

CBSE Class 12 Applied Mathematics 2024 - Set 465/S - Compartment



Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains 38 questions. All questions are compulsory.
- (ii) The question paper is divided into five sections: A, B, C, D and E.
- (iii) Section A: Questions 1–18 are multiple choice questions and Questions 19–20 are assertion–reason questions, carrying 1 mark each.
- (iv) Section B: Questions 21–25 are very short answer questions, carrying 2 marks each.
- (v) Section C: Questions 26–31 are short answer questions, carrying 3 marks each.
- (vi) Section D: Questions 32–35 are long answer questions, carrying 5 marks each.
- (vii) Section E: Questions 36–38 are case-study questions, carrying 4 marks each.
- (viii) There is no overall choice. Internal choice is provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.
- (ix) Use of calculators is not allowed.

Section A

1. In what ratio must water be mixed with milk to gain $16\frac{2}{3}\%$ on selling the mixture at cost price ?

- (A) 1 : 6 (B) 6 : 1
(C) 3 : 2 (D) 2 : 3

2. In a 100 m race, A can beat B by 25 m and B can beat C by 4 m. By how much can A beat C in the same race ?

- (A) 32 m (B) 28 m
(C) 24 m (D) 20 m

3. If $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $(A^2 - 6A)$ is equal to :

- (A) 3I (B) -5I
(C) 5I (D) -3I

4. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then the value of x is :

- (A) 1 (B) $\frac{1}{2}$
(C) $-\frac{1}{2}$ (D) 2

5. $\int 2^{2x} \cdot 3^x dx$ is equal to :

- (A) $\frac{12^x}{\log 12} + C$ (B) $\frac{2^{2x} \cdot 3^x}{\log 2 \cdot \log 3} + C$
(C) $\frac{4 \cdot 6^x}{\log 6} + C$ (D) $(12)^x \cdot \log 12 + C$

6. If $x + y = 8$, then the maximum value of (xy) is :

- (A) 12 (B) 16
(C) 20 (D) 24

7. The demand curve for a monopolist is given by $x = 100 - 4p$. The value of x for which $MR = 0$, is :

- (A) 25 (B) 30
(C) 45 (D) 50

8. A random variable X takes the values -1, 0, 1. If its mean is 0.6 and $P(X = 0) = 0.2$, then $P(X = 1)$ is :

- (A) 0.7 (B) 0.5
(C) 0.4 (D) 0.3

9. One hundred identical coins each with probability p showing up heads are tossed once. If $0 < p < 1$ and the probability of heads on 50 coins is equal to that of heads showing on 51 coins, then the value of p is :

- (A) $\frac{1}{2}$ (B) $\frac{49}{101}$
(C) $\frac{50}{101}$ (D) $\frac{51}{101}$

10. The probability that a bomb dropped from a plane strikes the target is $\frac{4}{5}$. What is the probability that out of 6 bombs dropped, exactly 2 bombs strike the target ?

- (A) $2 \left(\frac{4}{5}\right)^5$ (B) $1 - 2 \left(\frac{4}{5}\right)^5$
(C) $\frac{48}{5^5}$ (D) $\frac{64}{5^6}$

11. A specific characteristic of a sample is known as a :

- (A) population (B) parameter
(C) statistic (D) variance

12. The test statistic for a one sample t-test, denoted by t, is defined as :

where μ is the population mean and $\bar{x}$ is the sample mean.

- (A) $t = \frac{\bar{x} - \mu}{\left(\frac{S}{\sqrt{n}}\right)}$ (B) $t = \frac{\bar{x} - \mu}{\left(\frac{S}{n}\right)}$
(C) $t = \frac{\bar{x} - \mu}{\left(\frac{S^2}{n}\right)}$ (D) $t = \frac{\bar{x} - \mu}{\left(\frac{S}{n^2}\right)}$

13. If for a data, $n = 6$, $\Sigma y = 84$, $\Sigma xy = 108$, $\Sigma x^2 = 70$ and $\Sigma x = 0$, then the equation of the straight line trend is :

- (A) $y_c = 14 + 1.54x$ (B) $y_c = 1.54 + 14x$
(C) $y_c = 14 + 3.08x$ (D) $y_c = 3.08 + 14x$

14. At what rate of interest will the present value of a perpetuity of ₹ 500 payable at the end of each quarter be ₹ 40,000 ?

- (A) 1.25% p.a. (B) 2.5% p.a.
(C) 5% p.a. (D) 6% p.a.

15. If nominal rate is $r\%$ compounded k times in a year, then the effective rate of interest r_e is given by :

- (A) $r_e = \left(1 - \frac{r}{100k}\right)^k - 1$ (B) $r_e = \left(1 + \frac{r}{100k}\right)^k + 1$
(C) $r_e = \left(1 - \frac{r}{100k}\right)^k + 1$ (D) $r_e = \left(1 + \frac{r}{100k}\right)^k - 1$

16. If the annual depreciation of an asset is ₹ 40,000 and its scrap value after useful life of 15 years is ₹ 50,000, then the original cost of the asset is :

- (A) ₹ 7,60,000 (B) ₹ 7,20,000
(C) ₹ 6,50,000 (D) ₹ 6,30,000

17. The number of solutions of an L.P.P. to minimize $z = 3x + 2y$ under the constraints $x + y \geq 8$, $3x + 5y \leq 15$ and $x, y \geq 0$, is :

- (A) 2 (B) 5
(C) infinitely many (D) zero

18. An investment's starting value is ₹ 10,000 and it grows to ₹ 60,000 in 4 years. The CAGR is : (Given : $6^{1/4} = 1.56508$)

- (A) 1.56% (B) 5.65%
(C) 15.65% (D) 56.50%

19. Assertion (A) : The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is 3.

Reason (R) : The highest power of the highest order derivative involved in a differential equation, when it is written as a polynomial in derivatives, is called its degree.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

20. Assertion (A) : Minor of element a_{13} in the

matrix $\begin{bmatrix} 0 & 2 & 6 \\ 1 & 2 & -1 \\ 2 & 1 & 3 \end{bmatrix}$ is $\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$.

Reason (R) : Minor of an element a_{ij} of a matrix is the determinant obtained by deleting its j^{th} row and i^{th} column in which the element lies.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Section B

21(a). Evaluate $(137 + 995) \pmod{12}$.

OR

21(b). Find the unit's digit of 12^{12} .

22. If $y = (x + \sqrt{x^2 + 1})^p$, then prove that $(x^2 + 1)y_2 + xy_1 - p^2y = 0$; where $y_1 = \frac{dy}{dx}$ and $y_2 = \frac{d^2y}{dx^2}$.

23(a). If X is a normal variate with mean $(\mu) = 70$ and standard deviation $(\sigma) = 5$, then find $P(X > 75)$. (Given : $P(0 < Z < 1) = 0.3413$)

OR

23(b). If X is a Poisson variate such that $P(X = 0) = P(X = 1) = \alpha$, then show that $\alpha = e^{-1}$.

24. Find the trend values by taking five yearly moving averages for the following data :

Year	Annual Production (Million tons)
2012	16
2013	14
2014	20
2015	18
2016	22
2017	17
2018	19
2019	21
2020	20

25. Two tailors A and B earn ₹ 1500 and ₹ 2000 per day, respectively. Tailor A can stitch 6 shirts and 4 pants, while tailor B can stitch 10 shirts and 4 pants per day. Form a linear programming problem to minimize the labour cost to produce at least 60 shirts and 32 pants.

Section C

26(a). Solve $3x + 8 > 2$, when

- (i) x is an integer.
- (ii) x is a natural number.
- (iii) x is a whole number.

OR

26(b). A man goes 12 km downstream and comes back to the starting point by swimming non-stop in 3 hours. If the speed of the stream is 3 km/h, find the speed with which the man can swim in still water.

27(a). Evaluate :

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

OR

27(b). Find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}.$$

28. Ten students are selected at random from a college and their heights (in cm) are found to be 100, 104, 108, 110, 118, 120, 122, 124, 126 and 128. In the light of the data, discuss the conclusion that the mean height of the students of the college is 110 cm. [Given : $t_9(0.05) = 2.262$]

29. Given below is the data of workers welfare expenses (in lakh ₹) in steel industries during 2016 – 2020 :

Year	2016	2017	2018	2019	2020
Workers welfare expenses (in lakh ₹)	160	185	220	300	510

Find the best fitted trend line by the method of least squares and tabulate the trend values.

30. A machine costing ₹ 2,00,000 has effective life of 7 years and its scrap value is ₹ 30,000. What amount should the company put into a sinking fund earning 5% p.a. so that it can replace the machine after its useful life ? Assume that a new machine will cost ₹ 3,00,000 after 7 years. [Given : $(1.05)^7 = 1.407$]

31. The value of a car depreciates by 12.5% every year. By what percent will the value of the car decrease after 3 years and after 5 years ?

Section D

32. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of Vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of Vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimize the amount of Vitamin A? What is the minimum amount of Vitamin A?

Formulate the above problem as an L.P.P. and solve it graphically.

33(a). A wire of length 36 m is to be cut into two pieces. One of the pieces is to be made a square and the other, a circle. What would be the lengths of the two pieces, so that the combined area of the square and the circle is minimum?

OR

33(b). Find :

$$\int \frac{x^3}{x^4 + 3x^2 + 2} dx$$

34(a). An unbiased die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

OR

34(b). An aptitude test for selecting officers in a bank is conducted on 1000 candidates. The mean score obtained is 42 and the standard deviation of score is 24. Assuming normal distribution for the scores, find :

(i) the number of candidates whose scores exceed 60;

(ii) the number of candidates whose scores lie between 30 and 60.

[Given : $P(0 \leq Z \leq 0.75) = 0.2734$; $P(0 \leq Z \leq 0.5) = 0.1915$]

35. Mahesh purchased a house from a company for ₹ 70,00,000 and made a down payment of ₹ 15,00,000. He repays the balance in 25 years by monthly instalments at 9% p.a. compounded monthly.

(i) What is the amount of monthly payment?

(ii) What is the total interest payment?

[Given : $(1.0075)^{-300} = 0.1062878338$]

36. Case Study – 1

Rohini wants to give a rectangular plot of land for a school in her village. When she was asked to mention the dimensions of the plot, she told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area does not alter, but if its length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 sq m.

Based on the above information, answer the following questions :

36(i). Assuming x m and y m as the length and breadth of the plot respectively, write the system of linear equations in x and y .

36(ii). Write the system of linear equations obtained in (i) in the matrix equation $AX = B$.

36(iii)(a). Determine A^{-1} .

OR

36(iii)(b). Find the area of the plot.

37. Case Study – 2

Read the following passage and answer the questions given below :

“In an elliptical sports field, the authority wants to design a rectangular soccer field with the maximum possible area. The sports field is given by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.”

Based on the above information, answer the following questions :

37(i). If the length and breadth of the rectangular soccer field be $2x$ and $2y$ respectively, then find the area function $A(x)$ in terms of x .

37(ii). Find the critical point(s) of the area function $A(x)$.

37(iii)(a). Using first derivative test, find the length $2x$ and breadth $2y$ of the soccer field (in terms of a and b) that maximize the area.

OR

37(iii)(b). Using second derivative test, find the length $2x$ and breadth $2y$ of the soccer field (in terms of a and b) that maximize the area.

38. Case Study – 3

For providing water to the families of a colony, a large water tank with two inlet pipes A and B and an outlet pipe C, is installed. Pipes A and B can fill the tank in 10 hours and 12 hours respectively; whereas pipe C can empty the tank in 15 hours.

Based on the above information, answer the following questions :

38(i). If both pipes A and B are opened together, then find the time in which the tank will be filled completely.

38(ii). If both pipes A and C are opened together, then find the time in which the tank will be filled completely.

38(iii)(a). If all the three pipes A, B and C are opened together, then find the time in which the tank will be filled completely.

OR

38(iii)(b). Pipes A and B are opened together for some time and then pipe B is turned off after some time. If the tank is completely filled in 6 hours, then after how many hours is pipe B turned off ?

- End of Question Paper -