CBSE Class 12 Mathematics Question Paper 2024 with Solution PDF-Set 65-2-3

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions

(i) This question paper contains **38** questions. **All** questions are **compulsory**.

(ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.

(iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.

(iv) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.

(v) In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.

(vi) In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.

(vii) In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculators is not allowed.

Section A (20 marks)

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $y = \cos^{-1}(e^x)$, then $\frac{dy}{dx}$ is: [1 mark]

(A) $\frac{1}{\sqrt{e^{-2x}+1}}$

(B) $-\frac{1}{\sqrt{e^{-2x}+1}}$

(C) $\frac{1}{\sqrt{e^{-2x}-1}}$

(D) $-\frac{1}{\sqrt{e^{-2x}-1}}$

Correct Answer: (D) $-\frac{1}{\sqrt{e^{-2x}-1}}$

Solution:

Concept:

- For a differentiable function $u(x)$, $\frac{d}{dx}\cos^{-1} u = -\frac{u'}{\sqrt{1-u^2}}$ when $-1 < u < 1$.
- The chain rule differentiates the outer inverse cosine and then multiplies by the derivative of the inner function $u = e^x$.

Step 1: Apply the chain rule Here $u = e^x$ and $u' = e^x$. Since $e^x > 0$, the real function is defined for $x \leq 0$, and its derivative is finite for $x < 0$. Thus

$$\frac{dy}{dx} = -\frac{e^x}{\sqrt{1-e^{2x}}}.$$

Step 2: Simplify and compare the options Divide numerator and denominator by the positive number e^x:

$$\frac{dy}{dx} = -\frac{1}{\sqrt{e^{-2x}-1}}.$$

Options (A) and (B) contain an incorrect plus sign under the radical. Option (C) has the correct radical but misses the negative sign from differentiating inverse cosine.

Step 3: Conclusion $\frac{dy}{dx} = -\frac{1}{\sqrt{e^{-2x}-1}}$, for $x < 0$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the minus sign from inverse cosine, and divide the radical by e^x to match the printed options.

2. The degree and order of the differential equation $(y'')^2 + \log(y') = x^5$ respectively are: [1 mark]

(A) not defined, 5

(B) not defined, 2

(C) 5, not defined

(D) $2, 2$

Correct Answer: (B) not defined, 2

Solution:

Concept:

- The order is the order of the highest derivative occurring in a differential equation.
- The degree is defined only when the equation is polynomial in its derivatives; it is then the power of the highest-order derivative.

Step 1: Identify the highest derivative The equation contains the first derivative $y\prime$ and the second derivative $y\prime\prime$. Squaring $y\prime\prime$ changes its power, not its derivative order. Therefore, the order is 2; the power 5 of the independent variable x is irrelevant.

Step 2: Check whether the degree exists The term $\log(y\prime)$ is a logarithmic function of a derivative, so the equation is not polynomial in its derivatives. Exponentiating gives $y\prime = e^{x^5-(y\prime\prime)^2}$, which is still non-polynomial in derivatives. Thus the degree is not defined. Option (D) wrongly reads the square as the degree without checking this requirement; (A) and (C) misuse the exponent of x.

Step 3: Conclusion The degree is not defined and the order is 2, respectively. Hence, the correct option is (B).

Quick Tip to Solve: Check for derivatives inside logarithms before reading a power as the degree; powers of x never determine the order.

3. The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is: [1 mark]

(A) $2\hat{j}$

(B) $\hat{j}$

(C) $\dfrac{\hat{i} - \hat{k}}{\sqrt{2}}$

(D) $\dfrac{\hat{i} + \hat{k}}{\sqrt{2}}$

Correct Answer: (B) $\hat{j}$

Solution:

Concept:

- Two nonzero vectors are perpendicular when their scalar product is zero.
- A unit vector has magnitude 1; a cross product gives a vector perpendicular to both original vectors.

Quick Tip to Solve: For vectors lying in the xz-plane, test $\hat{j}$ immediately, but still check that the proposed vector has unit length.

Step 1: Find a perpendicular direction Let $\vec{a} = \hat{i} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{k}$. Using $\hat{i} \times \hat{k} = -\hat{j}$ and $\hat{k} \times \hat{i} = \hat{j}$,

$$\vec{a} \times \vec{b} = 2\hat{j}.$$

Its magnitude is 2, so division by 2 gives the unit vector $\hat{j}$.

Step 2: Test the listed choices Both dot products $\hat{j} \cdot (\hat{i} + \hat{k})$ and $\hat{j} \cdot (\hat{i} - \hat{k})$ are zero. Option (A) is perpendicular but has magnitude 2, not 1. Options (C) and (D) are unit vectors parallel to one of the original vectors, so neither is perpendicular to both. The opposite vector $-\hat{j}$ also qualifies mathematically but is not listed.

Step 3: Conclusion The listed unit vector perpendicular to both vectors is $\hat{j}$. Hence, the correct option is (B).

Quick Tip to Solve: For vectors lying in the xz-plane, test $\hat{j}$ immediately, but still check that the proposed vector has unit length.

4. Direction ratios of a vector parallel to the line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are: [1 mark]

(A) 2, -1, 6

(B) 2, 1, 6

(C) 2, 1, 3

(D) 2, -1, 3

Correct Answer: (D) 2, -1, 3

Solution:

Concept:

- For a line written as $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$, the numbers a, b, c are direction ratios.
- Equivalently, in a parametric representation, the coefficients of the common parameter in x, y, z form direction ratios.

Step 1: Introduce a common parameter Set each expression equal to t:

$$\frac{x-1}{2} = -y = \frac{2z+1}{6} = t.$$

Solving individually yields

$$x = 1 + 2t, \qquad y = -t, \qquad z = -\frac{1}{2} + 3t.$$

The coefficients of t, rather than the constant coordinates of a point, determine the direction.

Step 2: Read and check the direction ratios The displacement for a unit increase of t is $(2, -1, 3)$. In particular, the third ratio is 3, since the numerator contains $2z$. Option (A) fails to divide 6 by 2; options (B) and (C) give the wrong sign for the coefficient of y.

Step 3: Conclusion A set of direction ratios is $2, -1, 3$. Hence, the correct option is (D).

Quick Tip to Solve: Rewrite $(2z+1)/6$ as $(z+1/2)/3$, and rewrite $-y$ as $y/(-1)$, before reading denominators.

5. If for the matrix $A = \begin{pmatrix} \tan x & 1 \\ -1 & \tan x \end{pmatrix}$, $A + A' = 2\sqrt{3}\, I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is: [1 mark]

(A) 0
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{6}$

Correct Answer: (C) $\frac{\pi}{3}$

Solution:

Concept:

- The transpose $A\prime$ interchanges the rows and columns of A.
- Two matrices are equal when their corresponding entries agree; the identity matrix I has diagonal entries 1 and off-diagonal entries 0.

Step 1: Add the matrix and its transpose The off-diagonal entries cancel:

$$A\prime = \begin{pmatrix} \tan x & -1 \\ 1 & \tan x \end{pmatrix},$$

$$A + A\prime = \begin{pmatrix} 2\tan x & 0 \\ 0 & 2\tan x \end{pmatrix} = 2\tan x\, I.$$

Step 2: Compare the diagonal entries The given equality implies $2\tan x = 2\sqrt{3}$, hence $\tan x = \sqrt{3}$. In the stated interval this occurs at $x = \pi/3$. The endpoint $\pi/2$ is excluded by the domain of tangent. For options (A), (B) and (D), tangent is respectively $0, 1, 1/\sqrt{3}$, so none satisfies the equality.

Step 3: Conclusion $x = \frac{\pi}{3}$. Hence, the correct option is (C).

Quick Tip to Solve: In this matrix the opposite off-diagonal entries cancel in $A + A\prime$, so compare just one diagonal entry.

6. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is: [1 mark]

(A) 90°

(B) 120°

(C) 60°

(D) 0°

Correct Answer: (A) 90°

Solution:

Concept:

- If α, β, γ are the direction angles of a line, its direction cosines are $\cos\alpha, \cos\beta, \cos\gamma$.
- Direction cosines satisfy $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, with each direction angle between 0 and π.

Step 1: Substitute the two given angles Here $\cos 30^\circ = \sqrt{3}/2$ and $\cos 120^\circ = -1/2$. Thus

$$\cos^2\gamma = 1 - \left(\frac{\sqrt{3}}{2}\right)^2 - \left(-\frac{1}{2}\right)^2 = 1 - \frac{3}{4} - \frac{1}{4} = 0.$$

The negative cosine for 120° becomes positive after squaring.

Step 2: Find the angle and reject other choices Since $\cos\gamma = 0$ and $0^\circ \le \gamma \le 180^\circ$, we get $\gamma = 90^\circ$. Options (B) and (C) would each add $1/4$ to a sum already equal to 1; option (D) would add 1. None can satisfy the direction-cosine identity.

Step 3: Conclusion The angle with the positive z-axis is 90°. Hence, the correct option is (A).

Quick Tip to Solve: When the first two squared direction cosines already sum to 1, the third cosine is zero and its angle is 90°.

7. If the sum of all the elements of a 3×3 scalar matrix is 9, then the product of all its elements is: [1 mark]

(A) 0

(B) 9

(C) 27

(D) 729

Correct Answer: (A) 0

Solution:

Concept:

- A scalar matrix of order 3 has the form kI_3: all three diagonal entries equal k, and all six off-diagonal entries are zero.
- The product of all entries is zero if even one entry is zero; this differs from the determinant or the product of diagonal entries.

Quick Tip to Solve: For the product of all entries of a scalar matrix of order greater than 1, look for an off-diagonal zero before calculating.

Step 1: Use the sum of the entries Write the scalar matrix as

$$\begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}.$$

Its sum of entries is $3k$. Hence $3k = 9$, which gives $k = 3$. The six off-diagonal entries remain zero.

Step 2: Multiply all nine entries The requested product includes the zero entries, so

$$3 \times 0 \times 0 \times 0 \times 3 \times 0 \times 0 \times 0 \times 3 = 0.$$

Option (B) repeats the sum instead of the product; option (C) multiplies only the diagonal entries. Option (D) also ignores the forced zero factors. In fact the product would be zero for any scalar k.

Step 3: Conclusion The product of all elements is 0. Hence, the correct option is (A).

Quick Tip to Solve: For the product of all entries of a scalar matrix of order greater than 1, look for an off-diagonal zero before calculating.

8. Let $f : R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is: [1 mark]

(A) one-one

(B) onto

(C) bijective

(D) neither one-one nor onto

Correct Answer: (C) bijective

Solution:

Concept:

- A function is one-one if equal outputs force equal inputs; a strictly increasing function is one-one.
- A function is onto when its range equals its codomain. A function with both properties is called bijective.

Step 1: Check that the function is one-one On the given domain $x \geq 0$, the derivative is

$$f'(x) = 18x + 6 > 0.$$

Thus the function is strictly increasing throughout its domain, so distinct inputs cannot have the same output.

Step 2: Compare the range with the codomain The smallest value is $f(0) = -5$. Since this polynomial is continuous and tends to infinity as $x \to \infty$, it takes every value in $[-5, \infty)$. Thus it is also onto. Options (A) and (B) each name one property which does hold; (C) gives the complete classification by including both properties. Option (D) contradicts both checks.

Quick Tip to Solve: On a half-line, check the derivative's sign and the endpoint value to establish injectivity and range together.

Step 3: Conclusion The function is both one-one and onto, hence bijective. Hence, the correct option is (C).

Quick Tip to Solve: On a half-line, check the derivative's sign and the endpoint value to establish injectivity and range together.

9. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = k\,abc$**, then the value of** k **is:** [1 mark]

(A) 0

(B) 1

(C) 2

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- A factor common to an entire column can be taken outside a determinant. Here the respective column factors are a, b, c.
- Expand a determinant along its first row using the cofactor signs $+, -, +$.

Step 1: Take out the column factors Factoring directly from the columns, without dividing by any possibly zero variable, gives

$$\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = abc \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}.$$

Thus we only need the numerical determinant multiplying abc.

Step 2: Expand the numerical determinant Expanding along the first row gives

$$(-1)(1-1) - 1(-1-1) + 1(1+1) = 0 + 2 + 2 = 4.$$

Both nonzero cofactors contribute positively. The zero first minor does not make the determinant zero; keeping just one nonzero term would incorrectly give 2. Hence the coefficients 0, 1 and 2 in the other options do not match the determinant identity.

Step 3: Conclusion The determinant equals $4abc$, so $k = 4$. Hence, the correct option is (D).

Quick Tip to Solve: Factor each column first; the remaining determinant has only signs, making cofactor arithmetic much quicker.

10. The number of points of discontinuity of $f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$ **is:** [1 mark]

(A) 0

(B) 1

(C) 2

(D) infinite

Correct Answer: (B) 1

Solution:

Concept:

- Continuity at $x = a$ requires the left-hand limit, right-hand limit and $f(a)$ to be equal.
- Each given branch is continuous within its interval, so only the joining points -3 and 3 need checking.

Step 1: Check the join at negative three The left branch includes the endpoint and gives

$$\lim_{x \to -3^-} f(x) = |-3| + 3 = 6 = f(-3).$$

The middle branch gives $\lim_{x \to -3^+} f(x) = -2(-3) = 6$. Both limits equal the endpoint value, so this join is continuous despite the change in formula.

Step 2: Check the join at three At the other endpoint,

$$\lim_{x \to 3^-} f(x) = -6, \qquad \lim_{x \to 3^+} f(x) = 20 = f(3).$$

The unequal one-sided limits show a discontinuity here. There are no others, because the first branch equals $-x + 3$ on its interval and the remaining branches are linear. Thus neither zero, two nor infinitely many discontinuities are possible.

Step 3: Conclusion Only $x = 3$ is a point of discontinuity, so the number is 1. Hence, the correct option is (B).

Quick Tip to Solve: For piecewise linear functions, inspect the joining points rather than counting the number of displayed formulas.

11. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is: [1 mark]

(A) strictly decreasing on $\mathbb{R}$

(B) strictly increasing on $\mathbb{R}$

(C) neither strictly increasing nor strictly decreasing on $\mathbb{R}$

(D) strictly decreasing on $(-\infty, 0)$

Correct Answer: (B) strictly increasing on $\mathbb{R}$

Solution:

Concept:

- If $f'(x) > 0$ throughout an interval, the differentiable function f is strictly increasing there.
- Completing the square determines the sign of a quadratic for all real values of its variable.

Step 1: Differentiate the polynomial Apply the power rule term by term. The constant has zero derivative, so

$$f'(x) = 3x^2 - 6x + 12 = 3(x^2 - 2x + 4).$$

The sign of this derivative determines whether the original function increases or decreases.

Step 2: Establish the sign for every real input Complete the square inside the brackets:

$$f'(x) = 3\big((x-1)^2 + 3\big) \geq 9 > 0.$$

This is positive for every real x, including negative inputs. Consequently the function cannot be decreasing on the whole real line or on $(-\infty, 0)$, excluding (A) and (D). It has the same increasing behaviour everywhere, excluding (C).

Step 3: Conclusion The function is strictly increasing on $\mathbb{R}$. Hence, the correct option is (B).

Quick Tip to Solve: Complete the square in the derivative: a positive square plus a positive constant immediately settles global monotonicity.

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12. Anti-derivative of $\sqrt{1 + \sin 2x}$, $x \in \left[0, \frac{\pi}{4}\right]$ is: [1 mark]

(A) $\cos x + \sin x$

(B) $-\cos x + \sin x$

(C) $\cos x - \sin x$

(D) $-\cos x - \sin x$

Correct Answer: (B) $-\cos x + \sin x$

Solution:

Concept:

- The identity $1 + \sin 2x = (\sin x + \cos x)^2$ simplifies the radical.
- The principal square root of u^2 is $|u|$. An antiderivative is a function whose derivative equals the integrand.

Quick Tip to Solve: Rewrite $1 + \sin 2x$ as a perfect square, then use the stated interval to remove the absolute value correctly.

Step 1: Simplify the radical on the given interval On $0 \le x \le \pi/4$, sine is nonnegative and cosine is positive. Hence

$$\sqrt{1+\sin 2x} = |\sin x + \cos x| = \sin x + \cos x.$$

Checking this sign is essential before removing the square root.

Step 2: Integrate and test the listed functions Integrating the two terms gives

$$\int (\sin x + \cos x)\, dx = -\cos x + \sin x + C.$$

Option (B) is this antiderivative with $C = 0$. The derivatives of (A), (C) and (D) are respectively $\cos x - \sin x$, $-\sin x - \cos x$ and $\sin x - \cos x$; none equals the integrand throughout the interval.

Step 3: Conclusion An antiderivative is $-\cos x + \sin x$. Hence, the correct option is (B).

Quick Tip to Solve: Rewrite $1 + \sin 2x$ as a perfect square, then use the stated interval to remove the absolute value correctly.

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13. The differential equation $\dfrac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is: [1 mark]

(A) $\cos x - \sin\left(\dfrac{y}{x}\right)$

(B) $\dfrac{y}{x}$

(C) $\dfrac{x^2 + y^2}{xy}$

(D) $\cos^2\left(\dfrac{x}{y}\right)$

Correct Answer: (A) $\cos x - \sin\left(\dfrac{y}{x}\right)$

Solution:

Concept:

- The equation $dy/dx = F(x, y)$ is homogeneous when F is homogeneous of degree zero: $F(tx, ty) = F(x, y)$ wherever both sides are defined.
- An expression involving only the ratio y/x, or only x/y, satisfies this scaling test.

Step 1: Test the ratio-based expressions Option (B), y/x, is unchanged by multiplying both variables by the same nonzero number. Option (C) can be written

$$\frac{x^2 + y^2}{xy} = \frac{x}{y} + \frac{y}{x},$$

so it too depends only on ratios. Option (D), $\cos^2(x/y)$, is also unchanged by common scaling. Each gives a homogeneous equation on its domain.

Quick Tip to Solve: Apply $(x, y) \mapsto (tx, ty)$: ratios stay fixed, while a standalone trigonometric function of x exposes the exception.

Step 2: Identify the separate dependence on x In option (A), common scaling produces

$$F(tx, ty) = \cos(tx) - \sin(y/x).$$

The second term remains fixed but the first generally changes with t. Thus the isolated $\cos x$ term prevents degree-zero homogeneity; it cannot be absorbed into a function of the variable ratio alone.

Step 3: Conclusion The expression $\cos x - \sin(y/x)$ does not yield a homogeneous differential equation. Hence, the correct option is (A).

Quick Tip to Solve: Apply $(x, y) \mapsto (tx, ty)$: ratios stay fixed, while a standalone trigonometric function of x exposes the exception.

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14. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true? [1 mark]

(A) $\vec{a} \cdot \vec{b} \geq |\vec{a}|\,|\vec{b}|$
(B) $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}|$
(C) $\vec{a} \cdot \vec{b} \leq |\vec{a}|\,|\vec{b}|$
(D) $\vec{a} \cdot \vec{b} < |\vec{a}|\,|\vec{b}|$

Correct Answer: (C) $\vec{a} \cdot \vec{b} \leq |\vec{a}|\,|\vec{b}|$

Solution:

Concept:

- For nonzero vectors separated by an angle θ, their scalar product is $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$.
- Since $-1 \leq \cos\theta \leq 1$ and magnitudes are nonnegative, the scalar product is at most the product of the magnitudes.

Step 1: Bound the scalar product Multiply $\cos\theta \leq 1$ by the nonnegative number $|\vec{a}|\,|\vec{b}|$:

$$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta \leq |\vec{a}|\,|\vec{b}|.$$

If either vector is zero, both sides are zero, so the same inequality remains valid without assigning an angle to a zero vector.

Step 2: Check when equality is possible Two nonzero vectors in the same direction have $\theta = 0$, giving equality. This disproves the strict inequality in (D). Two nonzero perpendicular vectors have dot product zero but a positive product of magnitudes, disproving both the reversed inequality (A) and the universal equality (B). Only the non-strict upper bound survives both examples.

Step 3: Conclusion Always, $\vec{a} \cdot \vec{b} \leq |\vec{a}|\,|\vec{b}|$. Hence, the correct option is (C).

Quick Tip to Solve: Test parallel and perpendicular vectors to distinguish the universal non-strict bound from equality or a strict inequality.

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15. The coordinates of the foot of the perpendicular drawn from the point $(0, 1, 2)$ on the x-axis are given by: [1 mark]

(A) $(1, 0, 0)$

(B) $(2, 0, 0)$

(C) $(\sqrt{5}, 0, 0)$

(D) $(0, 0, 0)$

Correct Answer: (D) $(0, 0, 0)$

Solution:

Concept:

- A point on the x-axis has the form $F = (t, 0, 0)$, where t is its x-coordinate.
- Perpendicular vectors have zero dot product; the direction vector of the x-axis is $\hat{i} = (1, 0, 0)$.

Step 1: Express the perpendicular vector Let $P = (0, 1, 2)$ and let its foot on the x-axis be $F = (t, 0, 0)$. Then

$$\overrightarrow{PF} = (t, -1, -2).$$

The foot must lie on the axis and the segment joining it to P must be perpendicular to that axis.

Step 2: Apply the dot-product condition Perpendicularity gives

$$(t, -1, -2) \cdot (1, 0, 0) = t = 0.$$

Thus the projection retains the original x-coordinate and sets the other two coordinates to zero. The points in (A), (B) and (C) have nonzero x-coordinates, so their joining vectors have nonzero dot products with the axis direction. In particular, $\sqrt{5}$ is the distance from P to the axis, not the foot's coordinate.

Step 3: Conclusion The foot of the perpendicular is $\boxed{(0, 0, 0)}$. Hence, the correct option is (D).

Quick Tip to Solve: For projection onto a coordinate axis, retain the coordinate along that axis and replace the other two by zero.

16. The common region determined by all the constraints of a linear programming problem is called: [1 mark]

(A) an unbounded region

(B) an optimal region

(C) a bounded region

(D) a feasible region

Correct Answer: (D) a feasible region

Solution:

Concept:

- A constraint restricts the possible values of the decision variables in a linear programming problem.

Quick Tip to Solve: Associate constraints with feasibility; use the objective function only when deciding which feasible point is optimal.

- The feasible region consists of all points satisfying every constraint simultaneously, including any non-negativity restrictions.

Step 1: Identify the common set Each linear inequality determines an allowed half-plane. To satisfy all the constraints together, a point must belong to the intersection of these allowed sets. Every point in this common region is a feasible solution, so the set is called the feasible region.

Step 2: Distinguish the other terms A feasible region can be bounded or unbounded, depending on the constraints; therefore (A) and (C) do not name the common region in general. Optimality also involves the objective function: an optimal solution is a feasible point where the objective attains its required maximum or minimum. Merely satisfying the constraints does not establish optimality, so (B) is inappropriate.

Step 3: Conclusion The common region determined by all constraints is a $\boxed{\text{feasible region}}$. Hence, the correct option is (D).

Quick Tip to Solve: Associate constraints with feasibility; use the objective function only when deciding which feasible point is optimal.

17. Let E be an event of a sample space S of an experiment, then $P(S \mid E) =$ [1 mark]

(A) $P(S \cap E)$

(B) $P(E)$

(C) 1

(D) 0

Correct Answer: (C) 1

Solution:

Concept:

- Conditional probability is $P(A \mid E) = \dfrac{P(A \cap E)}{P(E)}$, provided $P(E) > 0$.
- Every event E is a subset of its sample space S, so $S \cap E = E$.

Step 1: Apply the definition of conditional probability Taking the event A to be the entire sample space gives

$$P(S \mid E) = \frac{P(S \cap E)}{P(E)}.$$

As usual when a conditional probability is specified, the conditioning event has positive probability, making this quotient defined.

Step 2: Simplify using the sample space Since $E \subseteq S$, the intersection in the numerator is just E. Therefore

$$P(S \mid E) = \frac{P(E)}{P(E)} = 1.$$

Options (A) and (B) both give $P(E)$, which need not equal one; they omit the conditional normalization. Option (D) would describe impossibility, whereas an outcome already in E certainly belongs to S.

Quick Tip to Solve: After conditioning on a positive-probability event, the entire sample space remains certain, so its conditional probability is one.

Step 3: Conclusion $\boxed{P(S \mid E) = 1}$. Hence, the correct option is (C).

Quick Tip to Solve: After conditioning on a positive-probability event, the entire sample space remains certain, so its conditional probability is one.

18. If $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = i - 3j$, then which of the following is false? [1 mark]

(A) $a_{11} < 0$

(B) $a_{12} + a_{21} = -6$

(C) $a_{13} > a_{31}$

(D) $a_{31} = 0$

Correct Answer: (C) $a_{13} > a_{31}$

Solution:

Concept:

- In $A = [a_{ij}]$, the first subscript i denotes the row and the second subscript j denotes the column.
- Substitute the indices directly into $a_{ij} = i - 3j$; the question asks for the false statement.

Step 1: Compute the entries in the options Using the given rule, the required entries are

$$a_{11} = 1 - 3 = -2, \quad a_{12} = 1 - 6 = -5,$$
$$a_{21} = 2 - 3 = -1, \quad a_{13} = 1 - 9 = -8,$$
$$a_{31} = 3 - 3 = 0.$$

Keeping the row and column indices in their printed order is essential because a_{13} and a_{31} need not be equal.

Step 2: Test all four statements Option (A) is true because $-2 < 0$. Option (B) is true because $a_{12} + a_{21} = -5 - 1 = -6$. Option (C) asserts $-8 > 0$, which is false; in fact $a_{13} < a_{31}$. Option (D) is true because $a_{31} = 0$. Thus exactly one of the listed statements fails.

Step 3: Conclusion The false statement is $a_{13} > a_{31}$. Hence, the correct option is (C).

Quick Tip to Solve: Write the row index first when substituting into $i - 3j$, and remember to select the false comparison here.

19. Assertion (A): For any symmetric matrix A, B'AB is a skew-symmetric matrix.

Reason (R): A square matrix P is skew-symmetric if $P' = -P$.

Select the correct answer from the codes (A), (B), (C) and (D): [1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (D) Assertion (A) is false, but Reason (R) is true.

Solution:

Concept:

- For a symmetric matrix A, $A' = A$, where the prime denotes transpose.
- Transposition reverses multiplication order: $(XYZ)' = Z'Y'X'$. A square matrix P is skew-symmetric precisely when $P' = -P$.

Step 1: Evaluate the assertion For any matrix B of compatible order, let $M = B'AB$. Then

$$M' = (B'AB)' = B'A'(B')' = B'AB = M.$$

Hence the product is symmetric, not necessarily skew-symmetric. For example, taking $A = B = I$ gives $M = I$, whose transpose is I, not $-I$. This counterexample disproves the assertion.

Step 2: Evaluate the reason The condition $P' = -P$ is exactly the definition of a skew-symmetric square matrix. Thus the reason is true. It differs from the condition $P' = P$ for a symmetric matrix.

Step 3: Check the explanatory relation A true definition cannot establish the false assertion. Options (A), (B) and (C) each require the assertion to be true, so all three are ruled out. Although the zero matrix is both symmetric and skew-symmetric, that special case cannot justify a claim made for every symmetric A.

Step 4: Conclusion The assertion is false, but the reason is true. Hence, the correct option is (D).

Quick Tip to Solve: To test a claim about $B'AB$, transpose the whole product and reverse the factor order before using $A' = A$.

20. Assertion (A): $(\vec{b}\cdot\vec{c})\vec{a}$ is a scalar quantity.

Reason (R): Dot product of two vectors is a scalar quantity.

Select the correct answer from the codes (A), (B), (C) and (D): [1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (D) Assertion (A) is false, but Reason (R) is true.

Solution:

Concept:

- The dot product $\vec{b}\cdot\vec{c}=|\vec{b}|\,|\vec{c}|\cos\theta$ is a scalar, where θ is the angle between the vectors.
- Multiplication of a vector $\vec{a}$ by a scalar k produces the vector $k\vec{a}$; it does not change the result into a scalar.

Step 1: Evaluate the assertion Put $k=\vec{b}\cdot\vec{c}$. The asserted expression then becomes

$$(\vec{b}\cdot\vec{c})\vec{a}=k\vec{a}.$$

This is a scalar multiple of the vector $\vec{a}$, so it is a vector quantity. Even if the multiplier is zero, the result is the zero vector. The assertion calling the expression a scalar is therefore false.

Step 2: Evaluate the reason A dot product assigns a scalar value to a pair of vectors. The formula $\vec{b}\cdot\vec{c}=|\vec{b}|\,|\vec{c}|\cos\theta$ confirms this directly, so the reason is true.

Step 3: Check the explanatory relation The reason identifies the type of the parenthesized factor only. Multiplying that scalar by $\vec{a}$ produces a vector, so the reason cannot support the assertion. Options (A), (B) and (C) all require a true assertion and are excluded.

Step 4: Conclusion The assertion is false, but the reason is true. Hence, the correct option is (D).

Quick Tip to Solve: Evaluate vector expressions from the brackets outward: a dot product followed by multiplication by a vector gives a vector.

Section B (10 marks)

This section comprises very short answer (VSA) type questions of 2 marks each.

21. Determine whether the function $f(x)=x^2-6x+3$ is increasing or decreasing in $[4,6]$. [2 marks]

Solution:

Concept:

- A differentiable function is strictly increasing on an interval if its derivative is positive throughout the interior of that interval.

Quick Tip to Solve: Factor the derivative as $2(x-3)$: every point of $[4,6]$ is to the right of its zero at 3, so the sign is positive throughout.

- The derivative measures the instantaneous rate of change: for $f(x) = x^2 - 6x + 3$, differentiate each term using the power rule.

Step 1: Differentiate the given function The function is a polynomial, so it is continuous on $[4, 6]$ and differentiable at every interior point. Its derivative is

$$f'(x) = 2x - 6 = 2(x - 3).$$

The derivative vanishes only at $x = 3$, which is outside the specified interval. To determine the behavior on $[4, 6]$, we need the sign of f' on that interval, rather than the sign of f itself.

Step 2: Check the derivative throughout the interval For $4 \le x \le 6$, subtracting three and multiplying by two gives

$$1 \le x - 3 \le 3, \qquad 2 \le f'(x) \le 6.$$

Thus $f'(x) > 0$ everywhere on the interval. More explicitly, if $4 \le u < v \le 6$, the mean value theorem provides $c \in (u, v)$ such that

$$f(v) - f(u) = f'(c)(v - u) > 0.$$

Consequently each larger input in this interval gives a larger function value, including when either input is an endpoint.

Step 3: Conclusion Since $f'(x) > 0$ on $[4, 6]$, the function $f(x) = x^2 - 6x + 3$ is strictly increasing on $\boxed{[4, 6]}$.

Quick Tip to Solve: Factor the derivative as $2(x - 3)$: every point of $[4, 6]$ is to the right of its zero at 3, so the sign is positive throughout.

22(a). Find the value of $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cot^{-1}\left(\tan\frac{4\pi}{3}\right)$. [2 marks]

Solution:

Concept:

- Principal values of $\sin^{-1} t$, $\cos^{-1} t$ and $\cot^{-1} t$ lie in $[-\pi/2, \pi/2]$, $[0, \pi]$ and $(0, \pi)$, respectively.
- Tangent has period π: $\tan(\pi + \theta) = \tan\theta$. Inverse functions return the angle in their own principal range.

Step 1: Evaluate the inverse sine and cosine The angle $-\pi/6$ lies in the principal range of inverse sine and has sine $-1/2$. For inverse cosine, the negative argument selects the second-quadrant angle $5\pi/6$, not $-\pi/6$. Thus

$$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}, \qquad \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}.$$

Step 2: Reduce the tangent and select the cotangent angle First evaluate the inner tangent by subtracting its period:

$$\tan\frac{4\pi}{3} = \tan\left(\pi + \frac{\pi}{3}\right) = \sqrt{3}.$$

Quick Tip to Solve: Evaluate the inner tangent first, then use the principal range of inverse cotangent; these two functions do not cancel each other.

Since $\cot(\pi/6) = \sqrt{3}$ and $\pi/6 \in (0, \pi)$, the final term is

$$\cot^{-1}\left(\tan\frac{4\pi}{3}\right) = \cot^{-1}\sqrt{3} = \frac{\pi}{6}.$$

Step 3: Conclusion The required value is $-\frac{\pi}{6} + \frac{5\pi}{6} + \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$.

Quick Tip to Solve: Evaluate the inner tangent first, then use the principal range of inverse cotangent; these two functions do not cancel each other.

OR

22(b). Find the domain of $f(x) = \cos^{-1}(1 - x^2)$. Also, find its range. [2 marks]

Solution:

Concept:

- For a real value of $\cos^{-1} u$, its argument must satisfy $-1 \le u \le 1$.
- The principal range of inverse cosine is $[0, \pi]$. To establish the range of a composite function, check which arguments its inner expression attains on the domain.

Step 1: Impose the condition on the cosine argument Here the inner expression is $u = 1 - x^2$, so the necessary and sufficient condition is

$$-1 \le 1 - x^2 \le 1 \quad \Longleftrightarrow \quad 0 \le x^2 \le 2.$$

The lower bound on x^2 holds for every real x. The upper bound gives $-\sqrt{2} \le x \le \sqrt{2}$. The endpoints are included because inverse cosine is defined at both -1 and 1.

Step 2: Determine all attained output values As x moves from 0 to $\sqrt{2}$, x^2 takes every value in $[0, 2]$, so $1 - x^2$ takes every value in $[-1, 1]$. Consequently its inverse cosine takes every value in $[0, \pi]$. In particular,

$$f(0) = \cos^{-1}(1) = 0, \qquad f(\pm\sqrt{2}) = \cos^{-1}(-1) = \pi.$$

No output outside this interval is possible for the principal inverse cosine.

Step 3: Conclusion The domain is $\boxed{[-\sqrt{2}, \sqrt{2}]}$, and the range is $\boxed{[0, \pi]}$.

Quick Tip to Solve: Bound the whole inverse-cosine argument between -1 and 1, then check whether that argument actually covers the full interval.

23(a). **If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \dfrac{\pi}{3}$.** [2 marks]

Solution:

Concept:

- The modulus satisfies $|u| = u$ when $u \geq 0$, and $|u| = -u$ when $u < 0$. Choose the appropriate local branch before differentiating.
- By the chain rule, $\dfrac{d}{dx}\tan(2x) = 2\sec^2(2x)$, wherever the tangent is defined.

Step 1: Identify the sign near the given point At $x = \pi/3$, the inner angle is $2\pi/3$, which lies in the second quadrant. Hence

$$\tan\left(2 \cdot \frac{\pi}{3}\right) = -\sqrt{3} < 0.$$

Tangent is continuous at this angle and remains negative for nearby angles. Thus, in a neighbourhood of $x = \pi/3$, the correct expression is $f(x) = -\tan(2x)$. The modulus does not cause a corner here because its argument is nonzero.

Step 2: Differentiate the local branch and substitute Differentiating the local expression gives

$$f'(x) = -2\sec^2(2x).$$

Since $\cos(2\pi/3) = -1/2$, its reciprocal squared is 4. Therefore

$$f'\left(\frac{\pi}{3}\right) = -2\left(\frac{1}{\cos^2(2\pi/3)}\right) = -2\left(\frac{1}{1/4}\right).$$

Step 3: Conclusion The required derivative is $\boxed{f'(\pi/3) = -8}$.

Quick Tip to Solve: Check the sign of the expression inside the modulus at the point; a negative value introduces a minus sign in the local derivative.

OR

23(b). **If $y = \operatorname{cosec}(\cot^{-1} x)$, then prove that $\sqrt{1+x^2}\,\dfrac{dy}{dx} - x = 0$.** [2 marks]

Solution:

Concept:

- If $\theta = \cot^{-1} x$, then $\cot\theta = x$ and $0 < \theta < \pi$. In this principal interval, $\sin\theta > 0$, hence $\operatorname{cosec}\theta > 0$.
- The identity $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ converts the inverse-trigonometric composite to an algebraic function, which can be differentiated using the chain rule.

Step 1: Express the composite function algebraically Put $\theta = \cot^{-1} x$, so $y = \operatorname{cosec}\theta$. The trigonometric identity gives

$$y^2 = 1 + \cot^2\theta = 1 + x^2.$$

Quick Tip to Solve: Simplify cosecant of inverse cotangent using $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$; the principal angle fixes the square-root sign.

Since $\theta \in (0, \pi)$, its cosecant is positive. We must therefore take the positive square root:

$$y = \sqrt{1 + x^2}.$$

This equality holds for every real x; the expression under the radical is always positive.

Step 2: Differentiate and rearrange Apply the chain rule to the algebraic expression:

$$\frac{dy}{dx} = \frac{1}{2}(1 + x^2)^{-1/2}(2x) = \frac{x}{\sqrt{1 + x^2}}.$$

Multiplication by the positive, nonzero factor $\sqrt{1 + x^2}$ gives

$$\sqrt{1 + x^2}\,\frac{dy}{dx} = x.$$

Step 3: Conclusion Therefore $\boxed{\sqrt{1 + x^2}\,\frac{dy}{dx} - x = 0}$, as required, for all real x.

Quick Tip to Solve: Simplify cosecant of inverse cotangent using $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$; the principal angle fixes the square-root sign.

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24. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ $(x \neq 0)$ respectively, find the value of $(M - m)$. [2 marks]

Solution:

Concept:

- At an interior stationary point of a differentiable function, $f'(x) = 0$. Only points in the domain can be candidates for local extrema.
- The second derivative test gives a local maximum when $f''(x) < 0$, and a local minimum when $f''(x) > 0$. Extremum values are the function values, not their locations.

Step 1: Locate the stationary points For $f(x) = x + x^{-1}$, with $x \neq 0$, differentiation gives

$$f'(x) = 1 - \frac{1}{x^2}, \qquad f''(x) = \frac{2}{x^3}.$$

Setting the first derivative equal to zero yields

$$1 - \frac{1}{x^2} = 0 \quad \Longrightarrow \quad x^2 = 1 \quad \Longrightarrow \quad x = -1 \text{ or } x = 1.$$

Both points belong to the domain. The excluded point $x = 0$ cannot be a local maximum or minimum.

Step 2: Classify the points and evaluate the function At $x = -1$, $f''(-1) = -2 < 0$, so the local maximum value is

$$M = f(-1) = -1 + \frac{1}{-1} = -2.$$

Quick Tip to Solve: Substitute the stationary points into the original function before finding $M - m$; M and m are values, not x-coordinates.

At $x = 1$, $f''(1) = 2 > 0$, so the local minimum value is

$$m = f(1) = 1 + \frac{1}{1} = 2.$$

These extrema occur on different parts of the domain. A local maximum value need not exceed a local minimum value elsewhere in the domain.

Step 3: Conclusion Therefore $\boxed{M - m = -2 - 2 = -4}$.

Quick Tip to Solve: Substitute the stationary points into the original function before finding $M - m$; M and m are values, not x-coordinates.

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25. Find: $\int \frac{e^{4x} - 1}{e^{4x} + 1}\, dx$. [2 marks]

Solution:

Concept:

- Rewrite a rational expression in an exponential so that the numerator contains the derivative of its denominator.
- The substitution rule gives $\int f'(x)/f(x)\, dx = \log|f(x)| + C$, where C is an arbitrary constant.

Step 1: Rearrange the numerator Let the required integral be I. Write $e^{4x} - 1 = 2e^{4x} - (e^{4x} + 1)$. Dividing by the denominator therefore gives

$$I = 2\int \frac{e^{4x}}{e^{4x} + 1}\, dx - \int 1\, dx.$$

This form is useful because the derivative of $e^{4x} + 1$ is a constant multiple of e^{4x}.

Step 2: Substitute the denominator Put $t = e^{4x} + 1$, so that $dt = 4e^{4x}\, dx$. The first integral becomes $\frac{1}{2}\int dt/t$, while the second is x. Since $e^{4x} + 1 > 0$ for every real x, no absolute-value sign is needed in its logarithm. As a check, differentiating $\frac{1}{2}\log(e^{4x} + 1) - x$ gives

$$\frac{2e^{4x}}{e^{4x} + 1} - 1 = \frac{e^{4x} - 1}{e^{4x} + 1},$$

which is exactly the given integrand.

Step 3: Conclusion

$$\boxed{\int \frac{e^{4x} - 1}{e^{4x} + 1}\, dx = \frac{1}{2}\log(e^{4x} + 1) - x + C.}$$

Quick Tip to Solve: Use $e^{4x} - 1 = 2e^{4x} - (e^{4x} + 1)$ to obtain a logarithmic derivative immediately; remember the factor 4 from differentiating e^{4x}.

Section C (18 marks)

This section comprises short answer (SA) type questions of 3 marks each.

26. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X. [3 marks]

Solution:

Concept:

- Two fair dice produce $6 \times 6 = 36$ equally likely ordered outcomes (a, b), where a and b denote the numbers on the first and second die.
- A discrete probability distribution lists every possible value k of a random variable and $P(X = k)$, the probability of that value. These probabilities must add to 1.

Step 1: List the possible absolute differences Here $X = |a - b|$. Equal numbers give the minimum difference 0, while 1 and 6 give the maximum difference 5. Every integer in between is possible, so the possible values are $0, 1, 2, 3, 4, 5$. Negative values are excluded by the absolute value.

Step 2: Count the outcomes for each value For $X = 0$, the outcomes are $(1, 1), (2, 2), \ldots, (6, 6)$, giving 6 outcomes. For a positive difference k, the outcomes with $a-b = k$ are $(k+1, 1), (k+2, 2), \ldots, (6, 6-k)$, giving $6 - k$ outcomes. Reversing the dice gives another $6 - k$ distinct outcomes with $b - a = k$. Hence the number for a positive difference is $2(6 - k)$. The counts for $k = 1, 2, 3, 4, 5$ are therefore $10, 8, 6, 4, 2$, respectively. We do not double the count for zero because reversing (a, a) gives the same outcome.

Step 3: Convert counts into probabilities Divide each count by 36, the total number of equally likely outcomes. The six counts sum to

$$6 + 10 + 8 + 6 + 4 + 2 = 36,$$

so the probabilities sum to 1, as required. Counting ordered outcomes is essential: for example, $(1, 2)$ and $(2, 1)$ are different outcomes even though both have absolute difference 1.

Step 4: Conclusion The required probability distribution is

x	0	1	2	3	4	5
$P(X = x)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

For every other value of x, $P(X = x) = 0$.

Quick Tip to Solve: For two dice, count 6 equal-number pairs separately; each positive absolute difference k has $2(6 - k)$ ordered outcomes.

27. Find: $\int x^2 \log(x^2-1)\,dx$. [3 marks]

Solution:

Concept:

- Integration by parts states $\int u\,dv = uv - \int v\,du$. Choose the logarithmic factor as u, since its derivative is a rational function.
- Polynomial division reduces an improper rational function to a polynomial and a proper fraction. Partial fractions then give logarithmic integrals.

Step 1: Apply integration by parts Let $I = \int x^2 \log(x^2-1)\,dx$. For a real logarithm the domain is $x^2 - 1 > 0$, that is, $x < -1$ or $x > 1$. Take

$$u = \log(x^2-1), \quad dv = x^2\,dx, \qquad du = \frac{2x}{x^2-1}\,dx, \quad v = \frac{x^3}{3}.$$

Hence

$$I = \frac{x^3}{3}\log(x^2-1) - \frac{2}{3}\int \frac{x^4}{x^2-1}\,dx.$$

Step 2: Divide and resolve the rational function Since $x^4 = (x^2-1)(x^2+1)+1$, polynomial division gives

$$\frac{x^4}{x^2-1} = x^2 + 1 + \frac{1}{x^2-1}.$$

Factoring the remaining denominator and resolving into partial fractions gives

$$\frac{1}{x^2-1} = \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right).$$

Thus all the remaining terms have standard antiderivatives.

Step 3: Integrate the separate terms Integrating the polynomial and the two simple fractions yields

$$\int \frac{x^4}{x^2-1}\,dx = \frac{x^3}{3} + x + \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C_1.$$

Substitute this expression into the integration-by-parts formula and absorb the constant multiple of C_1 into a single arbitrary constant C. To check the algebra, differentiation of the product term contributes the extra rational term $2x^4/[3(x^2-1)]$; the other terms contribute $-\frac{2}{3}(x^2+1+1/(x^2-1))$. These cancel by the division identity above, leaving precisely $x^2\log(x^2-1)$.

Step 4: Conclusion

$$\boxed{I = \frac{x^3}{3}\log(x^2-1) - \frac{2x^3}{9} - \frac{2x}{3} - \frac{1}{3}\log\left|\frac{x-1}{x+1}\right| + C.}$$

This holds on each interval of the real domain $x < -1$ or $x > 1$.

Quick Tip to Solve: After choosing the logarithm in integration by parts, divide x^4 by x^2-1 before using partial fractions; keep the negative factor $-2/3$ outside the integral.

28(a). If $y = (\log x)^2$, prove that $x^2y'' + xy' = 2$. [3 marks]

Solution:

Concept:

- The chain rule gives $\frac{d}{dx}(u^2) = 2u\frac{du}{dx}$. For the natural logarithm, $\frac{d}{dx}\log x = 1/x$, with $x > 0$.
- The product rule gives $\frac{d}{dx}(uv) = u'v + uv'$. Here y' and y'' denote the first and second derivatives of y with respect to x.

Step 1: Differentiate the given function The function $y = (\log x)^2$ is defined for $x > 0$. Apply the chain rule, treating $\log x$ as the inner function:

$$y' = 2\log x \cdot \frac{1}{x} = \frac{2\log x}{x}.$$

Multiplying both sides by x gives

$$xy' = 2\log x.$$

This rearrangement is helpful because differentiating its left-hand side will produce exactly the combination of derivatives in the question.

Step 2: Differentiate the rearranged identity Both x and y' depend on x, so the derivative of xy' must include both product-rule terms. Differentiate the two sides to obtain

$$1 \cdot y' + x \cdot y'' = \frac{2}{x}.$$

Multiplying the entire identity by x gives the required constant on the right. This operation is valid throughout the domain $x > 0$.

Step 3: Check by direct substitution As an independent algebraic check, write $y' = 2(\log x)x^{-1}$. Using the product rule again,

$$y'' = 2\left(\frac{1}{x}x^{-1} - (\log x)x^{-2}\right) = \frac{2(1 - \log x)}{x^2}.$$

Thus the two contributions to the stated expression are $x^2y'' = 2(1 - \log x)$ and $xy' = 2\log x$. Their logarithmic terms cancel exactly, confirming that the result does not depend on x.

Step 4: Conclusion

$$\boxed{x^2y'' + xy' = 2(1 - \log x) + 2\log x = 2}, \qquad x > 0.$$

Hence the required identity is proved.

Quick Tip to Solve: After finding y', multiply by x first: differentiating $xy' = 2\log x$ generates $xy'' + y'$ directly and avoids a lengthy quotient-rule calculation.

OR

28(b). If $y = \sin(\tan^{-1} e^x)$, then find $\dfrac{dy}{dx}$ at $x = 0$. [3 marks]

Solution:

Concept:

- The chain rule differentiates a composite function one layer at a time: if $y = \sin u$, then $\dfrac{dy}{dx} = \cos u \dfrac{du}{dx}$.
- For a differentiable function $v(x)$, $\dfrac{d}{dx}(\tan^{-1} v) = \dfrac{v'}{1+v^2}$. Here $v = e^x$, so $v' = e^x$.

Step 1: Differentiate the inner inverse tangent Put $u = \tan^{-1}(e^x)$, so that the given function is $y = \sin u$. Applying the inverse-tangent derivative and then differentiating the exponential gives

$$\frac{du}{dx} = \frac{1}{1+(e^x)^2}\frac{d(e^x)}{dx} = \frac{e^x}{1+e^{2x}}.$$

The factor e^x in the numerator is essential: the argument of the inverse tangent is itself a function of x.

Step 2: Apply the outer derivative Differentiating the sine now gives

$$\frac{dy}{dx} = \cos(\tan^{-1} e^x)\frac{e^x}{1+e^{2x}}.$$

Since $e^x > 0$, the principal angle $u = \tan^{-1} e^x$ lies in $(0, \pi/2)$. Thus $\cos u$ is positive; there is no sign ambiguity when evaluating the cosine.

Step 3: Substitute the specified value of the variable At $x = 0$, we have $e^0 = 1$, $e^{2\cdot 0} = 1$, and $\tan^{-1} 1 = \pi/4$. Therefore

$$\left.\frac{dy}{dx}\right|_{x=0} = \cos\frac{\pi}{4}\frac{1}{1+1} = \frac{1}{\sqrt{2}}\cdot\frac{1}{2}.$$

Substitution is made only after differentiation, since replacing x by zero in the original expression would merely give the value of y.

Step 4: Conclusion The required derivative is $\left.\dfrac{dy}{dx}\right|_{x=0} = \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}$.

Quick Tip to Solve: Follow the three layers sine, inverse tangent and exponential; then use $\tan^{-1} 1 = \pi/4$ at $x = 0$.

29(a). Evaluate: $\int_{-2}^{2} \sqrt{\frac{2-x}{2+x}}\, dx$. [3 marks]

Solution:

Concept:

- For radicals involving $2-x$ and $2+x$, the substitution $x = 2\cos\theta$ uses $1-\cos\theta = 2\sin^2(\theta/2)$ and $1+\cos\theta = 2\cos^2(\theta/2)$.
- A definite integral with an unbounded integrand at an endpoint is evaluated as a one-sided limit. The transformed limits and the sign of the differential must both be retained.

Step 1: Choose the substitution and transform the limits Let $I = \int_{-2}^{2} \sqrt{\frac{2-x}{2+x}}\, dx$. The integrand is real for $-2 < x \leq 2$, with a singularity at $x = -2$. Interpret the integral as the limit with lower endpoint approaching -2 from the right. Put $x = 2\cos\theta$. Then $dx = -2\sin\theta\, d\theta$. As x increases from -2 to 2, θ decreases from π to 0.

Step 2: Simplify the radical on the chosen interval For $0 < \theta < \pi$, both half-angle sine and cosine are positive. Hence

$$\sqrt{\frac{2-2\cos\theta}{2+2\cos\theta}} = \sqrt{\frac{2\sin^2(\theta/2)}{2\cos^2(\theta/2)}} = \tan\frac{\theta}{2}.$$

Reversing the limits absorbs the minus sign in the differential, giving

$$I = 2\int_{0}^{\pi} \sin\theta \tan\frac{\theta}{2}\, d\theta.$$

Also, $\sin\theta\tan(\theta/2) = 2\sin^2(\theta/2) = 1-\cos\theta$.

Step 3: Integrate the resulting elementary expression Consequently,

$$I = 2\int_{0}^{\pi} (1-\cos\theta)\, d\theta = 2[\theta - \sin\theta]_0^{\pi}.$$

Although the original radical is unbounded at $x = -2$, its product with the transformed differential simplifies to a continuous function at $\theta = \pi$. The endpoint limit therefore exists and is finite. Using $\sin 0 = \sin\pi = 0$ completes the evaluation.

Step 4: Conclusion The value of the definite integral is 2π.

Quick Tip to Solve: Set $x = 2\cos\theta$; the radical becomes $\tan(\theta/2)$, and its product with $\sin\theta$ is $1-\cos\theta$.

OR

29(b). Find: $\int \dfrac{1}{x\left[(\log x)^2 - 3\log x - 4\right]}\,dx$. [3 marks]

Solution:

Concept:

- The substitution $t = \log x$, where $\log$ denotes the natural logarithm, gives $dt = dx/x$ and turns a rational expression in $\log x$ into a rational function of t.
- A rational function with distinct linear factors can be split into partial fractions. The basic integral is $\int du/u = \log|u| + C$, for $u \neq 0$.

Step 1: Replace the logarithm by a new variable Put $t = \log x$, so $dt = dx/x$. The required integral becomes

$$I = \int \frac{dt}{t^2 - 3t - 4} = \int \frac{dt}{(t-4)(t+1)}.$$

The factorisation is checked by multiplying: $(t-4)(t+1) = t^2 - 3t - 4$. The original expression requires $x > 0$, and its denominator also excludes $x = e^4$ and $x = e^{-1}$.

Step 2: Determine the partial-fraction coefficients Write

$$\frac{1}{(t-4)(t+1)} = \frac{A}{t-4} + \frac{B}{t+1}.$$

Multiplying by $(t-4)(t+1)$ gives $1 = A(t+1) + B(t-4)$. Setting $t = 4$ yields $A = 1/5$, while setting $t = -1$ yields $B = -1/5$. Thus

$$I = \frac{1}{5}\int \left(\frac{1}{t-4} - \frac{1}{t+1}\right) dt.$$

Step 3: Integrate and return to the original variable Applying the logarithmic integral gives

$$I = \frac{1}{5}\big(\log|t-4| - \log|t+1|\big) + C.$$

Substituting $t = \log x$ restores the original variable. As a check, differentiation of this expression produces

$$\frac{1}{5x}\left(\frac{1}{\log x - 4} - \frac{1}{\log x + 1}\right) = \frac{1}{x[(\log x)^2 - 3\log x - 4]},$$

exactly the given integrand. The constant of integration is chosen separately on each interval of the domain.

Step 4: Conclusion

$$\boxed{I = \frac{1}{5}\log\left|\frac{\log x - 4}{\log x + 1}\right| + C}, \qquad x > 0, \quad x \neq e^{-1}, e^4.$$

Quick Tip to Solve: Whenever dx/x accompanies a polynomial in $\log x$, first set $t = \log x$, then factor the resulting polynomial.

30(a). Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2 \dfrac{dy}{dx} = 0$; $y = 2$, when $x = 1$. [3 marks]

Solution:

Concept:

- A first-order equation is homogeneous when its right-hand side depends only on y/x. Put $y = vx$, where v is a function of x.
- The product rule gives $dy/dx = v + x\,dv/dx$. After separating the variables, integrate and use the given point to determine the constant.

Step 1: Reduce the homogeneous equation Since the initial point has $x = 1$, work locally with $x > 0$. Dividing the given equation by $2x^2$ gives

$$\frac{dy}{dx} = \frac{y}{x} + \frac{1}{2}\left(\frac{y}{x}\right)^2.$$

Substitute $y = vx$ and differentiate using the product rule:

$$v + x\frac{dv}{dx} = v + \frac{v^2}{2}, \qquad x\frac{dv}{dx} = \frac{v^2}{2}.$$

Step 2: Separate and integrate At the specified point, $v = y/x = 2$, so the required solution is not the zero solution. Separating variables near this point yields

$$\frac{dv}{v^2} = \frac{dx}{2x}.$$

Therefore,

$$-\frac{1}{v} = \frac{1}{2}\ln x + C.$$

Substituting $x = 1$ and $v = 2$, we obtain $C = -1/2$. Thus the initial condition fixes the integration constant uniquely.

Step 3: Restore the original variables and check Since $v = y/x$, the relation becomes

$$-\frac{2x}{y} = \ln x - 1.$$

The denominator $1 - \ln x$ vanishes at $x = e$, so the solution interval containing the initial point is $0 < x < e$. For verification, differentiating $2x/(1 - \ln x)$ gives

$$\frac{2(2 - \ln x)}{(1 - \ln x)^2} = \frac{2}{1 - \ln x} + \frac{2}{(1 - \ln x)^2},$$

which equals $y/x + y^2/(2x^2)$; the value at $x = 1$ is 2.

Step 4: Conclusion The required particular solution is

$$\boxed{y = \frac{2x}{1 - \ln x}}, \qquad 0 < x < e.$$

Quick Tip to Solve: After putting $y = vx$, cancel the common v terms before separating variables; the initial condition becomes $v = 2$ at $x = 1$.

OR

30(b). Find the general solution of the differential equation: $y\,dx = (x + 2y^2)\,dy$. [3 marks]

Solution:

Concept:

- Treat x as the dependent variable and y as the independent variable. A linear equation has the form $dx/dy + P(y)x = Q(y)$.
- An integrating factor is $I(y) = e^{\int P(y)\,dy}$. Multiplication by it converts the left side into the derivative of $I(y)x$.

Step 1: Write the equation in linear form The differential relation is $y\,dx = (x + 2y^2)\,dy$. Taking x as a function of y and dividing by y, on an interval where $y \neq 0$, gives

$$\frac{dx}{dy} - \frac{x}{y} = 2y.$$

Hence $P(y) = -1/y$ and $Q(y) = 2y$. Choosing the dependent variable correctly avoids trying to force this equation into a linear equation for y in terms of x.

Step 2: Obtain and apply the integrating factor Since $\int(-1/y)\,dy = -\ln|y|$, the exponential integrating factor is $1/|y|$. On either interval of constant sign, it may be multiplied by a nonzero constant, so we may use $I(y) = 1/y$. Multiplication of the linear equation by this factor gives

$$\frac{1}{y}\frac{dx}{dy} - \frac{x}{y^2} = 2.$$

The quotient rule identifies the left side as a single derivative:

$$\frac{d}{dy}\left(\frac{x}{y}\right) = 2.$$

Step 3: Integrate and verify the family Integrating with respect to y introduces an arbitrary real constant C:

$$\frac{x}{y} = 2y + C.$$

To check, use $dx/dy = 4y + C$ for the resulting family. Then

$$y\frac{dx}{dy} = 4y^2 + Cy, \qquad x + 2y^2 = 4y^2 + Cy,$$

so both sides agree. No initial condition is specified, so the constant remains arbitrary.

Step 4: Conclusion The general solution, with x viewed as a function of y, is

$$\boxed{x = 2y^2 + Cy}, \qquad C \in \mathbb{R}.$$

Quick Tip to Solve: The coefficient of dx is y: divide by $y\,dy$ and solve linearly for $x(y)$, using $1/y$ as the integrating factor.

31. The position vectors of vertices of $\triangle ABC$ are $A(2\hat{i}-\hat{j}+\hat{k})$, $B(\hat{i}-3\hat{j}-5\hat{k})$ and $C(3\hat{i}-4\hat{j}-4\hat{k})$. Find all the angles of $\triangle ABC$. [3 marks]

Solution:

Concept:

- A vector from one vertex to another is the position vector of the endpoint minus that of the starting point. An interior angle uses two vectors directed away from the same vertex.
- For nonzero vectors $\vec{u}$ and $\vec{v}$, $\cos\theta = (\vec{u}\cdot\vec{v})/(|\vec{u}||\vec{v}|)$. A zero dot product means that the vectors are perpendicular.

Step 1: Find the side vectors and their magnitudes Subtract the given position vectors component by component:

$$\overrightarrow{AB} = -\hat{i}-2\hat{j}-6\hat{k},$$
$$\overrightarrow{AC} = \hat{i}-3\hat{j}-5\hat{k},$$
$$\overrightarrow{BC} = 2\hat{i}-\hat{j}+\hat{k}.$$

Their magnitudes, which are the corresponding side lengths, are

$$AB = \sqrt{1+4+36} = \sqrt{41}, \quad AC = \sqrt{1+9+25} = \sqrt{35}, \quad BC = \sqrt{4+1+1} = \sqrt{6}.$$

Step 2: Calculate the angles at A and B The two vectors starting at A give

$$\overrightarrow{AB}\cdot\overrightarrow{AC} = -1+6+30 = 35, \qquad \cos A = \frac{35}{\sqrt{41}\sqrt{35}} = \sqrt{\frac{35}{41}}.$$

At B, reverse $\overrightarrow{AB}$, so $\overrightarrow{BA} = \hat{i}+2\hat{j}+6\hat{k}$. Therefore,

$$\overrightarrow{BA}\cdot\overrightarrow{BC} = 2-2+6 = 6, \qquad \cos B = \frac{6}{\sqrt{41}\sqrt{6}} = \sqrt{\frac{6}{41}}.$$

Both dot products are positive, so both angles are acute.

Step 3: Calculate the angle at C and check At C, use $\overrightarrow{CA} = -\hat{i}+3\hat{j}+5\hat{k}$ and $\overrightarrow{CB} = -2\hat{i}+\hat{j}-\hat{k}$. Their dot product is

$$\overrightarrow{CA}\cdot\overrightarrow{CB} = 2+3-5 = 0.$$

Thus the sides meeting at C are perpendicular. As an independent check, the squared side lengths satisfy $AC^2+BC^2 = 35+6 = 41 = AB^2$, confirming that AB is the hypotenuse. Also $\cos^2 A+\cos^2 B = 1$, consistent with complementary acute angles.

Step 4: Conclusion The three interior angles are

$$\boxed{\angle A = \cos^{-1}\sqrt{\frac{35}{41}}, \qquad \angle B = \cos^{-1}\sqrt{\frac{6}{41}}, \qquad \angle C = 90^\circ.}$$

Quick Tip to Solve: The squared side lengths are $35, 6, 41$; spotting $35+6=41$ immediately identifies the right angle opposite AB, at vertex C.

Section D (20 marks)

This section comprises long answer (LA) type questions of 5 marks each.

32. If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$. [5 marks]

Solution:

Concept:

- The parabola $y^2 = 4x$ has branches $y = 2\sqrt{x}$ and $y = -2\sqrt{x}$, symmetric about the x-axis.
- For vertical strips, the enclosed area is $\int_a^b (y_{\text{upper}} - y_{\text{lower}})\, dx$, where a, b are the horizontal limits of the region.

Step 1: Identify the boundaries of the first region The first-quadrant branch is $y = 2\sqrt{x}$. It meets the x-axis at the origin and the line $x = 1$ at $(1, 2)$. Thus the required region extends over $0 \le x \le 1$, with upper boundary $y = 2\sqrt{x}$ and lower boundary $y = 0$.

Step 2: Integrate to find the first area A vertical strip has height $2\sqrt{x}$. Applying the power rule for integration,

$$A_1 = \int_0^1 2\sqrt{x}\, dx = \frac{4}{3}\left[x^{3/2}\right]_0^1$$
$$= \frac{4}{3}(1 - 0) = \frac{4}{3} \text{ square units.}$$

This is only the area above the axis, as required by the first-quadrant restriction.

Step 3: Identify the full second region The line $x = 4$ meets the parabola at $(4, 4)$ and $(4, -4)$. The region bounded by this line and the parabola extends above and below the x-axis. The first-quadrant restriction was stated only for A_1. Therefore, for A_2, use both branches over $0 \le x \le 4$. The vertical height is

$$2\sqrt{x} - (-2\sqrt{x}) = 4\sqrt{x}.$$

Step 4: Evaluate the second area Integrating the full vertical height gives

$$A_2 = \int_0^4 4\sqrt{x}\, dx = \frac{8}{3}\left[x^{3/2}\right]_0^4$$
$$= \frac{8}{3}(8 - 0) = \frac{64}{3} \text{ square units.}$$

This also equals twice the upper-half area, consistent with symmetry about the x-axis.

Step 5: Conclusion Hence the required ratio is

$$A_1 : A_2 = \frac{4}{3} : \frac{64}{3} = \boxed{1 : 16}.$$

Quick Tip to Solve: Include both branches for A_2: only A_1 is restricted to the first quadrant. Using only the upper branch would halve A_2.

33(a). Show that a function $f : R \to R$ defined as $f(x) = x^2 + x + 1$ is neither one-one nor onto. Also, find all the values of x for which $f(x) = 3$. [5 marks]

Solution:

Concept:

- A function is one-one if equal outputs imply equal inputs. Two distinct inputs with the same image disprove this property.
- A function $f : \mathbb{R} \to \mathbb{R}$ is onto when every real number has a real preimage. One codomain element outside the range disproves this property.
- Completing the square reveals the minimum of a quadratic, since the square of a real number is nonnegative.

Step 1: Find two distinct inputs with the same image Both 0 and -1 are in the domain $\mathbb{R}$. From the given rule,

$$f(0) = 0^2 + 0 + 1 = 1, \qquad f(-1) = (-1)^2 - 1 + 1 = 1.$$

Thus $f(0) = f(-1)$, although $0 \neq -1$. Equal outputs have arisen from different inputs, contradicting the requirement for a one-one function. This counterexample proves that f is not one-one on its stated domain.

Step 2: Find a lower bound for every output Completing the square gives

$$f(x) = x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}.$$

Since $(x + 1/2)^2 \geq 0$ for all real x, every output satisfies $f(x) \geq 3/4$. Equality occurs at $x = -1/2$, so $3/4$ is the minimum value. In particular, negative values and zero cannot occur as outputs.

Step 3: Disprove onto behaviour using the codomain The codomain is $\mathbb{R}$, so it contains 0. However, the bound $f(x) \geq 3/4$ shows that no real input has image 0. Equivalently, $f(x) = 0$ would require $(x + 1/2)^2 = -3/4$, impossible for a real square. Thus a codomain element has no preimage, and f is not onto $\mathbb{R}$.

Step 4: Find all inputs whose image is three Set the polynomial equal to 3, rearrange and factor:

$$x^2 + x + 1 = 3,$$
$$x^2 + x - 2 = 0,$$
$$(x + 2)(x - 1) = 0.$$

The zero-product property gives $x = -2$ or $x = 1$. Both lie in the domain, and substitution gives $f(-2) = 4 - 2 + 1 = 3$ and $f(1) = 1 + 1 + 1 = 3$. The factorization includes every root, so no further real values are possible.

Step 5: Conclusion The function is neither one-one nor onto. All values satisfying $f(x) = 3$ are $\boxed{x = -2 \text{ or } x = 1}$.

Quick Tip to Solve: Test 0 and -1 to disprove one-one behaviour, then complete the square and check whether 0 in the codomain has a preimage.

OR

33(b). A relation R is defined on $N \times N$ (where N is the set of natural numbers) as $(a, b)\, R\, (c, d) \iff \frac{a}{c} = \frac{b}{d}$. Show that R is an equivalence relation. [5 marks]

Solution:

Concept:

- A relation on a set is an equivalence relation if it is reflexive, symmetric and transitive. Here the elements of the underlying set are ordered pairs in $N \times N$.
- The defining condition is $(a, b)R(c, d) \iff a/c = b/d$. Natural numbers here are positive, so every denominator and every ratio used below is nonzero.

Step 1: Prove reflexivity for every ordered pair Let (a, b) be any element of $N \times N$. To compare this pair with itself, substitute $(c, d) = (a, b)$ in the defining condition. Since a and b are nonzero,

$$\frac{a}{a} = 1 = \frac{b}{b}.$$

Thus $(a, b)R(a, b)$ for every ordered pair in the set. This proves that R is reflexive; the claim concerns pairs, rather than individual natural numbers.

Step 2: Prove symmetry by reversing the related pairs Take any $(a, b), (c, d) \in N \times N$ and suppose $(a, b)R(c, d)$. By definition,

$$\frac{a}{c} = \frac{b}{d}.$$

All four natural numbers are positive, so taking reciprocals on both sides is valid and gives

$$\frac{c}{a} = \frac{d}{b}.$$

This is precisely the condition for $(c, d)R(a, b)$. Every related pair can therefore be reversed, proving that R is symmetric.

Step 3: Prove transitivity using a third ordered pair Let $(a, b), (c, d), (e, f) \in N \times N$, with $(a, b)R(c, d)$ and $(c, d)R(e, f)$. The two assumptions give

$$\frac{a}{c} = \frac{b}{d}, \qquad \frac{c}{e} = \frac{d}{f}.$$

Multiply the equalities and cancel the nonzero intermediate entries c and d:

$$\frac{a}{c} \cdot \frac{c}{e} = \frac{b}{d} \cdot \frac{d}{f} \quad \Longrightarrow \quad \frac{a}{e} = \frac{b}{f}.$$

Hence $(a, b)R(e, f)$, exactly the required implication for transitivity. The pairs were arbitrary, so the argument applies throughout $N \times N$.

Step 4: Conclusion The relation R is reflexive, symmetric and transitive on $N \times N$. Therefore, R is an equivalence relation.

Quick Tip to Solve: Keep each ordered pair intact: reverse both pairs for symmetry, and multiply the two given ratio equalities to cancel the middle pair for transitivity.

34. The vertices of $\triangle ABC$ are $A(1,1,0)$, $B(1,2,1)$ and $C(-2,2,-1)$. Find the equations of the medians through A and B. Use the equations so obtained to find the coordinates of the centroid. [5 marks]

Solution:

Concept:

- A median joins a vertex to the midpoint of the opposite side. A midpoint is found by averaging each pair of corresponding coordinates.
- A line through position vector $\vec{p}$, parallel to a nonzero vector $\vec{d}$, is $\vec{r} = \vec{p} + t\vec{d}$, $t \in \mathbb{R}$. The centroid is the common point of the medians.

Step 1: Find the median through A Let M be the midpoint of BC. Using the given coordinates,

$$M = \left(\frac{1-2}{2}, \frac{2+2}{2}, \frac{1-1}{2}\right) = \left(-\frac{1}{2}, 2, 0\right).$$

Thus $\overrightarrow{AM} = (-3/2, 1, 0)$, so we can use the proportional direction vector $(-3, 2, 0)$. The median through $A(1,1,0)$ is

$$\vec{r} = \hat{i} + \hat{j} + t(-3\hat{i} + 2\hat{j}), \qquad t \in \mathbb{R}.$$

Its coordinate equations are $x = 1 - 3t$, $y = 1 + 2t$, $z = 0$. The zero direction component means that every point of this median has $z = 0$.

Step 2: Find the median through B Let N be the midpoint of AC. Then

$$N = \left(\frac{1-2}{2}, \frac{1+2}{2}, \frac{0-1}{2}\right) = \left(-\frac{1}{2}, \frac{3}{2}, -\frac{1}{2}\right).$$

Subtracting $B(1,2,1)$ gives $\overrightarrow{BN} = (-3/2, -1/2, -3/2)$. Using the proportional direction vector $(-3, -1, -3)$, the median is

$$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + s(-3\hat{i} - \hat{j} - 3\hat{k}), \qquad s \in \mathbb{R}.$$

Its coordinate equations are $x = 1 - 3s$, $y = 2 - s$, $z = 1 - 3s$. Independent parameters s and t are used for the two lines.

Step 3: Use the equations to find their intersection At the common point, the first median requires $z = 0$. Hence the second median gives

$$1 - 3s = 0 \quad \Longrightarrow \quad s = \frac{1}{3}.$$

Substituting into its remaining coordinate equations,

$$x = 1 - 3\left(\frac{1}{3}\right) = 0, \qquad y = 2 - \frac{1}{3} = \frac{5}{3}.$$

Verify this point on the first median: $x = 0$ gives $t = 1/3$, and then $y = 1 + 2/3 = 5/3$, with $z = 0$. Thus both median equations give the same point, which is the centroid.

Quick Tip to Solve: The median from A has z-coordinate zero. Set the second median's z-coordinate equal to zero first to find its parameter immediately.

Step 4: Conclusion The required median equations are

$$AM: \quad \vec{r} = \hat{i} + \hat{j} + t(-3\hat{i} + 2\hat{j}),$$
$$BN: \quad \vec{r} = \hat{i} + 2\hat{j} + \hat{k} + s(-3\hat{i} - \hat{j} - 3\hat{k}).$$

Their intersection gives the centroid $\boxed{G(0, \frac{5}{3}, 0)}$.

Quick Tip to Solve: The median from A has z-coordinate zero. Set the second median's z-coordinate equal to zero first to find its parameter immediately.

35(a). Solve the following system of equations, using matrices: $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \quad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$**, where** $x, y, z \neq 0$**.** [5 marks]

Solution:

Concept:

- Reciprocal substitution converts this system into linear equations: put $u = 1/x$, $v = 1/y$ and $w = 1/z$. This is valid because the given variables are nonzero.
- A linear system has matrix form $AX = B$, where A is the coefficient matrix, X is the column of unknowns and B is the constant column. If $|A| \neq 0$, its unique solution is $X = A^{-1}B$, with $A^{-1} = \text{adj}\, A/|A|$.

Step 1: Express the reciprocal system in matrix form On making the substitution, the equations become

$$2u + 3v + 10w = 4, \qquad 4u - 6v + 5w = 1, \qquad 6u + 9v - 20w = 2.$$

Thus $AX = B$, where

$$A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}, \quad X = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad B = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}.$$

The columns of A are kept in the same order as the unknowns u, v, w.

Step 2: Check invertibility and compute the adjugate Expanding the determinant along the first row gives

$$|A| = 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$$
$$= 150 + 330 + 720 = 1200 \neq 0.$$

The matrix of signed cofactors is

$$C = \begin{pmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{pmatrix}.$$

Quick Tip to Solve: Solve for the reciprocals first; after computing $A^{-1}B$, invert each entry to obtain x, y, z, and check them in the original equations.

For example, $C_{12} = -[4(-20) - 5(6)] = 110$, while $C_{23} = -[2(9) - 3(6)] = 0$. Transposing the cofactor matrix gives

$$\operatorname{adj} A = C^T = \begin{pmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{pmatrix}.$$

Step 3: Multiply the inverse by the constant column Premultiplying $AX = B$ by A^{-1} yields

$$X = \frac{1}{1200}\begin{pmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$$
$$= \frac{1}{1200}\begin{pmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/3 \\ 1/5 \end{pmatrix}.$$

These entries are the reciprocals of the original unknowns. None is zero, so each can be inverted to recover a finite nonzero original variable.

Step 4: Verify the original equations Inserting the recovered values into the three left-hand sides gives

$$\frac{2}{2} + \frac{3}{3} + \frac{10}{5} = 1 + 1 + 2 = 4,$$
$$\frac{4}{2} - \frac{6}{3} + \frac{5}{5} = 2 - 2 + 1 = 1,$$
$$\frac{6}{2} + \frac{9}{3} - \frac{20}{5} = 3 + 3 - 4 = 2.$$

Thus all three original equations and the nonzero conditions are satisfied.

Step 5: Conclusion Since $1/x = 1/2$, $1/y = 1/3$ and $1/z = 1/5$, the unique solution is $\boxed{x = 2,\ y = 3,\ z = 5}$.

Quick Tip to Solve: Solve for the reciprocals first; after computing $A^{-1}B$, invert each entry to obtain x, y, z, and check them in the original equations.

OR

35(b). **If** $A = \begin{pmatrix} 1 & \cot x \\ -\cot x & 1 \end{pmatrix}$, **show that** $A'A^{-1} = \begin{pmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{pmatrix}$. [5 marks]

Solution:

Concept:

- The transpose A' is obtained by interchanging rows and columns. For $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, its inverse is $M^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, provided $ad - bc \neq 0$.

Quick Tip to Solve: Notice that $\operatorname{adj} A = A'$ here. Multiply the small matrix first, then multiply each ratio by $\sin^2 x / \sin^2 x$ to expose the double-angle identities.

- The identities $1+\cot^2 x=\csc^2 x$, $\cos 2x=\cos^2 x-\sin^2 x$ and $\sin 2x=2\sin x\cos x$ simplify the entries of the product.

Step 1: Find the transpose and check that the inverse exists Since the given matrix contains $\cot x$, it is defined for real x with $\sin x\neq 0$, equivalently $x\neq n\pi$ for every integer n. Transposing changes the positions of the two off-diagonal entries, so

$$A'=\begin{pmatrix}1 & -\cot x\\ \cot x & 1\end{pmatrix}.$$

The determinant of the original matrix is

$$|A|=1\cdot 1-(\cot x)(-\cot x)=1+\cot^2 x=\csc^2 x.$$

This is positive on the stated domain, hence the inverse exists wherever A is defined.

Step 2: Calculate the inverse and multiply in the required order Interchanging the diagonal entries and changing the signs of the off-diagonal entries gives the adjugate. Therefore

$$A^{-1}=\frac{1}{1+\cot^2 x}\begin{pmatrix}1 & -\cot x\\ \cot x & 1\end{pmatrix}.$$

Put $c=\cot x$ temporarily. Multiplication of rows by columns gives

$$\begin{aligned}A'A^{-1}&=\frac{1}{1+c^2}\begin{pmatrix}1 & -c\\ c & 1\end{pmatrix}\begin{pmatrix}1 & -c\\ c & 1\end{pmatrix}\\ &=\frac{1}{1+c^2}\begin{pmatrix}1-c^2 & -2c\\ 2c & 1-c^2\end{pmatrix}.\end{aligned}$$

In particular, the upper-right entry has two negative contributions, $-c-c$, while the lower-left entry has two positive contributions, $c+c$. This fixes the signs in the desired identity.

Step 3: Simplify the diagonal and off-diagonal entries Substitute $c=\cot x=\cos x/\sin x$ and multiply the numerator and denominator of each ratio by $\sin^2 x$. The diagonal ratio becomes

$$\frac{1-\cot^2 x}{1+\cot^2 x}=\frac{\sin^2 x-\cos^2 x}{\sin^2 x+\cos^2 x}=-\cos 2x.$$

Similarly, the positive off-diagonal ratio is

$$\frac{2\cot x}{1+\cot^2 x}=\frac{2\sin x\cos x}{\sin^2 x+\cos^2 x}=\sin 2x.$$

The upper-right ratio is its negative. These transformations are valid because $\sin x\neq 0$, and the final denominators equal 1.

Quick Tip to Solve: Notice that $\operatorname{adj}A=A'$ here. Multiply the small matrix first, then multiply each ratio by $\sin^2 x/\sin^2 x$ to expose the double-angle identities.

Step 4: Conclusion Substituting the simplified entries proves

$$\boxed{A'A^{-1} = \begin{pmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{pmatrix}}, \qquad x \neq n\pi \ (n \in \mathbb{Z}).$$

Quick Tip to Solve: Notice that $\operatorname{adj} A = A'$ here. Multiply the small matrix first, then multiply each ratio by $\sin^2 x / \sin^2 x$ to expose the double-angle identities.

Section E (12 marks)

This section comprises 3 case study based questions of 4 marks each.

36. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.

Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions: [4 marks]

36(i). Find the probability that the airplane will not crash. [1 mark]

Solution:

Concept:

- The events E_1 (crash) and E_2 (no crash) are complementary: $P(E_1) + P(E_2) = 1$.
- A percentage is converted to a probability by dividing by 100.

Step 1: Convert the crash percentage The given crash risk is 0.00001%, not the decimal probability 0.00001. Thus

$$P(E_1) = \frac{0.00001}{100} = 0.0000001 = 10^{-7}.$$

Step 2: Apply the complement rule Every journey either has a crash or has no crash, and these cases cannot occur together. Hence

$$P(E_2) = 1 - P(E_1) = 1 - 10^{-7} = 0.9999999.$$

The survival percentages are not needed for this unconditional event.

Step 3: Conclusion The probability that the airplane will not crash is $\boxed{0.9999999}$.

Quick Tip to Solve: First divide the printed crash percentage by 100; only then subtract the resulting probability from 1.

36(ii). Find $P(A \mid E_1) + P(A \mid E_2)$. [1 mark]

Solution:

Concept:

- $P(A \mid E_i)$ is the probability of survival under the specified condition E_i.
- A conditional certainty has probability 1; a stated percentage must be divided by 100.

Step 1: Read the two conditional probabilities Given a crash, the survival chance is 95%, so

$$P(A \mid E_1) = 0.95.$$

With no crash, all travellers survive by the stated assumption, giving

$$P(A \mid E_2) = 1.$$

Step 2: Add the requested quantities The question asks for the direct sum, without weighting either term by the likelihood of its condition. Therefore

$$P(A \mid E_1) + P(A \mid E_2) = 0.95 + 1 = 1.95.$$

This sum may exceed 1 because the probabilities refer to different conditioning events; it is not a single event probability.

Step 3: Conclusion The required sum is $\boxed{1.95}$.

Quick Tip to Solve: Use 0.95 after a crash and 1 after no crash; do not introduce crash probabilities into this direct sum.

36(iii)(a). Find $P(A)$. [2 marks]

Solution:

Concept:

- If E_1, E_2 partition the sample space, then $P(A) = P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)$.
- The crash probability is 10^{-7}, while conditional survival probabilities are 0.95 and 1.

Step 1: List the probabilities of both routes Converting the printed percentage gives

$$P(E_1) = \frac{0.00001}{100} = 10^{-7}, \qquad P(E_2) = 1 - 10^{-7}.$$

Survival can occur either after a crash or with no crash. These routes are disjoint and together include every possible survival outcome.

Step 2: Weight survival by each route Use the law of total probability with $P(A \mid E_1) = 0.95$ and $P(A \mid E_2) = 1$:

$$\begin{aligned} P(A) &= 10^{-7}(0.95) + (1 - 10^{-7})(1) \\ &= 1 - 0.05 \times 10^{-7} \\ &= 1 - 5 \times 10^{-9} = 0.999999995. \end{aligned}$$

Quick Tip to Solve: For this case study, compute non-survival as crash risk times 0.05, then subtract it from 1.

As a check, failure to survive requires a crash followed by non-survival, whose probability is $10^{-7} \times 0.05$. Its complement gives the same value.

Step 3: Conclusion The probability of survival is $\boxed{P(A) = 0.999999995}$.

Quick Tip to Solve: For this case study, compute non-survival as crash risk times 0.05, then subtract it from 1.

OR

36(iii)(b). Find $P(E_2 \mid A)$. [2 marks]

Solution:

Concept:

- Bayes theorem gives $P(E_2 \mid A) = P(E_2)P(A \mid E_2)/P(A)$, provided $P(A) > 0$.
- The total probability of survival includes survival after a crash and survival with no crash.

Step 1: Find the survival probability The crash probability is $P(E_1) = 0.00001/100 = 10^{-7}$; thus $P(E_2) = 1 - 10^{-7}$. Also $P(A \mid E_1) = 0.95$ and $P(A \mid E_2) = 1$. Therefore

$$\begin{aligned} P(A) &= (10^{-7})(0.95) + (1 - 10^{-7})(1) \\ &= 1 - 5 \times 10^{-9} = 0.999999995. \end{aligned}$$

Step 2: Condition the no-crash event on survival The event in the denominator is survival, because it is the information supplied after the conditioning bar. The numerator is the probability of both no crash and survival. Since survival is certain without a crash,

$$\begin{aligned} P(E_2 \mid A) &= \frac{P(E_2)P(A \mid E_2)}{P(A)} \\ &= \frac{0.9999999}{0.999999995} \\ &= \frac{199999980}{199999999}. \end{aligned}$$

It is slightly less than 1: some crash journeys also have survivors.

Step 3: Conclusion The required conditional probability is $\boxed{\frac{199999980}{199999999}}$.

Quick Tip to Solve: For no crash given survival, place total survival probability below the fraction and no-crash probability above it.

37. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.

The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

On the basis of the above information, answer the following questions: [4 marks]

37(i). Find F, when V = 40 km/h. [1 mark]

Solution:

Concept:

- The given model is $F(V) = V^2/500 - V/4 + 14$, where V is speed in km/h.
- F measures litres consumed per 100 km; evaluation requires substituting the given speed in every occurrence of V.

Step 1: Insert the specified speed At $V = 40$, the model becomes

$$F(40) = \frac{40^2}{500} - \frac{40}{4} + 14.$$

The squared speed must be evaluated before division.

Step 2: Simplify the three terms Here $40^2 = 1600$, so the quadratic contribution is $1600/500 = 3.2$. The linear contribution is $-40/4 = -10$. Combining them gives

$$F(40) = 3.2 - 10 + 14 = 7.2.$$

This is a consumption rate per 100 km, rather than a fuel quantity for an unspecified distance.

Step 3: Conclusion At 40 km/h, $\boxed{F = 7.2\,\text{l}/100\,\text{km}}$.

Quick Tip to Solve: Square 40 before dividing by 500, retain the minus sign on the linear term, and report litres per 100 km.

37(ii). Find $\frac{dF}{dV}$. [1 mark]

Solution:

Concept:

- For a power of speed, $\frac{d}{dV}V^n = nV^{n-1}$.
- Differentiate a polynomial term by term; constant multipliers remain and the derivative of a constant is zero.

Step 1: Differentiate each term The stated relation is $F = V^2/500 - V/4 + 14$. Keeping the denominators as constant factors gives

$$\frac{dF}{dV} = \frac{1}{500}\frac{d}{dV}(V^2) - \frac{1}{4}\frac{d}{dV}(V) + \frac{d}{dV}(14).$$

Quick Tip to Solve: Differentiate V squared to 2V, keep the coefficient -1/4, and remove the constant 14.

Step 2: Apply the power rule and simplify The three derivatives are $2V$, 1 and 0, respectively. Thus

$$\frac{dF}{dV} = \frac{2V}{500} - \frac{1}{4} + 0 = \frac{V}{250} - \frac{1}{4}.$$

The result describes the instantaneous change in consumption rate per unit increase in speed; no particular value of V is prescribed here.

Step 3: Conclusion The required derivative is $\boxed{\frac{dF}{dV} = \frac{V}{250} - \frac{1}{4}}$.

Quick Tip to Solve: Differentiate V squared to 2V, keep the coefficient -1/4, and remove the constant 14.

37(iii)(a). Find the speed V for which fuel consumption F is minimum. [2 marks]

Solution:

Concept:

- An interior minimum of a differentiable function occurs at a stationary point, where $F'(V) = 0$.
- The second derivative test confirms a strict minimum when $F''(V) > 0$ at the stationary point.

Step 1: Locate the stationary speed For the model $F(V) = V^2/500 - V/4 + 14$, differentiation gives

$$F'(V) = \frac{V}{250} - \frac{1}{4}.$$

Equating the derivative to zero yields

$$\frac{V}{250} = \frac{1}{4}, \qquad V = \frac{250}{4} = 62.5.$$

This speed is positive and therefore admissible for the vehicle model.

Step 2: Verify that fuel consumption is minimized Differentiate once more:

$$F''(V) = \frac{1}{250} > 0.$$

Thus the stationary point is a strict minimum. In fact the second derivative is positive for every speed, so the quadratic opens upwards and this is its unique global minimum. Equivalently, $F'(V) < 0$ for speeds below 62.5 and $F'(V) > 0$ above it: consumption decreases up to this speed and increases thereafter.

Step 3: Conclusion Fuel consumption is minimum at $\boxed{V = 62.5\,\text{km/h}}$.

Quick Tip to Solve: Set V/250-1/4 equal to zero, then use the positive constant second derivative to confirm the minimum.

OR

37(iii)(b). Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$. [2 marks]

Solution:

Concept:

- The derivative dF/dV measures the rate of change of fuel consumption F with speed V. Differentiate the given function and impose the specified rate to find the speed.
- Here F is measured in litres per 100 km. At a constant speed, fuel for a distance d km is $(d/100)F$ litres.

Step 1: Determine the speed from the rate condition Using the given relation and differentiating with respect to V,

$$F = \frac{V^2}{500} - \frac{V}{4} + 14, \qquad \frac{dF}{dV} = \frac{V}{250} - \frac{1}{4}.$$

Set this derivative equal to the specified value:

$$\frac{V}{250} - 0.25 = -0.01 \quad \Longrightarrow \quad V = 250(0.24) = 60\,\text{km/h}.$$

Step 2: Evaluate consumption and scale to the journey Substitute this speed into the original consumption function, since the derivative itself is not the amount of fuel consumed:

$$F(60) = \frac{3600}{500} - \frac{60}{4} + 14 = 7.2 - 15 + 14 = 6.2\,\text{litres per 100 km}.$$

The journey contains $600/100 = 6$ intervals of 100 km. Thus the required fuel is

$$6 \times 6.2 = 37.2\,\text{litres}.$$

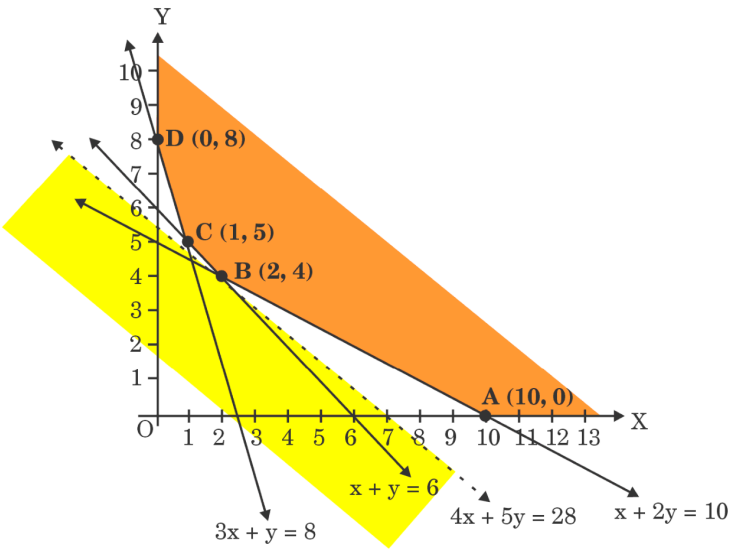
Step 3: Conclusion The speed is 60 km/h, and the quantity of fuel required for 600 km at this speed is $\boxed{37.2\,\text{litres}}$.

Quick Tip to Solve: First solve the derivative condition for speed, then use the original function for consumption; multiply by 6 because its unit is litres per 100 km.

38. The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg) which are available at the rate of ₹ 16/kg and ₹ 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

On the basis of the above information, answer the following questions: [4 marks]



38(i). Identify and write all the constraints which determine the given feasible region in Figure-2.

[2 marks]

Solution:

Concept:

- Each straight boundary gives a linear inequality; an interior point identifies the feasible side. Solid boundary lines are included.
- The food quantities x, y, in kg, require $x \geq 0, y \geq 0$.

Step 1: Identify the boundary equations The orange region has successive lower boundary segments DC, CB and BA, lying respectively on

$$3x + y = 8, \qquad x + y = 6, \qquad x + 2y = 10.$$

The labelled vertices $D(0, 8), C(1, 5), B(2, 4), A(10, 0)$ verify these equations.

Step 2: Select the shaded half-planes The interior point $(4, 4)$ gives $3x + y = 16 > 8$, $x + y = 8 > 6$ and $x + 2y = 12 > 10$. Therefore all three inequalities have the $\geq$ sign. Include the non-negativity conditions. The dotted $4x + 5y = 28$ is a cost line, not an additional edge of the feasible region.

Step 3: Conclusion The required constraints are

$$\boxed{3x + y \geq 8, \quad x + y \geq 6, \quad x + 2y \geq 10, \quad x \geq 0, \quad y \geq 0.}$$

Quick Tip to Solve: Trace the edges of the orange region and test an interior point for the inequality signs. The dotted cost line is not a separate constraint.

38(ii). If the objective is to minimize cost Z = 16x + 20y, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region. [2 marks]

Solution:

Concept:

- For a linear objective function, an optimum, when it exists on this feasible region, occurs at a corner point. Evaluate $Z = 16x + 20y$ at each vertex, where x, y are the food quantities in kg and Z is the cost in rupees.
- For an unbounded feasible region, check that no feasible point gives a value below the smallest corner value. A lower bound derived from the constraints verifies the minimum.

Step 1: Compare the costs at the corner points The figure gives the vertices $D(0, 8)$, $C(1, 5)$, $B(2, 4)$ and $A(10, 0)$. Substituting into the objective function,

$$Z(D) = 16(0) + 20(8) = 160,$$
$$Z(C) = 16(1) + 20(5) = 116,$$
$$Z(B) = 16(2) + 20(4) = 112,$$
$$Z(A) = 16(10) + 20(0) = 160.$$

The smallest of these values is 112, attained at B.

Step 2: Verify the lower bound throughout the region Every feasible point satisfies $x + y \geq 6$ and $x + 2y \geq 10$. Hence

$$Z = 12(x + y) + 4(x + 2y) \geq 12(6) + 4(10) = 112.$$

Equality holds when both inequalities are equalities. Solving $x + y = 6$ and $x + 2y = 10$ gives $y = 4$, $x = 2$. This point also satisfies $3x + y = 10 \geq 8$ and non-negativity, so it is feasible. Thus the unbounded part of the region cannot supply a lower cost.

Step 3: Conclusion The minimum cost is $\boxed{₹\,112}$, attained for $\boxed{x = 2\,\text{kg},\ y = 4\,\text{kg}}$, i.e. 2 kg of food X and 4 kg of food Y.

Quick Tip to Solve: After comparing the four vertices, write the objective as $12(x + y) + 4(x + 2y)$ to prove that no point in this unbounded region costs less than 112 rupees.