

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

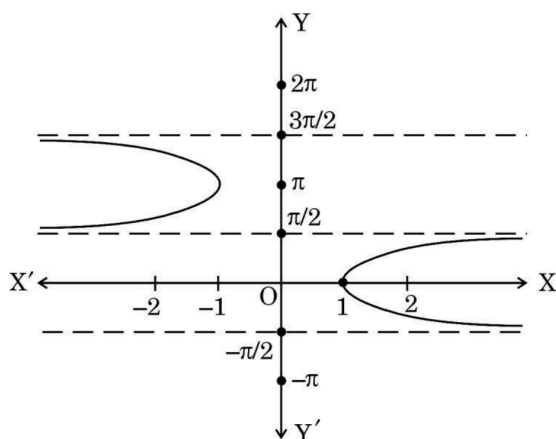
1. If $\tan^{-1}(x^2 - y^2) = a$, where a is a constant, then $\frac{dy}{dx}$ is:

- (A) $\frac{x}{y}$ (B) $-\frac{x}{y}$
(C) $\frac{y}{x}$ (D) $-\frac{y}{x}$

2. If $A = \begin{pmatrix} 0 & 0 & -5 \\ 0 & 3 & 0 \\ \frac{4}{3} & 0 & 0 \end{pmatrix}$, then A is a:

- (A) skew-symmetric matrix (B) scalar matrix
(C) diagonal matrix (D) square matrix

3. The graph shown below depicts



:

- (A) $y = \sec^{-1} x$ (B) $y = \sec x$
(C) $y = \csc^{-1} x$ (D) $y = \csc x$

4. $\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is equal to:

- (A) $\frac{11\pi}{12}$ (B) $\frac{5\pi}{12}$
(C) $-\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

5. Let both AB' and $B'A$ be defined for matrices A and B . If order of A is $n \times m$, then the order of B is:

- (A) $n \times n$ (B) $n \times m$
(C) $m \times m$ (D) $m \times n$

6. Sum of two skew-symmetric matrices of same order is always a/an:

- (A) skew-symmetric matrix (B) symmetric matrix
(C) null matrix (D) identity matrix

7. If $y = a \cos(\log x) + b \sin(\log x)$, then $x^2 y_2 + x y_1$ is:

- (A) $\cos(\log x)$ (B) y
(C) $-y$ (D) $\tan(\log x)$

8. If $f(x) = \begin{cases} \frac{\log(1+ax) + \log(1-bx)}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$

is continuous at $x = 0$, then the value of k is:

- (A) a (B) $a + b$
(C) $a - b$ (D) b

9. $f(x) = x^x$ has a critical point at:

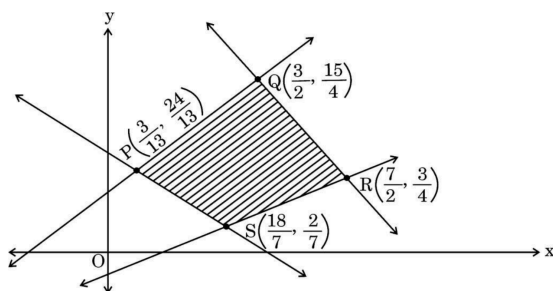
- (A) $x = e$ (B) $x = e^{-1}$
(C) $x = 0$ (D) $x = 1$

10. The solution for the differential equation

$\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is:

- (A) $3e^{-4y} + 4e^{3x} + C = 0$ (B) $e^{3x+4y} + C = 0$
(C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$

11. For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$. The feasible region $PQRS$ determined by the set of constraints is shown as a shaded region in the graph.



(Note: The figure is not to scale)

$$P = \left(\frac{3}{13}, \frac{24}{13}\right), Q = \left(\frac{3}{2}, \frac{15}{4}\right), R = \left(\frac{7}{2}, \frac{3}{4}\right), S = \left(\frac{18}{7}, \frac{2}{7}\right)$$

Which of the following statements is correct?

- (A) Z is minimum at $S\left(\frac{18}{7}, \frac{2}{7}\right)$ (B) Z is maximum at $R\left(\frac{7}{2}, \frac{3}{4}\right)$
 (C) (Value of Z at P) > (Value of Z at Q) (D) (Value of Z at Q) < (Value of Z at R)

12. The order and degree of the given differential equation $\left[\frac{d^2y}{dx^2} - 1\right]^2 = \frac{dy}{dx}$ are respectively:

- (A) 2, 2 (B) 2, not defined
 (C) 1, 2 (D) 1, not defined

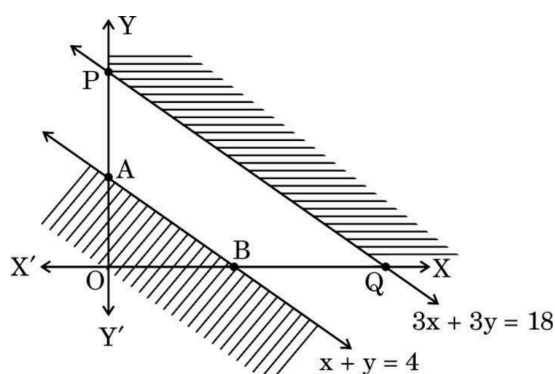
13. Let $f'(x) = 3(x^2 + 2x) - \frac{2}{x^3} + 5$, $f(1) = 0$. Then, $f(x)$ is:

- (A) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 1$ (B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$
 (C) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$ (D) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$

14. In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximised under the following constraints:

$$x + y \leq 4, 3x + 3y \geq 18, x, y \geq 0$$

Study the graph and select the correct option.



(Note: The figure is not to scale)

The solution of the given LPP:

- (A) lies in the shaded unbounded region. (B) lies in $\triangle AOB$.
 (C) does not exist. (D) lies in the combined region of $\triangle AOB$ and unbounded shaded region.

15. The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is:

- (A) $\frac{1}{3}$ sq units (B) $\frac{2}{3}$ sq units
(C) $\frac{3}{2}$ sq units (D) $\frac{4}{3}$ sq units

16. $\int \frac{x+5}{(x+6)^2} e^x dx$ is equal to:

- (A) $\log(x+6) + C$ (B) $e^x + C$
(C) $\frac{e^x}{x+6} + C$ (D) $\frac{e^x}{(x+6)^2} + C$

17. Let $|\vec{a}| = 5$ and $-2 \leq \lambda \leq 1$. Then, the range of $|\lambda \vec{a}|$ is:

- (A) $[5, 10]$ (B) $[-2, 5]$
(C) $[-2, 1]$ (D) $[-10, 5]$

18. A meeting will be held only if all three members A, B and C are present. The probability that member A does not turn up is 0.10, member B does not turn up is 0.20 and member C does not turn up is 0.05. The probability of the meeting being cancelled is:

- (A) 0.35 (B) 0.316
(C) 0.001 (D) 0.65

19. **Assertion (A):** If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 256$ and $|\vec{b}| = 8$, then $|\vec{a}| = 2$.

Reason (R): $\sin^2 \theta + \cos^2 \theta = 1$ and $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \theta$ and $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f+g)(x) = e^x + \log x$, where domain of $(f+g)$ is $\mathbb{R}$.

Reason (R): $\text{Dom}(f+g) = \text{Dom}(f) \cap \text{Dom}(g)$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Section B

21. (a) If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that $BC = 3BA$.

OR

(b) Vector $\vec{r}$ is inclined at equal angles to the three axes x , y and z . If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.

22. Find the domain of $f(x) = \sin^{-1}(-x^2)$.

23. Find the interval in which $f(x) = x + \frac{1}{x}$, $x \neq 0$ is always increasing.

24. (a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$, for $x > 0$.

OR

(b) If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.

25. Find the angle at which the given lines are inclined to each other:

$$l_1 : \frac{x-5}{2} = \frac{y+3}{1} = \frac{z-1}{-3}$$

$$l_2 : \frac{x}{3} = \frac{y-1}{2} = \frac{z+5}{-1}$$

Section C

26. Find the value of x , if

$$\begin{pmatrix} 1 & x & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ x \end{pmatrix} = O.$$

27. (a) Find the distance of the point $P(2, 4, -1)$ from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

OR

(b) Let the position vectors of the points A , B and C be $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC .

28. Consider the Linear Programming Problem, where the objective function $Z = (x + 4y)$ needs to be minimized subject to constraints

$$2x + y \geq 1000$$

$$x + 2y \geq 800$$

$$x, y \geq 0.$$

Draw a neat graph of the feasible region and find the minimum value of Z .

29. (a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in \mathbb{N}$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

OR

(b) Show that the function $f : \mathbb{N} \rightarrow \mathbb{N}$, where $\mathbb{N}$ is a set of natural numbers, given by $f(n) = \begin{cases} n-1, & \text{if } n \text{ is even} \\ n+1, & \text{if } n \text{ is odd} \end{cases}$ is a bijection.

30. (a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x , if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

OR

(b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x , when $x \in (0, 1)$.

31. Bag I contains 4 white and 5 black balls. Bag II contains 6 white and 7 black balls. A ball drawn randomly from bag I is transferred to bag II and then a ball is drawn randomly from bag II. Find the probability that the ball drawn is white.

Section D

32. (a) Solve the differential equation: $x^2y \, dx - (x^3 + y^3) \, dy = 0$.

OR

(b) Solve the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.

33. Using integration, find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ bounded between the lines $x = -\frac{a}{2}$ to $x = \frac{a}{2}$.

34. (a) Find: $\int \frac{x^2 + 1}{(x - 1)^2(x + 3)} \, dx$

OR

(b) Evaluate: $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} \, dx$

35. Show that the line passing through the points $A(0, -1, -1)$ and $B(4, 5, 1)$ intersects the line joining points $C(3, 9, 4)$ and $D(-4, 4, 4)$.

36. Case Study - 2

A ladder of fixed length ' h ' is to be placed along the wall such that it is free to move along the height of the wall.

Based upon the above information, answer the following questions:

(i) Express the distance (y) between the wall and foot of the ladder in terms of ' h ' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer. [1]

(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point. [1]

(iii) (a) Show that the area (A) of the right triangle is maximum at the critical point. [2]

OR

(iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall? [2]

37. Case Study - 3

A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.

A person buys a smartphone from this shop.

(i) Find the probability that it was defective. [2]

(ii) What is the probability that this defective smartphone was manufactured by company B? [2]

38. Case Study - 1

Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays -60. Rani buys 2 pens, 4 notepads and 6 erasers for -90. Sam pays -70 for 6 pens, 2 notepads and 3 erasers.

Based upon the above information, answer the following questions:

(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form $AX = B$. [1]

(ii) Find $|A|$ and confirm if it is possible to find A^{-1} . [1]

(iii) (a) Find A^{-1} , if possible, and write the formula to find X . [2]

OR

(iii) (b) Find $A^2 - 8I$, where I is an identity matrix. [2]

- End of Question Paper -