the prism angle is fixed and $\mu_R < \mu_Y < \mu_V$, the outgoing angles increase in the same order. Options (A) and (B) violate this ordering. Option (D) confuses the equal internal angles with the colour-dependent outgoing refraction angles.

Step 3: Conclusion The angles of refraction from the second surface obey $r_R < r_Y < r_V$. Hence, the correct option is (C).

Quick Tip to Solve: At minimum deviation, only the internal angles are fixed at $A/2$; outgoing angles still increase with the refractive index of the colour.

29(iv). A ray of light is incident normally on a prism ABC of refractive index $\sqrt{2}$, as shown in figure. After it strikes face AC, it will: [1 mark]



- (A) go straight undeviated
- (B) just graze along the face AC
- (C) refract and go out of the prism
- (D) undergo total internal reflection

Correct Answer: (D) undergo total internal reflection

Solution:

Concept:

- A ray incident normally at an interface enters without changing direction. Inside a prism, incidence angles are measured from the normal to each face.
- For a prism of index μ surrounded by air, $\sin C = 1/\mu$. Total internal reflection occurs at a glass-to-air face when the internal incidence angle exceeds the critical angle C .

Step 1: Determine the incidence angle at face AC The incoming ray enters AB normally, so it continues horizontally, parallel to BC. In the right-angled triangle, $\angle A = 60^\circ$ and $\angle B = 90^\circ$, giving $\angle C = 30^\circ$. Thus the ray makes 30° with face AC and

$$i = 90^\circ - 30^\circ = 60^\circ$$

with its normal. The angle to the surface itself must not be used as the incidence angle.

Step 2: Compare incidence with the critical angle For the given refractive index $\mu = \sqrt{2}$,

$$C = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ.$$

Since $60^\circ > 45^\circ$, the ray cannot emerge by refraction. Grazing emergence would require equality $i = C$, and an undeviated ray at AC would require normal incidence there. Thus options (A), (B) and (C) do not apply.

Step 3: Conclusion The ray undergoes total internal reflection at face AC. Hence, the correct option is (D).

Quick Tip to Solve: After normal entry at AB, use $i = A = 60^\circ$ at AC and compare it with $C = 45^\circ$; measure both angles from the normal.

30. Dielectrics play an important role in design of capacitors. The molecules of a dielectric may be polar or non-polar. When a dielectric slab is placed in an external electric field, opposite charges appear on the two surfaces of the slab perpendicular to electric field. Due to this an electric field is established inside the dielectric. The capacitance of a capacitor is determined by the dielectric constant of the material that fills the space between the plates. Consequently, the energy storage capacity of a capacitor is also affected. Like resistors, capacitors can also be arranged in series and/or parallel. [4 marks]

30(i). Which of the following is a polar molecule? [1 mark]

- (A) O_2
- (B) H_2
- (C) N_2
- (D) HCl

Correct Answer: (D) HCl

Solution:

Concept:

- A polar molecule has a permanent electric dipole moment: its centres of positive and negative charge do not coincide.
- For equal and opposite charges of magnitude q separated by a distance l , the dipole moment has magnitude $p = ql$.

Step 1: Compare the identical-atom molecules In O_2 , H_2 and N_2 , the two atoms are identical and share the bonding electrons symmetrically. Their charge centres coincide, so these molecules have no permanent dipole moment. Thus options (A), (B) and (C) describe non-polar molecules.

Step 2: Examine the unlike-atom molecule In HCl, chlorine attracts the shared electrons more strongly than hydrogen. The chlorine end is partially negative and the hydrogen end partially positive, giving separated charge centres and a permanent dipole moment even without an applied electric field.

Step 3: Conclusion HCl is a polar molecule. Hence, the correct option is (D).

Quick Tip to Solve: For these diatomic choices, first eliminate the three molecules made from identical atoms; the unlike atoms in HCl share charge unequally.

30(ii). Which of the following statements about dielectrics is correct?

[1 mark]

- (A) A polar dielectric has a net dipole moment in absence of an external electric field which gets modified due to the induced dipoles.
- (B) The net dipole moments of induced dipoles is along the direction of the applied electric field.
- (C) Dielectrics contain free charges.
- (D) The electric field produced due to induced surface charges inside a dielectric is along the external electric field.

Correct Answer: (B) The net dipole moments of induced dipoles is along the direction of the applied electric field.

Solution:

Concept:

- A dielectric is an insulator: its charges are bound, but an applied electric field can displace their positive and negative centres slightly, producing induced dipoles.
- Electric dipole moment points from negative to positive charge. The polarisation field inside the dielectric opposes the applied field.

Step 1: Determine the induced dipole direction The applied field shifts bound positive charge along the field and bound negative charge opposite to it. The resulting dipole moment therefore points along the applied field, which agrees with statement (B).

Step 2: Check the other statements In an unpolarised polar dielectric, individual molecules have permanent dipole moments, but random orientations give zero net macroscopic dipole moment without an external field; thus (A) confuses molecular and bulk behaviour. Dielectrics do not contain freely mobile charges as conductors do, so (C) is incorrect. Induced surface charges produce an opposing field inside the slab, not a field along the applied field, making (D) incorrect.

Step 3: Conclusion The net dipole moment of the induced dipoles is along the applied electric field. Hence, the correct option is (B).

Quick Tip to Solve: Separate the two directions: induced dipole moment follows the applied field, while the induced field inside the slab opposes it.

30(iii). When a dielectric slab is inserted between the plates of an isolated charged capacitor, the energy stored in it:

[1 mark]

- (A) increases and the electric field inside it also increases.
- (B) decreases and the electric field also decreases.
- (C) decreases and the electric field increases.
- (D) increases and the electric field decreases.

Correct Answer: (B) decreases and the electric field also decreases.

Solution:

Concept:

- An isolated capacitor has fixed free charge Q . Its stored energy is $U = Q^2/(2C)$, where C is its capacitance.
- A dielectric with dielectric constant $K > 1$ increases capacitance and reduces the electric field in the dielectric by polarisation.

Step 1: Keep the charge fixed Isolation means no charge can enter or leave the plates. Inserting the dielectric increases C ; since Q is unchanged, $U = Q^2/(2C)$ decreases. If the dielectric fills the gap, $C' = KC$ and $U' = U/K$. Using constant battery voltage here would describe a different physical situation.

Step 2: Determine the field change The dielectric develops bound surface charges whose field opposes the original field. The resultant field within the dielectric is therefore smaller; for a completely filled gap, $E' = E/K$. For a partly filled gap the field in the dielectric is reduced and the average field across the gap is also reduced, while the ideal air-region field stays unchanged. Options (A) and (D) incorrectly increase the energy, and (C) incorrectly increases the field.

Step 3: Conclusion The stored energy decreases and the electric field in the dielectric decreases. Hence, the correct option is (B).

Quick Tip to Solve: Underline the word isolated: use fixed charge and $U = Q^2/(2C)$, so increasing capacitance lowers the stored energy.

30(iv)(a). An air-filled capacitor with plate area A and plate separation d has capacitance C_0 . A slab of dielectric constant K , area A and thickness $\left(\frac{d}{5}\right)$ is inserted between the plates. The capacitance of the capacitor will become: [1 mark]

- (A) $\left[\frac{4K}{5K+1}\right] C_0$
 (B) $\left[\frac{K+5}{4}\right] C_0$
 (C) $\left[\frac{5K}{4K+1}\right] C_0$
 (D) $\left[\frac{K+4}{5K}\right] C_0$

Correct Answer: (C) $\left[\frac{5K}{4K+1}\right] C_0$

Solution:

Concept:

- An air-filled parallel-plate capacitor has $C_0 = \epsilon_0 A/d$, where A is plate area and d the separation.
- For a full-area dielectric slab of thickness t and dielectric constant K , the air and dielectric layers act in series: $C = \epsilon_0 A/(d - t + t/K)$.

Step 1: Identify the two layer thicknesses The printed slab thickness is $t = d/5$, so the remaining air thickness is $d - t = 4d/5$. The same displacement flux passes through both layers, while their potential differences add. Thus the dielectric contributes an effective air thickness $d/(5K)$.

Step 2: Calculate the capacitance ratio Substituting the layer thicknesses gives

$$C = \frac{\epsilon_0 A}{4d/5 + d/(5K)} = \frac{5K}{4K+1} \frac{\epsilon_0 A}{d} = \frac{5K}{4K+1} C_0.$$

Option (A) has the wrong layer coefficients and even gives less than C_0 for $K > 1$. Option (B) does not recover C_0 at $K = 1$. Option (D) decreases below C_0 for $K > 1$, contrary to dielectric insertion.

Step 3: Conclusion The new capacitance is $C = \frac{5K}{4K+1}C_0$. Hence, the correct option is (C).

Quick Tip to Solve: For layers along the field, add effective thicknesses: retain the air thickness and divide only the dielectric thickness by K .

OR

30(iv)(b). Two capacitors of capacitances $2C_0$ and $6C_0$ are first connected in series and then in parallel across the same battery. The ratio of energies stored in series combination to that in parallel is: [1 mark]

- (A) $\frac{1}{4}$
(B) $\frac{1}{6}$
(C) $\frac{2}{15}$
(D) $\frac{3}{16}$

Correct Answer: (D) $\frac{3}{16}$

Solution:

Concept:

- Two capacitances C_1 and C_2 have series equivalent $C_s = C_1C_2/(C_1 + C_2)$ and parallel equivalent $C_p = C_1 + C_2$.
- At fixed battery voltage V , stored energy is $U = \frac{1}{2}CV^2$; hence the energy ratio equals the equivalent-capacitance ratio.

Step 1: Calculate the equivalent capacitances Here $C_1 = 2C_0$ and $C_2 = 6C_0$. In series the same charge flows onto both capacitors, while their voltages add; in parallel both share the battery voltage. Therefore

$$C_s = \frac{(2C_0)(6C_0)}{2C_0 + 6C_0} = \frac{3}{2}C_0, \quad C_p = 2C_0 + 6C_0 = 8C_0.$$

Step 2: Compare energies at the same voltage Since the same battery is used for both combinations, V is unchanged and cancels:

$$\frac{U_s}{U_p} = \frac{\frac{1}{2}C_sV^2}{\frac{1}{2}C_pV^2} = \frac{3C_0/2}{8C_0} = \frac{3}{16}.$$

Option (A), $1/4$, would apply to two equal capacitances; these are unequal. Options (B), $1/6$, and (C), $2/15$, do not equal the ratio obtained from the series and parallel formulae.

Step 3: Conclusion The ratio of energy stored in series to that in parallel is $U_s/U_p = 3/16$. Hence, the correct option is (D).

Quick Tip to Solve: For the same battery, cancel $V^2/2$ first and use $U_s/U_p = C_1C_2/(C_1 + C_2)^2$ directly.

Section E (15 marks)

31(a)(i). A plane light wave propagating from a rarer into a denser medium, is incident at an angle i on the surface separating two media. Using Huygen's principle, draw the refracted wave and hence verify Snell's law of refraction. [5 marks total]

Solution:

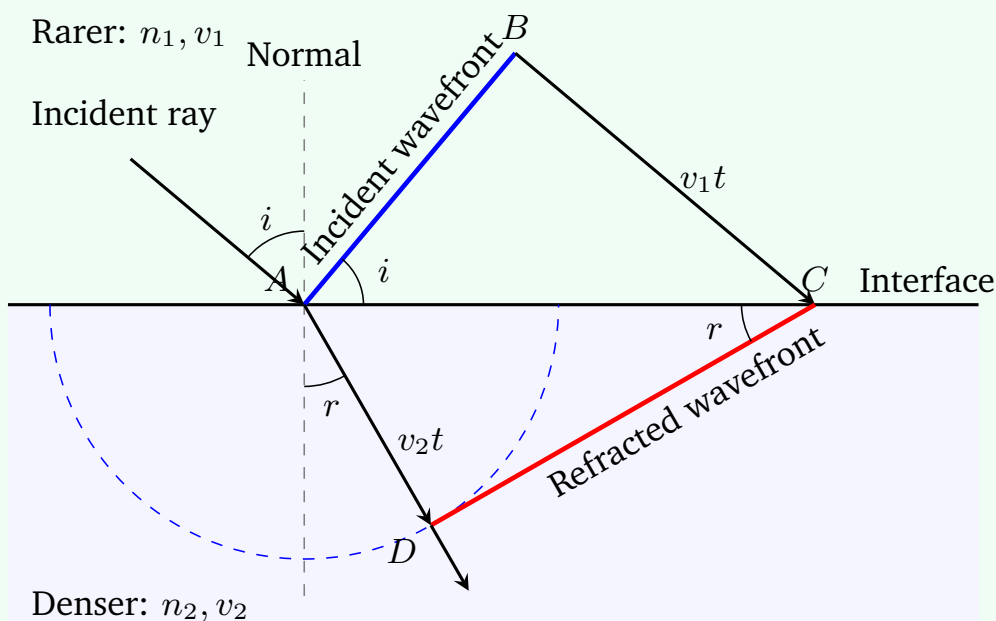
Concept:

- Huygens' principle: every point of a wavefront emits secondary wavelets; their forward common tangent gives the new wavefront. Rays are normal to wavefronts.
- In a medium of refractive index n , light travels at $v = c/n$. The incident and refracted angles i and r are measured from the interface normal.

Step 1: Construct the refracted wavefront Let the upper rarer medium have refractive index n_1 and speed v_1 , and the lower denser medium have index n_2 and speed v_2 . Thus $n_2 > n_1$ and $v_2 < v_1$. The plane incident wavefront AB first touches the interface at A . While its point B travels along the incident ray to C in time t , the secondary disturbance starting at A spreads in the denser medium through a distance v_2t .

$$BC = v_1t, \quad AD = v_2t.$$

Draw the secondary wavelet centred at A with radius AD , and draw its tangent CD through C . This tangent is the refracted plane wavefront at time t . The refracted ray AD is perpendicular to CD , just as BC is perpendicular to the incident wavefront AB .



Huygens' construction: AB is incident and CD is refracted; $BC = v_1t$, $AD = v_2t$. Dashed vertical lines show the normal at A .

Step 2: Verify Snell's law from the two right triangles Since ray and wavefront are perpendicular, the angle between AB and the interface AC equals i , and the angle between CD and the interface equals r . The right triangles ABC and ADC have the common hypotenuse AC . Therefore,

$$\sin i = \frac{BC}{AC} = \frac{v_1t}{AC}, \quad \sin r = \frac{AD}{AC} = \frac{v_2t}{AC}.$$

Dividing eliminates both the elapsed time and the construction length:

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}, \quad n_1 \sin i = n_2 \sin r.$$

This is Snell's law. Since $v_2 < v_1$, the construction also gives $\sin r < \sin i$, so $r < i$: the ray bends towards the normal on entering the denser medium.

Step 3: Conclusion The tangent CD to the secondary wavelet is the refracted wavefront. Huygens' construction gives $n_1 \sin i = n_2 \sin r$, which is Snell's law; $r < i$ for passage into the denser medium.

Quick Tip to Solve: Compare the two distances travelled in the same time: $BC = v_1 t$ and $AD = v_2 t$. Both right triangles have the same hypotenuse AC .

31(a)(ii). In a Young's double slit experiment, the slits are separated by 0.30 mm and the screen is kept 1.5 m away. The wavelength of light used is 600 nm . Calculate the distance between the central bright fringe and the 4^{th} dark fringe.

Solution:

Concept:

- In Young's double-slit experiment, the fringe width is $\beta = \lambda D/d$, where λ is the wavelength, D the screen distance and d the slit separation. The m th dark fringe from the central bright fringe occurs at $y_m = (m - \frac{1}{2})\beta$.
- Destructive interference requires path difference $(m - \frac{1}{2})\lambda$, where $m = 1, 2, 3, \dots$ counts dark fringes from the centre.

Step 1: Find the fringe width in SI units The given quantities are $d = 0.30 \text{ mm} = 3.0 \times 10^{-4} \text{ m}$, $D = 1.5 \text{ m}$, and $\lambda = 600 \text{ nm} = 6.0 \times 10^{-7} \text{ m}$. Hence,

$$\beta = \frac{\lambda D}{d} = \frac{(6.0 \times 10^{-7})(1.5)}{3.0 \times 10^{-4}} \text{ m} = 3.0 \times 10^{-3} \text{ m} = 3.0 \text{ mm}.$$

Step 2: Locate the fourth dark fringe Destructive interference occurs when the path difference is an odd half-multiple of the wavelength. Counting outward from the central bright fringe, the first dark fringe is at $\beta/2$; successive dark fringes are separated by β . Thus the fourth dark fringe corresponds to path difference $7\lambda/2$, giving

$$y_4 = \left(4 - \frac{1}{2}\right)\beta = \frac{7}{2}(3.0 \text{ mm}) = 10.5 \text{ mm}.$$

The pattern is symmetric: the two fourth dark fringes lie at signed coordinates $\pm 10.5 \text{ mm}$; the requested distance from the centre is positive.

Step 3: Conclusion The fourth dark fringe is at a distance 10.5 mm from the central bright fringe, on either side of the centre.

Quick Tip to Solve: The first dark fringe is at $\beta/2$, so the fourth is at 3.5β , not 4β . Convert nanometres and millimetres to metres before substitution.

31(b)(i). Discuss briefly diffraction of light from a single slit and draw the shape of the diffraction pattern. [5 marks total]

Solution:

Concept:

- Diffraction is the spreading of light at an aperture or edge. In single-slit diffraction, secondary wavelets from different points across a slit of width a interfere; λ denotes the wavelength.
- For a uniformly illuminated slit, minima occur at $a \sin \theta = n\lambda$, where $n = \pm 1, \pm 2, \dots$ and θ is measured from the straight-ahead direction.

Step 1: Explain the origin of the diffraction pattern Let monochromatic plane waves fall normally on a narrow slit. By Huygens' principle, every point of the illuminated slit acts as a source of secondary wavelets. Their superposition produces a diffraction pattern on a distant screen, or in the focal plane of a converging lens. Diffraction becomes appreciable when the slit is sufficiently narrow. In the straight-ahead direction, $\theta = 0$, the wavelets have equal optical path lengths and arrive in phase. Therefore they produce the brightest, central maximum. For an observation direction θ , the path difference between wavelets from the two edges is $a \sin \theta$.

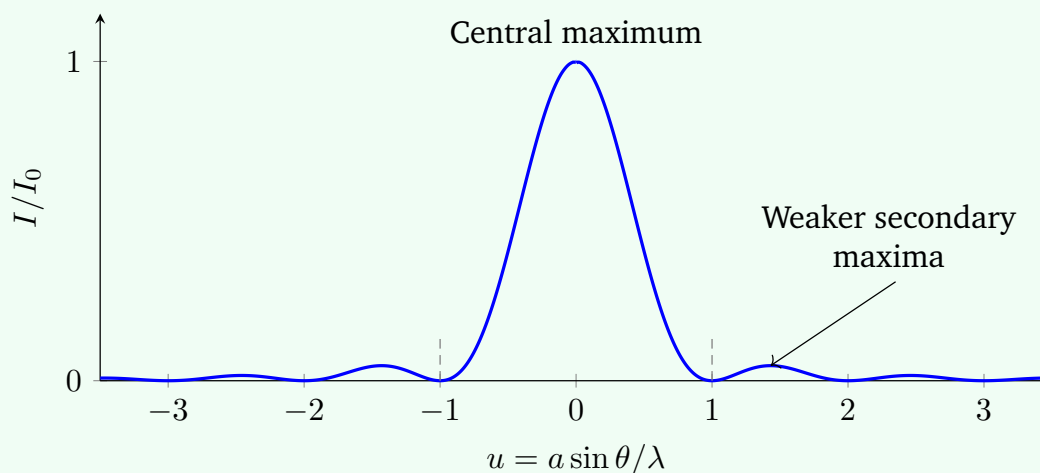
Step 2: Locate the minima and sketch the intensity distribution At $a \sin \theta = \lambda$, divide the slit into two equal halves. Corresponding points in the two halves have path difference $\lambda/2$, so their contributions cancel pairwise, producing the first minimum. More generally, at $a \sin \theta = n\lambda$, the slit can be divided into $2n$ equal strips whose contributions cancel in pairs.

$$a \sin \theta_n = \pm n\lambda, \quad n = 1, 2, 3, \dots$$

Between successive minima are weaker secondary maxima, with intensity decreasing away from the centre. The central maximum extends between the first minima on either side. For small angles its angular width is $2\lambda/a$; on a screen at distance D , its linear width is $2D\lambda/a$. It is twice as wide as the interval between successive minima on either side. The symmetric intensity curve is

$$I(\theta) = I_0 \left(\frac{\sin \beta}{\beta} \right)^2, \quad \beta = \frac{\pi a \sin \theta}{\lambda},$$

with the limiting value $I(0) = I_0$.



Single-slit intensity pattern: the central maximum spans $-1 < u < 1$, where $u = a \sin \theta / \lambda$; successive side minima are one unit apart.

Step 3: Conclusion Single-slit diffraction produces a broad, bright central maximum with narrower, progressively weaker secondary maxima on both sides. The dark minima satisfy $a \sin \theta = \pm n\lambda$.

Quick Tip to Solve: In the intensity sketch, place the first minima at $\pm \lambda/a$ for small angles. The central maximum is twice as wide as the interval between neighbouring side minima.

31(b)(ii). An object is placed between the pole and the focus of a concave mirror. Using mirror formula, prove mathematically that it produces a virtual and an enlarged image.

Solution:

Concept:

- The mirror formula is $1/f = 1/v + 1/u$, and transverse magnification is $m = -v/u$, using the Cartesian sign convention.
- With the Cartesian sign convention, a real object in front of a concave mirror has $u < 0$ and the mirror has $f < 0$; $v > 0$ denotes a virtual image behind the mirror.

Step 1: Apply the mirror formula with the correct signs Choose the pole as origin and the direction of incident light as positive. For a concave mirror write $f = -F$, where $F > 0$ is the magnitude of the focal length. Let the real object be at $u = -x$. Since it lies between the pole and focus, $0 < x < F$. Substitution gives

$$\begin{aligned} -\frac{1}{F} &= \frac{1}{v} - \frac{1}{x}, \\ \frac{1}{v} &= \frac{1}{x} - \frac{1}{F} = \frac{F-x}{Fx}, \\ v &= \frac{Fx}{F-x}. \end{aligned}$$

Both numerator and denominator are positive, so $v > 0$. The image is behind the mirror, where the backward extensions of reflected rays meet; it is therefore virtual.

Step 2: Establish the size and orientation of the image Using the signed object and image distances,

$$m = -\frac{v}{u} = -\frac{Fx/(F-x)}{-x} = \frac{F}{F-x}.$$

Since $0 < x < F$, the denominator obeys $0 < F - x < F$. Thus $m > 1$. The positive sign means that the image is erect, and its magnitude greater than unity means that the image height exceeds the object height. This proves the result for every object position strictly between the pole and focus.

Step 3: Conclusion For $0 < x < F$, $v = Fx/(F - x) > 0$ and $m = F/(F - x) > 1$. Thus an object between the pole and focus of a concave mirror produces a virtual, erect and enlarged image.

Quick Tip to Solve: Set $u = -x$, $f = -F$ and state $0 < x < F$ first; then the signs of v and $m - 1$ follow directly from the denominator $F - x$.

32(a). (i) Draw equipotential surfaces for an electric dipole.

(ii) Two point charges q_1 and q_2 are located at $\vec{r}_1$ and $\vec{r}_2$ respectively in an external electric field $\vec{E}$. Obtain an expression for the potential energy of the system.

(iii) The dipole moment of a molecule is 10^{-30} Cm. It is placed in an electric field $\vec{E}$ of 10^5 V/m such that its axis is along the electric field. The direction of $\vec{E}$ is suddenly changed by 60° at an instant. Find the change in the potential energy of the dipole, at that instant. [5 marks]

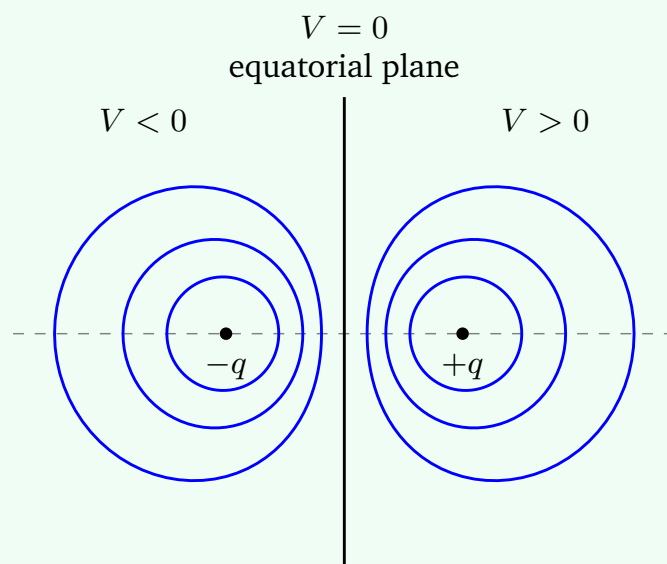
Solution:

Concept:

- An equipotential surface has constant potential. For charges $+q$ and $-q$, $V = kq(1/r_+ - 1/r_-)$, where $k = 1/(4\pi\epsilon_0)$ and r_+ , r_- are distances from the two charges.
- Work done in assembling fixed charges is stored as electrostatic potential energy. The external-field contribution is qV_{ext} ; each mutual pair interaction is counted once.
- The potential energy of a permanent electric dipole is $U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$, where θ is the angle between the dipole moment and the field.

Step 1: (i) Draw and interpret the equipotential surfaces The figure shows sections of the equipotential surfaces in a plane containing the dipole axis. Rotating the closed contours about the horizontal axis gives the corresponding three-dimensional surfaces. The contours near each point charge are approximately circular; farther away they are distorted by the other charge. Surfaces surrounding $+q$ have positive potential and those surrounding $-q$ have negative potential, with zero potential at infinity as the reference.

At every point of the perpendicular bisector, $r_+ = r_-$, so the potentials cancel. The entire equatorial plane is therefore an equipotential surface with $V = 0$. Distinct equipotential surfaces never intersect, and the electric field is normal to each surface.



Rotate the contours about the dipole axis.

Meridional sections of electric-dipole equipotential surfaces; the vertical line represents the zero-potential equatorial plane.

Step 2: (ii) Bring the first charge into the external field Let $V_{\text{ext}}(\vec{r})$ denote the potential produced by the fixed sources of the external field, with a chosen reference zero. Bring q_1 to $\vec{r}_1$ quasistatically. The work done against the external electric force is

$$W_1 = q_1 V_{\text{ext}}(\vec{r}_1).$$

We exclude the unchanged self-energy of the external sources and the self-energies of the point charges.

Step 3: (ii) Bring the second charge and add the work Write $r_{12} = |\vec{r}_2 - \vec{r}_1|$. After the first charge is placed, the potential at $\vec{r}_2$ is the sum of the external potential and kq_1/r_{12} . Hence the work to bring q_2 is

$$W_2 = q_2 \left[V_{\text{ext}}(\vec{r}_2) + \frac{kq_1}{r_{12}} \right].$$

Adding the two contributions gives

$$U = q_1 V_{\text{ext}}(\vec{r}_1) + q_2 V_{\text{ext}}(\vec{r}_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}.$$

The last term is the mutual interaction energy and is included only once. For a uniform field, $V_{\text{ext}}(\vec{r}) = -\vec{E} \cdot \vec{r} + C$; the chosen constant sets the energy reference.

Step 4: (iii) Compare the energies immediately before and after the change Initially the dipole moment is along the field, so $\theta_i = 0^\circ$. At the instant the field turns, the molecule has not yet rotated; therefore the angle becomes $\theta_f = 60^\circ$, while p and the field magnitude remain unchanged. Thus

$$\begin{aligned} pE &= (10^{-30})(10^5) = 10^{-25} \text{ J}, \\ U_i &= -pE = -10^{-25} \text{ J}, \\ U_f &= -pE \cos 60^\circ = -5 \times 10^{-26} \text{ J}, \\ \Delta U &= U_f - U_i = pE(1 - \cos 60^\circ). \end{aligned}$$

The final orientation is less favourable energetically than the initial aligned orientation. Any later rotation of the molecule is irrelevant to the change requested at that instant.

Step 5: Conclusion The dipole equipotentials are shown above, including the zero-potential equatorial plane. The two-charge energy is $U = q_1 V_{\text{ext}}(\vec{r}_1) + q_2 V_{\text{ext}}(\vec{r}_2) + q_1 q_2 / (4\pi\epsilon_0 r_{12})$. The instantaneous change in the molecule's potential energy is $\Delta U = +5 \times 10^{-26} \text{ J}$, an increase.

Quick Tip to Solve: Count the mutual charge-pair energy once. When the field direction changes suddenly, keep the dipole direction fixed and use the new relative angle in $-pE \cos \theta$.

OR

32(b). (i) A thin spherical shell of radius R has a uniform surface charge density σ . Using Gauss' law, deduce an expression for electric field (i) outside and (ii) inside the shell.

(ii) Two long straight thin wires AB and CD have linear charge densities $10 \mu\text{C/m}$ and $-20 \mu\text{C/m}$, respectively. They are kept parallel to each other at a distance 1 m. Find magnitude and direction of the net electric field at a point midway between them. [5 marks]

Solution:

Concept:

- Gauss' law is $\oint \vec{E} \cdot d\vec{S} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the charge enclosed by the chosen closed surface.
- A uniformly charged spherical shell has spherical symmetry: its electric field is radial, with constant magnitude on a concentric sphere.
- The field of a very long straight wire at perpendicular distance r has magnitude $E = |\lambda|/(2\pi\epsilon_0 r)$, where λ is the linear charge density. Fields add vectorially.

Step 1: (i) Choose an exterior Gaussian sphere The shell carries total charge $Q = 4\pi R^2 \sigma$. Choose a concentric spherical Gaussian surface of radius $r > R$. The radial field is constant in magnitude on this surface. Hence

$$E_r(4\pi r^2) = \frac{4\pi R^2 \sigma}{\epsilon_0}, \quad \vec{E}(r) = \frac{\sigma R^2}{\epsilon_0 r^2} \hat{r}.$$

The field equals that of a central point charge Q : outward for positive σ , inward for negative σ .

Step 2: (i) Apply Gauss' law inside the shell For a concentric Gaussian sphere of radius $r < R$, all the charge lies outside the Gaussian surface, so $Q_{\text{enc}} = 0$. By the same spherical symmetry,

$$E_r(4\pi r^2) = 0 \Rightarrow \vec{E} = 0.$$

The field is also zero at the centre by symmetry. Zero enclosed charge together with spherical symmetry gives zero field. Just outside the shell, $\vec{E}(R^+) = \sigma \hat{r}/\epsilon_0$; just inside, it is zero.

Step 3: (ii) Establish the directions at the midpoint The midpoint P is 0.50 m from each wire. The field of positive AB points away from AB, towards CD. The field of negative CD points towards CD. Both therefore act from AB to CD, so their magnitudes add.

Step 4: (ii) Substitute the two linear charge densities Using $1/(4\pi\epsilon_0) = 9 \times 10^9 \text{ N m}^2\text{C}^{-2}$, we have

$1/(2\pi\epsilon_0) = 18 \times 10^9 \text{ N m}^2\text{C}^{-2}$. Thus

$$E_{AB} = \frac{18 \times 10^9 \times 10 \times 10^{-6}}{0.50} = 3.6 \times 10^5 \text{ N C}^{-1},$$

$$E_{CD} = \frac{18 \times 10^9 \times 20 \times 10^{-6}}{0.50} = 7.2 \times 10^5 \text{ N C}^{-1},$$

$$E_{\text{net}} = E_{AB} + E_{CD} = 1.08 \times 10^6 \text{ N C}^{-1}.$$

Step 5: Conclusion The shell's field is $\vec{E} = \sigma R^2 \hat{r} / (\epsilon_0 r^2)$ outside and zero inside. The midpoint field is $1.08 \times 10^6 \text{ N C}^{-1}$, perpendicular to the wires, directed from positively charged AB towards negatively charged CD.

Quick Tip to Solve: Between oppositely charged parallel wires, draw each field arrow first: both arrows point from the positive wire to the negative wire, so add the magnitudes.

33(a). (i) You are given three circuit elements X, Y and Z. They are connected one by one across a given ac source. It is found that V and I are in phase for element X. V leads I by $\left(\frac{\pi}{4}\right)$ for element Y while I leads V by $\left(\frac{\pi}{4}\right)$ for element Z. Identify elements X, Y and Z.

(ii) Establish the expression for impedance of circuit when elements X, Y and Z are connected in series to an ac source. Show the variation of current in the circuit with the frequency of the applied ac source.

(iii) In a series LCR circuit, obtain the conditions under which (i) impedance is minimum and (ii) wattless current flows in the circuit.

[5 marks]

Solution:

Concept:

- In a resistor, voltage and current are in phase. In an ideal inductor, voltage leads current by $\pi/2$; in an ideal capacitor, current leads voltage by $\pi/2$.
- In a series circuit, the same current flows through every component. Voltage phasors add, with $V_R = IR$, $V_L = I\omega L$, and $V_C = I/(\omega C)$, where $\omega = 2\pi f$.
- For a sinusoidal supply, the mean power is $P = V_{\text{rms}} I_{\text{rms}} \cos \phi = I_{\text{rms}}^2 R$. Here ϕ is the phase by which the supply voltage leads the current.

Step 1: (i) Interpret the printed phase differences Element X is a resistor. The phase lead for Y shows inductive behaviour, while the current lead for Z shows capacitive behaviour. However, the paper prints $\pi/4$, not $\pi/2$, in both cases. Consequently Y and Z cannot be an ideal inductor and an ideal capacitor individually: those ideal elements have phase differences of $\pi/2$. A simple model consistent with the printed data is a resistor r_Y in series with an inductor L for Y, and a resistor r_Z in series with a capacitor C for Z. At the given source frequency,

$$\tan(\pi/4) = \frac{\omega L}{r_Y} = 1, \quad \frac{1}{\omega C r_Z} = 1.$$

Thus $r_Y = \omega L$ and $r_Z = 1/(\omega C)$. The phase information identifies resistive-inductive and resistive-capacitive behaviour, but does not uniquely determine the internal circuits. The usual identification Y = ideal inductor and Z = ideal capacitor would require different phase values from those printed.

Step 2: (ii) Add the series voltage phasors For the series LCR model, let R denote the total series

resistance, including any resistance within Y and Z. Choose the common current as the reference phasor. The resistive voltage lies along the current; the inductive voltage is perpendicular above it and the capacitive voltage is perpendicular below it. Therefore the net perpendicular component is $V_L - V_C$, giving

$$V^2 = V_R^2 + (V_L - V_C)^2,$$

$$V^2 = I^2 \left[R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2 \right],$$

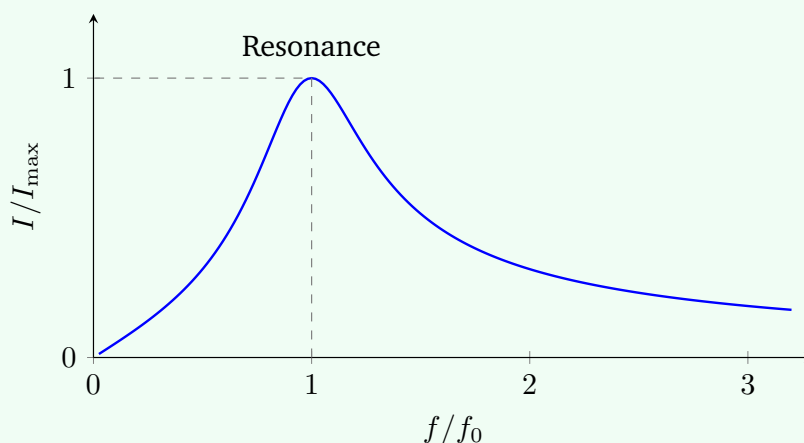
$$Z_{\text{imp}} = \frac{V}{I} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}.$$

All voltage and current magnitudes here may consistently be rms values. We use Z_{imp} to distinguish impedance from the element named Z. Also $\tan \phi = (\omega L - 1/(\omega C))/R$.

Step 3: (ii) Describe the current-frequency curve For fixed rms supply voltage V and frequency-independent R , L and C ,

$$I(f) = \frac{V}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC} \right)^2}}.$$

At low frequency capacitive reactance is very large, so current tends to zero. At high frequency inductive reactance is very large, so current again tends to zero. Between these limits, the current peaks when the two reactances cancel. The graph shows this resonance curve using dimensionless axes; its precise width depends on R , L and C .



Series-resonance current curve at fixed supply voltage; both axes are dimensionless.

Step 4: (iii) Obtain the minimum-impedance condition Since the squared reactance difference is non-negative, impedance is minimum at series resonance:

$$\omega_0 L = \frac{1}{\omega_0 C}, \quad f_0 = \frac{1}{2\pi\sqrt{LC}}, \quad Z_{\text{min}} = R, \quad I_{\text{max}} = \frac{V}{R}.$$

At resonance voltage and current are in phase, so the power factor is unity. This finite peak assumes $R > 0$.

Step 5: (iii) Obtain the wattless-current condition Wattless current means nonzero ac current with zero average power. Therefore

$$P = VI \cos \phi = 0 \quad \Rightarrow \quad \cos \phi = 0, \quad \phi = \pm \frac{\pi}{2}.$$

For the ideal series LCR expression this requires $R = 0$ and nonzero net reactance $\omega L - 1/(\omega C)$. The circuit is then purely reactive: it stores energy during part of a cycle and returns it during the rest. With any positive series resistance, a nonzero current dissipates power $I^2 R$, so the current is not exactly wattless. Resonance with $R > 0$ is not a wattless condition. The ideal zero-resistance resonant case has zero impedance and no finite steady current under a nonzero ideal voltage source.

Step 6: Conclusion X is resistive; the printed $\pi/4$ values make Y resistive-inductive and Z resistive-capacitive, rather than pure ideal L and C. For a series LCR model, $Z_{\text{imp}} = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$. Impedance is minimum and current maximum at $f_0 = 1/(2\pi\sqrt{LC})$. Nonzero wattless current requires purely reactive impedance, so $R = 0$ away from resonance.

Quick Tip to Solve: Read the printed phase angle before naming an ideal element: 45° implies equal resistive and reactive contributions, while a pure inductor or capacitor gives 90° .

OR

- 33(b).** (i) Describe the construction and working of a transformer and hence obtain the relation for $\left(\frac{v_s}{v_p}\right)$ in terms of number of turns of primary and secondary.
- (ii) Discuss four main causes of energy loss in a real transformer. [5 marks]

Solution:

Concept:

- A transformer works on mutual induction: a changing current in the primary produces a changing magnetic flux linked with the secondary.
- Faraday's law gives the induced emf $e = -N d\Phi/dt$, where N is the number of turns and Φ is the magnetic flux through each turn.
- For an ideal transformer, both windings link the same flux, winding resistance and core losses are negligible, and input power equals output power.

Step 1: (i) Describe the construction A transformer consists of two electrically insulated copper coils wound on a common closed core of laminated soft iron. The primary coil, of N_p turns, is connected to an alternating-voltage source; the secondary coil, of N_s turns, supplies the load. The coils are insulated from each other and from the core. The high-permeability core provides an easy path for magnetic flux and ensures strong magnetic coupling. Its thin insulated laminations reduce eddy-current heating.

Step 2: Explain the working The alternating current in the primary sets up an alternating magnetic flux $\Phi(t)$ in the core. This flux links both coils and induces an emf in each. If the secondary circuit is closed, its induced emf drives an alternating current through the load. Energy is thus transferred magnetically from the primary circuit to the secondary circuit without a direct electrical connection between them. A steady direct current cannot sustain transformer action because, after the switching transient, its flux does not change.

Step 3: Derive the voltage-to-turns relation Applying Faraday's law to coils that link the same flux per turn, with consistent winding reference directions, gives

$$e_p = -N_p \frac{d\Phi}{dt}, \quad e_s = -N_s \frac{d\Phi}{dt}.$$

Dividing these equations yields $e_s/e_p = N_s/N_p$. For ideal windings, the primary applied voltage balances the primary induced emf, while the secondary terminal voltage equals its induced emf. Therefore the magnitudes (or rms values) satisfy

$$\frac{v_s}{v_p} = \frac{N_s}{N_p}.$$

Here the voltage ratio refers to magnitudes; terminal polarities depend on the winding connections. A greater number of secondary turns gives a step-up transformer; fewer secondary turns gives a step-down transformer. Ideally $V_p I_p = V_s I_s$, so increasing voltage decreases the available current in the same ratio.

Step 4: (ii) Discuss copper and eddy-current losses **Copper loss:** The primary and secondary windings have finite resistances, so electrical energy is dissipated as heat. The total loss is $I_p^2 R_p + I_s^2 R_s$, with currents expressed as rms values. Thick, low-resistance copper wire reduces this loss.

Eddy-current loss: The changing flux induces circulating currents within the conducting iron core. Their Joule heating wastes energy. Dividing the core into thin sheets insulated from one another increases the resistance of these circulating paths and suppresses the currents.

Step 5: Discuss hysteresis and flux-leakage losses **Hysteresis loss:** The core is magnetised and demagnetised repeatedly by the alternating field. Energy is spent in reversing its magnetisation during each cycle and appears as heat. A magnetically soft core with a narrow hysteresis loop reduces this loss.

Flux leakage: Some flux produced by the primary closes through paths outside the common core and does not link the secondary. Consequently, magnetic coupling and useful voltage transfer are reduced. This is conventionally listed among transformer losses, although leakage itself is not a direct heat-dissipation mechanism. A closed high-permeability core and closely coupled windings minimise leakage.

Step 6: Conclusion A transformer transfers energy by mutual induction, with the ideal voltage ratio $v_s/v_p = N_s/N_p$. The four principal practical loss mechanisms discussed are copper heating, eddy currents, hysteresis and flux leakage.

Quick Tip to Solve: Derive the turns ratio by writing Faraday's law for both coils with the same flux; pair each practical loss with its cause and its remedy.