

CBSE Class 12 Physics Question Paper 2025 with Solution PDF-Set 55/4/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All questions are compulsory.**
- (ii) This question paper is divided into **five** sections – **Sections A, B, C, D and E.**
- (iii) In **Section A** – Questions no. **1 to 16** are Multiple Choice type questions. Each question carries **1** mark.
- (iv) In **Section B** – Questions no. **17 to 21** are Very Short Answer type questions. Each question carries **2** marks.
- (v) In **Section C** – Questions no. **22 to 28** are Short Answer type questions. Each question carries **3** marks.
- (vi) In **Section D** – Questions no. **29 and 30** are case study-based questions. Each question carries **4** marks.
- (vii) In **Section E** – Questions no. **31 to 33** are Long Answer type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is **not** allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. A body acquires charge 8.0×10^{-12} C. The mass of the body :

[1 mark]

- (A) increases by 4.5×10^{-7} kg
- (B) decreases by 1.0×10^{-6} kg
- (C) decreases by 4.55×10^{-23} kg
- (D) increases by 9.1×10^{-23} kg

Correct Answer: (C) decreases by 4.55×10^{-23} kg

Solution:

Concept:

- Charge is quantised: $Q = ne$, where n is the number of electrons transferred and $e = 1.6 \times 10^{-19}$ C is the magnitude of electronic charge.
- A positive charge is acquired by losing electrons. The corresponding decrease in mass is $\Delta m = nm_e$, where $m_e = 9.1 \times 10^{-31}$ kg.

Step 1: Count the electrons removed The stated charge 8.0×10^{-12} C is positive, so the body has a deficit of electrons. The number removed is

$$n = \frac{Q}{e} = \frac{8.0 \times 10^{-12}}{1.6 \times 10^{-19}} = 5.0 \times 10^7.$$

Step 2: Calculate the mass decrease and compare the options Each removed electron carries its mass away from the body. Thus,

$$\Delta m = (5.0 \times 10^7)(9.1 \times 10^{-31}) = 4.55 \times 10^{-23} \text{ kg}.$$

Options (A) and (D) predict an increase in mass and therefore have the wrong direction of change. Option (B) predicts a decrease, but its magnitude 1.0×10^{-6} kg does not equal the mass of the removed electrons.

Step 3: Conclusion The mass of the body decreases by 4.55×10^{-23} kg. Hence, the correct option is (C).

Quick Tip to Solve: Decide the sign first: positive charging removes electrons. Then use the one-step relation $\Delta m = (Q/e)m_e$.

2. A current flows through a cylindrical conductor of radius R . The current density at a point in the conductor is $j = \alpha r$ (along its axis), here α is a constant and r is distance from the axis of the conductor. The current flowing through the portion of the conductor from $r = 0$ to $r = \frac{R}{2}$ is proportional to : [1 mark]

- (A) R
- (B) R^2
- (C) R^3
- (D) R^4

Correct Answer: (C) R^3

Solution:

Concept:

- Current through a cross-section is $I = \int \vec{j} \cdot d\vec{A}$. Here the current density is along the cylinder's axis, so $dI = j dA$.
- At distance r from the axis, a thin annulus of radial thickness dr has area $dA = 2\pi r dr$. The given density is $j = \alpha r$, with α constant.

Step 1: Write the current through a thin annulus The density varies with radius, so a single value of j cannot be multiplied by the whole area. Within a thin annulus it is effectively constant, giving

$$dI = (\alpha r)(2\pi r dr) = 2\pi\alpha r^2 dr.$$

Step 2: Integrate over the specified inner portion The required cross-section extends from the axis to $R/2$. Therefore,

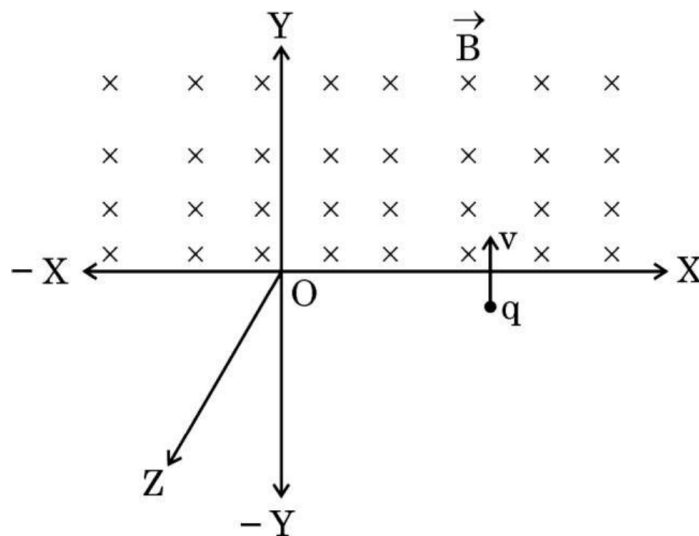
$$I = 2\pi\alpha \int_0^{R/2} r^2 dr = \frac{2\pi\alpha}{3} \left(\frac{R}{2}\right)^3 = \frac{\pi\alpha R^3}{12}.$$

For fixed α , the coefficient is independent of R . An R^2 dependence would describe uniform current density; the extra radial factor in j raises the power to three. Neither R nor R^4 matches the integral.

Step 3: Conclusion The current through the portion $0 \leq r \leq R/2$ is proportional to R^3 . Hence, the correct option is (C).

Quick Tip to Solve: Combine the powers before integrating: $j \propto r$ and $dA \propto r dr$, so $I \propto \int r^2 dr \propto R^3$.

3. A particle having charge $+q$ enters a uniform magnetic field $\vec{B}$ as shown in the figure. The particle will describe : [1 mark]



- (A) a circular path in XZ plane
- (B) a semicircular path in XY plane
- (C) a helical path with its axis parallel to Y -axis
- (D) a semicircular path in YZ plane

Correct Answer: (B) a semicircular path in XY plane

Solution:

Concept:

- A moving charge experiences the magnetic force $\vec{F} = q(\vec{v} \times \vec{B})$. This force is perpendicular to velocity and does no work, so it changes direction but not speed.
- When $\vec{v} \perp \vec{B}$, magnetic force supplies centripetal force: $qvB = mv^2/r$, giving radius $r = mv/(qB)$. The circular motion lies in the plane perpendicular to $\vec{B}$.

Step 1: Identify the plane and direction of deflection The crosses show the field into the page, perpendicular to the XY plane. The positive charge enters upward, along $+Y$, so its velocity is perpendicular to the field. The right-hand rule for $\vec{v} \times \vec{B}$ gives an initial force towards $-X$. The trajectory therefore curves left within the XY plane.

Step 2: Use the boundary of the magnetic-field region The field occupies the region above the X -axis. The particle enters at this straight boundary and follows a circular arc at constant speed. After half a revolution it reaches the boundary again with velocity towards $-Y$ and leaves the field. Its path inside the field is thus a semicircle. Options (A) and (D) give the wrong plane. A helix as in (C) needs a nonzero velocity component parallel to the field, which is absent here.

Step 3: Conclusion In the magnetic-field region, the particle describes a semicircular path in the XY plane. Hence, the correct option is (B).

Quick Tip to Solve: First choose the plane perpendicular to the field; then inspect the field boundary to distinguish a full circle from a semicircle.

4. A bar magnet is initially at right angles to a uniform magnetic field. The magnet is rotated till the torque acting on it becomes one-half of its initial value. The angle through which the bar magnet is rotated is : [1 mark]

- (A) 30°
- (B) 45°
- (C) 60°
- (D) 75°

Correct Answer: (C) 60°

Solution:

Concept:

- The torque magnitude on a bar magnet in a uniform magnetic field is $\tau = mB \sin \theta$, where m is its magnetic dipole moment and θ is the angle between the moment and the field.
- Torque is maximum at $\theta = 90^\circ$. The angle through which a magnet rotates is the change in its orientation angle, not the final angle with the field.

Step 1: Relate the final torque to the initial torque Initially the magnet is perpendicular to the field, so $\tau_i = mB$. If it turns through an angle ϕ towards the field, its final orientation is $\theta = 90^\circ - \phi$. Hence,

$$\frac{\tau_f}{\tau_i} = \sin(90^\circ - \phi) = \cos \phi.$$

Step 2: Find the rotation needed to halve the torque The required ratio is one-half, giving

$$\cos \phi = \frac{1}{2} \quad \Rightarrow \quad \phi = 60^\circ.$$

The final angle with the field is 30° , which explains the tempting option (A), but the question asks for the rotation from 90° . Rotations of 30° , 45° and 75° give torque ratios $\sqrt{3}/2$, $1/\sqrt{2}$ and $\cos 75^\circ$, respectively, none equal to one-half. Turning initially away from the field also reaches half the torque magnitude after a 60° rotation.

Step 3: Conclusion The bar magnet must be rotated through 60° . Hence, the correct option is (C).

Quick Tip to Solve: From an initially perpendicular position, use $\tau_f/\tau_i = \cos \phi$, where ϕ is the rotation being asked for.

5. Which one out of the following materials is not paramagnetic ?

[1 mark]

- (A) Aluminium
- (B) Sodium Chloride
- (C) Calcium
- (D) Copper Chloride

Correct Answer: (B) Sodium Chloride

Solution:

Concept:

- Paramagnetic materials have a small positive magnetic susceptibility and are weakly attracted by an applied magnetic field.
- Diamagnetic materials have a small negative magnetic susceptibility and are weakly repelled. Closed-shell ions, whose electrons are all paired, provide no permanent spin magnetic moment.

Step 1: Examine sodium chloride Sodium chloride consists of Na^+ and Cl^- ions. The sodium ion has the closed-shell configuration of neon, while the chloride ion has the closed-shell configuration of argon. All their electrons are paired. Consequently, sodium chloride shows diamagnetism and is not paramagnetic.

Step 2: Compare the other listed materials Aluminium and calcium are standard examples of weakly paramagnetic metals. Copper chloride, in the usual copper(II) form CuCl_2 , is also paramagnetic: its Cu^{2+} ion has a $3d^9$ configuration with an unpaired electron. Thus options (A), (C) and (D) belong to the paramagnetic group, whereas sodium chloride is the exception requested by the word “not”.

Step 3: Conclusion Sodium chloride is diamagnetic and therefore not paramagnetic. Hence, the correct option is (B).

Quick Tip to Solve: For sodium chloride, consider the ions rather than the neutral atoms: both Na^+ and Cl^- have closed electron shells.

6. An ammeter connected in series in an ac circuit reads 10 A. The maximum value of current at any instant in the circuit is :

[1 mark]

- (A) $10\sqrt{2}$ A
- (B) $\frac{10}{\sqrt{2}}$ A
- (C) $\frac{10}{\pi}$ A
- (D) $\frac{10}{\sqrt{2}\pi}$ A

Correct Answer: (A) $10\sqrt{2}$ A

Solution:

Concept:

- An ac ammeter reads the root mean square (rms) current, the steady current that would produce the same heating effect in a given resistance.
- For the standard sinusoidal alternating current $i = I_0 \sin \omega t$, the rms and peak values satisfy $I_{\text{rms}} = I_0/\sqrt{2}$, where I_0 is the maximum instantaneous current.

Step 1: Identify the quantity measured by the ammeter The reading 10 A is the rms value, not the

maximum value and not the average over a complete cycle. For a sinusoidal current, $\langle \sin^2 \omega t \rangle = 1/2$, so

$$I_{\text{rms}} = \sqrt{\langle i^2 \rangle} = \frac{I_0}{\sqrt{2}}.$$

Step 2: Convert the rms value to the peak value Rearranging the rms relation and substituting the reading gives

$$I_0 = \sqrt{2} I_{\text{rms}} = 10\sqrt{2} \text{ A} \approx 14.14 \text{ A}.$$

Option (B) divides by $\sqrt{2}$, reversing the required conversion. Options (C) and (D) incorrectly introduce π ; such a factor does not occur in the sinusoidal rms-to-peak relation. All three alternatives are smaller than the rms reading, whereas the peak current must be larger.

Step 3: Conclusion The maximum instantaneous current is $10\sqrt{2} \text{ A}$. Hence, the correct option is (A).

Quick Tip to Solve: An ac meter gives rms current. To obtain the sinusoidal peak, multiply its reading by $\sqrt{2}$ and check that the result increases.

7. The amplitude of electric field in an electromagnetic wave in free space is 1000 V m^{-1} . The amplitude of the magnetic field in this electromagnetic wave is : [1 mark]

- (A) $3.0 \times 10^{-3} \text{ T}$
- (B) $3.33 \times 10^{-8} \text{ T}$
- (C) $3.0 \times 10^{11} \text{ T}$
- (D) $3.33 \times 10^{-6} \text{ T}$

Correct Answer: (D) $3.33 \times 10^{-6} \text{ T}$

Solution:

Concept:

- In an electromagnetic wave travelling through free space, the electric and magnetic field amplitudes obey $E_0 = cB_0$.
- Here E_0 is the electric field amplitude, B_0 is the magnetic field amplitude, and $c = 3.0 \times 10^8 \text{ m s}^{-1}$ is the speed of light in vacuum.

Step 1: Express the magnetic amplitude using the electric amplitude The problem gives the amplitude $E_0 = 1000 \text{ V m}^{-1}$, so it is related directly to B_0 ; no rms conversion is needed. Rearranging the free-space wave relation gives

$$B_0 = \frac{E_0}{c}.$$

Step 2: Substitute and check the power of ten Write $1000 = 10^3$ and divide by the speed of light:

$$B_0 = \frac{10^3}{3.0 \times 10^8} = \frac{1}{3} \times 10^{-5} = 3.33 \times 10^{-6} \text{ T}.$$

The units are tesla because $(\text{V m}^{-1})/(\text{m s}^{-1}) = \text{V s m}^{-2} = \text{T}$. Options (A) and (B) have incorrect

exponents. Option (C), 3.0×10^{11} T, would arise from multiplying the numerical electric amplitude by c , whereas the required operation is division.

Step 3: Conclusion The magnetic field amplitude is 3.33×10^{-6} T. Hence, the correct option is (D).

Quick Tip to Solve: Use $B_0 = E_0/c$: $10^3/10^8 = 10^{-5}$, then convert $(1/3) \times 10^{-5}$ to 3.33×10^{-6} .

8. The magnification produced by a spherical mirror is -2.0 . The mirror used and the nature of the image formed will be : [1 mark]

- (A) Convex and virtual
- (B) Concave and real
- (C) Concave and virtual
- (D) Convex and real

Correct Answer: (B) Concave and real

Solution:

Concept:

- Mirror magnification is $m = h_i/h_o = -v/u$, where h_i, h_o are image and object heights and v, u are their signed distances from the pole.
- For a real object, negative magnification means an inverted, real image. A convex mirror always gives an erect, virtual and diminished image.

Step 1: Interpret the sign and magnitude The given value $m = -2.0$ has a negative sign, so the image is inverted. Its height is twice the object's height because $|m| = 2.0$. Thus a virtual image, which would have positive magnification for a real object, cannot satisfy the given value.

Step 2: Identify the possible mirror A concave mirror can produce an inverted magnified image when the object lies between its focus and centre of curvature. A convex mirror cannot produce this image: its magnification is positive and less than one. Hence options (A) and (D) fail because they specify a convex mirror, while (C) fails because it specifies a virtual image.

Step 3: Conclusion The mirror is concave and the image is real, inverted and magnified. Hence, the correct option is (B).

Quick Tip to Solve: Read the sign of magnification before its size: a negative value identifies an inverted real image, and then a concave mirror.

9. Choose the correct statement :

[1 mark]

- (A) Photons of light show diffraction whereas electrons do not show diffraction.
- (B) Electrons have momentum whereas photons do not have momentum.
- (C) Photons of light and electrons both exhibit dual nature.
- (D) All electromagnetic radiations do not have photons.

Correct Answer: (C) Photons of light and electrons both exhibit dual nature.

Solution:

Concept:

- Light exhibits wave behaviour in interference and diffraction, and particle behaviour in the photoelectric effect. A photon has energy $E = h\nu$ and momentum $p = h/\lambda$.
- Electrons also exhibit dual nature: an electron of momentum p has de Broglie wavelength $\lambda = h/p$, where h is Planck's constant.

Step 1: Check the wave behaviour Both light and electrons can produce diffraction patterns. Electron diffraction demonstrates that matter also has wave properties. Therefore option (A), which denies diffraction of electrons, is false. The existence of particle and wave behaviour for each supports the statement in (C).

Step 2: Check the particle statements Photons carry momentum despite having zero rest mass, so option (B) is false. The photon description applies throughout the electromagnetic spectrum, including radio waves, visible light and X-rays. Thus option (D), which denies photons for electromagnetic radiation, is also false.

Step 3: Conclusion Photons of light and electrons both exhibit dual nature. Hence, the correct option is (C).

Quick Tip to Solve: Use $p = h/\lambda$ for both a photon and a moving electron; zero photon rest mass does not mean zero momentum.

10. A beam of red light and a beam of blue light have equal intensities. Which of the following statements is true ?

[1 mark]

- (A) The blue beam has more number of photons than the red beam.
- (B) The red beam has more number of photons than the blue beam.
- (C) Wavelength of red light is lesser than wavelength of blue light.
- (D) The blue light beam has lesser energy per photon than that in the red light beam.

Correct Answer: (B) The red beam has more number of photons than the blue beam.

Solution:

Concept:

- Intensity I is energy crossing unit area per unit time. If $\dot{n}$ photons cross unit area per unit time, then $I = \dot{n}E_\gamma$.
- A photon's energy is $E_\gamma = hc/\lambda$, where h is Planck's constant, c is the speed of light and λ is its wavelength.

Step 1: Compare the photon energies Red light has a longer wavelength than blue light, so $\lambda_r > \lambda_b$ and $E_r < E_b$. Thus one blue photon carries more energy than one red photon. Options (C) and (D)

reverse these relations and are false.

Step 2: Compare the photon rates at equal intensity Equal intensities mean equal energies crossing equal areas in equal times. The beam with less energy per photon needs more photons to carry that same energy. Quantitatively,

$$\frac{\dot{n}_r}{\dot{n}_b} = \frac{E_b}{E_r} = \frac{\lambda_r}{\lambda_b} > 1.$$

Therefore the red beam has the greater photon rate per unit area, contrary to option (A). The comparison of photon numbers is understood for equal areas and equal time intervals.

Step 3: Conclusion The red beam contains more photons for equal sampled area and time. Hence, the correct option is (B).

Quick Tip to Solve: At fixed intensity, photon rate is proportional to wavelength: the longer-wavelength beam needs more photons.

11. Which of the following is an electrical conductor at room temperature ?

[1 mark]

- (A) Sn
- (B) Mica
- (C) Si
- (D) C

Correct Answer: (A) Sn

Solution:

Concept:

- A metal conducts because many electrons can move through its crystal under an applied electric field. In band language, available nearby electron states permit this motion.
- Insulators have a large forbidden energy gap; intrinsic semiconductors have a smaller gap and far fewer mobile carriers than metals at room temperature.

Step 1: Identify the metallic material Sn is tin. At ordinary room temperature it is a metal with mobile conduction electrons, so it is classified as an electrical conductor.

Step 2: Compare the other materials Mica is an electrical insulator. Si, silicon, is classified as a semiconductor, even though it has a small nonzero conductivity at room temperature. In the usual textbook band-theory comparison, C refers to diamond carbon, an insulator with a large energy gap. Carbon's allotrope is not specified in the printed option: graphite does conduct, so carbon should not be described as universally insulating. The intended material-classification answer is the metal tin.

Step 3: Conclusion Tin, Sn, is the metallic electrical conductor at room temperature. Hence, the correct option is (A).

Quick Tip to Solve: Identify Sn as the metal and Si as the semiconductor; for carbon, remember that conductivity depends on its allotrope.

12. A long straight wire is held vertically and carries a steady current in upward direction. The shape of magnetic field lines produced by the current-carrying wire are : [1 mark]

- (A) horizontal straight lines directed radially out from the wire.
- (B) straight lines parallel to the current-carrying wire.
- (C) concentric horizontal circles around the wire.
- (D) coaxial helices around the wire.

Correct Answer: (C) concentric horizontal circles around the wire.

Solution:

Concept:

- A long straight current-carrying wire produces circular magnetic field lines centred on the wire in planes perpendicular to it.
- The field magnitude is $B = \mu_0 I / (2\pi r)$, where I is current and r is perpendicular distance from the wire. The right-hand thumb rule gives its direction.

Step 1: Determine the plane and shape The wire is vertical, so planes perpendicular to it are horizontal. The magnetic field is tangential around the wire, and at a fixed distance its magnitude is the same at all points. Therefore the field lines form concentric circles in horizontal planes, with the wire passing through their centres.

Step 2: Check the direction and alternatives Point the right thumb upward along the current: the curled fingers show anticlockwise circulation when viewed from above. The field is neither radially outward as in (A), nor parallel to the wire as in (B). There is no component along the wire to make the lines helical, so (D) also fails. Upward current determines the circulation, while the straight vertical wire determines the circular horizontal geometry.

Step 3: Conclusion The field lines are concentric horizontal circles around the wire. Hence, the correct option is (C).

Quick Tip to Solve: First place each circular field line in a plane perpendicular to the wire; then use the right-hand thumb rule to add its direction.

13. Assertion (A): n-type semiconductor is not negatively charged.

Reason (R): Neutral pentavalent impurity atom doped in intrinsic semiconductor (neutral) donates its fifth unpaired electron to the crystal lattice and becomes a positive donor. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- In an n-type semiconductor, electrons are the majority mobile charge carriers. The term n-type identifies the carrier type, not a net negative charge of the whole crystal.
- Donor ionisation creates one conduction electron and one fixed positive donor ion. Equal and opposite charges preserve the neutrality of an initially neutral sample.

Step 1: Evaluate Assertion (A) An ordinary n-type semiconductor is electrically neutral overall. Although the mobile electron concentration exceeds the hole concentration, the crystal also contains positively charged donor ions. Thus the assertion that it is not negatively charged is true.

Step 2: Evaluate Reason (R) A neutral pentavalent impurity atom contributes four valence electrons to covalent bonds with neighbouring host atoms. Its fifth electron is weakly bound and can become a conduction electron, leaving a fixed positive donor ion behind. Therefore the reason is true.

Step 3: Test the explanation The extra mobile electron has charge $-e$, while its donor ion has charge $+e$, where e is the elementary charge magnitude. Their sum is zero. This donation redistributes charge within the crystal without adding a net negative charge, so the reason directly explains the assertion. Options (B), (C) and (D) therefore fail.

Step 4: Conclusion Both A and R are true, and R correctly explains A. Hence, the correct option is (A).

Quick Tip to Solve: Count the fixed donor ion as well as the donated electron: one $+e$ balances one $-e$, even though electrons carry most of the current.

14. Assertion (A): A series LCR circuit behaves as a pure resistive circuit at resonance.

Reason (R): At resonance, $X_L = X_C$ gives $\omega = \frac{1}{\sqrt{LC}}$.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- For a series LCR circuit, the impedance magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, with $X_L = \omega L$ and $X_C = 1/(\omega C)$.
- Here R, L, C are resistance, inductance and capacitance, and ω is the applied angular frequency. The phase angle satisfies $\tan \phi = (X_L - X_C)/R$.

Step 1: Evaluate Assertion (A) At resonance the net reactance of the series circuit vanishes. Its impedance magnitude is then $Z = R$, and the current is in phase with the applied voltage. These are the characteristics of a purely resistive circuit, so the assertion is true.

Step 2: Evaluate Reason (R) Equating the inductive and capacitive reactances gives

$$\omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}.$$

The positive root is the resonant angular frequency. Hence the reason is true.

Step 3: Test the explanation The equality $X_L = X_C$ stated in the reason is precisely what makes $X_L - X_C = 0$. The inductive and capacitive effects cancel in the series combination, giving $Z = R$ and $\phi = 0$. Therefore the reason explains the assertion; option (B) wrongly denies this link, and options (C) and (D) wrongly judge true statements false.

Step 4: Conclusion Both A and R are true, and R correctly explains A. Hence, the correct option is (A).

Quick Tip to Solve: In a series resonance question, substitute $X_L - X_C = 0$ into both impedance and phase-angle formulae to see the resistive behaviour immediately.

15. Assertion (A): In double slit experiment if one slit is closed, diffraction pattern due to the other slit will appear on the screen.

Reason (R): For interference, at least two waves are required.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- Two-slit interference requires the superposition of light waves from both slits. A sustained fringe pattern requires a fixed phase relationship between them.
- A single slit of finite width also produces diffraction: secondary wavelets from different parts of its opening superpose, giving a broad central maximum and weaker side maxima.

Step 1: Evaluate the assertion Closing one slit removes the contribution from that slit, so the two-slit interference fringes disappear. Light still passes through the remaining finite-width slit, and its single-slit diffraction pattern is obtained on the screen. Assertion (A) is therefore true.

Step 2: Evaluate the reason Interference involves superposition of at least two waves, so Reason (R) is true. These waves need not originate from two separate slits: in diffraction they can come from different regions of the same wavefront.

Step 3: Test the explanatory link The reason accounts for the loss of interference between the two slits, but it does not explain why a diffraction pattern remains. That requires the superposition of secondary wavelets across the open slit. Thus option (A) incorrectly treats the reason as sufficient; options (C) and (D) incorrectly classify true statements as false.

Step 4: Conclusion Both statements are true, but the reason does not correctly explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: In a slit-closing question, distinguish the disappearance of two-slit fringes from the appearance of the remaining slit's diffraction pattern before testing the reason.

16. Assertion (A): For monochromatic incident radiation, the emitted photoelectrons from a given metal have speed ranging from zero to a certain maximum value.

Reason (R): Each metal has a definite work function.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = \frac{1}{2}m_e v_{\max}^2$, where ν is the incident frequency, ϕ the work function and m_e the electron mass.
- The work function is the minimum energy needed to remove an electron from a given metal surface. Electrons emerging after different energy losses can have different kinetic energies.

Step 1: Evaluate the assertion Monochromatic radiation supplies photons of the same energy $h\nu$. However, electrons originate at different depths and may lose different amounts of energy before escaping. Some emerge with almost no kinetic energy, while the most energetic have $K_{\max} = h\nu - \phi$. Thus the emitted speeds range from zero to $v_{\max}$, and Assertion (A) is true.

Step 2: Evaluate the reason A specified metal surface has a definite work function ϕ , which fixes its threshold frequency through $\nu_0 = \phi/h$. Reason (R) is therefore true.

Step 3: Test the explanatory link A definite work function determines the maximum kinetic energy at a given frequency; it does not explain the range of emitted energies. The spread arises from the different circumstances and energy losses of escaping electrons. Option (A) therefore asserts an incorrect explanatory link, while options (C) and (D) wrongly declare true statements false.

Step 4: Conclusion Both statements are true, but the reason does not correctly explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Treat $h\nu - \phi$ as the maximum photoelectron energy, not the energy of every emitted electron; a fixed photon energy does not imply one emitted speed.

Section B (10 marks)

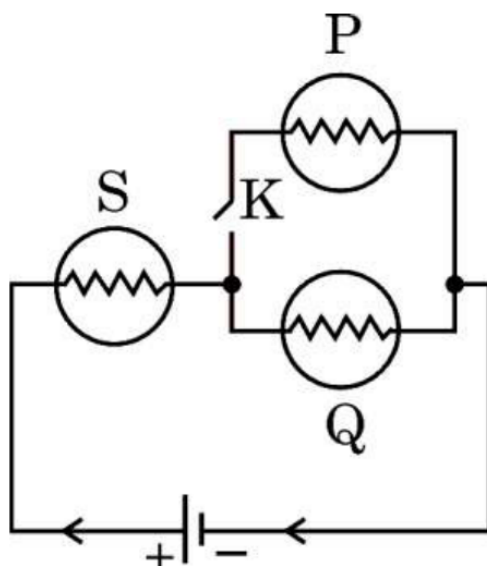
17(a). In the given figure, three identical bulbs P , Q and S are connected to a battery.

(i) Compare the brightness of bulbs P and Q with that of bulb S when key K is closed.

(ii) Compare the brightness of the bulbs S and Q when the key K is opened.

Justify your answer in both cases.

[2 marks]



Solution:

Concept:

- For identical bulbs treated as equal resistances R , brightness is compared using the electrical power $\mathcal{P} = I^2 R$, where I is the current through a bulb.
- Series elements carry the same current. Identical parallel branches carry equal currents, and their currents add at the junction according to Kirchhoff's junction law.

Step 1: Compare the currents when the key is closed In part (i), closing K connects P and Q in parallel, while S remains in series with their combination. If the total current through S is I , the two equal-resistance branches divide it equally:

$$I_S = I, \quad I_P = I_Q = \frac{I}{2}.$$

Comparing the dissipated powers gives

$$\begin{aligned} \mathcal{P}_S &= I^2 R, \\ \mathcal{P}_P &= \mathcal{P}_Q = \left(\frac{I}{2}\right)^2 R = \frac{I^2 R}{4}. \end{aligned}$$

The equality of branch currents makes P and Q equally bright; the greater current through S makes it brighter than either parallel bulb.

Step 2: Compare the currents when the key is opened In part (ii), opening K disconnects the branch containing P , so P carries no current. The remaining closed path contains only S and Q in series. Let

their common current be I' ; then

$$I_S = I_Q = I', \quad \mathcal{P}_S = \mathcal{P}_Q = (I')^2 R.$$

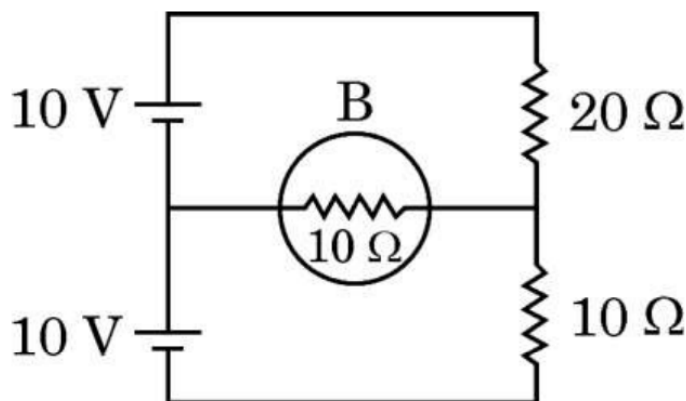
Their identical resistances and common current give equal powers, which is the required reason for their equal brightness.

Step 3: Conclusion With K closed, P and Q are equally bright and each is dimmer than S ; the ideal equal-resistance power ratio is $\mathcal{P}_S : \mathcal{P}_P : \mathcal{P}_Q = 4 : 1 : 1$. With K open, S and Q are equally bright, while P is off.

Quick Tip to Solve: Trace the surviving current paths after changing the key: current divides between P and Q only when the key is closed. Compare powers using the square of each bulb's current.

OR

17(b). Two cells of emf 10 V each, two resistors of $20\ \Omega$ and $10\ \Omega$ and a bulb of $10\ \Omega$ resistance are connected together as shown in the figure. Find the current that flows through the bulb. [2 marks]



Solution:

Concept:

- Potential rises by the emf across an ideal cell from its negative plate to its longer, positive plate.
- Ohm's law gives current from node a to node b as $(V_a - V_b)/R$. By Kirchhoff's junction law, the algebraic sum of currents leaving a junction is zero.

Step 1: Assign the potentials fixed by the cells Both positive plates face upwards. Taking the bottom wire as zero potential, the bulb's left end (the cells' junction) is at 10 V and the top wire is at 20 V. Let the potential at the bulb's right end be V .

Step 2: Apply the junction law at the bulb's right end Write the currents leaving this junction through the upper resistor, lower resistor and bulb, respectively:

$$\frac{V - 20\text{ V}}{20\ \Omega} + \frac{V}{10\ \Omega} + \frac{V - 10\text{ V}}{10\ \Omega} = 0.$$

Multiply by $20\ \Omega$ and collect terms:

$$\begin{aligned}V - 20V + 2V + 2(V - 10V) &= 0, \\5V &= 40V, \\V &= 8V.\end{aligned}$$

Step 3: Calculate the bulb current The bulb's left end is 2 V above its right end. Hence, taking left-to-right as positive,

$$I_B = \frac{10V - 8V}{10\ \Omega} = 0.20\text{ A}.$$

Check: 0.60 A enters from above and 0.80 A leaves below; the bulb supplies their difference.

Step 4: Conclusion The current through bulb B is 0.20 A , flowing from the junction of the two cells towards the junction of the two resistors, that is, from left to right in the figure.

Quick Tip to Solve: Fix the three left-side node potentials from the cell polarities first. Only the right junction potential remains unknown, so one junction equation determines the bulb current.

18. Find the angle of diffraction (in degrees) for first secondary maximum of the pattern due to diffraction at a single slit. The width of the slit and wavelength of light used are 0.55 mm and 550 nm, respectively. [2 marks]

Solution:

Concept:

- For a single slit of width a , diffraction minima satisfy $a \sin \theta = n\lambda$, where λ is the wavelength and θ is measured from the central direction.
- The first secondary maximum lies approximately midway between the first two minima. Its school-level position is therefore $a \sin \theta \simeq 3\lambda/2$; for a small angle, $\sin \theta \simeq \theta$ in radians.

Step 1: Convert the slit width and wavelength to metres The supplied width and wavelength must use the same unit before forming their ratio:

$$\begin{aligned}a &= 0.55\text{ mm} = 5.5 \times 10^{-4}\text{ m}, \\ \lambda &= 550\text{ nm} = 5.50 \times 10^{-7}\text{ m}.\end{aligned}$$

Thus $\lambda/a = 1.00 \times 10^{-3}$. The first secondary maximum is outside the central maximum, between the minima with path differences λ and 2λ .

Step 2: Calculate the approximate diffraction angle Use the secondary-maximum condition, rather than the first-minimum condition $a \sin \theta = \lambda$:

$$\sin \theta \simeq \frac{3\lambda}{2a} = \frac{3(5.50 \times 10^{-7})}{2(5.5 \times 10^{-4})} = 1.50 \times 10^{-3}.$$

This is small enough to use $\theta \simeq \sin \theta$. Convert the resulting radians to the degrees requested:

$$\theta \simeq 1.50 \times 10^{-3} \times \frac{180}{\pi} = 0.0859^\circ.$$

The midpoint condition is itself an approximation for the position of a secondary maximum, as used in this school-level calculation.

Step 3: Conclusion The first secondary maximum occurs at an angular distance of approximately 0.086° from the central direction, on either side of the central maximum.

Quick Tip to Solve: Use the factor $3/2$ for the first secondary maximum, and multiply radians by $180/\pi$ before reporting an angle in degrees.

19. An equiconvex lens is made of glass of refractive index 1.55. If the focal length of the lens is 15.0 cm, calculate the radius of curvature of its surfaces. [2 marks]

Solution:

Concept:

- For a thin lens in air, the lens-maker's formula is $\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$, where μ is the refractive index of glass and R_1, R_2 are signed radii of curvature.
- An equiconvex lens has equal radii in magnitude. With light travelling from left to right and the Cartesian sign convention, $R_1 = +R$, $R_2 = -R$, and its focal length in air is positive.

Step 1: Substitute the signed radii in the lens-maker's formula The first surface has its centre of curvature to the right, whereas the second has its centre to the left. Therefore the two curvature terms add in magnitude:

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{2(\mu - 1)}{R}.$$

Rearranging gives $R = 2(\mu - 1)f$. Equal curvature magnitudes do not mean equal signed radii.

Step 2: Insert the refractive index and focal length Using $\mu = 1.55$ and $f = +15.0$ cm,

$$R = 2(1.55 - 1)(15.0 \text{ cm}) = 2(0.55)(15.0 \text{ cm}) = 16.5 \text{ cm}.$$

Since refractive index is dimensionless, the radius has the same length unit as the given focal length.

Step 3: Conclusion The radius of curvature of each surface is 16.5 cm in magnitude; the signed radii are $R_1 = +16.5 \text{ cm}$ and $R_2 = -16.5 \text{ cm}$.

Quick Tip to Solve: For an equiconvex thin lens in air, use $R = 2(\mu - 1)f$; assign opposite signs to the two surface radii.

20. Calculate the mass of an α -particle in atomic mass unit (u). Given,

Mass of a normal helium atom = 4.002603 u

Mass of carbon atom = $1.9926 \times 10^{-26} \text{ kg}$

[2 marks]

Solution:

Concept:

- An α -particle is the nucleus of a helium atom. A neutral helium atom contains this nucleus and two electrons, so $m_\alpha \simeq m_{\text{He}} - 2m_e$, neglecting the very small electronic binding-energy correction.
- One atomic mass unit is one-twelfth of the mass of a carbon-12 atom: $1 \text{ u} = m(^{12}\text{C})/12$. All masses must be expressed in the same unit before subtraction.

Step 1: Convert the electron mass to atomic mass units The given carbon atom mass provides the

conversion factor:

$$1 \text{ u} = \frac{1.9926 \times 10^{-26}}{12} \text{ kg} = 1.6605 \times 10^{-27} \text{ kg}.$$

Using the electron mass $m_e = 9.1 \times 10^{-31} \text{ kg}$ supplied in the paper,

$$m_e = \frac{9.1 \times 10^{-31}}{1.6605 \times 10^{-27}} \text{ u} \simeq 5.4803 \times 10^{-4} \text{ u}.$$

Step 2: Subtract the masses of the two atomic electrons A neutral helium atom has atomic number two, so two electron masses must be removed to obtain the nuclear mass. Thus,

$$\begin{aligned} m_\alpha &\simeq 4.002603 \text{ u} - 2(5.4803 \times 10^{-4} \text{ u}) \\ &\simeq 4.002603 \text{ u} - 0.00109606 \text{ u} \\ &\simeq 4.00150694 \text{ u}. \end{aligned}$$

The result is slightly less than the neutral atomic mass, as expected after removing the electrons.

Step 3: Conclusion The mass of the α -particle is approximately 4.0015 u (about 4.001507 u before rounding).

Quick Tip to Solve: To obtain a nuclear mass from a neutral atomic mass, subtract Z electron masses after converting them to the same unit.

21. In an intrinsic semiconductor, carrier's concentration is $5 \times 10^8 \text{ m}^{-3}$. On doping with impurity atoms, the hole concentration becomes $8 \times 10^{12} \text{ m}^{-3}$.

(a) Identify (i) the type of dopant and (ii) the extrinsic semiconductor so formed.

(b) Calculate the electron concentration in the extrinsic semiconductor.

[2 marks]

Solution:

Concept:

- In an intrinsic semiconductor, electron and hole concentrations are equal: $n_e = n_h = n_i$, where n_i is the intrinsic carrier concentration.
- At the same temperature in thermal equilibrium, the mass-action law gives $n_e n_h = n_i^2$. An acceptor impurity increases the hole concentration, producing a p-type semiconductor.

Step 1: (a) Identify the dopant and the resulting semiconductor The printed intrinsic concentration is $n_i = 5 \times 10^8 \text{ m}^{-3}$, whereas after doping the hole concentration is $n_h = 8 \times 10^{12} \text{ m}^{-3}$. Thus the hole concentration has increased by a factor

$$\frac{n_h}{n_i} = \frac{8 \times 10^{12}}{5 \times 10^8} = 1.6 \times 10^4.$$

Holes are therefore the majority carriers. The dopant is a **trivalent acceptor impurity**, such as boron, and the resulting extrinsic semiconductor is **p-type**. Acceptor atoms provide vacant electron states, creating holes in a tetravalent semiconductor.

Step 2: (b) Apply the mass-action law to find the electron concentration Using $n_e n_h = n_i^2$ at the same

temperature,

$$n_e = \frac{n_i^2}{n_h} = \frac{(5 \times 10^8)^2}{8 \times 10^{12}} \text{ m}^{-3} \\ = \frac{25 \times 10^{16}}{8 \times 10^{12}} \text{ m}^{-3} = 3.125 \times 10^4 \text{ m}^{-3}.$$

This is much smaller than the hole concentration, confirming that electrons are minority carriers in this doped sample.

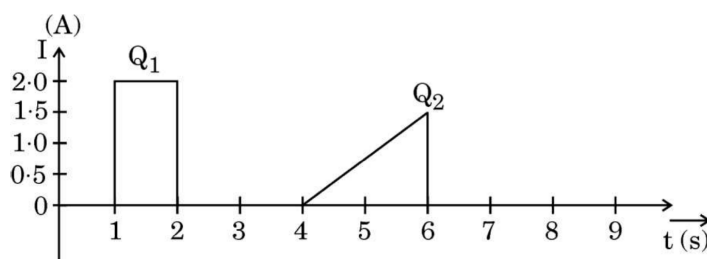
Step 3: Conclusion (a) The dopant is a **trivalent acceptor**, and the semiconductor is **p-type**. (b) The electron concentration is $n_e = 3.125 \times 10^4 \text{ m}^{-3}$.

Quick Tip to Solve: Compare the doped hole concentration with the printed n_i first; then square n_i , including its exponent, before using $n_e = n_i^2/n_h$.

Section C (21 marks)

22(a). (i) Derive an expression for the resistivity of a conductor in terms of number density of free electrons and relaxation time.

(ii) The figure shows the plot of current through a cross-section of wire over two different time intervals. Compare the charges (Q_1 and Q_2) that pass through the cross-section during these time intervals. [3 marks]



Solution:

Concept:

- Relaxation time τ is the mean time between successive collisions of a conduction electron. In a field $\mathbf{E}$, an electron of mass m and charge $-e$, with $e > 0$, has drift velocity $\mathbf{v}_d = -e\tau\mathbf{E}/m$.
- For free-electron number density n , current density is $\mathbf{J} = -nev_d$. The microscopic form of Ohm's law is $\mathbf{E} = \rho\mathbf{J}$, where ρ is resistivity.
- Charge passing through a cross-section is $Q = \int I dt$, the area under the current-time graph; $1 \text{ As} = 1 \text{ C}$.

Step 1: (i) Obtain the drift velocity and current density The electric force on a free electron is $\mathbf{F} = -e\mathbf{E}$, so its acceleration between collisions is $\mathbf{a} = -e\mathbf{E}/m$. Random thermal velocities average to zero. Over the mean relaxation time, the field produces the average drift velocity

$$\mathbf{v}_d = \mathbf{a}\tau = -\frac{e\tau}{m}\mathbf{E}.$$

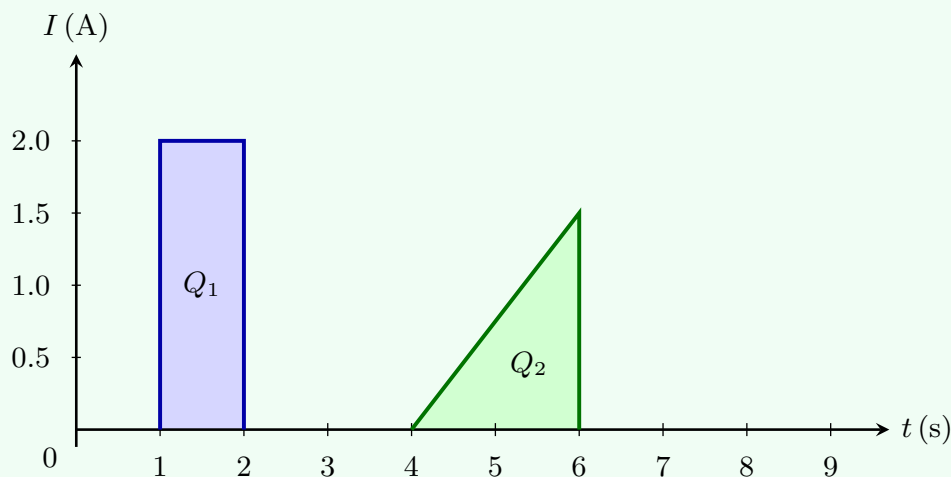
Electrons drift opposite to the field, while conventional current is along the field. Hence

$$\mathbf{J} = -nev_d = \frac{ne^2\tau}{m}\mathbf{E}.$$

Step 2: (i) Identify the resistivity Rearranging the current-density relation and comparing it with $\mathbf{E} = \rho \mathbf{J}$ gives

$$\mathbf{E} = \frac{m}{ne^2\tau} \mathbf{J}, \quad \rho = \frac{m}{ne^2\tau}.$$

Thus resistivity decreases when the free-electron density or relaxation time increases. Its SI unit is $\Omega \text{ m}$; it is a material property, independent of the conductor's length and cross-sectional area.



The shaded rectangle and triangle represent Q_1 and Q_2 , respectively.

Step 3: (ii) Calculate the areas under the two current pulses For the first pulse, current is constant at 2.0 A from $t = 1 \text{ s}$ to $t = 2 \text{ s}$. Its area is rectangular:

$$Q_1 = 2.0(2 - 1) \text{ A s} = 2.0 \text{ C}.$$

For the second pulse, current rises linearly from zero at 4 s to 1.5 A at 6 s. Its area is triangular:

$$Q_2 = \frac{1}{2}(6 - 4)(1.5) \text{ A s} = 1.5 \text{ C}.$$

Vertical changes in current take zero time in the ideal graph and contribute no area. The comparison follows from $Q_1/Q_2 = 2.0/1.5 = 4/3$.

Step 4: Conclusion (i) $\rho = \frac{m}{ne^2\tau}$. (ii) $Q_1 = 2.0 \text{ C}$, $Q_2 = 1.5 \text{ C}$, so $Q_1 : Q_2 = 4 : 3$ and $Q_1 > Q_2$.

Quick Tip to Solve: Compare areas under an I -versus- t graph: the first pulse is a rectangle, but the rising second pulse requires the triangle factor $1/2$.

22(b). (i) A battery of emf E and internal resistance r is connected to a variable external resistance R .

(I) Obtain the expression for current I in the circuit and the value of maximum current the battery can supply.

(II) Obtain the terminal voltage V across the battery and its maximum possible value.

(ii) The above battery sends a current I_1 when $R = R_1$ and a current I_2 when $R = R_2$. Obtain the internal resistance of the battery in terms of I_1 , I_2 , R_1 and R_2 . [3 marks]

Solution:

Concept:

- For a discharging battery, the emf E drives current through the external resistance R and internal resistance r in series. Kirchhoff's loop law gives $E = I(R + r)$.
- The terminal voltage is $V = E - Ir = IR$. The internal voltage drop Ir vanishes when the battery supplies no current.

Step 1: (i)(I) Express the current and identify its maximum The total circuit resistance is $R + r$, so applying the loop law gives

$$E - IR - Ir = 0 \Rightarrow I = \frac{E}{R + r}.$$

For a fixed battery with $r > 0$, decreasing the nonnegative external resistance increases the current. Its greatest value occurs in the short-circuit limit $R = 0$:

$$I_{\max} = \frac{E}{r}.$$

Step 2: (i)(II) Express the terminal voltage and find its maximum Subtract the voltage drop inside the battery from its emf:

$$V = E - Ir = E - \frac{Er}{R + r} = \frac{ER}{R + r}.$$

As R increases, the current and hence the internal drop decrease. For an open circuit, $R \rightarrow \infty$ and $I \rightarrow 0$, so the maximum possible terminal voltage is $V_{\max} = E$. For any finite load with positive current, $V < E$.

Step 3: (ii) Eliminate the emf using the two load conditions The same battery has the same emf and internal resistance in both observations. Thus

$$E = I_1(R_1 + r), \quad E = I_2(R_2 + r).$$

Equating these expressions and collecting the terms in r ,

$$\begin{aligned} I_1 R_1 + I_1 r &= I_2 R_2 + I_2 r, \\ (I_1 - I_2)r &= I_2 R_2 - I_1 R_1, \\ r &= \frac{I_2 R_2 - I_1 R_1}{I_1 - I_2}. \end{aligned}$$

The two measurements must give distinct currents to determine r by this expression; identical load conditions do not supply independent information.

Step 4: Conclusion $I = E/(R + r)$, with $I_{\max} = E/r$ at $R = 0$; $V = ER/(R + r)$, with $V_{\max} = E$ for

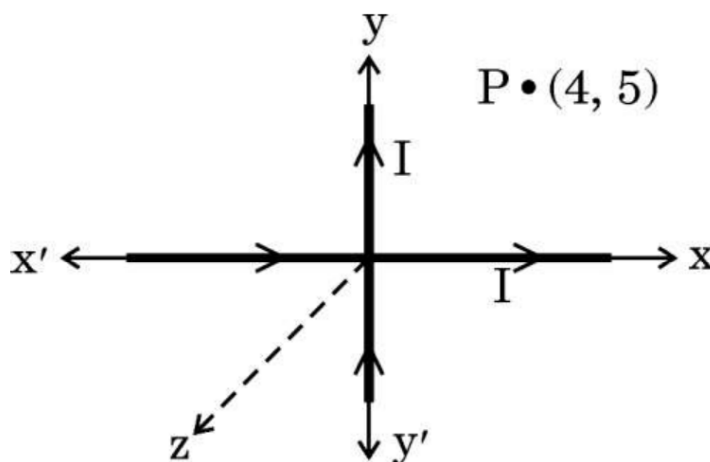
an open circuit. The internal resistance is $r = \frac{I_2 R_2 - I_1 R_1}{I_1 - I_2}$.

Quick Tip to Solve: Use the two extremes: a short circuit maximises current, whereas an open circuit maximises terminal voltage. For two loads, equate the two expressions for emf before solving for r .

23. (a) Write vector form of Biot-Savart law.

(b) Two insulated long straight wires, each carrying 2.0 A current are kept along xx' and yy' axis as shown in the figure. Find the magnitude and direction of resultant magnetic field at point $P(4\text{ m}, 5\text{ m})$.

[3 marks]



Solution:

Concept:

- Biot-Savart law gives the magnetic field of a current element: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$, where $\hat{r}$ points from the element towards the observation point.
- An infinitely long straight wire carrying current I produces $B = \mu_0 I / (2\pi d)$ at perpendicular distance d . Use the right-hand rule for direction and vector superposition for the resultant.

Step 1: (a) State the vector form and define its directions For a small current element $I d\vec{\ell}$, the vector form of Biot-Savart law is

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \vec{r}}{r^3}.$$

Here $d\vec{\ell}$ points along the conventional current, $\vec{r}$ is drawn from that element to the observation point, and μ_0 is the permeability of free space. The cross product makes $d\vec{B}$ perpendicular to both $d\vec{\ell}$ and $\vec{r}$.

Step 2: (b) Find the contribution from the horizontal wire The figure shows current along $+x$. At $P(4\text{ m}, 5\text{ m})$, the perpendicular distance from the x -axis is 5 m. Taking $+z$ as the outward normal to the xy -plane, the right-hand rule gives $\hat{x} \times \hat{y} = \hat{z}$. Therefore,

$$\vec{B}_x = \frac{4\pi \times 10^{-7} \times 2.0}{2\pi \times 5} \hat{z} = 8.0 \times 10^{-8} \hat{z} \text{ T}.$$

Step 3: Find the contribution from the vertical wire and add vectors Current in the vertical wire is along $+y$, and the perpendicular distance to P is 4 m. Since $\hat{y} \times \hat{x} = -\hat{z}$, its field is directed into the

plane:

$$\vec{B}_y = -\frac{4\pi \times 10^{-7} \times 2.0}{2\pi \times 4} \hat{z} = -1.0 \times 10^{-7} \hat{z} \text{ T.}$$

Although the wires are perpendicular, their fields at this point are oppositely directed along the common normal. Thus subtract their magnitudes:

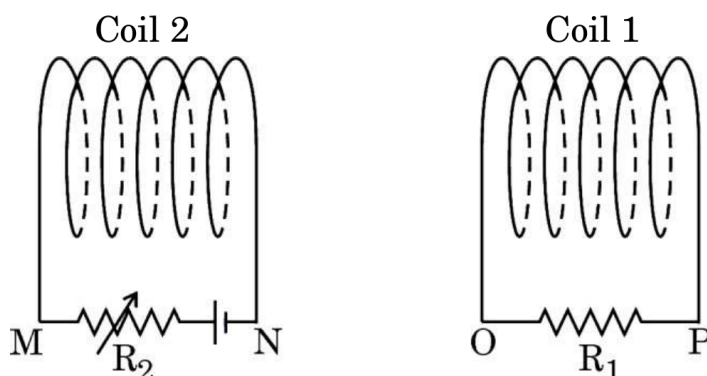
$$\vec{B} = \vec{B}_x + \vec{B}_y = (8.0 \times 10^{-8} - 1.0 \times 10^{-7}) \hat{z} \text{ T} = -2.0 \times 10^{-8} \hat{z} \text{ T.}$$

Step 4: Conclusion The vector law is $d\vec{B} = \frac{\mu_0 I}{4\pi r^2} (d\vec{\ell} \times \hat{r})$. At P , the resultant magnetic field has magnitude $2.0 \times 10^{-8} \text{ T}$, directed along $-z$, into the plane of the paper.

Quick Tip to Solve: Distance from the horizontal wire is the y -coordinate, and distance from the vertical wire is the x -coordinate. Determine field directions before adding: here the stronger field is due to the nearer vertical wire.

24. Two coils '1' and '2' are placed close to each other as shown in the figure. Find the direction of induced current in coil '1' in each of the following situations, justifying your answers :

- Coil '2' is moving towards coil '1'.
- Coil '2' is moving away from coil '1'.
- The resistance connected with coil '2' is increased keeping both the coils stationary. [3 marks]



Solution:

Concept:

- Faraday's law gives the induced emf, $\varepsilon_1 = -N_1 d\Phi_{21}/dt$, where Φ_{21} is the flux per turn of coil 1 due to coil 2 and N_1 is the number of turns in coil 1.
- By Lenz's law, the induced current produces a magnetic field that opposes the change in linked flux. Increasing flux induces an opposing field; decreasing flux induces a field in the original direction.

Step 1: Read the battery polarity and fix the current convention The longer battery plate is positive and lies on the side connected to R_2 . Conventional current therefore passes through R_2 to M , then through the winding of coil 2 from M to N . Both coils are wound in the same sense in the printed figure. Hence a current through the winding of coil 1 from O to P produces an axial field in the same direction as that of coil 2; winding current from P to O produces the opposite field. The current in the resistor R_1 is opposite to the current through the winding between these same endpoints.

Step 2: (a) Oppose the increase in flux as coil 2 approaches When coil 2 moves towards coil 1, the magnetic flux linked with coil 1 increases. The induced field must oppose the field of coil 2, so the

induced winding current is from P to O . It returns through R_1 from O to P . The two facing ends acquire like magnetic poles, giving repulsion that opposes the approach.

Step 3: (b) Support the flux as coil 2 recedes As coil 2 moves away, the linked magnetic flux decreases. The induced field must be in the same direction as the original field to oppose this decrease. Thus the winding current in coil 1 is from O to P , and the current through R_1 is from P to O . The facing ends have unlike poles, giving attraction that opposes the separation.

Step 4: (c) Support the flux while the resistance is increased Increasing R_2 reduces the current in coil 2 and hence reduces its magnetic field and the flux linked with stationary coil 1. This is again a decreasing-flux situation, so the winding current is from O to P and the current through R_1 is from P to O . This induced current exists during the change; once the source current becomes steady at its new value, the linked flux is constant and the induced emf vanishes.

Step 5: Conclusion Directions through the resistor R_1 are: (a) $O \rightarrow P$; (b) $P \rightarrow O$; (c) $P \rightarrow O$. Through the winding of coil 1, the directions are respectively $P \rightarrow O$, $O \rightarrow P$, and $O \rightarrow P$.

Quick Tip to Solve: First decide whether the linked flux increases or decreases, then apply Lenz's law. Always specify whether an O -to- P direction refers to the winding or to the resistor, since their directions are opposite.

25. (a) State any three characteristics of electromagnetic waves.

(b) Briefly explain how and where the displacement current exists during the charging of a capacitor.

[3 marks]

Solution:

Concept:

- An electromagnetic wave consists of oscillating electric and magnetic fields, $\vec{E}$ and $\vec{B}$, which sustain one another as the wave propagates.
- Maxwell's displacement current is associated with a changing electric flux. In vacuum, $I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$, where $\Phi_E = \int \vec{E} \cdot d\vec{A}$ and ε_0 is the permittivity of free space.

Step 1: (a) State three characteristics of electromagnetic waves **1. Transverse nature:** The electric and magnetic fields are mutually perpendicular, and each is perpendicular to the direction of propagation. The two fields oscillate in the same phase.

2. No material medium required: Electromagnetic waves can propagate through vacuum; they do not require a material medium to carry their disturbance.

3. Definite speed in vacuum: All electromagnetic waves travel in vacuum at the same speed

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \simeq 3.0 \times 10^8 \text{ m s}^{-1},$$

where μ_0 is the permeability of free space. Their field amplitudes obey $E_0/B_0 = c$.

Step 2: (b) Locate and explain displacement current during charging During charging, conduction current in the connecting wires supplies charges to the capacitor plates. The increasing plate charge produces a changing electric field, and hence a changing electric flux, in the insulating gap between the plates. This changing flux constitutes displacement current in that gap, even though free charges do not flow across it.

For an ideal parallel-plate capacitor with vacuum between plates of area A , neglecting edge effects,

$$E = \frac{Q}{\epsilon_0 A}, \quad \Phi_E = EA = \frac{Q}{\epsilon_0}.$$

Therefore,

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} = I_c,$$

where I_c is the conduction current in the wires. Thus displacement current provides current continuity through the capacitor gap and produces a magnetic field. When charging is complete and the electric field is constant, the displacement current becomes zero.

Step 3: Conclusion Electromagnetic waves are transverse, can travel through vacuum and have the vacuum speed c . During capacitor charging, the changing electric flux between the plates gives a displacement current equal to the conduction current in the leads.

Quick Tip to Solve: For a charging capacitor, use $\Phi_E = Q/\epsilon_0$ to obtain $I_d = dQ/dt$; a current can exist in the gap without charges crossing it.

26. A double slit set-up was initially placed in a tank filled with water and the interference pattern was obtained using a laser light. When water is replaced by a transparent liquid of refractive index $n > n_{\text{water}}$, what will be the effect on the following ?

(a) Speed, frequency and wavelength of the light of laser beam.

(b) The fringe width, shape of interference fringes and shift in the position of central maximum.

[3 marks]

Solution:

Concept:

- In a transparent medium of refractive index n , light has speed $v = c/n$ and wavelength $\lambda = v/\nu$. Its frequency ν is fixed by the laser source and does not change when the medium changes.
- In Young's double-slit experiment, the fringe width is $\beta = \lambda D/d$, where D is the slit-to-screen distance and d is the slit separation. The central maximum occurs where the optical path difference is zero.

Step 1: (a) Compare speed, frequency and wavelength Let n_w be the refractive index of water, and let subscripts w and l refer to water and the replacement liquid. Since $n > n_w$,

$$v_w = \frac{c}{n_w}, \quad v_l = \frac{c}{n}, \quad \frac{v_l}{v_w} = \frac{n_w}{n} < 1.$$

Hence the speed decreases. The same laser supplies the light, so the frequency remains unchanged: $\nu_l = \nu_w$. Using $\lambda = v/\nu$,

$$\frac{\lambda_l}{\lambda_w} = \frac{n_w}{n} < 1.$$

The wavelength therefore decreases by the same factor as the speed.

Step 2: (b) Determine the fringe width and shape The entire setup is immersed uniformly, and neither D nor d changes. Thus

$$\frac{\beta_l}{\beta_w} = \frac{\lambda_l}{\lambda_w} = \frac{n_w}{n} < 1.$$

The fringe width decreases: adjacent bright fringes, or adjacent dark fringes, become closer together. The geometrical arrangement and symmetry of the two slits remain the same, so the shape of the interference fringes is unchanged. In the usual small-angle arrangement, they remain approximately straight bands parallel to the slits.

Step 3: (b) Check the position of the central maximum At the original central point, the distances from the two slits are equal, say $r_1 = r_2$. After uniform replacement of water, the optical path difference there is

$$\Delta = n(r_2 - r_1) = 0.$$

The waves therefore still arrive in phase at this point. There is no shift of the central maximum; the other fringes move closer to it symmetrically as their spacing decreases.

Step 4: Conclusion Speed, wavelength and fringe width decrease by the factor n_w/n . Frequency and fringe shape remain unchanged, and the central maximum does not shift.

Quick Tip to Solve: For uniform immersion, scale both λ and β by the old refractive index divided by the new one; equal optical paths still locate the same central maximum.

27. Explain the following observations using Einstein's photoelectric equation :

(a) Photoelectric emission does not occur from a surface when the frequency of the light incident on it is less than a certain minimum value.

(b) It is the frequency, and not the intensity of the incident light which affects the maximum kinetic energy of the photoelectrons.

(c) The cut-off voltage (V_0) versus frequency (ν) of the incident light curve is a straight line with a slope $\frac{h}{e}$. [3 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $h\nu = \phi + K_{\max}$, where h is Planck's constant, ν is the incident frequency, ϕ is the work function of the surface, and $K_{\max}$ is the maximum photoelectron kinetic energy.
- The threshold frequency satisfies $\phi = h\nu_0$. The magnitude V_0 of the stopping potential obeys $eV_0 = K_{\max}$, where e is the magnitude of the electronic charge.

Step 1: (a) Explain the minimum frequency for emission An electron absorbs the energy $h\nu$ of a single photon. It needs at least the work function ϕ to escape from the surface, leaving

$$K_{\max} = h\nu - \phi = h(\nu - \nu_0).$$

Since kinetic energy cannot be negative, emission requires $h\nu \geq \phi$, or $\nu \geq \nu_0 = \phi/h$. For $\nu < \nu_0$, a photon supplies insufficient energy, so no photoelectrons are emitted. At threshold, electrons just escape with zero maximum kinetic energy.

Step 2: (b) Distinguish the effects of frequency and intensity For a given surface, ϕ is fixed, so Einstein's equation gives $\Delta K_{\max} = h \Delta \nu$. Increasing frequency above threshold increases each photon's energy and hence the maximum photoelectron kinetic energy. Increasing intensity at fixed frequency increases the number of incident photons per unit area per unit time, but leaves each photon's energy $h\nu$ unchanged. It can therefore increase the photoelectric current, when emission is

possible, without changing $K_{\max}$.

Step 3: (c) Derive the slope of the cut-off voltage graph At the stopping potential, the retarding electric field just stops even the most energetic photoelectrons. Thus

$$eV_0 = K_{\max} = h\nu - \phi,$$
$$V_0 = \frac{h}{e}\nu - \frac{\phi}{e} = \frac{h}{e}(\nu - \nu_0).$$

Comparing with $y = mx + b$, the V_0 -versus- ν graph is a straight line of slope h/e . Its frequency-axis intercept is ν_0 , and its extrapolated vertical intercept is $-\phi/e$. The physical relation applies where emission occurs.

Step 4: Conclusion Emission requires $\nu \geq \phi/h$. Above threshold, frequency determines $K_{\max}$, while intensity controls photon number. The relation $V_0 = (h/e)\nu - \phi/e$ has slope h/e .

Quick Tip to Solve: Start all three explanations from $h\nu = \phi + K_{\max}$; then replace $K_{\max}$ by eV_0 to read the graph's slope directly.

28. (a) What are majority and minority charge carriers of p-type and n-type semiconductors ?

(b) Explain briefly the formation of diffusion current and drift current in a p-n junction diode.

[3 marks]

Solution:

Concept:

- Majority carriers are the more numerous mobile charge carriers in a semiconductor; minority carriers are the less numerous ones. Doping determines which kind is in the majority.
- Diffusion is carrier motion caused by a concentration gradient. Drift is carrier motion caused by an electric field; conventional current follows positive-charge motion and opposes electron motion.

Step 1: Identify the carriers in each type (a) In a p-type semiconductor, acceptor doping supplies holes: holes are the majority carriers and electrons are the minority carriers. In an n-type semiconductor, donor doping supplies electrons: electrons are the majority carriers and holes are the minority carriers. Thermal generation supplies the minority carriers in both cases. The crystal as a whole remains electrically neutral; the words p-type and n-type identify the majority mobile carriers, not a net charge on the crystal.

Step 2: Explain diffusion across the junction (b) When p-type and n-type regions form a junction, the large difference in carrier concentrations causes holes to diffuse from p to n and electrons to diffuse from n to p. Both motions produce conventional diffusion current from p to n. Carriers recombine near the junction, leaving behind immobile positive donor ions on the n side and immobile negative acceptor ions on the p side. The region depleted of mobile carriers is called the depletion region.

Step 3: Explain drift and equilibrium The uncovered ions set up a junction electric field directed from n to p. This field drives minority holes in the n region towards p and minority electrons in the p region towards n. Both contributions give conventional drift current from n to p, opposite to the diffusion current. At thermal equilibrium in an unbiased diode, the two currents have equal magnitudes, so the net junction current is zero:

$$|I_{\text{diff}}| = |I_{\text{drift}}|.$$

Step 4: Conclusion p-type: majority holes, minority electrons; n-type: majority electrons, minority holes. Concentration differences cause diffusion current from p to n, while the junction field causes drift current from n to p. These currents balance at equilibrium.

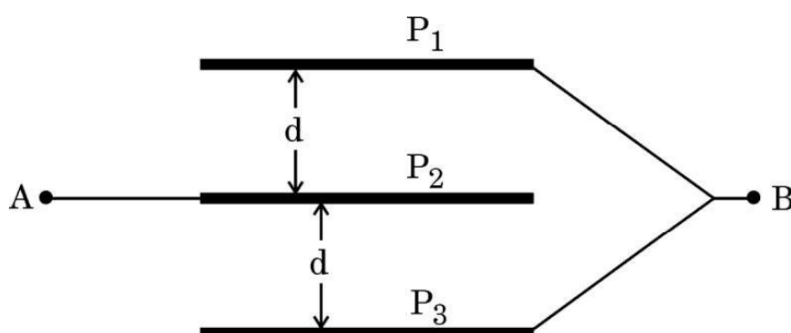
Quick Tip to Solve: First draw the junction field from the positive n-side ions to the negative p-side ions. Hole drift follows this field, electron drift opposes it, but both give the same conventional drift-current direction.

Section D (8 marks)

Questions number 29 and 30 are Case Study-based questions. Read the following paragraphs and answer the questions that follow.

29. A parallel plate capacitor consists of two conducting plates kept generally parallel to each other at a distance. When the capacitor is charged, the charge resides on the inner surfaces of the plates and an electric field is set up between them. Thus, electrostatic energy is stored in the capacitor. The figure shows three large square metallic plates, each of side ' L ' held parallel and equidistant from each other. The space between P_1 and P_2 and P_2 and P_3 is completely filled with mica sheets of dielectric constant ' K '. The plate P_2 is connected to point A and other plates P_1 and P_3 are connected to point B . Point A is maintained at a positive potential with respect to point B and the potential difference between A and B is V .

[4 marks]



29(i). (i) The capacitance of the system between A and B will be :

[1 mark]

- (A) $\frac{\epsilon_0 K L^2}{d}$
 (B) $\frac{\epsilon_0 K L^2}{2d}$
 (C) $\frac{2\epsilon_0 K L^2}{d}$
 (D) $\frac{2\epsilon_0 K d}{L^2}$

Correct Answer: (C) $\frac{2\epsilon_0 K L^2}{d}$

Solution:

Concept:

- A dielectric-filled parallel plate capacitor has capacitance $C_0 = \epsilon_0 K S / d$, where S is the facing plate area, d the gap and K the relative permittivity.
- Capacitors connected between the same two electrical nodes are in parallel, so their capacitances add.

Step 1: Identify the two capacitors The facing pairs P_1, P_2 and P_2, P_3 each form a capacitor of area $S = L^2$ and separation d . Both contain mica, so each has capacitance

$$C_0 = \frac{\epsilon_0 K L^2}{d}.$$

The central plate is connected to A , while both outer plates are connected to B . Thus each gap has the same potential difference V .

Step 2: Add the parallel capacitances The two capacitors are connected in parallel between A and B ,

giving

$$C_{AB} = C_0 + C_0 = \frac{2\varepsilon_0 K L^2}{d}.$$

Option (A) counts only one gap; (B) incorrectly combines them in series. Option (D) also has the wrong dependence on area and separation and does not have the dimensions of capacitance.

Step 3: Conclusion $C_{AB} = 2\varepsilon_0 K L^2/d$. Hence, the correct option is (C).

Quick Tip to Solve: Label the electrical nodes before combining plates: both outer plates share B, so the two gaps are in parallel even though the plates look stacked.

29(ii). (ii) The charge on plate P_1 is :

[1 mark]

- (A) $\frac{\varepsilon_0 V K L^2}{2d}$
(B) $\frac{\varepsilon_0 V K L^2}{d}$
(C) $\frac{2\varepsilon_0 V K L^2}{d}$
(D) $\frac{\varepsilon_0 V K L^2}{4d}$

Correct Answer: (B) $\frac{\varepsilon_0 V K L^2}{d}$

Solution:

Concept:

- The magnitude of charge on either plate of a capacitor is $q = CV$, where V is its potential difference. The plate at lower potential carries negative charge.
- Each outer plate faces the central plate across one mica-filled gap, with capacitance $C_0 = \varepsilon_0 K L^2/d$. The two outer plates together form the negative terminal.

Step 1: Use the single gap adjoining P_1 Plate P_1 is connected to B, and the facing plate P_2 is connected to A. Their separation is d , area is L^2 , and potential difference is V . Hence the magnitude on P_1 is

$$|Q_1| = C_0 V = \frac{\varepsilon_0 V K L^2}{d}.$$

Step 2: Check the sign and compare the options Since B is at lower potential than A, the signed charge is $Q_1 = -\varepsilon_0 V K L^2/d$. The positive expressions in the choices represent charge magnitudes, for which (B) agrees. Option (C) is the magnitude of the total charge on the central plate, which has two charged faces. Options (A) and (D) incorrectly reduce the charge for this single gap by factors of two and four.

Step 3: Conclusion Plate P_1 carries negative charge of magnitude $\varepsilon_0 V K L^2/d$. Hence, the correct option is (B).

Quick Tip to Solve: For an outer plate count one active face, but for the central plate count two. Determine the sign separately from the terminal potential.

29(iii). (iii) The electric field in the region between P_1 and P_2 is :

[1 mark]

- (A) $\frac{V}{d}$
- (B) $\frac{2V}{d}$
- (C) $\frac{V}{2d}$
- (D) $\frac{d}{V}$

Correct Answer: (A) $\frac{V}{d}$

Solution:

Concept:

- Between large parallel plates, neglecting edge effects, the field is uniform and its magnitude is $E = \Delta V/s$, where s is their separation.
- Electric field points from higher to lower potential. If a source fixes the voltage, a dielectric changes the stored charge while the ratio $\Delta V/s$ remains fixed.

Step 1: Identify the voltage across the upper gap Plate P_2 is at the potential of A and plate P_1 is at the potential of B . Thus the potential difference across this gap is the full applied voltage V , and the separation shown in the figure is d , not $2d$.

Step 2: Calculate the field and test the alternatives For this uniform gap,

$$E = \frac{V_A - V_B}{d} = \frac{V}{d}.$$

It points from P_2 towards P_1 . Option (B) incorrectly doubles the field, while (C) assigns the outer-plate separation $2d$ to this single gap. Option (D) has inverse electric-field units and cannot represent E .

Step 3: Conclusion The electric-field magnitude is V/d , directed from P_2 to P_1 . Hence, the correct option is (A).

Quick Tip to Solve: Apply $E = \text{voltage across the chosen gap} / \text{that gap's width}$. Each gap here receives the full voltage V because both are connected across A and B .

29(iv). (iv) (a) The separation between the plates of same area (L^2) of a parallel plate air capacitor having capacitance equal to that of this system, will be : [1 mark]

- (A) $\frac{d}{K}$
- (B) $\frac{2d}{K}$
- (C) $\frac{d}{2K}$
- (D) $\frac{d}{4K}$

Correct Answer: (C) $\frac{d}{2K}$

Solution:

Concept:

- The two mica-filled gaps are in parallel, giving total capacitance $C_{AB} = 2\varepsilon_0 K L^2 / d$.
- An air capacitor of plate area S and separation x has capacitance $C_{\text{air}} = \varepsilon_0 S / x$, taking the relative permittivity of air as unity.

Step 1: Equate the two capacitances Let x be the required air gap. The new capacitor has the same area L^2 , so equality of capacitances requires

$$\frac{\varepsilon_0 L^2}{x} = \frac{2\varepsilon_0 K L^2}{d}.$$

The factor two on the right comes from the two parallel gaps of the original system.

Step 2: Solve for the air gap Cancel the common factor $\varepsilon_0 L^2$ and rearrange:

$$\frac{1}{x} = \frac{2K}{d}, \quad x = \frac{d}{2K}.$$

Option (A) would match only one mica-filled gap. Option (B) corresponds to treating the original two gaps as a series combination; option (D) gives twice the required capacitance. A smaller air gap compensates both for removing mica and for replacing the two parallel capacitors by one.

Step 3: Conclusion The required separation is $d/(2K)$. Hence, the correct option is (C).

Quick Tip to Solve: At fixed plate area, capacitance varies inversely with air-gap width. A capacitance enhancement of $2K$ therefore requires reducing the width by $2K$.

29(iv). (b) If the source of potential difference applied between A and B is removed, and then A and B are connected by a conducting wire, the net charge on the system will be : [1 mark]

- (A) $\frac{\varepsilon_0 V K L^2}{4d}$
 (B) $\frac{\varepsilon_0 V K L^2}{2d}$
 (C) $\frac{\varepsilon_0 V K L^2}{d}$
 (D) Zero

Correct Answer: (D) Zero

Solution:

Concept:

- Charging a capacitor transfers charge between its terminals: their charges are equal and opposite, so the total charge of the complete capacitor system remains zero.
- After removing the source, joining the terminals by a conducting wire makes their potentials equal and allows the capacitor to discharge. Charge is redistributed, not destroyed.

Step 1: Sum the charges before connecting the wire Write $q = \varepsilon_0 K L^2 V / d$ for the magnitude on either outer plate. Initially the central plate has charge $+2q$, while each outer plate has charge $-q$. Therefore the algebraic total is

$$Q_{\text{net}} = 2q - q - q = 0.$$

Step 2: Apply conservation during discharge The removed source can no longer supply charge. On connecting A and B , excess electrons flow from the negatively charged outer plates towards the positive central plate until their potential difference becomes zero. The system discharges, while its total charge stays zero. Options (A), (B) and (C) are nonzero positive quantities and confuse an individual stored-charge magnitude with the algebraic net charge of the complete system.

Step 3: Conclusion The net charge on the system is zero. Hence, the correct option is (D).

Quick Tip to Solve: For net charge, add the signed charges on all three plates. The equal positive and negative terminal charges cancel even before the wire is connected.

30. A hydrogen atom consists of an electron revolving in a circular orbit of radius r with certain velocity v around a proton located at the nucleus of the atom. The electrostatic force of attraction between the revolving electron and the proton provides the requisite centripetal force to keep it in the orbit. According to Bohr's model, an electron can revolve only in certain stable orbits. The angular momentum of the electron in these orbits is some integral multiple of $\frac{h}{2\pi}$, where h is the Planck's constant. Further, when an electron makes a transition from one orbit of higher energy to that of lower energy, a photon is emitted having energy equal to the difference between energies of the initial and final states. Assuming the mass and charge of an electron as m and e respectively, answer the following questions. [4 marks]

30(i). (i) The expression for the speed of electron v in terms of radius of the orbit (r) and physical constant $\left(K = \frac{1}{4\pi\epsilon_0}\right)$ is : [1 mark]

- (A) $\frac{Ke^2}{mr}$
 (B) $\frac{Ke^2}{mr^2}$
 (C) $\sqrt{\frac{Ke^2}{mr}}$
 (D) $\sqrt{\frac{Ke^2}{mr^2}}$

Correct Answer: (C) $\sqrt{\frac{Ke^2}{mr}}$

Solution:

Concept:

- The electrostatic attraction supplies the centripetal force: $F = Ke^2/r^2 = mv^2/r$, where m is the electron mass, e is the charge magnitude, and $K = 1/(4\pi\epsilon_0)$.
- The centripetal-force equation first gives the square of the speed. The speed v is its positive square root.

Step 1: Equate the radial forces For the electron's circular orbit of radius r , Coulomb attraction acts towards the proton. Thus,

$$\frac{mv^2}{r} = \frac{Ke^2}{r^2} \Rightarrow v^2 = \frac{Ke^2}{mr}.$$

Step 2: Check the form and dimensions of each option Taking the positive square root gives the speed. Option (A) is the expression for v^2 , while option (B) is the centripetal acceleration, $Ke^2/(mr^2)$. Option (D) incorrectly takes the square root of that acceleration; it retains an extra factor of r in the denominator. Only the square-root expression in option (C) has both the required radius dependence and dimensions of speed.

Step 3: Conclusion The electron's speed is $v = \sqrt{\frac{Ke^2}{mr}}$. Hence, the correct option is (C).

Quick Tip to Solve: Cancel one factor of r when equating Coulomb and centripetal forces, then take the square root to obtain speed.

30(ii). (ii) The total energy of the atom in terms of r and physical constant K is :

[1 mark]

- (A) $\frac{Ke^2}{r}$
(B) $-\frac{Ke^2}{2r}$
(C) $\frac{Ke^2}{2r}$
(D) $\frac{3Ke^2}{2r}$

Correct Answer: (B) $-\frac{Ke^2}{2r}$

Solution:

Concept:

- Taking zero potential energy at infinite separation, an electron and proton have potential energy $U = -Ke^2/r$, where e is the charge magnitude and $K = 1/(4\pi\epsilon_0)$.
- The atom's total energy is $E = T + U$, with electron kinetic energy $T = mv^2/2$. The nucleus is treated as fixed in the stated Bohr model.

Step 1: Express the kinetic energy using the orbit radius Coulomb attraction supplies the required centripetal force. Therefore,

$$\frac{mv^2}{r} = \frac{Ke^2}{r^2} \Rightarrow T = \frac{1}{2}mv^2 = \frac{Ke^2}{2r}.$$

Step 2: Add the kinetic and potential energies The attractive interaction gives negative potential energy, whose magnitude is twice the kinetic energy. Hence,

$$E = T + U = \frac{Ke^2}{2r} - \frac{Ke^2}{r} = -\frac{Ke^2}{2r}.$$

Option (A) is only the positive magnitude of the potential energy, and option (C) gives the kinetic energy alone. Option (D) would result from adding kinetic energy to the magnitude of potential energy, losing the negative sign of the attractive potential. All three are positive and cannot describe the total energy of this bound orbit.

Step 3: Conclusion The total energy is $E = -\frac{Ke^2}{2r}$. Hence, the correct option is (B).

Quick Tip to Solve: In a circular Coulomb orbit, use $U = -2T$, so the total energy is immediately $E = -T = U/2$.

30(iii). (iii) A photon of wavelength 500 nm is emitted when an electron makes a transition from one state to the other state in an atom. The change in the total energy of the electron and change in its kinetic energy in eV as per Bohr's model, respectively will be : [1 mark]

(A) 2.48, -2.48

(B) -1.24, 1.24

(C) -2.48, 2.48

(D) 1.24, -1.24

Correct Answer: (C) -2.48, 2.48

Solution:

Concept:

- An emitted photon carries energy $E_\gamma = hc/\lambda$, where h is Planck's constant, c is the speed of light and λ is the wavelength. The atom loses this energy.
- In a Bohr orbit, total energy E and kinetic energy T obey $E = -T$. Thus, for changes defined as final minus initial, $\Delta T = -\Delta E$.

Step 1: Calculate the emitted photon energy Use $hc \simeq 1240 \text{ eV nm}$ and the given wavelength $\lambda = 500 \text{ nm}$:

$$E_\gamma = \frac{1240 \text{ eV nm}}{500 \text{ nm}} = 2.48 \text{ eV}.$$

This is the positive energy carried away by the photon; it is not the signed change in the atom's energy.

Step 2: Assign the signs of both changes Emission lowers the total energy of the electron, so its final-minus-initial change is negative. Since $E = -T$ in each orbit, the kinetic energy increases by the same amount:

$$\Delta E = -E_\gamma = -2.48 \text{ eV}, \quad \Delta T = -\Delta E = +2.48 \text{ eV}.$$

Option (A) reverses both signs, as for absorption. Options (B) and (D) incorrectly halve the photon energy; (D) also has the signs for absorption. The relation $E = -T$ does not introduce an additional factor of one half into the energy changes.

Step 3: Conclusion The changes in total energy and kinetic energy are respectively -2.48 eV and +2.48 eV. Hence, the correct option is (C).

Quick Tip to Solve: For emission, first write $\Delta E = -hc/\lambda$; then reverse its sign to obtain the kinetic-energy change in a Bohr orbit.

30(iv)(a). (iv) (a) In Bohr's model of hydrogen atom, the frequency of revolution of electron in its n^{th} orbit is proportional to :

[1 mark]

- (A) n
- (B) $\frac{1}{n}$
- (C) $\frac{1}{n^2}$
- (D) $\frac{1}{n^3}$

Correct Answer: (D) $\frac{1}{n^3}$

Solution:

Concept:

- Bohr's angular-momentum condition is $mv_n r_n = n\hbar$, with $\hbar = h/(2\pi)$, for the orbit of principal quantum number n .
- The revolution frequency is the number of complete orbits per second: $f_n = v_n/(2\pi r_n)$. It is different from the frequency of light emitted in a transition.

Step 1: Establish how radius and speed depend on the orbit number Combining Coulomb attraction with the centripetal-force equation gives $mv_n^2 r_n = Ke^2$. Using $v_n = n\hbar/(mr_n)$ from angular-momentum quantisation then gives

$$r_n = \frac{n^2 \hbar^2}{mKe^2} \propto n^2, \quad v_n = \frac{Ke^2}{n\hbar} \propto \frac{1}{n}.$$

Step 2: Divide speed by the orbital circumference One revolution covers the circumference $2\pi r_n$. Therefore,

$$f_n = \frac{v_n}{2\pi r_n} \propto \frac{n^{-1}}{n^2} = n^{-3}.$$

Option (B) gives the dependence of speed alone and ignores the increasing circumference. Option (C) includes only the inverse-radius dependence and ignores the decrease in speed. Option (A) predicts an increase, whereas the larger orbit and lower speed both make the frequency decrease.

Step 3: Conclusion The electron's frequency of revolution is proportional to $\frac{1}{n^3}$. Hence, the correct option is (D).

Quick Tip to Solve: Remember $v_n \propto n^{-1}$ and $r_n \propto n^2$; dividing speed by circumference subtracts the exponents to give n^{-3} .

30(iv)(b). (b) An electron makes a transition from -3.4 eV state to the ground state in hydrogen atom. Its radius of orbit changes by : (radius of orbit of electron in ground state = $0.53 \text{ \AA}$) [1 mark]

- (A) $0.53 \text{ \AA}$
- (B) $1.06 \text{ \AA}$
- (C) $1.59 \text{ \AA}$
- (D) $2.12 \text{ \AA}$

Correct Answer: (C) $1.59 \text{ \AA}$

Solution:

Concept:

- Hydrogen energy levels obey $E_n = -13.6/n^2$ eV, where n is the principal quantum number; the ground state has $n = 1$.
- The radius of the n th Bohr orbit is $r_n = n^2 r_1$, where the given ground-state radius is $r_1 = 0.53 \text{ \AA}$.

Step 1: Identify the initial energy level Set the stated initial energy equal to the Bohr energy expression:

$$-\frac{13.6}{n_i^2} = -3.4 \Rightarrow n_i^2 = 4 \Rightarrow n_i = 2.$$

The electron therefore moves from the second orbit to the first orbit, so the radius decreases.

Step 2: Calculate both radii and their difference The initial and final radii are

$$r_i = 2^2(0.53) = 2.12 \text{ \AA}, \quad r_f = 0.53 \text{ \AA}.$$

Thus the signed change is

$$\Delta r = r_f - r_i = 0.53 - 2.12 = -1.59 \text{ \AA}.$$

The positive choices ask for the amount of change. Option (A) is the final radius and option (D) is the initial radius, rather than their difference. Option (B) is twice the ground-state radius; it does not follow the required square dependence on n .

Step 3: Conclusion The orbital radius decreases by $1.59 \text{ \AA}$, corresponding to $\Delta r = -1.59 \text{ \AA}$. Hence, the correct option is (C).

Quick Tip to Solve: Use the energy to identify $n_i = 2$, then calculate the radius decrease directly as $(2^2 - 1^2)r_1 = 3r_1$.

Section E (15 marks)

31(a). (i) Two point charges $+q$ and $-q$ are held at $(a, 0)$ and $(-a, 0)$ in x - y plane. Obtain an expression for the net electric field due to the charges at a point $(0, y)$. Hence, find electric field at a far off point ($y \gg a$).

(ii) Three point charges of -2 nC , -1 nC , and $+5 \text{ nC}$ are kept at the vertices A , B and C of an equilateral triangle of side 0.2 m . Find the total amount of work done in shifting the charges from A to A_1 , B to B_1 and C to C_1 . Here A_1 , B_1 and C_1 are the midpoints of sides AB , BC and CA , respectively. [5 marks]

Solution:

Concept:

- By superposition, the net electric field is the vector sum of the individual fields. For a charge Q , $\vec{E} = kQ\vec{R}/R^3$, where $\vec{R}$ points from the charge to the observation point and $k = 1/(4\pi\epsilon_0)$.
- The electric dipole moment points from the negative charge to the positive charge. Here $\vec{p} = 2qa\hat{x}$, and $(0, y)$ is on the perpendicular bisector, or equatorial line, of the dipole.
- The electrostatic potential energy of three point charges is $U = k(q_Aq_B/r_{AB} + q_Bq_C/r_{BC} + q_Cq_A/r_{CA})$, with each distinct pair counted once.
- For a slow rearrangement with no change in kinetic energy, the work done by the external agent is $W_{\text{ext}} = U_f - U_i$. The work done by the electric field has the opposite sign.

Step 1: (i) Write the two electric-field vectors Let the observation point be $P = (0, y)$. Its distance from each charge is $R = \sqrt{a^2 + y^2}$. The displacement from $+q$ at $(a, 0)$ to P is $-a\hat{x} + y\hat{y}$, whereas the displacement from $-q$ at $(-a, 0)$ is $a\hat{x} + y\hat{y}$. Hence,

$$\begin{aligned}\vec{E}_+ &= \frac{kq}{(a^2 + y^2)^{3/2}}(-a\hat{x} + y\hat{y}), \\ \vec{E}_- &= \frac{-kq}{(a^2 + y^2)^{3/2}}(a\hat{x} + y\hat{y}).\end{aligned}$$

The negative sign in the second expression makes that field point towards the negative charge.

Step 2: (i) Add the fields and take the far-field limit The equal and opposite y -components cancel, while both x -components point along the negative x -axis. Therefore,

$$\vec{E}(0, y) = -\frac{2kqa}{(a^2 + y^2)^{3/2}}\hat{x} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{(a^2 + y^2)^{3/2}}.$$

At a far-off point with $y \gg a$, the ratio a^2/y^2 is negligible, so

$$(a^2 + y^2)^{3/2} = y^3 \left(1 + \frac{a^2}{y^2}\right)^{3/2} \simeq y^3.$$

Thus the equatorial field varies inversely as the cube of the distance:

$$\vec{E}_{\text{far}} \simeq -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{y^3}.$$

Step 3: (ii) Calculate the initial potential energy Initially all three pair separations equal $s = 0.2 \text{ m}$.

Converting nanocoulombs to coulombs and keeping the signs of the charges,

$$\begin{aligned} q_A q_B + q_B q_C + q_C q_A &= [(-2)(-1) + (-1)(5) + (5)(-2)]10^{-18} \text{ C}^2 \\ &= (2 - 5 - 10)10^{-18} \text{ C}^2 \\ &= -13 \times 10^{-18} \text{ C}^2. \end{aligned}$$

Using $k = 9 \times 10^9 \text{ N m}^2 \text{C}^{-2}$,

$$U_i = \frac{k}{s}(q_A q_B + q_B q_C + q_C q_A) = \frac{9 \times 10^9 (-13 \times 10^{-18})}{0.2} = -5.85 \times 10^{-7} \text{ J}.$$

Step 4: (ii) Use the midpoint geometry to find the final energy Joining the midpoints of the sides of an equilateral triangle gives another equilateral triangle with half the original side. Thus the separations of the relocated charges are

$$A_1 B_1 = B_1 C_1 = C_1 A_1 = \frac{s}{2} = 0.1 \text{ m}.$$

Every pair product is unchanged, and every denominator is halved. Therefore,

$$U_f = \frac{k}{s/2}(q_A q_B + q_B q_C + q_C q_A) = 2U_i = -1.17 \times 10^{-6} \text{ J}.$$

Since electrostatic forces are conservative, only these initial and final configurations enter the work calculation.

Step 5: (ii) Find the work done in shifting the charges The external work required to rearrange the charges quasistatically is

$$\begin{aligned} W_{\text{ext}} &= U_f - U_i \\ &= -1.17 \times 10^{-6} - (-5.85 \times 10^{-7}) \\ &= -5.85 \times 10^{-7} \text{ J}. \end{aligned}$$

The negative sign means that the system releases energy during this rearrangement. The electric field does positive work of $5.85 \times 10^{-7} \text{ J}$, and the external agent removes that energy if the kinetic energy is kept unchanged.

Step 6: Conclusion (i) $\vec{E}(0, y) = -\frac{2qa}{4\pi\epsilon_0(a^2 + y^2)^{3/2}}\hat{x}$. For $y \gg a$, $\vec{E} \simeq -\frac{\vec{p}}{4\pi\epsilon_0 y^3}$, opposite to the dipole moment. (ii) The work done by the external agent is $\boxed{-5.85 \times 10^{-7} \text{ J}}$; the work done by the electric field is $+5.85 \times 10^{-7} \text{ J}$.

Quick Tip to Solve: At an equatorial point, cancel the components perpendicular to the dipole axis first. For the midpoint rearrangement, all pair distances halve, so use $U_f = 2U_i$ and $W_{\text{ext}} = U_i$.

31(b). (i) Show that Gauss's theorem is consistent with Coulomb's law. Using it, derive an expression for the electric field due to a uniformly charged thin spherical shell of radius r at a point at a distance y from the centre of the shell such that (I) $y > r$, and (II) $y < r$.

(ii) A point charge of $+2 \text{ nC}$ is kept at the origin of a three-dimensional coordinate system. Find the type and magnitude of the charge which should be kept at $(0, 0, -6 \text{ m})$ so that the potential due to the system becomes zero at $(0, 0, 2 \text{ m})$. [5 marks]

Solution:

Concept:

- Gauss's theorem states that the net outward electric flux through any closed surface is $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the enclosed charge.
- For a spherically symmetric charge distribution, the electric field at distance y from its centre is radial and has the same magnitude everywhere on a concentric spherical Gaussian surface of area $4\pi y^2$.
- The potential of a point charge q , with zero potential at infinity, is $V = kq/d$, where d is the positive distance from the charge and $k = 1/(4\pi\epsilon_0)$. Potentials add algebraically as scalars.

Step 1: (i) Establish consistency with Coulomb's law Consider a point charge Q at the centre of a spherical Gaussian surface of radius R . Coulomb's law gives a radial field with signed radial component $E_R = Q/(4\pi\epsilon_0 R^2)$. Since this component is constant over the sphere,

$$\oint \vec{E} \cdot d\vec{A} = E_R(4\pi R^2) = \frac{Q}{4\pi\epsilon_0 R^2}(4\pi R^2) = \frac{Q}{\epsilon_0}.$$

This is the flux required by Gauss's theorem. Conversely, using spherical symmetry with Gauss's theorem gives

$$E_R(4\pi R^2) = \frac{Q}{\epsilon_0} \Rightarrow \vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{\mathbf{R}}.$$

The force on a test charge q_0 is then $\vec{F} = q_0 \vec{E}$, reproducing the inverse-square Coulomb force, with its sign giving attraction or repulsion. Thus the two laws give consistent results.

Step 2: (i)(I) Apply Gauss's theorem outside the shell Let Q be the total charge uniformly distributed over a thin spherical shell of radius r . If its surface charge density is σ , then $Q = 4\pi r^2 \sigma$. To find the field at $y > r$, choose a concentric spherical Gaussian surface of radius y . It encloses the entire charge Q . Spherical symmetry makes the field radial and its radial component E_y constant on this surface, so

$$\oint \vec{E} \cdot d\vec{A} = E_y(4\pi y^2) = \frac{Q}{\epsilon_0},$$

$$\vec{E}(y) = \frac{Q}{4\pi\epsilon_0 y^2} \hat{\mathbf{y}}_{\text{rad}} = \frac{\sigma r^2}{\epsilon_0 y^2} \hat{\mathbf{y}}_{\text{rad}}, \quad y > r.$$

Here $\hat{\mathbf{y}}_{\text{rad}}$ denotes the outward radial unit vector, not a Cartesian coordinate direction. Thus the external field equals the field of a point charge Q placed at the centre; it is outward for positive Q and inward for negative Q .

Step 3: (i)(II) Apply Gauss's theorem inside the shell Choose a concentric Gaussian sphere with $0 < y < r$. All the charge lies on the shell at radius r , so the enclosed charge is zero. Using the same

spherical symmetry,

$$E_y(4\pi y^2) = \frac{0}{\epsilon_0} = 0 \Rightarrow \vec{E} = \vec{0}.$$

The field at the centre also vanishes by symmetry. Hence the electric field is zero everywhere in the interior. Zero enclosed charge alone would imply only zero net flux in general; the spherical symmetry here is what allows us to infer zero field at every interior point.

Step 4: (ii) Calculate the distances to the observation point Let $P = (0, 0, 2 \text{ m})$, and let the required charge be q at $(0, 0, -6 \text{ m})$. The given $+2 \text{ nC}$ charge is at the origin. The two positive distances from these charges to P are

$$d_1 = 2 \text{ m}, \quad d_2 = |2 - (-6)| \text{ m} = 8 \text{ m}.$$

The second distance is 8 m , because the charge and the observation point lie on opposite sides of the origin. A negative coordinate does not make the distance in the potential formula negative.

Step 5: (ii) Set the total potential equal to zero Apply scalar superposition of the potentials at P :

$$V(P) = k \left(\frac{2 \times 10^{-9} \text{ C}}{2 \text{ m}} + \frac{q}{8 \text{ m}} \right) = 0,$$
$$q = -(2 \times 10^{-9} \text{ C}) \frac{8}{2} = -8 \times 10^{-9} \text{ C}.$$

The required charge is negative because it must cancel the positive potential of the charge at the origin. Its magnitude is four times larger because its distance from P is four times greater. Numerically the potentials are $+9 \text{ V}$ and -9 V , which add to zero.

Step 6: Conclusion (i) Gauss's theorem gives the Coulomb field $\vec{E} = Q\hat{\mathbf{R}}/(4\pi\epsilon_0 R^2)$. For the thin spherical shell, the field magnitude is $|Q|/(4\pi\epsilon_0 y^2)$ for $y > r$, with direction set by the sign of Q , and 0 for $y < r$. (ii) A -8 nC charge, negative with magnitude 8 nC , must be placed at $(0, 0, -6 \text{ m})$.

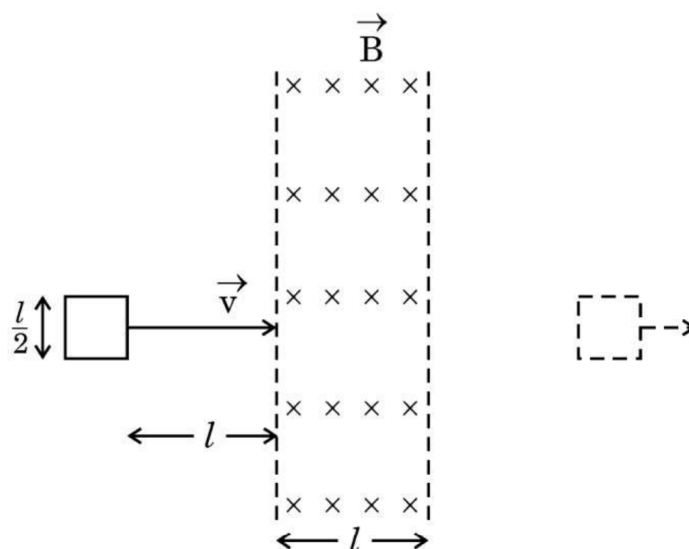
Quick Tip to Solve: For the shell, combine the enclosed charge with spherical symmetry. For zero potential, use signed q/d terms and positive separations: here the distances are 2 m and 8 m , so the charge magnitudes must be in the ratio $1 : 4$.

32(a). (i) State Lenz's law and explain how this law is a consequence of conservation of energy principle.

(ii) A square shaped loop of side $\frac{\ell}{2}$ is initially lying outside a region of uniform magnetic field $\vec{B}$ as shown in the figure. The loop is moved towards right with a constant velocity $\vec{v}$ till it goes out of the region of magnetic field.

(I) What will be the directions of induced current when the loop enters the field and when it leaves the field ?

(II) Draw the plots showing the variation of magnetic flux ϕ linked with the loop with time t and variation of induced emf E with time t . Mark the relevant values of E , ϕ and t on the graphs. [5 marks]



Solution:

Concept:

- Lenz's law: the induced current flows so that its magnetic effect opposes the change in magnetic flux that produces it.
- Faraday's law for this single-turn loop is $E = -d\phi/dt$. In a perpendicular uniform field, $\phi = BA_{\text{in}}$, where A_{in} is the area of the loop lying inside the field.
- The printed loop side is $a = \ell/2$. During entry or exit, $|E| = Bav = B\ell v/2$, because the overlap area changes at the rate av .

Step 1: (i) Explain the connection with conservation of energy Lenz's law states that the direction of the induced current is such that it opposes the change in magnetic flux responsible for inducing it. For example, when a magnet approaches a conducting loop, the induced current produces a magnetic field that opposes the increase of flux. Consequently, the loop repels the approaching magnet, and an external agent must do work to continue the motion.

This mechanical work supplies the electrical energy dissipated as heat in the loop. If the induced current instead aided the change, the magnet would be accelerated towards the loop while electrical energy was also produced, without a corresponding energy supply. This would violate conservation of energy. The opposing direction specified by Lenz's law therefore follows from that principle.

Step 2: (ii)(I) Determine the induced-current directions The crosses in the figure show that the applied magnetic field points into the page. As the loop enters, the flux into the page increases. The induced field must point out of the page to oppose this increase; the right-hand rule therefore gives an **anticlockwise current**.

As the loop leaves, the flux into the page decreases. The induced field must now point into the page to oppose this decrease; the induced current is **clockwise**. When the loop is wholly inside the uniform field, or wholly outside it, the flux is constant and there is no induced emf or current.

Step 3: (ii)(II) Locate the four transition times Take $t = 0$ at the initial position shown. The leading (right) edge is a distance ℓ from the near boundary, and the field region is also of width ℓ . The loop side is $a = \ell/2$. Dividing each required travel distance by the constant speed v gives

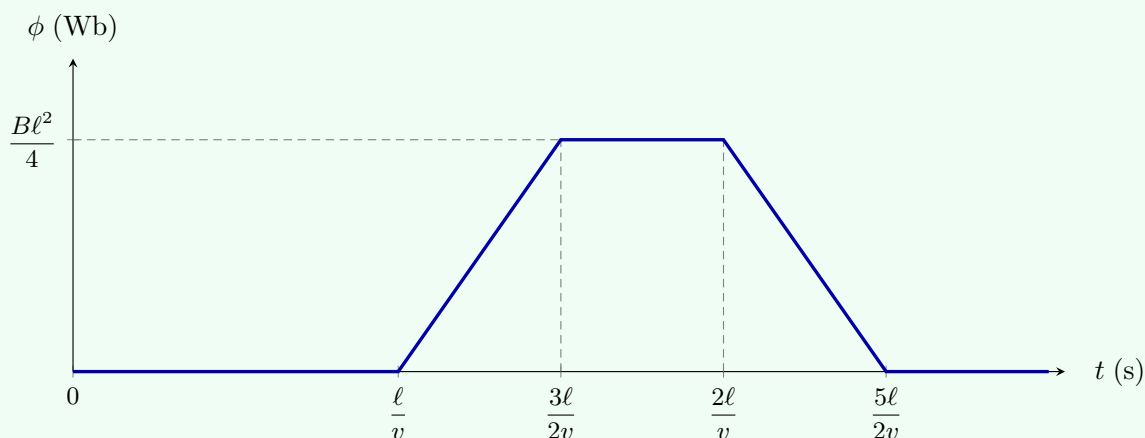
$$\begin{aligned} t_1 &= \frac{\ell}{v} && \text{(leading edge enters),} \\ t_2 &= \frac{\ell + a}{v} = \frac{3\ell}{2v} && \text{(trailing edge enters),} \\ t_3 &= \frac{2\ell}{v} && \text{(leading edge leaves),} \\ t_4 &= \frac{2\ell + a}{v} = \frac{5\ell}{2v} && \text{(trailing edge leaves).} \end{aligned}$$

Thus entry, complete immersion and exit each last $\ell/(2v)$, following the initial delay ℓ/v .

Step 4: Draw the flux-time graph Choose the area normal into the page, so the applied flux is positive. During entry, the overlap width is $vt - \ell$; during exit, it is $5\ell/2 - vt$. Multiplying this width by the side $\ell/2$ and by B gives

$$\phi(t) = \begin{cases} 0, & 0 \leq t \leq \ell/v, \\ \frac{B\ell}{2}(vt - \ell), & \ell/v \leq t \leq 3\ell/(2v), \\ \frac{B\ell^2}{4}, & 3\ell/(2v) \leq t \leq 2\ell/v, \\ \frac{B\ell}{2}\left(\frac{5\ell}{2} - vt\right), & 2\ell/v \leq t \leq 5\ell/(2v), \\ 0, & t \geq 5\ell/(2v). \end{cases}$$

The maximum flux is $Ba^2 = B\ell^2/4$. The graph therefore rises linearly during entry, remains horizontal while the loop is fully inside, and falls linearly during exit.



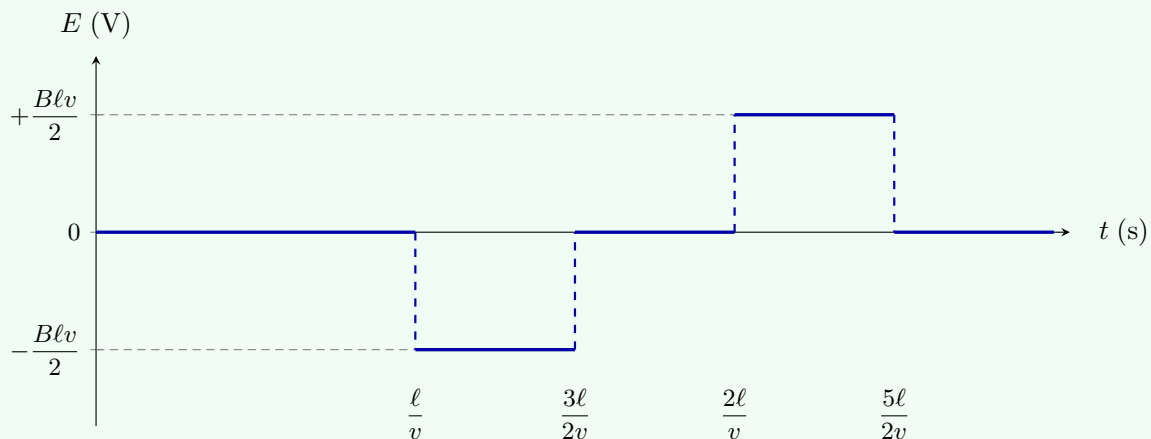
Flux versus time, with $t = 0$ at the shown initial position and flux into the page positive.

Step 5: Draw the signed emf-time graph With the chosen inward area normal, a clockwise circuit direction is positive. Apply $E = -d\phi/dt$: the positive slope on entry produces a negative emf, while

the negative slope on exit produces a positive emf.

$$E(t) = \begin{cases} -\frac{B\ell v}{2}, & \ell/v < t < 3\ell/(2v), \\ +\frac{B\ell v}{2}, & 2\ell/v < t < 5\ell/(2v), \\ 0, & \text{in the remaining time intervals.} \end{cases}$$

The pulses have equal magnitudes and durations but opposite signs. At the ideal sharp field boundaries, the slope of the flux changes abruptly, giving the indicated jumps in emf. Reversing both the flux normal and the positive circuit direction reverses both graphs' signs, while the physical current directions remain the same.



Emf versus time; clockwise emf is positive. Dashed vertical lines indicate ideal jumps.

Step 6: Conclusion Lenz's law expresses opposition to the flux change and ensures conservation of energy. The induced current is anticlockwise on entry and clockwise on exit. From the initial position shown, the flux has maximum value $B\ell^2/4$, and the signed emf pulses are $-B\ell v/2$ on entry and $+B\ell v/2$ on exit, with transition times ℓ/v , $3\ell/(2v)$, $2\ell/v$, $5\ell/(2v)$.

Quick Tip to Solve: Track the leading and trailing edges separately, using the printed side $\ell/2$. First sketch the overlap area versus time; multiply by B for flux and take the negative slope for emf.

- 32(b).** (i) Differentiate between peak and rms values of alternating current. How are they related ?
 (ii) A current element X is connected across an ac source of emf $V = V_0 \sin 2\pi\nu t$. It is found that the voltage leads the current in phase by $\frac{\pi}{2}$ radian. If element X was replaced by element Y , the voltage lags behind the current in phase by $\frac{\pi}{2}$ radian.
 (I) Identify elements X and Y by drawing phasor diagrams.
 (II) Obtain the condition of resonance when both elements X and Y are connected in series to the source and obtain expression for resonant frequency. What is the impedance value in this case ?

[5 marks]

Solution:**Concept:**

- The peak current I_0 is the maximum magnitude of an alternating current. Its rms value $I_{\text{rms}} = \sqrt{\langle i^2 \rangle}$ is the equivalent steady current producing the same heating in a given resistor over a complete cycle.
- For angular frequency $\omega = 2\pi\nu$, inductive reactance is $X_L = \omega L$ and capacitive reactance is $X_C = 1/(\omega C)$, where L is inductance and C is capacitance.
- In a series circuit the same current passes through all elements. Resonance occurs when the opposite inductive and capacitive voltage phasors cancel, so $X_L = X_C$.

Step 1: (i) Distinguish peak and rms current and derive their relation Write a sinusoidal current as $i = I_0 \sin \omega t$, with period T . Its instantaneous values range from $-I_0$ to $+I_0$, so I_0 specifies the largest magnitude reached. The rms value specifies the effective current for heating: in a resistance R , the mean power is $I_{\text{rms}}^2 R$. It is positive even though the mean current over a cycle is zero.

$$\begin{aligned} I_{\text{rms}}^2 &= \frac{1}{T} \int_0^T i^2 dt = \frac{I_0^2}{T} \int_0^T \sin^2 \omega t dt \\ &= \frac{I_0^2}{2T} \int_0^T (1 - \cos 2\omega t) dt = \frac{I_0^2}{2}. \end{aligned}$$

The cosine term integrates to zero over a complete cycle. Taking the positive square root gives $I_{\text{rms}} = I_0/\sqrt{2}$, or approximately $0.707I_0$. This numerical relation applies to a sinusoidal alternating current.

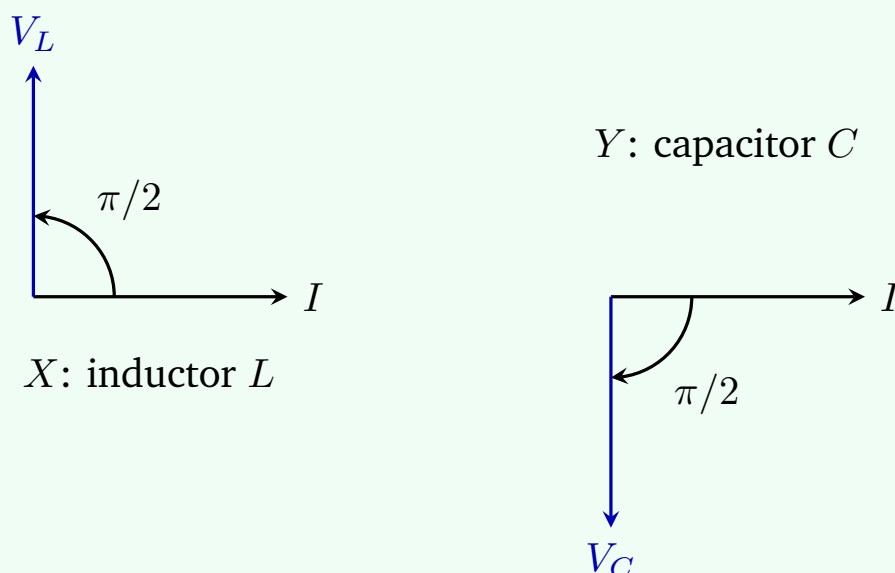
Step 2: (ii)(I) Identify the elements from their phase relations For an ideal inductor, $V_L = L di/dt$; differentiation advances the phase of a sinusoidal current by $\pi/2$. Thus voltage leads current by $\pi/2$, identifying X as an inductor. With the applied voltage $V = V_0 \sin \omega t$, its steady alternating current is

$$i_L = \frac{V_0}{\omega L} \sin \left(\omega t - \frac{\pi}{2} \right).$$

For an ideal capacitor, $i_C = C dV/dt$, so current leads voltage by $\pi/2$. Thus Y is a capacitor, with

$$i_C = \omega C V_0 \sin \left(\omega t + \frac{\pi}{2} \right).$$

In the phasor diagrams, take the current phasor along the horizontal reference direction and positive phase counterclockwise. The inductive voltage lies 90° ahead of current; the capacitive voltage lies 90° behind it.



Current-reference phasors: voltage leads in $X = L$ and lags in $Y = C$.

Step 3: (ii)(II) Obtain the resonance condition and frequency Connect L and C in series. Since the current is common, the voltage amplitudes are $V_{L0} = I_0\omega L$ and $V_{C0} = I_0/(\omega C)$. Their phasors point in opposite directions. They cancel when their magnitudes are equal:

$$\begin{aligned} X_L &= X_C, \\ \omega_0 L &= \frac{1}{\omega_0 C}, \\ \omega_0^2 &= \frac{1}{LC}, \quad \nu_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}. \end{aligned}$$

This is the series resonance condition: the net reactance vanishes at ν_0 .

Step 4: State the impedance at resonance For the ideal inductor and capacitor specified here, with no resistance included, the impedance magnitude is

$$Z = \left| \omega L - \frac{1}{\omega C} \right|.$$

Therefore its resonant value is zero. If a real circuit has total series resistance R , then $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and its resonant value is R . Thus the practical current is limited by resistance; the zero-impedance result refers to the ideal LC limit.

Step 5: Conclusion $I_{\text{rms}} = I_0/\sqrt{2}$ for sinusoidal current. Element X is an inductor and Y is a capacitor. Resonance requires $X_L = X_C$, giving $\nu_0 = 1/(2\pi\sqrt{LC})$. At resonance, the ideal series LC impedance is $Z = 0$; with total series resistance R , it is $Z = R$.

Quick Tip to Solve: Draw current to the right: inductive voltage points up and capacitive voltage points down. At resonance their equal, opposite phasors cancel.

33(a). (i) An object is placed 30 cm from a thin convex lens of focal length 10 cm. The lens forms a sharp image on a screen. If a thin concave lens is placed in contact with the convex lens, the sharp image on the screen is formed when the screen is moved by 45 cm from its initial position. Calculate the focal length of the concave lens.

(ii) Calculate the angle of minimum deviation of an equilateral prism. The refractive index of the prism is $\sqrt{3}$. Calculate the angle of incidence for this case of minimum deviation also. [5 marks]

Solution:

Concept:

- For a thin lens, $1/f = 1/v - 1/u$, where u , v and f are signed object distance, image distance and focal length. With light travelling left to right, distances to the left of the optical centre are negative.
- Thin lenses in contact have equivalent focal length F given by $1/F = 1/f_1 + 1/f_2$. Their powers add because their separation is negligible.
- At minimum deviation through a prism in air, the path is symmetric: $i = e$ and $r_1 = r_2 = A/2$. Here A is the prism angle, i , e are the incidence and emergence angles, and r_1 , r_2 are the internal refraction angles.
- The refractive index is $\mu = \sin i / \sin(A/2)$ at minimum deviation, and $\delta_{\min} = 2i - A$.

Step 1: (i) Locate the image formed by the convex lens alone Use the Cartesian sign convention. The real object lies on the incident side of the convex lens, so $u = -30$ cm, while its focal length is $f_1 = +10$ cm. Apply the thin-lens formula:

$$\begin{aligned}\frac{1}{v_1} &= \frac{1}{f_1} + \frac{1}{u} \\ &= \frac{1}{10} - \frac{1}{30} = \frac{1}{15} \text{ cm}^{-1}, \\ v_1 &= +15 \text{ cm}.\end{aligned}$$

The positive sign agrees with the stated sharp image on a screen: the image is real and is formed on the other side of the lens.

Step 2: Determine the new image distance and equivalent focal length The object and the convex lens remain fixed when the concave lens is placed in contact. The concave lens reduces the converging power, so the real image moves farther from the combination. The given screen displacement is 45 cm; hence

$$v_2 = 15 + 45 = +60 \text{ cm}.$$

Subtracting the displacement would give $v_2 = -30$ cm, a virtual-image position, inconsistent with an image caught on the screen. Treat the contact combination as a single lens:

$$\begin{aligned}\frac{1}{F} &= \frac{1}{v_2} - \frac{1}{u} \\ &= \frac{1}{60} + \frac{1}{30} = \frac{1}{20} \text{ cm}^{-1}, \\ F &= +20 \text{ cm}.\end{aligned}$$

Step 3: Calculate the focal length of the concave lens Let the concave lens have focal length f_2 . Use

the formula for thin lenses in contact and solve for its reciprocal focal length:

$$\begin{aligned}\frac{1}{f_2} &= \frac{1}{F} - \frac{1}{f_1} \\ &= \frac{1}{20} - \frac{1}{10} = -\frac{1}{20} \text{ cm}^{-1}, \\ f_2 &= -20 \text{ cm}.\end{aligned}$$

The negative sign confirms that this lens is diverging. The combined focal length is positive and longer than that of the original convex lens, as expected when its converging power is reduced.

Step 4: (ii) Use the symmetric ray path at minimum deviation An equilateral prism has angle $A = 60^\circ$. At minimum deviation, the internal refraction angles are equal and their sum is A , so $r_1 = r_2 = 30^\circ$. Apply Snell's law at the first face, taking the refractive index of air as unity:

$$\begin{aligned}\sin i &= \mu \sin r_1 = \sqrt{3} \sin 30^\circ = \frac{\sqrt{3}}{2}, \\ i &= 60^\circ.\end{aligned}$$

Choose the physical incidence angle between 0° and 90° . The emergence angle is also $e = 60^\circ$. The total deviation is the sum of the incidence and emergence angles minus the prism angle:

$$\delta_{\min} = i + e - A = 60^\circ + 60^\circ - 60^\circ = 60^\circ.$$

Step 5: Conclusion (i) The concave lens has focal length $f_2 = -20 \text{ cm}$. (ii) The prism's angle of minimum deviation is $\delta_{\min} = 60^\circ$, and the corresponding angle of incidence is $i = 60^\circ$.

Quick Tip to Solve: First find the original screen distance before adding its displacement. For an equilateral prism at minimum deviation, immediately use $r_1 = r_2 = 30^\circ$.

OR

33(b). (i) A physics teacher wants to demonstrate interference with the help of double slit experiment using a laser beam of 633 nm wavelength. Since the hall is large enough, interference pattern is formed on the wall 5.0 m from the slits. For clear and comfortable view by all the students they want the fringe width 5 mm.

(I) Find the slit separation for obtaining the desired interference pattern.

(II) How far will the first minimum be from the central maximum ?

(ii) A parallel beam of light of wavelength 650 nm passes through a slit of width 0.6 mm. The diffraction pattern is obtained on a screen kept 60 cm away from the slit. Find the distance between first order minima on both sides of the central maximum.

[5 marks]

Solution:

Concept:

- In Young's double-slit experiment, the path difference at a point a distance y from the central maximum is approximately $\Delta = dy/D$. Here d is the slit separation and D is the distance of the screen from the slits; the approximation applies for small angles.
- Successive bright fringes have path differences differing by one wavelength λ , so their separation

(the fringe width) is $\beta = \lambda D/d$. A dark fringe occurs when $\Delta = (m + \frac{1}{2})\lambda$, where $m = 0, 1, 2, \dots$

- For a single slit of width a , diffraction minima satisfy $a \sin \theta = n\lambda$, with $n = \pm 1, \pm 2, \dots$. At small angles the first minima are at $y = \pm \lambda D/a$, so the width of the central maximum is $2\lambda D/a$.

Step 1: (i)(I) Determine the slit separation Write the given quantities in SI units before using the fringe-width relation:

$$\lambda = 633 \times 10^{-9} \text{ m}, \quad D = 5.0 \text{ m}, \quad \beta = 5 \times 10^{-3} \text{ m}.$$

At the central maximum the path difference is zero; at the adjacent bright fringe it is λ . Thus $d\beta/D = \lambda$. Rearranging and substituting gives

$$\begin{aligned} d &= \frac{\lambda D}{\beta} \\ &= \frac{(633 \times 10^{-9})(5.0)}{5 \times 10^{-3}} \text{ m} \\ &= 6.33 \times 10^{-4} \text{ m} = 0.633 \text{ mm}. \end{aligned}$$

The slit separation is much smaller than the screen distance, consistent with the usual Young's experiment geometry.

Step 2: (i)(II) Locate the first interference minimum The first dark fringe on either side of the central maximum has path-difference magnitude $\lambda/2$, rather than λ . Hence

$$\begin{aligned} \frac{d|y_1|}{D} &= \frac{\lambda}{2}, \\ |y_1| &= \frac{\lambda D}{2d} = \frac{\beta}{2} = \frac{5}{2} \text{ mm} = 2.5 \text{ mm}. \end{aligned}$$

Thus the two first dark fringes are symmetrically placed at $y = +2.5 \text{ mm}$ and $y = -2.5 \text{ mm}$, each measured from the central maximum.

Step 3: (ii) Find the first single-slit diffraction minima This part uses the width of one slit, not the separation of two slits. The new data are

$$\lambda = 650 \times 10^{-9} \text{ m}, \quad a = 0.6 \times 10^{-3} \text{ m}, \quad D = 60 \times 10^{-2} \text{ m} = 0.60 \text{ m}.$$

For the first minimum, $a \sin \theta_1 = \lambda$: pairing points in the two halves of the slit gives a path difference $\lambda/2$ for each pair, so their contributions cancel. Since $\lambda/a \simeq 1.08 \times 10^{-3}$, the angle is small and $\tan \theta_1 \simeq \sin \theta_1$. Therefore the distance of each first minimum from the centre is

$$\begin{aligned} |y_1| &= D \tan \theta_1 \simeq \frac{D\lambda}{a} \\ &= \frac{(0.60)(650 \times 10^{-9})}{0.6 \times 10^{-3}} \text{ m} \\ &= 6.50 \times 10^{-4} \text{ m} = 0.65 \text{ mm}. \end{aligned}$$

Step 4: (ii) Calculate the separation of the two minima The minima lie on opposite sides of the central maximum, at $+y_1$ and $-y_1$. The requested separation is consequently twice the centre-to-minimum distance:

$$W = (+y_1) - (-y_1) = 2|y_1| = 2(0.65) \text{ mm} = 1.30 \text{ mm}.$$

This separation is the width of the central diffraction maximum.

Step 5: Conclusion (i)(I) The required slit separation is 0.633 mm . (i)(II) The first interference minimum is 2.5 mm from the central maximum on either side. (ii) The distance between the first diffraction minima is 1.30 mm .

Quick Tip to Solve: For double-slit interference, the first dark fringe is half a fringe width from the centre. For single-slit diffraction, first find $\lambda D/a$, then double it when the question asks for the distance between the minima on opposite sides.