

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. If $\vec{p}$ and $\vec{q}$ are unit vectors, then which of the following values of $\vec{p} \cdot \vec{q}$ is not possible?

- (A) $\frac{-1}{2}$ (B) $\frac{1}{\sqrt{2}}$
(C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{3}$

2. Which of the following can be both a symmetric and skew-symmetric matrix?

- (A) Unit Matrix (B) Diagonal Matrix
(C) Null Matrix (D) Row Matrix

3. If $\int_0^a x dx < \frac{a^2}{2} + 6$, then

- (A) $-4 < a < 3$ (B) $a > 4, a < -3$
(C) $-3 < a < 4$ (D) $-3 < a < 0$

4. If A and B are square matrices of same order, then $(AB^T - BA^T)$ is a

- (A) symmetric matrix (B) skew-symmetric matrix
(C) null matrix (D) unit matrix

5. The value of $\cos\left(\frac{\pi}{3} + \cot^{-1} \sqrt{3}\right)$ is

- (A) -1 (B) $\frac{-\sqrt{3}}{2}$
(C) 0 (D) 1

6. If p and q are respectively the order and degree of the differential equation

$$\frac{d}{dx} \left(\left(\frac{dy}{dx} \right)^3 \right) = 0, \text{ then } (p - q) \text{ is}$$

- (A) 0 (B) 1
(C) 2 (D) 3

7. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval

- (A) $(0, 2)$ (B) $(-\infty, 2]$
(C) $[1, 2]$ (D) $[2, \infty)$

8. The line $x = 1 + 5\mu, y = -5 + \mu, z = -6 - 3\mu$ passes through which of the following point?

- (A) $(1, -5, 6)$ (B) $(1, 5, 6)$
(C) $(1, -5, -6)$ (D) $(-1, -5, 6)$

9. The area of the shaded region represented by the curves $y = x^2, 0 \leq x \leq 2$ and the y -axis is given by

- (A) $\int_0^2 x^2 dx$ (B) $\int_0^2 \sqrt{y} dy$
(C) $\int_0^4 x^2 dx$ (D) $\int_0^4 \sqrt{y} dy$

10. If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(\bar{E}/\bar{F})$ is

- (A) $\frac{P(E)}{P(F)}$ (B) $1 - P(E/F)$
(C) $1 - P(\bar{E}/F)$ (D) $\frac{1 - P(E \cup F)}{P(\bar{F})}$

11. The probability distribution of a random variable X is given by:

X	-4	-3	-2	-1	0
P(X)	0.1	0.2	0.3	0.2	0.2

Then $E(X)$ of distribution is

- (A) -1.8 (B) -1
(C) 1 (D) 1.8

12. If projection of $\vec{a} = a\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units, then a is

- (A) -13 (B) -5
(C) 13 (D) 5

13. The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point $(4, -3, 7)$ is

- (A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$ (B) $x = 3t + 4, y = t + 3, z = 2t + 7$
 (C) $x = 3t + 4, y = t - 3, z = 2t + 7$ (D) $x = 3t + 4, y = t + 3, z = 2t + 7$

14. If a line makes angles of $\frac{3\pi}{4}, \frac{\pi}{3}$ and θ with the positive directions of x, y and z -axis respectively, then θ is

- (A) $\frac{-\pi}{3}$ only (B) $\frac{\pi}{3}$ only
 (C) $\frac{\pi}{6}$ (D) $\pm \frac{\pi}{3}$

15. A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct?

- (A) The objective function maximizes the difference of the profit earned from products X and Y. (B) The objective function measures the total production of products X and Y.
 (C) The objective function maximizes the combined profit earned from selling X and Y. (D) The objective function ensures the company produces more of product X than product Y.

16. If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set A and B?



- (A) A inside B (B) B inside A
 (C) A and B overlapping (D) A and B disjoint

17. Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify $4AB + 3(AB + BA) - 4BA$, where A and B are both matrices of order 2×2 . It is known that $A \neq B \neq I$ and $A^{-1} \neq B$. Their answers are given as: Abhay: $6AB$, Bina: $7AB - BA$, Chhaya: $8AB$, Devesh: $7BA - AB$. Who answered it correctly?

- (A) Abhay (B) Bina
 (C) Chhaya (D) Devesh

18. If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct?

- (A) $A = B$ (B) $AB = BA$
 (C) $A = 0$ or $B = 0$ (D) $A = I$ or $B = I$

19. **Assertion (A):** Every point of the feasible region of a Linear Programming Problem is an optimal solution.

Reason (R): The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region.

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
 (C) Assertion (A) is true but Reason (R) is false. (D) Assertion (A) is false but Reason (R) is true.

20. **Assertion (A):** $A = \text{diag} [3 \ 5 \ 2]$ is a scalar matrix of order 3×3 .

Reason (R): If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
 (C) Assertion (A) is true but Reason (R) is false. (D) Assertion (A) is false but Reason (R) is true.

Section B

21. Find the values of 'a' for which $f(x) = \sin x - ax + b$ is increasing on $\mathbb{R}$.

22. Find: $\int 2x^3 e^{x^2} dx$.

23. (a) If $x^x = e^y$, then prove that $\frac{dy}{dx} = \frac{x-y}{x \log x}$.

OR

(b) If $f(x) = \begin{cases} 2x-3, & -3 \leq x < -2 \\ x+1, & -2 \leq x < 0 \end{cases}$, check the differentiability of $f(x)$ at $x = -2$.

24. If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, then evaluate $|\vec{a} + 2\vec{b}|$.

25. (a) A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$.

OR

(b) If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $QR = \frac{3}{2}QP$.

Section C

26. (a) If $y = \log \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2$, then show that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.

OR

(b) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, $-1 < x < 1$, $x \neq y$, then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.

27. Let R be a relation on set of real numbers $\mathbb{R}$ defined as $R = \{(x, y) : x - y + \sqrt{3} \text{ is an irrational number, } x, y \in \mathbb{R}\}$. Verify R for reflexivity, symmetry and transitivity.

28. Solve the following linear programming problem graphically:

Minimise $Z = 2x + y$ subject to the constraints:

$$3x + y \geq 9$$

$$x + y \geq 7$$

$$x + 2y \geq 8$$

$$x, y \geq 0$$

29. (a) A die with number 1 to 6 is biased such that $P(2) = \frac{3}{10}$ and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the dice, if the dice is thrown twice.

OR

(b) Two dice are thrown. Defined are the following two events A and B :

$A = \{(x, y) : x + y = 9\}$, $B = \{(x, y) : x \neq 3\}$, where (x, y) denote a point in the sample space.

Check if events A and B are independent or mutually exclusive.

30. (a) Solve the differential equation $2(y+3) - xy \frac{dy}{dx} = 0$; given $y(1) = -2$.

OR

(b) Solve the following differential equation: $(1+x^2) \frac{dy}{dx} + 2xy = 4x^2$.

31. If $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$, then find the values of a and b .

Section D

32. (a) Find the shortest distance between the lines:

$$\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3} \text{ and } \frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}.$$

OR

(b) Find the image A' of the point $A(2, 1, 2)$ in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA' . Find the foot of perpendicular from point A on the line l .

33. Find: $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$.

34. Using integration, find the area of the region bounded by the line $y = 5x + 2$, the x-axis and the ordinates $x = -2$ and $x = 2$.

35. (a) Given $A = \begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}$, find AB . Hence, solve the system of linear equations:

$$\begin{aligned} x - y + z &= 4 \\ x - 2y - 2z &= 9 \\ 2x + y + 3z &= 1 \end{aligned}$$

OR

(b) If $A = \begin{pmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{pmatrix}$, then find A^{-1} .

Hence, solve the system of linear equations:

$$\begin{aligned} x - 2y &= 10 \\ 2x - y - z &= 8 \\ -2y + z &= 7 \end{aligned}$$

36. A school is organizing a debate competition with participants as speakers $S = \{S1, S2, S3, S4\}$ and these are judged by judges $J = \{J1, J2, J3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$.

Based on the above, answer the following:

- (i) How many relations can be there from S to J ?
- (ii) A student identifies a function from S to J as $f = \{(S1, J1), (S2, J2), (S3, J2), (S4, J3)\}$. Check if it is bijective.
- (iii) (a) How many one-one functions can be there from set S to set J ?

OR

- (iii) (b) Another student considers a relation $R1 = \{(S1, S2), (S2, S4)\}$ in set S . Write minimum ordered pairs to be included in $R1$ so that $R1$ is reflexive but not symmetric.

37. Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated).

Based on the above, answer the following:

- (i) (a) What is the probability that a randomly selected car is an electric car?

OR

- (i) (b) What is the probability that a randomly selected car is a petrol car?
- (ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet?
- (iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi?

38. A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres.

Based on the above, answer the following:

- (i) Find the intervals on which $f(x)$ is increasing or decreasing, $x \in [0, \pi]$.
- (ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion.

- End of Question Paper -