

CBSE Class 12 Physics Question Paper 2024 with Solution PDF-Set 55/5/3



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are compulsory.
- (ii) Question paper is divided into **FIVE** sections – Section **A, B, C, D** and **E**.
- (iii) In **Section A** : Question number **1** to **16** are Multiple Choice (MCQ) type questions carrying **1** mark each.
- (iv) In **Section B** : Question number **17** to **21** are Very Short Answer (VSA) type questions carrying **2** marks each.
- (v) In **Section C** : Question number **22** to **28** are Short Answer (SA) type questions carrying **3** marks each.
- (vi) In **Section D** : Question number **29 & 30** are Long Answer (LA) type questions carrying **4** marks each.
- (vii) In **Section E** : Question number **31** to **33** are Case-Based questions carrying **5** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in **1** question in Section–**B**, **1** question in Section–**C**, **2** questions in Section–**D** and **3** questions in Section–**E**.
- (ix) Use of calculators is NOT allowed.

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ J s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Section A (16 marks)

1. A heater coil rated as (P, V) is cut into two equal parts. One of the parts is then connected to a battery of V volt. The power consumed by it will be: [1 mark]

- (A) P
- (B) $\frac{P}{2}$
- (C) $\frac{P}{4}$
- (D) $2P$

Correct Answer: (D) $2P$

Solution:

Concept:

- For a resistor of resistance R at voltage V , electrical power is $P = V^2/R$.
- A uniform wire has resistance $R = \rho L/A$, where ρ is resistivity, L is length and A is cross-sectional area.

Step 1: Determine the resistance of one half The original rating gives $R = V^2/P$. Cutting the uniform coil into equal lengths halves its resistance, since its material and cross-sectional area are unchanged. Thus the connected part has $R' = R/2$.

Step 2: Apply the same voltage to the shorter coil At the stated battery voltage, the power ratio is

$$\frac{P'}{P} = \frac{V^2/(R/2)}{V^2/R} = 2.$$

Option (A) would require unchanged resistance. Options (B) and (C) incorrectly predict less power: at fixed voltage, a lower resistance draws more current and consumes more power.

Step 3: Conclusion The power consumed is $2P$. Hence, the correct option is (D).

Quick Tip to Solve: At fixed voltage, power varies inversely with wire length; halving the length doubles the power.

2. Two insulated concentric coils, each of radius R , placed at right angles to each other, carry currents I and $\sqrt{3}I$ respectively. The magnitude of the net magnetic field at their common centre will be:

[1 mark]

- (A) $\frac{\mu_0 I}{R}$
(B) $\frac{\mu_0 I}{2R}$
(C) $\frac{\mu_0 I}{4R}$
(D) $\frac{2\mu_0 I}{R}$

Correct Answer: (A) $\frac{\mu_0 I}{R}$

Solution:

Concept:

- A circular coil of radius R , carrying current I , produces a magnetic field $B = \mu_0 I / (2R)$ at its centre, normal to the coil's plane.
- Magnetic fields add as vectors. Perpendicular components B_1 and B_2 give magnitude $B = \sqrt{B_1^2 + B_2^2}$.

Step 1: Find the two perpendicular fields The planes of the coils are at right angles, so their normals and hence their fields are perpendicular. With the stated currents,

$$B_1 = \frac{\mu_0 I}{2R}, \quad B_2 = \frac{\sqrt{3}\mu_0 I}{2R}.$$

Step 2: Add the fields vectorially Using the Pythagorean relation,

$$B = \frac{\mu_0 I}{2R} \sqrt{1 + 3} = \frac{\mu_0 I}{R}.$$

Option (B) counts only the first coil. Option (C) is smaller than either component and cannot be the resultant of perpendicular fields. Option (D) is twice the calculated magnitude; scalar addition or an extra factor of two is not justified.

Step 3: Conclusion The net field has magnitude $\mu_0 I / R$. Hence, the correct option is (A).

Quick Tip to Solve: For perpendicular coil currents in the ratio $1 : \sqrt{3}$, the resultant field is twice the smaller field.

3. Which of the following material has its magnetic susceptibility χ in the range $0 < \chi < \epsilon$, where ϵ is positive and small? [1 mark]

- (A) Aluminium
- (B) Water
- (C) Gadolinium
- (D) Bismuth

Correct Answer: (A) Aluminium

Solution:

Concept:

- Magnetic susceptibility χ relates magnetisation M to applied magnetic field strength H through $M = \chi H$.
- A paramagnetic substance has small positive susceptibility; diamagnetic substances have negative susceptibility, while ferromagnets have large positive susceptibility.

Step 1: Identify the magnetic class The condition $0 < \chi < \epsilon$, with ϵ positive and small, means the induced magnetisation is weak and parallel to the applied field. This is the characteristic behaviour of a paramagnetic material.

Step 2: Compare the listed substances Aluminium is paramagnetic and satisfies the required small positive susceptibility. Water and bismuth are diamagnetic, so their susceptibility is negative and fails the given inequality. Gadolinium is the ferromagnetic example in the standard classification used here, so it is not the small-positive-susceptibility choice.

Step 3: Conclusion Aluminium has the required susceptibility. Hence, the correct option is (A).

Quick Tip to Solve: Check the sign first: negative means diamagnetic, small positive means paramagnetic, and large positive means ferromagnetic.

4. A galvanometer of resistance $100\ \Omega$ gives full scale deflection for a current of $1.0\ \text{mA}$. It is converted into an ammeter of range $(0-1\ \text{A})$. The resistance of the ammeter will be close to: [1 mark]

- (A) $0.1\ \Omega$
- (B) $0.8\ \Omega$
- (C) $1.0\ \Omega$
- (D) $10\ \Omega$

Correct Answer: (A) $0.1\ \Omega$

Solution:

Concept:

- A galvanometer is converted into an ammeter by connecting a small shunt resistance in parallel, so most of the measured current bypasses the galvanometer.
- At full scale the terminal voltage is $V = I_g G$, where I_g is galvanometer full-scale current and G is its resistance. The effective ammeter resistance is $R_A = V/I$.

Step 1: Find the full-scale terminal voltage Convert the galvanometer current to amperes: $I_g =$

1.0×10^{-3} A. With $G = 100 \Omega$, the voltage across both parallel branches is

$$V = I_g G = (1.0 \times 10^{-3})(100) = 0.10 \text{ V}.$$

Step 2: Use the total ammeter current The full-scale current through the complete instrument is $I = 1$ A, hence

$$R_A = \frac{0.10 \text{ V}}{1 \text{ A}} = 0.10 \Omega.$$

This is the parallel combination's resistance, not the original 100Ω . The values 0.8Ω , 1.0Ω and 10Ω would each give too large a terminal voltage at 1 A, overdriving the galvanometer.

Step 3: Conclusion The resistance of the ammeter is 0.1Ω . Hence, the correct option is (A).

Quick Tip to Solve: Find the galvanometer full-scale voltage first, then divide it by the desired ammeter range current.

5. The mutual inductance of two coils, in a given orientation is 50 mH. If the current in one of the coils changes as $i = 1.0 \sin\left(100\pi t + \frac{\pi}{3}\right)$ A, the peak value of emf (in volt) induced in the other coil will be: [1 mark]

- (A) $\frac{\pi}{5}$
- (B) 5π
- (C) 0.5π
- (D) 0.05π

Correct Answer: (B) 5π

Solution:

Concept:

- Mutual induction gives induced emf $\varepsilon = -M di/dt$, where M is mutual inductance and i is the current in the other coil.
- For $i = I_0 \sin(\omega t + \phi)$, the maximum magnitude of di/dt is ωI_0 ; the constant phase ϕ changes the timing but not the amplitude.

Step 1: Differentiate the printed current The printed current has $I_0 = 1.0$ A, $\omega = 100\pi \text{ rad s}^{-1}$ and phase $\pi/3$. Therefore

$$\frac{di}{dt} = 100\pi \cos\left(100\pi t + \frac{\pi}{3}\right) \text{ A s}^{-1}.$$

Step 2: Calculate the emf amplitude Use $M = 50 \text{ mH} = 0.050 \text{ H}$ and the maximum absolute cosine value of one:

$$\varepsilon_0 = M\omega I_0 = (0.050)(100\pi)(1.0) = 5\pi \text{ V}.$$

Options (C) and (D) are too small by factors of ten and one hundred. Option (A), $\pi/5$, also fails the product $M\omega I_0$. The negative sign in Faraday's law specifies polarity, not a negative peak magnitude.

Step 3: Conclusion The peak induced emf is $5\pi \text{ V}$. Hence, the correct option is (B).

Quick Tip to Solve: For sinusoidal current, multiply mutual inductance in henry by current amplitude and angular frequency; ignore the phase for the peak value.

6. The potential energy between two nucleons inside a nucleus is minimum at a distance of about:

[1 mark]

- (A) 0.8 fm
- (B) 1.6 fm
- (C) 2.0 fm
- (D) 2.8 fm

Correct Answer: (A) 0.8 fm

Solution:

Concept:

- Nuclear interaction has a short-range repulsive core and an attractive region at slightly greater nucleon separation r .
- The force and potential energy are related by $F_r = -dU/dr$. At the bottom of the potential well, U is minimum and the force changes between repulsion and attraction.

Step 1: Locate the minimum of the nuclear potential In the standard nuclear potential-energy curve, the attractive well reaches its lowest value at a nucleon separation of approximately 0.8 fm, with $1 \text{ fm} = 10^{-15} \text{ m}$. At shorter separation, the steep repulsive core makes the potential energy rise.

Step 2: Distinguish the well minimum from the force range Beyond the minimum, the potential energy rises towards zero as the separation increases. The listed distances 1.6 fm, 2.0 fm and 2.8 fm are farther along this outer part of the curve; none represents its bottom. A distance over which attraction acts is not necessarily the separation of minimum energy.

Step 3: Conclusion The potential energy is minimum at about 0.8 fm. Hence, the correct option is (A).

Quick Tip to Solve: For the nuclear potential curve, identify the bottom of the well near 0.8 fm, not the outer range of attraction.

7. A pure Si crystal having 5×10^{28} atoms m^{-3} is doped with 1 ppm concentration of antimony. If the concentration of holes in the doped crystal is found to be $4.5 \times 10^9 \text{ m}^{-3}$, the concentration (in m^{-3}) of intrinsic charge carriers in Si crystal is about: [1 mark]

- (A) 1.2×10^{15}
- (B) 1.5×10^{16}
- (C) 3.0×10^{15}
- (D) 2.0×10^{16}

Correct Answer: (B) 1.5×10^{16}

Solution:

Concept:

- Pentavalent antimony is a donor in silicon. With donors ionised, the majority electron concentration is approximately the donor concentration: $n \simeq N_D$.
- At thermal equilibrium, the mass-action law is $np = n_i^2$, where p is hole concentration and n_i is the intrinsic electron or hole concentration.

Step 1: Calculate the donor concentration One part per million corresponds to a fraction 10^{-6} of the silicon atom concentration. Since each antimony atom supplies one donor electron,

$$n \simeq N_D = (5 \times 10^{28})(10^{-6}) = 5 \times 10^{22} \text{ m}^{-3}.$$

The given hole concentration is negligible compared with this value, so this approximation is consistent.

Step 2: Apply the mass-action law With $p = 4.5 \times 10^9 \text{ m}^{-3}$,

$$n_i = \sqrt{np} = \sqrt{(5 \times 10^{22})(4.5 \times 10^9)} = \sqrt{2.25 \times 10^{32}} \text{ m}^{-3}.$$

The square of option (B) is exactly $2.25 \times 10^{32} \text{ m}^{-6}$. Options (A) and (C) have the wrong order of magnitude, while option (D) squares to $4.0 \times 10^{32} \text{ m}^{-6}$. Do not add intrinsic electrons and holes: n_i denotes the concentration of either species.

Step 3: Conclusion The intrinsic carrier concentration is $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$. Hence, the correct option is (B).

Quick Tip to Solve: Convert ppm to 10^{-6} , find the majority-carrier density, then take the square root of its product with the hole density.

8. The energy of an electron in the ground state of hydrogen atom is -13.6 eV . The kinetic and potential energy of the electron in the first excited state will be: [1 mark]

(A) -13.6 eV , 27.2 eV

(B) -6.8 eV , 13.6 eV

(C) 3.4 eV , -6.8 eV

(D) 6.8 eV , -3.4 eV

Correct Answer: (C) 3.4 eV , -6.8 eV

Solution:

Concept:

- In hydrogen, the total energy in level n is $E_n = -13.6/n^2\text{ eV}$; the first excited state has $n = 2$.
- For an electron bound by the Coulomb force, kinetic energy $K = -E_n$ and potential energy $U = 2E_n$, so $U = -2K$.

Step 1: Identify the excited energy level The ground state is $n = 1$, so use $n = 2$ for the first excited state. Its total energy is

$$E_2 = \frac{-13.6}{2^2} = -3.4\text{ eV}.$$

Step 2: Separate kinetic and potential energies The kinetic energy is positive, $K = -E_2 = 3.4\text{ eV}$, whereas the attractive potential gives $U = 2E_2 = -6.8\text{ eV}$. Their sum is -3.4 eV , as required. Options (A) and (B) incorrectly assign negative kinetic energies; (D) fails both the total-energy check and $U = -2K$.

Step 3: Conclusion The kinetic and potential energies are 3.4 eV and -6.8 eV , respectively. Hence, the correct option is (C).

Quick Tip to Solve: For a hydrogen level, first find its negative total energy; reverse its sign for kinetic energy and double it for potential energy.

9. The electromagnetic waves used to purify water are: [1 mark]

(A) Infrared rays

(B) Ultraviolet rays

(C) X-rays

(D) Gamma rays

Correct Answer: (B) Ultraviolet rays

Solution:

Concept:

- Different regions of the electromagnetic spectrum have characteristic applications determined by their interaction with matter.
- Ultraviolet radiation has a germicidal action: it damages the genetic material of microorganisms, preventing their reproduction and enabling water disinfection.

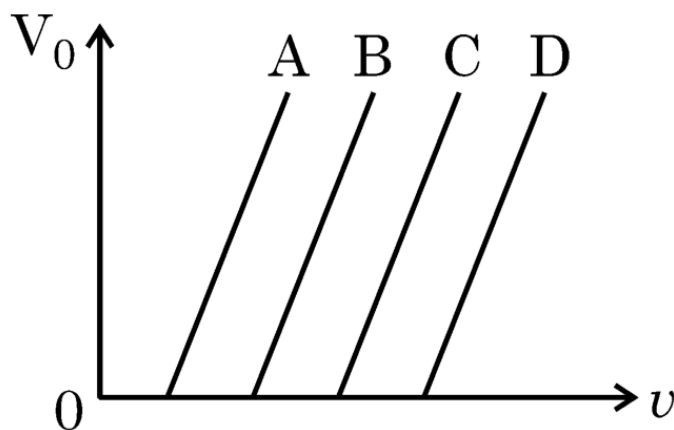
Step 1: Identify the required application The question refers to the standard use of electromagnetic radiation for water purification by destroying or inactivating harmful microorganisms. Ultraviolet rays are used for this disinfection process.

Step 2: Compare the other spectral regions Infrared rays are chiefly associated with heating and are not the germicidal radiation intended here. X-rays are chiefly used in diagnostic imaging, while gamma rays are used for applications such as cancer treatment and sterilisation of medical equipment. Neither (C) nor (D) names the standard water-purification application asked in this question.

Step 3: Conclusion Ultraviolet rays are used to purify water by their germicidal action. Hence, the correct option is (B).

Quick Tip to Solve: Associate water disinfection with ultraviolet rays; do not confuse this application with infrared heating or X-ray imaging.

10. The variation of the stopping potential (V_0) with the frequency (ν) of the incident radiation for four metals A, B, C and D is shown in the figure. For the same frequency of incident radiation producing photo-electrons in all metals, the kinetic energy of photo-electrons will be maximum for metal [1 mark]



- (A) A
- (B) B
- (C) C
- (D) D

Correct Answer: (A) A

Solution:

Concept:

- Einstein's photoelectric equation gives maximum kinetic energy $K_{\max} = h\nu - \phi = eV_0$, where ϕ is the work function and V_0 the stopping potential.
- A stopping-potential graph has slope h/e and frequency-axis intercept $\nu_0 = \phi/h$, the threshold frequency of the metal.

Step 1: Read the threshold frequencies The four parallel lines intersect the frequency axis in the order A, B, C, D from left to right. Thus metal A has the smallest threshold frequency and hence the smallest work function.

Step 2: Compare energies at a common frequency At a frequency high enough to eject electrons from all four metals, $h\nu$ is common. Subtracting the smallest work function leaves the largest $K_{\max}$. Equivalently, extend a vertical line at this frequency: line A gives the highest stopping potential.

Metals B, C and D have successively larger work functions and therefore smaller maximum kinetic energies; the rightmost line does not represent the largest energy at fixed frequency.

Step 3: Conclusion The maximum kinetic energy of emitted photoelectrons is greatest for metal A. Hence, the correct option is (A).

Quick Tip to Solve: At a common incident frequency, the leftmost threshold intercept corresponds to the greatest stopping potential and maximum kinetic energy.

11. The focal lengths of the objective and the eyepiece of a compound microscope are 1 cm and 2 cm respectively. If the tube length of the microscope is 10 cm, the magnification obtained by the microscope for most suitable viewing by relaxed eye is: [1 mark]

- (A) 250
- (B) 200
- (C) 150
- (D) 125

Correct Answer: (D) 125

Solution:

Concept:

- For a compound microscope in normal adjustment, the final image is at infinity, allowing observation with a relaxed eye.
- The standard approximate magnification magnitude is $M = (L/f_o)(D/f_e)$, where L is tube length, f_o and f_e are objective and eyepiece focal lengths, and $D = 25$ cm.

Step 1: Choose the relaxed-eye expression The objective provides linear magnification approximately L/f_o , and the eyepiece provides angular magnification D/f_e for the final image at infinity. We therefore multiply these two factors. The expression $1 + D/f_e$ applies when the final image is at the near point and does not describe a relaxed eye.

Step 2: Substitute consistent lengths All given lengths are in centimetres, so the ratios are dimensionless:

$$M = \frac{10}{1} \times \frac{25}{2} = 10 \times 12.5 = 125.$$

The image is inverted, but the listed options give magnification magnitudes. Values 250, 200 and 150 do not follow from the normal-adjustment formula with the stated focal lengths.

Step 3: Conclusion The magnification magnitude for relaxed-eye viewing is 125. Hence, the correct option is (D).

Quick Tip to Solve: Read “relaxed eye” as final image at infinity and use the eyepiece factor D/f_e , without adding 1.

12. A point object is kept 60 cm in front of a spherical convex surface ($n = 1.5$, radius of curvature 40 cm). The image formed is:

[1 mark]

- (A) real, at a distance 1.8 m from the surface.
- (B) virtual, at a distance 1.8 m from the surface.
- (C) real, at a distance 3.6 m from the surface.
- (D) virtual, at a distance 3.6 m from the surface.

Correct Answer: (D) virtual, at a distance 3.6 m from the surface.

Solution:

Concept:

- Refraction at a spherical surface obeys $n_2/v - n_1/u = (n_2 - n_1)/R$, with distances measured from the surface pole.
- In the Cartesian sign convention, distances along incident light are positive. Here the object is in air, so $n_1 = 1$, $n_2 = 1.5$, $u = -60$ cm, and the convex surface has $R = +40$ cm.

Step 1: Substitute with the correct signs The centre of curvature lies inside the refracting medium, on the positive side. Substituting in the spherical-refraction formula gives

$$\frac{1.5}{v} - \frac{1}{-60} = \frac{1.5 - 1}{40}.$$

Step 2: Solve for the image distance Rearranging in centimetres,

$$\frac{1.5}{v} = \frac{1}{80} - \frac{1}{60} = -\frac{1}{240}, \quad v = -360 \text{ cm} = -3.6 \text{ m}.$$

The negative sign places the image on the incident side: it is virtual. Thus (A) and (C) have the wrong image nature, and (B) has the wrong distance. Retaining the refractive-index factor multiplying $1/v$ is essential.

Step 3: Conclusion The image is virtual and lies 3.6 m from the surface on the object's side. Hence, the correct option is (D).

Quick Tip to Solve: Assign $u < 0$ and $R > 0$ before substitution; a negative result for v immediately identifies a virtual image on the incident side.

13. Assertion (A): In a Young's double-slit experiment, interference pattern is not observed when two coherent sources are infinitely close to each other.

Reason (R): The fringe width is proportional to the separation between the two sources. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (C) Assertion (A) is true but Reason (R) is false.

Solution:

Concept:

- In Young's double-slit experiment, fringe width is $\beta = \lambda D/d$, where λ is wavelength, D is screen distance and d is source separation.
- A visible fringe pattern requires appreciable variation of path difference across the screen; for a screen coordinate y , the paraxial path difference is $\Delta = dy/D$.

Step 1: Evaluate the assertion As the two coherent sources become infinitely close, $d \rightarrow 0$, so $\beta \rightarrow \infty$. Across a finite screen, alternating bright and dark fringes can no longer be resolved. At coincident sources, the path difference no longer varies with screen position. The assertion is therefore true.

Step 2: Evaluate the reason Holding λ and D fixed, the formula gives $\beta \propto 1/d$, not $\beta \propto d$. The reason states direct proportionality and is therefore false.

Step 3: Check the explanatory link A false reason cannot explain the true assertion. The correct explanation is the unbounded increase of fringe width as separation decreases. Options (A) and (B) require a true reason, while (D) requires a false assertion; neither condition applies here.

Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: In $\beta = \lambda D/d$, source separation is in the denominator: bringing the slits closer spreads the fringes farther apart.

14. Assertion (A): An alpha particle is moving towards a gold nucleus. The impact parameter is maximum for the scattering angle of 180° .

Reason (R): The impact parameter in an alpha particle scattering experiment does not depend upon the atomic number of the target nucleus. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (D) Both Assertion (A) and Reason (R) are false.

Solution:

Concept:

- The impact parameter b is the perpendicular distance between the initial direction of an alpha particle and a parallel line through the nucleus.
- Rutherford scattering gives $b = [kZe^2/K] \cot(\theta/2)$ for an alpha particle of charge $2e$ and incident kinetic energy K , with $k = 1/(4\pi\epsilon_0)$, target atomic number Z , and scattering angle θ .

Step 1: Evaluate the assertion A scattering angle of 180° corresponds to a head-on approach. Substitution gives $\cot(180^\circ/2) = \cot 90^\circ = 0$, hence $b = 0$. The impact parameter is minimum, not maximum, so the assertion is false.

Step 2: Evaluate the reason For a fixed incident energy and a specified non-head-on scattering angle, the formula contains Z as a multiplicative factor: $b \propto Z$. A nucleus with a greater positive charge produces the same deflection at a larger impact parameter. Thus the claimed independence from atomic number is false.

Step 3: Test the assertion-reason relationship Neither statement is true, so the reason cannot provide a valid explanation of the assertion. Options (A), (B) and (C) all require a true assertion and are ruled out by the head-on scattering condition.

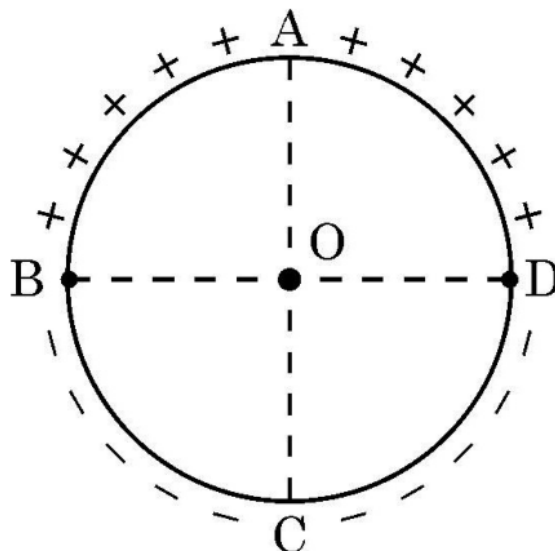
Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: Remember that complete backscattering means a head-on approach with $b = 0$; at fixed energy and deflection, use $b \propto Z$.

15. Assertion (A): Equal amount of positive and negative charges are distributed uniformly on two halves of a thin circular ring as shown in figure. The resultant electric field at the centre O of the ring is along OC.

Reason (R): It is so because the net potential at O is not zero.

[1 mark]



- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).
 (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
 (C) Assertion (A) is true but Reason (R) is false.
 (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (C) Assertion (A) is true but Reason (R) is false.

Solution:

Concept:

- The electric field is a vector directed away from positive charge and towards negative charge; contributions add by vector superposition.
- Potential is a scalar: $V = k \sum_i q_i / r_i$, where $k = 1/(4\pi\epsilon_0)$. At the centre of a ring every charge element is the same distance R away.

Step 1: Evaluate Assertion (A) In the positive upper semicircle, elements symmetrically placed about AO have cancelling horizontal field components at O. Their downward components add along OC. In the negative lower semicircle, the field at O points towards the charges: horizontal components again cancel and downward components again add. Hence the total field is along OC, so Assertion (A) is true.

Step 2: Evaluate Reason (R) Let the total charges on the two halves be $+Q$ and $-Q$. Since every element lies at radius R , the potential at O, relative to infinity, is

$$V_O = \frac{kQ}{R} + \frac{k(-Q)}{R} = 0.$$

Therefore the statement that the net potential at O is not zero is false.

Step 3: Check the explanation and the alternatives The field direction follows from vector addition, whereas the zero potential follows from scalar cancellation. Zero potential at one point does not imply

zero field there. Options (A) and (B) incorrectly treat the reason as true, and (D) incorrectly treats the assertion as false.

Step 4: Conclusion The assertion is true but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: At a ring's centre, add signed charges to find potential, but resolve field directions before adding them.

16. Assertion (A): The energy of a charged particle moving in a magnetic field does not change.

Reason (R): It is because the work done by the magnetic force on the charge moving in a magnetic field is zero. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).

Solution:

Concept:

- The magnetic force on a charge q moving with velocity $\vec{v}$ is $\vec{F}_B = q(\vec{v} \times \vec{B})$, perpendicular to its velocity.
- Instantaneous power is $P = \vec{F} \cdot \vec{v}$, and the work-energy theorem gives $\Delta K = W$, where K is kinetic energy.

Step 1: Evaluate Assertion (A) A magnetic force can change the direction of a moving particle's velocity, but it cannot change its speed. Thus its kinetic energy remains constant when the magnetic force alone acts. In the stated magnetic-field context, Assertion (A) is true.

Step 2: Evaluate Reason (R) The displacement in a short interval is $d\vec{r} = \vec{v} dt$, perpendicular to the magnetic force. Consequently,

$$dW_B = \vec{F}_B \cdot d\vec{r} = q(\vec{v} \times \vec{B}) \cdot \vec{v} dt = 0.$$

This also holds if the velocity is parallel to the field, when the magnetic force itself is zero. Reason (R) is true.

Step 3: Check the explanatory link Applying the work-energy theorem to the zero magnetic work gives $\Delta K = 0$. The reason directly explains the assertion, so option (B), which denies this link, is incorrect. Options (C) and (D) are excluded because neither statement is false. Any change due to an additional electric force would be a separate effect.

Step 4: Conclusion Both statements are true, and the reason correctly explains the assertion. Hence, the correct option is (A).

Quick Tip to Solve: Use the scalar product of magnetic force and velocity: a perpendicular force has zero power and preserves speed.

Section B (10 marks)

17. Derive an expression for magnetic force $\vec{F}$ acting on a straight conductor of length L carrying current I in an external magnetic field $\vec{B}$. Is it valid when the conductor is in zig-zag form? Justify.

[2 marks]

Solution:

Concept:

- A moving charge experiences magnetic force $\vec{f} = q(\vec{v}_d \times \vec{B})$, where $\vec{v}_d$ is its drift velocity.
- For a wire of cross-sectional area A and carrier number density n , conventional current obeys $I\hat{\ell} = nqA\vec{v}_d$, with $\hat{\ell}$ directed along the current.

Step 1: Add the forces on the charge carriers Consider a straight wire segment of length L in a uniform external magnetic field. It contains $N = nAL$ mobile charge carriers. The forces on the drifting carriers are transmitted to the conductor, giving

$$\vec{F} = nALq(\vec{v}_d \times \vec{B}).$$

Using $nqA\vec{v}_d = I\hat{\ell}$ and defining $\vec{L} = L\hat{\ell}$, this becomes

$$\vec{F} = I(\vec{L} \times \vec{B}), \quad F = ILB \sin \theta,$$

where θ is the angle between conventional current and the field. This direction convention also handles negative charge carriers, whose drift is opposite to the current.

Step 2: Add the forces on the zig-zag segments Split a zig-zag conductor into straight segments with current-directed length vectors $\vec{L}_1, \vec{L}_2, \dots$. If the same current flows throughout and the field is uniform,

$$\begin{aligned} \vec{F}_{\text{net}} &= \sum_j I(\vec{L}_j \times \vec{B}) \\ &= I \left(\sum_j \vec{L}_j \right) \times \vec{B} = I(\vec{L}_{\text{end}} \times \vec{B}). \end{aligned}$$

Here $\vec{L}_{\text{end}}$ joins the starting end to the finishing end along the current's overall displacement. It is not the total bent length of wire. In a non-uniform field, use $\vec{F} = I \int d\vec{\ell} \times \vec{B}$ instead.

Step 3: Conclusion For a straight conductor, $\vec{F} = I(\vec{L} \times \vec{B})$. The same vector form gives the net force on a zig-zag conductor in a uniform field when $\vec{L}$ is the end-to-end displacement directed along the current.

Quick Tip to Solve: For a bent wire in a uniform magnetic field, replace its path by the current-directed vector joining its ends.

18(a). Four point charges of $1\ \mu\text{C}$, $-2\ \mu\text{C}$, $1\ \mu\text{C}$ and $-2\ \mu\text{C}$ are placed at the corners A, B, C and D respectively, of a square of side 30 cm. Find the net force acting on a charge of $4\ \mu\text{C}$ placed at the centre of the square. [2 marks]

Solution:

Concept:

- Coulomb's law gives $F = k|qQ|/r^2$, where r is the separation and $k = 9 \times 10^9\ \text{N m}^2\ \text{C}^{-2}$.
- Forces add as vectors. Equal charges at opposite vertices of a square exert equal and opposite forces on a charge at its centre.

Step 1: Find the common centre-to-vertex distance Denote the centre by O and the side by $a = 0.30\ \text{m}$. Half the square's diagonal is the distance from every corner to O:

$$r = \frac{a}{\sqrt{2}}, \quad r^2 = \frac{(0.30)^2}{2} = 0.045\ \text{m}^2.$$

The test charge is $Q = 4 \times 10^{-6}\ \text{C}$, and A and C are opposite corners, as are B and D.

Step 2: Cancel the forces from the opposite positive charges The equal $+1\ \mu\text{C}$ charges at A and C both repel the central positive charge. Each force has magnitude

$$F_A = F_C = \frac{9 \times 10^9 (1 \times 10^{-6})(4 \times 10^{-6})}{0.045} = 0.80\ \text{N}.$$

They act along diagonal AC in opposite directions, so $\vec{F}_A + \vec{F}_C = \vec{0}$.

Step 3: Cancel the forces from the opposite negative charges The equal $-2\ \mu\text{C}$ charges at B and D attract the central positive charge towards their respective corners. Each force has magnitude

$$F_B = F_D = \frac{9 \times 10^9 (2 \times 10^{-6})(4 \times 10^{-6})}{0.045} = 1.60\ \text{N}.$$

These forces are opposite along diagonal BD, so $\vec{F}_B + \vec{F}_D = \vec{0}$. The two cancelling pairs need not have the same individual force magnitude.

Step 4: Conclusion The resultant force on the central charge is $\vec{F}_{\text{net}} = \vec{0}$, with magnitude 0 N.

Quick Tip to Solve: Pair opposite corners first: equal charges in each opposite pair cancel at the centre even when the pairs differ.

OR

18(b). Three point charges, 1 pC each, are kept at the vertices of an equilateral triangle of side 10 cm. Find the net electric field at the centroid of triangle. [2 marks]

Solution:

Concept:

- A point charge q produces electric field of magnitude $E = k|q|/r^2$, directed away from a positive charge.
- At the centroid of an equilateral triangle, the three equal vertex charges produce equal field vectors separated by 120° ; their vector sum is zero.

Step 1: Calculate the field from each vertex Each printed charge is $q = 1 \text{ pC} = 10^{-12} \text{ C}$, and the side is $a = 10 \text{ cm} = 0.10 \text{ m}$. The centroid O lies two-thirds of the way down each median, so its distance from a vertex is

$$r = \frac{2}{3} \frac{\sqrt{3}a}{2} = \frac{a}{\sqrt{3}}.$$

Thus each field has the same magnitude

$$E_0 = \frac{kq}{a^2/3} = \frac{3(9 \times 10^9)(10^{-12})}{(0.10)^2} = 2.7 \text{ N C}^{-1}.$$

Step 2: Add the three fields as vectors Take the positive x-axis along the field due to one vertex charge. The other two fields make angles 120° and 240° with this axis because the triangle is equilateral. Their components give

$$E_x = E_0(1 + \cos 120^\circ + \cos 240^\circ) = 0,$$

$$E_y = E_0(0 + \sin 120^\circ + \sin 240^\circ) = 0.$$

The scalar sum $3E_0$ is not the resultant: the three fields point in different directions. Their threefold symmetry causes exact cancellation at the centroid.

Step 3: Conclusion The net electric field at the centroid is $\vec{E}_{\text{net}} = \vec{0}$, with magnitude 0 N C^{-1} .

Quick Tip to Solve: At the centre of an equilateral triangle, three identical vertex charges give equal fields at 120° degrees and cancel.

19. A thin converging lens of focal length 10 cm is placed coaxially in contact with a thin diverging lens of focal length 15 cm. How will the combination behave? Justify your answer. [2 marks]

Solution:

Concept:

- For thin lenses placed coaxially in contact, their powers add: $P = P_1 + P_2$, or $1/F = 1/f_1 + 1/f_2$, where F is the equivalent focal length.
- A converging lens has positive focal length and a diverging lens has negative focal length. A positive net power means the combination converges a parallel incident beam.

Step 1: Assign the signs of the focal lengths Using the Cartesian sign convention, the converging lens has $f_1 = +10 \text{ cm}$, while the diverging lens has $f_2 = -15 \text{ cm}$. The second value must be negative even though the question gives its magnitude as 15 cm .

Step 2: Add the powers of the two lenses Since the lenses are in contact, no separation term is needed. Thus,

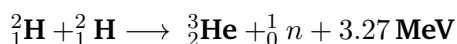
$$\frac{1}{F} = \frac{1}{10} - \frac{1}{15} = \frac{3-2}{30} = \frac{1}{30} \text{ cm}^{-1}.$$

Equivalently, the lens powers are $+10 \text{ D}$ and -6.67 D . The positive lens has the larger power magnitude, so its converging effect dominates the diverging effect.

Step 3: Conclusion The combination behaves as a **converging lens** of focal length $F = +30 \text{ cm}$, with net power $+3.33 \text{ D}$. Its positive focal length justifies this behaviour.

Quick Tip to Solve: Give the diverging focal length a negative sign before adding reciprocals; never add the focal lengths directly.

20. Deuterium undergoes the following fusion reaction:



How long an electric bulb of 200 W will glow by using the energy released in 2 g of deuterium?

[2 marks]

Solution:

Concept:

- Each printed fusion reaction consumes two deuterium nuclei and releases 3.27 MeV. Thus, if N nuclei fuse completely, the number of reactions is $N/2$.
- The energy supplied to a bulb of constant power P for time t is $E = Pt$. Use $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$ to convert nuclear energy to electrical energy units.

Step 1: Count the available fusion reactions Taking the molar mass of deuterium nuclei as 2 g mol^{-1} , the sample contains one mole of nuclei. With Avogadro's constant $N_A = 6.023 \times 10^{23} \text{ mol}^{-1}$,

$$N = \frac{2}{2} N_A = 6.023 \times 10^{23}, \quad N_{\text{reactions}} = \frac{N}{2} = 3.0115 \times 10^{23}.$$

The factor of two is necessary because the reaction uses a pair of deuterium nuclei, not a single nucleus.

Step 2: Convert the released energy and divide by power Assuming complete fusion and full conversion of released energy into energy supplied to the bulb,

$$\begin{aligned} E &= (3.0115 \times 10^{23})(3.27)(1.6 \times 10^{-13}) \\ &= 1.5756 \times 10^{11} \text{ J}, \\ t &= \frac{E}{P} = \frac{1.5756 \times 10^{11}}{200} = 7.878 \times 10^8 \text{ s}. \end{aligned}$$

A watt is one joule per second, so this division gives the duration in seconds.

Step 3: Conclusion The 200 W bulb can glow for approximately $7.9 \times 10^8 \text{ s}$, or about 25 years, under the assumed complete energy conversion.

Quick Tip to Solve: Divide the number of deuterium nuclei by two before multiplying by the energy released per fusion reaction.

21. The electron in hydrogen atom is revolving with the speed of $2.2 \times 10^6 \text{ m/s}$ in an orbit of radius $0.53 \text{ \AA}$. Calculate the initial frequency of light emitted by the electron using classical physics. [2 marks]

Solution:

Concept:

- In classical electrodynamics, an accelerating charge radiates electromagnetic waves. For an electron initially in a circular orbit, the emitted radiation has the initial orbital frequency.
- One revolution covers a distance $2\pi r$, so the period is $T = 2\pi r/v$ and the frequency is $\nu = 1/T = v/(2\pi r)$, where r is the orbit radius and v the electron's speed.

Step 1: Express the radius in SI units An angstrom equals 10^{-10} m. Therefore,

$$r = 0.53 \times 10^{-10} \text{ m} = 5.3 \times 10^{-11} \text{ m}.$$

The initial speed is $v = 2.2 \times 10^6 \text{ m s}^{-1}$. Both quantities are now in consistent SI units.

Step 2: Calculate the initial orbital frequency Divide the speed by the circumference of the orbit:

$$\nu = \frac{2.2 \times 10^6}{2\pi(5.3 \times 10^{-11})} \simeq 6.61 \times 10^{15} \text{ s}^{-1}.$$

This is an initial classical prediction. Classical energy loss would subsequently change the orbit and hence its frequency. The question does not ask for a quantum transition frequency, so no difference between two energy levels is used here.

Step 3: Conclusion According to classical physics, the initial frequency of the emitted light is $\nu \simeq 6.6 \times 10^{15} \text{ Hz}$.

Quick Tip to Solve: Use the electron speed, not the speed of light, in $v/(2\pi r)$; first convert the orbit radius from angstroms to metres.

Section C (21 marks)

22(a). (i) State Lenz's Law. In a closed circuit, the induced current opposes the change in magnetic flux that produced it as per the law of conservation of energy. Justify.

(ii) A metal rod of length 2 m is rotated with a frequency 60 rev/s about an axis passing through its centre and perpendicular to its length. A uniform magnetic field of 2 T perpendicular to its plane of rotation is switched-on in the region. Calculate the e.m.f. induced between the centre and the end of the rod.

[3 marks]

Solution:

Concept:

- Lenz's law states that the direction of induced current is such that the magnetic effect of that current opposes the change in magnetic flux producing it. This is represented by the minus sign in $\varepsilon = -d\Phi_B/dt$.
- For a rod rotating in a uniform magnetic field perpendicular to its plane, an element dr at distance r from the axis has speed $v = \omega r$ and contributes motional emf $d\varepsilon = B\omega r dr$. Here $\omega = 2\pi f$ is angular speed.

Step 1: State and justify Lenz's law in part (i) The induced current opposes the *change* in flux, rather than necessarily opposing the existing field. For example, when a magnet approaches a conducting loop, the induced current produces a magnetic force opposing the approach. External work must therefore be done to maintain the motion; that work supplies the electrical energy dissipated as heat in the loop.

If the induced current instead aided the flux change, it would attract the approaching magnet, speeding it up. The increasing rate of flux change would increase the induced current and further accelerate the magnet while also producing heat, without an external energy supply. Such a self-amplifying gain of energy violates conservation of energy. Thus opposition to the change is required by energy conservation.

Step 2: Identify the rotation radius in part (ii) The total rod length is $L = 2\text{ m}$, but the requested emf is between its centre and one end. The relevant radial length is therefore $a = L/2 = 1\text{ m}$. The rotation frequency gives

$$\omega = 2\pi f = 2\pi(60) = 120\pi \text{ rad s}^{-1}.$$

Step 3: Integrate the motional emf along half the rod Speed increases linearly from zero at the centre to ωa at the end, so the emf must be summed along the rod:

$$\begin{aligned} |\varepsilon| &= \int_0^a B\omega r \, dr = \frac{1}{2}B\omega a^2 \\ &= \frac{1}{2}(2)(120\pi)(1)^2 = 120\pi \text{ V}. \end{aligned}$$

This is the motional emf while the rod rotates in the established uniform field. The problem gives no switching duration for calculating a separate transient emf. Since neither field direction nor rotation sense is specified, only the magnitude, rather than the end's polarity, is determined.

Step 4: Conclusion (i) Lenz's law follows conservation of energy because external work against the induced opposition supplies the electrical energy. (ii) The induced emf between the centre and either end is $120\pi \text{ V} \simeq 377 \text{ V}$.

Quick Tip to Solve: For rotation about the centre, use half the total rod length in $\varepsilon = \frac{1}{2}B\omega a^2$; using the full length makes the answer four times too large.

OR

22(b). (i) State and explain Ampere's circuital law.

(ii) Two long straight parallel wires separated by 20 cm, carry 5 A and 10 A current respectively, in the same direction. Find the magnitude and direction of the net magnetic field at a point midway between them.

[3 marks]

Solution:

Concept:

- Ampere's circuital law for steady currents relates magnetic circulation around a closed path to the algebraic current threading its bounded surface: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$.
- A long straight wire carrying current I produces $B = \mu_0 I / (2\pi r)$ at perpendicular distance r . The right-hand thumb rule gives its direction.

Step 1: (i) State and explain the circuital law The line integral of the magnetic field around any closed path equals μ_0 times the net current enclosed by that path:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \sum I_{\text{enc}}.$$

Here $d\vec{l}$ is a small directed path element and μ_0 is the permeability of free space. The dot product selects the component of field tangent to the path. Currents through the enclosed surface are added algebraically: curl the right-hand fingers along the chosen path direction and the thumb gives the positive current direction. For a circular path about a straight wire, symmetry gives $B(2\pi r) = \mu_0 I$.

Step 2: (ii) Calculate the fields at the midpoint The separation is 20 cm, so the midpoint is $r =$

10 cm = 0.10 m from each wire. Thus

$$B_5 = \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 0.10} = 1.0 \times 10^{-5} \text{ T},$$
$$B_{10} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.10} = 2.0 \times 10^{-5} \text{ T}.$$

Although the currents flow in the same direction, their fields at a point between the wires are opposite. The right-hand thumb rule must be applied separately to each wire.

Step 3: Combine the opposite fields and specify the direction The resultant magnitude follows by subtraction:

$$B_{\text{net}} = B_{10} - B_5 = (2.0 - 1.0) \times 10^{-5} \text{ T}.$$

Its direction is that of the field due to the 10 A wire, perpendicular to the plane containing both wires. For example, if the 5 A wire is on the left, the 10 A wire on the right, and both currents are upward, the resultant is out of the page. Reversing both currents reverses this direction.

Step 4: Conclusion Ampere's law is $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$. The midpoint field is $1.0 \times 10^{-5} \text{ T}$, in the direction of the 10 A wire's field, normal to the plane of the wires.

Quick Tip to Solve: At the midpoint of parallel currents in the same direction, subtract the two field magnitudes; use the stronger current to select the resultant direction.

23. The threshold frequency for a metal is $3.0 \times 10^{14} \text{ Hz}$. A beam of frequency $9.0 \times 10^{14} \text{ Hz}$ is incident on the metal. Calculate (i) the work function (in eV) of the metal and (ii) the maximum speed of photoelectrons. [3 marks]

Solution:

Concept:

- The work function ϕ is the minimum energy needed to liberate an electron from a metal. At the threshold frequency ν_0 , $\phi = h\nu_0$, where h is Planck's constant.
- Einstein's photoelectric equation is $K_{\text{max}} = h\nu - \phi = h(\nu - \nu_0)$. For the emitted electron, $K_{\text{max}} = \frac{1}{2}m_e v_{\text{max}}^2$, with m_e its mass.

Step 1: (i) Calculate the work function in joules The given threshold frequency is $\nu_0 = 3.0 \times 10^{14} \text{ Hz}$. Using $h = 6.63 \times 10^{-34} \text{ J s}$,

$$\phi = h\nu_0 = (6.63 \times 10^{-34})(3.0 \times 10^{14}) = 1.989 \times 10^{-19} \text{ J}.$$

Step 2: Express the work function in electron volts Since $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$, divide the energy in joules by this conversion factor:

$$\phi = \frac{1.989 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 1.243 \text{ eV}.$$

Step 3: (ii) Find the maximum kinetic energy Only the photon energy in excess of the work function

becomes kinetic energy. With $\nu = 9.0 \times 10^{14}$ Hz,

$$\begin{aligned}K_{\max} &= h(\nu - \nu_0) \\&= (6.63 \times 10^{-34})(9.0 - 3.0) \times 10^{14} \\&= 3.978 \times 10^{-19} \text{ J.}\end{aligned}$$

Step 4: Convert kinetic energy into speed Using $m_e = 9.1 \times 10^{-31}$ kg,

$$\begin{aligned}v_{\max} &= \sqrt{\frac{2K_{\max}}{m_e}} \\&= \sqrt{\frac{2(3.978 \times 10^{-19})}{9.1 \times 10^{-31}}} \\&= \sqrt{8.743 \times 10^{11}} \text{ m s}^{-1}.\end{aligned}$$

Step 5: Conclusion The work function is $\phi \approx 1.24 \text{ eV}$, and the maximum photoelectron speed is

$$v_{\max} \approx 9.35 \times 10^5 \text{ m s}^{-1}.$$

Quick Tip to Solve: Subtract the threshold frequency before calculating electron kinetic energy; the full incident photon energy cannot be used in $\frac{1}{2}m_e v^2$.

24. (a) Name the parts of the electromagnetic spectrum which are (i) also known as ‘heat waves’ and (ii) absorbed by ozone layer in the atmosphere.

(b) Write briefly one method each, of the production and detection of these radiations.

[3 marks]

Solution:

Concept:

- Different regions of the electromagnetic spectrum are identified by their wavelengths and characteristic interactions with matter. Infrared radiation lies beyond the red end of visible light, while ultraviolet radiation lies beyond the violet end.
- Production means generating the radiation; detection means converting its absorption into an observable signal, such as heating or visible fluorescence.

Step 1: (a) Identify the two spectral regions (i) The radiations commonly called heat waves are **infrared rays**. Their absorption increases the internal energy of matter and causes heating. This name describes a prominent effect of infrared radiation; it does not imply that other absorbed radiations cannot heat an object.

(ii) The radiation absorbed by the ozone layer is **ultraviolet radiation**. Ozone absorbs strongly in parts of the ultraviolet spectrum, shielding life at the Earth’s surface from much of the harmful solar ultraviolet radiation.

Step 2: (b) Give production and detection methods for infrared rays **Production:** Use a hot body, for example an electrically heated filament. Thermal motion of its atoms and molecules produces radiation, including infrared radiation.

Detection: Use a thermopile with a blackened absorbing surface. Incident infrared radiation heats its absorbing junctions relative to the reference junctions, producing a thermoelectric emf that can be measured. Thus the radiation is detected through its heating effect.

Step 3: Give production and detection methods for ultraviolet rays **Production:** Pass an electric discharge through mercury vapour in a quartz lamp. Excited mercury atoms emit ultraviolet radiation on returning to lower energy states; quartz permits this radiation to emerge.

Detection: Use a fluorescent screen. The material absorbs incident ultraviolet photons and re-emits part of their energy as visible light. The resulting visible fluorescence reveals the presence of ultraviolet radiation even though the ultraviolet itself cannot be seen by the eye.

Step 4: Conclusion Heat waves are **infrared rays**, produced by hot bodies and detected with a thermopile. Radiation absorbed by ozone is **ultraviolet radiation**, produced by a mercury-vapour discharge and detected by a fluorescent screen.

Quick Tip to Solve: Pair each radiation with a detector using its characteristic effect: infrared gives a thermal signal, while ultraviolet can produce visible fluorescence.

25. (a) Explain the characteristics of a p-n junction diode that makes it suitable for its use as a rectifier.

(b) With the help of a circuit diagram, explain the working of a full wave rectifier.

[3 marks]

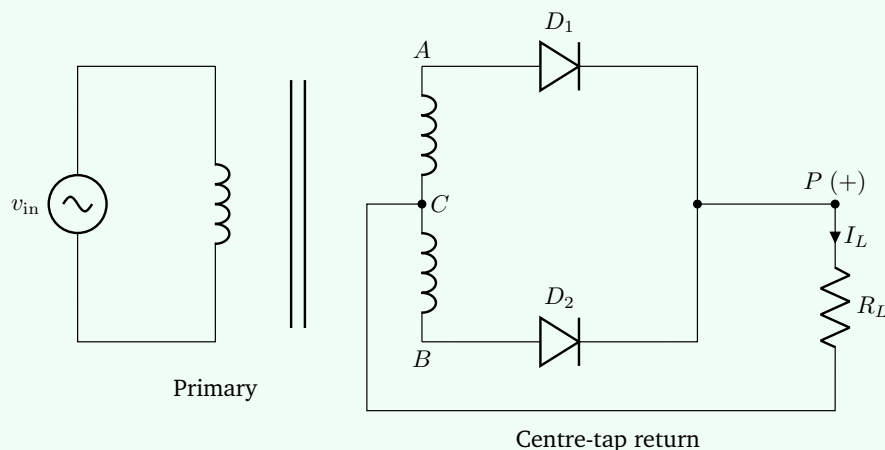
Solution:

Concept:

- Rectification converts alternating current into unidirectional current. A p-n junction conducts readily in forward bias and offers a very high resistance in reverse bias.
- In a centre-tapped full-wave rectifier, the two ends of the transformer secondary have opposite instantaneous polarities relative to its centre tap. Two diodes conduct alternately through the same load.

Step 1: (a) Identify the useful diode characteristics In forward bias, the junction barrier is reduced and the diode offers low resistance after its threshold voltage is crossed; an appreciable current then flows. In reverse bias, the barrier increases and the diode offers very high resistance, allowing only a negligible reverse current before breakdown. Thus it acts approximately as a one-way switch and can select the direction of current through a load.

Step 2: (b) Connect the full-wave rectifier Connect the ends A and B of a centre-tapped transformer secondary to the anodes of D_1 and D_2 . Join their cathodes at P . Connect the load resistance R_L between P and the centre tap C . The primary is supplied by an alternating voltage. The two secondary halves have equal numbers of turns.



Centre-tapped full-wave rectifier; load current is downward in both half-cycles.

Step 3: Follow both half-cycles through the load When A is positive relative to C , B is negative. Diode D_1 is forward biased and D_2 is reverse biased. Conventional current follows $A \rightarrow D_1 \rightarrow P \rightarrow R_L \rightarrow C$. In the next half-cycle, B becomes positive and A negative relative to C . Now D_2 conducts and D_1 blocks; current follows $B \rightarrow D_2 \rightarrow P \rightarrow R_L \rightarrow C$. In both cases it flows through the load from P towards C . Both input half-cycles therefore contribute to the output.

Step 4: Conclusion The diode's unequal forward and reverse resistances enable rectification. The two-diode circuit gives a unidirectional, pulsating output across R_L , with ripple frequency $2f$ for input frequency f ; a filter is needed for a smoother dc output.

Quick Tip to Solve: Trace each conducting diode back to the centre tap: the two paths must pass through the load in the same direction.

26. Explain the following, giving reasons:

- A doped semiconductor is electrically neutral.
- In a p-n junction under equilibrium, there is no net current.
- In a diode, the reverse current is practically not dependent on the applied voltage.

[3 marks]

Solution:

Concept:

- Doping changes the concentration of mobile charge carriers, but it does not supply a net electric charge to an initially neutral crystal.
- Diffusion is carrier motion due to a concentration gradient, while drift is carrier motion due to an electric field. At junction equilibrium their currents balance.
- Below breakdown, reverse diode current is controlled mainly by the supply of thermally generated minority carriers and hence by temperature.

Step 1: (a) Account for both mobile and fixed charges A donor impurity atom added to form an n-type semiconductor is initially neutral. On donating its extra electron, it becomes a fixed positive donor ion; its positive charge balances the negative charge of the donated mobile electron. In p-type material, an acceptor atom captures an electron and becomes a fixed negative ion, leaving a positive mobile hole. The opposite charges again balance. Thus having electrons or holes as majority carriers does not imply a net charge on the doped semiconductor as a whole.

Step 2: (b) Balance the two junction currents Because of their concentration differences, electrons diffuse from the n-side to the p-side and holes from the p-side to the n-side. The uncovered fixed ions create a depletion layer and a built-in electric field directed from n to p. This field drives a drift current opposite to the diffusion current. At thermal equilibrium their magnitudes are equal, so

$$I_{\text{net}} = I_{\text{diffusion}} + I_{\text{drift}} = 0.$$

Carrier motion has not stopped; the opposing currents cancel and there is no net current across the junction.

Step 3: (c) Identify what limits reverse current Reverse bias suppresses majority-carrier diffusion and sweeps minority carriers across the junction. Once the reverse field is sufficient to collect essentially all available minority carriers, increasing the voltage does not appreciably increase their supply. At a fixed temperature, the rate of thermal generation is nearly fixed, so the reverse current approaches a small saturation value. This explanation applies before reverse breakdown; at breakdown the current increases sharply. Raising temperature can also increase reverse current significantly.

Step 4: Conclusion A doped crystal remains neutral because mobile-carrier charges are balanced by ionised impurity charges. Junction equilibrium has equal opposing drift and diffusion currents. Reverse current is nearly voltage-independent below breakdown because minority-carrier generation limits it.

Quick Tip to Solve: For neutrality count impurity ions as well as carriers; for equilibrium balance currents; for reverse bias ask what supplies the minority carriers.

27. An electron is moving with a velocity $\vec{v} = (3 \times 10^6 \text{ m/s})\hat{i}$. It enters a region of magnetic field $\vec{B} = (91 \text{ mT})\hat{k}$.

(a) Calculate the magnetic force $\vec{F}_B$ acting on electron and the radius of its path.

(b) Trace the path described by it.

[3 marks]

Solution:

Concept:

- The magnetic force is $\vec{F}_B = q(\vec{v} \times \vec{B})$. For an electron $q = -e$, where $e = 1.6 \times 10^{-19} \text{ C}$, so its force is opposite to $\vec{v} \times \vec{B}$.
- For velocity perpendicular to a uniform magnetic field, magnetic force supplies the centripetal force: $evB = mv^2/r$, hence $r = mv/(eB)$. Here $m = 9.1 \times 10^{-31} \text{ kg}$ is the electron mass.

Step 1: (a) Evaluate the vector product and force The printed speed is $v = 3 \times 10^6 \text{ m s}^{-1}$, and $B = 91 \text{ mT} = 91 \times 10^{-3} \text{ T}$. Since $\hat{i} \times \hat{k} = -\hat{j}$, the electron's negative charge reverses this direction:

$$\begin{aligned}\vec{F}_B &= (-e)(v\hat{i} \times B\hat{k}) = evB\hat{j}, \\ evB &= (1.6 \times 10^{-19})(3 \times 10^6)(91 \times 10^{-3}) \\ &= 4.368 \times 10^{-14} \text{ N}.\end{aligned}$$

Thus the initial deflection is towards the positive y -axis. The stated force direction refers to entry; it subsequently turns with the electron's position.

Step 2: Determine the orbit radius Since $\vec{v} \perp \vec{B}$, the entire magnetic force is perpendicular to the velocity. It does no work and leaves the speed unchanged, so equate its magnitude to the required

centripetal force:

$$evB = \frac{mv^2}{r},$$

$$r = \frac{mv}{eB} = \frac{(9.1 \times 10^{-31})(3 \times 10^6)}{(1.6 \times 10^{-19})(91 \times 10^{-3})}$$

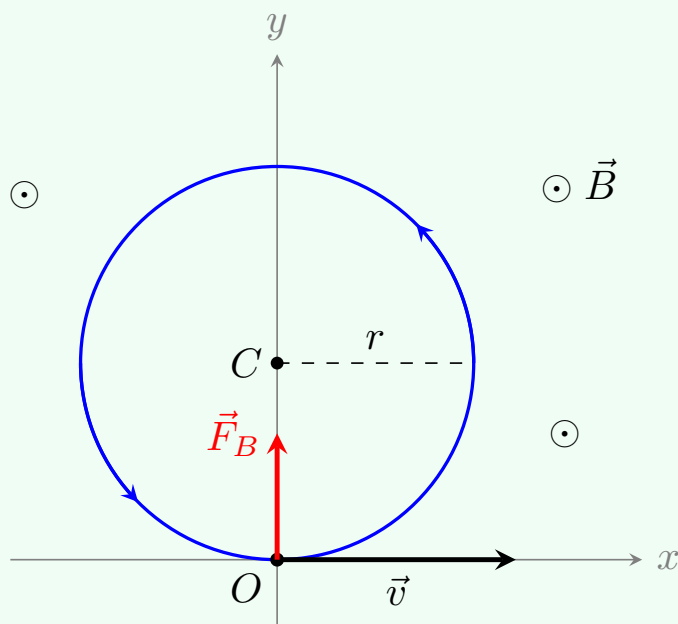
$$= 1.875 \times 10^{-4} \text{ m} = 0.1875 \text{ mm}.$$

There is no component of velocity parallel to the field, so no helical advance occurs along z .

Step 3: (b) Trace the path in the perpendicular plane Taking the entry point as $O = (0, 0)$, the initial velocity is along $+x$ and the initial force along $+y$. Therefore the circle's centre is $C = (0, r)$, and its equation is

$$x^2 + (y - r)^2 = r^2.$$

The electron follows a circular path in the xy -plane, turning counterclockwise when viewed from the positive z -axis towards the origin. In the sketch $\odot$ indicates $\vec{B}$ directed out of the page. This circular motion continues while the electron remains in the uniform-field region.



Electron orbit viewed from the positive z -axis; the field points towards the viewer.

Step 4: Conclusion At entry, $\vec{F}_B = +4.368 \times 10^{-14} \hat{j} \text{ N}$. The path is a circle of radius $1.875 \times 10^{-4} \text{ m}$ in the xy -plane, initially curving towards $+y$, counterclockwise as viewed from $+z$.

Quick Tip to Solve: Compute the cross product first, then reverse it for the electron; the initial force points directly from the entry point to the circle's centre.

28. A potential difference of 1.0 V is applied across a conductor of length 5.0 m and area of cross-section 1.0 mm^2 . When current of 4.25 A is passed through the conductor, calculate (i) the drift speed and (ii) relaxation time, of electrons. (Given number density of electrons in the conductor, $n = 8.5 \times 10^{28} \text{ m}^{-3}$). [3 marks]

Solution:

Concept:

- The current magnitude is $I = neAv_d$, where n is electron number density, e is the electron charge magnitude, A is the cross-sectional area and v_d is drift speed.
- In the Drude model, $v_d = eE\tau/m_e$, where $E = V/L$ is the electric field and τ is the mean relaxation time between collisions.

Step 1: Convert the area and find the field The given area must be expressed in square metres before using the number density in SI units. Since the conversion factor is squared,

$$A = 1.0 \text{ mm}^2 = 1.0 \times 10^{-6} \text{ m}^2.$$

The uniform electric field along the conductor has magnitude

$$E = \frac{V}{L} = \frac{1.0}{5.0} = 0.20 \text{ V m}^{-1}.$$

Step 2: Calculate the electron drift speed In a time interval Δt , the electrons drifting through a section occupy a volume $Av_d\Delta t$. Their charge magnitude is $neAv_d\Delta t$, giving $I = neAv_d$. Using $e = 1.6 \times 10^{-19} \text{ C}$,

$$v_d = \frac{4.25}{(8.5 \times 10^{28})(1.6 \times 10^{-19})(1.0 \times 10^{-6})} = 3.125 \times 10^{-4} \text{ m s}^{-1}.$$

This is the small directed drift speed, not the much larger random thermal speed. Electrons drift opposite to the electric field and conventional current.

Step 3: Determine the relaxation time The field gives an electron acceleration of magnitude eE/m_e . Averaging the acceleration over the mean time between collisions gives the drift relation. Rearranging it and using $m_e = 9.1 \times 10^{-31} \text{ kg}$,

$$\tau = \frac{m_e v_d}{eE} = \frac{(9.1 \times 10^{-31})(3.125 \times 10^{-4})}{(1.6 \times 10^{-19})(0.20)} = 8.89 \times 10^{-15} \text{ s}.$$

Step 4: Conclusion (i) The drift speed is $v_d \simeq 3.1 \times 10^{-4} \text{ m s}^{-1}$. (ii) The relaxation time is $\tau \simeq 8.9 \times 10^{-15} \text{ s}$.

Quick Tip to Solve: Convert square millimetres to square metres first; then find v_d from current and τ from the field V/L .

Section D (8 marks)

29. A prism is an optical medium bounded by three refracting plane surfaces. A ray of light suffers successive refractions on passing through its two surfaces and deviates by a certain angle from its original path. The refractive index of the material of the prism is given by $\mu = \sin \left(\frac{A + \delta_m}{2} \right) / \sin \frac{A}{2}$.

If the angle of incidence on the second surface is greater than an angle called critical angle, the ray will not be refracted from the second surface and is totally internally reflected. [4 marks]

29(i). The critical angle for glass is θ_1 and that for water is θ_2 . The critical angle for glass-water surface would be (given ${}_a\mu_g = 1.5$, ${}_a\mu_w = 1.33$): [1 mark]

- (A) less than θ_2
- (B) between θ_1 and θ_2
- (C) greater than θ_2
- (D) less than θ_1

Correct Answer: (C) greater than θ_2

Solution:

Concept:

- For light passing from a denser medium of index n_1 into a rarer medium of index n_2 , the critical angle satisfies $\sin C = n_2/n_1$.
- The inverse sine increases with its argument between zero and one, so the critical angles can be compared using their sines.

Step 1: Compare the original critical angles The stated glass and water critical angles refer to their interfaces with air:

$$\sin \theta_1 = \frac{1}{1.5} = 0.667, \quad \sin \theta_2 = \frac{1}{1.33} = 0.752.$$

Thus $\theta_1 < \theta_2$.

Step 2: Evaluate the glass-water ratio For light travelling from glass to water,

$$\sin C_{gw} = \frac{1.33}{1.5} = 0.887 > \sin \theta_2.$$

Therefore $C_{gw} \approx 62.5^\circ$, whereas $\theta_2 \approx 48.8^\circ$. Options (A) and (B) wrongly place it below θ_2 ; (D) places it still lower, below θ_1 .

Step 3: Conclusion The glass-water critical angle is greater than θ_2 . Hence, the correct option is (C).

Quick Tip to Solve: Use the ratio of the two absolute indices for a glass-water boundary; do not use the glass-air ratio $1/1.5$.

29(ii). When a ray of light of wavelength λ and frequency ν is refracted into a denser medium:

[1 mark]

- (A) λ and ν both increase.
- (B) λ increases but ν is unchanged.
- (C) λ decreases but ν is unchanged.
- (D) λ and ν both decrease.

Correct Answer: (C) λ decreases but ν is unchanged.

Solution:

Concept:

- The frequency ν of light is set by its source and remains unchanged on refraction at a stationary boundary.
- Wave speed, wavelength and frequency obey $v = \nu\lambda$. In an optically denser medium, the wave speed is smaller.

Step 1: Keep the frequency fixed The incident and transmitted waves must oscillate at the same frequency at the interface. Changing the medium therefore changes the propagation speed, but not ν . This immediately excludes (A) and (D), both of which change the frequency.

Step 2: Determine the wavelength change As the ray enters the denser medium, $v_2 < v_1$. Since ν is unchanged,

$$\frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1} < 1.$$

Thus the crests are closer together in the denser medium. Option (B), which increases the wavelength at fixed frequency, would instead require an increase in speed.

Step 3: Conclusion The wavelength λ decreases while the frequency ν remains unchanged. Hence, the correct option is (C).

Quick Tip to Solve: At a stationary refracting surface, keep frequency fixed and let wavelength change in the same proportion as wave speed.

29(iii)(a). The critical angle for a ray of light passing from glass to water is minimum for:

[1 mark]

- (A) red colour
- (B) blue colour
- (C) yellow colour
- (D) violet colour

Correct Answer: (D) violet colour

Solution:

Concept:

- At a glass-water interface, the critical angle satisfies $\sin C = n_w/n_g = 1/\mu_{gw}$, where μ_{gw} is the refractive index of glass relative to water.
- Under the usual normal-dispersion ordering used here, the relative refractive index is greatest for violet light and least for red light.

Step 1: Relate the critical angle to colour A larger relative refractive index gives a smaller value of

$\sin C$, and hence a smaller critical angle. The inverse relation must be applied to the index of glass relative to water, since the ray does not emerge into air.

Step 2: Compare the four colours Violet has the largest relative index among red, yellow, blue and violet in the normal-dispersion model. Therefore their critical angles obey

$$C_V < C_B < C_Y < C_R.$$

Red gives the largest angle, so (A) fails. Blue and yellow give intermediate values, excluding (B) and (C).

Step 3: Conclusion The critical angle is minimum for violet light. Hence, the correct option is (D).

Quick Tip to Solve: For critical-angle comparisons, invert the relative-index ordering: a larger relative index means a smaller critical angle.

OR

29(iii)(b). Three beams of red, yellow and violet colours are passed through a prism, one by one under the same condition. When the prism is in the position of minimum deviation, the angles of refraction from the second surface are r_R , r_Y and r_V respectively. Then: [1 mark]

(A) $r_V < r_Y < r_R$

(B) $r_Y < r_R < r_V$

(C) $r_R < r_Y < r_V$

(D) $r_R = r_Y = r_V$

Correct Answer: (C) $r_R < r_Y < r_V$

Solution:

Concept:

- At minimum deviation, the internal angles at the two refracting faces are equal to $A/2$, where A is the prism angle.
- Refraction from the second face into air obeys $\sin e = \mu \sin(A/2)$, where e is the emergent angle and μ is the prism's refractive index for that colour.

Step 1: Identify the angles asked for The printed question asks for the angles of refraction *from the second surface*; these are the emergent angles in air, denoted here by r_R, r_Y, r_V . They are distinct from the internal incidence angles, which are $A/2$ for each colour when it is at minimum deviation.

Step 2: Apply dispersion and Snell's law For normal dispersion, $\mu_R < \mu_Y < \mu_V$. For each colour, Snell's law at the exit face gives

$$\sin r_R = \mu_R \sin(A/2), \quad \sin r_Y = \mu_Y \sin(A/2), \quad \sin r_V = \mu_V \sin(A/2).$$

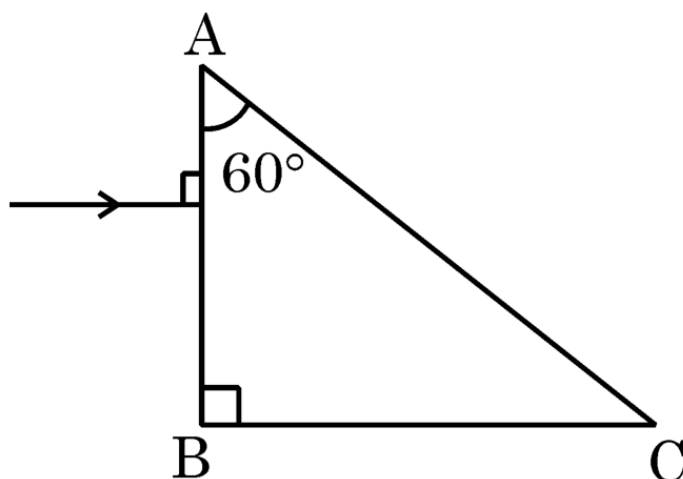
The common geometric factor is positive, so the emergent angles increase from red through yellow to

violet. Options (A) and (B) have the wrong order. Option (D) confuses the equal internal angles with the unequal emergent angles.

Step 3: Conclusion The angles of refraction from the second surface satisfy $r_R < r_Y < r_V$. Hence, the correct option is (C).

Quick Tip to Solve: At minimum deviation, internal angles equal $A/2$, but exit angles in air still depend on colour through Snell's law.

29(iv). A ray of light is incident normally on a prism ABC of refractive index $\sqrt{2}$, as shown in figure. After it strikes face AC, it will: [1 mark]



- (A) go straight undeviated
- (B) just graze along the face AC
- (C) refract and go out of the prism
- (D) undergo total internal reflection

Correct Answer: (D) undergo total internal reflection

Solution:

Concept:

- A ray incident normally on a face enters without bending. The angle of incidence at any later face is measured from its normal.
- For a prism-air interface, $\sin C = 1/\mu$; total internal reflection occurs when the internal incidence angle exceeds the critical angle C .

Step 1: Determine the incidence angle at AC The ray enters AB normally, so it travels horizontally inside the prism, parallel to BC. Since $\angle A = 60^\circ$ and $\angle B = 90^\circ$, face AC makes 30° with BC. The ray therefore makes 30° with AC, giving the angle from its normal as

$$i = 90^\circ - 30^\circ = 60^\circ.$$

Step 2: Compare with the critical angle With $\mu = \sqrt{2}$,

$$\sin C = \frac{1}{\sqrt{2}}, \quad C = 45^\circ.$$

Since $i = 60^\circ > C$, a transmitted ray cannot emerge into air. Grazing emergence in (B) would require $i = C$; ordinary refraction in (C) requires $i < C$. Undeviated transmission in (A) is inconsistent with incidence beyond the critical angle.

Step 3: Conclusion The ray undergoes total internal reflection at face AC. Hence, the correct option is (D).

Quick Tip to Solve: The angle between the ray and face AC is 30° , but the incidence angle from the normal is its complement, 60° .

30. Dielectrics play an important role in design of capacitors. The molecules of a dielectric may be polar or non-polar. When a dielectric slab is placed in an external electric field, opposite charges appear on the two surfaces of the slab perpendicular to electric field. Due to this an electric field is established inside the dielectric. The capacitance of a capacitor is determined by the dielectric constant of the material that fills the space between the plates. Consequently, the energy storage capacity of a capacitor is also affected. Like resistors, capacitors can also be arranged in series and/or parallel.

[4 marks]

30(i). Which of the following is a polar molecule?

[1 mark]

- (A) O_2
- (B) H_2
- (C) N_2
- (D) HCl

Correct Answer: (D) HCl

Solution:

Concept:

- A polar molecule has a permanent electric dipole moment because its centres of positive and negative charge do not coincide.
- A homonuclear diatomic molecule has symmetric charge distribution and zero permanent dipole moment.

Step 1: Compare the identical-atom molecules In O_2 , H_2 and N_2 , both atoms have the same electronegativity. The bonding electrons are shared symmetrically, so the positive and negative charge centres coincide. These three molecules are non-polar, eliminating options (A), (B) and (C).

Step 2: Examine the unlike-atom bond In HCl, chlorine attracts the bonding electrons more strongly than hydrogen. Chlorine acquires a partial negative charge and hydrogen a partial positive charge. Their separated charge centres give a permanent dipole moment even without an applied electric

field.

Step 3: Conclusion HCl is a polar molecule. Hence, the correct option is (D).

Quick Tip to Solve: Eliminate the three homonuclear diatomic molecules first; their identical atoms cannot create a permanent bond dipole.

30(ii). Which of the following statements about dielectrics is correct?

[1 mark]

- (A) A polar dielectric has a net dipole moment in absence of an external electric field which gets modified due to the induced dipoles.
- (B) The net dipole moments of induced dipoles is along the direction of the applied electric field.
- (C) Dielectrics contain free charges.
- (D) The electric field produced due to induced surface charges inside a dielectric is along the external electric field.

Correct Answer: (B) The net dipole moments of induced dipoles is along the direction of the applied electric field.

Solution:

Concept:

- Polarisation is the electric dipole moment per unit volume. In an ordinary dielectric, the induced polarisation is along the applied electric field.
- Bound charges can shift slightly within molecules, while an ideal dielectric has no freely mobile charge carriers.

Step 1: Identify the direction of induced dipoles An applied field shifts positive bound charge along the field and negative bound charge oppositely. Since electric dipole moment points from negative to positive charge, the induced dipole moments and their net sum point along the applied field. This agrees with statement (B).

Step 2: Test the other statements Statement (A) confuses a molecule with the bulk dielectric: individual polar molecules have permanent dipoles, but their random orientations normally give zero net dipole moment without an external field. Statement (C) is false because dielectric charges are bound. Statement (D) is false because the induced surface charges produce an internal field opposite to the applied field, reducing the resultant field.

Step 3: Conclusion The net dipole moment of induced dipoles is along the applied electric field. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish the directions: polarisation is along the external field, while the field of induced surface charges opposes it.

30(iii). When a dielectric slab is inserted between the plates of an isolated charged capacitor, the energy stored in it:

[1 mark]

- (A) increases and the electric field inside it also increases.
- (B) decreases and the electric field also decreases.
- (C) decreases and the electric field increases.
- (D) increases and the electric field decreases.

Correct Answer: (B) decreases and the electric field also decreases.

Solution:

Concept:

- An isolated charged capacitor retains its free plate charge Q ; no battery supplies additional charge when a dielectric is inserted.
- Stored energy is $U = Q^2/(2C)$. A dielectric increases capacitance C and its polarisation reduces the field within it.

Step 1: Hold the free charge constant Inserting the dielectric increases capacitance. Because Q is fixed, the energy varies inversely with capacitance, not directly. For a dielectric of constant $K > 1$ completely filling the gap,

$$C' = KC, \quad U' = \frac{Q^2}{2KC} = \frac{U}{K}.$$

Thus the energy decreases. Using $U = CV^2/2$ with a fixed voltage would be inappropriate because the capacitor is isolated.

Step 2: Determine the field change The bound surface charges of the dielectric produce a field opposite to the original plate field. For a completely filled gap, $E' = E/K$; hence the field inside the dielectric decreases as well. Options (A) and (D) incorrectly predict increased energy, while option (C) incorrectly predicts an increased field.

Step 3: Conclusion Both stored energy and the electric field inside the dielectric decrease. Hence, the correct option is (B).

Quick Tip to Solve: Underline isolated: keep Q fixed and use $U = Q^2/(2C)$ before deciding how the energy changes.

30(iv)(a). An air-filled capacitor with plate area A and plate separation d has capacitance C_0 . A slab of dielectric constant K , area A and thickness $\left(\frac{d}{5}\right)$ is inserted between the plates. The capacitance of the capacitor will become:

[1 mark]

- (A) $\left[\frac{4K}{5K+1}\right] C_0$
 (B) $\left[\frac{K+5}{4}\right] C_0$
 (C) $\left[\frac{5K}{4K+1}\right] C_0$
 (D) $\left[\frac{K+4}{5K}\right] C_0$

Correct Answer: (C) $\left[\frac{5K}{4K+1}\right] C_0$

Solution:

Concept:

- The initial air capacitance is $C_0 = \epsilon_0 A/d$, where A is plate area and d is plate separation.
- Dielectric and air layers stacked along the electric field act as capacitors in series; a dielectric thickness t contributes the effective air thickness t/K .

Step 1: Find the two layer thicknesses The printed slab thickness is $t = d/5$. Since its area equals the plate area, it covers the entire area but only one-fifth of the separation. The remaining air thickness is $d - t = 4d/5$. The field crosses both layers, so they must be combined in series rather than in parallel.

Step 2: Add the effective thicknesses The potential differences across the layers add at the same free plate charge. Therefore,

$$\frac{1}{C} = \frac{4d/5}{\epsilon_0 A} + \frac{d/5}{K\epsilon_0 A},$$

$$C = \frac{\epsilon_0 A}{4d/5 + d/(5K)} = \frac{5K}{4K+1} C_0.$$

This matches (C). As a check, $K = 1$ must restore C_0 , whereas (A) gives $2C_0/3$ and (B) gives $3C_0/2$. Option (D) decreases below C_0 for $K > 1$, which is inconsistent with inserting a dielectric.

Step 3: Conclusion The new capacitance is $\frac{5K}{4K+1} C_0$. Hence, the correct option is (C).

Quick Tip to Solve: Replace only the dielectric thickness $d/5$ by $d/(5K)$; retain the remaining $4d/5$ of air unchanged.

30(iv)(b). Two capacitors of capacitances $2C_0$ and $6C_0$ are first connected in series and then in parallel across the same battery. The ratio of energies stored in series combination to that in parallel is: [1 mark]

- (A) $\frac{1}{4}$
 (B) $\frac{1}{6}$
 (C) $\frac{2}{15}$
 (D) $\frac{3}{16}$

Correct Answer: (D) $\frac{3}{16}$

Solution:

Concept:

- For two capacitors, the series equivalent capacitance is $C_s = C_1 C_2 / (C_1 + C_2)$ and the parallel equivalent is $C_p = C_1 + C_2$.
- The total stored energy across a battery of voltage V is $U = \frac{1}{2} C_{eq} V^2$. At the same voltage, the energy ratio equals the equivalent-capacitance ratio.

Step 1: Calculate both equivalent capacitances Substitute $C_1 = 2C_0$ and $C_2 = 6C_0$ into the combination rules:

$$C_s = \frac{(2C_0)(6C_0)}{2C_0 + 6C_0} = \frac{3}{2}C_0, \quad C_p = 2C_0 + 6C_0 = 8C_0.$$

Step 2: Form the requested energy ratio The same battery places the same total voltage V across either combination. Thus the factor $V^2/2$ cancels:

$$\frac{U_s}{U_p} = \frac{C_s}{C_p} = \frac{(3/2)C_0}{8C_0} = \frac{3}{16}.$$

This is smaller than unity, as expected because the series capacitance is smaller. Options (A), (B) and (C) do not equal this ratio. In particular, $1/4$ is the result for two equal capacitors and does not apply to the unequal pair here.

Step 3: Conclusion The series-to-parallel energy ratio is $\frac{3}{16}$. Hence, the correct option is (D).

Quick Tip to Solve: For the same battery, use $U_s/U_p = C_1 C_2 / (C_1 + C_2)^2$; no separate voltage calculation is needed.

Section E (15 marks)

- 31(a).** (i) You are given three circuit elements X, Y and Z. They are connected one by one across a given ac source. It is found that V and I are in phase for element X. V leads I by $\left(\frac{\pi}{4}\right)$ for element Y while I leads V by $\left(\frac{\pi}{4}\right)$ for element Z. Identify elements X, Y and Z.
- (ii) Establish the expression for impedance of circuit when elements X, Y and Z are connected in series to an ac source. Show the variation of current in the circuit with the frequency of the applied ac source.
- (iii) In a series LCR circuit, obtain the conditions under which (i) impedance is minimum and (ii) wattless current flows in the circuit.

[5 marks]

Solution:

Concept:

- The phase angle of voltage relative to current is $\tan \phi = X/R$, where R is resistance and X is net reactance. Positive reactance is inductive; negative reactance is capacitive.
- For a series LCR circuit, $X_L = \omega L$, $X_C = 1/(\omega C)$ and $\omega = 2\pi f$. The same current flows through all components.
- Average power is $P = V_{\text{rms}} I_{\text{rms}} \cos \phi = I_{\text{rms}}^2 R$. Reactive components store energy and return it to the source.

Step 1: (i) Identify the nature of each element Since voltage and current are in phase for X, it is resistive. For Y, voltage leads current, so Y is inductive; for Z, current leads voltage, so Z is capacitive. The printed phase angles are $\pi/4$, not $\pi/2$. Thus Y and Z cannot be a pure ideal inductor and a pure ideal capacitor: those would have phase differences of $\pi/2$. A simple realization consistent with the printed values is a coil with resistance r_Y and inductance L , and a capacitor C with series resistance r_Z . At the stated source frequency ω_s ,

$$\frac{\omega_s L}{r_Y} = \tan \frac{\pi}{4} = 1, \quad \frac{1}{\omega_s C r_Z} = 1.$$

The phase information specifies their resistive, inductive and capacitive nature, but not a unique internal circuit. In the following standard series-LCR model, R includes all series resistances.

Step 2: (ii) Add the voltages as phasors Take the current phasor as the reference. The resistive voltage $V_R = IR$ is in phase with current. The inductive voltage $V_L = IX_L$ points 90° ahead of current, whereas the capacitive voltage $V_C = IX_C$ points 90° behind it. Therefore the net reactive voltage is $V_L - V_C$, perpendicular to V_R . By the right-triangle relation,

$$\begin{aligned} V^2 &= V_R^2 + (V_L - V_C)^2 \\ &= I^2 [R^2 + (X_L - X_C)^2], \\ Z = \frac{V}{I} &= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}, \\ \tan \phi &= \frac{\omega L - 1/(\omega C)}{R}. \end{aligned}$$

Here all voltages and currents are rms values, or all are amplitudes; the ratio is the same in either convention.

Step 3: (ii) Show the current-frequency variation Keep the applied rms voltage and the circuit

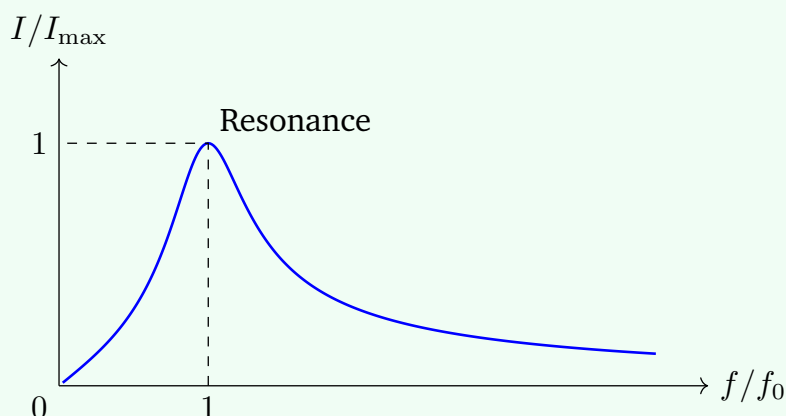
parameters fixed while varying frequency. Then

$$I_{\text{rms}}(f) = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}.$$

At low frequency, capacitive reactance is large and current is small. As frequency rises, the reactances approach equality and the current increases. At resonance,

$$f_0 = \frac{1}{2\pi\sqrt{LC}}, \quad I_{\text{max}} = \frac{V_{\text{rms}}}{R}.$$

Above resonance, inductive reactance dominates, so the current falls again. The graph shows this single resonance peak; it assumes $R > 0$.



Current resonance at fixed applied voltage, shown using normalized axes.

Step 4: (iii) Obtain the minimum-impedance condition Since the squared reactance term in Z^2 is nonnegative, impedance has its minimum value when it vanishes:

$$X_L = X_C \implies \omega_0 L = \frac{1}{\omega_0 C}, \quad \omega_0 = \frac{1}{\sqrt{LC}}, \quad Z_{\text{min}} = R.$$

This is series resonance. Voltage and current are then in phase and the power factor is unity.

Step 5: (iii) Obtain the wattless-current condition A nonzero alternating current is wattless when its average power is zero:

$$P = V_{\text{rms}} I_{\text{rms}} \cos \phi = 0 \implies \phi = \pm \frac{\pi}{2}.$$

Equivalently, $P = I_{\text{rms}}^2 R$ requires zero resistance for a nonzero current. Thus the circuit must be purely reactive: $R = 0$ and $X_L \neq X_C$ for a finite steady current under a nonzero ideal applied voltage. Current lags in the net-inductive case and leads in the net-capacitive case. A real series LCR circuit with $R > 0$ cannot carry exactly wattless nonzero current; resonance in such a circuit gives maximum power, not zero power.

Step 6: Conclusion X is resistive, Y inductive and Z capacitive; the printed $\pi/4$ requires resistive contributions in Y and Z. The series impedance is $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$. It is minimum at $\omega_0 = 1/\sqrt{LC}$; wattless nonzero current requires a purely reactive circuit with $\cos \phi = 0$.

Quick Tip to Solve: Use the sign of the phase difference to identify inductive or capacitive behaviour, but check its magnitude before calling an element pure. At resonance subtract the reactances, never the resistance.

- 31(b).** (i) Describe the construction and working of a transformer and hence obtain the relation for $\left(\frac{V_s}{V_p}\right)$ in terms of number of turns of primary and secondary.
- (ii) Discuss four main causes of energy loss in a real transformer. [5 marks]

Solution:

Concept:

- A transformer works by mutual induction: changing current in its primary winding creates a changing magnetic flux that links its secondary winding and induces an emf.
- Faraday's law gives $e = -N d\Phi/dt$, where N is the number of turns and Φ is the magnetic flux through each turn.
- For an ideal transformer both windings link the same flux, winding resistance is negligible and the core has no energy loss. Thus $V_s/V_p = N_s/N_p$.

Step 1: (i) Describe the construction A transformer consists of two electrically insulated coils, the primary and secondary windings, wound on a common closed soft-iron core. The primary has N_p turns and is connected to an alternating-voltage source. The secondary has N_s turns and supplies the load. The coils are electrically insulated from one another and from the core, but are magnetically coupled by the core. The high-permeability closed core provides a path for magnetic flux. It is made of thin insulated laminations to reduce eddy-current heating.

Step 2: (i) Explain its working by mutual induction The alternating voltage applied to the primary drives an alternating magnetizing current. This produces a time-varying flux $\Phi(t)$ in the core. The same changing flux links the secondary and induces an alternating emf in it. When a load is connected, the secondary emf drives a current and energy is transferred from the source to the load through the magnetic field. A steady direct current cannot sustain transformer action, because its constant flux produces no continuous induced secondary emf.

Step 3: (i) Derive the voltage-turns relation With negligible flux leakage, every turn of both coils links the same core flux. The induced emfs are

$$e_p = -N_p \frac{d\Phi}{dt}, \quad e_s = -N_s \frac{d\Phi}{dt}.$$

Dividing their magnitudes cancels the common rate of change of flux. For sinusoidal flux $\Phi = \Phi_0 \sin \omega t$, the rms values explicitly are

$$E_p = \frac{N_p \omega \Phi_0}{\sqrt{2}}, \quad E_s = \frac{N_s \omega \Phi_0}{\sqrt{2}}.$$

In an ideal transformer the applied primary rms voltage balances the primary back emf, and there is no internal voltage drop in either winding. Hence

$$\frac{V_s}{V_p} = \frac{E_s}{E_p} = \frac{N_s}{N_p}.$$

These are voltage magnitudes; terminal polarity depends on winding orientation. If $N_s > N_p$, the transformer steps voltage up; if $N_s < N_p$, it steps voltage down. Ideal power conservation also gives $V_p I_p = V_s I_s$, so the current ratio is inverse to the turns ratio.

Step 4: (ii) Explain four main causes of loss **Copper loss:** The primary and secondary wires have finite resistance. They dissipate heat at a rate $I_p^2 R_p + I_s^2 R_s$. Thick, low-resistivity copper windings reduce this heating.

Eddy-current loss: The alternating flux induces circulating currents in the conducting core. Their resistance causes heating. Thin core laminations insulated from one another interrupt large current loops and increase their resistance, reducing the loss.

Hysteresis loss: During each ac cycle the core undergoes repeated magnetization and demagnetization. Energy is dissipated as heat in each hysteresis cycle. A suitable soft magnetic material with a narrow hysteresis loop reduces this loss.

Flux leakage: Some of the primary magnetic flux does not link the secondary, so magnetic coupling is incomplete and the secondary voltage under load is reduced. A closed high-permeability core and well-coupled windings reduce leakage. Strictly, leakage flux itself represents imperfect coupling and leakage reactance, rather than an independent irreversible heat loss; it is conventionally listed among the main causes of nonideal transformer performance.

Step 5: Conclusion A transformer transfers ac energy by mutual induction through a common laminated magnetic core. For ideal coupling and negligible internal losses, $V_s/V_p = N_s/N_p$. The main practical loss or nonideality mechanisms are copper heating, eddy currents, hysteresis and flux leakage.

Quick Tip to Solve: Apply Faraday's law separately to each winding and cancel the common flux derivative. Match each practical defect to its remedy: thick wire, insulated laminations, a narrow hysteresis loop and tight magnetic coupling.

32(a). (i) A plane light wave propagating from a rarer into a denser medium, is incident at an angle i on the surface separating two media. Using Huygen's principle, draw the refracted wave and hence verify Snell's law of refraction.

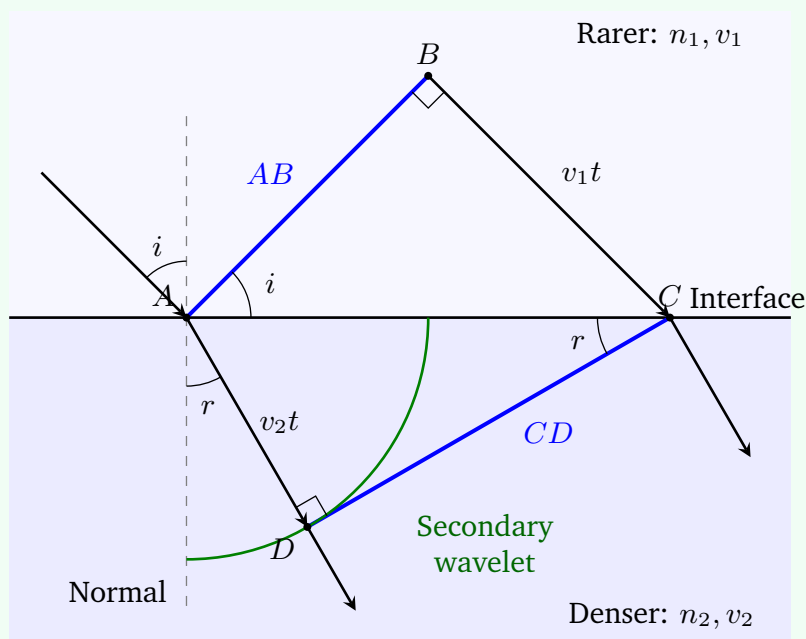
(ii) In a Young's double slit experiment, the slits are separated by 0.30 mm and the screen is kept 1.5 m away. The wavelength of light used is 600 nm. Calculate the distance between the central bright fringe and the 4th dark fringe. [5 marks]

Solution:

Concept:

- Huygens' principle states that each point on a wavefront acts as a source of secondary wavelets. Their forward common tangent gives the new wavefront; rays are perpendicular to wavefronts.
- The speed in a medium of refractive index n is $v = c/n$. A denser optical medium has a larger refractive index and a smaller wave speed.
- In Young's double-slit experiment, the fringe width is $\beta = \lambda D/d$, where λ is the wavelength, D the slit-to-screen distance and d the slit separation. The m th dark fringe is at $y_m = (m - \frac{1}{2})\beta$, counting the nearest dark fringe as $m = 1$.

Step 1: (i) Construct the refracted wavefront Let the upper, rarer medium have refractive index n_1 and wave speed v_1 ; let the lower, denser medium have $n_2 > n_1$ and speed $v_2 < v_1$. The incident plane wavefront AB reaches the interface first at A . While its point B travels along the incident ray to C , taking time t , the secondary wavelet from A spreads a distance $AD = v_2 t$ into the denser medium. Thus $BC = v_1 t$. Draw the tangent CD from C to that wavelet. This tangent is the refracted plane wavefront, and AD , perpendicular to CD , gives the refracted ray direction.



Huygens' construction: AB is the incident wavefront, CD the refracted wavefront, $BC = v_1 t$ and $AD = v_2 t$. The diagram is schematic.

Step 2: (i) Obtain the geometrical relations Let i and r be the incident and refracted ray angles measured from the normal. Since wavefronts are perpendicular to their rays, the angle between AB and the interface AC is also i ; the angle between CD and CA is r . The triangles ABC and ADC are right-angled at B and D , respectively. Their common hypotenuse is AC , so

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}, \quad \sin r = \frac{AD}{AC} = \frac{v_2 t}{AC}.$$

Dividing these equations and using $v_1 = c/n_1$ and $v_2 = c/n_2$ gives

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{n_2}{n_1}, \quad n_1 \sin i = n_2 \sin r.$$

Because $v_2 < v_1$, the construction has $AD < BC$ and $r < i$, consistent with bending towards the normal.

Step 3: (ii) Calculate the fringe width in SI units Convert the supplied quantities before substituting:

$$d = 0.30 \times 10^{-3} \text{ m}, \quad D = 1.5 \text{ m}, \quad \lambda = 600 \times 10^{-9} \text{ m}.$$

The separation is much smaller than the screen distance, so the usual small-angle fringe formula applies:

$$\beta = \frac{\lambda D}{d} = \frac{(600 \times 10^{-9})(1.5)}{0.30 \times 10^{-3}} = 3.0 \times 10^{-3} \text{ m} = 3.0 \text{ mm}.$$

Step 4: (ii) Locate the fourth dark fringe Destructive interference occurs when the path difference is an odd multiple of $\lambda/2$. Hence the first, second, third and fourth dark fringes on either side of the centre occur at $\beta/2$, $3\beta/2$, $5\beta/2$ and $7\beta/2$. For the required fourth dark fringe,

$$y_4 = \left(4 - \frac{1}{2}\right) \beta = \frac{7}{2}(3.0 \text{ mm}).$$

Counting 4β would locate the fourth bright fringe instead, because the centre itself is bright, not dark.

Step 5: Conclusion (i) Huygens' construction verifies Snell's law, $n_1 \sin i = n_2 \sin r$, with the refracted ray bending towards the normal. (ii) The distance between the central bright fringe and the fourth dark fringe is 10.5 mm , on either side of the centre.

Quick Tip to Solve: For the n th dark fringe counted from the centre, multiply the fringe width by $n - \frac{1}{2}$; the nearest dark fringe is already half a fringe width away.

OR

32(b). (i) Discuss briefly diffraction of light from a single slit and draw the shape of the diffraction pattern.

(ii) An object is placed between the pole and the focus of a concave mirror. Using mirror formula, prove mathematically that it produces a virtual and an enlarged image. [5 marks]

Solution:

Concept:

- Diffraction is the spreading of light into the geometrical shadow when it passes through a narrow aperture. Different points of a single slit act as coherent secondary sources, and their waves interfere.
- For slit width a , wavelength λ and diffraction angle θ , dark fringes occur at $a \sin \theta = n\lambda$, where $n = \pm 1, \pm 2, \dots$
- The mirror formula is $1/f = 1/v + 1/u$, and transverse magnification is $m = -v/u$. Distances are measured from the pole using the Cartesian sign convention.

Step 1: (i) Explain the formation of the single-slit pattern Consider monochromatic plane light incident normally on a slit of width a . A distant screen, or a screen in the focal plane of a converging lens, receives waves from all points of the slit. Along the forward direction $\theta = 0$, their path differences vanish, so they arrive in phase and form the intense central maximum.

At angle θ , the path difference between waves from the two extreme edges is $a \sin \theta$. When this is λ , divide the slit into two equal halves. Each point in one half has a corresponding point in the other half with path difference $\lambda/2$; these pairs cancel, producing the first minimum. More generally, dividing the slit into $2n$ equal strips gives pairwise cancellation whenever

$$a \sin \theta = n\lambda, \quad n = \pm 1, \pm 2, \dots$$

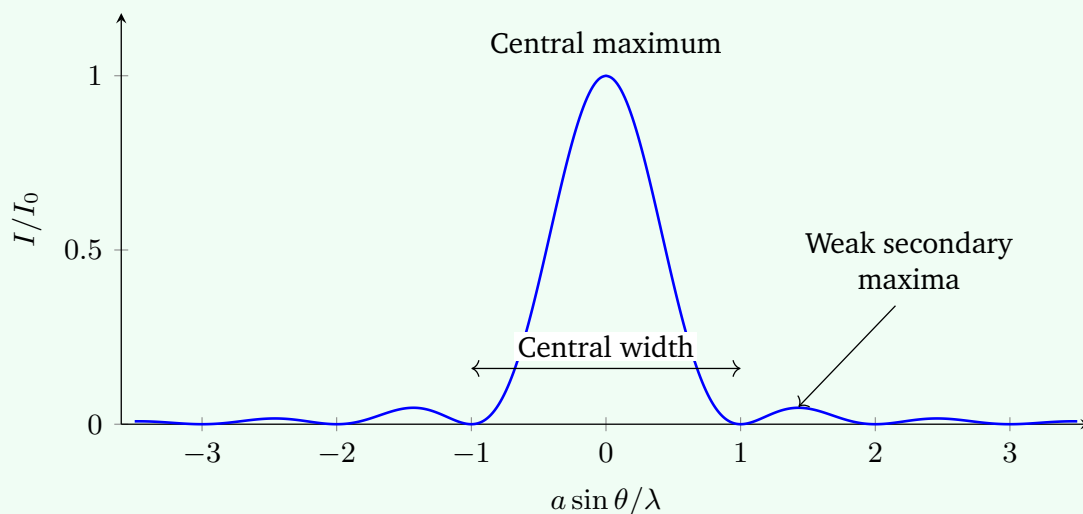
Between consecutive minima, incomplete cancellation produces secondary maxima whose intensity decreases away from the centre.

Step 2: (i) Draw and interpret the intensity distribution The intensity distribution is

$$I(\theta) = I_0 \left(\frac{\sin \beta}{\beta} \right)^2, \quad \beta = \frac{\pi a \sin \theta}{\lambda},$$

with limiting value I_0 at $\theta = 0$. The sketch below uses the dimensionless coordinate $a \sin \theta / \lambda$; its integer nonzero values are the dark minima. The central maximum extends between the first minima on either side. For small angles its angular width is $2\lambda/a$, while the width between adjacent minima around a secondary maximum is approximately λ/a . Thus the central maximum is approximately twice as wide as a secondary maximum and is much brighter. On a screen a distance D away, with

D large enough for the small-angle approximation, its linear width is $2D\lambda/a$. Narrowing the slit broadens the pattern.



Single-slit intensity pattern: the central maximum lies between the first minima at -1 and $+1$.

Step 3: (ii) Apply the mirror formula with signed distances Write the focal length of the concave mirror as $f = -F$, where $F > 0$. If the object lies between the pole and focus, write $u = -d$, where $0 < d < F$. Substituting in the mirror formula gives

$$\begin{aligned} -\frac{1}{F} &= \frac{1}{v} - \frac{1}{d}, \\ \frac{1}{v} &= \frac{1}{d} - \frac{1}{F} = \frac{F-d}{Fd}, \\ v &= \frac{Fd}{F-d}. \end{aligned}$$

Since $F - d > 0$, the image distance is positive. It is therefore behind the mirror, where the backward extensions of reflected rays meet; the image is virtual.

Step 4: (ii) Establish the size and orientation mathematically The magnification is

$$m = -\frac{v}{u} = -\frac{Fd/(F-d)}{-d} = \frac{F}{F-d}.$$

Because $0 < F - d < F$, this ratio is greater than unity. Hence the image height has greater magnitude than the object height. Moreover, $m > 0$, so their heights have the same sign and the image is erect.

Step 5: Conclusion (i) Single-slit diffraction gives a broad, intense central maximum between $a \sin \theta = \pm \lambda$, with weaker secondary maxima on both sides. (ii) For $u = -d$, $f = -F$ and $0 < d < F$, $v = Fd/(F-d) > 0$ and $m = F/(F-d) > 1$: the concave mirror forms a virtual, erect and enlarged image.

Quick Tip to Solve: Place the first diffraction minima at $\pm \lambda/a$ before drawing the central peak. For a concave mirror, use $f = -F$ and $u = -d$ with $0 < d < F$ to make the image signs and magnification inequality immediate.

33(a). (i) Draw equipotential surfaces for an electric dipole.

(ii) Two point charges q_1 and q_2 are located at $\vec{r}_1$ and $\vec{r}_2$ respectively in an external electric field $\vec{E}$. Obtain an expression for the potential energy of the system.

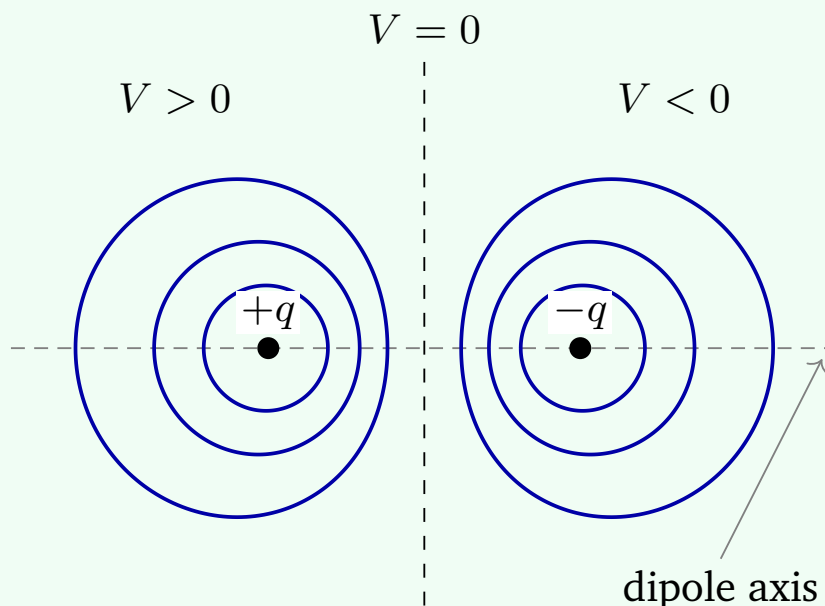
(iii) The dipole moment of a molecule is 10^{-30} Cm. It is placed in an electric field $\vec{E}$ of 10^5 V/m such that its axis is along the electric field. The direction of $\vec{E}$ is suddenly changed by 60° at an instant. Find the change in the potential energy of the dipole, at that instant. [5 marks]

Solution:

Concept:

- An equipotential surface has constant potential. For charges $+q$ and $-q$, $V = kq(1/r_+ - 1/r_-)$, where $k = 1/(4\pi\epsilon_0)$ and r_+, r_- are distances from the two charges.
- The energy needed to assemble charges equals the external work done against electrostatic forces. In a prescribed external potential V_{ext} , each charge contributes qV_{ext} , and each mutual pair interaction is counted once.
- A dipole of moment $\vec{p}$ in a uniform field has potential energy $U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$, with θ the angle between the dipole moment and the field.

Step 1: (i) Sketch the dipole equipotentials The sketch shows a section through the dipole axis. Rotating each closed curve about that axis produces an equipotential surface. The surfaces are nearly spherical close to either charge and become distorted farther away because both charges contribute to the potential. Positive-potential surfaces surround $+q$, while corresponding negative-potential surfaces surround $-q$. At every point in the perpendicular bisector plane, $r_+ = r_-$, so the two potentials cancel and $V = 0$. Its section is the straight dashed line through the midpoint. The equipotentials do not intersect, and the electric field is normal to each surface. Curves are symmetric about the dipole axis; the labels indicate their signs, not equally spaced potential values.



Axial section of dipole equipotential surfaces; the dashed vertical line represents the zero-potential plane.

Step 2: (ii) Bring in the first charge Let $V_{\text{ext}}(\vec{r})$ be the potential produced only by the sources of the given external electric field, with a consistent reference for energy. Neglect the self-energies of the point charges and keep the external sources fixed. Bringing q_1 to $\vec{r}_1$ in the absence of q_2 requires

external work

$$W_1 = q_1 V_{\text{ext}}(\vec{r}_1).$$

Step 3: (ii) Add the second charge and the mutual interaction At $\vec{r}_2$, the potential is the sum of the prescribed external potential and the potential due to the first charge. Writing $r_{12} = |\vec{r}_2 - \vec{r}_1|$, the additional work is

$$W_2 = q_2 \left[V_{\text{ext}}(\vec{r}_2) + \frac{q_1}{4\pi\epsilon_0 r_{12}} \right].$$

Adding the two assembly stages gives

$$U = q_1 V_{\text{ext}}(\vec{r}_1) + q_2 V_{\text{ext}}(\vec{r}_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}.$$

The last term is counted only once: it represents one pair of charges, not two different interactions. For a uniform external field and an origin chosen to have zero potential, $V_{\text{ext}}(\vec{r}) = -\vec{E} \cdot \vec{r}$; substitution gives the equivalent expression in terms of $\vec{E}$.

Step 4: (iii) Compare the instantaneous orientations Initially the dipole moment is along the electric field, so $\theta_i = 0^\circ$. Immediately after the field direction changes, the molecule has not yet rotated. Thus the angle between the unchanged dipole moment and the new field is $\theta_f = 60^\circ$, with the field magnitude unchanged.

$$\begin{aligned} U_i &= -pE = -10^{-30} \times 10^5 = -10^{-25} \text{ J}, \\ U_f &= -pE \cos 60^\circ = -5 \times 10^{-26} \text{ J}, \\ \Delta U &= U_f - U_i = pE(1 - \cos 60^\circ) \\ &= 10^{-30} \times 10^5 \times \frac{1}{2} = 5 \times 10^{-26} \text{ J}. \end{aligned}$$

The positive change means the dipole's potential energy increases at that instant. Its later rotation under the new torque is a separate process.

Step 5: Conclusion The equipotentials form symmetric surfaces around the opposite charges, with the perpendicular bisector plane at $V = 0$. The two-charge system has $U = q_1 V_{\text{ext}}(\vec{r}_1) + q_2 V_{\text{ext}}(\vec{r}_2) + q_1 q_2 / (4\pi\epsilon_0 r_{12})$. The molecular dipole's instantaneous energy change is $\Delta U = +5 \times 10^{-26} \text{ J}$.

Quick Tip to Solve: Count the mutual charge-pair energy once. For a sudden field rotation, hold the dipole direction fixed at that instant and use $\Delta U = pE(\cos \theta_i - \cos \theta_f)$.

OR

33(b). (i) A thin spherical shell of radius R has a uniform surface charge density σ . Using Gauss' law, deduce an expression for electric field (i) outside and (ii) inside the shell.

(ii) Two long straight thin wires AB and CD have linear charge densities $10 \mu\text{C/m}$ and $-20 \mu\text{C/m}$, respectively. They are kept parallel to each other at a distance 1 m. Find magnitude and direction of the net electric field at a point midway between them. [5 marks]

Solution:

Concept:

- Gauss' law states $\oint_S \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the charge enclosed by a closed Gaussian surface and ϵ_0 is the permittivity of free space.

- A uniformly charged spherical shell has spherical symmetry: its electric field is radial and its magnitude depends only on the distance r from the centre. Its total charge is $Q = 4\pi R^2\sigma$.
- An effectively infinite straight wire with linear charge density λ produces a field of magnitude $E = |\lambda|/(2\pi\epsilon_0 r)$. The field points away from a positive wire and towards a negative wire; electric fields add vectorially.

Step 1: (i) Choose a spherical Gaussian surface outside the shell Take a sphere of radius $r > R$, concentric with the charged shell. By spherical symmetry, $\vec{E}$ is parallel to the outward area vector at every point for positive σ , and its radial component E_r is constant on this surface. The surface encloses the entire shell charge. Hence,

$$\oint_S \vec{E} \cdot d\vec{A} = E_r(4\pi r^2) = \frac{4\pi R^2\sigma}{\epsilon_0},$$

$$E_r = \frac{\sigma R^2}{\epsilon_0 r^2}.$$

Thus the outside field is the same as that of a point charge Q at the centre. It is radially outward for positive σ and inward for negative σ .

Step 2: (i) Apply Gauss' law inside the shell Now choose a concentric spherical Gaussian surface of radius $r < R$. All the charge is on the shell at radius R , so this smaller surface encloses no charge. Spherical symmetry again allows the radial field to be taken outside the flux integral:

$$E_r(4\pi r^2) = \frac{0}{\epsilon_0} = 0 \Rightarrow E_r = 0.$$

The field is zero everywhere inside, including the centre by symmetry. Zero enclosed charge alone would imply only zero net flux; here spherical symmetry is what makes it sufficient to determine the field itself.

Step 3: (ii) Determine the two field directions at the midpoint The midpoint is $r = 1/2 = 0.50$ m from each wire. The field of the positively charged wire AB points away from AB, towards CD. The field of the negatively charged wire CD points towards CD. Therefore both fields have the same direction, perpendicular to the wires from AB to CD, and their magnitudes must be added.

Step 4: (ii) Calculate and add the field magnitudes Convert the charge densities to SI units: $\lambda_{AB} = 10 \times 10^{-6} \text{ C m}^{-1}$ and $\lambda_{CD} = -20 \times 10^{-6} \text{ C m}^{-1}$. Using $1/(4\pi\epsilon_0) = 9 \times 10^9 \text{ N m}^2\text{C}^{-2}$,

$$E_{AB} = \frac{2(9 \times 10^9)(10 \times 10^{-6})}{0.50} = 3.6 \times 10^5 \text{ N C}^{-1},$$

$$E_{CD} = \frac{2(9 \times 10^9)(20 \times 10^{-6})}{0.50} = 7.2 \times 10^5 \text{ N C}^{-1},$$

$$E_{\text{net}} = E_{AB} + E_{CD} = 1.08 \times 10^6 \text{ N C}^{-1}.$$

Step 5: Conclusion (i) For the shell, $\vec{E} = \frac{\sigma R^2}{\epsilon_0 r^2} \hat{r}$ outside ($r > R$), while $\vec{E} = 0$ inside ($r < R$), where $\hat{r}$ points radially outward. (ii) The midpoint field is $1.08 \times 10^6 \text{ N C}^{-1}$, directed from the positive wire AB towards the negative wire CD, perpendicular to the wires.

Quick Tip to Solve: For a spherical shell, first ask whether your Gaussian sphere encloses the shell charge. Between oppositely charged parallel wires, draw the field directions before calculating: their fields reinforce, so add magnitudes.