

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. The projection vector of vector $\vec{a}$ on vector $\vec{b}$ is

- (A) $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b}$ (B) $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$
 (C) $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{b}$ (D) $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \left(\frac{\vec{a}}{|\vec{a}|} \right)$

2. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval

- (A) $(0, 2)$ (B) $(-\infty, 2]$
 (C) $[1, 2]$ (D) $[2, \infty)$

3. If $f(2a - x) = f(x)$, then $\int_0^{2a} f(x) dx$ is

- (A) $\int_0^{2a} f(x) dx$ (B) $\int_0^a f(x) dx$
 (C) $2 \int_a^0 f(x) dx$ (D) $2 \int_0^a f(x) dx$

4. If $A = \begin{pmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{pmatrix}$ is a symmetric matrix, then $(2x + y)$ is

- (A) -8 (B) 0
 (C) 6 (D) 8

5. If $y = \sin^{-1} x$, $-1 \leq x < 0$, then the range of y is

- (A) $\left[-\frac{\pi}{2}, 0\right]$ (B) $\left[-\frac{\pi}{2}, 0\right)$
 (C) $\left(-\frac{\pi}{2}, 0\right]$ (D) $\left(-\frac{\pi}{2}, 0\right)$

6. If a line makes angles of $\frac{\pi}{3}$, $\frac{\pi}{2}$ and θ with the positive directions of x , y and z -axis respectively, then θ is

- (A) only $\frac{\pi}{6}$ (B) only $\frac{\pi}{3}$
 (C) $\frac{\pi}{6}$ (D) $\pm \frac{\pi}{3}$

7. If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(\bar{E}/\bar{F})$ is

- (A) $\frac{P(E)}{P(F)}$ (B) $1 - P(E/F)$
 (C) $1 - P(\bar{E}/F)$ (D) $\frac{1 - P(E \cup F)}{P(\bar{F})}$

8. Which of the following can be both a symmetric and skew-symmetric matrix?

- (A) Unit Matrix (B) Diagonal Matrix
 (C) Null Matrix (D) Row Matrix

9. The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point $(4, -3, 7)$ is

- (A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$ (B) $x = 3t + 4, y = t + 3, z = 2t + 7$
 (C) $x = 3t + 4, y = t - 3, z = 2t + 7$ (D) $x = 3t + 4, y = t + 3, z = 2t + 7$

10. Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify $4AB + 3(AB + BA) - 4BA$, where A and B are both matrices of order 2×2 . It is known that $A \neq B \neq I$ and $A^{-1} \neq B$.

Their answers are:

Abhay: $6AB$

Bina: $7AB - BA$

Chhaya: $8AB$

Devesh: $7BA - AB$

Who answered it correctly?

- (A) Abhay (B) Bina
(C) Chhaya (D) Devesh

11. A cylindrical tank of radius 10 cm is being filled with sugar at the rate of $100\pi \text{ cm}^3/\text{s}$. The rate, at which the height of the sugar inside the tank is increasing, is

- (A) 0.1 cm/s (B) 0.5 cm/s
(C) 1 cm/s (D) 1.1 cm/s

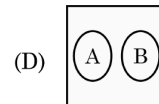
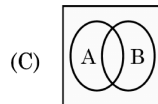
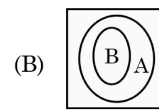
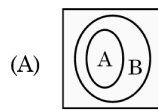
12. Let $\vec{p}$ and $\vec{q}$ be two unit vectors and α be the angle between them. Then $(\vec{p} + \vec{q})$ will be a unit vector for what value of α ?

- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$
(C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$

13. The line $x = 1 + 3\mu$, $y = -5 + \mu$, $z = -6 - 3\mu$ passes through which of the following point?

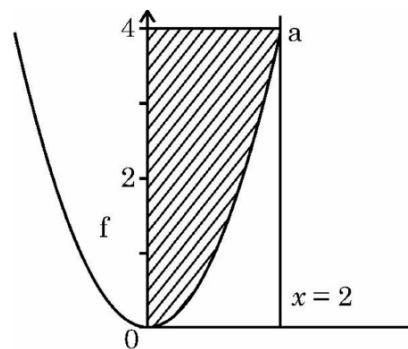
- (A) $(1, -5, 6)$ (B) $(1, 5, 6)$
(C) $(1, -5, -6)$ (D) $(-1, -5, 6)$

14. If A denotes the set of continuous functions and B denotes the set of differentiable functions, then which of the following depicts the correct relation between set A and B ?



- (A) B is a subset inside A (B) A is a subset inside B
(C) A and B intersect partially (D) A and B are disjoint

15. The area of the shaded region represented by the curves $y = x^2$, $0 \leq x \leq 2$ and the y -axis is given by



- (A) $\int_0^2 x^2 dx$ (B) $\int_0^2 \sqrt{y} dy$
(C) $\int_0^4 x^2 dx$ (D) $\int_0^4 \sqrt{y} dy$

16. A factory produces two products X and Y . The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct?

- | | |
|--|--|
| (A) The objective function maximizes the difference of the profit earned from products X and Y . | (B) The objective function measures the total production of products X and Y . |
| (C) The objective function maximizes the combined profit earned from selling X and Y . | (D) The objective function ensures the company produces more of product X than product Y . |

17. If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct?

- | | |
|------------------------|------------------------|
| (A) $A = B$ | (B) $AB = BA$ |
| (C) $A = O$ or $B = O$ | (D) $A = I$ or $B = I$ |

18. If p and q are respectively the order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$, then $(p - q)$ is

- | | |
|-------|-------|
| (A) 0 | (B) 1 |
| (C) 2 | (D) 3 |

19. **Assertion (A):** $A = \text{diag}[3 \ 5 \ 2]$ is a scalar matrix of order 3×3 .

Reason (R): If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.

- | | |
|--|---|
| (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). | (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). |
| (C) Assertion (A) is true but Reason (R) is false. | (D) Assertion (A) is false but Reason (R) is true. |

20. **Assertion (A):** Every point of the feasible region of a Linear Programming Problem is an optimal solution.

Reason (R): The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region.

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|--|---|
| (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). | (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). |
| (C) Assertion (A) is true but Reason (R) is false. | (D) Assertion (A) is false but Reason (R) is true. |

Section B

21. (a) A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$.

OR

(b) If $\vec{a}$ and $\vec{b}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $\vec{QR} = \frac{3}{2}\vec{QP}$.

22. Evaluate: $\int_0^{\pi/4} \sqrt{1 + \sin 2x} \, dx$

23. Find the values of 'a' for which $f(x) = \sin x - ax + b$ is increasing on $\mathbb{R}$.

24. If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors, then find x , such that $\vec{\alpha} = (x - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (3 + 2x)\vec{a} - 2\vec{b}$ are collinear.

25. (a) If $x^x = e^y$, then prove that $\frac{dy}{dx} = \frac{x - y}{x \log x}$.

OR

(b) If $f(x) = \begin{cases} 2x - 3, & -3 \leq x < -2 \\ x + 1, & -2 \leq x < 0 \end{cases}$, check the differentiability of $f(x)$ at $x = -2$.

Section C

26. (a) Solve the differential equation $2(y + 3) - xy \frac{dy}{dx} = 0$; given $y(1) = -2$.

OR

(b) Solve the following differential equation:
 $(1 + y^2) \frac{dx}{dy} + 2xy = 4x^2$.

27. Let R be a relation defined over $\mathbb{N}$, where $\mathbb{N}$ is set of natural numbers, defined as " mRn if and only if m is a multiple of n , $m, n \in \mathbb{N}$ ". Find whether R is reflexive, symmetric and transitive or not.

28. Solve the following linear programming problem graphically:

Minimise $Z = x - 5y$ subject to the constraints:

$$x - y \geq 0$$

$$-x + 2y \geq 2$$

$$x \geq 3, y \leq 4, y \geq 0$$

29. (a) If $y = \log \left(x + \frac{1}{x} \right)^2$, then show that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.

OR

(b) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, $-1 < x < 1$, $x \neq y$, then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.

30. (a) A die with number 1 to 6 is biased such that $P(2) = \frac{1}{4}$ and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the dice, if the dice is thrown twice.

OR

(b) Two dice are thrown. Defined are the following two events A and B :

$A = \{(x, y) : x + y = 9\}$, $B = \{(x, y) : x \neq 3\}$, where (x, y) denote a point in the sample space.

Check if events A and B are independent or mutually exclusive.

31. Find: $\int \frac{1}{x\sqrt{x-a}} dx$.

Section D

32. Using integration, find the area of the region bounded by the line $y = 5x + 2$, the x -axis and the ordinates $x = -2$ and $x = 2$.

33. Find: $\int \frac{x^2}{(x+2)(x^2+1)} dx$.

34. (a) Find the shortest distance between the lines:

$$\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3} \text{ and } \frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}.$$

OR

(b) Find the image A' of the point $A(2, 1, 2)$ in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA' . Find the foot of perpendicular from point A on the line l .

35. (a) Given $A = \begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}$, find AB . Hence, solve the system of linear equations:

$$\begin{aligned} x - y + z &= 4 \\ x - 2y - 2z &= 9 \\ 2x + y + 3z &= 1 \end{aligned}$$

OR

(b) If $A = \begin{pmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{pmatrix}$, then find A^{-1} .

Hence, solve the system of linear equations:

$$\begin{aligned} x - 2y &= 10 \\ 2x - y - z &= 8 \\ -2y + z &= 7 \end{aligned}$$

36. A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$.

Based on the above, answer the following:

(i) How many relations can be there from S to J ? [1]

(ii) A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_3), (S_4, J_2)\}$. Check if it is bijective. [1]

(iii) (a) How many one-one functions can be there from set S to set J ? [2]

OR

(iii) (b) Another student considers a relation $R_1 = \{(S_1, S_2), (S_2, S_1)\}$ in set S . Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. [2]

37. Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated).

Based on the above, answer the following:

(i) (a) What is the probability that a randomly selected car is an electric car? [2]

OR

(i) (b) What is the probability that a randomly selected car is a petrol car? [2]

(ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet? [1]

(iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi? [1]

38. A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres.

Based on the above, answer the following:

(i) Find the intervals on which the $f(x)$ is increasing or decreasing, $x \in [0, \pi]$. [2]

(ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion. [2]

- End of Question Paper -