

CBSE Class 12 Physics Question Paper 2025 with Solution PDF-Set 55/4/2



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – **Sections A, B, C, D** and **E**.
- (iii) In **Section A** – Questions no. **1** to **16** are Multiple Choice type questions. Each question carries **1** mark.
- (iv) In **Section B** – Questions no. **17** to **21** are Very Short Answer type questions. Each question carries **2** marks.
- (v) In **Section C** – Questions no. **22** to **28** are Short Answer type questions. Each question carries **3** marks.
- (vi) In **Section D** – Questions no. **29** and **30** are case study-based questions. Each question carries **4** marks.
- (vii) In **Section E** – Questions no. **31** to **33** are Long Answer type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is **not** allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. An electric dipole of dipole moment 1.0×10^{-12} Cm lies along x -axis. An electric field of magnitude 2.0×10^4 N C $^{-1}$ is switched on at an instant in the region. The unit vector along the electric field is $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$. The magnitude of the torque acting on the dipole at that instant is : [1 mark]

- (A) 0.5×10^{-6} Nm
(B) 1.0×10^{-8} Nm
(C) 2.0×10^{-8} Nm
(D) 4.0×10^{-8} Nm

Correct Answer: (B) 1.0×10^{-8} Nm

Solution:

Concept:

- The torque on an electric dipole in a uniform electric field is $\vec{\tau} = \vec{p} \times \vec{E}$, where $\vec{p}$ is the dipole moment.
- Its magnitude is $\tau = pE \sin \theta$, where θ is the angle between the dipole axis and the field. Only the field component perpendicular to the dipole contributes.

Step 1: Identify the perpendicular field component The dipole lies along the x -axis. From the printed field direction, the perpendicular component is along y :

$$E_y = E \left(\frac{1}{2} \right) = 1.0 \times 10^4 \text{ N C}^{-1}.$$

The x -component, $E\sqrt{3}/2$, is parallel to the dipole and produces no torque. Equivalently, $\sin \theta = 1/2$.

Step 2: Substitute in the torque formula Using the perpendicular component gives

$$\tau = pE_y = (1.0 \times 10^{-12})(1.0 \times 10^4) = 1.0 \times 10^{-8} \text{ N m}.$$

Option (C) equals pE and incorrectly assumes a perpendicular field. Options (A) and (D) exceed the maximum possible torque $pE = 2.0 \times 10^{-8}$ N m, so neither is possible.

Step 3: Conclusion The torque magnitude is 1.0×10^{-8} N m. Hence, the correct option is (B).

Quick Tip to Solve: For a dipole along the x -axis, use the perpendicular field component directly; here the $\hat{j}$ coefficient gives $\tau = pE/2$.

2. When the switch of the circuit is turned on, the filament of the bulb glows instantaneously because :

[1 mark]

- (A) the electrons coming from the power source move fast through the initially empty filament.
- (B) the filament may be old having low resistance.
- (C) electric field is established instantaneously across the filament which pushes the electrons.
- (D) free electrons in the filament travel with the speed of light.

Correct Answer: (C) electric field is established instantaneously across the filament which pushes the electrons.

Solution:

Concept:

- A metal filament already contains mobile conduction electrons before the switch is closed. An electric field gives them a small net drift velocity.
- The electromagnetic signal establishing the field travels through the circuit very rapidly, whereas the electron drift speed is much smaller. Heating begins as current flows through the filament.

Step 1: Explain the prompt response Closing the switch rapidly establishes an electric field throughout the circuit, including the filament. Electrons already present in the filament respond locally to this field, producing current and resistive heating. In the wording of the question, this rapid response is described as instantaneous; physically, signal propagation and heating take finite times.

Step 2: Eliminate the other explanations Option (A) is incorrect because the filament is never initially empty of electrons and does not wait for electrons to arrive from the source. Option (B) gives no explanation for the rapid establishment of current; the age of the filament is irrelevant. Option (D) confuses the high propagation speed of the electromagnetic signal with the much smaller drift speed of conduction electrons.

Step 3: Conclusion The field is established very rapidly across the filament and drives its existing free electrons. Hence, the correct option is (C).

Quick Tip to Solve: Separate signal speed from electron drift speed: a bulb responds when the field reaches its existing electrons, without waiting for electrons from the battery.

3. A particle with charge q moving with velocity $\vec{v} = v_0\hat{i}$ enters a region with magnetic field $\vec{B} = B_1\hat{j} + B_2\hat{k}$. The magnitude of force experienced by the particle is :

[1 mark]

- (A) $qv_0(B_1 + B_2)$
- (B) $q\sqrt{v_0(B_1 + B_2)}$
- (C) $qv_0\sqrt{B_1^2 + B_2^2}$
- (D) $q\sqrt{v_0(B_1^2 + B_2^2)}$

Correct Answer: (C) $qv_0\sqrt{B_1^2 + B_2^2}$

Solution:

Concept:

- The magnetic force on a moving charge is $\vec{F} = q(\vec{v} \times \vec{B})$. Its magnitude is $|q|vB \sin \theta$, with θ the angle between velocity and field.
- Perpendicular vector components combine in quadrature. Here both magnetic-field components

are perpendicular to $\vec{v} = v_0 \hat{i}$.

Step 1: Evaluate the vector product Using $\hat{i} \times \hat{j} = \hat{k}$ and $\hat{i} \times \hat{k} = -\hat{j}$,

$$\begin{aligned}\vec{F} &= qv_0 \hat{i} \times (B_1 \hat{j} + B_2 \hat{k}) \\ &= qv_0 (B_1 \hat{k} - B_2 \hat{j}).\end{aligned}$$

Thus the two force components lie along perpendicular axes; their magnitudes cannot simply be added.

Step 2: Calculate the magnitude and compare the options Squaring and adding the perpendicular components yields

$$F = \sqrt{(qv_0 B_1)^2 + (qv_0 B_2)^2} = |q|v_0 \sqrt{B_1^2 + B_2^2}.$$

In the printed options, q denotes the charge magnitude. Option (A) adds perpendicular components arithmetically. Options (B) and (D) incorrectly place v_0 inside a square root; magnetic force must be proportional to speed, so both fail.

Step 3: Conclusion The force magnitude is $qv_0 \sqrt{B_1^2 + B_2^2}$, taking q as the charge magnitude. Hence, the correct option is (C).

Quick Tip to Solve: When the velocity points along x , combine the y - and z -components of the magnetic field using Pythagoras, then multiply by $|q|v$.

4. A bar magnet is initially at right angles to a uniform magnetic field. The magnet is rotated till the torque acting on it becomes one-half of its initial value. The angle through which the bar magnet is rotated is :

[1 mark]

- (A) 30°
- (B) 45°
- (C) 60°
- (D) 75°

Correct Answer: (C) 60°

Solution:

Concept:

- A bar magnet of magnetic dipole moment m in a uniform magnetic field B experiences torque of magnitude $\tau = mB \sin \theta$, where θ is the angle between its magnetic axis and the field.
- Torque is maximum at $\theta = 90^\circ$. The angle through which the magnet rotates is the change in orientation, not its final angle to the field.

Step 1: Find the final orientation Initially the magnet is perpendicular to the field, so $\tau_i = mB$. The final torque has half this magnitude:

$$mB \sin \theta_f = \frac{mB}{2} \Rightarrow \sin \theta_f = \frac{1}{2}.$$

For rotation towards alignment, the final angle with the field is $\theta_f = 30^\circ$.

Step 2: Calculate the rotation from the initial position The required rotation is the difference between

the initial and final angles:

$$\Delta\theta = 90^\circ - 30^\circ = 60^\circ.$$

Rotation in the opposite sense to 150° also first halves the torque magnitude after 60° . Option (A) confuses the final orientation with the rotation; rotations of 45° and 75° give torque ratios $\cos 45^\circ$ and $\cos 75^\circ$, neither equal to one-half.

Step 3: Conclusion The magnet is rotated through 60° . Hence, the correct option is (C).

Quick Tip to Solve: If the magnet starts perpendicular to the field and rotates by α , use $\tau/\tau_{\max} = \cos \alpha$ directly.

5. The materials having negative magnetic susceptibility are :

[1 mark]

- (A) diamagnetic
- (B) paramagnetic
- (C) ferromagnetic
- (D) non-magnetic

Correct Answer: (A) diamagnetic

Solution:

Concept:

- Magnetic susceptibility χ_m relates magnetisation $\vec{M}$ to the applied magnetising field $\vec{H}$: $\vec{M} = \chi_m \vec{H}$.
- A negative susceptibility means that the induced magnetisation points opposite to the applied field. This is the defining magnetic response of a diamagnetic material.

Step 1: Interpret the negative sign In a diamagnetic substance, the applied magnetic field induces atomic magnetic moments that oppose it. The resulting magnetisation is antiparallel to $\vec{H}$, so the proportionality factor χ_m is negative. Such substances are weakly repelled by a non-uniform magnetic field.

Step 2: Compare the remaining materials Paramagnetic substances have a small positive susceptibility because their magnetisation is along the applied field. Ferromagnetic substances have a very large positive magnetic response, rather than the small negative response asked for here. An ideal non-magnetic material has zero susceptibility, so option (D) does not describe negative susceptibility.

Step 3: Conclusion Diamagnetic materials have negative magnetic susceptibility. Hence, the correct option is (A).

Quick Tip to Solve: Read the sign of χ_m as a direction test: negative means magnetisation opposes the applied field, identifying diamagnetism.

6. When current in a coil changes at a steady rate from 8 A to 6 A in 4 ms, an emf of 1.5 V is induced in it. The value of self-inductance of the coil is : [1 mark]

- (A) 6 mH
- (B) 12 mH
- (C) 3 mH
- (D) 9 mH

Correct Answer: (C) 3 mH

Solution:

Concept:

- Self-induction produces an emf $\varepsilon = -L dI/dt$, where L is the self-inductance and I is the current. The minus sign represents opposition to the change in current.
- For a steady rate of current change, $|\varepsilon| = L|\Delta I|/\Delta t$. Use amperes and seconds to obtain L in henries.

Step 1: Calculate the rate of change of current The current falls from 8 A to 6 A, so its change is -2 A. Convert 4 ms to 4×10^{-3} s:

$$\left| \frac{\Delta I}{\Delta t} \right| = \frac{2}{4 \times 10^{-3}} = 500 \text{ A s}^{-1}.$$

The magnitude of the change, rather than the initial or final current alone, determines the induced emf.

Step 2: Use the induced emf to find the inductance Rearranging the magnitude form of the induction law gives

$$L = \frac{|\varepsilon|}{|\Delta I/\Delta t|} = \frac{1.5}{500} = 3.0 \times 10^{-3} \text{ H}.$$

As $1 \text{ mH} = 10^{-3} \text{ H}$, this is 3 mH. The alternatives 6, 12, and 9 mH would produce emfs of 3, 6, and 4.5 V, respectively, at the given rate and therefore do not match the data.

Step 3: Conclusion The self-inductance is 3 mH. Hence, the correct option is (C).

Quick Tip to Solve: Use $L = |\varepsilon|\Delta t/|\Delta I|$; substituting time in milliseconds gives the numerical inductance directly in millihenries.

7. The electric field in space between the plates of a parallel plate capacitor (each of area $2.5 \times 10^{-3} \text{ m}^2$) is changing at the rate of $4 \times 10^6 \text{ V m}^{-1} \text{ s}^{-1}$. The displacement current between the plates of the capacitor is : [1 mark]

- (A) $1.8 \times 10^{-5} \text{ A}$
- (B) $3.47 \times 10^{-6} \text{ A}$
- (C) $8.85 \times 10^{-8} \text{ A}$
- (D) $6.32 \times 10^{-4} \text{ A}$

Correct Answer: (C) $8.85 \times 10^{-8} \text{ A}$

Solution:

Concept:

- The displacement current is $I_d = \epsilon_0 d\Phi_E/dt$, where Φ_E is the electric flux and $\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$.
- For a uniform field perpendicular to parallel capacitor plates of fixed area A , $\Phi_E = EA$, giving $I_d = \epsilon_0 A dE/dt$.

Step 1: Find the rate of change of electric flux Each plate has area $A = 2.5 \times 10^{-3} \text{ m}^2$. Use the area of one plate because the flux is evaluated through a single surface spanning the gap, not through the combined area of both plates:

$$\frac{d\Phi_E}{dt} = A \frac{dE}{dt} = (2.5 \times 10^{-3})(4 \times 10^6) = 1.0 \times 10^4 \text{ V m s}^{-1}.$$

Step 2: Multiply by the permittivity The displacement-current magnitude is

$$I_d = (8.85 \times 10^{-12})(1.0 \times 10^4) = 8.85 \times 10^{-8} \text{ A}.$$

No conduction of charge across the ideal capacitor gap is required: the changing electric flux contributes the displacement current. Options (A), (B), and (D) do not match the product of the stated area, field-change rate, and permittivity; the exponent follows from $-12 - 3 + 6 = -9$, with the coefficient 88.5.

Step 3: Conclusion The displacement current is $8.85 \times 10^{-8} \text{ A}$. Hence, the correct option is (C).

Quick Tip to Solve: Multiply the area by the field-change rate first: here it equals 10^4 , so multiplying by ϵ_0 gives the answer immediately.

8. A long straight wire is held vertically and carries a steady current in upward direction. The shape of magnetic field lines produced by the current-carrying wire are : [1 mark]

- (A) horizontal straight lines directed radially out from the wire.
- (B) straight lines parallel to the current-carrying wire.
- (C) concentric horizontal circles around the wire.
- (D) coaxial helixes around the wire.

Correct Answer: (C) concentric horizontal circles around the wire.

Solution:

Concept:

- The magnetic field of a long straight current-carrying wire is tangential to circles centred on the wire in planes perpendicular to it.
- The right-hand thumb rule gives the direction: the thumb points along the current and the curled fingers point along the magnetic field.

Step 1: Identify the planes of the field lines The wire is vertical, so planes perpendicular to it are horizontal. Rotational symmetry about the wire gives circular field lines whose centres lie on the wire. Their radius is the perpendicular distance from the wire.

Step 2: Apply the direction rule and check the alternatives Point the right thumb upward. The curled fingers indicate anticlockwise circulation as seen from above. Radial straight lines in (A) and parallel straight lines in (B) do not follow this circulation. A helix in (D) would require a magnetic-field component along the wire; the field of the given straight wire has no such component.

Step 3: Conclusion The magnetic field lines are concentric horizontal circles around the wire. Hence, the correct option is (C).

Quick Tip to Solve: First choose the plane perpendicular to the wire; then curl your right-hand fingers around it to determine the field direction.

9. Which of the following is an electrical conductor at room temperature? [1 mark]

- (A) Sn
- (B) Mica
- (C) Si
- (D) C

Correct Answer: (A) Sn

Solution:

Concept:

- A metallic conductor has mobile charge carriers at room temperature; its conduction band is partially filled or overlaps an occupied band.
- An insulator has a large forbidden energy gap, whereas an intrinsic semiconductor has a smaller gap and a limited number of thermally generated carriers.

Step 1: Identify the metallic material The symbol Sn denotes tin, a metal. At room temperature tin contains freely mobile electrons that drift under an applied electric field, so it is classified as an electrical conductor.

Step 2: Compare the other choices Mica is an electrical insulator and Si (silicon) is an intrinsic semiconductor. Carbon requires care because its behaviour depends on its allotrope: diamond is insulating, while graphite conducts. In the usual band-classification context, carbon in the diamond structure is the insulating comparison; Sn is the intended, unequivocal metallic conductor among the listed materials.

Step 3: Conclusion Tin (Sn) is the intended electrical conductor at room temperature. Hence, the correct option is (A).

Quick Tip to Solve: Classify Sn as a metal, Si as a semiconductor and mica as an insulator; do not assume all allotropes of carbon have identical conductivity.

10. The magnification produced by a spherical mirror is -2.0 . The mirror used and the nature of the image formed will be : [1 mark]

- (A) Convex and virtual
- (B) Concave and real
- (C) Concave and virtual
- (D) Convex and real

Correct Answer: (B) Concave and real

Solution:

Concept:

- The transverse magnification of a spherical mirror is $m = h_i/h_o = -v/u$, where h_i, h_o are image and object heights, and v, u are their signed distances.
- For a real object, negative magnification indicates an inverted real image; a convex mirror always forms an erect, diminished virtual image.

Step 1: Interpret the sign and magnitude Here $m = -2.0$. The negative sign means that the image is inverted relative to the object, while $|m| = 2.0$ means that its height is twice the object's height. Thus the image is real and enlarged.

Step 2: Identify the mirror and eliminate the alternatives A concave mirror forms a real enlarged image when the object is between its focus and centre of curvature. For a convex mirror and a real object, $0 < m < 1$, excluding both convex-mirror alternatives (A) and (D). A virtual image formed by a concave mirror is erect and has positive magnification, so (C) is excluded as well.

Step 3: Conclusion The mirror is concave and the image is real, inverted and enlarged. Hence, the correct option is (B).

Quick Tip to Solve: Read the minus sign first: it identifies inversion and a real image. Then use $|m| > 1$ to identify enlargement.

11. Choose the correct statement :

[1 mark]

- (A) Photons of light show diffraction whereas electrons do not show diffraction.
- (B) Electrons have momentum whereas photons do not have momentum.
- (C) Photons of light and electrons both exhibit dual nature.
- (D) All electromagnetic radiations do not have photons.

Correct Answer: (C) Photons of light and electrons both exhibit dual nature.

Solution:

Concept:

- Light exhibits wave behaviour in interference and diffraction, and particle behaviour through photons of energy $E = h\nu$ and momentum $p = h/\lambda$.
- A moving electron has a de Broglie wavelength $\lambda = h/p$, where h is Planck's constant and p is the electron's momentum, so it also exhibits wave behaviour.

Step 1: Compare the behaviour of photons and electrons Photon detection and the photoelectric effect demonstrate the particle aspect of light, while diffraction demonstrates its wave aspect. Electrons have mass and momentum, yet an electron beam also produces diffraction patterns. Both therefore possess particle and wave characteristics.

Step 2: Test the remaining statements Statement (A) is false because electrons do show diffraction. Statement (B) is false because photons carry momentum even though their rest mass is zero. Statement (D) is false because the photon description applies to all electromagnetic radiation, including radio waves, visible light and X-rays; photon energy varies with frequency.

Step 3: Conclusion Photons of light and electrons both exhibit dual nature. Hence, the correct option is (C).

Quick Tip to Solve: Zero rest mass does not mean zero momentum: use $p = h/\lambda$ for a photon and $\lambda = h/p$ for a moving electron.

12. A beam of red light and a beam of blue light have equal intensities. Which of the following statements is true ?

[1 mark]

- (A) The blue beam has more number of photons than the red beam.
- (B) The red beam has more number of photons than the blue beam.
- (C) Wavelength of red light is lesser than wavelength of blue light.
- (D) The blue light beam has lesser energy per photon than that in the red light beam.

Correct Answer: (B) The red beam has more number of photons than the blue beam.

Solution:

Concept:

- The energy of a photon is $E = h\nu = hc/\lambda$, where ν is frequency, λ is wavelength and c is the speed of light in vacuum.
- Intensity I is energy incident per unit area per unit time. If F is the photon flux (photons per unit area per unit time), then $I = FE$.

Step 1: Compare the photon energies Red light has a longer wavelength than blue light, so $\lambda_r > \lambda_b$. Therefore $E_r = hc/\lambda_r < E_b = hc/\lambda_b$: each red photon carries less energy than a blue photon. This

directly rules out (C) and (D).

Step 2: Use the equality of intensities Equal intensities mean equal energy crosses equal areas in equal times. More lower-energy red photons are required to supply that same energy:

$$\frac{F_r}{F_b} = \frac{I/E_r}{I/E_b} = \frac{E_b}{E_r} = \frac{\lambda_r}{\lambda_b} > 1.$$

Thus the red beam has the larger photon count for the same area and time interval; (A) reverses this comparison.

Step 3: Conclusion The red beam has more photons than the blue beam for equal area and time at equal intensity. Hence, the correct option is (B).

Quick Tip to Solve: At fixed intensity, photon flux is proportional to wavelength: the longer-wavelength beam needs more photons to carry the same energy.

13. Assertion (A): For monochromatic incident radiation, the emitted photoelectrons from a given metal have speed ranging from zero to a certain maximum value.

Reason (R): Each metal has a definite work function.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- Einstein's photoelectric equation gives the maximum kinetic energy: $K_{\max} = h\nu - \phi = \frac{1}{2}m_e v_{\max}^2$, where ϕ is the work function and ν is incident frequency.
- The work function is the minimum energy required to remove an electron from a metal surface. Emitted electrons can have less than the maximum kinetic energy because their initial energies and energy losses differ.

Step 1: Judge the assertion Monochromatic radiation supplies photons of one energy, $h\nu$, but the electrons in the metal do not all emerge under identical conditions. Some lose energy through interactions before reaching the surface. Their emitted kinetic energies extend from nearly zero to $K_{\max}$, giving speeds from zero up to $v_{\max}$. The assertion is true.

Step 2: Judge the reason For a given metal surface under fixed conditions, the work function has a definite characteristic value. It sets the minimum energy for emission and the maximum kinetic energy for a specified incident frequency. The reason is true.

Step 3: Test whether the reason explains the assertion A definite work function fixes the upper limit of the electron energy, but does not explain why emitted electrons have a range of lower energies. That range arises from their differing initial states and energy losses before escape. Thus the reason

does not explain the assertion; (A) incorrectly treats the energy threshold as the cause of the spread, while (C) and (D) misjudge the truth values.

Step 4: Conclusion Both the assertion and the reason are true, but the reason does not correctly explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Use the work function to find the maximum photoelectron energy; use differing electron energies and losses to explain the spread below it.

14. Assertion (A): In double slit experiment if one slit is closed, diffraction pattern due to the other slit will appear on the screen.

Reason (R): For interference, at least two waves are required.

[1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- Diffraction arises from the superposition of secondary wavelets from different parts of a finite aperture. A single slit can therefore produce a diffraction pattern.
- Interference requires the superposition of at least two waves; a stable interference pattern requires a constant phase difference between the contributing waves.

Step 1: Judge the assertion Closing one slit removes the two-slit interference fringes. Light still passes through the other slit, whose finite width allows secondary wavelets from its different parts to superpose. The screen therefore shows that slit's diffraction pattern. The assertion is true.

Step 2: Judge the reason Interference is produced by the superposition of two or more waves. Hence at least two waves are necessary, although their mere presence is not sufficient for stable fringes unless their phase relationship is maintained. The reason is true.

Step 3: Check the explanatory link The reason explains why interference between the two slits ceases when one slit is blocked. It does not explain why the remaining finite slit produces diffraction; that requires wavelets from different parts of the same aperture. Two separate slits are not required for those wavelets. Thus (A) gives an incorrect explanatory link, while (C) and (D) incorrectly classify true statements as false.

Step 4: Conclusion Both the assertion and the reason are true, but the reason does not correctly explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish two slits from two waves: one finite slit still supplies many secondary wavelets and can produce diffraction.

15. Assertion (A): A series LCR circuit behaves as a pure resistive circuit at resonance.

Reason (R): At resonance, $X_L = X_C$ gives $\omega = \frac{1}{\sqrt{LC}}$.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- In a series LCR circuit, the impedance magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where R is resistance, $X_L = \omega L$ is inductive reactance and $X_C = 1/(\omega C)$ is capacitive reactance.
- The phase angle between the supply voltage and current obeys $\tan \phi = (X_L - X_C)/R$. Equal and opposite reactances therefore give purely resistive behaviour.

Step 1: Judge the assertion At series resonance the net reactance vanishes, so $Z = R$ and $\phi = 0$. The supply voltage and current are in phase, just as in a circuit containing only resistance. Thus Assertion (A) is true; the individual inductor and capacitor need not be absent.

Step 2: Judge the reason Setting the two reactances equal gives

$$\omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}.$$

This is the resonance angular frequency, so Reason (R) is true.

Step 3: Check whether the reason explains the assertion The condition $X_L = X_C$ stated in the reason directly cancels the reactive part of the circuit impedance. It therefore explains the purely resistive behaviour in the assertion. Option (B) incorrectly denies this explanatory link, while (C) and (D) incorrectly classify a true statement as false.

Step 4: Conclusion Both Assertion (A) and Reason (R) are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: For series resonance, insert $X_L - X_C = 0$ into the impedance and phase equations before judging the assertion-reason link.

16. Assertion (A): n-type semiconductor is not negatively charged.

Reason (R): Neutral pentavalent impurity atom doped in intrinsic semiconductor (neutral) donates its fifth unpaired electron to the crystal lattice and becomes a positive donor. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- In an n-type semiconductor, pentavalent donor atoms supply electrons as the majority mobile carriers. The letter n identifies the carrier type, not the net charge of the specimen.
- Ionisation of each initially neutral donor produces one mobile electron of charge $-e$ and one fixed positive donor ion of charge $+e$, where e is the elementary charge.

Step 1: Judge the assertion An initially neutral semiconductor doped with neutral donor atoms remains electrically neutral overall. Although electrons greatly outnumber holes among its mobile carriers, fixed positive donor ions balance the extra electronic charge. Hence Assertion (A), that an n-type semiconductor is not negatively charged, is true.

Step 2: Judge the reason Four valence electrons of a pentavalent impurity participate in covalent bonding with neighbouring semiconductor atoms. Its weakly bound fifth electron can become a conduction electron, leaving the donor as a positive ion. This is the donation process described in Reason (R), so the reason is true.

Step 3: Check the charge balance and the explanatory link The charge change associated with one donor is $(-e) + (+e) = 0$. Thus the mobile negative charge is accompanied by an equal fixed positive charge within the same crystal. This directly explains the assertion. Option (B) overlooks that charge balance, while (C) and (D) wrongly treat true statements as false.

Step 4: Conclusion Both Assertion (A) and Reason (R) are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: When deciding a doped crystal's net charge, count the fixed impurity ions as well as the mobile electrons and holes.

Section B (10 marks)

17. In an intrinsic semiconductor, carrier's concentration is $5 \times 10^8 \text{ m}^{-3}$. On doping with impurity atoms, the hole concentration becomes $8 \times 10^{12} \text{ m}^{-3}$.

(a) Identify (i) the type of dopant and (ii) the extrinsic semiconductor so formed.

(b) Calculate the electron concentration in the extrinsic semiconductor.

[2 marks]

Solution:

Concept:

- In an intrinsic semiconductor at a fixed temperature, the electron and hole concentrations are equal: $n = p = n_i$, where n_i is the intrinsic carrier concentration.
- At thermal equilibrium, the mass-action law is $np = n_i^2$. A trivalent acceptor impurity increases the hole concentration and forms a p-type semiconductor.

Step 1: (a) Identify the dopant and semiconductor The given intrinsic concentration is $n_i = 5 \times 10^8 \text{ m}^{-3}$, whereas the hole concentration after doping is $p = 8 \times 10^{12} \text{ m}^{-3}$. Since $p \gg n_i$, doping has introduced holes as the majority carriers. Therefore (i) the dopant is a trivalent, or acceptor, impurity, such as boron, and (ii) the resulting extrinsic semiconductor is p-type.

Step 2: (b) Apply the mass-action law At the same temperature, the product of the electron and hole concentrations remains n_i^2 . Hence the electron concentration is

$$\begin{aligned} n &= \frac{n_i^2}{p} = \frac{(5 \times 10^8)^2}{8 \times 10^{12}} \text{ m}^{-3} \\ &= \frac{25 \times 10^{16}}{8 \times 10^{12}} \text{ m}^{-3} = 3.125 \times 10^4 \text{ m}^{-3}. \end{aligned}$$

This is much smaller than the hole concentration, consistent with electrons being minority carriers in the p-type material. The concentration unit is m^{-3} , since n_i^2/p has units $\text{m}^{-6}/\text{m}^{-3}$.

Step 3: Conclusion (a) (i) Trivalent acceptor dopant; (ii) p-type semiconductor. (b) The electron concentration is $n = 3.125 \times 10^4 \text{ m}^{-3}$.

Quick Tip to Solve: Identify the majority carrier first, then square the intrinsic concentration before dividing by the stated majority-carrier concentration.

18. In a double slit experiment, the two slits are 1.5 mm apart. The slits are illuminated by a mixture of lights of wavelengths of 600 nm and 400 nm and the interference pattern is observed on a screen 1.5 m away from the slits. Find the minimum distance of the point from the central maximum at which bright fringes of the interference patterns of the two wavelengths coincide. [2 marks]

Solution:

Concept:

- In Young's double-slit experiment, a bright fringe satisfies the path-difference condition $\Delta = m\lambda$, where m is an integer and λ is the wavelength.
- For small angles, the position of the m -th bright fringe is $y_m = m\lambda D/d$, where D is the slit-to-screen distance and d is the slit separation. Coincident bright fringes have the same path difference.

Step 1: Find the smallest nonzero coincident orders Let m be the bright-fringe order for $\lambda_1 = 600 \text{ nm}$

and n that for $\lambda_2 = 400 \text{ nm}$. At a common bright fringe,

$$m\lambda_1 = n\lambda_2 \Rightarrow 600m = 400n \Rightarrow 3m = 2n.$$

The central maximum corresponds to $m = n = 0$. For the nearest coincidence away from it, the smallest positive integer orders are $m = 2$ and $n = 3$. Their common path difference is $2(600 \text{ nm}) = 3(400 \text{ nm}) = 1200 \text{ nm}$.

Step 2: Calculate the distance from the central maximum Convert the slit separation to metres: $d = 1.5 \times 10^{-3} \text{ m}$, while $D = 1.5 \text{ m}$. Using the second bright fringe of the longer wavelength,

$$\begin{aligned} y_{\min} &= \frac{2\lambda_1 D}{d} = \frac{2(600 \times 10^{-9})(1.5)}{1.5 \times 10^{-3}} \text{ m} \\ &= 1.2 \times 10^{-3} \text{ m} = 1.2 \text{ mm}. \end{aligned}$$

As a check, the fringe widths are 0.6 mm and 0.4 mm ; twice the first and three times the second both give 1.2 mm .

Step 3: Conclusion The nearest bright-fringe coincidence away from the central maximum is at 1.2 mm on either side. It joins the second-order 600 nm fringe and the third-order 400 nm fringe.

Quick Tip to Solve: Reduce the wavelength ratio first: $600 : 400 = 3 : 2$, so the smallest coincident fringe orders are the reversed ratio $2 : 3$.

19. Find the focal length of plano-convex lens of refractive index 1.5 and radius of curvature 10 cm when it is immersed in a liquid of refractive index 1.25. [2 marks]

Solution:

Concept:

- For a thin lens immersed in a medium, the lens-maker formula is $\frac{1}{f} = \left(\frac{n_\ell}{n_m} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$, where n_ℓ and n_m are the refractive indices of the lens and surrounding medium.
- A plane surface has infinite radius of curvature and contributes zero to $1/f$. Use the refractive index of the lens relative to its surrounding medium, together with signed radii.

Step 1: Calculate the relative refractive index and choose signed radii The refractive index of the lens relative to the liquid is

$$n_{\text{rel}} = \frac{n_\ell}{n_m} = \frac{1.5}{1.25} = 1.2.$$

Take light to travel from left to right and enter through the convex surface first. Its centre of curvature lies to the right, so $R_1 = +10 \text{ cm}$. The second surface is plane, giving $R_2 = \infty$ and $1/R_2 = 0$.

Step 2: Substitute in the lens-maker formula Applying the formula in the liquid gives

$$\begin{aligned} \frac{1}{f} &= (1.2 - 1) \left(\frac{1}{10} - 0 \right) \text{ cm}^{-1} \\ &= 0.02 \text{ cm}^{-1}, \\ f &= +50 \text{ cm} = +0.50 \text{ m}. \end{aligned}$$

The positive focal length is consistent with a converging lens because its refractive index is greater than the liquid's. Reversing the lens gives $R_1 = \infty$, $R_2 = -10 \text{ cm}$, and the same result, as expected.

Step 3: Conclusion The focal length of the plano-convex lens in the liquid is $f = +50 \text{ cm}$, or $+0.50 \text{ m}$.

Quick Tip to Solve: For a plano-convex lens in a liquid, use $f = R/(n_\ell/n_m - 1)$; replacing the relative index with the glass index gives the wrong focal length.

20. Find the ratio of minimum to maximum wavelength of radiations emitted when electron jumps from higher energy state into ground state of hydrogen atom. [2 marks]

Solution:

Concept:

- For a hydrogen transition from level n to the ground state $n = 1$, the Rydberg formula is $\lambda^{-1} = R_H(1 - 1/n^2)$, where R_H is the Rydberg constant and $n \geq 2$.
- A photon has energy $E = hc/\lambda$. Thus the largest energy drop gives the shortest wavelength, while the smallest allowed energy drop gives the longest wavelength.

Step 1: Identify the longest wavelength Among transitions ending in the ground state, the smallest energy difference is for $n = 2 \rightarrow 1$. This first line of the Lyman series therefore has the maximum wavelength:

$$\frac{1}{\lambda_{\max}} = R_H \left(1 - \frac{1}{4}\right) = \frac{3R_H}{4}, \quad \lambda_{\max} = \frac{4}{3R_H}.$$

Step 2: Identify the shortest wavelength As the initial level increases, the energy released increases. The limiting largest energy drop occurs for $n \rightarrow \infty$ to $n = 1$, giving the series-limit wavelength:

$$\frac{1}{\lambda_{\min}} = R_H(1 - 0) = R_H, \quad \lambda_{\min} = \frac{1}{R_H}.$$

Form the requested ratio in the order minimum to maximum; the common Rydberg constant cancels.

Step 3: Conclusion

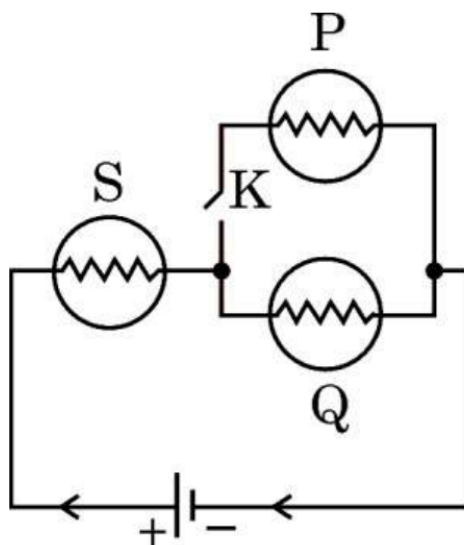
$$\frac{\lambda_{\min}}{\lambda_{\max}} = \frac{1/R_H}{4/(3R_H)} = \frac{3}{4}.$$

Hence, $\lambda_{\min} : \lambda_{\max} = 3 : 4$.

Quick Tip to Solve: For all transitions to the ground state, pair $2 \rightarrow 1$ with the longest wavelength and $\infty \rightarrow 1$ with the shortest; wavelength reverses the energy ordering.

21(a). In the given figure, three identical bulbs P , Q and S are connected to a battery.
 (i) Compare the brightness of bulbs P and Q with that of bulb S when key K is closed.
 (ii) Compare the brightness of the bulbs S and Q when the key K is opened.
 Justify your answer in both cases.

[2 marks]



Solution:

Concept:

- Brightness increases with power $\mathcal{P} = I^2 R$. Taking each identical bulb to have resistance R , compare their current squared.
- Series elements carry the same current. Identical parallel branches have the same potential difference and divide the incoming current equally.

Step 1: Compare the bulbs with the key closed With K closed, P and Q are parallel, in series with S . Current I through S splits equally:

$$I_P = I_Q = \frac{I}{2}, \quad I_S = I.$$

The corresponding powers are

$$\mathcal{P}_P = \mathcal{P}_Q = \frac{I^2 R}{4}, \quad \mathcal{P}_S = I^2 R.$$

Thus P and Q have equal brightness; S dissipates four times the power of either and is brighter than each.

Step 2: Compare the bulbs with the key open Opening K disconnects P , which does not glow. The remaining bulbs S and Q are in series, carrying the same current I' through equal resistances:

$$\mathcal{P}_S = I'^2 R = \mathcal{P}_Q.$$

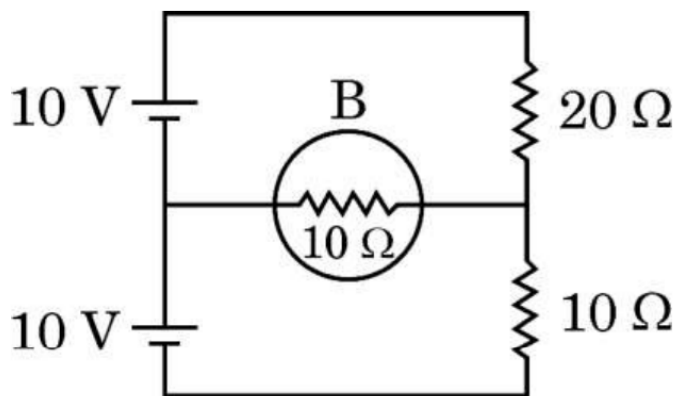
Equal powers give S and Q equal brightness.

Step 3: Conclusion With K closed, the brightness ordering is $S > P = Q$, and the powers satisfy $\mathcal{P}_S : \mathcal{P}_P : \mathcal{P}_Q = 4 : 1 : 1$. With K open, S and Q have equal brightness and P is off.

Quick Tip to Solve: Trace the closed path after changing the switch: S carries the sum of the branch currents when K is closed, but the same current as Q when K is open.

OR

21(b). Two cells of emf 10 V each, two resistors of $20\ \Omega$ and $10\ \Omega$ and a bulb B of $10\ \Omega$ resistance are connected together as shown in the figure. Find the current that flows through the bulb. [2 marks]



Solution:

Concept:

- Kirchhoff's junction law states that the algebraic sum of currents leaving a junction is zero. The current from a node at potential V_1 to a node at V_2 through resistance R is $(V_1 - V_2)/R$.
- The longer plate of a cell is its positive terminal. An ideal cell fixes its positive terminal at a potential higher than its negative terminal by its emf.

Step 1: Set the potentials fixed by the cells In the printed circuit, both cells have their positive terminals above their negative terminals. Choose the bottom connecting wire as 0 V . The lower cell then fixes the left end of the bulb at 10 V , and the upper cell fixes the top wire at 20 V . Let the potential at the right end of the bulb, where the two external resistors meet, be V .

Step 2: Apply the junction law at the right end of the bulb This junction connects to 20 V through $20\ \Omega$, to 0 V through $10\ \Omega$, and to 10 V through the $10\ \Omega$ bulb. Using SI units, the sum of the currents leaving the junction gives

$$\frac{V - 20}{20} + \frac{V}{10} + \frac{V - 10}{10} = 0.$$

Multiplying by 20 and collecting terms,

$$V - 20 + 2V + 2(V - 10) = 0, \quad 5V = 40, \quad V = 8\text{ V}.$$

The bulb's left end is therefore at the higher potential. Its current is determined by the 2 V potential difference across its own resistance.

Step 3: Conclusion

$$I_B = \frac{10 - 8}{10} = 0.20 \text{ A.}$$

The current through the bulb is 0.20 A, flowing from left to right, from the junction of the cells toward the junction of the resistors.

Quick Tip to Solve: Mark the three potentials fixed by the cells first; the bulb connects the two midpoints, so it is not simply in series with either external resistor.

Section C (21 marks)

22. Explain the following observations using Einstein's photoelectric equation :

- (a) Photoelectric emission does not occur from a surface when the frequency of the light incident on it is less than a certain minimum value.
- (b) It is the frequency, and not the intensity of the incident light which affects the maximum kinetic energy of the photoelectrons.
- (c) The cut-off voltage (V_0) versus frequency (ν) of the incident light curve is a straight line with a slope $\frac{h}{e}$. [3 marks]

Solution:

Concept:

- Einstein's photoelectric equation expresses energy conservation for absorption of one photon by one electron: $h\nu = \phi + K_{\max}$, where h is Planck's constant, ν is the incident frequency, ϕ is the metal's work function, and $K_{\max}$ is the maximum electron kinetic energy.
- The work function is the minimum energy needed to free an electron from the surface. At the stopping or cut-off voltage of magnitude V_0 , even the fastest emitted electrons stop, so $eV_0 = K_{\max}$, where e is the magnitude of electron charge.

Step 1: Explain the threshold frequency in part (a) An electron can escape only if the energy $h\nu$ supplied by a photon is at least the work function. At the limiting frequency ν_0 , the fastest electrons just escape with zero kinetic energy:

$$h\nu_0 = \phi, \quad \nu_0 = \frac{\phi}{h}.$$

For $\nu < \nu_0$, a photon has insufficient energy to release an electron. Formally, $h\nu - \phi < 0$; a negative outgoing kinetic energy is impossible, so no photoelectric emission occurs. Increasing the intensity at this same frequency cannot supply the missing energy to an electron in the usual photoelectric process.

Step 2: Explain the role of frequency and intensity in part (b) For a fixed metal, ϕ is constant, and Einstein's equation gives

$$K_{\max} = h\nu - \phi.$$

Increasing frequency increases the energy of each photon and therefore the maximum kinetic energy of the emitted electrons. At a fixed frequency above threshold, increasing intensity increases the

number of incident photons per unit area per unit time. This can increase the number of emitted electrons and the photoelectric current, but leaves the photon energy $h\nu$ and hence $K_{\max}$ unchanged.

Step 3: Derive the straight-line relation in part (c) Substitute the stopping-potential relation into Einstein's equation:

$$eV_0 = h\nu - \phi, \quad V_0 = \frac{h}{e}\nu - \frac{\phi}{e}.$$

For frequencies at or above threshold, this has the form $y = mx + c$, with $y = V_0$ and $x = \nu$. The slope is h/e ; the extrapolated vertical intercept is $-\phi/e$, and the frequency-axis intercept is $\nu_0 = \phi/h$. Below threshold there is no emission and hence no measured stopping potential for photoelectrons.

Step 4: Conclusion Einstein's equation gives a threshold $\nu_0 = \phi/h$, a maximum kinetic energy determined by frequency through $K_{\max} = h\nu - \phi$, and a straight-line stopping-voltage relation $V_0 = (h/e)\nu - \phi/e$ with slope h/e .

Quick Tip to Solve: Use $K_{\max} = h\nu - \phi$ for energy questions and replace $K_{\max}$ by eV_0 for stopping-voltage questions; intensity changes the photon count at a fixed frequency.

23. (a) A charged particle q moving with a velocity $\vec{v}$ is subjected to a uniform magnetic field $\vec{B}$ acting perpendicular to $\vec{v}$. If a uniform electric field $\vec{E}$ is also set up in the region along the direction of $\vec{B}$, describe the path followed by the particle and draw its shape.

(b) How will the magnetic field inside a long solenoid be affected when :

(i) the radius of the turns of the solenoid is increased,

(ii) the length of solenoid as well as the total number of its turns are doubled ?

[3 marks]

Solution:

Concept:

- The Lorentz force is $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. A magnetic force changes the direction of motion, while an electric force can change its speed.
- For motion perpendicular to a uniform magnetic field, $r = mv_{\perp}/(|q|B)$ and $T = 2\pi m/(|q|B)$, where m is the particle mass and T the revolution period.
- Inside a long air-core solenoid, $B = \mu_0 nI = \mu_0 NI/L$, where N is its total number of turns, L its length and I its current.

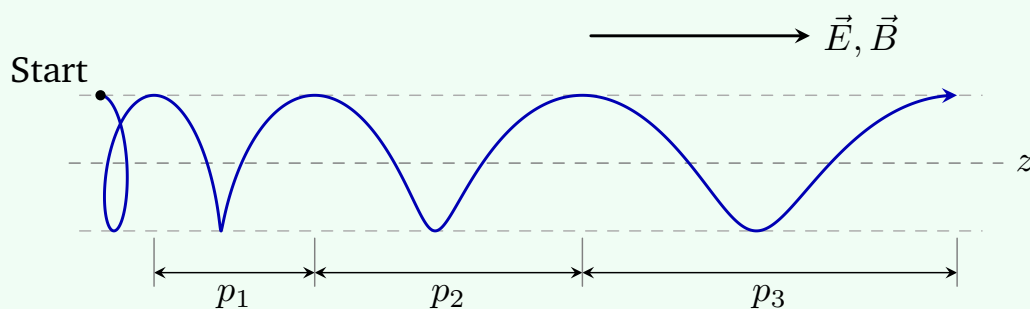
Step 1: (a) Resolve the motion perpendicular and parallel to the fields Take the common field direction as the z -axis. Initially $v_{\parallel} = 0$ and $v_{\perp} = v$. The magnetic force provides the centripetal force in the transverse plane. The electric field has no transverse component, so $v_{\perp}$, the radius r , and the period T remain constant in the usual non-relativistic treatment. Along the field, the magnetic force is zero and the electric force gives constant acceleration:

$$a_{\parallel} = \frac{qE}{m}, \quad v_{\parallel} = \frac{qE}{m}t, \quad z = \frac{qE}{2m}t^2.$$

Thus the particle winds around the field direction with a constant radius, while its axial speed increases. It follows a helix of increasing pitch, rather than a helix of uniform pitch. The magnitude of the axial distance travelled during a turn starting at time t is

$$p(t) = \frac{|q|E}{m} \left(tT + \frac{T^2}{2} \right),$$

which grows with time. For a positive charge the axial motion is along $\vec{E}$; for a negative charge it is opposite to $\vec{E}$, and the sense of winding also reverses.



Schematic projection for $q > 0$: constant radius and increasing pitch, with $p_1 < p_2 < p_3$.

Step 2: (b)(i) Change the radius of the turns The expression $B = \mu_0 NI/L$ contains no radius. Therefore, increasing the turn radius leaves the interior magnetic field unchanged, provided the current and turns per unit length stay fixed and the solenoid remains long compared with its radius. This is the ideal long-solenoid approximation.

Step 3: (b)(ii) Double the length and the total number of turns With $N' = 2N$ and $L' = 2L$, the number of turns per unit length is

$$n' = \frac{N'}{L'} = \frac{2N}{2L} = \frac{N}{L} = n.$$

Hence, at the same current, $B' = \mu_0 n' I = \mu_0 n I = B$.

Step 4: Conclusion (a) The path is a constant-radius helix whose pitch increases with time, as shown. (b) The magnetic field remains unchanged in both cases, at fixed current in the long-solenoid approximation.

Quick Tip to Solve: Resolve velocity along and across the field first: parallel electric acceleration changes the pitch, while fixed transverse speed fixes the radius. For a long solenoid, compare N/L .

24. (a) Differentiate between magnetic flux through an area and magnetic field at a point.

(b) A bar magnet is held with its length along the axis of a closed coil. Initially the south pole of the magnet faces the coil. If the magnet is moved towards the coil, explain how a current is induced in the coil and in what direction.

[3 marks]

Solution:

Concept:

- Magnetic flux through a surface is $\Phi_B = \int \vec{B} \cdot d\vec{A}$; for a uniform field through a plane area, $\Phi_B = BA \cos \theta$, where θ is the angle between the field and the area normal.
- Faraday's law gives the induced emf in an N -turn coil as $\mathcal{E} = -N d\Phi_B/dt$. Lenz's law gives its direction: the induced magnetic effect opposes the change in flux.

Step 1: (a) Distinguish the local field from flux through an area The magnetic field $\vec{B}$ describes the magnetic influence at a particular point. It is a vector, specifying both magnitude and direction, and determines the force on a moving charge through $\vec{F}_B = q\vec{v} \times \vec{B}$. Its SI unit is tesla, T. Magnetic flux Φ_B describes how much magnetic field passes through a chosen surface. It is a scalar with a sign fixed

by the chosen surface normal; its SI unit is weber, Wb. Unlike the field at a point, it depends on the area and its orientation as well as the field over the surface. For example, with the same uniform B , rotating a plane surface from a normal parallel to the field to a normal perpendicular to the field changes the flux from BA to zero. The units are related by $1 \text{ Wb} = 1 \text{ T m}^2$.

Step 2: (b) Apply Faraday's law to the approaching magnet Outside a bar magnet, magnetic field lines enter its south pole. Hence, at the coil, the magnet's axial field is directed towards the approaching south pole. As the magnet moves closer, the magnitude of magnetic flux linked with the coil increases. This changing flux produces an emf by Faraday's law; because the coil forms a closed conducting path, an induced current flows.

Step 3: Determine the current direction using Lenz's law The induced field must oppose the increase in the magnet's flux, so at the coil it points away from the magnet. Equivalently, the coil face nearest the approaching south pole becomes a south pole, repelling the approach. By the right-hand rule for a current loop, a face is south when the current is clockwise as viewed from that face. Thus an observer on the magnet side, looking towards the coil, sees a clockwise induced current. The same current appears anticlockwise when viewed from the opposite side.

Step 4: Conclusion (a) Magnetic field is a vector at a point, measured in tesla; magnetic flux is a scalar through a surface, measured in weber. (b) The changing flux induces a current that is clockwise when viewed from the approaching magnet, making the near coil face a south pole.

Quick Tip to Solve: First make the near coil face repel an approaching pole. Then use the face rule: clockwise current makes a south face, and anticlockwise current makes a north face.

25. (a) State any three characteristics of electromagnetic waves.

(b) Briefly explain how and where the displacement current exists during the charging of a capacitor.

[3 marks]

Solution:

Concept:

- An electromagnetic wave consists of oscillating electric and magnetic fields. In vacuum its speed is $c = 1/\sqrt{\mu_0 \epsilon_0}$, where μ_0 and ϵ_0 are the permeability and permittivity of free space.
- Maxwell's displacement current is associated with changing electric flux: $I_d = \epsilon_0 d\Phi_E/dt$, where $\Phi_E = \int \vec{E} \cdot d\vec{A}$ for a surface in vacuum or approximately in air.

Step 1: (a) State three characteristics of electromagnetic waves (i) Transverse nature: The electric and magnetic fields are perpendicular to each other, and both are perpendicular to the direction of wave propagation. The propagation direction is along $\vec{E} \times \vec{B}$.

(ii) Propagation in vacuum: These waves need no material medium. In vacuum they travel with speed

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \simeq 3.0 \times 10^8 \text{ m s}^{-1}.$$

(iii) Phase relation: The electric and magnetic fields of a travelling electromagnetic wave oscillate in the same phase: their corresponding maxima, minima and zeros occur together. In vacuum their amplitudes satisfy $E_0/B_0 = c$.

Step 2: (b) Locate the changing electric field during charging While a capacitor charges, conduction current carries charge onto one plate and removes charge from the other. The magnitude of charge Q

on the plates increases, so the electric field in the gap between them changes with time. Although charge does not pass across the insulating gap as conduction current, this changing electric field constitutes a displacement current there. It produces a magnetic effect and maintains continuity of the total current in the charging circuit.

Step 3: Relate the displacement current to the charging current For an ideal parallel-plate air or vacuum capacitor of plate area A , neglect fringing. The field and electric flux through the gap are

$$E = \frac{Q}{\epsilon_0 A}, \quad \Phi_E = EA = \frac{Q}{\epsilon_0}.$$

Therefore,

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} = I_c,$$

where I_c is the conduction current in the wires. For consistently oriented surfaces, their directions also agree. After charging is complete in a steady DC circuit, $dE/dt = 0$, and the displacement current becomes zero even though an electric field remains between the plates.

Step 4: Conclusion Electromagnetic waves are transverse, propagate through vacuum at c , and have electric and magnetic fields oscillating in phase. During capacitor charging, displacement current exists in the interplate gap because the electric flux changes; it equals the charging current, $I_d = I_c$.

Quick Tip to Solve: In a capacitor gap, ask whether the electric field is changing: a changing field gives displacement current, while a constant nonzero field gives none.

26. (a) Define ‘wavefront’ of a light wave. A plane wavefront is refracted from a convex lens. Draw the shape of the refracted wavefront.

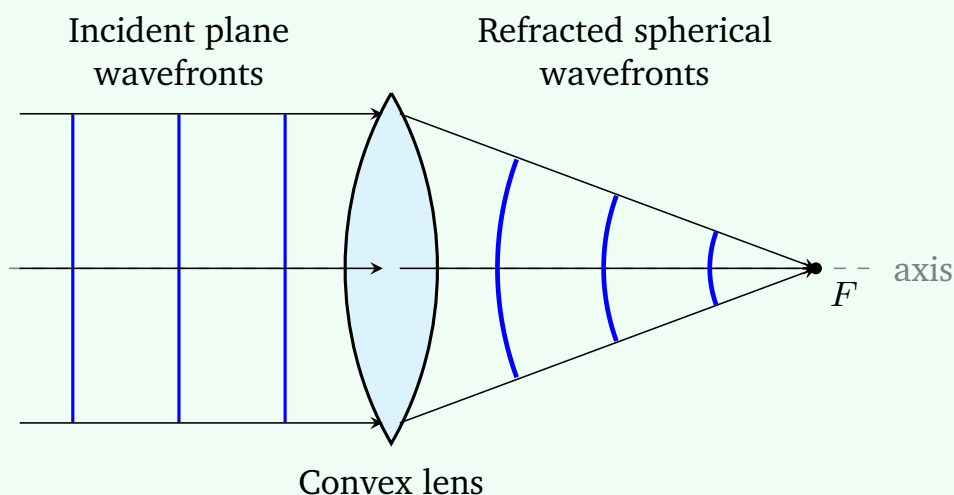
(b) A plane wave travelling in a medium is incident on a plane surface separating this medium from a rarer medium. Draw a diagram to show refraction of the wave. Hence, verify Snell’s law. [3 marks]

Solution:

Concept:

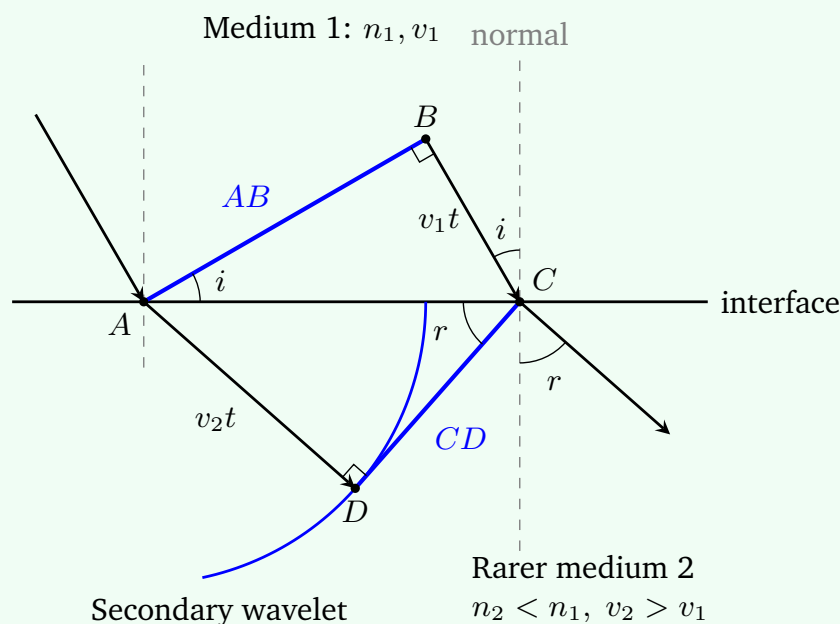
- A wavefront is the locus of points of a light wave that vibrate in the same phase at a given instant. In an isotropic medium, light rays are perpendicular to the wavefront.
- Huygens’ principle: each point on a wavefront acts as a source of secondary wavelets; their forward common tangent gives the new wavefront.
- In a medium of refractive index n , light travels with speed $v = c/n$, where c is its speed in vacuum. Refraction into a rarer medium therefore increases its speed.

Step 1: (a) Define the wavefront and draw the lens output A **wavefront** is a surface joining points of equal phase at an instant. Consider a plane wavefront incident normally on the principal axis of a convex lens. The emerging rays converge towards its principal focus F ; hence the refracted wavefront is a **converging spherical wavefront**, with its centre at F , in the paraxial approximation. The curved lines below are sections of these spherical wavefronts before they reach the focus.



Plane wavefronts become converging spherical wavefronts, centred at F .

Step 2: (b) Construct the refracted wavefront Let AB be an incident plane wavefront in medium 1, with speed v_1 and refractive index n_1 . Point A first reaches the interface. During time t , point B travels along the incident ray to C , so $BC = v_1t$. In the rarer medium 2, the secondary wavelet from A has radius $AD = v_2t$, with $v_2 > v_1$. Draw the tangent CD from C to this wavelet. Then CD is the refracted wavefront and AD , perpendicular to CD , is a refracted ray. The construction assumes incidence below the critical angle so that a transmitted wave exists.



Huygens' construction: $BC = v_1t$, $AD = v_2t$, and CD is tangent to the secondary wavelet.

Step 3: (b) Use the two right triangles to verify Snell's law Let i and r be the angles of incidence and refraction measured from the normal. Since rays are normal to wavefronts, $\angle BAC = i$ and $\angle ACD = r$. Triangles ABC and ADC are right-angled at B and D , respectively. Thus,

$$\sin i = \frac{BC}{AC} = \frac{v_1t}{AC}, \quad \sin r = \frac{AD}{AC} = \frac{v_2t}{AC}.$$

Dividing and using $v_1 = c/n_1$, $v_2 = c/n_2$,

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{n_2}{n_1} \implies n_1 \sin i = n_2 \sin r.$$

This is Snell's law. Since $n_2 < n_1$, the construction gives $r > i$ for oblique incidence: the ray bends away from the normal.

Step 4: Conclusion A convex lens converts the incident plane wavefront into a converging spherical wavefront. Huygens' construction gives $n_1 \sin i = n_2 \sin r$, verifying Snell's law for refraction into the rarer medium.

Quick Tip to Solve: In Huygens' construction, use the common hypotenuse AC : the opposite sides are $v_1 t$ for i and $v_2 t$ for r , so the time cancels immediately.

27. (a) What are majority and minority charge carriers of p-type and n-type semiconductors ?
(b) Explain briefly the formation of diffusion current and drift current in a p-n junction diode.

[3 marks]

Solution:

Concept:

- Majority carriers are the charge carriers present in greater concentration in a doped semiconductor; minority carriers are those present in much smaller concentration.
- Diffusion is carrier motion caused by a concentration gradient. Drift is carrier motion caused by an electric field; conventional current follows positive-charge motion and is opposite to electron motion.

Step 1: (a) Identify the carriers in each semiconductor In a **p-type semiconductor**, acceptor doping produces many holes, so holes are the majority carriers and electrons are the minority carriers. In an **n-type semiconductor**, donor doping supplies many free electrons, so electrons are the majority carriers and holes are the minority carriers. The minority carriers arise from thermal generation of electron-hole pairs. These names refer to carrier concentrations, not to a net charge on the bulk semiconductor.

Step 2: (b) Explain the diffusion current When p-type and n-type regions form a junction, the electron concentration is greater on the n-side and the hole concentration is greater on the p-side. Electrons therefore diffuse from n to p, while holes diffuse from p to n. Both motions produce a conventional **diffusion current from p to n**. Near the junction, recombination removes mobile carriers and leaves fixed positive donor ions on the n-side and negative acceptor ions on the p-side, forming the depletion region.

Step 3: (b) Explain the drift current The uncovered ions establish a built-in electric field across the depletion region directed from the n-side to the p-side. This field opposes further majority-carrier diffusion. It sweeps minority electrons from p to n and minority holes from n to p. Both motions contribute to a conventional **drift current from n to p**, opposite to the diffusion current. In an

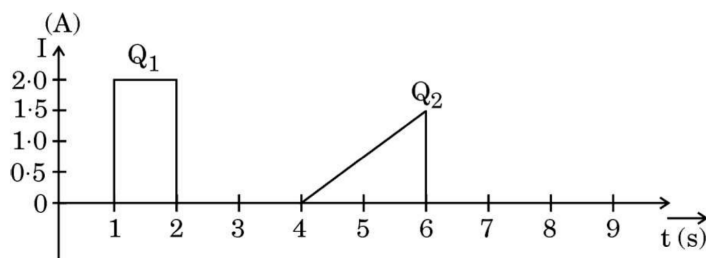
unbiased junction at thermal equilibrium, drift and diffusion currents have equal magnitudes, so the net current is zero even though both processes continue.

Step 4: Conclusion P-type material has majority holes and minority electrons; n-type material has majority electrons and minority holes. Carrier concentration differences create diffusion current, whereas the junction electric field creates drift current in the opposite direction.

Quick Tip to Solve: First draw the carrier motion arrows, then reverse only the electron arrows to obtain conventional current: diffusion is p to n and drift is n to p.

28(a). (i) Derive an expression for the resistivity of a conductor in terms of number density of free electrons and relaxation time.

(ii) The figure shows the plot of current through a cross-section of wire over two different time intervals. Compare the charges (Q_1 and Q_2) that pass through the cross-section during these time intervals. [3 marks]



Solution:

Concept:

- In the relaxation-time model, the drift speed is $v_d = e\mathcal{E}\tau/m$, where e is the magnitude of electron charge, m its mass, $\mathcal{E}$ the electric field and τ the mean time between collisions.
- The current density magnitude is $J = nev_d$, where n is the number of free electrons per unit volume; Ohm's law gives $\mathcal{E} = \rho J$.
- Charge crossing a section is $Q = \int I dt$, equal to the area under the current-time graph.

Step 1: Relate drift speed to the electric field (i) In a uniform electric field, the force on an electron is opposite to the field and its acceleration has magnitude $e\mathcal{E}/m$. Random thermal velocities average to zero. Averaging the velocity acquired between successive collisions therefore gives the drift-speed magnitude

$$v_d = \frac{e\mathcal{E}\tau}{m}.$$

For a conductor with cross-sectional area A , the charge crossing a section in time dt is $neAv_d dt$. Thus $I = neAv_d$; conventional current flows along the electric field, opposite to electron drift.

Step 2: Identify the resistivity Dividing the current by the area and substituting the drift speed,

$$J = nev_d = \frac{ne^2\tau}{m}\mathcal{E}.$$

Comparing with $J = \mathcal{E}/\rho$ yields

$$\rho = \frac{m}{ne^2\tau}.$$

Hence a larger free-electron number density or a longer relaxation time increases conductivity and

reduces resistivity. The SI unit of ρ is $\Omega \text{ m}$.

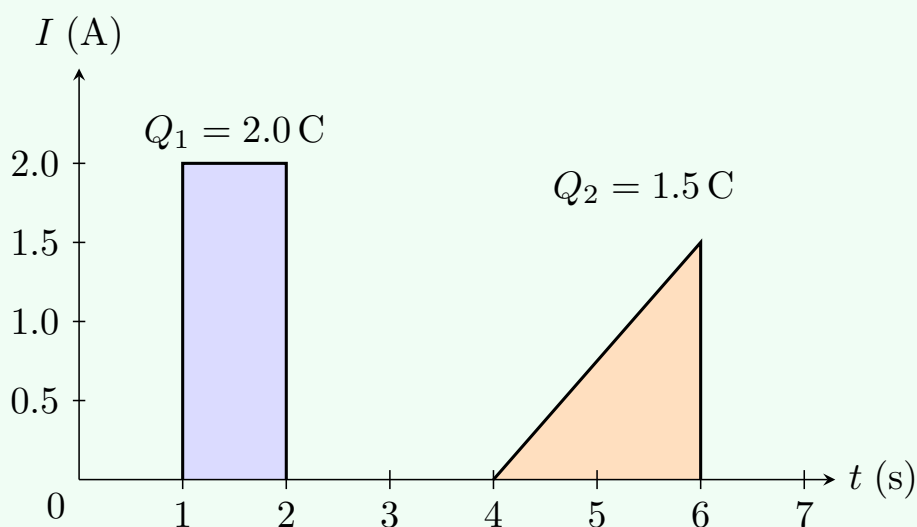
Step 3: Evaluate both areas under the graph (ii) In the first interval, the current is 2.0 A from $t = 1 \text{ s}$ to $t = 2 \text{ s}$. Its rectangular area is

$$Q_1 = (2.0)(2 - 1) = 2.0 \text{ C}.$$

In the second interval, the current rises linearly from zero at 4 s to 1.5 A at 6 s. Its triangular area is

$$Q_2 = \frac{1}{2}(6 - 4)(1.5) = 1.5 \text{ C}.$$

The full width of the second interval is 2 s, but its average current is half its peak value. Both plotted currents are positive, so these areas give positive transferred charges.



Shaded areas under the current–time graph give the two charges.

Step 4: Conclusion (i) $\rho = m/(ne^2\tau)$. (ii) $Q_1 = 2.0 \text{ C}$, $Q_2 = 1.5 \text{ C}$, hence $Q_1 : Q_2 = 4 : 3$; the first interval transfers more charge.

Quick Tip to Solve: Compare areas, not peak currents: use width times height for the rectangular pulse and half of that product for the triangular pulse.

28(b). (i) A battery of emf E and internal resistance r is connected to a variable external resistance R .

(I) Obtain the expression for current I in the circuit and the value of maximum current the battery can supply.

(II) Obtain the terminal voltage V across the battery and its maximum possible value.

(ii) The above battery sends a current I_1 when $R = R_1$ and a current I_2 when $R = R_2$. Obtain the internal resistance of the battery in terms of I_1 , I_2 , R_1 and R_2 . [3 marks]

Solution:

Concept:

- A battery of emf E and internal resistance r supplies a load resistance R in series with r . Kirchhoff's loop law gives $E = I(R + r)$.
- For a discharging battery, the terminal voltage is $V = E - Ir = IR$. The internal voltage drop Ir reduces the voltage delivered to the load.

Step 1: Find the current and its maximum (i)(I) The external and internal resistances carry the same current. Applying the loop rule around the complete circuit,

$$E - IR - Ir = 0 \implies I = \frac{E}{R + r}.$$

For fixed positive E and r , decreasing the non-negative load resistance R decreases the denominator and increases the current. Its maximum occurs for a short circuit, $R = 0$:

$$I_{\max} = \frac{E}{r}.$$

This finite maximum assumes a battery with nonzero internal resistance and the stated constant-emf model.

Step 2: Find the terminal voltage and its maximum (i)(II) The voltage across the battery terminals equals the voltage across the external resistance. Substitution of the current gives

$$V = IR = \frac{ER}{R + r} = E - Ir.$$

As R increases, the current and the internal voltage drop decrease, so the terminal voltage rises. In the open-circuit limit $R \rightarrow \infty$, $I \rightarrow 0$, and

$$V_{\max} = E.$$

For every finite positive load and $r > 0$, $V < E$. In particular, the short-circuit condition that gives maximum current gives zero terminal voltage, so the two maxima occur under different load conditions.

Step 3: Eliminate the emf between the two load conditions (ii) The same battery has the same emf and internal resistance in both experiments. Therefore

$$E = I_1(R_1 + r) = I_2(R_2 + r).$$

Expanding and collecting the terms containing r ,

$$\begin{aligned}I_1 R_1 + I_1 r &= I_2 R_2 + I_2 r, \\(I_1 - I_2)r &= I_2 R_2 - I_1 R_1.\end{aligned}$$

Thus the internal resistance can be determined without knowing the emf, provided the measured currents differ. Both numerator and denominator must use the same ordering; reversing just one would give an incorrect sign.

Step 4: Conclusion $I = E/(R + r)$, with $I_{\max} = E/r$ at $R = 0$; $V = ER/(R + r)$, with $V_{\max} = E$ on open circuit. For the two measurements,

$$r = \frac{I_2 R_2 - I_1 R_1}{I_1 - I_2}.$$

Quick Tip to Solve: Write $E = I(R + r)$ separately for each load and equate the two expressions; this removes the unknown emf in one step.

Section D (8 marks)

Questions number 29 and 30 are Case Study-based questions. Read the following paragraphs and answer the questions that follow.

29. A hydrogen atom consists of an electron revolving in a circular orbit of radius r with certain velocity v around a proton located at the nucleus of the atom. The electrostatic force of attraction between the revolving electron and the proton provides the requisite centripetal force to keep it in the orbit. According to Bohr's model, an electron can revolve only in certain stable orbits. The angular momentum of the electron in these orbits is some integral multiple of $\frac{h}{2\pi}$, where h is the Planck's constant. Further, when an electron makes a transition from one orbit of higher energy to that of lower energy, a photon is emitted having energy equal to the difference between energies of the initial and final states. Assuming the mass and charge of an electron as m and e respectively, answer the following questions. [4 marks]

29(i). (i) The expression for the speed of electron v in terms of radius of the orbit (r) and physical constant $\left(K = \frac{1}{4\pi\epsilon_0}\right)$ is : [1 mark]

- (A) $\frac{Ke^2}{mr}$
(B) $\frac{Ke^2}{mr^2}$
(C) $\sqrt{\frac{Ke^2}{mr}}$
(D) $\sqrt{\frac{Ke^2}{mr^2}}$

Correct Answer: (C) $\sqrt{\frac{Ke^2}{mr}}$

Solution:

Concept:

- The electrostatic attraction between the proton and electron provides the centripetal force: $F = Ke^2/r^2$, where $K = 1/(4\pi\epsilon_0)$.
- For an electron of mass m moving with speed v in a circular orbit of radius r , the required centripetal force is mv^2/r .

Step 1: Equate the attractive and centripetal forces Both force magnitudes refer to the same circular orbit. Therefore

$$\frac{mv^2}{r} = \frac{Ke^2}{r^2} \implies v^2 = \frac{Ke^2}{mr}.$$

One factor of r cancels; retaining r^2 in the denominator after this step would be incorrect.

Step 2: Select the positive speed and compare the options Speed is non-negative, so take the positive square root of the expression for v^2 . Option (A) gives the square of the speed instead of the speed. Option (B) gives $Ke^2/(mr^2)$, which has dimensions of acceleration, and option (D) is the square root

of that acceleration. Only option (C) has the required form and dimensions of speed.

Step 3: Conclusion $v = \sqrt{Ke^2/(mr)}$. Hence, the correct option is (C).

Quick Tip to Solve: First cancel one power of r in the force equation; then take a square root because centripetal force contains v^2 .

29(ii). (ii) The total energy of the atom in terms of r and physical constant K is :

[1 mark]

- (A) $\frac{Ke^2}{r}$
(B) $-\frac{Ke^2}{2r}$
(C) $\frac{Ke^2}{2r}$
(D) $\frac{3Ke^2}{2r}$

Correct Answer: (B) $-\frac{Ke^2}{2r}$

Solution:

Concept:

- For the hydrogen electron, force balance gives $mv^2/r = Ke^2/r^2$, so its kinetic energy is $T = \frac{1}{2}mv^2 = Ke^2/(2r)$.
- Taking electrostatic potential energy as zero at infinite separation, the attractive electron–proton system has $U = -Ke^2/r$; its total energy is $E = T + U$.

Step 1: Determine the two energy contributions The kinetic energy is positive, while the potential energy is negative because the charges attract. Their magnitudes obey $U = -2T$. Thus the magnitude of the potential energy is twice the kinetic energy, not equal to it.

Step 2: Add the energies and check the signs of the options Adding both contributions gives

$$E = \frac{Ke^2}{2r} - \frac{Ke^2}{r} = -\frac{Ke^2}{2r}.$$

A negative total energy is consistent with a bound electron: energy must be supplied to remove it to infinity. Option (A) is the magnitude of the potential energy and option (C) is the kinetic energy alone. Option (D) adds the magnitudes of the two energies instead of their signed values. None of those positive quantities is the total energy of this bound atom.

Step 3: Conclusion The total energy is $E = -Ke^2/(2r)$. Hence, the correct option is (B).

Quick Tip to Solve: For a Bohr orbit, remember $U = -2T$, so $E = -T = U/2$; the bound-state total energy must be negative.

29(iii). (iii) A photon of wavelength 500 nm is emitted when an electron makes a transition from one state to the other state in an atom. The change in the total energy of the electron and change in its kinetic energy in eV as per Bohr's model, respectively will be : [1 mark]

(A) 2.48, -2.48

(B) -1.24, 1.24

(C) -2.48, 2.48

(D) 1.24, -1.24

Correct Answer: (C) -2.48, 2.48

Solution:

Concept:

- A photon of wavelength λ carries energy $E_\gamma = hc/\lambda$, where h is Planck's constant and c is the speed of light.
- During emission, the atom loses energy: $\Delta E = E_f - E_i = -E_\gamma$. In a Bohr orbit the kinetic energy T obeys $T = -E$, so $\Delta T = -\Delta E$.

Step 1: Calculate the photon energy in electron volts Use the standard conversion $hc \simeq 1240 \text{ eV nm}$, which incorporates the conversion from joules to electron volts. With the wavelength expressed in nanometres,

$$E_\gamma = \frac{1240 \text{ eV nm}}{500 \text{ nm}} = 2.48 \text{ eV}.$$

This is the positive energy carried away by the photon; the sign of the electron's energy change must be assigned separately.

Step 2: Apply the emission sign and the Bohr energy relation The final total energy is lower than the initial total energy because a photon is emitted. Hence $\Delta E = -2.48 \text{ eV}$. Since $T = -E$ in each orbit, the kinetic energy increases by $\Delta T = +2.48 \text{ eV}$. Option (A) reverses both signs and corresponds to absorption; options (B) and (D) incorrectly use half the photon energy, and (D) also reverses the emission signs.

Step 3: Conclusion The changes in total and kinetic energies, respectively, are $(-2.48, +2.48) \text{ eV}$. Hence, the correct option is (C).

Quick Tip to Solve: For emission, assign a minus sign to the total-energy change first; in a Bohr orbit the kinetic-energy change has the opposite sign.

29(iv)(a). In Bohr's model of hydrogen atom, the frequency of revolution of electron in its n^{th} orbit is proportional to :

[1 mark]

- (A) n
- (B) $\frac{1}{n}$
- (C) $\frac{1}{n^2}$
- (D) $\frac{1}{n^3}$

Correct Answer: (D) $\frac{1}{n^3}$

Solution:

Concept:

- Bohr's quantization condition is $mv_n r_n = nh/(2\pi)$, where n is the principal quantum number and r_n, v_n are the orbital radius and speed.
- For hydrogen, $r_n \propto n^2$ and $v_n \propto n^{-1}$. The revolution frequency is speed divided by circumference:
 $f_n = v_n/(2\pi r_n)$.

Step 1: Combine the speed and radius dependences One revolution covers the circumference $2\pi r_n$. Therefore the period is $T_n = 2\pi r_n/v_n$, so

$$f_n = \frac{1}{T_n} = \frac{v_n}{2\pi r_n} \propto \frac{n^{-1}}{n^2} = n^{-3}.$$

Step 2: Compare the proposed powers Option (A) would make the frequency increase in larger, slower orbits. Option (B) includes only the speed dependence; option (C) includes only the inverse-radius dependence. The revolution frequency must include both effects, giving the inverse cube in option (D).

Step 3: Conclusion The frequency of revolution is proportional to $1/n^3$. Hence, the correct option is (D).

Quick Tip to Solve: Write $f = v/(2\pi r)$ first, then subtract the power of r_n from the power of v_n : $-1 - 2 = -3$.

29(iv)(b). An electron makes a transition from -3.4 eV state to the ground state in hydrogen atom. Its radius of orbit changes by : (radius of orbit of electron in ground state = $0.53 \text{ \AA}$) [1 mark]

- (A) $0.53 \text{ \AA}$
- (B) $1.06 \text{ \AA}$
- (C) $1.59 \text{ \AA}$
- (D) $2.12 \text{ \AA}$

Correct Answer: (C) $1.59 \text{ \AA}$

Solution:

Concept:

- The energy of hydrogen in orbit n is $E_n = -13.6/n^2$ eV, with ground state $n = 1$.
- Bohr radii obey $r_n = n^2 r_1$, where the given ground-state radius is $r_1 = 0.53 \text{ \AA}$.

Step 1: Identify the initial orbit Substituting the initial energy into the hydrogen energy formula gives

$$-3.4 = -\frac{13.6}{n_i^2}, \quad n_i^2 = 4, \quad n_i = 2.$$

The transition is consequently from the second orbit to the first orbit.

Step 2: Calculate the decrease in radius The initial radius is $r_2 = 4(0.53) = 2.12 \text{ \AA}$. The signed change is

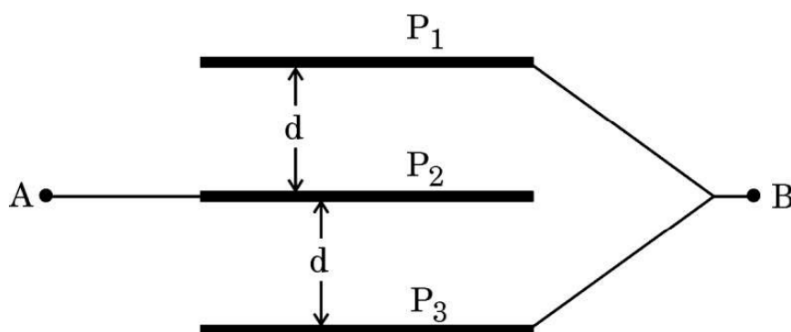
$$\Delta r = r_1 - r_2 = 0.53 - 2.12 = -1.59 \text{ \AA}.$$

Thus the requested amount of change is $1.59 \text{ \AA}$. Option (A) gives the final radius, option (D) the initial radius, and option (B) incorrectly scales the radius linearly with n .

Step 3: Conclusion The orbital radius decreases by $1.59 \text{ \AA}$. Hence, the correct option is (C).

Quick Tip to Solve: Determine n^2 from the energy, then use $|\Delta r| = r_1 |n_i^2 - n_f^2|$; do not subtract the quantum numbers themselves.

30. A parallel plate capacitor consists of two conducting plates kept generally parallel to each other at a distance. When the capacitor is charged, the charge resides on the inner surfaces of the plates and an electric field is set up between them. Thus, electrostatic energy is stored in the capacitor. The figure shows three large square metallic plates, each of side ' L ' held parallel and equidistant from each other. The space between P_1 and P_2 and P_2 and P_3 is completely filled with mica sheets of dielectric constant ' K '. The plate P_2 is connected to point A and other plates P_1 and P_3 are connected to point B . Point A is maintained at a positive potential with respect to point B and the potential difference between A and B is V . [4 marks]



30(i). The capacitance of the system between A and B will be :

[1 mark]

(A) $\frac{\epsilon_0 K L^2}{d}$

(B) $\frac{\epsilon_0 K L^2}{2d}$

(C) $\frac{2\epsilon_0 K L^2}{d}$

(D) $\frac{2\epsilon_0 K d}{L^2}$

Correct Answer: (C) $\frac{2\epsilon_0 K L^2}{d}$

Solution:

Concept:

- A fully dielectric-filled parallel plate capacitor has capacitance $C_0 = K\epsilon_0 A/d$, where A is plate area, d the separation, and K the relative permittivity.
- Capacitors connected across the same two potentials are in parallel; their capacitances add.

Step 1: Identify the two capacitors Plates P_1, P_2 form one capacitor, and P_2, P_3 form another. Each has area L^2 , gap d , and mica dielectric, so each has capacitance $C_0 = K\epsilon_0 L^2/d$.

Step 2: Combine the parallel branches Both gaps are between terminal A , connected to P_2 , and terminal B , connected to both outer plates. Each therefore has voltage V , making their connection

parallel:

$$C_{AB} = C_0 + C_0 = \frac{2K\epsilon_0 L^2}{d}.$$

Option (A) counts only one gap; option (B) treats the gaps as series. Option (D) reverses the area and gap factors and does not even have the dimensions of capacitance.

Step 3: Conclusion The system has capacitance $2K\epsilon_0 L^2/d$. Hence, the correct option is (C).

Quick Tip to Solve: Mark the potential at every plate: equal terminal pairs mean parallel capacitors, even when the plates are stacked geometrically.

30(ii). The charge on plate P_1 is :

[1 mark]

(A) $\frac{\epsilon_0 VKL^2}{2d}$

(B) $\frac{\epsilon_0 VKL^2}{d}$

(C) $\frac{2\epsilon_0 VKL^2}{d}$

(D) $\frac{\epsilon_0 VKL^2}{4d}$

Correct Answer: (B) $\frac{\epsilon_0 VKL^2}{d}$

Solution:

Concept:

- The charge magnitude on either plate of one capacitor is $q = C_0 V$, with $C_0 = K\epsilon_0 L^2/d$ for either dielectric-filled gap.
- P_1 is an outer plate at the lower potential B ; it carries negative charge. The central plate P_2 has positive charge on both faces.

Step 1: Select the capacitor containing the requested plate The printed question asks for P_1 . This plate belongs to the single gap between P_1 and P_2 , across which the potential difference is V . It therefore has charge magnitude

$$|Q_1| = C_0 V = \frac{K\epsilon_0 L^2 V}{d}.$$

Step 2: Distinguish magnitude from sign and total charge Since P_1 is at the lower potential, its signed charge is $Q_1 = -K\epsilon_0 L^2 V/d$. The printed choices give positive magnitudes, so the matching magnitude is (B). Option (C) is the magnitude of the central plate's total charge from both gaps; (A) and (D) incorrectly reduce the single-gap charge by factors of two and four.

Step 3: Conclusion Plate P_1 carries negative charge of magnitude $K\epsilon_0 L^2 V/d$. Hence, the correct option is (B).

Quick Tip to Solve: For an outer plate, count one charged face; only the central plate collects the positive charges from both capacitor gaps.

30(iii). The electric field in the region between P_1 and P_2 is :

[1 mark]

- (A) $\frac{V}{d}$
- (B) $\frac{2V}{d}$
- (C) $\frac{V}{2d}$
- (D) $\frac{d}{V}$

Correct Answer: (A) $\frac{V}{d}$

Solution:

Concept:

- Between large parallel plates, neglecting edge effects, the field is uniform and its magnitude is $E = \Delta V/d$, where d is the gap width.
- P_2 is connected to A and P_1 to B ; their potential difference is the full applied value V .

Step 1: Read the voltage and separation of this gap The diagram labels the distance from P_1 to P_2 as d . These plates are directly connected to opposite supply terminals, so this gap has potential difference V , rather than $V/2$.

Step 2: Apply the uniform-field relation Using $\Delta V = Ed$, the field magnitude is

$$E = \frac{V}{d}.$$

It points from positive P_2 toward negative P_1 . At fixed applied voltage the dielectric changes the required free charge, not this voltage gradient. Option (B) doubles the field unnecessarily, (C) incorrectly divides the voltage between parallel gaps, and (D) has inverse-field dimensions.

Step 3: Conclusion The electric field magnitude between P_1 and P_2 is V/d . Hence, the correct option is (A).

Quick Tip to Solve: Use the voltage and spacing of the one gap being asked about; two parallel gaps each receive the full supply voltage.

30(iv). (a) The separation between the plates of same area (L^2) of a parallel plate air capacitor having capacitance equal to that of this system, will be :

[1 mark]

- (A) $\frac{d}{K}$
- (B) $\frac{2d}{K}$
- (C) $\frac{d}{2K}$
- (D) $\frac{d}{4K}$

Correct Answer: (C) $\frac{d}{2K}$

Solution:

Concept:

- Each mica-filled gap has capacitance $K\epsilon_0 L^2/d$; the two gaps are in parallel, so $C_{AB} = 2K\epsilon_0 L^2/d$.
- A single parallel plate air capacitor of area L^2 and separation x has capacitance $C_{\text{air}} = \epsilon_0 L^2/x$.

Step 1: Equate the required capacitances The replacement must retain the capacitance of both original gaps together. Therefore

$$\frac{\epsilon_0 L^2}{x} = \frac{2K\epsilon_0 L^2}{d}.$$

The common area and vacuum permittivity cancel; the dielectric and the two parallel gaps remain relevant.

Step 2: Solve for the air-gap separation Rearranging gives $1/x = 2K/d$, hence $x = d/(2K)$. Option (A) compensates for the dielectric alone and misses the two parallel capacitors. Option (B) would give only one-quarter of the required capacitance, while option (D) would give twice the required capacitance.

Step 3: Conclusion The required separation of the air capacitor is $d/(2K)$. Hence, the correct option is (C).

Quick Tip to Solve: At fixed plate area, capacitance is inversely proportional to gap: the factor $2K$ enhancement requires a gap reduced by $2K$.

30(iv). (b) If the source of potential difference applied between A and B is removed, and then A and B are connected by a conducting wire, the net charge on the system will be : [1 mark]

- (A) $\frac{\epsilon_0 V K L^2}{4d}$
 (B) $\frac{\epsilon_0 V K L^2}{2d}$
 (C) $\frac{\epsilon_0 V K L^2}{d}$
 (D) Zero

Correct Answer: (D) Zero

Solution:

Concept:

- Charging a capacitor transfers charge from one conductor to another, so the equal positive and negative charges sum to zero for the whole initially neutral system.
- After the supply is removed, a conducting wire between the terminals allows redistribution until the potential difference vanishes; the total charge is conserved.

Step 1: Sum the charges before the short circuit Let $q = K\epsilon_0 L^2 V/d$ be the charge magnitude associated with either gap. The central plate P_2 carries $+2q$, while each outer plate carries $-q$. Thus

$$Q_{\text{net}} = (+2q) + (-q) + (-q) = 0.$$

Step 2: Follow the redistribution after connection With the source disconnected, the wire joins A and B , making all three plates equipotential. The separated charges neutralize as the system discharges, and no external charge is supplied. The net charge consequently remains zero. All three nonzero options retain a multiple of the original separated charge and cannot describe the net charge of the whole system.

Step 3: Conclusion The net charge on the system is zero. Hence, the correct option is (D).

Quick Tip to Solve: Add plate charges with their signs before using CV : a capacitor may store separated charge and energy while its total charge is zero.

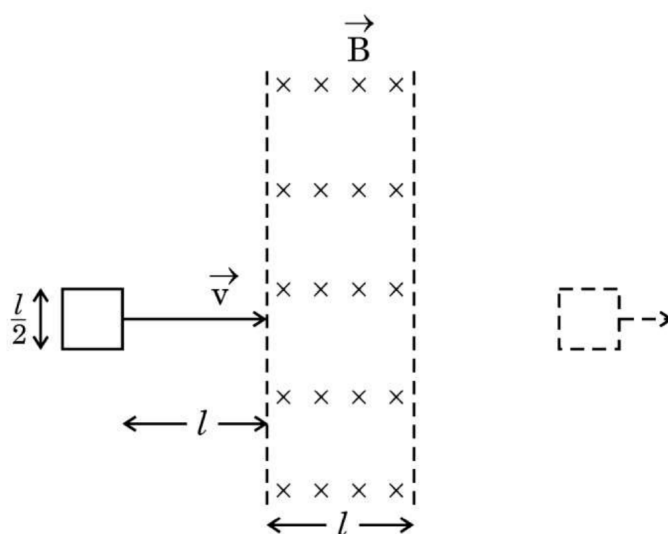
Section E (15 marks)

31(a). (i) State Lenz's law and explain how this law is a consequence of conservation of energy principle.

(ii) A square shaped loop of side $\ell/2$ is initially lying outside a region of uniform magnetic field $\vec{B}$ as shown in the figure. The loop is moved towards right with a constant velocity $\vec{v}$ till it goes out of the region of magnetic field.

(I) What will be the directions of induced current when the loop enters the field and when it leaves the field?

(II) Draw the plots showing the variation of magnetic flux ϕ linked with the loop with time t and variation of induced emf E with time t . Mark the relevant values of E , ϕ and t on the graphs. [5 marks]



Solution:

Concept:

- Lenz's law: the induced current flows in a direction such that its magnetic effect opposes the change in magnetic flux that produces it.
- Faraday's law for one loop is $E = -d\phi/dt$, where $\phi = BA_{\text{in}}$ for a perpendicular uniform field and A_{in} is the loop area inside that field.
- The printed loop side is $a = \ell/2$. During entry or exit, its area in the field changes at rate av , so $|E| = Bav = B\ell v/2$.

Step 1: (i) Relate Lenz's law to conservation of energy Suppose a magnet is moved towards a conducting loop. The induced current produces a magnetic field opposing the increase of flux, so the loop exerts a force opposing the magnet's approach. An external agent must therefore do work to maintain the motion; this mechanical work supplies the electrical energy, which can appear as Joule heat in the loop. Likewise, withdrawing the magnet is opposed by attraction. If the induced current instead aided the change, the magnetic force would speed up the approaching magnet, causing still more induction and further acceleration. Both kinetic energy and electrical energy would then increase without an external energy supply. This contradicts conservation of energy. The opposing direction expressed by Lenz's law prevents this contradiction and gives the negative sign in Faraday's law.

Step 2: (ii)(I) Determine the current directions The crosses in the printed figure represent a magnetic

field directed into the page. While the loop enters, the inward flux increases. The induced field must point out of the page, requiring an **anticlockwise** current as viewed on the page. While the loop leaves, the inward flux decreases. The induced field must point into the page to oppose that decrease, requiring a **clockwise** current. While the loop is fully within the uniform field, its flux is constant and the net induced emf and current are zero.

Step 3: (ii)(II) Locate the four transition times Take $t = 0$ at the initial position shown. The leading (right) edge is a distance ℓ from the field, the field region has width ℓ , and the loop itself has width $\ell/2$. With constant speed v , first contact, complete entry, the start of exit and complete exit occur respectively at

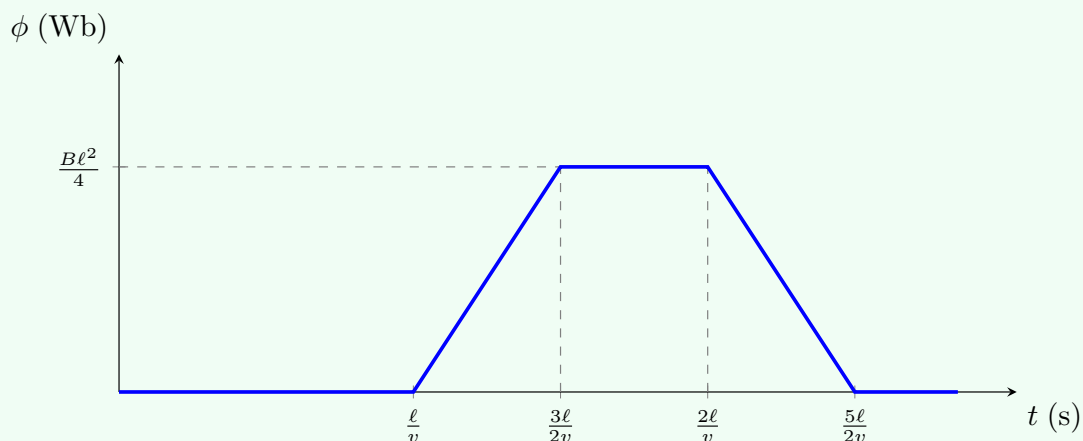
$$t_1 = \frac{\ell}{v}, \quad t_2 = \frac{3\ell}{2v}, \quad t_3 = \frac{2\ell}{v}, \quad t_4 = \frac{5\ell}{2v}.$$

Entry and exit each take $\ell/(2v)$; the loop is wholly inside for a further $\ell/(2v)$.

Step 4: Plot the linked magnetic flux Choose the positive area normal into the page, so the inward flux is positive. Let $\phi_0 = B\ell^2/4$, the flux through the entire square. During entry, the overlap width is $vt - \ell$; during exit it is $5\ell/2 - vt$. Multiplying by the side $\ell/2$ and by B gives

$$\phi(t) = \begin{cases} 0, & 0 \leq t \leq t_1, \\ \frac{B\ell}{2}(vt - \ell), & t_1 \leq t \leq t_2, \\ \frac{B\ell^2}{4}, & t_2 \leq t \leq t_3, \\ \frac{B\ell}{2}\left(\frac{5\ell}{2} - vt\right), & t_3 \leq t \leq t_4, \\ 0, & t \geq t_4. \end{cases}$$

Thus the flux rises linearly, remains at its maximum while the whole loop is inside, and falls linearly to zero.



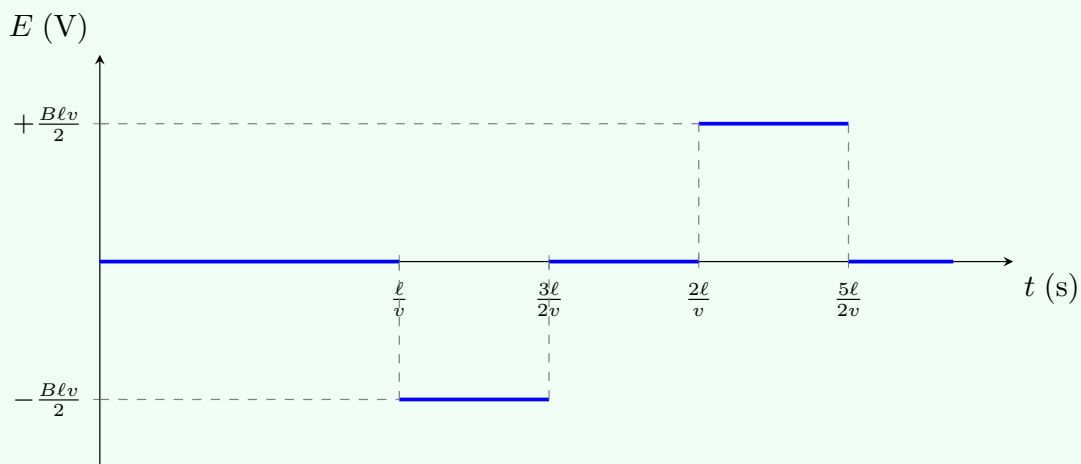
Flux into the page versus time, measured from the initial position shown.

Step 5: Plot the induced emf with a consistent sign convention The chosen inward normal makes clockwise circulation positive. Differentiating the straight sections of the flux graph gives a negative

emf on entry, zero emf while fully inside, and a positive emf on exit:

$$E(t) = \begin{cases} 0, & 0 < t < t_1, \\ -\frac{B\ell v}{2}, & t_1 < t < t_2, \\ 0, & t_2 < t < t_3, \\ +\frac{B\ell v}{2}, & t_3 < t < t_4, \\ 0, & t > t_4. \end{cases}$$

The abrupt changes occur at the ideal sharp boundaries; the dashed vertical lines indicate jumps between the constant levels. Reversing the area normal reverses both signed graphs and leaves the physical current directions unchanged.



Induced emf versus time; positive emf drives clockwise current.

Step 6: Conclusion Lenz's law enforces conservation of energy by opposing the flux change. The current is anticlockwise during entry and clockwise during exit. With inward flux positive, the flux reaches $B\ell^2/4$, while the emf is $-B\ell v/2$ on entry and $+B\ell v/2$ on exit, with zero emf during the other intervals.

Quick Tip to Solve: First mark the leading-edge and trailing-edge crossings. Here the loop side is $\ell/2$, while both the initial gap and field width are ℓ ; keeping these three lengths distinct fixes every graph breakpoint.

- 31(b).** (i) Differentiate between peak and rms values of alternating current. How are they related ?
 (ii) A current element X is connected across an ac source of emf $V = V_0 \sin 2\pi\nu t$. It is found that the voltage leads the current in phase by $\frac{\pi}{2}$ radian. If element X was replaced by element Y , the voltage lags behind the current in phase by $\frac{\pi}{2}$ radian.
 (I) Identify elements X and Y by drawing phasor diagrams.
 (II) Obtain the condition of resonance when both elements X and Y are connected in series to the source and obtain expression for resonant frequency. What is the impedance value in this case ?

[5 marks]

Solution:**Concept:**

- The peak current I_0 is the largest magnitude of the instantaneous alternating current. Its rms value is $I_{\text{rms}} = \sqrt{\langle i^2 \rangle}$, where the average is taken over a complete cycle.
- For an ideal inductor, $v = L di/dt$; for an ideal capacitor, $i = C dv/dt$. These relations determine the relative phases of voltage and current.
- Inductive and capacitive reactances are $X_L = \omega L$ and $X_C = 1/(\omega C)$, where $\omega = 2\pi\nu$. In a series circuit they oppose each other in the voltage phasor sum.

Step 1: (i) Distinguish the peak and rms currents Peak current is the maximum instantaneous magnitude attained during a cycle; it is the amplitude I_0 . The rms current is the value of steady direct current that produces the same heat in a given resistor in the same time as the alternating current. It therefore measures the heating effect of ac rather than its largest instantaneous value. For a sinusoidal current $i = I_0 \sin \omega t$, the average of $\sin^2 \omega t$ over a complete period T is $1/2$. Hence

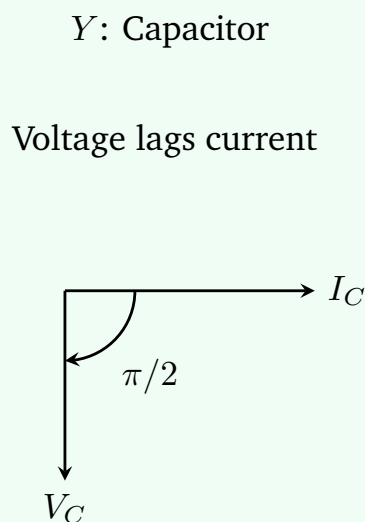
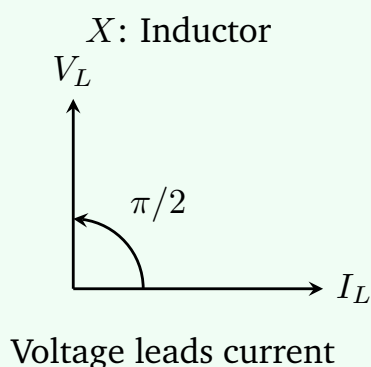
$$I_{\text{rms}}^2 = \frac{1}{T} \int_0^T I_0^2 \sin^2 \omega t dt = \frac{I_0^2}{2}, \quad I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \simeq 0.707 I_0.$$

The ordinary cycle-average current is zero, but the rms current is nonzero because squaring makes the heating contribution positive in both half-cycles.

Step 2: (ii)(I) Identify the elements from their phases Across element X , voltage leads current by $\pi/2$. This is the behaviour of a pure inductor of inductance L . Taking the current as the horizontal reference phasor, the inductor voltage points vertically upward and leads the current by 90° . Across element Y , voltage lags current by $\pi/2$, which identifies a pure capacitor of capacitance C . Its voltage phasor points vertically downward when current is taken along the same horizontal reference. With the given voltage $v = V_0 \sin \omega t$, the corresponding currents are

$$i_L = \frac{V_0}{\omega L} \sin(\omega t - \pi/2), \quad i_C = \omega C V_0 \sin(\omega t + \pi/2).$$

The phasors below use the usual counterclockwise phase convention; their lengths are illustrative.



Voltage-current phasors for the inductor X and capacitor Y ; current is the common phase reference.

Step 3: (ii)(II) Derive the resonance condition and frequency When L and C are in series, the same current flows through both. Their voltage phasors are opposite, so the net reactive voltage has magnitude $I|X_L - X_C|$. Resonance occurs when the inductive and capacitive reactances are equal, making their reactive voltages cancel:

$$X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C},$$

$$\omega_0^2 = \frac{1}{LC},$$

$$\nu_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}.$$

This condition depends on the values of L and C , not on the applied voltage amplitude.

Step 4: Evaluate the impedance at resonance For the ideal series combination of only the two stated elements,

$$Z = \left| \omega L - \frac{1}{\omega C} \right|, \quad Z(\omega_0) = 0.$$

If the coil, connections or source contribute a resistance R , the series impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, which becomes R at resonance. Thus zero impedance is the ideal LC result; a real circuit has its minimum impedance equal to its total resistance. The zero-resistance idealization does not give a finite steady-state resonant current.

Step 5: Conclusion $I_{\text{rms}} = I_0/\sqrt{2}$ for sinusoidal ac. Element X is an inductor and Y is a capacitor. Series resonance requires $X_L = X_C$, giving $\nu_0 = 1/(2\pi\sqrt{LC})$. The ideal LC impedance is zero; with total resistance R , the resonant impedance is R .

Quick Tip to Solve: Take current as the horizontal reference: inductor voltage points up and capacitor voltage down. At resonance these opposite voltages cancel because their reactances are equal.

32(a). (i) An object is placed 30 cm from a thin convex lens of focal length 10 cm. The lens forms a sharp image on a screen. If a thin concave lens is placed in contact with the convex lens, the sharp image on the screen is formed when the screen is moved by 45 cm from its initial position. Calculate the focal length of the concave lens.

(ii) Calculate the angle of minimum deviation of an equilateral prism. The refractive index of the prism is $\sqrt{3}$. Calculate the angle of incidence for this case of minimum deviation also. [5 marks]

Solution:

Concept:

- The thin-lens formula in the Cartesian sign convention is $1/v - 1/u = 1/f$, where u , v and f are signed object distance, image distance and focal length. A real object on the incident side has $u < 0$; a real image on the transmitted side has $v > 0$.
- For two thin lenses in contact, powers add: $1/F = 1/f_1 + 1/f_2$. Here F is the equivalent focal length, while f_1 and f_2 are the individual focal lengths.
- For a prism in air at minimum deviation, the path is symmetric: $i = e$, $r_1 = r_2 = A/2$, and $\mu = \sin[(A + \delta_m)/2] / \sin(A/2)$. Here A is the prism angle and δ_m its minimum deviation.

Step 1: (i) Locate the image formed by the convex lens The object lies 30 cm in front of the convex lens, so $u = -30$ cm. Its focal length is positive: $f_1 = +10$ cm. Applying the lens formula gives

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15} \text{ cm}^{-1},$$

$$v_1 = +15 \text{ cm}.$$

The positive image distance confirms that the image can be obtained on the screen behind the lens.

Step 2: Determine the screen position after adding the concave lens Adding the concave lens reduces the net converging power. Since a real image is still obtained, the new image lies farther from the lens. Thus the screen moves away from the lens by 45 cm, and

$$v_2 = 15 + 45 = 60 \text{ cm}.$$

Choosing $15 - 45 = -30$ cm would instead describe a virtual image on the object side, which cannot be received on a screen. Because the lenses are thin and in contact, the object distance for the equivalent lens remains -30 cm. Therefore

$$\frac{1}{F} = \frac{1}{v_2} - \frac{1}{u} = \frac{1}{60} + \frac{1}{30} = \frac{1}{20} \text{ cm}^{-1}, \quad F = 20 \text{ cm}.$$

Step 3: Calculate the focal length of the concave lens Subtract the known convex-lens power from the total power:

$$\frac{1}{f_2} = \frac{1}{F} - \frac{1}{f_1} = \frac{1}{20} - \frac{1}{10} = -\frac{1}{20} \text{ cm}^{-1},$$

$$f_2 = -20 \text{ cm}.$$

The negative sign is required for a concave lens. As a check, the lens powers in dioptres are $+10$ D and -5 D, whose sum $+5$ D corresponds to the equivalent focal length $+0.20$ m.

Step 4: (ii) Apply the minimum-deviation condition to the prism An equilateral prism has refracting angle $A = 60^\circ$. At minimum deviation the refractions are equal, so $r_1 = r_2 = 30^\circ$, and the angle of

incidence equals the angle of emergence. Snell's law at the first face gives

$$\sin i = \mu \sin r_1 = \sqrt{3} \sin 30^\circ = \frac{\sqrt{3}}{2}.$$

The physical incident angle measured from the normal lies between 0° and 90° , hence $i = 60^\circ$. Using the prism deviation relation,

$$\delta_m = i + e - A = 2i - A = 2(60^\circ) - 60^\circ = 60^\circ.$$

These values also satisfy the minimum-deviation formula: $\sin[(60^\circ + 60^\circ)/2]/\sin 30^\circ = \sqrt{3}$.

Step 5: Conclusion (i) The concave lens has focal length $f_2 = -20$ cm. (ii) The prism's angle of minimum deviation is $\delta_m = 60^\circ$, and the corresponding angle of incidence is $i = 60^\circ$.

Quick Tip to Solve: Find the original image distance before applying the screen shift; a screen requires the new image distance to remain positive. For a prism at minimum deviation, start with $r = A/2$ and $i = e$.

OR

32(b). (i) A physics teacher wants to demonstrate interference with the help of double slit experiment using a laser beam of 633 nm wavelength. Since the hall is large enough, interference pattern is formed on the wall 5.0 m from the slits. For clear and comfortable view by all the students they want the fringe width 5 mm.

(I) Find the slit separation for obtaining the desired interference pattern.

(II) How far will the first minimum be from the central maximum ?

(ii) A parallel beam of light of wavelength 650 nm passes through a slit of width 0.6 mm. The diffraction pattern is obtained on a screen kept 60 cm away from the slit. Find the distance between first order minima on both sides of the central maximum. [5 marks]

Solution:

Concept:

- In Young's double-slit experiment, the fringe width is $\beta = \lambda D/d$, where λ is the wavelength, D the screen distance and d the slit separation. This uses the small-angle approximation.
- Destructive interference occurs when the path difference is $(n + \frac{1}{2})\lambda$, with $n = 0, 1, 2, \dots$. The first dark fringe is therefore at a distance $\beta/2$ from the central bright fringe.
- For a single slit of width b , diffraction minima satisfy $b \sin \theta = m\lambda$, where $m = \pm 1, \pm 2, \dots$. The distance between the first minima is $2\lambda D/b$.

Step 1: (i)(I) Convert the data and find the slit separation For the interference experiment, express the wavelength and fringe width in metres:

$$\lambda = 633 \times 10^{-9} \text{ m}, \quad D = 5.0 \text{ m}, \quad \beta = 5.0 \times 10^{-3} \text{ m}.$$

The path difference at a transverse screen displacement x is approximately dx/D . Successive bright

fringes differ in path difference by one wavelength, giving $d\beta/D = \lambda$. Hence,

$$d = \frac{\lambda D}{\beta} = \frac{(633 \times 10^{-9})(5.0)}{5.0 \times 10^{-3}} \text{ m} \\ = 6.33 \times 10^{-4} \text{ m} = 0.633 \text{ mm}.$$

This separation is much smaller than the screen distance, as required for the usual fringe-width formula.

Step 2: (i)(II) Locate the first interference minimum At the central maximum the path difference is zero. The nearest minimum occurs when it becomes $\lambda/2$; it lies halfway between the central and first neighbouring bright fringes. If its distance from the centre is x_1 ,

$$\frac{dx_1}{D} = \frac{\lambda}{2}, \quad x_1 = \frac{\lambda D}{2d} = \frac{\beta}{2} = \frac{5.0}{2} \text{ mm} = 2.5 \text{ mm}.$$

The same distance applies on either side of the centre; the two nearest dark fringes are at signed positions $x = \pm 2.5 \text{ mm}$.

Step 3: (ii) Calculate the separation of the diffraction minima This part concerns a single slit, so use its width rather than the double-slit separation obtained above. The new data are

$$\lambda' = 650 \times 10^{-9} \text{ m}, \quad b = 0.6 \times 10^{-3} \text{ m}, \quad D' = 0.60 \text{ m}.$$

For the first minimum, $b \sin \theta = \lambda'$. Since $\lambda'/b \simeq 1.08 \times 10^{-3}$, the angle is small, and $\sin \theta \simeq \tan \theta = x/D'$. Thus the distance of either first minimum from the centre is

$$x = \frac{\lambda' D'}{b} = \frac{(650 \times 10^{-9})(0.60)}{0.6 \times 10^{-3}} \text{ m} = 6.50 \times 10^{-4} \text{ m} = 0.650 \text{ mm}.$$

The minima are on opposite sides of the central maximum, so their separation is

$$\Delta x = x - (-x) = 2x = 1.30 \text{ mm}.$$

Step 4: Conclusion (i)(I) The required slit separation is 0.633 mm. (i)(II) The first interference minimum is 2.5 mm from the centre on either side. (ii) The first diffraction minima are separated by 1.30 mm, the width of the central diffraction maximum.

Quick Tip to Solve: For the first interference minimum use half the fringe width. For the full central diffraction width, double the distance $\lambda D/b$ to one minimum.

33(a). (i) Two point charges $+q$ and $-q$ are held at $(a, 0)$ and $(-a, 0)$ in x - y plane. Obtain an expression for the net electric field due to the charges at a point $(0, y)$. Hence, find electric field at a far off point ($y \gg a$).

(ii) Three point charges of -2 nC , -1 nC , and $+5 \text{ nC}$ are kept at the vertices A , B and C of an equilateral triangle of side 0.2 m . Find the total amount of work done in shifting the charges from A to A_1 , B to B_1 and C to C_1 . Here A_1 , B_1 and C_1 are the midpoints of sides AB , BC and CA , respectively. [5 marks]

Solution:

Concept:

- Coulomb's law in vector form is $\vec{E} = kQ\vec{R}/R^3$, where $\vec{R}$ points from the source charge to the observation point and $k = 1/(4\pi\epsilon_0)$. Electric fields add vectorially.
- The dipole moment $\vec{p}$ points from the negative charge to the positive charge. Here $\vec{p} = 2qa\hat{i}$, and $(0, y)$ lies on the equatorial line of the dipole.
- The potential energy of a system of point charges is the sum over distinct pairs: $U = k \sum_{i < j} q_i q_j / r_{ij}$. For a slow rearrangement with no change in kinetic energy, the external work is $W_{\text{ext}} = U_f - U_i$.

Step 1: (i) Add the fields of the two charges at the equatorial point Let the observation point be $P(0, y)$. Its distance from each charge is $R = \sqrt{a^2 + y^2}$. The displacement vectors from $+q$ at $(a, 0)$ and $-q$ at $(-a, 0)$ to P are respectively $-a\hat{i} + y\hat{j}$ and $a\hat{i} + y\hat{j}$. Therefore,

$$\vec{E}_+ = \frac{kq}{(a^2 + y^2)^{3/2}}(-a\hat{i} + y\hat{j}),$$

$$\vec{E}_- = \frac{-kq}{(a^2 + y^2)^{3/2}}(a\hat{i} + y\hat{j}).$$

Their vertical components cancel, while both horizontal components point towards the negative charge, along the negative x -direction. Adding them gives

$$\vec{E} = \vec{E}_+ + \vec{E}_- = -\frac{2kqa}{(a^2 + y^2)^{3/2}}\hat{i} = -\frac{k\vec{p}}{(a^2 + y^2)^{3/2}}.$$

Step 2: (i) Take the far-field limit At a distant point with $y \gg a$, the ratio a^2/y^2 is negligible. Thus

$$(a^2 + y^2)^{3/2} = y^3 \left(1 + \frac{a^2}{y^2}\right)^{3/2} \simeq y^3,$$

and the field becomes

$$\vec{E}_{\text{far}} \simeq -\frac{k\vec{p}}{y^3} = -\frac{1}{4\pi\epsilon_0} \frac{2qa}{y^3}\hat{i}.$$

Its magnitude decreases as the inverse cube of distance. Its direction remains opposite to the dipole moment because the point is on the equatorial line.

Step 3: (ii) Determine the initial and final pair separations Initially, each pair of charges is separated by $r_i = 0.20 \text{ m}$. The segment joining midpoints of two sides of a triangle is parallel to the third side and half its length. Hence $A_1B_1C_1$ is an equilateral triangle of side $r_f = 0.10 \text{ m}$. All three final pair separations are therefore half their initial values. With $q_A = -2 \times 10^{-9} \text{ C}$, $q_B = -1 \times 10^{-9} \text{ C}$ and

$q_C = 5 \times 10^{-9} \text{ C}$, the sum of charge products is

$$\begin{aligned} S &= q_A q_B + q_B q_C + q_C q_A \\ &= [(-2)(-1) + (-1)(5) + (5)(-2)] 10^{-18} \text{ C}^2 \\ &= -13 \times 10^{-18} \text{ C}^2. \end{aligned}$$

Count each pair once; the energy is a property of the entire configuration, so the three individual displacements need not be treated as independent motions in fixed fields.

Step 4: (ii) Calculate the change in potential energy and the work Using $k = 9 \times 10^9 \text{ N m}^2 \text{C}^{-2}$,

$$\begin{aligned} U_i &= \frac{kS}{r_i} = \frac{(9 \times 10^9)(-13 \times 10^{-18})}{0.20} \text{ J} = -5.85 \times 10^{-7} \text{ J}, \\ U_f &= \frac{kS}{r_f} = \frac{(9 \times 10^9)(-13 \times 10^{-18})}{0.10} \text{ J} = -1.17 \times 10^{-6} \text{ J}. \end{aligned}$$

Thus the external work required for the rearrangement is

$$W_{\text{ext}} = U_f - U_i = -1.17 \times 10^{-6} - (-5.85 \times 10^{-7}) = -5.85 \times 10^{-7} \text{ J}.$$

The negative sign means that the system releases energy during the slow rearrangement. The electric field does positive work $W_{\text{field}} = -\Delta U = +5.85 \times 10^{-7} \text{ J}$.

Step 5: Conclusion (i) The exact field is $\vec{E} = -\frac{1}{4\pi\epsilon_0} \frac{2qa}{(a^2 + y^2)^{3/2}} \hat{\mathbf{i}}$. For $y \gg a$, $\vec{E} \simeq -\frac{1}{4\pi\epsilon_0} \frac{2qa}{y^3} \hat{\mathbf{i}}$, opposite to $\vec{p}$. (ii) The work done by the external agent is $-5.85 \times 10^{-7} \text{ J}$; the electric field does the corresponding positive work.

Quick Tip to Solve: Identify $(0, y)$ as an equatorial point before using a dipole formula. Since the midpoint triangle halves every pair separation, $U_f = 2U_i$ and the external work equals U_i , including its sign.

OR

33(b). (i) Show that Gauss's theorem is consistent with Coulomb's law. Using it, derive an expression for the electric field due to a uniformly charged thin spherical shell of radius r at a point at a distance y from the centre of the shell such that (I) $y > r$, and (II) $y < r$.

(ii) A point charge of $+2 \text{ nC}$ is kept at the origin of a three-dimensional coordinate system. Find the type and magnitude of the charge which should be kept at $(0, 0, -6 \text{ m})$ so that the potential due to the system becomes zero at $(0, 0, 2 \text{ m})$. [5 marks]

Solution:

Concept:

- Gauss's theorem states $\oint_S \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the algebraic charge enclosed by the closed surface S .
- For a spherically symmetric charge distribution, the electric field is radial and its radial component E_r has the same value everywhere on a concentric sphere. Thus its flux is $E_r 4\pi y^2$.
- Coulomb's law gives the force on a test charge q_0 as $\vec{F} = q_0 \vec{E}$, with $\vec{E} = q\hat{r}/(4\pi\epsilon_0 s^2)$ for a point charge q at separation s .

- Electric potential is a scalar and obeys superposition: $V = k \sum_i q_i/d_i$, where $k = 1/(4\pi\epsilon_0)$, d_i is the distance from charge q_i to the observation point, and potential is zero at infinity.

Step 1: (i) Check Gauss's theorem against Coulomb's law Place a point charge q at the centre of a spherical Gaussian surface of radius s . Coulomb's law gives a radial field with constant radial component $q/(4\pi\epsilon_0 s^2)$ on this sphere. The outward area element is also radial, so the flux is

$$\Phi = \oint_S \vec{E} \cdot d\vec{A} = \frac{q}{4\pi\epsilon_0 s^2} \oint_S dA = \frac{q}{4\pi\epsilon_0 s^2} (4\pi s^2) = \frac{q}{\epsilon_0}.$$

This is precisely Gauss's theorem. Conversely, applying Gauss's theorem with spherical symmetry gives

$$E_r(4\pi s^2) = \frac{q}{\epsilon_0}, \quad \vec{F} = q_0 \vec{E} = \frac{qq_0}{4\pi\epsilon_0 s^2} \hat{r},$$

which is Coulomb's inverse-square force law, including attraction or repulsion through the sign of qq_0 . Thus the two laws give consistent results.

Step 2: (i)(I) Choose a Gaussian surface outside the shell Let the uniformly charged thin shell have total charge Q , radius r , and surface charge density $\sigma = Q/(4\pi r^2)$. For a point at distance $y > r$, choose a concentric spherical Gaussian surface of radius y . Symmetry eliminates tangential components and makes E_r constant on this surface. It encloses the entire shell charge, so

$$\oint_S \vec{E} \cdot d\vec{A} = E_r(4\pi y^2) = \frac{Q}{\epsilon_0},$$

$$\vec{E}(y) = \frac{Q}{4\pi\epsilon_0 y^2} \hat{r} = \frac{\sigma r^2}{\epsilon_0 y^2} \hat{r}, \quad y > r.$$

Here $\hat{r}$ points away from the centre. The field is outward for $Q > 0$ and inward for $Q < 0$. Outside the shell, its field is the same as that of a point charge Q placed at its centre.

Step 3: (i)(II) Choose a Gaussian surface inside the shell For $0 < y < r$, take a concentric Gaussian sphere of radius y . All charge lies on the shell at radius r , so this Gaussian sphere encloses no charge. Spherical symmetry still makes the radial component constant over the Gaussian surface; hence

$$E_r(4\pi y^2) = \frac{0}{\epsilon_0} = 0 \quad \Rightarrow \quad \vec{E}(y) = \vec{0}, \quad y < r.$$

The field at the centre is also zero by symmetry. The inference uses both zero enclosed charge and spherical symmetry: zero net flux alone would not generally imply zero field.

Step 4: (ii) Find both distances to the zero-potential point Write the observation point as $P = (0, 0, 2 \text{ m})$. The known charge $q_1 = +2 \text{ nC}$ is at the origin, whereas the unknown charge q_2 is at $A = (0, 0, -6 \text{ m})$. Their distances from P are

$$d_1 = |2 - 0| = 2 \text{ m}, \quad d_2 = |2 - (-6)| = 8 \text{ m}.$$

The second denominator is 8 m, because the potential is evaluated at P ; the distance of A from the origin is not the required distance.

Step 5: (ii) Apply scalar superposition of potential The zero-potential condition is

$$\begin{aligned}V_P &= k \left(\frac{q_1}{d_1} + \frac{q_2}{d_2} \right) = 0, \\ \frac{2 \times 10^{-9} \text{ C}}{2 \text{ m}} + \frac{q_2}{8 \text{ m}} &= 0, \\ q_2 &= -(2 \times 10^{-9} \text{ C}) \frac{8}{2} = -8 \times 10^{-9} \text{ C}.\end{aligned}$$

The charge must be negative so that its potential cancels the positive potential of q_1 . As a check, using $k = 9 \times 10^9 \text{ N m}^2\text{C}^{-2}$, the contributions at P are $+9 \text{ V}$ and -9 V , respectively.

Step 6: Conclusion Gauss's theorem reproduces Coulomb's law. For a shell of total charge Q , $\vec{E} = Q\hat{r}/(4\pi\epsilon_0 y^2)$ when $y > r$, and $\vec{E} = \vec{0}$ when $y < r$. The required charge at $(0, 0, -6 \text{ m})$ is negative, of magnitude 8 nC , i.e. $q_2 = -8 \text{ nC}$.

Quick Tip to Solve: Use a concentric Gaussian sphere for the shell. For zero potential from two charges, use $q_2 = -q_1 d_2/d_1$, measuring both distances from the specified observation point.