

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. The principal branch of $\cos^{-1} x$ is:

- (A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (B) $[\pi, 2\pi]$
(C) $[0, \pi]$ (D) $[2\pi, 3\pi]$

2. If A and B are square matrices of same order such that $AB = A$ and $BA = B$, then $A^2 + B^2$ is equal to:

- (A) $A + B$ (B) BA
(C) $2(A + B)$ (D) $2BA$

3. For real x , let $f(x) = x^3 + 5x + 1$. Then:

- (A) f is one-one but not onto on $\mathbb{R}$ (B) f is onto on $\mathbb{R}$ but not one-one
(C) f is one-one and onto on $\mathbb{R}$ (D) f is neither one-one nor onto on $\mathbb{R}$

4. If $f(x) = \begin{cases} \frac{1 - \sin^3 x}{3 \cos^2 x}, & \text{for } x \neq \frac{\pi}{2} \\ k, & \text{for } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then the value of k is:

- (A) $\frac{1}{2}$ (B) $\frac{1}{6}$
(C) $\frac{2}{3}$ (D) 1

5. The values of λ so that $f(x) = \sin x - \cos x - \lambda x + C$ decreases for all real values of x are:

- (A) $1 < \lambda < \sqrt{2}$ (B) $\lambda \geq 1$
(C) $\lambda \geq \sqrt{2}$ (D) $\lambda < 1$

6. If the direction cosines of a line are $\lambda, \lambda, \lambda$, then λ is equal to:

- (A) $-\frac{1}{\sqrt{3}}$ (B) 1
(C) $\frac{1}{\sqrt{3}}$ (D) $\pm \frac{1}{\sqrt{3}}$

7. If $\begin{vmatrix} -1 & 2 & 4 \\ 1 & x & 1 \\ 0 & 3 & 3x \end{vmatrix} = -57$, then the product of the possible values of x is:

- (A) -24 (B) -16
(C) 16 (D) 24

8. The matrix $\begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & -7 \\ 2 & 7 & 0 \end{pmatrix}$ is a:

- (A) diagonal matrix (B) symmetric matrix
(C) skew symmetric matrix (D) scalar matrix

9. If $f(x) = \begin{cases} 3x - 2, & 0 < x < 1 \\ 2x^2 + ax, & 1 \leq x < 2 \end{cases}$ is continuous for $x \in (0, 2)$, then a is equal to:

- (A) -4 (B) $-\frac{7}{2}$
(C) -2 (D) -1

10. If $f: \mathbb{N} \rightarrow W$ is defined as $f(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}$, then f is:

- (A) injective only (B) surjective only
(C) a bijection (D) neither surjective nor injective

11. If $f(x) = 2x + \cos x$, then $f(x)$:

- (A) has a maxima at $x = \pi$ (B) has a minima at $x = \pi$
(C) is an increasing function (D) is a decreasing function

12. If the sides AB and AC of $\triangle ABC$ are represented by vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ respectively, then the length of the median through A on BC is:

- (A) $2\sqrt{2}$ units (B) $\sqrt{18}$ units
(C) $\frac{\sqrt{34}}{2}$ units (D) $\frac{\sqrt{48}}{2}$ units

13. The function f defined by $f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$ is not continuous at:

- (A) $x = 0$ (B) $x = 1$
(C) $x = 2$ (D) $x = 5$

14. $\int e^x (\cos x - \sin x) dx$ is equal to:

- (A) $e^x \sin x + C$ (B) $-e^x \sin x + C$
(C) $-e^x \cos x + C$ (D) $e^x \cos x + C$

15. The area of the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 0$ and $x = 4$ and the x -axis is:

- (A) $\frac{16}{3}$ sq. units (B) $\frac{32}{3}$ sq. units
(C) $\frac{8}{3}$ sq. units (D) $\frac{4}{3}$ sq. units

16. The integrating factor of the differential equation $\frac{dy}{dx} + y \tan x - \sec x = 0$ is:

- (A) $-\cos x$ (B) $\sec x$
(C) $\log \sec x$ (D) $e^{\sec x}$

17. The corner points of the feasible region of a Linear Programming Problem are $(0, 2)$, $(3, 0)$, $(6, 0)$, $(6, 8)$ and $(0, 5)$. If $Z = ax + by$; ($a, b > 0$) be the objective function, and maximum value of Z is obtained at $(0, 2)$ and $(3, 0)$, then the relation between a and b is:

- (A) $a = b$ (B) $a = 3b$
(C) $b = 6a$ (D) $3a = 2b$

18. The value of $\int_0^1 \frac{1}{e^x + e^{-x}} dx$ is:

- (A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$
(C) $\tan^{-1} e - \frac{\pi}{4}$ (D) $\tan^{-1} e$

19. **Assertion (A):** If A and B are two events such that $P(A \cap B) = 0$, then A and B are independent events.

Reason (R): Two events are independent if the occurrence of one does not effect the occurrence of the other.

Select the correct answer from the codes given below:

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** In a Linear Programming Problem, if the feasible region is empty, then the Linear Programming Problem has no solution.

Reason (R): A feasible region is defined as the region that satisfies all the constraints.

Select the correct answer from the codes given below:

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

Section B

21. Using matrices and determinants, find the value(s) of k for which the pair of equations $5x - ky = 2$; $7x - 5y = 3$ has a unique solution.

22. (a) Simplify $\sin^{-1} \left(\frac{x}{\sqrt{1+x^2}} \right)$.

OR

(b) Find the domain of $\sin^{-1} \sqrt{x-1}$.

23. Calculate the area of the region bounded by the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the x -axis using integration.

24. (a) Find the least value of ' a ' so that $f(x) = 2x^2 - ax + 3$ is an increasing function on $[2, 4]$.

OR

(b) If $f(x) = x + \frac{1}{x}$, $x \geq 1$, show that f is an increasing function.

25. Find the local maxima and local minima of the function $f(x) = x^3 - 12x^2 + 18x + 5$.

Section C

26. (a) Find the probability distribution of the number of boys in families having three children, assuming equal probability for a boy and a girl.

OR

(b) A coin is tossed twice. Let X be a random variable defined as number of heads minus number of tails. Obtain the probability distribution of X and also find its mean.

27. (a) If $f : \mathbb{R}^+ \rightarrow \mathbb{R}$, $f(x) = \log_a x$ ($a > 0$ and $a \neq 1$) is defined, then prove that f is a bijection. ($\mathbb{R}^+$ is the set of all positive real numbers.)

OR

(b) Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. A relation R from A to B is defined as $R = \{(x, y) : x + y = 6, x \in A, y \in B\}$.

(i) Write all elements of R .

(ii) Is R a function? Justify.

(iii) Determine the domain and range of R .

28. Find: $\int \frac{\cos x}{1 + \cos x + \sin x} dx$.

29. (a) Consider the experiment of tossing a coin. If the coin shows head, toss it again; but if it shows a tail, then throw a die. Find the conditional probability of the event A : 'the die shows a number greater than 3' given that B : 'there is at least one tail'.

OR

(b) The probability distribution of a random variable X is given as:

X	1	2	3	2λ	3λ	4λ
$P(X)$	$\frac{1}{30}$	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{15}$	$\frac{1}{10}$

(i) Calculate λ , if $E(X) = 3.2$.

(ii) Find $P(X > 1)$.

30. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.

31. Solve the following Linear Programming Problem graphically:

Minimise $Z = 3x + 5y$, subject to the constraints

$$x + 2y \geq 10$$

$$x + y \geq 6$$

$$3x + y \geq 8$$

$$x, y \geq 0.$$

Section D

32. The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight.

(i) Find the rate of growth of the plant with respect to sunlight.

(ii) In how many days will the plant attain its maximum height? What is the maximum height?

33. If A is a 3×3 invertible matrix, show that for any scalar $k \neq 0$, $(kA)^{-1} = \frac{1}{k}A^{-1}$. Hence calculate $(3A)^{-1}$, where $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

34. (a) Evaluate: $\int_0^{\pi/4} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$.

OR

(b) Find: $\int \frac{dx}{x\sqrt{x^2+1} [\log(x^2+1) - 2\log x]}$.

35. (a) Show that the area of a parallelogram whose diagonals are represented by $\vec{a}$ and $\vec{b}$ is given by $\frac{1}{2}|\vec{a} \times \vec{b}|$. Also find the area of a parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.

OR

(b) Find the equation of a line in vector and cartesian form which passes through the point $(1, 2, -4)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

36. Case Study - 1

Some students are having a misconception while comparing decimals. For example, a student may mention that $78.56 > 78.9$ as $7856 > 789$. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question:

In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table:

Ajay: 47.7 m; Bijoy: 47.07 m; Kartik: 43.09 m; Dinesh: 43.9 m; Devesh: 45.2 m.

The students were asked to identify who has thrown the javelin the farthest.

Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions:

- (i) What is the probability of a student not having misconception but still answers Bijoy in the test?
- (ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test?
- (iii) (a) What is the probability that a student who answered as Bijoy is having misconception?

OR

- (iii) (b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception?

37. Case Study - 2

An engineer is designing a new metro rail network in a city.

Initially, two metro lines, Line A and Line B, each consisting of multiple stations are designed. The track for Line A is represented by $l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4}$, while the track for Line B is represented by $l_2 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}$.

Based on the above information, answer the following questions:

- (i) Find whether the two metro tracks are parallel.
- (ii) Solar panels are to be installed on the rooftop of the metro stations. Determine the equation of the line representing the placement of solar panels on the rooftop of Line A's stations, given that panels are to be positioned parallel to Line A's track (l_1) and pass through the point $(1, -2, -3)$.
- (iii) (a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point $(3, 2, 1)$. Determine the equation of the pedestrian walkway.

OR

- (iii) (b) Find the shortest distance between Line A and Line B.

38. Case Study - 3

During a heavy gaming session, the temperature of a student's laptop processor increases significantly. After the session, the processor begins to cool down, and the rate of cooling is proportional to the difference between the processor's temperature and the room temperature (25°C). Initially the processor's temperature is 85°C . The rate of cooling is defined by the equation $\frac{d}{dt}(T(t)) = -k(T(t) - 25)$, where $T(t)$ represents the temperature of the processor at time t (in minutes) and k is a constant.

Based on the above information, answer the following questions:

- (i) Find the expression for the temperature of the processor, $T(t)$, given that $T(0) = 85^{\circ}\text{C}$.
- (ii) How long will it take for the processor's temperature to reach 40°C ? Given that $k = 0.03$, $\log_e 4 = 1.3863$.

- End of Question Paper -