

CBSE Class 12 Mathematics 2025 - 65-S-3 - Compartment

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) There is no overall choice. Internal choice is provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
- (viii) Use of calculators is not allowed.

Section A

1. The principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is:

- (A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$
(C) $\frac{3\pi}{4}$ (D) $\frac{2\pi}{3}$

2. The matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ is:

- (A) an identity matrix (B) a scalar matrix
(C) a symmetric matrix (D) a skew-symmetric matrix

3. If $(3x - 5) \begin{pmatrix} x \\ -15 \end{pmatrix} = 0$, then the value of x is:

- (A) zero (B) 5
(C) $2\sqrt{5}$ (D) ± 5

4. If $A = \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix}$ and $|3A| = k|A|$, then the value of k is:

- (A) 3 (B) 9
(C) 6 (D) 27

5. The area of a triangle with vertices $(4, -1)$, $(1, k)$ and $(-2, -1)$ is 9 sq units. The value of k is:

- (A) 8 (B) -8
(C) 2 (D) 6

6. If $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$, then the value of $|\text{adj } A|$ is:

- (A) 5 (B) 25
(C) -5 (D) -25

7. If $\frac{d}{dx}f(x) = 3x^2 - \frac{3}{x^4}$ such that $f(1) = 0$, then $f(x)$ is:

- (A) $6x + \frac{12}{x^5}$ (B) $x^4 - \frac{1}{x^3} + 2$
(C) $x^3 + \frac{1}{x^3} - 2$ (D) $x^3 + \frac{1}{x^3} + 2$

8. If $x = \sin t$ and $y = \cos t$, then $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$ is:

- (A) -1 (B) 2
(C) $2\sqrt{2}$ (D) $-2\sqrt{2}$

9. The area bounded by the line $y = x$, y -axis and the lines $y = 0$ and $y = 4$ is:

- (A) 16 sq units (B) 8 sq units
(C) 4 sq units (D) 2 sq units

10. The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 1$, $x, y \geq 0$ is:

- (A) 3 (B) 4
(C) 7 (D) 0

11. In an LPP, corner points of the feasible region determined by the system of linear constraints are $(1, 1)$, $(3, 0)$ and $(0, 3)$. If $Z = ax + by$, where $a, b > 0$ is to be minimized, the condition on a and b , so that the minimum of Z occurs at $(3, 0)$ and $(1, 1)$, will be:

- (A) $a = 2b$ (B) $a = \frac{b}{2}$
(C) $a = 3b$ (D) $a = b$

12. If $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 2$, then the value of $|\vec{a} + \vec{b}|$ is:

- (A) 9 (B) 3
(C) -3 (D) 2

13. $\int \frac{\cot \sqrt{x}}{\sqrt{x}} dx$ is equal to:

- (A) $\log \sin \sqrt{x} + C$ (B) $\frac{\log \sin \sqrt{x}}{2\sqrt{x}} + C$
(C) $2 \log \sin \sqrt{x} + C$ (D) $\frac{\log \sin \sqrt{x}}{\sqrt{x}} + C$

14. If the rate of change of volume of a sphere is twice the rate of change of its radius, then the surface area of the sphere is:

- (A) 1 sq unit (B) 2 sq units
(C) 3 sq units (D) 4 sq units

15. If 'm' and 'n' are the degree and order respectively of the differential equation $1 + \left(\frac{dy}{dx}\right)^3 = \frac{d^2y}{dx^2}$, then the value of $(m + n)$ is:

- (A) 4 (B) 3
(C) 2 (D) 5

16. A coin is tossed three times. The probability of getting at least two heads is:

- (A) $\frac{1}{2}$ (B) $\frac{3}{8}$
(C) $\frac{1}{8}$ (D) $\frac{1}{4}$

17. Two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$. The angle between the two vectors is:

- (A) 30° (B) 60°
(C) 45° (D) 90°

18. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is:

- (A) $\log x$ (B) $-\log x$
(C) $e^x - x$ (D) $\frac{e^x}{x}$

19. **Assertion (A):** $f(x) = [x]$, $x \in \mathbb{R}$, the greatest integer function, is not differentiable at $x = 2$.

Reason (R): The greatest integer function is not continuous at any integral value.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

20. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$, defined as $f(x) = x^3$.

Assertion (A): $f(x)$ is a one-one function.

Reason (R): $f(x)$ is a one-one function, if co-domain = range.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Section B

21. If $e^y(x+1) = 1$, prove that $\frac{dy}{dx} = -e^y$.

22(a). Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

OR

22(b). Prove that: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$, $x \in [0, 1]$.

23. A spherical balloon has a variable diameter $\frac{2}{3}(2t+1)$. Find the rate of change of its volume with respect to t .

24. Find the angle between the lines $\vec{r} = (3+2\lambda)\hat{i} - (2-2\lambda)\hat{j} + (6+2\lambda)\hat{k}$ and $\vec{r} = (2\hat{j} - 5\hat{k}) + \mu(6\hat{i} + 3\hat{j} + 2\hat{k})$.

25(a). Find the value of λ , if the points $(-1, -1, 2)$, $(2, 8, \lambda)$ and $(3, 11, 6)$ are collinear.

OR

25(b). $\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$. Find the area of the parallelogram.

Section C

26. Find the intervals in which the function f , given by $f(x) = -2x^3 - 9x^2 - 12x + 1$, is: (i) strictly increasing, (ii) strictly decreasing.

27(a). Find: $\int \sqrt{4x^2 - 4x + 10} dx$

OR

27(b). Evaluate: $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$

28. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a} \cdot \hat{b} = \hat{a} \cdot \hat{c} = 0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a} = \pm 2(\hat{b} \times \hat{c})$.

29. Solve the following linear programming problem graphically:

Maximize $Z = 2x + 3y$

subject to the constraints

$$x + 4y \leq 8,$$

$$2x + 3y \leq 12,$$

$$3x + y \leq 9,$$

$$x \geq 0, y \geq 0.$$

30(a). Four students of Class XII are given a problem to solve independently. Their chances of solving the problem respectively are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$ and $\frac{1}{5}$. Find the probability that at most one of them will solve the problem.

OR

30(b). The probability distribution of a random variable X is given below:

X	1	2	4	2k	3k	5k
P(X)	1/2	1/5	3/25	1/10	1/25	1/25

If $E(X) = 2.94$, find k and also find $P(X \leq 4)$.

31(a). Find the general solution of the differential equation $(2x^2 + y) dx = x dy$.

OR

31(b). For the differential equation $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$, find the particular solution, given that $y = 0$ when $x = 1$.

Section D

32(a). If $y = \cos x^2 + \cos^2 x + \cos^2(x^2) + \cos(x^x)$, find $\frac{dy}{dx}$.

OR

32(b). Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is:
(i) strictly increasing, (ii) strictly decreasing.

33. If $A = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$, find BA and use this to solve the system of equations: $y + 2z = 8$, $x - y = -1$, $2x + 3y + 4z = 20$.

34(a). Find the shortest distance between the lines l_1 and l_2 given by: $l_1 : \vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{i} + 6\hat{j} + 12\hat{k})$ and $l_2 : \vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(6\hat{i} + 9\hat{j} + 18\hat{k})$.

OR

34(b). Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Also, find their point of intersection.

35. Using integration, find the area of the region $\{(x, y) : 0 \leq y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 3\}$.

Section E

36(i). A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth x and y metres respectively. If the perimeter of the window is 12 m, find the relation between x and y .

36(ii). Using the expression obtained in (i), write an expression for the area of the window as a function of x only.

36(iii)(a). Find the dimensions of the rectangle that will allow maximum light through the window (using the expression obtained in (ii)).

OR

36(iii)(b). If it is given that the area of the window is 50 m^2 , find an expression for the perimeter of the window in terms of x .

37(i). During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park, which traced the path of a parabola $x^2 = y$ while swinging.

Let $f : \mathbb{N} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. What will be its range?

37(ii). Let $f : \mathbb{N} \rightarrow \mathbb{N}$ be defined by $f(x) = x^2$. Check whether the function is injective or not.

37(iii)(a). Let $f : \{1, 2, 3, 4, \dots\} \rightarrow \{1, 4, 9, 16, \dots\}$ be defined by $f(x) = x^2$. Prove that the function is bijective.

OR

37(iii)(b). Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. Show that f is neither injective nor surjective.

38(i). Two persons are competing for a position on the Managing Committee of an organisation. The probabilities that the first and the second person will be appointed are 0.5 and 0.6 respectively. Also, if the first person gets appointed, then the probability of introducing a waste treatment plant is 0.7, and the corresponding probability is 0.4 if the second person gets appointed.

What is the probability that the waste treatment plant is introduced?

38(ii). After the selection, if the waste treatment plant is introduced, what is the probability that the first person had introduced it?

- End of Question Paper -