

CBSE Class 12 Applied Mathematics 2024 - Set 465



Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains 38 questions. All questions are compulsory.
- (ii) The question paper is divided into five sections: A, B, C, D and E.
- (iii) Section A: Questions 1–18 are multiple choice questions and Questions 19–20 are assertion–reason questions, carrying 1 mark each.
- (iv) Section B: Questions 21–25 are very short answer questions, carrying 2 marks each.
- (v) Section C: Questions 26–31 are short answer questions, carrying 3 marks each.
- (vi) Section D: Questions 32–35 are long answer questions, carrying 5 marks each.
- (vii) Section E: Questions 36–38 are case-study questions, carrying 4 marks each.
- (viii) There is no overall choice. Internal choice is provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.
- (ix) Use of calculators is not allowed.

Section A

1. In a 1 km race, player P beats player Q by 18 metres or 9 seconds. What is P's time to complete the race?

- (A) 512 seconds (B) 502 seconds
(C) 491 seconds (D) 481 seconds

2. If $x > y$ and $z < 0$, then :

- (A) $xz > yz$ (B) $xz \geq yz$
(C) $\frac{x}{z} > \frac{y}{z}$ (D) $\frac{x}{z} < \frac{y}{z}$

3. If $AB = A$ and $BA = B$, then $(B^2 + B)$ equals :

- (A) $2A$ (B) O
(C) $2I$ (D) $2B$

4. The value of $\Delta = \begin{vmatrix} 42 & 2 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$ is :

- (A) 0 (B) 1
(C) -3 (D) -15

5. If $y = e^{-2x}$, then $\frac{d^3y}{dx^3}$ is equal to :

- (A) $2e^{-2x}$ (B) e^{-4x}
(C) $4e^{-4x}$ (D) $-8e^{-2x}$

6. The function $f(x) = x^2 - x + 1$ is :

- (A) increasing in $(0, 1)$ (B) decreasing in $(0, 1)$
(C) increasing in $(0, \frac{1}{2})$ and decreasing in $(\frac{1}{2}, 1)$ (D) increasing in $(\frac{1}{2}, 1)$ and decreasing in $(0, \frac{1}{2})$

7. The order and the degree of the differential equation

$$y dx + x \log\left(\frac{y}{x}\right) dy - 2x dy = 0$$

are respectively :

- (A) 1, 1 (B) 1, 2
(C) 2, 1 (D) 1, not defined

8. A fair coin is tossed twice and outcomes are noted. If the random variable X represents the number of heads that appeared in the experiment, then the mathematical expectation of X is :

- (A) 1 (B) $\frac{1}{2}$
(C) $\frac{1}{4}$ (D) $1\frac{1}{2}$

9. What time will it be after 1275 hours, if the present time is 9:00 p.m. ?

- (A) 11 p.m. (B) 12 p.m.
(C) 9 p.m. (D) 9 a.m.

10. If for a Poisson variate X, $P(X = k) = P(X = k + 1)$, then the variance of X is :

- (A) $k - 1$ (B) k
(C) $k + 1$ (D) $k + 2$

11. If the calculated value of $|t| < t_\nu(\alpha)$ (critical value of t), then the null hypothesis :

- (A) is rejected. (B) is accepted.
(C) is neither accepted nor rejected. (D) cannot be determined.

12. For testing the significance of difference between the means of two independent samples, the degree of freedom (ν) is taken as :

- (A) $n_1 - n_2 + 2$ (B) $n_1 - n_2 - 2$
(C) $n_1 + n_2 - 2$ (D) $n_1 + n_2 + 2$

13. For the given values 23, 32, 40, 47, 58, 33, 42; the 5-yearly moving averages are :

- (A) 38, 40, 42 (B) 40, 42, 44
(C) 40, 42, 46 (D) 42, 44, 46

14. Using flat rate method, the EMI to repay a loan of ₹ 20,000 in $2\frac{1}{2}$ years at an interest rate of 8% p.a. is :

- (A) ₹ 700 (B) ₹ 800
(C) ₹ 900 (D) ₹ 100

15. A mobile phone costs ₹ 12,000 and its scrap value after a useful life of 3 years is ₹ 3,000. Then, the book value of the mobile phone at the end of 2 years is :

- (A) ₹ 3,000 (B) ₹ 6,000
(C) ₹ 5,000 (D) ₹ 7,000

16. What sum of money should be deposited at the end of every 6 months to accumulate ₹ 50,000 in 8 years, if money is worth 6% p.a. compounded semi-annually ? [Given : $(1.03)^{16} = 1.6047$]

- (A) ₹ 3,432.53 (B) ₹ 2,783.08
(C) ₹ 2,480.57 (D) ₹ 2,149.93

17. The graph of the inequation $2x + 3y > 6$ is the :

- (A) entire XOY-plane (B) half-plane that contains the origin
(C) half-plane that neither contains the origin nor the points on the line $2x + 3y = 6$ (D) whole XOY-plane excluding the points on the line $2x + 3y = 6$

18. In an LPP, if the objective function $Z = ax + by$ has same maximum value on two corner points of the feasible region, then the number of points at which maximum value of Z occurs is :

- (A) 0 (B) 2
(C) finite (D) infinite

19. Assertion (A) : The function $f(x) = x^2 - x + 1$ is strictly increasing on $(-1, 1)$.

Reason (R) : If $f(x)$ is continuous on $[a, b]$ and derivable on (a, b) , then $f(x)$ is strictly increasing on $[a, b]$ if $f'(x) > 0$ for all $x \in (a, b)$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

20. In a binomial distribution, $n = 200$ and $p = 0.04$. Taking Poisson distribution as an approximation to the binomial distribution :

Assertion (A) : Mean of Poisson distribution = 8.

Reason (R) : $P(X = 4) = \frac{512}{3e^8}$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

Section B

21(a). If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find the value of k such that $A^2 - 8A + kI = 0$.

OR

21(b). If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, find the values of x, y, z and w .

22. Using Cramer's rule, solve the following system of equations :

$$2x_1 + 3x_2 = 5$$

$$11x_1 - 5x_2 = 6$$

23. Find the solution to the following linear programming problem (if it exists) graphically :

$$\text{Maximize } Z = x + y$$

subject to the constraints

$$x - y \leq -1$$

$$-x + y \leq 0$$

$$x, y \geq 0.$$

24. At 6% p.a., compounded quarterly, find the present value of a perpetuity of ₹ 600 payable at the end of each quarter.

25(a). Assume an investment's starting value is ₹ 20,000 and it grows to ₹ 50,000 in 3 years. Calculate CAGR (Compounded Annual Growth Rate) [Use : $(2.5)^{1/3} = 1.355$]

OR

25(b). A man bought an item for ₹ 12,000. At the end of the year, he decided to sell it for ₹ 15,000. If the inflation rate was 6%, find the nominal and real rate of return.

Section C

26. A container has 50 litres of juice in it. 5 litres of juice is taken out and is replaced by 5 litres of water. This process is repeated 4 more times. Determine the quantity of juice in the container after final replacement. [Use $(0.9)^5 = 0.59049$]

27(a). Evaluate : $\int_0^2 x^2 dx$ and hence show the region on the graph whose area it represents.

OR

27(b). Evaluate : $\int_0^1 \frac{e^{-x}}{1+e^x} dx$

28. Find the differential equation of all circles in the first quadrant which touches both the coordinate axes.

29. Given that the scores of a set of candidates on an IQ test are normally distributed. If the IQ test has a mean of 100 and a standard deviation of 10, determine the probability that a candidate who takes the test will score between 90 and 110. [Given $P(Z < 1) = 0.8413$ and $P(Z < -1) = 0.1587$]

30. The mean weekly sales of a 4-wheeler was 50 units per agency in 20 agencies. After an advertising campaign, the mean weekly sales increased to 55 units per agency with standard deviation of 10 units. Test whether the advertising campaign was successful.

[Given $\sqrt{5} = 2.24$, $t_{19}(0.05) = 1.729$]

31(a). A recent accounting graduate opened a new business and installed a computer system that costs ₹ 45,200. The computer system will be depreciated linearly over 3 years and will have a scrap value of ₹ 0.

(i) What is the rate of depreciation ?

(ii) Give a linear equation that describes the computer system's book value at the end of t^{th} year, where $0 \leq t \leq 3$.

(iii) What will be the computer system's book value at the end of the first year and a half ?

OR

31(b). Find the effective rate which is equivalent to normal rate of 10% p.a. compounded :

(i) semi-annually.

(ii) quarterly.

[Given $(1.05)^2 = 1.1025$, $(1.025)^4 = 1.1038$]

32. A cistern has three pipes A, B and C. Pipes A and B are inlet pipes whereas C is an outlet pipe. Pipes A and B can fill the cistern separately in 3 hours and 4 hours respectively; while pipe C can empty the completely filled cistern in 1 hour. If the pipes A, B and C are opened in order at 5, 6 and 7 a.m. respectively, at what time will the cistern be empty ?

33(a). Find all the points of local maxima and local minima of the function :

$$f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105.$$

OR

33(b). Find the intervals in which the following function f is strictly increasing or strictly decreasing :

$$f(x) = 20 - 9x + 6x^2 - x^3.$$

34(a). Let X denote the number of hours a Class 12 student studies during a randomly selected school day. The probability that X can take the values x_i , for an unknown constant 'k' :

$$P(X = k) = \begin{cases} 0.1 & \text{if } x_i = 0 \\ kx_i & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i) & \text{if } x_i = 3 \text{ or } 4 \end{cases}$$

(i) Find the value of k .

(ii) Determine the probability that the student studied for at least 2 hours.

(iii) Determine the probability that the student studied for at most 2 hours.

OR

34(b). A river near a small town floods and overflows twice in every 10 years on an average. Assuming that the Poisson distribution is appropriate, what is the mean expectation ? Also, calculate the probability of 3 or less overflows and floods in a 10-year interval. [Given : $e^{-2} = 0.13534$]

35. Amrita buys a car for which she makes a down payment of ₹ 2,50,000 and the balance is to be paid in 2 years by monthly instalments of ₹ 25,448 each. If the financier charges interest at the rate of 20% p.a, find the actual price of the car. [Given $\left(\frac{61}{60}\right)^{-24} = 0.67253$]

36. Case Study – 1

On her birthday, Prema decides to donate some money to children of an orphanage home.



If there are 8 children less, everyone gets ₹ 10 more. However, if there are 16 children more, everyone gets ₹ 10 less.

Let the number of children in the orphanage home be x and the amount to be donated to each child be ₹ y .

Based on the above information, answer the following questions :

36(i). Write the system of linear equations in x and y formed of the given situation.

36(ii). Write the system of linear equations, obtained in (i) above, in matrix form $AX = B$.

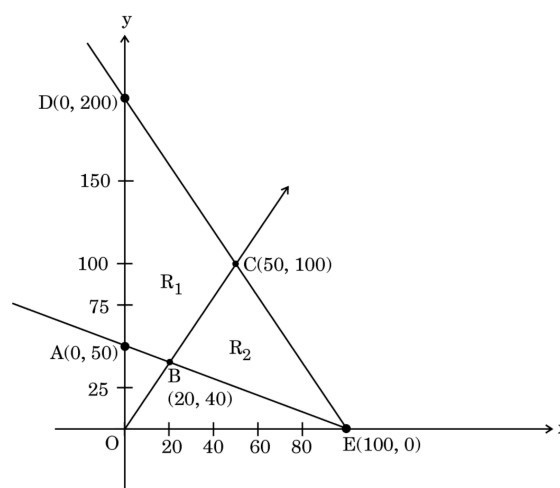
36(iii)(a). Find the inverse of matrix A .

OR

36(iii)(b). Determine the values of x and y .

37. Case Study – 2

In number theory, it is often important to find factors of an integer N . The number N has two trivial factors, namely 1 and N . Any other factor, if exists, is called non-trivial factor of N . Naresh has plotted a graph of some constraints (linear inequations) with points $A(0, 50)$, $B(20, 40)$, $C(50, 100)$, $D(0, 200)$ and $E(100, 0)$. This graph is constructed using three non-trivial constraints and two trivial constraints. One of the non-trivial constraints is $x + 2y \geq 100$.



Based on the above information, answer the following questions :

37(i). What are the two trivial constraints ?

37(ii)(a). If R_1 is the feasible region, then what are the other two non-trivial constraints ?

OR

37(ii)(b). If R_2 is the feasible region, then what are the other two non-trivial constraints ?

37(iii). If R_1 is the feasible region, then find the maximum value of the objective function $z = 5x + 2y$.

38. Case Study – 3

When observed over a long period of time, a time series data can predict trends that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production.

The table below shows the sale of an item in a district during 1996 – 2001 :

Year :	1996	1997	1998	1999	2000	2001
Sales (in lakh ₹) :	6.5	5.3	4.3	6.1	5.6	7.8

Based on the above information, answer the following questions :

38(i). Determine the equation of the straight-line trend.

38(ii)(a). Tabulate the trend values of the years and also compute expected sales trend for the year 2002.

OR

38(ii)(b). Fit a straight-line trend by the method of least squares for the following data :

Year :	Profit (₹ '000)
2004	114
2005	130
2006	126
2007	144
2008	138
2009	156
2010	164

- End of Question Paper -