

CBSE Class 12 Mathematics 2024 - Set 65-S-2 - Compartment



Question Paper

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains 38 questions. All questions are compulsory.
- (ii) This question paper is divided into five Sections – A, B, C, D and E.
- (iii) In Section A, Questions no. 1 to 18 are Multiple Choice Questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
- (iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
- (v) In Section C, Questions no. 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
- (vi) In Section D, Questions no. 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
- (vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
- (ix) Use of calculator is not allowed.

Section A

1. If A and B are two square matrices of order 2 and $|A| = 2$ and $|B| = 5$, then $|-3AB|$ is:

- (A) -90 (B) -30
(C) 30 (D) 90

2. $\int \frac{x-3}{(x-1)^3} e^x dx$ is equal to:

- (A) $\frac{2e^x}{(x-1)^3} + C$ (B) $\frac{-2e^x}{(x-1)^2} + C$
(C) $\frac{e^x}{(x-1)} + C$ (D) $\frac{e^x}{(x-1)^2} + C$

3. The area (in sq. units) of the region bounded by the curve $y = x$, the x-axis, $x = 0$ and $x = 2$ is:

- (A) $\frac{3}{2}$ (B) $\frac{1}{2} \log 2$
(C) 2 (D) 4

4. What is the value of $\frac{\text{projection of } \vec{a} \text{ on } \vec{b}}{\text{projection of } \vec{b} \text{ on } \vec{a}}$ for vectors $\vec{a} = 2\hat{i} - 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$?

- (A) $\frac{3}{7}$ (B) $\frac{7}{3}$
(C) $\frac{4}{3}$ (D) $\frac{4}{7}$

5. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a} \cdot \vec{b} > 0$ and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is:

- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$
(C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{4}$

6. For two matrices A and B, given that $A^{-1} = \frac{1}{4}B$, then inverse of $(4A)$ is:

- (A) $4B$ (B) B
(C) $\frac{1}{4}B$ (D) $\frac{1}{16}B$

7. If $x = at^2$ and $y = 2at$, then $\frac{dy}{dx}$ is equal to:

- (A) $2at$ (B) $\frac{1}{t}$
(C) $-\frac{1}{t^2}$ (D) $-\frac{1}{2at^3}$

8. The function $f(x) = |x| - x$ where $x \in \mathbb{R}$, at $x = 0$ is:

- (A) continuous and differentiable (B) continuous but not differentiable
(C) not continuous but differentiable (D) neither continuous nor differentiable

9. If X, Y and XY are matrices of order 2×3 , $m \times n$ and 2×5 respectively, then the number of elements in matrix Y is:

- (A) 6 (B) 10
(C) 15 (D) 35

10. The number of discontinuities of the function

f given by $f(x) = \begin{cases} x+2, & x < 0 \\ e^x, & 0 \leq x \leq 1 \\ 2-x, & x > 1 \end{cases}$ is:

- (A) 0 (B) 1
(C) 2 (D) 3

11. $\int e^{3 \log x} (x^4 + 1)^{-1} dx$ is equal to:

- (A) $\frac{1}{4} \log(x^4 + 1) + C$ (B) $\frac{1}{4} \log\left(\frac{1}{x^4 + 1}\right) + C$
(C) $\frac{x^3}{x^4 + 1} + C$ (D) $\frac{e^x}{x^4 + 1} + C$

12. Let $y = f\left(\frac{1}{x}\right)$ and $f'(x) = x^3$. What is the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}$?

- (A) $-\frac{1}{64}$ (B) $-\frac{1}{32}$
(C) -32 (D) -64

13. If $y = \log \sqrt{\sec \sqrt{x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi^2}{16}$ is:

- (A) $\frac{1}{\pi}$ (B) π
(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

14. A particular solution of the differential equation $x \frac{dy}{dx} + y = 0$, when $x = 1$ and $y = 1$, is:

- (A) $y = x$ (B) $y = e^x$
(C) $y = \frac{1}{x}$ (D) $y = \log x$

15. The greatest integer function defined by $f(x) = [x]$, $1 < x < 3$ is not differentiable at $x =$:

- (A) 0 (B) 1
(C) 2 (D) $\frac{3}{2}$

16. A vector makes equal angles with the positive directions of the x, y and z axes. The direction cosines of the vector are:

- (A) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (B) $\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$
(C) $\frac{-1}{2}, \frac{1}{2}, \frac{-1}{2}$ (D) $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}$

17. The vector equation of the line passing through the points $(0, 0, 2)$ and $(3, -2, 5)$ is:

- (A) $\vec{r} = 2\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 5\hat{k})$ (B) $\vec{r} = 2\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 5\hat{k})$
(C) $\vec{r} = 2\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 3\hat{k})$ (D) $\vec{r} = 3\hat{i} - 2\hat{j} + 5\hat{k} + \lambda(2\hat{k})$

18. If the radius of a circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is:

- (A) $\frac{2\pi}{3}$ cm/s (B) π cm/s
(C) $\frac{4\pi}{3}$ cm/s (D) 2π cm/s

19. **Assertion (A):** If R and S are two events such that $P(R | S) = 1$ and $P(S) > 0$, then $S \subset R$.

Reason (R): If two events A and B are such that $P(A \cap B) = P(B)$, then $A \subset B$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** $\cos^{-1}\left(\cos \frac{13\pi}{6}\right)$ is equal to $\frac{\pi}{6}$.

Reason (R): The range of the principal value branch of the function $y = \cos^{-1} x$ is $[0, \pi]$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Section B

21. Evaluate: $\sin^{-1}\left(\sin \frac{13\pi}{6}\right) + \cos^{-1}\left(\cos \frac{\pi}{3}\right) + \tan^{-1}(\sqrt{3})$.

22. Given that $f(x) = \frac{\log x}{x}$, find the point of local maximum of $f(x)$.

23(a). Find: $\int \frac{x^3 - 1}{x^3 - x} dx$.

OR

23(b). Evaluate: $\int_{-4}^0 |x + 2| dx$.

24(a). If $y = (\sin^{-1} x)^2$, then find $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx}$.

OR

24(b). If $y^x = x^y$, then find $\frac{dy}{dx}$.

25. Find the value of k so that the lines joining the points $(1, -1, 2)$ and $(3, 4, k)$ is perpendicular to the line joining the points $(0, 3, 2)$ and $(3, 5, 6)$.

Section C

26. It is known that 20% of the students in a school have above 90% attendance and 80% of the students are irregular. Past year results show that 80% of students who have above 90% attendance and 20% of irregular students get an 'A' grade in their annual examination. At the end of a year, a student is chosen at random from the school and is found to have an 'A' grade. What is the probability that the student is irregular?

27(a). If $y = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$, $0 < x < 1$, then find $\frac{dy}{dx}$.

OR

27(b). If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$.

28(a). If $A \begin{pmatrix} 4 & 0 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 17 & 10 \\ 0 & -16 \end{pmatrix}$, find matrix A. Also, find A^{-1} .

OR

28(b). Given that A is a square matrix of order 3 such that $A^2 = \begin{pmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{pmatrix}$, show that $A^3 = A^{-1}$.

29. Find the particular solution of the differential equation $x \cos y \, dy = (x e^x \log x + e^x) \, dx$, given that $y = \frac{\pi}{2}$ when $x = 1$.

30. Show that the vectors $3\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} - 8\hat{k}$ and $4\hat{i} - 2\hat{j} - 7\hat{k}$ form the vertices of a right triangle.

31(a). Find: $\int \frac{dx}{\cos x \sqrt{\cos 2x}}$.

OR

31(b). Find: $\int \frac{5x-3}{\sqrt{1+4x-2x^2}} \, dx$.

Section D

32. Check whether the relation S in the set of real numbers $\mathbb{R}$, defined as $S = \{(a, b) : a \leq b^2\}$, is reflexive, symmetric or transitive. Also, determine all $x \in \mathbb{R}$ for which $(x, x) \in S$.

33. Solve the following linear programming problem graphically:

Minimise $Z = 6x + 7y$

subject to constraints

$$x + 2y \geq 240,$$

$$3x + 4y \leq 620,$$

$$2x + y \geq 180,$$

$$x, y \geq 0.$$

34(a). Using integration, find the area of the region bounded by the curve $y = \sqrt{4 - x^2}$, the lines $x = -\sqrt{2}$ and $x = \sqrt{3}$, and the x-axis.

OR

34(b). Using integration, evaluate the area of the region bounded by the curve $y = x^2$, the lines $y = 1$ and $y = 3$ and the y-axis.

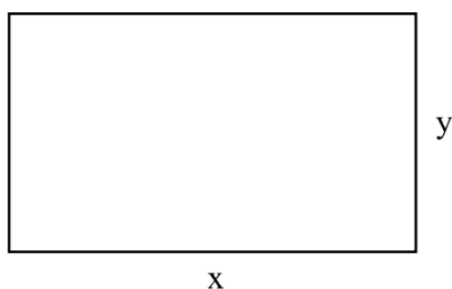
35(a). Find the shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

OR

35(b). Find the point of intersection of the lines $\vec{r} = \hat{i} - \hat{j} + 6\hat{k} + \lambda(3\hat{i} - \hat{k})$ and $\vec{r} = -3\hat{j} + 3\hat{k} + \mu(\hat{i} + 2\hat{j} - \hat{k})$. Also, find the vector equation of the line passing through the point of intersection of the given lines and perpendicular to both the lines.

Section E

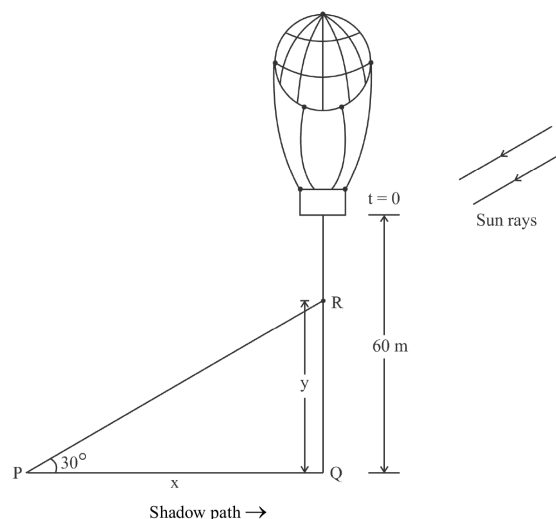
36. An architect is developing a plot of land for a commercial complex. When asked about the dimensions of the plot, he said that if the length is decreased by 25 m and the breadth is increased by 25 m, then its area increases by 625 m^2 . If the length is decreased by 20 m and the breadth is increased by 10 m, then its area decreases by 200 m^2 .



On the basis of the above information, answer the following questions:

- (i) Formulate the linear equations in x and y to represent the given information.
- (ii) Find the dimensions of the plot of land by the matrix method.

37. A sandbag is dropped from a balloon at a height of 60 metres. When the angle of elevation of the sun is 30° , the position of the sandbag is given by the equation $y = 60 - 4.9t^2$, where y is the height of the sandbag above the ground and t is the time in seconds.



On the basis of the above information, answer the following questions:

- (i) Find the relation between x and y , where x is the distance of the shadow at P from the point Q and y is the height of the sandbag above the ground.
- (ii) After how much time will the sandbag be 35 metres above the ground?
- (iii)(a) Find the rate at which the shadow of the sandbag is travelling along the ground when the sandbag is at a height of 35 metres.

OR

37(iii)(b). How fast is the height of the sandbag decreasing when 2 seconds have elapsed?

38. A salesman receives a commission for each sale he makes together with a fixed daily income. The number of sales he makes in a day along with their probabilities are given in the table below:

X	0	1	2	3	4	5
P(X)	0.42	3k	0.3	0.05	2k	0.03

His daily income Y (in ₹) is given by $Y = 800X + 50$.



On the basis of the above information, answer the following questions:

- (i) Find the value of k .
- (ii) Evaluate $P(X \geq 3)$.
- (iii)(a) Calculate the expected weekly income of the salesman assuming he works five days per week.

OR

38(iii)(b). Calculate the expected weekly income of the salesman assuming he works only three days a week.

- End of Question Paper -