

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct?

- (A) It will only have a maximum value. (B) It will only have a minimum value.
(C) It will have both maximum and minimum values. (D) It will have neither maximum nor minimum value.

2. The unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ is

- (A) $\hat{k}$ (B) $-\hat{k} + \hat{j}$
(C) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

3. If $\int_0^1 \frac{e^x}{1+x} dx = \alpha$, then $\int_0^1 \frac{e^x}{(1+x)^2} dx$ is equal to

- (A) $\alpha - 1 + \frac{e}{2}$ (B) $\alpha + 1 - \frac{e}{2}$
(C) $\alpha - 1 - \frac{e}{2}$ (D) $\alpha + 1 + \frac{e}{2}$

4. If $\int \frac{2^x}{x^2} dx = k \cdot 2^x + C$, then k is equal to

- (A) $\frac{1}{\log 2}$ (B) $-\log 2$
(C) -1 (D) $\frac{1}{2}$

5. If $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then A^{-1} is

- (A) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$
(C) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

6. If $\begin{pmatrix} x+y & 3y \\ 3x & x+3 \end{pmatrix} = \begin{pmatrix} 9 & 4x+y \\ x+6 & y \end{pmatrix}$, then $(x-y)$ is equal to

- (A) -7 (B) -3
(C) 3 (D) 7

7. Let M and N be two events such that $P(M) = 0.6$, $P(N) = 0.2$ and $P(M \cap N) = 0.5$, then $P(M'/N')$ is

- (A) $\frac{7}{8}$ (B) $\frac{2}{5}$
(C) $\frac{1}{2}$ (D) $\frac{2}{3}$

8. Which of the following is not a homogeneous function of x and y ?

- (A) $y^2 - xy$ (B) $x - 3y$
(C) $\sin^2 \frac{y}{x} + \frac{y}{x}$ (D) $\tan x - \sec y$

9. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = \sqrt{37}$, $|\vec{b}| = 3$ and $|\vec{c}| = 4$, then angle between $\vec{b}$ and $\vec{c}$ is

- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

10. If $f(x) = |x| + |x-1|$, then which of the following is correct?

- (A) $f(x)$ is both continuous and differentiable, at $x=0$ and $x=1$. (B) $f(x)$ is differentiable but not continuous, at $x=0$ and $x=1$.
(C) $f(x)$ is continuous but not differentiable, at $x=0$ and $x=1$. (D) $f(x)$ is neither continuous nor differentiable, at $x=0$ and $x=1$.

11. A system of linear equations is represented as $AX = B$, where A is coefficient matrix, X is variable matrix and B is the constant matrix. Then the system of equations is

- (A) Consistent, if $|A| \neq 0$, solution is given by $X = BA^{-1}$. (B) Inconsistent if $|A| = 0$ and $(\text{adj } A)B = 0$.
(C) Inconsistent if $|A| \neq 0$. (D) May or may not be consistent if $|A| = 0$ and $(\text{adj } A)B = 0$.

12. The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is:

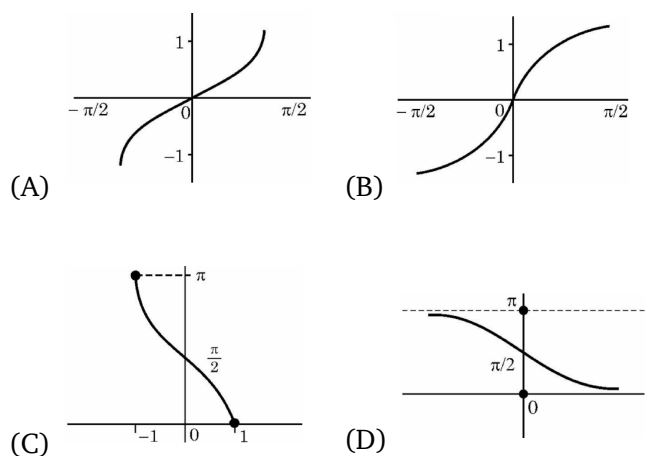
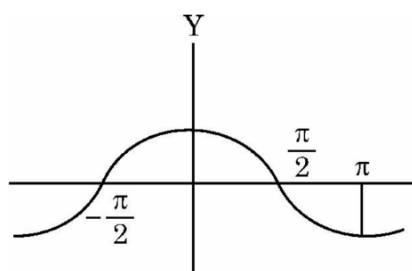
- (A) 0 (B) 2
(C) 4 (D) 5

13. The order and degree of differential equation

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2} \text{ are}$$

- (A) order 1, degree 1 (B) order 1, degree 2
(C) order 2, degree 1 (D) order 2, degree 2

14. The graph of a trigonometric function is as shown. Which of the following will represent the graph of its inverse?



15. The corner points of the feasible region in graphical representation of an L.P.P are $(2, 72)$, $(15, 20)$ and $(40, 15)$. If $Z = 18x + 9y$ is the objective function, then

- (A) Z is maximum at $(2, 72)$, minimum at $(15, 20)$ (B) Z is maximum at $(15, 20)$, minimum at $(40, 15)$
(C) Z is maximum at $(40, 15)$, minimum at $(15, 20)$ (D) Z is maximum at $(40, 15)$, minimum at $(2, 72)$

16. Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$ and $C = [9 \ 8 \ 7]$, then which of the following is defined?

- (A) Only AB (B) Only AC
(C) Only BA (D) All AB, AC and BA

17. If A and B are invertible matrices, then which of the following is not correct?

- (A) $(A + B)^{-1} = B^{-1} + A^{-1}$ (B) $(AB)^{-1} = B^{-1}A^{-1}$
(C) $\text{adj}(A) = |A|A^{-1}$ (D) $|A|^{-1} = |A^{-1}|$

18. The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x -axis is given by:

- (A) $\int_0^4 x \, dx$ (B) $\int_0^2 y^2 \, dy$
(C) $2 \int_0^4 \sqrt{x} \, dx$ (D) $\int_0^4 \sqrt{x} \, dx$

19. **Assertion (A):** Let Z be the set of integers. A function $f : Z \rightarrow Z$ defined as $f(x) = 3x - 5$, $\forall x \in Z$ is bijective.

Reason (R): A function which is one-one and onto both is called bijective.

- (A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true, but R is not the correct explanation of A.
(C) A is true, but R is false. (D) A is false, but R is true.

20. **Assertion (A):** $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$.

Reason (R): A function f is continuous at $x = a$ if $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$.

- (A) Both A and R are true and R is the correct explanation of A. (B) Both A and R are true, but R is not the correct explanation of A.
(C) A is true, but R is false. (D) A is false, but R is true.

Section B

21. Find the area of a parallelogram whose diagonals are given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$.

22. Find the values of a for which $f(x) = 4x^3 \sin x - \cos x - 2ax + b$ is decreasing on $\mathbb{R}$.

23. (a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kite strings, assuming there is no slack in the strings.

OR

(b) Find a vector of magnitude 21 units in the direction opposite to that of $\vec{AB}$ where $A(2, 1, 3)$ and $B(8, -1, 0)$.

24. Solve for x : $2 \tan^{-1} x + \sin^{-1} \left(\frac{2x}{1+x^2} \right) = \frac{4\pi}{3}$.

25. (a) Differentiate $2^{\cos^2 x}$ w.r.t. $\cos^2 x$.

OR

(b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.

Section C

26. Solve the following linear programming problem graphically:

Maximize $Z = 8x + 9y$

Subject to the constraints:

$$2x + 3y \leq 6$$

$$3x - 2y \leq 6$$

$$y \leq 1$$

$$x \geq 0, y \geq 0$$

27. (a) Find: $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$

OR

(b) Evaluate: $\int_0^5 (|x-1| + |x-2| + |x-5|) dx$

28. A spherical medicine ball when dropped in water dissolves in such a way that the rate of decrease of volume at any instant is proportional to its surface area. Calculate the rate of decrease of its radius.

29. Sketch the graph of $y = |x+3|$ and find the area of the region enclosed by the curve and the x -axis, between $x = -6$ and $x = 0$, using integration.

30. (a) Verify that lines given by $\vec{r} = (1-\lambda)\hat{i} + (\lambda-2)\hat{j} + (3-2\lambda)\hat{k}$ and $\vec{r} = (\mu+1)\hat{i} + (2\mu-1)\hat{j} - (2\mu+1)\hat{k}$ are skew lines. Hence, find the shortest distance between the lines.

OR

(b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicket keeper divides the line segment joining the bowler and the leg slip fielder.

31. (a) The probability distribution for the number of students being absent in a class on a Saturday is as follows:

X	0	2	4	5
$P(X)$	p	$2p$	$3p$	p

where X is the number of students absent.

(i) Find the value of p . (1)

(ii) Find the mean of the number of students absent on Saturday. (2)

OR

(b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two-third of the total applicants were females and the rest were males. Selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in the written test is 0.4 and that of a female getting a distinction is 0.35. Find the probability that a candidate chosen at random will have a distinction in the written test.

Section D

32. A school wants to allocate students into three clubs: Sports, Music and Drama, under the following conditions:

- The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club.
- The number of students in Music club should be 20 more than half the number of students in Sports club.
- The total number of students to be allocated in all three clubs is 180.

Find the number of students allocated to different clubs, using matrix method.

33. Find: $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$.

34. (a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

(b) If $x = a \left(\cos \theta + \log \tan \frac{\theta}{2} \right)$ and $y = a \sin \theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

35. (a) Find the image A' of the point $A(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A' .

OR

(b) Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point $Q(2, 4, -1)$ is 7 units. Also, find the equation of the line joining P and Q .

Section E

36. A class-room teacher is keen to assess the learning of her students on the concept of "relations" taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$:

$$R_1 = \{(2, 3), (3, 2)\}$$

$$R_2 = \{(1, 2), (1, 3), (3, 2)\}$$

$$R_3 = \{(1, 2), (2, 1), (1, 1)\}$$

$$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$$

$$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$$

The students are asked to answer the following questions about the above relations:

- (i) Identify the relation which is reflexive, transitive but not symmetric. (1)
- (ii) Identify the relation which is reflexive and symmetric but not transitive. (1)
- (iii)(a) Identify the relations which are symmetric but neither reflexive nor transitive. (2)

OR

- (iii)(b) What pairs should be added to the relation R_2 to make it an equivalence relation? (2)

37. A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails a loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing a loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively.

Based on the above information, answer the following:

- (i) What is the probability that a customer after availing the loan will default on the loan repayment? (2)
- (ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest? (2)

38. A technical company is designing a rectangular solar panel installation on a roof, using 300 metres of boundary material. The design includes a partition running parallel to one side, dividing the panel into two parts. Let the side perpendicular to the partition have length x metres, and the side parallel to it have length y metres.

Based on the above information, answer the following:

- (i) Form the equation in x and y using the total boundary material used in the boundary and the parallel partition. (1)
- (ii) Write the area of the solar panel as a function of x . (1)
- (iii)(a) Find the critical point of the area function. Using the second derivative, find the critical point at which the area is maximum. Also find the maximum area. (2)

OR

- (iii)(b) Using the first-derivative test, find the maximum area enclosed using 300 metres of boundary material, where the parallel partition is also taken into account. (2)

- End of Question Paper -