

CBSE Class 12 Mathematics 2025 - 65-S-1 - Compartment

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) There is no overall choice. Internal choice is provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
- (viii) Use of calculators is not allowed.

Section A

1. If the matrix $A = [a_{ij}]_{2 \times 2}$ is such that $a_{ij} = \begin{cases} 1, & i \neq j \\ 0, & i = j \end{cases}$, then $A + A^2$ is equal to:

- (A) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
 (C) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

2. The domain of $f(x) = \cos^{-1}(2x)$ is:

- (A) $[-1, 1]$ (B) $\left[0, \frac{1}{2}\right]$
 (C) $[-2, 2]$ (D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

3. The value of the determinant

$$\begin{vmatrix} \cos 75^\circ & \sin 75^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix} \text{ is:}$$

- (A) 1 (B) zero
 (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

4. For a non-singular matrix X , if $X^2 = I$, then X^{-1} is equal to:

- (A) X (B) X^2
 (C) I (D) O

5. The cofactor of the element a_{32} in the

determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$ is:

- (A) ± 5 (B) -5
 (C) 5 (D) 0

6. If A is an identity matrix of order n , then $A(\text{Adj } A)$ is a/an:

- (A) identity matrix (B) row matrix
 (C) zero matrix (D) skew symmetric matrix

7. If $x = t^3$ and $y = t^2$, then $\frac{d^2y}{dx^2}$ at $t = 1$ is:

- (A) $\frac{3}{2}$ (B) $-\frac{2}{9}$
 (C) $-\frac{3}{2}$ (D) $-\frac{2}{3}$

8. The area bounded by the parabola $x^2 = y$ and the line $y = 1$ is:

- (A) $\frac{2}{3}$ sq unit (B) $\frac{1}{3}$ sq unit
 (C) $\frac{4}{3}$ sq units (D) 2 sq units

9. If the rate of change of volume of a sphere is twice the rate of change of its radius, then the surface area of the sphere is:

- (A) 1 sq unit (B) 2 sq units
 (C) 3 sq units (D) 4 sq units

10. $\int \frac{3 \cos \sqrt{x}}{\sqrt{x}} dx$ is equal to:

- (A) $-6 \sin \sqrt{x} + C$ (B) $-6 \cos \sqrt{x} + C$
 (C) $6 \cos \sqrt{x} + C$ (D) $6 \sin \sqrt{x} + C$

11. If $\frac{d}{dx} f(x) = 3x^2 - \frac{3}{x^4}$ such that $f(1) = 0$, then $f(x)$ is:

- (A) $6x + \frac{12}{x^5}$ (B) $x^4 - \frac{1}{x^3} + 2$
 (C) $x^3 + \frac{1}{x^3} - 2$ (D) $x^3 + \frac{1}{x^3} + 2$

12. In an LPP, corner points of the feasible region determined by the system of linear constraints are $(1, 1)$, $(3, 0)$ and $(0, 3)$. If $Z = ax + by$, where $a, b > 0$ is to be minimized, the condition on a and b , so that the minimum of Z occurs at $(3, 0)$ and $(1, 1)$, will be:

- (A) $a = 2b$ (B) $a = \frac{b}{2}$
(C) $a = 3b$ (D) $a = b$

13. The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 1$, $x, y \geq 0$ is:

- (A) 3 (B) 4
(C) 7 (D) 0

14. The general solution of the differential equation $\frac{dy}{dx} = 2x \cdot e^{x^2+y}$ is:

- (A) $e^{x^2+y} = C$ (B) $e^{x^2} + e^{-y} = C$
(C) $e^{x^2} = e^y + C$ (D) $e^{x^2-y} = C$

15. If 'm' and 'n' are the degree and order respectively of the differential equation $1 + \left(\frac{dy}{dx}\right)^3 = \frac{d^2y}{dx^2}$, then the value of $(m + n)$ is:

- (A) 4 (B) 3
(C) 2 (D) 5

16. If $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 2$, then the value of $|\vec{a} + \vec{b}|$ is:

- (A) 9 (B) 3
(C) -3 (D) 2

17. Two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$. The angle between the two vectors is:

- (A) 30° (B) 60°
(C) 45° (D) 90°

18. A coin is tossed three times. The probability of getting at least two heads is:

- (A) $\frac{1}{2}$ (B) $\frac{3}{8}$
(C) $\frac{1}{8}$ (D) $\frac{1}{4}$

19. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$, defined as $f(x) = x^3$.

Assertion (A): $f(x)$ is a one-one function.

Reason (R): $f(x)$ is a one-one function, if co-domain = range.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** $f(x) = [x]$, $x \in \mathbb{R}$, the greatest integer function, is not differentiable at $x = 2$.

Reason (R): The greatest integer function is not continuous at any integral value.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.

Section B

21(a). Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

OR

21(b). Prove that: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$, $x \in [0, 1]$.

22. If $e^y(x+1) = 1$, prove that $\frac{dy}{dx} = -e^y$.

23. A ladder 13 m long is leaning against the wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 12 m away from the wall?

24(a). Find the value of λ , if the points $(-1, -1, 2)$, $(2, 8, \lambda)$ and $(3, 11, 6)$ are collinear.

OR

24(b). $\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$. Find the area of the parallelogram.

25. Find the angle between the lines $\vec{r} = (3 + 2\lambda)\hat{i} - (2 - 2\lambda)\hat{j} + (6 + 2\lambda)\hat{k}$ and $\vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$.

Section C

26. Find the maximum slope of the curve $y = -x^3 + 3x^2 + 9x - 30$.

27(a). Find: $\int \sqrt{4x^2 - 4x + 10} \, dx$

OR

27(b). Evaluate: $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} \, dx$

28. Solve the following linear programming problem graphically:

Maximize $Z = 2x + 3y$

subject to the constraints

$$x + 4y \leq 8,$$

$$2x + 3y \leq 12,$$

$$3x + y \leq 9,$$

$$x \geq 0, y \geq 0.$$

29(a). Find the general solution of the differential equation $(2x^2 + y) \, dx = x \, dy$.

OR

29(b). For the differential equation $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$, find the particular solution, given that $y = 0$ when $x = 1$.

30. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a} \cdot \hat{b} = \hat{a} \cdot \hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a} = \pm 2(\hat{b} \times \hat{c})$.

31(a). Four students of Class XII are given a problem to solve independently. Their chances of solving the problem respectively are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$ and $\frac{1}{5}$. Find the probability that at most one of them will solve the problem.

OR

31(b). The probability distribution of a random variable X is given below:

X	1	2	4	2k	3k	5k
P(X)	1/2	1/5	3/25	1/10	1/25	1/25

If $E(X) = 2.94$, find k and also find $P(X \leq 4)$.

Section D

32. If $A = \begin{pmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{pmatrix}$, find A^{-1} . Using A^{-1} ,

solve the given system of equations $3x + 4y + 7z = 14$; $2x - y + 3z = 4$; $x + 2y - 3z = 0$.

33(a). If $y = \cos x^2 + \cos^2 x + \cos^2(x^2) + \cos(x^x)$, find $\frac{dy}{dx}$.

OR

33(b). Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is:
(i) strictly increasing, (ii) strictly decreasing.

34. Using integration, find the area of the region $\{(x, y) : 0 \leq y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 3\}$.

35(a). Find the shortest distance between the lines l_1 and l_2 given by: $l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$ and $l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$.

OR

35(b). Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Also, find their point of intersection.

Section E

36(i). A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth x and y metres respectively. If the perimeter of the window is 12 m, find the relation between x and y .

36(ii). Using the expression obtained in (i), write an expression for the area of the window as a function of x only.

36(iii)(a). Find the dimensions of the rectangle that will allow maximum light through the window (using the expression obtained in (ii)).

OR

36(iii)(b). If it is given that the area of the window is 50 m^2 , find an expression for the perimeter of the window in terms of x .

37(i). During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park, which traced the path of a parabola $x^2 = y$ while swinging.

Let $f : \mathbb{N} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. What will be its range?

37(ii). Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be defined by $f(x) = x^2$. Check whether the function is injective or not.

37(iii)(a). Let $f : \{1, 2, 3, 4, \dots\} \rightarrow \{1, 4, 9, 16, \dots\}$ be defined by $f(x) = x^2$. Prove that the function is bijective.

OR

37(iii)(b). Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. Show that f is neither injective nor surjective.

38(i). Two persons are competing for a position on the Managing Committee of an organisation. The probabilities that the first and the second person will be appointed are 0.5 and 0.6 respectively. Also, if the first person gets appointed, then the probability of introducing a waste treatment plant is 0.7, and the corresponding probability is 0.4 if the second person gets appointed.

What is the probability that the waste treatment plant is introduced?

38(ii). After the selection, if the waste treatment plant is introduced, what is the probability that the first person had introduced it?

- End of Question Paper -