

CBSE Class 12 Mathematics Question Paper 2024 with Solution PDF-Set 65-5-1

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 80

General Instructions

- (i) This Question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.
- (iii) In **Section A** – Questions Number **1** to **18** are Multiple Choice Questions (MCQs) type and Questions Number **19 & 20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B** – Questions Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C** – Questions Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D** – Questions Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.
- (vii) In **Section E** – Questions Number **36** to **38** are case study based questions, carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, **2** questions in Section – **D** and **2** questions in Section – **E**.
- (ix) Use of calculators is **NOT** allowed.

Section A (20 marks)

This section has 20 multiple choice questions of 1 mark each.

1. A function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is:

[1 mark]

- (A) injective but not surjective.
- (B) surjective but not injective.
- (C) both injective and surjective.
- (D) neither injective nor surjective.

Correct Answer: (D) neither injective nor surjective.

Solution:

Concept:

- Injectivity requires distinct inputs to have distinct outputs.
- Surjectivity onto $\mathbb{R}$ requires every real number to be an output.

Step 1: Test injectivity Complete the square: $f(x) = (x - 2)^2 + 1$. In particular, $f(1) = 2 = f(3)$, although $1 \neq 3$. Thus the function is not injective, eliminating options (A) and (C).

Step 2: Test surjectivity Since $(x - 2)^2 \geq 0$, we have $f(x) \geq 1$. Conversely, each $t \geq 1$ is attained at $x = 2 + \sqrt{t - 1}$. Its range is therefore $[1, \infty)$, which omits real numbers such as 0. It is not surjective onto its stated codomain $\mathbb{R}$, eliminating (B).

Step 3: Conclusion The function is neither injective nor surjective. Hence, the correct option is (D).

Quick Tip to Solve: Complete the square first: symmetry about $x = 2$ disproves injectivity, and the minimum 1 disproves surjectivity.

2. If $A = \begin{pmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{pmatrix}$ is a skew-symmetric matrix, then the value of $2a - (b + c)$ is:

[1 mark]

- (A) 0
- (B) 1
- (C) -10
- (D) 10

Correct Answer: (A) 0

Solution:

Concept:

- A real skew-symmetric matrix satisfies $A^T = -A$.
- Its diagonal entries are zero and opposite off-diagonal entries sum to zero.

Step 1: Use the diagonal condition The entry in row 1, column 1 gives $a = -a$, hence $2a = 0$ and $a = 0$. The other diagonal entries are already zero, as required.

Quick Tip to Solve: For a skew-symmetric matrix, set diagonal entries to zero and sum each opposite pair before evaluating the expression.

Step 2: Use opposite entries The entries in positions $(1, 2)$ and $(2, 1)$ give $c = -b$, hence $b + c = 0$. The remaining pairs, $-1, 1$ and $5, -5$, already satisfy skew-symmetry. Substitution gives $2a - (b + c) = 2(0) - 0 = 0$. Options (B), (C) and (D) are nonzero and violate these conditions.

Step 3: Conclusion The value is 0. Hence, the correct option is (A).

Quick Tip to Solve: For a skew-symmetric matrix, set diagonal entries to zero and sum each opposite pair before evaluating the expression.

3. If A is a square matrix of order 3 such that the value of $|\text{adj } A| = 8$, then the value of $|A^T|$ is: [1 mark]

- (A) $\sqrt{2}$
- (B) $-\sqrt{2}$
- (C) 8
- (D) $2\sqrt{2}$

Correct Answer: (D) $2\sqrt{2}$

Solution:

Concept:

- For a square matrix of order n , $|\text{adj } A| = |A|^{n-1}$.
- Transposition preserves the determinant: $|A^T| = |A|$.

Step 1: Apply the adjugate identity Here $n = 3$, so $|\text{adj } A| = |A|^2 = 8$. Taking square roots gives $|A| = \pm\sqrt{8} = \pm 2\sqrt{2}$. The vertical bars denote a determinant, not an absolute value, so the sign cannot be determined from the given information.

Step 2: Compare all printed choices We also have $|A^T| = \pm 2\sqrt{2}$. Options (A) and (B) would square to 2, not 8, and (C) would square to 64. Option (D), $2\sqrt{2}$, is the only listed value consistent with the condition. The other possible value, $-2\sqrt{2}$, is not listed; the data alone do not force a positive determinant.

Step 3: Conclusion Among the printed options, the correct choice is (D), $2\sqrt{2}$. In general the determinant may be $\pm 2\sqrt{2}$.

Quick Tip to Solve: Use the exponent $n - 1 = 2$, and retain both square-root signs before comparing the printed options.

4. If inverse of matrix $\begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ is the matrix $\begin{pmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{pmatrix}$, then value of λ is: [1 mark]

- (A) -4
 (B) 1
 (C) 3
 (D) 4

Correct Answer: (D) 4

Solution:

Concept:

- If $B = A^{-1}$, then $AB = I$.
- A product entry is the dot product of the corresponding row and column.

Step 1: Find the decisive product entry Use row 2 of the given matrix, $(-1, 1, 0)$, and column 2 of its stated inverse, $(3, \lambda, 3)^T$. Their dot product is $-3 + \lambda$. This is the diagonal entry $(2, 2)$ of I , so $-3 + \lambda = 1$.

Step 2: Check the resulting inverse The equation yields $\lambda = 4$. With this value, direct multiplication gives

$$\begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix} = I_3.$$

The alternatives $-4, 1, 3$ give diagonal entries $-7, -2, 0$, respectively, rather than 1 .

Step 3: Conclusion $\lambda = 4$. Hence, the correct option is (D).

Quick Tip to Solve: Use the row and column containing the unknown to form one identity-matrix entry instead of calculating the full inverse.

5. If $\begin{bmatrix} x & 2 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ x \end{bmatrix}$, then value of x is: [1 mark]

- (A) -1
 (B) 0
 (C) 1
 (D) 2

Correct Answer: (A) -1

Solution:

Concept:

- A row matrix times a column matrix is their scalar dot product.
- Equality of the two 1×1 products gives an ordinary linear equation.

Quick Tip to Solve: Compute each row-column dot product separately, then move the two variable terms to the same side.

Step 1: Evaluate both products The left side equals $x(5) + 2(-1) + 0(x) = 5x - 2$. The right side equals $3(-2) + 1(x) = x - 6$. Keep the right-hand matrix product; it contributes both the constant -6 and the variable x .

Step 2: Solve and test the options Equating the products gives $5x - 2 = x - 6$, hence $4x = -4$ and $x = -1$. At $x = -1$, both sides are -7 . For the other printed choices $x = 0, 1, 2$, the left minus right is $4x + 4 = 4, 8, 12$, respectively, so none satisfies the matrix equality.

Step 3: Conclusion $x = -1$. Hence, the correct option is (A).

Quick Tip to Solve: Compute each row-column dot product separately, then move the two variable terms to the same side.

6. Find the matrix A^2 , where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = \max(i, j) - \min(i, j)$: [1 mark]

(A) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Correct Answer: (C) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Solution:

Concept:

- The entry rule is $a_{ij} = \max(i, j) - \min(i, j) = |i - j|$.
- Matrix squaring uses row-column multiplication, not entrywise squaring.

Step 1: Construct the matrix For $i, j \in \{1, 2\}$, the diagonal entries are $a_{11} = 1 - 1 = 0$ and $a_{22} = 2 - 2 = 0$. The off-diagonal entries are $a_{12} = a_{21} = 2 - 1 = 1$. Thus $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

Step 2: Multiply the matrix by itself

$$A^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Each diagonal entry is 1 and each off-diagonal entry is 0. Option (A) is the zero matrix; option (B) is A itself; option (D) incorrectly makes the off-diagonal entries 1.

Step 3: Conclusion $A^2 = I_2$. Hence, the correct option is (C).

Quick Tip to Solve: The matrix exchanges the two coordinates; applying this exchange twice gives the identity matrix.

7. If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is:

[1 mark]

(A) -1

(B) 1

(C) $-e$

(D) $-\frac{1}{e}$

Correct Answer: (A) -1

Solution:

Concept:

- Implicit differentiation uses the product rule on xe^y , with y a function of x .
- The chain rule gives $\frac{d}{dx}e^y = e^y \frac{dy}{dx}$.

Step 1: Differentiate implicitly Differentiating $xe^y = 1$ gives $e^y + xe^y \frac{dy}{dx} = 0$. Since $e^y > 0$, divide by e^y to obtain $1 + x \frac{dy}{dx} = 0$, hence $\frac{dy}{dx} = -\frac{1}{x}$.

Step 2: Evaluate at the specified input At $x = 1$, the original relation gives $e^y = 1$, so $y = 0$. Substituting $x = 1$ in the derivative gives -1 . Option (B) has the wrong sign. Options (C) and (D) incorrectly retain a factor of e , although the exponential factor cancels during implicit differentiation.

Step 3: Conclusion $\left. \frac{dy}{dx} \right|_{x=1} = -1$. Hence, the correct option is (A).

Quick Tip to Solve: Cancel the common nonzero factor e^y after differentiation; only $-1/x$ remains.

8. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is:

[1 mark]

(A) $\sin x e^{\sin^2 x}$

(B) $\cos x e^{\sin^2 x}$

(C) $-2 \cos x e^{\sin^2 x}$

(D) $-2 \sin^2 x \cos x e^{\sin^2 x}$

Correct Answer: (C) $-2 \cos x e^{\sin^2 x}$

Solution:

Concept:

- For $u = \cos x$, write $\sin^2 x = 1 - u^2$.
- The chain rule gives $\frac{d}{du}e^{g(u)} = e^{g(u)} g'(u)$.

Step 1: Express the function in the new variable Put $u = \cos x$. The identity $\sin^2 x + \cos^2 x = 1$ rewrites the given expression as $e^{\sin^2 x} = e^{1-u^2}$. Thus differentiation with respect to $\cos x$ means differentiation of this expression with respect to u .

Quick Tip to Solve: For differentiation with respect to $\cos x$, rewrite $\sin^2 x$ as $1 - \cos^2 x$ before applying the chain rule.

Step 2: Apply the chain rule

$$\frac{d}{du} e^{1-u^2} = e^{1-u^2} (-2u) = -2 \cos x e^{\sin^2 x}.$$

Options (A) and (B) omit the factor -2 and do not differentiate the exponent correctly. Option (D) adds an unnecessary $\sin^2 x$ factor. The substitution also avoids dividing by $-\sin x$ at points where it is zero.

Step 3: Conclusion The derivative is $-2 \cos x e^{\sin^2 x}$. Hence, the correct option is (C).

Quick Tip to Solve: For differentiation with respect to $\cos x$, rewrite $\sin^2 x$ as $1 - \cos^2 x$ before applying the chain rule.

9. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to:

[1 mark]

- (A) 2
- (B) 1
- (C) 0
- (D) -2

Correct Answer: (A) 2

Solution:

Concept:

- A stationary point satisfies $f'(x) = 0$, provided the function is defined there.
- The second derivative test gives a local minimum when $f''(x) > 0$.

Step 1: Find the stationary points For $x \neq 0$, differentiate the printed function:

$$f'(x) = \frac{1}{2} - \frac{2}{x^2}.$$

Thus $f'(x) = 0$ gives $x^2 = 4$, so the only stationary points are $x = 2$ and $x = -2$.

Step 2: Classify and compare the options Now $f''(x) = 4/x^3$. At $x = 2$ this is positive, whereas at $x = -2$ it is negative, giving a local maximum there. At $x = 1$, the derivative is $-3/2$, so it is not stationary. The value $x = 0$ lies outside the domain and cannot give a local extremum.

Step 3: Conclusion The local minimum occurs at $x = 2$. Hence, the correct option is (A).

Quick Tip to Solve: After finding both stationary points, use the sign of $4/x^3$ to distinguish the minimum from the maximum.

10. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$, is: [1 mark]

- (A) -60 units/sec
- (B) 60 units/sec
- (C) -70 units/sec
- (D) -140 units/sec

Correct Answer: (A) -60 units/sec

Solution:

Concept:

- The slope of a curve is $m = dy/dx$.
- When x depends on time, $dm/dt = (dm/dx)(dx/dt)$.

Step 1: Find the slope function Differentiating $y = 7x - x^3$ with respect to x gives $m = 7 - 3x^2$. Therefore $dm/dx = -6x$. The question asks for the change in this slope, so differentiating the ordinate with respect to time would calculate a different quantity.

Step 2: Apply the chain rule Using $dx/dt = 2$, we obtain

$$\frac{dm}{dt} = (-6x)(2) = -12x.$$

At $x = 5$, this is -60 units/sec. Option (B) loses the negative sign. Options (C) and (D) do not follow from differentiating the slope and applying the given rate; the constant term 7 disappears from its derivative.

Step 3: Conclusion The rate of change of slope is -60 units/sec. Hence, the correct option is (A).

Quick Tip to Solve: Write the slope as a separate function before using the time chain rule; the constant 7 then drops out.

11. $\int \frac{1}{x(\log x)^2} dx$ is equal to: [1 mark]

- (A) $2 \log(\log x) + c$
- (B) $-\frac{1}{\log x} + c$
- (C) $\frac{(\log x)^3}{3} + c$
- (D) $\frac{3}{(\log x)^3} + c$

Correct Answer: (B) $-\frac{1}{\log x} + c$

Solution:

Concept:

- Use substitution $u = \log x$, so $du = dx/x$.
- The power rule gives $\int u^{-2} du = -u^{-1} + c$.

Quick Tip to Solve: Whenever a logarithmic power is multiplied by $1/x$, substitute $u = \log x$ before integrating.

Step 1: Substitute the logarithm For the real integrand, $x > 0$ and $x \neq 1$. Put $u = \log x$, with $du = dx/x$. The integral becomes

$$\int \frac{dx}{x(\log x)^2} = \int u^{-2} du = -\frac{1}{u} + c.$$

This substitution absorbs precisely the factor $1/x$.

Step 2: Check the printed alternatives Replacing u by $\log x$ gives $-1/\log x + c$, whose derivative is $1/[x(\log x)^2]$. Option (A) differentiates to $2/(x \log x)$, option (C) to $(\log x)^2/x$, and option (D) to $-9/[x(\log x)^4]$. None of these three equals the given integrand.

Step 3: Conclusion The integral is $-\frac{1}{\log x} + c$. Hence, the correct option is (B).

Quick Tip to Solve: Whenever a logarithmic power is multiplied by $1/x$, substitute $u = \log x$ before integrating.

12. The value of $\int_{-1}^1 x|x| dx$ is:

[1 mark]

- (A) $\frac{1}{6}$
- (B) $\frac{1}{3}$
- (C) $-\frac{1}{6}$
- (D) 0

Correct Answer: (D) 0

Solution:

Concept:

- An odd function satisfies $g(-x) = -g(x)$.
- The integral of an integrable odd function over $[-a, a]$ is zero.

Step 1: Identify the symmetry Let $g(x) = x|x|$. Since $|-x| = |x|$, we have $g(-x) = (-x)|x| = -g(x)$. The integrand is continuous and odd, while the limits -1 and 1 are symmetric about zero. Its negative and positive contributions therefore cancel.

Step 2: Verify by splitting at zero For $x < 0$, $x|x| = -x^2$; for $x > 0$, $x|x| = x^2$. Hence

$$\int_{-1}^1 x|x| dx = -\int_{-1}^0 x^2 dx + \int_0^1 x^2 dx = -\frac{1}{3} + \frac{1}{3} = 0.$$

Options (A), (B) and (C) are nonzero, so none can represent the complete signed integral.

Step 3: Conclusion The value of the integral is 0. Hence, the correct option is (D).

Quick Tip to Solve: Check parity before splitting an absolute-value integral with symmetric limits; here $x|x|$ is odd.

13. Area of the region bounded by curve $y^2 = 4x$ and the X -axis between $x = 0$ and $x = 1$ is: [1 mark]

- (A) $\frac{2}{3}$
- (B) $\frac{8}{3}$
- (C) 3
- (D) $\frac{4}{3}$

Correct Answer: (D) $\frac{4}{3}$

Solution:

Concept:

- Area between a branch $y = f(x)$ and the X -axis is $\int_a^b |f(x)| dx$.
- For $y^2 = 4x$, either branch has magnitude $|y| = 2\sqrt{x}$.

Step 1: Choose the branch bounding the region The parabola has branches $y = 2\sqrt{x}$ and $y = -2\sqrt{x}$. Between $x = 0$ and $x = 1$, either branch and the X -axis enclose congruent regions. For the region bounded by the axis and a branch, the height is $2\sqrt{x}$. Counting both branches would instead give the region between the branches.

Step 2: Integrate and compare the choices The required area is

$$A = \int_0^1 2\sqrt{x} dx = \left[\frac{4}{3}x^{3/2} \right]_0^1 = \frac{4}{3}.$$

Option (A) omits the factor 2 in the height. Option (B) doubles the area by counting both congruent regions on opposite sides of the axis. Option (C), namely 3, does not equal the evaluated integral.

Step 3: Conclusion The required area is $\frac{4}{3}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Use the distance from one branch to the X -axis, $2\sqrt{x}$; doubling it counts both regions.

14. The order of the differential equation $\frac{d^4y}{dx^4} - \sin\left(\frac{d^2y}{dx^2}\right) = 5$ is: [1 mark]

- (A) 4
- (B) 3
- (C) 2
- (D) not defined

Correct Answer: (A) 4

Solution:

Concept:

- The order is the order of the highest derivative actually present in a differential equation.
- Degree is a separate concept requiring polynomial dependence on derivatives.

Quick Tip to Solve: For order, scan only for the highest differentiation index; do not apply the polynomial test used for degree.

Step 1: Identify the derivatives present The printed equation contains d^4y/dx^4 and d^2y/dx^2 , with the latter inside a sine function. The highest derivative present is the fourth derivative. The value 5 on the right-hand side is a constant and does not affect the order.

Step 2: Distinguish order from degree The sine of a derivative does not prevent us from identifying the highest derivative. It affects the polynomial condition used when defining degree, whereas the question asks only for order. Option (B) would require a highest third derivative, and option (C) overlooks the fourth derivative. Option (D) confuses order with degree.

Step 3: Conclusion The order of the differential equation is 4. Hence, the correct option is (A).

Quick Tip to Solve: For order, scan only for the highest differentiation index; do not apply the polynomial test used for degree.

15. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio $3 : 1$ and S is the mid-point of line segment PR . The position vector of S is: [1 mark]

- (A) $\frac{\vec{p} + 3\vec{q}}{4}$
(B) $\frac{\vec{p} + 3\vec{q}}{8}$
(C) $\frac{5\vec{p} + 3\vec{q}}{4}$
(D) $\frac{5\vec{p} + 3\vec{q}}{8}$

Correct Answer: (D) $\frac{5\vec{p} + 3\vec{q}}{8}$

Solution:

Concept:

- For $PR : RQ = m : n$, the internal section formula is $\vec{r} = (n\vec{p} + m\vec{q})/(m + n)$.
- The midpoint of positions $\vec{a}, \vec{b}$ has position $(\vec{a} + \vec{b})/2$.

Step 1: Find the position of R The ratio is $PR : RQ = 3 : 1$, so the internal section formula gives

$$\vec{r} = \frac{\vec{p} + 3\vec{q}}{4}.$$

The coefficient of $\vec{q}$ is larger because R is nearer Q , consistent with the stated division.

Step 2: Take the midpoint of PR Since S bisects PR ,

$$\vec{s} = \frac{\vec{p} + \vec{r}}{2} = \frac{1}{2} \left(\vec{p} + \frac{\vec{p} + 3\vec{q}}{4} \right) = \frac{5\vec{p} + 3\vec{q}}{8}.$$

Option (A) gives R , while (B) halves its vector without including $\vec{p}$. Option (C) omits the final factor of 2 from taking the midpoint.

Step 3: Conclusion The position vector is $\frac{5\vec{p} + 3\vec{q}}{8}$. Hence, the correct option is (D).

Quick Tip to Solve: Find R first, then average its position with $\vec{p}$; do not average with the origin.

16. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y -axis is: [1 mark]

- (A) $\frac{5\pi}{6}$
 (B) $\frac{3\pi}{4}$
 (C) $\frac{5\pi}{4}$
 (D) $\frac{7\pi}{4}$

Correct Answer: (B) $\frac{3\pi}{4}$

Solution:

Concept:

- Direction ratios (a, b, c) give the direction cosine with the positive Y -axis as $b/\sqrt{a^2 + b^2 + c^2}$.
- A coordinate direction angle lies in $[0, \pi]$.

Step 1: Read the direction ratios The symmetric form gives direction ratios $(1, -1, 0)$. The zero third ratio means the direction has no z -component. Its magnitude is $\sqrt{1^2 + (-1)^2 + 0^2} = \sqrt{2}$. Thus the direction cosine with the positive Y -axis is $-1/\sqrt{2}$.

Step 2: Select the coordinate direction angle Writing this angle as β , we have $\cos \beta = -1/\sqrt{2}$. In the allowed interval $0 \leq \beta \leq \pi$, this gives $\beta = 3\pi/4$. Option (A) has cosine $-\sqrt{3}/2$. Although (C) has the same cosine, $5\pi/4$ is outside the direction-angle interval; (D) is also outside it and has positive cosine.

Step 3: Conclusion The required angle is $\frac{3\pi}{4}$. Hence, the correct option is (B).

Quick Tip to Solve: Use the signed Y -direction ratio and restrict the coordinate direction angle to $[0, \pi]$.

17. The Cartesian equation of the line passing through the point $(1, -3, 2)$ and parallel to the line $\vec{r} = (2 + \lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$ is: [1 mark]

- (A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$
 (B) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$
 (C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$
 (D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

Correct Answer: (D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

Solution:

Concept:

- A line through (x_0, y_0, z_0) with direction ratios (a, b, c) has symmetric form $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$.
- Parallel lines have proportional direction ratios; these are coefficients of the parameter, not

Quick Tip to Solve: Read the direction from the coefficients of λ , then subtract the given point coordinates.

coordinates of a point.

Step 1: Extract the direction Write the given line as $\vec{r} = (2\hat{i} - \hat{k}) + \lambda(\hat{i} + \hat{j} + 2\hat{k})$. Thus its direction ratios are $(1, 1, 2)$, so the required parallel line has the same direction.

Step 2: Use the point and test the options For $(1, -3, 2)$, the numerators must be $x - 1, y + 3, z - 2$. Option (D) has these numerators and direction ratios $(1, 1, 2)$. Option (A) uses the correct point but the wrong direction $(2, 0, -1)$; (B) uses the correct direction but passes through $(-1, 3, -2)$; (C) has both the wrong point and the wrong direction.

Step 3: Conclusion The required equation is $\boxed{\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}}$. Hence, the correct option is (D).

Quick Tip to Solve: Read the direction from the coefficients of λ , then subtract the given point coordinates.

18. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then:

[1 mark]

- (A) $A \subset B$, but $A \neq B$
- (B) $A = B$
- (C) $A \cap B = \phi$
- (D) $P(A) = P(B)$

Correct Answer: (D) $P(A) = P(B)$

Solution:

Concept:

- Conditional probabilities satisfy $P(A | B) = P(A \cap B)/P(B)$ and $P(B | A) = P(A \cap B)/P(A)$.
- The given nonzero conditional probabilities imply $P(A \cap B) > 0$, allowing cancellation.

Step 1: Substitute the definitions The hypothesis gives

$$\frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)} \neq 0.$$

Both denominators are positive because the conditional probabilities are defined.

Step 2: Cancel and assess alternatives Cancel the positive common numerator to obtain $1/P(B) = 1/P(A)$. Disjointness in (C) would make both conditional probabilities zero. Equality of probabilities does not require equal events or a strict containment: on equally likely outcomes $\{1, 2, 3\}$, events $A = \{1, 2\}$, $B = \{2, 3\}$ satisfy the hypothesis but neither (A) nor (B).

Step 3: Conclusion $\boxed{P(A) = P(B)}$. Hence, the correct option is (D).

Quick Tip to Solve: The condition $\neq 0$ is what permits cancelling the intersection probability.

19. Assertion (A): Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.

Reason (R): The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- The principal inverse cosine has domain $[-1, 1]$ and range $[0, \pi]$.
- The value $\cos^{-1} 0 = \pi/2$ belongs to the principal range.

Step 1: Judge the assertion Cosine restricted to $[0, \pi]$ takes every value in $[-1, 1]$ exactly once. Its inverse therefore accepts precisely the inputs $-1 \leq x \leq 1$. The assertion is true, including both endpoints.

Step 2: Judge the reason The reason incorrectly removes $\pi/2$ from the range. Taking $x = 0$ gives $y = \cos^{-1} 0 = \pi/2$, directly disproving that exclusion. The reason is false.

Step 3: Check the explanation A false statement cannot explain the true domain statement. Options (A) and (B) both require a true reason, while (D) requires a false assertion; these truth assignments do not match the results above.

Step 4: Conclusion The assertion is true and the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Test any proposed excluded inverse-trigonometric value by substituting its cosine as the input.

20. Assertion (A): The vectors $\vec{a} = 6\hat{i} + 2\hat{j} - 8\hat{k}$, $\vec{b} = 10\hat{i} - 2\hat{j} - 6\hat{k}$ and $\vec{c} = 4\hat{i} - 4\hat{j} + 2\hat{k}$ represent the sides of a right angled triangle.

Reason (R): Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- Three nonzero, noncollinear vectors can form triangle sides when their directed sum is zero, or when one equals the sum of the other two.
- A right angle requires the dot product of the corresponding side directions to be zero.

Step 1: Judge the assertion Here $\vec{a} + \vec{c} = (10, -2, -6) = \vec{b}$, so the vectors form a triangle. Further,

$$\vec{a} \cdot \vec{c} = 6(4) + 2(-4) + (-8)(2) = 24 - 8 - 16 = 0.$$

The nonzero vectors $\vec{a}$ and $\vec{c}$ are perpendicular. Together with the closure relation this establishes a right angled triangle, so the assertion is true.

Step 2: Judge the reason The reason is the standard vector closure condition for a triangle. When one vector is the sum of two others, reversing that vector makes the three directed sides sum to zero. The nonzero and noncollinear conditions prevent a degenerate triangle. Thus the reason is true.

Step 3: Check the explanation The reason establishes triangle closure, but imposes no perpendicularity condition. It therefore does not explain why this particular triangle is right angled. That additional fact follows from the zero dot product calculated above. Both statements are true, excluding (C) and (D), but the missing right-angle justification excludes (A).

Step 4: Conclusion Both the assertion and the reason are true, but the reason is not the correct explanation of the assertion. Hence, the correct option is (B).

Quick Tip to Solve: After checking triangle closure, use dot products separately to test the claim of a right angle.

Section B (10 marks)

This section has 5 Very Short Answer questions of 2 marks each.

21. Find value of k if $\sin^{-1} \left[k \tan \left(2 \cos^{-1} \frac{\sqrt{3}}{2} \right) \right] = \frac{\pi}{3}$.

[2 marks]

Solution:

Concept:

- The principal value $\cos^{-1}(\sqrt{3}/2) = \pi/6$ lies in $[0, \pi]$.
- If $\sin^{-1} u = \theta$ with $\theta \in [-\pi/2, \pi/2]$, then $u = \sin \theta$, provided $u \in [-1, 1]$.

Step 1: Evaluate the inner angle Using the principal inverse cosine,

$$2 \cos^{-1} \frac{\sqrt{3}}{2} = 2 \left(\frac{\pi}{6} \right) = \frac{\pi}{3}.$$

Therefore the tangent inside the inverse sine is $\tan(\pi/3) = \sqrt{3}$, and the printed equation reduces to $\sin^{-1}(k\sqrt{3}) = \pi/3$.

Step 2: Apply sine and solve The angle $\pi/3$ belongs to the principal inverse-sine range, so applying sine is valid:

$$k\sqrt{3} = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

Dividing by the nonzero factor $\sqrt{3}$ gives the required value. This linear equation has only one solution.

Step 3: Verify the domain and original equation At the computed value, the inverse-sine argument is $\sqrt{3}/2$, which lies in $[-1, 1]$. Its principal inverse sine is exactly $\pi/3$; hence the result satisfies the original equation without a branch ambiguity.

Step 4: Conclusion $k = \frac{1}{2}$.

Quick Tip to Solve: Evaluate the inverse cosine first, and check the inverse-sine argument lies between -1 and 1 .

22(a). Verify whether the function f defined by

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is continuous at $x = 0$ or not.

[2 marks]

Solution:

Concept:

- Continuity at $x = 0$ requires $\lim_{x \rightarrow 0} f(x) = f(0)$.
- The bound $|\sin t| \leq 1$ for every real t gives a squeeze estimate even when the sine factor oscillates.

Quick Tip to Solve: For an oscillating sine multiplied by x , bound the absolute value by $|x|$ and apply the squeeze theorem.

Step 1: Read the value at the origin From the second branch, $f(0) = 0$. For every nonzero x , the first branch gives $f(x) = x \sin(1/x)$. Direct substitution into this branch is not allowed at zero because $1/x$ is undefined there.

Step 2: Bound the oscillating expression Since $|\sin(1/x)| \leq 1$,

$$\left| x \sin \frac{1}{x} \right| \leq |x|.$$

Equivalently, $-|x| \leq f(x) \leq |x|$ for $x \neq 0$. Both bounding functions tend to zero as $x \rightarrow 0$, from either side. The squeeze theorem therefore gives $\lim_{x \rightarrow 0} f(x) = 0$.

Step 3: Compare the limit and function value The two-sided limit exists and equals the separately defined value $f(0) = 0$. Although $\sin(1/x)$ itself has no limit at zero, multiplying by x forces the amplitude to tend to zero, so the product has the limit required for continuity.

Step 4: Conclusion f is continuous at $x = 0$, since $\lim_{x \rightarrow 0} f(x) = f(0) = 0$.

Quick Tip to Solve: For an oscillating sine multiplied by x , bound the absolute value by $|x|$ and apply the squeeze theorem.

OR

22(b). Check for differentiability of the function f defined by $f(x) = |x - 5|$, at the point $x = 5$.

[2 marks]

Solution:

Concept:

- The derivative at a point exists only when the left-hand and right-hand difference quotients have the same finite limit.
- For a real number u , $|u| = u$ when $u \geq 0$, and $|u| = -u$ when $u < 0$.

Step 1: Compute the left-hand derivative Here $f(5) = |5 - 5| = 0$. For $h < 0$, $|h| = -h$, so the left-hand difference quotient is

$$f'_{-}(5) = \lim_{h \rightarrow 0^{-}} \frac{f(5+h) - f(5)}{h} = \lim_{h \rightarrow 0^{-}} \frac{|h|}{h} = \lim_{h \rightarrow 0^{-}} (-1) = -1.$$

Thus the slope on the left of the specified point is negative.

Step 2: Compute and compare the right-hand derivative For $h > 0$, $|h| = h$. Hence

$$f'_{+}(5) = \lim_{h \rightarrow 0^{+}} \frac{f(5+h) - f(5)}{h} = \lim_{h \rightarrow 0^{+}} \frac{|h|}{h} = 1.$$

The two limits are finite but unequal. Although the function is continuous at 5, since $|x - 5| \rightarrow 0 = f(5)$, continuity does not guarantee differentiability at a corner.

Step 3: Conclusion Since $f'_{-}(5) = -1 \neq 1 = f'_{+}(5)$, $f(x) = |x - 5|$ is not differentiable at $x = 5$.

Quick Tip to Solve: At the zero of a modulus, evaluate the difference quotient separately for positive and negative increments.

23. The area of the circle is increasing at a uniform rate of $2 \text{ cm}^2/\text{sec}$. How fast is the circumference of the circle increasing when the radius $r = 5 \text{ cm}$? [2 marks]

Solution:

Concept:

- For a circle of radius r , its area is $A = \pi r^2$ and its circumference is $C = 2\pi r$.
- When the radius changes with time t , apply the chain rule: $dA/dt = 2\pi r dr/dt$ and $dC/dt = 2\pi dr/dt$.

Step 1: Find the instantaneous rate of change of radius The given uniform rate is $dA/dt = 2 \text{ cm}^2/\text{s}$. Differentiating the area relation with respect to time gives

$$2 = 2\pi r \frac{dr}{dt}.$$

At the instant when $r = 5 \text{ cm}$,

$$\frac{dr}{dt} = \frac{2}{2\pi(5)} = \frac{1}{5\pi} \text{ cm/s}.$$

The radius grows at this instantaneous rate; a constant area rate does not mean a constant radius rate.

Step 2: Use the circumference relation Differentiate $C = 2\pi r$ with respect to the same time variable and substitute the radius rate:

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt} = 2\pi \left(\frac{1}{5\pi} \right) = \frac{2}{5} \text{ cm/s}.$$

The positive sign indicates an increasing circumference, as required by the increasing area.

Step 3: Conclusion The circumference is increasing at $\boxed{\frac{2}{5} \text{ cm/s}}$ when the radius is 5 cm .

Quick Tip to Solve: Divide the differentiated circumference and area equations to get $dC/dt = (1/r)(dA/dt)$.

24(a). Find: $\int \cos^3 x e^{\log \sin x} dx$.

[2 marks]

Solution:

Concept:

- The exponential and natural logarithm are inverse functions: $e^{\log u} = u$ for $u > 0$.
- Use substitution when a factor is the derivative of another: for $u = \cos x$, $du = -\sin x dx$.

Step 1: Simplify the exponential factor For the given real integrand, $\log \sin x$ is defined on intervals where $\sin x > 0$. On such an interval, $e^{\log \sin x} = \sin x$. Therefore the integral becomes

$$I = \int \cos^3 x \sin x dx.$$

The exponent on the cosine remains three; the logarithm-exponential simplification affects only its own factor.

Quick Tip to Solve: Simplify $e^{\log \sin x}$ first, then pair $\sin x dx$ with the derivative of $\cos x$.

Step 2: Substitute and integrate the power Put $u = \cos x$. Then $du = -\sin x \, dx$, so

$$I = - \int u^3 \, du = -\frac{u^4}{4} + C = -\frac{\cos^4 x}{4} + C.$$

To check the sign and power, differentiation gives

$$\frac{d}{dx} \left(-\frac{\cos^4 x}{4} \right) = \cos^3 x \sin x,$$

which equals the original integrand wherever its logarithm is defined.

Step 3: Conclusion $\int \cos^3 x e^{\log \sin x} \, dx = -\frac{\cos^4 x}{4} + C$, on each interval where $\sin x > 0$.

Quick Tip to Solve: Simplify $e^{\log \sin x}$ first, then pair $\sin x \, dx$ with the derivative of $\cos x$.

OR

24(b). Find: $\int \frac{1}{5 + 4x - x^2} \, dx$.

[2 marks]

Solution:

Concept:

- Completing the square rewrites $5 + 4x - x^2$ as $9 - (x - 2)^2$.
- For $a > 0$, $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \log \left| \frac{a+u}{a-u} \right| + C$; this also follows by partial fractions.

Step 1: Complete the square and change variable Write

$$5 + 4x - x^2 = 9 - (x^2 - 4x + 4) = 9 - (x - 2)^2.$$

Put $u = x - 2$, so $du = dx$. The required integral is therefore

$$I = \int \frac{du}{9 - u^2}.$$

Its denominator vanishes when $x = -1$ or $x = 5$, so an antiderivative is understood on an interval avoiding these points.

Step 2: Integrate by partial fractions Since $9 - u^2 = (3 - u)(3 + u)$,

$$\frac{1}{9 - u^2} = \frac{1}{6} \left(\frac{1}{3 + u} + \frac{1}{3 - u} \right).$$

Integrating the second term requires a minus sign because the derivative of $3 - u$ is -1 . Hence

$$I = \frac{1}{6} [\log |3 + u| - \log |3 - u|] + C.$$

Quick Tip to Solve: Factor the denominator as $(5 - x)(x + 1)$; remember the minus sign when integrating $1/(5 - x)$.

Substitute $u = x - 2$, obtaining $3 + u = x + 1$ and $3 - u = 5 - x$.

Step 3: Conclusion $\int \frac{dx}{5 + 4x - x^2} = \frac{1}{6} \log \left| \frac{x+1}{5-x} \right| + C$, for $x \neq -1, 5$.

Quick Tip to Solve: Factor the denominator as $(5 - x)(x + 1)$; remember the minus sign when integrating $1/(5 - x)$.

25. Find the vector equation of the line passing through the point $(2, 3, -5)$ and making equal angles with the co-ordinate axes. [2 marks]

Solution:

Concept:

- Direction cosines l, m, n are the cosines of the angles a line makes with the positive coordinate axes and satisfy $l^2 + m^2 + n^2 = 1$.
- A line through position vector $\vec{a}$ and parallel to nonzero vector $\vec{b}$ has equation $\vec{r} = \vec{a} + \lambda \vec{b}$, with $\lambda \in \mathbb{R}$.

Step 1: Find a direction vector from the equal angles Let the three equal angles be α . Their direction cosines are equal, so $l = m = n$. Using the direction-cosine identity,

$$3l^2 = 1, \quad l = m = n = \pm \frac{1}{\sqrt{3}}.$$

Thus a direction vector is proportional to $(1, 1, 1)$. Taking the common negative sign only reverses the direction along the same line, so it does not produce a different geometric line.

Step 2: Use the given point to form the equation The position vector of $(2, 3, -5)$ is $\vec{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}$. Choose $\vec{b} = \hat{i} + \hat{j} + \hat{k}$. Substitution in the vector equation gives

$$\vec{r} = (2\hat{i} + 3\hat{j} - 5\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k}).$$

At $\lambda = 0$ it passes through the specified point. Its normalized direction vector has three equal components $1/\sqrt{3}$, verifying the angle condition.

Step 3: Conclusion The required vector equation is $\vec{r} = 2\hat{i} + 3\hat{j} - 5\hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k}), \quad \lambda \in \mathbb{R}$.

Quick Tip to Solve: Equal angles give equal direction cosines, so use the simple direction ratios $1 : 1 : 1$.

Section C (18 marks)

There are 6 short answer questions in this section. Each is of 3 marks.

26(a). If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$.

[3 marks]

Solution:

Concept:

- Logarithmic differentiation converts a variable power into a product: $\log(u^v) = v \log u$, for $u > 0$. Work on a real interval where the logarithms exist.
- For dependent $y = y(x)$, use the chain rule $\frac{d}{dx} \log(\cos y) = -\tan y \frac{dy}{dx}$, along with the product rule.

Step 1: Take logarithms of the given relation The bases and exponents both depend on the variables, so direct use of the ordinary power rule would miss terms. Taking logarithms on both sides gives

$$y \log(\cos x) = x \log(\cos y).$$

This form allows us to differentiate each side as a product, keeping the derivative of the dependent variable explicit.

Step 2: Differentiate implicitly and collect derivative terms Write $y' = \frac{dy}{dx}$. Differentiating with respect to x , we obtain

$$y' \log(\cos x) - y \tan x = \log(\cos y) - x \tan y y'.$$

The minus signs arise because the derivative of $\cos x$ is $-\sin x$. Move the last term on the right to the left, and move $-y \tan x$ to the right:

$$y' [\log(\cos x) + x \tan y] = \log(\cos y) + y \tan x.$$

Finally divide by the coefficient of y' , wherever that coefficient is nonzero. At points where it vanishes the undivided implicit relation must be considered separately.

Step 3: Conclusion Hence,

$$\frac{dy}{dx} = \frac{\log(\cos y) + y \tan x}{\log(\cos x) + x \tan y}.$$

Quick Tip to Solve: Take logarithms before differentiating a variable raised to a variable power; retain both product-rule terms on each side.

26(b). If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

[3 marks]

Solution:**Concept:**

- Let $A = \sqrt{1-x^2}$ and $B = \sqrt{1-y^2}$. On the interior real domain $|x| < 1$, $|y| < 1$, both radicals are positive and differentiable.
- Use $\frac{dA}{dx} = -\frac{x}{A}$ and $\frac{dB}{dx} = -\frac{y}{B} \frac{dy}{dx}$. The original relation helps simplify the coefficient of the derivative.

Step 1: Obtain an identity from the original equation The given relation is $A + B = a(x-y)$. Multiply it by $B - A$. Since

$$(B + A)(B - A) = B^2 - A^2 = x^2 - y^2,$$

we get

$$a(x-y)(B-A) = (x-y)(x+y).$$

In the interior domain, $x = y$ would force $A + B = 0$, contradicting positivity. Thus cancellation gives $a(B-A) = x+y$, or equivalently

$$aB - y = aA + x.$$

Step 2: Differentiate and simplify using the identity With $y' = \frac{dy}{dx}$, differentiation of $A + B = a(x-y)$ gives

$$-\frac{x}{A} - \frac{y}{B} y' = a(1 - y').$$

Collecting derivative terms,

$$y' \left(a - \frac{y}{B} \right) = a + \frac{x}{A}.$$

Therefore

$$y' = \frac{B}{A} \frac{aA + x}{aB - y} = \frac{B}{A}.$$

The last equality uses the identity found above. Its common factor is nonzero: if both $aA + x = 0$ and $aB - y = 0$, then $x = -aA$, $y = aB$, and the original relation gives $A + B = -a^2(A + B)$, impossible for real a and $A + B > 0$.

Step 3: Conclusion Thus, at differentiable interior points,

$$\boxed{\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} = \sqrt{\frac{1-y^2}{1-x^2}}}.$$

Hence proved.

Quick Tip to Solve: Before simplifying the differentiated equation, multiply the original equation by the difference of its two radicals.

27. If $x = a \sin^3 \theta$, $y = b \cos^3 \theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

[3 marks]

Solution:

Concept:

- For a parametrically defined curve, $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$, provided $dx/d\theta \neq 0$.
- The second derivative requires another division by $dx/d\theta$: $\frac{d^2y}{dx^2} = \frac{d}{d\theta} \left(\frac{dy}{dx} \right) / \frac{dx}{d\theta}$. The constants a, b remain fixed.

Step 1: Find the first derivative Differentiate the two parametric equations separately:

$$\frac{dx}{d\theta} = 3a \sin^2 \theta \cos \theta, \quad \frac{dy}{d\theta} = -3b \cos^2 \theta \sin \theta.$$

Taking their ratio and cancelling the common factors near $\theta = \pi/4$,

$$\frac{dy}{dx} = -\frac{b \cos \theta}{a \sin \theta} = -\frac{b}{a} \cot \theta.$$

The negative sign comes from differentiating $\cos \theta$. We assume $a \neq 0$, as required for the derivative with respect to x at the specified parameter value.

Step 2: Differentiate again and evaluate Now differentiate the first derivative with respect to θ , then divide by $dx/d\theta$:

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{(b/a) \csc^2 \theta}{3a \sin^2 \theta \cos \theta} \\ &= \frac{b}{3a^2 \sin^4 \theta \cos \theta}. \end{aligned}$$

At $\theta = \pi/4$, $\sin \theta = \cos \theta = 1/\sqrt{2}$. Thus $\sin^4 \theta = 1/4$, and substitution gives

$$\left. \frac{d^2y}{dx^2} \right|_{\theta=\pi/4} = \frac{b}{3a^2(1/4)(1/\sqrt{2})}.$$

This uses the second-derivative chain rule; differentiating the slope with respect to the parameter alone would give a different quantity.

Step 3: Conclusion The required value is

$$\boxed{\left. \frac{d^2y}{dx^2} \right|_{\theta=\pi/4} = \frac{4\sqrt{2}b}{3a^2}}.$$

Quick Tip to Solve: For a parametric second derivative, divide by $dx/d\theta$ again after differentiating the first derivative with respect to θ .

28(a). Evaluate: $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx.$

[3 marks]

Solution:

Concept:

- The reflection property of definite integrals is $\int_0^a f(x) dx = \int_0^a f(a-x) dx$. Here the reflecting endpoint is $a = \pi$.
- Use $\cos(\pi - x) = -\cos x$. The original and reflected fractions have the same denominator and complementary numerators.

Step 1: Reflect the integral about the midpoint Let

$$I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx.$$

The denominator is strictly positive for every real x , so the integrand is continuous on the whole interval. Applying the reflection property and simplifying the cosine gives

$$\begin{aligned} I &= \int_0^{\pi} \frac{e^{\cos(\pi-x)}}{e^{\cos(\pi-x)} + e^{-\cos(\pi-x)}} dx \\ &= \int_0^{\pi} \frac{e^{-\cos x}}{e^{-\cos x} + e^{\cos x}} dx. \end{aligned}$$

The substitution reverses the integration limits initially, and its negative differential restores the displayed order.

Step 2: Add the two expressions for the same integral Add the original and reflected expressions. Since both integrals have the same limits, their integrands may be added:

$$\begin{aligned} 2I &= \int_0^{\pi} \left(\frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} + \frac{e^{-\cos x}}{e^{\cos x} + e^{-\cos x}} \right) dx \\ &= \int_0^{\pi} 1 dx = [x]_0^{\pi} = \pi. \end{aligned}$$

Thus the symmetry removes the exponential terms completely. Dividing by two gives the value of the original integral, without needing an antiderivative of its individual fraction.

Step 3: Conclusion Therefore,

$$\boxed{\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx = \frac{\pi}{2}}.$$

Quick Tip to Solve: When reflection changes the numerator into the other term of the denominator, add the reflected integral to the original one.

28(b). Find: $\int \frac{2x+1}{(x+1)^2(x-1)} dx.$

[3 marks]

Solution:

Concept:

- For a repeated linear factor, partial fractions must include both $1/(x+1)$ and $1/(x+1)^2$. Each distinct denominator receives its own constant coefficient.
- Use $\int dx/(x-a) = \ln|x-a| + C$ and $\int (x-a)^{-2} dx = -(x-a)^{-1} + C$. The absolute values make logarithms valid on each interval avoiding the poles.

Step 1: Decompose the rational function Write

$$\frac{2x+1}{(x+1)^2(x-1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{D}{(x+1)^2}.$$

Multiplication by the common denominator gives

$$2x+1 = A(x+1)^2 + B(x+1)(x-1) + D(x-1).$$

At $x = 1$, $3 = 4A$, so $A = 3/4$. At $x = -1$, $-1 = -2D$, so $D = 1/2$. Comparing the coefficients of x^2 gives $A + B = 0$, hence $B = -3/4$. These values also give the required linear and constant coefficients.

Step 2: Integrate and check the signs Therefore

$$\begin{aligned} I &= \frac{3}{4} \int \frac{dx}{x-1} - \frac{3}{4} \int \frac{dx}{x+1} + \frac{1}{2} \int \frac{dx}{(x+1)^2} \\ &= \frac{3}{4} \ln|x-1| - \frac{3}{4} \ln|x+1| - \frac{1}{2(x+1)} + C. \end{aligned}$$

Differentiating the last rational term gives $+1/[2(x+1)^2]$, which confirms its negative sign. Combining all three derivatives over the common denominator recovers the numerator $2x+1$.

Step 3: Conclusion $I = \frac{3}{4} \ln \left| \frac{x-1}{x+1} \right| - \frac{1}{2(x+1)} + C$, on intervals with $x \neq -1, 1$.

Quick Tip to Solve: For a repeated factor, include every power in the partial fractions; substitute its root before comparing coefficients.

29(a). Find the particular solution of the differential equation $\frac{dy}{dx} - 2xy = 3x^2e^{x^2}; y(0) = 5.$ [3 marks]

Solution:

Concept:

- A first-order linear equation $y' + P(x)y = Q(x)$ has integrating factor $e^{\int P(x)dx}$. Multiplication by this factor turns the left-hand side into a product derivative.

Quick Tip to Solve: Choose the integrating factor to cancel the exponential on the right, and apply the initial condition only after integration.

- The initial value determines the integration constant after integration; it must be applied to the complete solution, including the integrating factor.

Step 1: Find the integrating factor The equation is already in linear form:

$$\frac{dy}{dx} - 2xy = 3x^2e^{x^2}.$$

Here $P(x) = -2x$ and $Q(x) = 3x^2e^{x^2}$. Thus

$$\text{I.F.} = e^{\int -2x dx} = e^{-x^2}.$$

This integrating factor is positive for every real x , so multiplication by it does not discard any solution.

Step 2: Integrate the product derivative Multiply the equation by e^{-x^2} :

$$e^{-x^2} \frac{dy}{dx} - 2xe^{-x^2}y = 3x^2.$$

By the product rule, the expression on the left is $d(ye^{-x^2})/dx$. Consequently,

$$\frac{d}{dx}(ye^{-x^2}) = 3x^2 \implies ye^{-x^2} = x^3 + C.$$

Equivalently, $y = (x^3 + C)e^{x^2}$.

Step 3: Use the initial condition and verify At $x = 0$, the condition gives $5 = (0 + C)e^0 = C$. Hence the required function is $y = (x^3 + 5)e^{x^2}$. Its derivative is

$$y' = 3x^2e^{x^2} + 2x(x^3 + 5)e^{x^2}.$$

Subtracting $2xy$ leaves precisely $3x^2e^{x^2}$; also $y(0) = 5$, verifying both requirements.

Step 4: Conclusion The particular solution is $y = (x^3 + 5)e^{x^2}$.

Quick Tip to Solve: Choose the integrating factor to cancel the exponential on the right, and apply the initial condition only after integration.

OR

29(b). Solve the following differential equation: $x^2 dy + y(x + y) dx = 0$.

[3 marks]

Solution:

Concept:

- The equation is homogeneous because $dy/dx = -y/x - (y/x)^2$ depends only on y/x . Set $y = vx$, where v is a function of x , so $dy/dx = v + x dv/dx$.
- After separation, integrate using partial fractions. Any values divided out during separation must be checked separately for additional solutions.

Quick Tip to Solve: When separating a homogeneous equation, record constant values of the substituted variable before dividing by its factors.

Step 1: Make the homogeneous substitution For $x \neq 0$, divide the given equation by $x^2 dx$:

$$\frac{dy}{dx} = -\frac{y}{x} - \frac{y^2}{x^2}.$$

Putting $y = vx$ gives

$$v + x \frac{dv}{dx} = -v - v^2 \implies x \frac{dv}{dx} = -v(v+2).$$

Thus, when $v \neq 0, -2$, the variables separate as $dv/[v(v+2)] = -dx/x$.

Step 2: Integrate and return to the original variables Since

$$\frac{1}{v(v+2)} = \frac{1}{2} \left(\frac{1}{v} - \frac{1}{v+2} \right),$$

integration yields

$$\frac{1}{2} \ln \left| \frac{v}{v+2} \right| = -\ln|x| + K.$$

On an interval where the factors do not vanish, exponentiating and absorbing the sign into a constant gives

$$\frac{v}{v+2} = \frac{k}{x^2}, \quad \frac{y}{y+2x} = \frac{k}{x^2}.$$

Solving algebraically, $y(x^2 - k) = 2kx$. For $k \neq 0$, rename $1/k = C$ to obtain $y = 2x/(Cx^2 - 1)$.

Step 3: Recover divided-out solutions and verify The constant values $v = 0$ and $v = -2$ give $y = 0$ and $y = -2x$. Both satisfy the original equation; $y = -2x$ is already included by $C = 0$ in the displayed family. For that family,

$$y' = \frac{-2(Cx^2 + 1)}{(Cx^2 - 1)^2}, \quad -\frac{y}{x} - \frac{y^2}{x^2} = \frac{-2(Cx^2 + 1)}{(Cx^2 - 1)^2}.$$

Thus the family satisfies the equation wherever its denominator is nonzero.

Step 4: Conclusion The solutions are $y = \frac{2x}{Cx^2 - 1}$, with $Cx^2 \neq 1$, together with $y = 0$.

Quick Tip to Solve: When separating a homogeneous equation, record constant values of the substituted variable before dividing by its factors.

30. Find a vector of magnitude 4 units perpendicular to each of the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$ and hence verify your answer. [3 marks]

Solution:

Concept:

- The cross product $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$. Dividing a nonzero vector by its magnitude gives a unit vector in the same direction.
- A required vector of magnitude m is $\pm m(\vec{a} \times \vec{b})/|\vec{a} \times \vec{b}|$. Verify perpendicularity with two zero dot

Quick Tip to Solve: Compute a cross product, normalize it, then scale; check both dot products as well as the final magnitude.

products and verify magnitude separately.

Step 1: Compute a common perpendicular Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$. Their cross product is

$$\begin{aligned}\vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} \\ &= \hat{i}(1 - 1) - \hat{j}(-2 - 1) + \hat{k}(2 + 1) \\ &= 3\hat{j} + 3\hat{k}.\end{aligned}$$

It is nonzero, and its magnitude is $\sqrt{3^2 + 3^2} = 3\sqrt{2}$. The zero $\hat{i}$ -component is consistent with the equal and opposite second and third components of the given vectors.

Step 2: Scale to the specified magnitude A unit vector along this common perpendicular is

$$\frac{3\hat{j} + 3\hat{k}}{3\sqrt{2}} = \frac{\hat{j} + \hat{k}}{\sqrt{2}}.$$

Multiplying by 4 gives $\vec{c} = 2\sqrt{2}\hat{j} + 2\sqrt{2}\hat{k}$. Reversing its direction also preserves its magnitude and its perpendicularity, so $-\vec{c}$ is equally valid.

Step 3: Verify all the required conditions Compute the two dot products explicitly:

$$\begin{aligned}\vec{c} \cdot \vec{a} &= 0 \cdot 2 + (2\sqrt{2})(-1) + (2\sqrt{2})(1) = 0, \\ \vec{c} \cdot \vec{b} &= 0 \cdot 1 + (2\sqrt{2})(1) + (2\sqrt{2})(-1) = 0.\end{aligned}$$

Also $|\vec{c}| = \sqrt{0^2 + (2\sqrt{2})^2 + (2\sqrt{2})^2} = \sqrt{16} = 4$. Thus the vector has precisely the requested magnitude and is perpendicular to each given vector.

Step 4: Conclusion One required vector is $\boxed{2\sqrt{2}\hat{j} + 2\sqrt{2}\hat{k}}$ units; its negative is also a valid answer.

Quick Tip to Solve: Compute a cross product, normalize it, then scale; check both dot products as well as the final magnitude.

31. The random variable X has the following probability distribution where a and b are some constants:

X	1	2	3	4	5
$P(X)$	0.2	a	a	0.2	b

If the mean $E(X) = 3$, then find values of a and b and hence determine $P(X \geq 3)$.

[3 marks]

Solution:

Concept:

- The probabilities in a discrete distribution are non-negative and sum to one: $\sum_x P(X = x) = 1$.
- The mean is $E(X) = \sum_x xP(X = x)$. Add the probabilities of mutually exclusive values to find the probability of an event.

Quick Tip to Solve: Subtract the mean equation $a + b = 0.4$ from the normalization equation $2a + b = 0.6$ to isolate a immediately.

Step 1: Form the total-probability equation The values 1, 2, 3, 4, 5 exhaust all possibilities. Therefore,

$$0.2 + a + a + 0.2 + b = 1 \implies 2a + b = 0.6. \quad (1)$$

The probability a occurs twice, at $X = 2$ and $X = 3$, so both entries must be counted.

Step 2: Use the mean to determine the constants Multiply each value of X by its corresponding probability:

$$\begin{aligned} E(X) &= 1(0.2) + 2a + 3a + 4(0.2) + 5b \\ &= 1 + 5a + 5b = 3. \end{aligned}$$

Thus $a + b = 0.4$. Subtracting this from (1) yields $a = 0.2$, and substitution gives $b = 0.4 - 0.2 = 0.2$. All the probabilities are consequently non-negative and their sum is $5(0.2) = 1$. As a check, the mean is $0.2(1 + 2 + 3 + 4 + 5) = 3$, as required.

Step 3: Evaluate the required event probability The event $X \geq 3$ consists of the three mutually exclusive outcomes $X = 3, 4, 5$. In particular, equality includes $X = 3$. Hence

$$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5) = a + 0.2 + b = 0.6.$$

Alternatively, its complement comprises $X = 1, 2$, giving $1 - (0.2 + a) = 0.6$, which agrees.

Step 4: Conclusion $a = 0.2, \quad b = 0.2, \quad P(X \geq 3) = 0.6$.

Quick Tip to Solve: Subtract the mean equation $a + b = 0.4$ from the normalization equation $2a + b = 0.6$ to isolate a immediately.

Section D (20 marks)

There are 4 long answer questions in this section. Each question is of 5 marks.

32(a). If $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{pmatrix}$, then find A^{-1} and hence solve the following system of equations:

$$x + 2y - 3z = 1$$

$$2x - 3z = 2$$

$$x + 2y = 3$$

[5 marks]

Solution:

Concept:

- A square matrix A is invertible if $|A| \neq 0$, and $A^{-1} = \frac{\text{adj } A}{|A|}$.
- The cofactor $C_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is the minor obtained by deleting row i and column j . The adjugate is the transpose of the cofactor matrix.
- A linear system written as $AX = B$, with $X = (x, y, z)^T$, has the unique solution $X = A^{-1}B$ when A is invertible.

Quick Tip to Solve: Keep the cofactor matrix and its transpose separate, then multiply the inverse by the constants column in the same equation order.

Step 1: Calculate the determinant Expanding along the first row of the given matrix,

$$\begin{aligned}|A| &= 1\{0(0) - (-3)(2)\} - 2\{2(0) - (-3)(1)\} \\ &\quad + (-3)\{2(2) - 0(1)\} \\ &= 6 - 6 - 12 = -12.\end{aligned}$$

Since this is non-zero, the inverse exists and the corresponding system has a unique solution.

Step 2: Find all cofactors and transpose Applying the alternating cofactor signs gives

$$\begin{aligned}C_{11} &= 0(0) - (-3)(2) = 6, & C_{12} &= -\{2(0) - (-3)(1)\} = -3, \\ C_{13} &= 2(2) - 0(1) = 4, & C_{21} &= -\{2(0) - (-3)(2)\} = -6, \\ C_{22} &= 1(0) - (-3)(1) = 3, & C_{23} &= -\{1(2) - 2(1)\} = 0, \\ C_{31} &= 2(-3) - (-3)(0) = -6, & C_{32} &= -\{1(-3) - (-3)(2)\} = -3, \\ C_{33} &= 1(0) - 2(2) = -4.\end{aligned}$$

Thus

$$C = \begin{pmatrix} 6 & -3 & 4 \\ -6 & 3 & 0 \\ -6 & -3 & -4 \end{pmatrix}, \quad \text{adj } A = C^T = \begin{pmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{pmatrix}.$$

Step 3: Construct the inverse Divide every entry of the adjugate by the determinant:

$$A^{-1} = -\frac{1}{12} \begin{pmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \\ -\frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}.$$

The transpose in the adjugate formula is essential; dividing the cofactor matrix directly would generally give the wrong inverse.

Step 4: Solve using the inverse matrix The equations are $AX = B$, where $B = (1, 2, 3)^T$. Multiplying by A^{-1} gives

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \\ -\frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} + 1 + \frac{3}{2} \\ \frac{1}{4} - \frac{1}{2} + \frac{3}{4} \\ -\frac{1}{3} + 1 \end{pmatrix} = \begin{pmatrix} 2 \\ \frac{1}{2} \\ \frac{2}{3} \end{pmatrix}.$$

Substitution verifies each equation: $2 + 2(\frac{1}{2}) - 3(\frac{2}{3}) = 1$, $2(2) - 3(\frac{2}{3}) = 2$, and $2 + 2(\frac{1}{2}) = 3$.

Step 5: Conclusion

$$A^{-1} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \\ -\frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}, \quad \boxed{x = 2, \quad y = \frac{1}{2}, \quad z = \frac{2}{3}}.$$

Quick Tip to Solve: Keep the cofactor matrix and its transpose separate, then multiply the inverse by the constants column in the same equation order.

32(b). Find the product of the matrices $\begin{pmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{pmatrix} \begin{pmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{pmatrix}$ and hence solve the system

of linear equations:

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

[5 marks]

Solution:

Concept:

- Each entry of a matrix product is the scalar product of a row of the first matrix with a column of the second matrix.
- If square matrices satisfy $AB = kI$, where $k \neq 0$, then $A^{-1} = B/k$. A system $AX = C$ is solved by $X = A^{-1}C$.

Step 1: Calculate the first row of the product Let the first printed matrix be A and the second be B . Multiplying the first row of A by each column of B gives

$$(AB)_{11} = -6 + 28 + 45 = 67,$$

$$(AB)_{12} = 17 + 10 - 27 = 0,$$

$$(AB)_{13} = 13 - 16 + 3 = 0.$$

The order of the factors matters, so we calculate the product in the order stated in the question.

Step 2: Complete the matrix product For the second and third rows, the same row-by-column rule gives

$$(AB)_{21} = -12 + 42 - 30 = 0,$$

$$(AB)_{22} = 34 + 15 + 18 = 67,$$

$$(AB)_{23} = 26 - 24 - 2 = 0,$$

$$(AB)_{31} = -18 - 42 + 60 = 0,$$

$$(AB)_{32} = 51 - 15 - 36 = 0,$$

$$(AB)_{33} = 39 + 24 + 4 = 67.$$

Thus $AB = \begin{pmatrix} 67 & 0 & 0 \\ 0 & 67 & 0 \\ 0 & 0 & 67 \end{pmatrix} = 67I$. Since 67 is nonzero, A is invertible and $A^{-1} = B/67$.

Step 3: Write and solve the matrix equation The coefficients of the three equations form exactly the matrix A . Therefore

$$AX = C, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad C = \begin{pmatrix} -4 \\ 2 \\ 11 \end{pmatrix}.$$

Quick Tip to Solve: When a supplied matrix product is a nonzero scalar times the identity, divide the second matrix by that scalar to obtain the inverse directly.

Multiplying on the left by A^{-1} ,

$$\begin{aligned} X &= \frac{1}{67} BC \\ &= \frac{1}{67} \begin{pmatrix} 24 + 34 + 143 \\ -56 + 10 - 88 \\ 60 + 18 - 11 \end{pmatrix} \\ &= \frac{1}{67} \begin{pmatrix} 201 \\ -134 \\ 67 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}. \end{aligned}$$

The inverse guarantees that this solution is unique.

Step 4: Verify in all three equations Substitute the coordinates:

$$\begin{aligned} 3 + 2(-2) - 3(1) &= -4, \\ 2(3) + 3(-2) + 2(1) &= 2, \\ 3(3) - 3(-2) - 4(1) &= 11. \end{aligned}$$

All three right-hand sides agree, confirming both the variable order and the negative sign of the second coordinate.

Step 5: Conclusion The product is $\boxed{67I}$; hence $\boxed{x = 3, y = -2, z = 1}$.

Quick Tip to Solve: When a supplied matrix product is a nonzero scalar times the identity, divide the second matrix by that scalar to obtain the inverse directly.

33. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

[5 marks]

Solution:

Concept:

- The curve $4x^2 + y^2 = 36$ is the ellipse $x^2/9 + y^2/36 = 1$, symmetric about both coordinate axes.
- The area under a nonnegative ordinate $y = f(x)$ between $x = a$ and $x = b$ is $\int_a^b f(x) dx$. Symmetry gives the total enclosed area as four times its first-quadrant area.

Step 1: Identify the region and limits Setting $y = 0$ gives $x = \pm 3$, while setting $x = 0$ gives $y = \pm 6$. In the first quadrant the upper boundary is

$$y = \sqrt{36 - 4x^2} = 2\sqrt{9 - x^2}, \quad 0 \leq x \leq 3.$$

The lower boundary of this quarter of the enclosed region is the x -axis. Its right endpoint is $(3, 0)$. The four quarters have equal area because the equation involves only squares of both coordinates.

Step 2: Set up the definite integral Let S denote the total enclosed area. A vertical strip in the first quadrant has height $2\sqrt{9 - x^2}$ and width dx . Summing these strips by integration and then using

Quick Tip to Solve: For an ellipse symmetric about both axes, integrate its positive ordinate over the first quadrant and multiply by four; keep the ordinate factor separate.

symmetry gives

$$S = 4 \int_0^3 2\sqrt{9-x^2} dx = 8 \int_0^3 \sqrt{9-x^2} dx.$$

The factor four accounts for all four quadrants; the additional factor two comes from the ordinate, not from symmetry. The integrand is nonnegative throughout the interval, so this integral represents geometric area directly.

Step 3: Evaluate using a trigonometric substitution Put $x = 3 \sin \theta$. Then $dx = 3 \cos \theta d\theta$, and θ increases from 0 to $\pi/2$ as x increases from 0 to 3. On this interval $\cos \theta \geq 0$, hence $\sqrt{9-x^2} = 3 \cos \theta$. Therefore

$$\begin{aligned} S &= 72 \int_0^{\pi/2} \cos^2 \theta d\theta \\ &= 36 \int_0^{\pi/2} (1 + \cos 2\theta) d\theta \\ &= 36 \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\pi/2} \\ &= 36 \left(\frac{\pi}{2} + 0 - 0 \right) = 18\pi. \end{aligned}$$

Here the identity $\cos^2 \theta = (1 + \cos 2\theta)/2$ reduces the integral to elementary terms. The substitution covers exactly the first-quadrant interval, with no sign change in the square root.

Step 4: Check the scale of the area The ellipse has horizontal semi-axis 3 and vertical semi-axis 6. The value obtained by integration agrees with $\pi(3)(6)$. It is also smaller than the area $6 \times 12 = 72$ of the enclosing rectangle, as it should be. This provides an independent check on the symmetry factor and the integration limits.

Step 5: Conclusion The area enclosed by $4x^2 + y^2 = 36$ is 18π square units.

Quick Tip to Solve: For an ellipse symmetric about both axes, integrate its positive ordinate over the first quadrant and multiply by four; keep the ordinate factor separate.

34(a). Find the co-ordinates of the foot of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also, find the perpendicular distance of the given point from the line. [5 marks]

Solution:

Concept:

- A general point on a line is $\vec{a} + t\vec{b}$, where $\vec{a}$ locates a fixed point and $\vec{b}$ is a direction vector.
- For the perpendicular foot H from P , the condition is $\overrightarrow{PH} \cdot \vec{b} = 0$. Its length gives the distance from the point to the line.

Step 1: Parameterise the line Put all three ratios equal to t :

$$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3} = t.$$

Solving for the individual coordinates gives $x = 4 - 2t$, $y = 6t$ and $z = 1 - 3t$. Thus a general

Quick Tip to Solve: First solve the ratios for coordinates; this prevents replacing the correct direction vector $(-2, 6, -3)$ by $(2, 6, 3)$.

point is $H = (4 - 2t, 6t, 1 - 3t)$, with direction vector $\vec{b} = (-2, 6, -3)$. The first and third signs are negative because their printed numerators are $4 - x$ and $1 - z$. At $t = 0$, the fixed point on the line is $A = (4, 0, 1)$.

Step 2: Locate the perpendicular foot For the given point $P = (2, 3, -8)$,

$$\overrightarrow{PH} = (2 - 2t, 6t - 3, 9 - 3t).$$

Perpendicularity requires

$$\begin{aligned} -2(2 - 2t) + 6(6t - 3) - 3(9 - 3t) &= 0, \\ -4 + 4t + 36t - 18 - 27 + 9t &= 0, \\ 49t - 49 &= 0. \end{aligned}$$

Hence $t = 1$, so the point is $H = (2, 6, -2)$. Substitution into the printed ratios gives $(4 - 2)/2 = 6/6 = (1 + 2)/3 = 1$, verifying that this point belongs to the line.

Step 3: Find and verify the distance At $t = 1$, the joining vector is $\overrightarrow{PH} = (0, 3, 6)$, whose dot product with $(-2, 6, -3)$ is $0 + 18 - 18 = 0$. Therefore the segment is perpendicular, as required. The distance formula yields

$$PH = \sqrt{(2 - 2)^2 + (6 - 3)^2 + (-2 + 8)^2} = \sqrt{45} = 3\sqrt{5}.$$

For any other point Q on the line, triangle PHQ is right-angled at H , and $PQ^2 = PH^2 + HQ^2$. Consequently this perpendicular length is indeed the minimum distance from P to the line, not merely a distance to an arbitrary point.

Step 4: Conclusion The foot is $(2, 6, -2)$, and the perpendicular distance is $3\sqrt{5}$ units.

Quick Tip to Solve: First solve the ratios for coordinates; this prevents replacing the correct direction vector $(-2, 6, -3)$ by $(2, 6, 3)$.

OR

34(b). Find the shortest distance between the lines L_1 and L_2 given below:

L_1 : The line passing through $(2, -1, 1)$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$.

L_2 : $\vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k}$.

[5 marks]

Solution:

Concept:

- For nonparallel lines $\vec{r} = \vec{a}_1 + t\vec{b}_1$ and $\vec{r} = \vec{a}_2 + s\vec{b}_2$, the shortest distance is $d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$.
- The cross product gives a common normal to both directions. Projecting a vector joining the lines onto its unit vector gives their perpendicular separation.

Step 1: Extract points and directions The line L_1 passes through $(2, -1, 1)$ and is parallel to $x/1 = y/1 = z/3$. Hence choose

$$\vec{a}_1 = (2, -1, 1), \quad \vec{b}_1 = (1, 1, 3).$$

Quick Tip to Solve: Expand the second vector equation into a fixed vector and a parameter vector before computing the cross product; its direction is $(0, 2, -1)$.

The second line is

$$\vec{r} = \hat{i} + (2\mu + 1)\hat{j} - (\mu + 2)\hat{k} = (\hat{i} + \hat{j} - 2\hat{k}) + \mu(2\hat{j} - \hat{k}).$$

Its fixed point and direction vector are therefore $\vec{a}_2 = (1, 1, -2)$ and $\vec{b}_2 = (0, 2, -1)$. In particular, its x -coordinate is constantly 1, so the first direction component is zero.

Step 2: Compute the common normal Taking the cross product in the same order as the distance formula,

$$\begin{aligned}\vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 3 \\ 0 & 2 & -1 \end{vmatrix} \\ &= (-1 - 6)\hat{i} - (-1 - 0)\hat{j} + (2 - 0)\hat{k} \\ &= -7\hat{i} + \hat{j} + 2\hat{k}.\end{aligned}$$

Its magnitude is $\sqrt{49 + 1 + 4} = \sqrt{54} = 3\sqrt{6}$. The cross product is nonzero, so the lines are not parallel and the nonparallel-lines formula is applicable. As a check, $(-7, 1, 2) \cdot (1, 1, 3) = 0$ and $(-7, 1, 2) \cdot (0, 2, -1) = 0$.

Step 3: Project the joining vector onto the normal The vector joining the chosen fixed points is

$$\vec{a}_2 - \vec{a}_1 = (1 - 2, 1 + 1, -2 - 1) = (-1, 2, -3).$$

Its scalar product with the normal is

$$(-1, 2, -3) \cdot (-7, 1, 2) = 7 + 2 - 6 = 3.$$

Hence the perpendicular separation is

$$d = \frac{|3|}{3\sqrt{6}} = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}.$$

The nonzero scalar triple product also establishes that the lines do not intersect; combined with nonparallel directions, this confirms they are skew. The absolute value in the formula ensures that reversing the normal or the joining vector cannot change the positive distance.

Step 4: Conclusion The shortest distance between L_1 and L_2 is $\boxed{\frac{1}{\sqrt{6}} \text{ units}}$, equivalently $\boxed{\frac{\sqrt{6}}{6} \text{ units}}$.

Quick Tip to Solve: Expand the second vector equation into a fixed vector and a parameter vector before computing the cross product; its direction is $(0, 2, -1)$.

35. Solve the following L.P.P. graphically:

Maximise $Z = 60x + 40y$

Subject to $x + 2y \leq 12$

$2x + y \leq 12$

$4x + 5y \geq 20$

$x, y \geq 0$.

[5 marks]

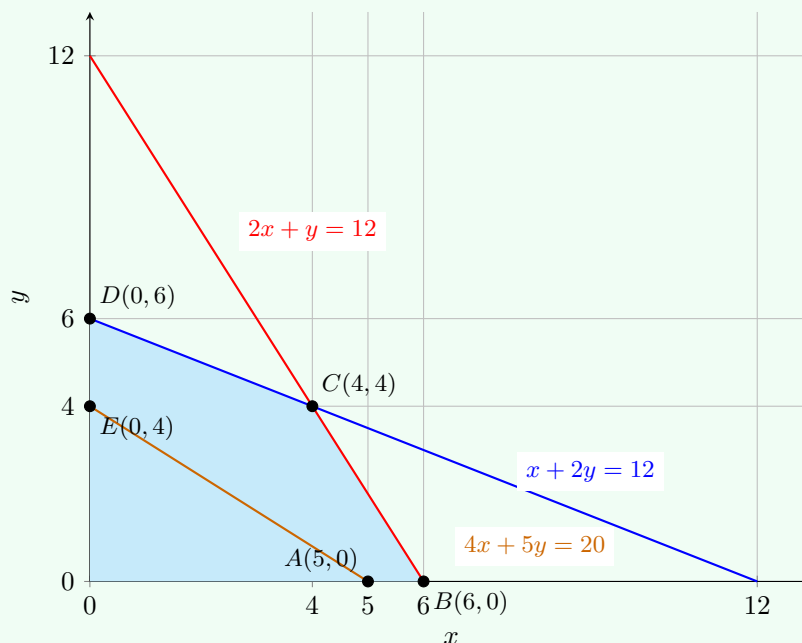
Solution:

Concept:

- The boundary lines divide the plane into half-planes. Their common region with $x, y \geq 0$ is the feasible region.
- For a linear objective $Z = 60x + 40y$, a maximum on a nonempty bounded polygon occurs at a vertex. Here x and y are the decision variables.

Step 1: Draw the three boundary lines The line $x + 2y = 12$ passes through $(12, 0)$ and $(0, 6)$. The line $2x + y = 12$ passes through $(6, 0)$ and $(0, 12)$. The line $4x + 5y = 20$ passes through $(5, 0)$ and $(0, 4)$. Plot these intercepts and join the respective pairs. Restrict attention to the first quadrant, since both variables must be nonnegative.

Step 2: Identify and shade the feasible region The origin satisfies each of the first two inequalities, so take the sides of their boundary lines containing the origin. It fails $4x + 5y \geq 20$, so take the opposite side of the third line. Their common region is the closed pentagon $ABCDE$, including its edges. The lower intercepts are $A = (5, 0)$ and $E = (0, 4)$; the upper intercepts are $B = (6, 0)$ and $D = (0, 6)$. This region is bounded, and all five vertices satisfy every constraint.



The shaded pentagon $ABCDE$ is the feasible region.

Step 3: Find the intersection of the upper boundary lines At vertex C , solve the two equations

Quick Tip to Solve: Plot the intercepts first; the constraint with the greater-than sign removes the origin and creates two extra vertices on the axes.

simultaneously:

$$x + 2y = 12,$$

$$2x + y = 12.$$

Subtracting gives $y = x$, and substitution gives $3x = 12$. Thus $C = (4, 4)$. The third constraint is satisfied here because $4(4) + 5(4) = 36 \geq 20$. Its intersections with the two upper lines outside the first quadrant do not add feasible vertices.

Step 4: Evaluate the objective at every vertex Using $Z = 60x + 40y$, the corner values are

$$Z(A) = 60(5) + 40(0) = 300,$$

$$Z(B) = 60(6) + 40(0) = 360,$$

$$Z(C) = 60(4) + 40(4) = 400,$$

$$Z(D) = 60(0) + 40(6) = 240,$$

$$Z(E) = 60(0) + 40(4) = 160.$$

Comparing all five values applies the corner-point method to the complete feasible polygon. The largest value occurs only at C ; no boundary edge joins two vertices having that same value.

Step 5: Conclusion $Z_{\max} = 400$ at $x = y = 4$.

Quick Tip to Solve: Plot the intercepts first; the constraint with the greater-than sign removes the origin and creates two extra vertices on the axes.

Section E (12 marks)

In this section there are 3 case study questions of 4 marks each.

36. Students of a school are taken to a railway museum to learn about railways heritage and its history.

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by $R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$.

On the basis of the above information, answer the following questions:

[4 marks]

36(a)(i). Find whether the relation R is symmetric or not.

[1 mark]

Solution:

Concept:

- A relation R on L is symmetric if $(l_1, l_2) \in R$ implies $(l_2, l_1) \in R$ for every pair of elements.
- Here membership in R means that the two rail lines have parallel directions.

Step 1: Take an arbitrary related pair Let $(l_1, l_2) \in R$. By the given definition, l_1 is parallel to l_2 . The lines are arbitrary members of L , so the argument is not restricted to any one pair shown at the museum.

Step 2: Reverse the order of the lines Parallelism is unchanged on reversing the order: if $l_1 \parallel l_2$, then $l_2 \parallel l_1$. Consequently $(l_2, l_1) \in R$. This proves the required implication for every related pair; it does

Quick Tip to Solve: To test symmetry, reverse a related pair and check that the defining property still holds.

not depend on whether unrelated lines occur elsewhere in L .

Step 3: Conclusion R is symmetric.

Quick Tip to Solve: To test symmetry, reverse a related pair and check that the defining property still holds.

36(a)(ii). Find whether the relation R is transitive or not.

[1 mark]

Solution:

Concept:

- A relation is transitive when $(l_1, l_2) \in R$ and $(l_2, l_3) \in R$ imply $(l_1, l_3) \in R$.
- Parallel rail lines have the same direction; lines parallel to a common line are parallel to one another.

Step 1: Start with two linked pairs Let $(l_1, l_2) \in R$ and $(l_2, l_3) \in R$, where the three rail lines belong to L . Thus $l_1 \parallel l_2$ and $l_2 \parallel l_3$. Both hypotheses are required in the definition of transitivity.

Step 2: Use the common direction Since each of l_1 and l_3 has the direction of l_2 , they have the same direction as each other. Hence $l_1 \parallel l_3$, which means $(l_1, l_3) \in R$. The implication holds for arbitrary rail lines satisfying the hypotheses, establishing transitivity.

Step 3: Conclusion R is transitive.

Quick Tip to Solve: To test transitivity, use the shared middle line to connect the first and third lines.

36(a)(iii). If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

[2 marks]

Solution:

Concept:

- A nonvertical straight line $y = mx + c$ has slope m and vertical intercept c .
- Parallel lines have equal slopes. The required set contains only lines belonging to the given set L .

Step 1: Find the direction of the specified line Compare $y = 3x + 2$ with the slope-intercept form $y = mx + c$. Its slope is $m = 3$, and its vertical intercept is 2. A rail line related to this line by R must be parallel to it. Therefore its slope must also be 3; the intercept alone does not determine the direction of a line.

Step 2: Describe all related rail lines Write a candidate line as $l_c : y = 3x + c$, where $c \in \mathbb{R}$. Changing c shifts the line vertically without changing its slope, so each such line has the required parallel direction. Conversely, every nonvertical line parallel to $y = 3x + 2$ has slope 3, and hence has this form. Because the relation is on L , retain precisely those members of this family which actually belong to L . Thus the set is a family of lines, rather than a set of points on the original line.

Step 3: Conclusion The set of related rail lines is $\{l_c \in L : l_c \text{ has equation } y = 3x + c, c \in \mathbb{R}\}$.

Quick Tip to Solve: Keep the coefficient of x fixed to preserve parallelism; vary the intercept and retain the lines in L .

36(b). Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$, check whether the relation S is symmetric and transitive. [4 marks]

Solution:

Concept:

- Symmetry of S requires $(l_1, l_2) \in S \Rightarrow (l_2, l_1) \in S$ for every related pair.
- Transitivity requires $(l_1, l_2) \in S$ and $(l_2, l_3) \in S$ to imply $(l_1, l_3) \in S$. One counterexample disproves this universal implication.
- Perpendicular lines meet at a right angle. Two distinct coplanar lines perpendicular to the same line are parallel, rather than perpendicular to each other.

Step 1: Test symmetry using an arbitrary pair Suppose $(l_1, l_2) \in S$. The definition gives $l_1 \perp l_2$, so the angle between the lines is 90° . Interchanging their order does not change that angle. Hence $l_2 \perp l_1$, and therefore $(l_2, l_1) \in S$. This proves the required reverse-pair implication for every pair in the relation. Thus the relation satisfies symmetry.

Step 2: Construct linked pairs for the transitivity test Consider the straight lines $l_1 : y = 0$, $l_2 : x = 0$, and $l_3 : y = 1$. The first and third are horizontal, while the middle line is vertical. The vertical line intersects each horizontal line at a right angle. Consequently $l_1 \perp l_2$ and $l_2 \perp l_3$, which give both hypotheses $(l_1, l_2) \in S$ and $(l_2, l_3) \in S$. Thus these three lines provide a direct test of the implication, rather than merely a pair of unrelated lines.

Step 3: Check the first and third lines The lines $y = 0$ and $y = 1$ have the same direction and are distinct. They are parallel and do not meet, so they are not perpendicular. Therefore $(l_1, l_3) \notin S$, despite both linked pairs belonging to S . The required implication fails, proving that S is not transitive. In particular, perpendicularity to a common line does not pass perpendicularity on to the two outer lines; it gives them parallel directions in this example.

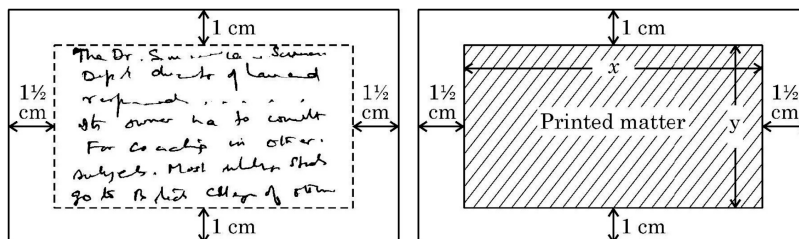
Step 4: Conclusion S is symmetric but not transitive.

Quick Tip to Solve: For transitivity of perpendicularity, test two horizontal lines with one shared vertical line.

37. A rectangular visiting card is to contain 24 sq.cm of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be $1\frac{1}{2}$ cm as shown below:

On the basis of the above information, answer the following questions:

[4 marks]



37(i). Write the expression for the area of the visiting card in terms of x .

[2 marks]

Solution:

Concept:

- The area of a rectangle is the product of its length and breadth. Here x and y denote the horizontal and vertical dimensions, in centimetres, of the printed region.
- The printed area is fixed, so $xy = 24$, with $x > 0$ and $y > 0$. The card dimensions include the margins on both sides.

Step 1: Express the printed height The figure labels the width of the printed matter as x and its height as y . Therefore

$$xy = 24 \implies y = \frac{24}{x}.$$

The variable x is the width of the printed region, rather than the full width of the card.

Step 2: Include all four margins The left and right margins each measure $1\frac{1}{2}$ cm, so the full width is $x + 3$ cm. The top and bottom margins each measure 1 cm, so the full height is $y + 2$ cm. Thus the card area is

$$A = (x + 3)(y + 2) = (x + 3) \left(\frac{24}{x} + 2 \right).$$

Expanding the product separates the fixed printed area, the margin contributions and the corner areas:

$$A = 24 + 2x + \frac{72}{x} + 6.$$

Step 3: Conclusion The area of the visiting card is

$$A(x) = 2x + 30 + \frac{72}{x} \text{ cm}^2, \quad x > 0.$$

Quick Tip to Solve: Use the printed width for x ; add both horizontal margins before multiplying by the full height.

37(ii). Obtain the dimensions of the card of minimum area.

[2 marks]

Solution:

Concept:

- For a differentiable area function, an interior minimum can occur where $A'(x) = 0$. A positive second derivative confirms a strict minimum.
- The printed dimensions satisfy $xy = 24$; the complete card measures $(x + 3)$ cm by $(y + 2)$ cm.

Step 1: Form and differentiate the area From the printed area, $y = 24/x$, where $x > 0$. Including the margins gives

$$A(x) = (x + 3) \left(\frac{24}{x} + 2 \right) = 2x + 30 + \frac{72}{x}.$$

Hence

$$A'(x) = 2 - \frac{72}{x^2}.$$

Setting this equal to zero gives $2x^2 = 72$, or $x^2 = 36$. Since a dimension is positive, the admissible critical value is $x = 6$ cm.

Step 2: Verify the minimum and find the full card dimensions The second derivative is

$$A''(x) = \frac{144}{x^3} > 0 \quad (x > 0).$$

Thus the critical point is a strict minimum. In fact the area decreases for $0 < x < 6$ and increases for $x > 6$, so it is the minimum over the entire physical domain. At $x = 6$, the printed height is $y = 24/6 = 4$ cm. Adding the left and right margins gives the full width $6 + 3 = 9$ cm, and adding the top and bottom margins gives the full height $4 + 2 = 6$ cm. The resulting area is 54 cm^2 .

Step 3: Conclusion The card of minimum area has dimensions

$9 \text{ cm} \times 6 \text{ cm}.$

Quick Tip to Solve: After minimizing in the printed dimensions, add the margins to report the dimensions of the card.

38. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions:

[4 marks]

38(i). Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time. Find $P(E_1)$, $P(E_2)$.

[1 mark]

Solution:

Concept:

- An event and its complement have probabilities adding to one: $P(E) + P(E^c) = 1$.
- Convert a percentage to a probability by dividing it by 100.

Step 1: Read the first-month payment probability The statement says that 70% of customers pay their first month bill in time. Since E_1 denotes exactly this event,

$$P(E_1) = \frac{70}{100} = 0.7.$$

Step 2: Use the complementary event For the same customer and the same first month, paying in time and not paying in time are mutually exclusive and exhaustive. Thus $E_2 = E_1^c$, so

$$P(E_2) = 1 - P(E_1) = 1 - 0.7 = 0.3.$$

The next-month transition probabilities are unnecessary for these first-month events.

Step 3: Conclusion $P(E_1) = 0.7, \quad P(E_2) = 0.3.$

Quick Tip to Solve: First identify the month of the event; the first-month probabilities come directly from 70 percent.

38(ii). Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$.

[1 mark]

Solution:

Concept:

- $P(A | E)$ is the probability of event A when event E is known to have occurred.
- Here A refers to timely payment in the second month, while E_1 and E_2 describe the first month.

Step 1: Condition on timely first-month payment The store found that a customer who pays in time has probability 0.8 of paying in time the next month. When the first-month event E_1 is known, this is precisely

$$P(A | E_1) = 0.8.$$

Step 2: Condition on late first-month payment A customer who does not pay in time has probability

Quick Tip to Solve: Read the phrase after given as the first-month condition; do not multiply when only a conditional probability is asked.

0.4 of paying in time the next month. Conditioning on E_2 therefore gives

$$P(A \mid E_2) = 0.4.$$

These are conditional probabilities already supplied in the statement; multiplying by the first-month probabilities would instead produce joint probabilities.

Step 3: Conclusion $P(A \mid E_1) = 0.8, \quad P(A \mid E_2) = 0.4.$

Quick Tip to Solve: Read the phrase after given as the first-month condition; do not multiply when only a conditional probability is asked.

38(iii)(a). Find the probability of customer paying second month's bill in time.

[2 marks]

Solution:

Concept:

- The law of total probability states that $P(A) = P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)$ for mutually exclusive, exhaustive events E_1, E_2 .
- Here A is timely second-month payment, E_1 is timely first-month payment and E_2 is its complement.

Step 1: Identify the two possible first-month histories The first-month probabilities are $P(E_1) = 0.7$ and $P(E_2) = 0.3$. The respective second-month conditional probabilities are $P(A \mid E_1) = 0.8$ and $P(A \mid E_2) = 0.4$. Every timely second-month payment occurs through one of these two disjoint histories.

Step 2: Add the probabilities of the disjoint routes Using the multiplication rule,

$$P(A \cap E_1) = 0.7 \times 0.8 = 0.56, \quad P(A \cap E_2) = 0.3 \times 0.4 = 0.12.$$

The total probability is therefore

$$\begin{aligned} P(A) &= P(A \cap E_1) + P(A \cap E_2) \\ &= 0.56 + 0.12 = 0.68. \end{aligned}$$

This is a weighted average of the two conditional probabilities, with weights determined by the first-month payment pattern; simply averaging 0.8 and 0.4 would ignore the unequal groups.

Step 3: Conclusion The probability of paying the second month bill in time is $\boxed{0.68}$.

Quick Tip to Solve: Multiply each next-month probability by its first-month group probability, then add the two routes.

38(iii)(b). Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time. [2 marks]

Solution:

Concept:

- Let E_1 be the event that the first month's bill is paid in time, E_2 its complement, and A the event that the second month's bill is paid in time.
- Bayes' theorem gives $P(E_1 | A) = \frac{P(E_1)P(A | E_1)}{P(E_1)P(A | E_1) + P(E_2)P(A | E_2)}$, where the denominator includes both possible first-month histories.

Step 1: Record the given probabilities The two first-month events are mutually exclusive and exhaustive. From the given information,

$$P(E_1) = 0.7, \quad P(E_2) = 1 - 0.7 = 0.3, \quad P(A | E_1) = 0.8, \quad P(A | E_2) = 0.4.$$

We require $P(E_1 | A)$, since payment in the second month is already known. This reverses the conditioning in the given value $P(A | E_1)$.

Step 2: Apply total probability and Bayes' theorem The probability of timely payment in both months is $P(E_1 \cap A) = 0.7 \times 0.8 = 0.56$. Customers who missed timely payment in the first month contribute $0.3 \times 0.4 = 0.12$ to second-month timely payments. Hence,

$$P(A) = 0.56 + 0.12 = 0.68,$$

$$P(E_1 | A) = \frac{P(E_1 \cap A)}{P(A)} = \frac{0.56}{0.68} = \frac{56}{68} = \frac{14}{17}.$$

Step 3: Conclusion The required probability is $\boxed{\frac{14}{17}}$.

Quick Tip to Solve: Identify the known event after the word "if": second-month payment belongs in the conditioning denominator, so use Bayes' theorem.