

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

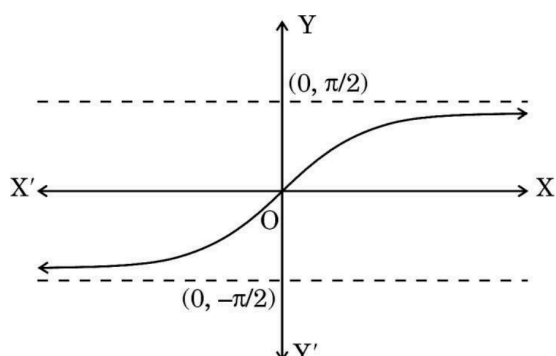
Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. The given graph (with horizontal asymptote $y = -\frac{\pi}{2}$) illustrates:



- (A) $y = \tan^{-1} x$ (B) $y = \operatorname{cosec}^{-1} x$
(C) $y = \cot^{-1} x$ (D) $y = \sec^{-1} x$

2. Domain of $f(x) = \cos^{-1} x + \sin x$ is:

- (A) $\mathbb{R}$ (B) $(-1, 1)$
(C) $[-1, 1]$ (D) φ

3. What is the total number of possible matrices of order 3×3 with each entry as $\sqrt{2}$ or $\sqrt{3}$?

- (A) 9 (B) 512
(C) 615 (D) 64

4. The matrix $A = \begin{pmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{pmatrix}$ is a/an:

- (A) scalar matrix (B) identity matrix
(C) null matrix (D) symmetric matrix

5. If A and B are two square matrices each of order 3 with $|A| = 3$ and $|B| = 5$, then $|2AB|$ is:

- (A) 30 (B) 120
(C) 15 (D) 225

6. Let A be a square matrix of order 3. If $|A| = 5$, then $|\operatorname{adj} A|$ is:

- (A) 5 (B) 125
(C) 25 (D) -5

7. If $\begin{vmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{vmatrix} = \begin{vmatrix} x+3 & 12 \\ 0 & 35 \end{vmatrix}$, then the value of $(x-y)$ is:

- (A) 2 or 10 (B) -2 or 10
(C) 2 or -10 (D) -2 or -10

8. If $f(x) = \begin{cases} 1, & \text{if } x \leq 3 \\ ax+b, & \text{if } 3 < x < 5 \\ 7, & \text{if } 5 \leq x \end{cases}$ is continuous in $\mathbb{R}$, then the values of a and b are:

- (A) $a = 3, b = -8$ (B) $a = 3, b = 8$
(C) $a = -3, b = -8$ (D) $a = -3, b = 8$

9. If $f(x) = -2x^3$, then the correct statement is:

- (A) f is a decreasing function on $\mathbb{R}$ (B) f is an increasing function on $\mathbb{R}$
(C) f has a maximum at $x = 0$ (D) f has a minimum at $x = 0$

10. A spherical ball has a variable diameter $2(3x+1)$. The rate of change of its volume w.r.t. x , when $x = 1$, is:

- (A) 225π (B) 300π
(C) 375π (D) 125π

11. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined as $f(x) = 2x - \sin x$, then f is:

- (A) a decreasing function (B) an increasing function
(C) maximum at $x = 5$ (D) maximum at $x = 0$

12. $\int \frac{e^{2 \log x} - e^{3 \log x}}{e^{6 \log x} - e^{5 \log x}} dx$ is equal to:

- (A) $x + C$ (B) $\frac{x^2}{2} + C$
(C) $\frac{x^4}{4} + C$ (D) $\frac{x^3}{3} + C$

13. For a function $f(x)$, which of the following holds true?

- (A) $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$
 (B) $\int_{-a}^a f(x) dx = 0$, if f is an even function
 (C) $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$, if f is an odd function
 (D) $\int_0^{2a} f(x) dx = \int_0^a f(x) dx - \int_0^a f(2a+x) dx$

14. $\int \frac{e^x}{\sqrt{4-e^{2x}}} dx$ is equal to:

- (A) $\frac{1}{2} \cos^{-1}(e^x) + C$
 (B) $\frac{1}{2} \sin^{-1}(e^x) + C$
 (C) $\sin^{-1}\left(\frac{e^x}{2}\right) + C$
 (D) $\sin^{-1}\left(\frac{e^{-x}}{2}\right) + C$

15. A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{i} + 15\hat{j} + 6\hat{k}$ and the other is along the vector $2\hat{i} + 10\hat{j} + \lambda\hat{k}$, then the value of λ is:

- (A) 6
 (B) 1
 (C) $\frac{1}{4}$
 (D) 4

16. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ for any two vectors, then vectors $\vec{a}$ and $\vec{b}$ are:

- (A) orthogonal vectors
 (B) parallel to each other
 (C) unit vectors
 (D) collinear vectors

17. If $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P(A | B)$ is:

- (A) $\frac{6}{7}$
 (B) $\frac{3}{8}$
 (C) $\frac{4}{9}$
 (D) $\frac{1}{2}$

18. A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is:

- (A) $\frac{2}{13}$
 (B) $\frac{3}{26}$
 (C) $\frac{19}{26}$
 (D) $\frac{3}{13}$

19. Assertion (A): $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$.

Reason (R): When $x \rightarrow 0$, $\sin \frac{1}{x}$ is a finite value between -1 and 1 .

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
 (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
 (C) Assertion (A) is true, but Reason (R) is false.
 (D) Assertion (A) is false, but Reason (R) is true.

20. Assertion (A): Set of values of $\sec^{-1}\left(\frac{1}{2}\right)$ is a null set.

Reason (R): $\sec^{-1} x$ is defined for $x \in \mathbb{R} - (-1, 1)$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
 (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
 (C) Assertion (A) is true, but Reason (R) is false.
 (D) Assertion (A) is false, but Reason (R) is true.

Section B

21. Let $f : A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3}$, where $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Discuss the bijectivity of the function.

22. If $A = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$, then show that $A^2 - 4A + 7I = 0$.

23. (a) Differentiate $\frac{5x}{x^2}$ with respect to x .

OR

(b) If $-2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.

24. In a Linear Programming Problem, the objective function $Z = 5x + 4y$ needs to be maximised under constraints $3x + y \leq 6$, $x \leq 1$, $x, y \geq 0$. Express the LPP on the graph and shade the feasible region and mark the corner points.

25. (a) 10 identical blocks are marked with '0' on two of them, '1' on three of them, '2' on four of them and '3' on one of them and put in a box. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean.

OR

(b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village?

Section C

26. (a) Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 4x^3 - 5$, $\forall x \in \mathbb{R}$ is one-one and onto.

OR

(b) Let R be a relation defined on a set $\mathbb{N}$ of natural numbers such that $R = \{(x, y) : xy \text{ is a square of a natural number, } x, y \in \mathbb{N}\}$. Determine if the relation R is an equivalence relation.

27. (a) Let $2x + 5y - 1 = 0$ and $3x + 2y - 7 = 0$ represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.

OR

(b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each such subject book is Rs. 150 (Chemistry), Rs. 175 (Physics) and Rs. 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is Rs. 35,000, what profit did he earn after the sale of two days?

28. Differentiate $y = \sqrt{\log(\sin \sqrt{x})}$ with respect to x .

29. Amongst all pairs of positive integers with product as 289, find which of the two numbers add up to the least.

30. In the Linear Programming Problem for objective function $Z = 18x + 10y$ subject to constraints $4x + y \geq 20$, $2x + 3y \geq 30$, $x, y \geq 0$, find the minimum value of Z .

31. (a) The scalar product of the vector $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ with a unit vector along sum of vectors $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$ is equal to 1. Find the value of λ .

OR

(b) Find the shortest distance between the lines:

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

$$\vec{r} = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k})$$

Section D

32. (a) Find: $\int \frac{x^2 + 1}{(x^2 + 2)(2x^2 + 1)} dx$.

OR

(b) Evaluate: $\int_0^{\pi/4} \frac{x \tan x}{\sec x + \tan x} dx$.

33. A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x -axis. Find the area of the region enclosed by the x -axis, the scratch and the circular table top in the first quadrant, using integration.

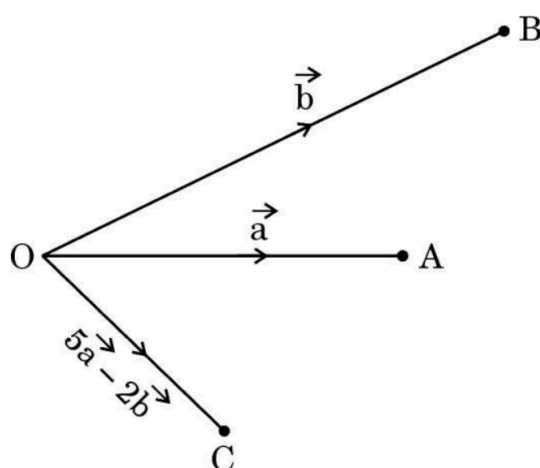
34. Solve the differential equation $\frac{dy}{dx} = \cos x - 2y$.

35. (a) Find the point Q on the line $\frac{2x + 4}{6} = \frac{y + 1}{2} = \frac{-2z + 6}{-4}$ at a distance of $3\sqrt{2}$ from the point $P(1, 2, 3)$.

OR

(b) Find the image of the point $(-1, 5, 2)$ in the line $\frac{2x - 4}{2} = \frac{y}{2} = \frac{z - 2}{-4}$. Find the length of the line segment joining the points (given point and the image point).

36. Case Study - 1: Three friends A , B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his predecided destination, following straight paths from A to C and B to C in such a way that $\vec{OA} = \vec{a}$, $\vec{OB} = \vec{b}$ and $\vec{OC} = 5\vec{a} - 2\vec{b}$ respectively.



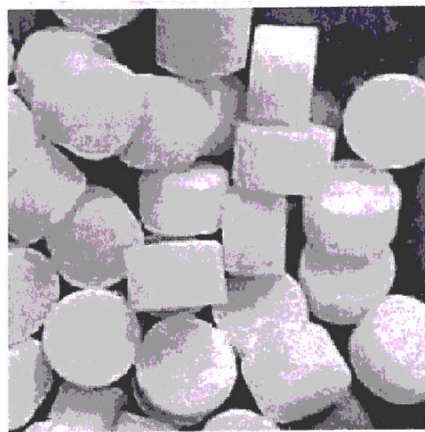
Based upon the above information, answer the following questions:

- (i) Complete the given figure to explain their entire movement plan along the respective vectors.
- (ii) Find vectors $\vec{AC}$ and $\vec{BC}$.
- (iii) (a) If $\vec{a} \cdot \vec{b} = 1$, distance of O to A is 1 km and that from O to B is 2 km, then find the angle between $\vec{OA}$ and $\vec{OB}$. Also, find $|\vec{a} \times \vec{b}|$.

OR

- (iii) (b) If $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a unit vector perpendicular to $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$.

37. Case Study - 2: Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.



A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, $\frac{dV}{dt} = kS$ is the differential equation, where V is the volume, S is the surface area and t is the time in hours.

Based upon the above information, answer the following questions:

- (i) Write the order and degree of the given differential equation.
- (ii) Substituting $V = \pi r^3$ and $S = 2\pi r^2$, we get the differential equation $\frac{dr}{dt} = \frac{2}{3}k$. Solve it, given that $r(0) = 5$ mm.
- (iii) (a) If it is given that $r = 3$ mm when $t = 1$ hour, find the value of k . Hence, find t for $r = 0$ mm.

OR

- (iii) (b) If it is given that $r = 1$ mm when $t = 1$ hour, find the value of k . Hence, find t for $r = 0$ mm.

38. Case Study - 3: Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let A_1 : People with good health,

A_2 : People with average health,

and A_3 : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category A_1 , A_2 and A_3 are 25%, 35% and 50%, respectively.

Based upon the above information, answer the following questions:

(i) A person was tested randomly. What is the probability that he/she has contracted the disease?

(ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_1 ?

- End of Question Paper -