

## Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

### General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

## Section A

1. If  $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$  is continuous at  $x = 0$ , then the value of  $a$  is:

- (A) 1 (B) -1  
(C)  $\pm 1$  (D) 0

2. The principal value of  $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$  is:

- (A)  $-\frac{\pi}{3}$  (B)  $-\frac{2\pi}{3}$   
(C)  $\frac{\pi}{3}$  (D)  $\frac{2\pi}{3}$

3. If  $A$  and  $B$  are two square matrices of the same order, then  $(A + B)(A - B)$  is equal to:

- (A)  $A^2 - AB + BA - B^2$  (B)  $A^2 + AB - BA - B^2$   
(C)  $A^2 - AB - BA - B^2$  (D)  $A^2 - B^2 + AB + BA$

4. If  $A = [a_{ij}]$  is a  $3 \times 3$  diagonal matrix such that  $a_{11} = 1$ ,  $a_{22} = 5$  and  $a_{33} = -2$ , then  $|A|$  is:

- (A) 0 (B) -10  
(C) 10 (D) 1

5. If  $A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ , then  $A^3$  is:

- (A)  $3 \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$  (B)  $\begin{pmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{pmatrix}$   
(C)  $\begin{pmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{pmatrix}$  (D)  $\begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

6. If  $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$ , then the value of  $x$  is:

- (A) 3 (B) 7  
(C)  $\pm 7$  (D)  $\pm 3$

7. If  $P(A \cup B) = 0.9$  and  $P(A \cap B) = 0.4$ , then  $P(\overline{A}) + P(\overline{B})$  is:

- (A) 0.3 (B) 1  
(C) 1.3 (D) 0.7

8. If a matrix  $A$  is both symmetric and skew-symmetric, then  $A$  is a:

- (A) diagonal matrix (B) zero matrix  
(C) non-singular matrix (D) scalar matrix

9. The slope of the curve  $y = -x^3 + 3x^2 + 8x - 20$  is maximum at:

- (A)  $(1, -10)$  (B)  $(1, 10)$   
(C)  $(10, 1)$  (D)  $(-10, 1)$

10. The area of the region enclosed between the curve  $y = x|x|$ ,  $x$ -axis,  $x = -2$  and  $x = 2$  is:

- (A)  $\frac{8}{3}$  (B)  $\frac{16}{3}$   
(C) 0 (D) 8

11.  $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$  is equal to:

- (A)  $\cot x + \tan x + C$  (B)  $-(\cot x + \tan x) + C$   
(C)  $-\cot x + \tan x + C$  (D)  $\cot x - \tan x + C$

12. If  $\int_0^a \frac{1}{1 + 4x^2} dx = \frac{\pi}{8}$ , then the value of  $a$  is:

- (A)  $\frac{1}{4}$  (B)  $\frac{1}{2}$   
(C)  $\frac{1}{8}$  (D) 4

13. If  $f(x) = \{[x], x \in \mathbb{R}\}$  is the greatest integer function, then the correct statement is:

- (A)  $f$  is continuous but not differentiable at  $x = 2$ .  
 (B)  $f$  is neither continuous nor differentiable at  $x = 2$ .  
 (C)  $f$  is continuous as well as differentiable at  $x = 2$ .  
 (D)  $f$  is not continuous but differentiable at  $x = 2$ .

14. The integrating factor of the differential equation  $\frac{dx}{dy} = \frac{x \log x}{\frac{2}{x} \log x - y}$  is:

- (A)  $\frac{1}{\log x}$   
 (B)  $\frac{1}{x}$   
 (C)  $e^{\log x}$   
 (D)  $\log x$

15. Let  $\vec{a}$  be a position vector whose tip is the point  $(2, -3)$ . If  $\overrightarrow{AB} = \vec{a}$ , where coordinates of  $A$  are  $(-4, 5)$ , then the coordinates of  $B$  are:

- (A)  $(-2, -2)$   
 (B)  $(2, -2)$   
 (C)  $(-2, 2)$   
 (D)  $(2, 2)$

16. If  $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 512$  and  $|\vec{a}| = 3|\vec{b}|$ , then the respective values of  $|\vec{a}|$  and  $|\vec{b}|$  are:

- (A) 48 and 16  
 (B) 3 and 1  
 (C) 24 and 8  
 (D) 6 and 2

17. For a Linear Programming Problem (LPP), the given objective function  $Z = 3x + 2y$  is subject to constraints:  $x + 2y \leq 10$ ,  $3x + y \leq 15$ ,  $x, y \geq 0$ . The correct feasible region is:

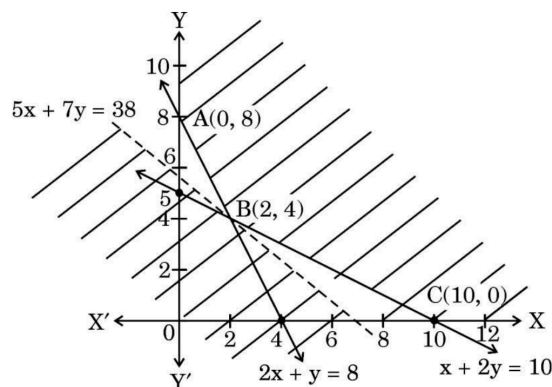
- (A)  $ABC$   
 (B)  $AOEC$   
 (C)  $CED$   
 (D) Open unbounded region  $BCD$

18. The sum of the order and degree of the differential equation  $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$  is:

- (A) 2  
 (B)  $\frac{5}{2}$   
 (C) 3  
 (D) 4

19. **Assertion (A):** The shaded portion of the graph represents the feasible region for the LPP: minimize  $Z = 50x + 70y$  subject to  $2x + y \geq 8$ ,  $x + 2y \geq 10$ ,  $x, y \geq 0$ , and  $Z$  has a minimum value = 380 at  $B(2, 4)$ .

**Reason (R):** The region representing  $50x + 70y < 380$  does not have any point common with the feasible region.



- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).  
 (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).  
 (C) Assertion (A) is true, but Reason (R) is false.  
 (D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** Let  $A = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$ . If  $f : A \rightarrow A$  is defined as  $f(x) = x^2$ , then  $f$  is not an onto function.

**Reason (R):** If  $y = -1 \in A$ , then  $x = \pm\sqrt{-1} \notin A$ .

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).  
 (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).  
 (C) Assertion (A) is true, but Reason (R) is false.  
 (D) Assertion (A) is false, but Reason (R) is true.

## Section B

21. Find the domain of  $\sec^{-1}(2x + 1)$ .

22. The radius of a cylinder is decreasing at a rate of 2 cm/s and the altitude is increasing at the rate of 3 cm/s. Find the rate of change of volume of this cylinder when its radius is 4 cm and altitude is 6 cm.

23. (a) Find a vector of magnitude 5 which is perpendicular to both the vectors  $3\hat{i} - 2\hat{j} + \hat{k}$  and  $4\hat{i} + 3\hat{j} - 2\hat{k}$ .

OR

(b) Let  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  be three vectors such that  $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$  and  $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ ,  $\vec{a} \neq \vec{0}$ . Show that  $\vec{b} = \vec{c}$ .

24. A man needs to hang two lanterns on a straight wire whose end points have coordinates  $A(4, 1, -2)$  and  $B(6, 2, -3)$ . He wants the lanterns to be hung at the points that trisect the wire  $AB$ . Find the coordinates of the points where he hangs the two lanterns.

25. (a) Differentiate  $\frac{\sin x}{\sqrt{\cos x}}$  with respect to  $x$ .

OR

(b) If  $y = 5 \cos x - 3 \sin x$ , prove that  $\frac{d^2y}{dx^2} + y = 0$ .

## Section C

26. Show that  $f(x) = \tan^{-1}(\sin x + \cos x)$  is an increasing function in  $\left(0, \frac{\pi}{4}\right)$ .

27. (a) The probability that a student buys a colouring book is 0.7 and that she buys a box of colours is 0.2. The probability that she buys a colouring book, given that she buys a box of colours, is 0.3. Find the probability that the student:

(i) Buys both the colouring book and the box of colours.

(ii) Buys a box of colours given that she buys the colouring book.

OR

(b) A person has a fruit box that contains 6 apples and 4 oranges. He picks out a fruit three times, one after the other, after replacing the previous one in the box. Find:

(i) The probability distribution of the number of oranges he draws.

(ii) The expectation of the random variable (number of oranges).

28. Find the particular solution of the differential equation  $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$ ; given that  $y = 0$ , when  $x = 1$ .

29. (a) Find:  $\int \frac{2x}{(x^2 + 3)(x^2 - 5)} dx$ .

OR

(b) Evaluate:  $\int_1^4 (|x - 2| + |x - 4|) dx$ .

30. In the Linear Programming Problem (LPP), find the point/points giving maximum value for  $Z = 5x + 10y$  subject to constraints  $x + 2y \leq 120$ ,  $x + y \geq 60$ ,  $x - 2y \geq 0$ ,  $x, y \geq 0$ .

31. (a) If  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$  such that  $|\vec{a}| = 3$ ,  $|\vec{b}| = 5$ ,  $|\vec{c}| = 7$ , then find the angle between  $\vec{a}$  and  $\vec{b}$ .

OR

(b) If  $\vec{a}$  and  $\vec{b}$  are unit vectors inclined with each other at an angle  $\theta$ , then prove that  $\frac{1}{2}|\vec{a} - \vec{b}| = \sin \frac{\theta}{2}$ .

## Section D

**32.** Draw a rough sketch of the curve  $y = \sqrt{x}$ . Using integration, find the area of the region bounded by the curve  $y = \sqrt{x}$ ,  $x = 4$  and  $x$ -axis, in the first quadrant.

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**33.** An amount of Rs. 10,000 is put into three investments at the rate of 10%, 12% and 15% per annum. The combined annual income of all three investments is Rs. 1,310, however the combined annual income of the first and the second investments is Rs. 190 short of the income from the third. Use matrix method and find the investment amount in each at the beginning of the year.

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**34.** (a) Find the foot of the perpendicular drawn from the point  $(1, 1, 4)$  on the line  $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$ .

OR

(b) Find the point on the line  $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-4}{6}$  at a distance of  $2\sqrt{14}$  units from the point  $(-1, -1, 2)$ .

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**35.** (a) For a positive constant  $a$ , differentiate  $a^{t+\frac{1}{t}}$  with respect to  $\left(t + \frac{1}{t}\right)^a$ , where  $t$  is a non-zero real number.

OR

(b) Find  $\frac{dy}{dx}$  if  $y^x + x^y + x^x = a^b$ , where  $a$  and  $b$  are constants.

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**36. Case Study – 3.** A gardener wanted to plant vegetables in his garden. Hence he bought 10 seeds of brinjal plant, 12 seeds of cabbage plant and 8 seeds of radish plant. The shopkeeper assured him of germination probabilities of brinjal, cabbage and radish to be 25%, 35% and 40% respectively. But before he could plant the seeds, they got mixed up in the bag and he had to sow them randomly.



Radish



Cabbage



Brinjal

Based upon the above information, answer the following questions:

- Calculate the probability of a randomly chosen seed to germinate.
- What is the probability that it is a cabbage seed, given that the chosen seed germinates?

**37. Case Study – 1.** A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.

On the basis of the above information, answer the following questions:

- Taking length = breadth =  $x$  m and height =  $y$  m, express the surface area ( $S$ ) of the box in terms of  $x$  and its volume ( $V$ ), which is constant.
- Find  $\frac{dS}{dx}$ .
- (a) Find a relation between  $x$  and  $y$  such that the surface area ( $S$ ) is minimum.

OR

- (b) If surface area ( $S$ ) is constant, the volume ( $V$ ) =  $\frac{1}{2}(Sx - 2x^3)$ ,  $x$  being the edge of base.

Show that volume ( $V$ ) is maximum for  $x = \sqrt{\frac{S}{6}}$ .

**38. Case Study – 2.** Let  $A$  be the set of 30 students of class XII in a school. Let  $f : A \rightarrow \mathbb{N}$ ,  $\mathbb{N}$  is a set of natural numbers such that function  $f(x)$  = Roll Number of student  $x$ .

On the basis of the given information, answer the following:

- Is  $f$  a bijective function?
- Give reasons to support your answer to (i).
- (a) Let  $R$  be a relation defined by the teacher to plan the seating arrangement of students in pairs, where  $R = \{(x, y) :$   
 $x, y$  are Roll Numbers of students such that  $y = 3x\}$ . List the elements of  $R$ . Is the relation  $R$  reflexive, symmetric and transitive? Justify your answer.

OR

- (b) Let  $R$  be a relation defined by  $R = \{(x, y) :$   
 $x, y$  are Roll Numbers of students such that  $y = x^3\}$ . List the elements of  $R$ . Is  $R$  a function? Justify your answer.

- End of Question Paper -