

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{1}{4}$, then $P(E/F)$ is equal to:

- (A) $\frac{1}{6}$ (B) $\frac{1}{5}$
(C) $\frac{2}{3}$ (D) $\frac{7}{9}$

2. If $\vec{\alpha} = \hat{i} - 4\hat{j} + 9\hat{k}$ and $\vec{\beta} = 2\hat{i} - 8\hat{j} + \lambda\hat{k}$ are two mutually parallel vectors, then λ is equal to:

- (A) -18 (B) 18
(C) $-\frac{34}{9}$ (D) $\frac{34}{9}$

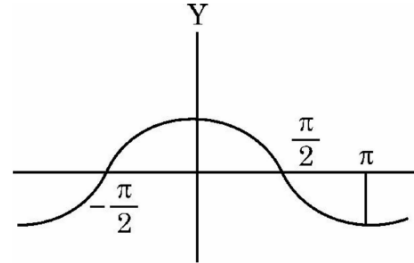
3. $\int \frac{1 - 2 \sin x}{\cos^2 x} dx$ is equal to:

- (A) $\tan x - 2 \sec x + C$ (B) $-\tan x + 2 \sec x + C$
(C) $-\tan x - 2 \sec x + C$ (D) $\tan x + 2 \sec x + C$

4. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = \sqrt{37}$, $|\vec{b}| = 3$ and $|\vec{c}| = 4$, then angle between $\vec{b}$ and $\vec{c}$ is

- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

5. The graph of a trigonometric function is as shown. Which of the following will represent the graph of its inverse?



- (A)
- (B)
- (C)
- (D)

6. If A is a square matrix of order 3 such that $\det(A) = 9$, then $\det(9A^{-1})$ is equal to

- (A) 9 (B) 9^2
(C) 9^3 (D) 9^4

7. If $f(x) = |x| + |x - 1|$, then which of the following is correct?

- (A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$.
(B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$.
(C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.
(D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.

8. Which of the following is not a homogeneous function of x and y ?

- (A) $y^2 - xy$
(B) $x - 3y$
(C) $\sin^2 \frac{y}{x} + \frac{y}{x}$
(D) $\tan x - \sec y$

9. Let A be a matrix of order $m \times n$ and B is a matrix such that $A^T B$ and BA^T are defined. Then, the order of B is:

- (A) $m \times m$
(B) $n \times n$
(C) $m \times n$
(D) $n \times m$

10. If the feasible region of a linear programming problem with objective function $Z = ax + by$ is bounded, then which of the following is correct?

- (A) It will only have a maximum value.
(B) It will only have a minimum value.
(C) It will have both maximum and minimum values.
(D) It will have neither maximum nor minimum value.

11. If $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then A^{-1} is

- (A) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$
(B) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$
(C) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
(D) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

12. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

- (A) xe^x
(B) $\frac{e^x}{x}$
(C) $\frac{x}{e^x}$
(D) xe^{-x}

13. Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = j - 2i$. Then which of the following is true?

- (A) $a_{12} > 0$
(B) All $a_{ij} < 0$
(C) $a_{13} + a_{31} = -6$
(D) $a_{23} > a_{32}$

14. The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is:

- (A) 0
(B) 2
(C) 4
(D) 5

15. If $\int \frac{2^x}{x^2} dx = k \cdot 2^x + C$, then k is equal to
- (A) $\frac{1}{\log 2}$ (B) $-\log 2$
 (C) -1 (D) $\frac{1}{2}$

16. If A and B are invertible matrices, then which of the following is not correct?
- (A) $(A + B)^{-1} = B^{-1} + A^{-1}$ (B) $(AB)^{-1} = B^{-1}A^{-1}$
 (C) $\text{adj}(A) = |A| A^{-1}$ (D) $|A^{-1}| = |A|^{-1}$

17. The corner points of the feasible region in graphical representation of an L.P.P. are $(2, 72)$, $(15, 20)$ and $(40, 15)$. If $Z = 18x + 9y$ be the objective function, then
- (A) Z is maximum at $(2, 72)$, minimum at $(15, 20)$ (B) Z is maximum at $(15, 20)$, minimum at $(40, 15)$
 (C) Z is maximum at $(40, 15)$, minimum at $(15, 20)$ (D) Z is maximum at $(40, 15)$, minimum at $(2, 72)$

18. The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x -axis is given by
- (A) $\int_0^4 x dx$ (B) $\int_0^2 y^2 dy$
 (C) $2 \int_0^4 \sqrt{x} dx$ (D) $\int_0^4 \sqrt{x} dx$

19. **Assertion (A):** Let Z be the set of integers. A function $f : Z \rightarrow Z$ defined as $f(x) = 3x - 5$, $\forall x \in Z$ is bijective.

Reason (R): A function is bijective if it is both surjective and injective.

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
 (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$.

Reason (R): For a function f to be continuous at $x = a$, $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$.

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
 (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

Section B

21. (a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings.

OR

(b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A(2, 1, 3) and B(8, -1, 0) respectively.

22. Find the values of a for which $f(x) = x^2 - 2ax + b$ is an increasing function for $x > 0$.

23. (a) Differentiate $2^{\cos^2 x}$ w.r.t. $\cos^2 x$.

OR

(b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.

24. Evaluate: $\sin^{-1}\left[\sin \frac{3\pi}{5}\right]$.

25. The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Find the area of the parallelogram.

26. (a) Verify that lines given by $\vec{r} = (1 - \lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$ and $\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$ are skew lines. Hence, find the shortest distance between the lines.

OR

(b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.

27. Solve the following linear programming problem graphically:

Maximise $Z = 20x + 30y$

Subject to the constraints:

$$x + y \leq 80$$

$$2x + 3y \geq 100$$

$$x \geq 14$$

$$y \geq 14$$

28. The area of an expanding rectangle is increasing at the rate of $48 \text{ cm}^2/\text{s}$. The length of the rectangle is always square of its breadth. At what rate the length of rectangle increasing at an instant, when breadth = 4.5 cm?

29. (a) The probability distribution for the number of students being absent in a class on a Saturday is as follows:

X	0	2	4	5
P(X)	p	2p	3p	p

Where X is the number of students absent.

(i) Calculate p. (1 mark)

(ii) Calculate the mean of the number of absent students on Saturday. (2 marks)

OR

(b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.

30. (a) Find: $\int \frac{\cos x}{(\sin x + \cos x)^2} dx$

OR

(b) Evaluate: $\int_0^{\pi/2} \frac{\sin^2 x \cos x + \sin x \cos^2 x}{\sin x + \cos x} dx$

31. Sketch the graph of $y = |x + 3|$ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.

32. (a) Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ w.r.t. $\cos^{-1}(2x\sqrt{1-x^2})$, $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$.

OR

(b) Find $\frac{dy}{dx}$, if $y = x^{\tan x} + \frac{\sqrt{x^2+1}}{2}$.

33. Find the absolute maximum and absolute minimum of function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$.

34. A school wants to allocate students into three clubs: Sports, Music and Drama, under following conditions:

- The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club.
- The number of students in Music club should be 20 more than half the number of students in Sports club.
- The total number of students to be allocated in all three clubs are 180.

Find the number of students allocated to different clubs, using matrix method.

35. (a) Find the image A' of the point $A(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A' .

OR

(b) Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point $Q(2, 4, -1)$ is 7 units. Also, find the equation of line joining P and Q .

36(i). A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively.

Based on the above information, answer the following: What is the probability that a customer after availing the loan will default on the loan repayment?

36(ii). (Continuing from the bank loan case study above.) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest?

37(i). A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections. Let the length of the side perpendicular to the partition be x metres and parallel to the partition be y metres.

Based on this information, write the equation for the total boundary material used in the boundary and parallel to the partition, in terms of x and y .

37(ii). (Continuing from the solar panel case study above.) Write the area of the solar panel as a function of x .

37(iii). (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area.

OR

(b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition.

38(i). A class teacher is keen to assess the learning of the concept of “relations” taught to her students. She writes the following five relations, each defined on the set $A = \{1, 2, 3\}$:

$$R_1 = \{(2,3), (3,2)\}$$

$$R_2 = \{(1,2), (1,3), (3,2)\}$$

$$R_3 = \{(1,2), (2,1), (1,1)\}$$

$$R_4 = \{(1,1), (1,2), (3,3), (2,2)\}$$

$$R_5 = \{(1,1), (1,2), (3,3), (2,2), (2,1), (2,3), (3,2)\}$$

Students are asked to answer the following questions about the above relations: Identify the relation which is reflexive and transitive but not symmetric.

38(ii). (Continuing from the relations case study above.) Identify the relation which is reflexive and symmetric but not transitive.

38(iii). (a) Identify the relation which is symmetric but neither reflexive nor transitive.

OR

(b) State the pairs which need to be added to relation R_2 to make it an equivalence relation.

- End of Question Paper -