

CBSE Class 12 Physics Question Paper 2025 with Solution PDF-Set 55/4/3



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – **Sections A, B, C, D** and **E**.
- (iii) In **Section A** – Questions no. **1** to **16** are Multiple Choice type questions. Each question carries **1** mark.
- (iv) In **Section B** – Questions no. **17** to **21** are Very Short Answer type questions. Each question carries **2** marks.
- (v) In **Section C** – Questions no. **22** to **28** are Short Answer type questions. Each question carries **3** marks.
- (vi) In **Section D** – Questions no. **29** and **30** are case study-based questions. Each question carries **4** marks.
- (vii) In **Section E** – Questions no. **31** to **33** are Long Answer type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is **not** allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. Two point charges Q and $-q$ are held ' r ' distance apart in free space. A uniform electric field $\vec{E}$ is applied in the region perpendicular to the line joining the two charges. Which one of the following angles will the direction of the net force acting on charge $-q$ make with the line joining Q and $-q$? [1 mark]

(A) $\tan^{-1}\left(\frac{4\pi\epsilon_0 Er^2}{Q}\right)$

(B) $\cot^{-1}\left(\frac{4\pi\epsilon_0 Er^2}{Q}\right)$

(C) $\tan^{-1}\left(\frac{QE}{4\pi\epsilon_0 r^2}\right)$

(D) $\cot^{-1}\left(\frac{QE}{4\pi\epsilon_0 r^2}\right)$

Correct Answer: (A) $\tan^{-1}\left(\frac{4\pi\epsilon_0 Er^2}{Q}\right)$

Solution:

Concept:

- Coulomb's law gives the attractive force magnitude $F_C = Qq/(4\pi\epsilon_0 r^2)$, where r is the separation and ϵ_0 is the permittivity of free space.
- The force on charge $-q$ in the applied field has magnitude $F_E = qE$ and points opposite to the field. It is perpendicular to the line joining the charges.

Step 1: Resolve the two perpendicular forces The electrostatic attraction acts from $-q$ towards Q , along the joining line. The applied-field force acts perpendicular to this line. If θ is the angle of the resultant with the joining line towards Q , its adjacent component is F_C and its opposite component is F_E .

Step 2: Form the ratio of the components Divide the perpendicular component by the component along the joining line:

$$\tan \theta = \frac{F_E}{F_C} = \frac{qE}{Qq/(4\pi\epsilon_0 r^2)} = \frac{4\pi\epsilon_0 Er^2}{Q}.$$

The charge magnitude q cancels. Option (B) gives the complementary angle by using inverse cotangent of the same ratio. The arguments in options (C) and (D) do not equal the force ratio and are not dimensionless.

Step 3: Conclusion The required angle is $\theta = \tan^{-1}\left(\frac{4\pi\epsilon_0 Er^2}{Q}\right)$. Hence, the correct option is (A).

Quick Tip to Solve: For an angle measured from the joining line, place the perpendicular force qE in the numerator of $\tan \theta$.

2. Three wires A , B and C of the same material have lengths and area of cross-sections as $\left(2l, \frac{A}{2}\right)$, (l, A) and $\left(\frac{l}{2}, 2A\right)$, respectively. If the resistances of these wires are R_A , R_B and R_C respectively, then :

[1 mark]

(A) $R_A > R_B > R_C$

(B) $R_B > R_C > R_A$

(C) $R_C > R_A > R_B$

(D) $R_A > R_C > R_B$

Correct Answer: (A) $R_A > R_B > R_C$

Solution:

Concept:

- The resistance of a uniform wire is $R = \rho L/S$, where ρ is resistivity, L is length and S is cross-sectional area.
- For the same material under the same conditions, ρ is common. Resistance increases with length and decreases with cross-sectional area.

Step 1: Express each resistance using a common reference Let $R_0 = \rho l/A$. Substitute the given length and area for each wire:

$$R_A = \frac{\rho(2l)}{A/2} = 4R_0,$$

$$R_B = \frac{\rho l}{A} = R_0,$$

$$R_C = \frac{\rho(l/2)}{2A} = \frac{R_0}{4}.$$

Both changes in wire A increase its resistance; both changes in wire C reduce its resistance.

Step 2: Compare the three positive values Their ratio is $R_A : R_B : R_C = 16 : 4 : 1$. Options (B) and (C) incorrectly place a wire above A, even though A is longest and thinnest. Option (D) incorrectly places C above B, although C is shorter and has a larger area.

Step 3: Conclusion The resistances satisfy $R_A > R_B > R_C$. Hence, the correct option is (A).

Quick Tip to Solve: Compare the factor L/S : doubling length and halving area multiplies resistance by four, while the reverse divides it by four.

3. A particle of mass m and charge q moves along y -axis in a region in which a uniform magnetic field $\vec{B}$ is pointing along x -axis. The Lorentz force acting on the charge will point along : [1 mark]

- (A) x -axis
- (B) y -axis
- (C) z -axis
- (D) negative z -axis

Correct Answer: (D) negative z -axis

Solution:

Concept:

- The magnetic part of the Lorentz force is $\vec{F} = q(\vec{v} \times \vec{B})$, where q is charge, $\vec{v}$ is velocity and $\vec{B}$ is magnetic field.
- Cross products follow the cyclic order $\hat{i} \times \hat{j} = \hat{k}$. Reversing their order changes the sign, so $\hat{j} \times \hat{i} = -\hat{k}$.

Step 1: Write the velocity and field as vectors With the usual convention of the stated axes denoting their positive directions, write $\vec{v} = v\hat{j}$ and $\vec{B} = B\hat{i}$. The charge denoted by q is taken as positive in this question. The particle's mass does not enter the magnetic-force direction.

Step 2: Apply the cross product in the correct order Substituting the vectors gives

$$\vec{F} = qvB(\hat{j} \times \hat{i}) = -qvB\hat{k}.$$

The force is perpendicular to both velocity and magnetic field, excluding options (A) and (B). Option (C), the positive z -direction, results from reversing the cross-product order; a negative charge would also reverse the force, but the stated charge is q , not $-q$.

Step 3: Conclusion The force points along the negative z -axis. Hence, the correct option is (D).

Quick Tip to Solve: Keep velocity first in $\vec{v} \times \vec{B}$: going from y to x reverses the usual x, y, z cyclic order.

4. A bar magnet is initially at right angles to a uniform magnetic field. The magnet is rotated till the torque acting on it becomes one-half of its initial value. The angle through which the bar magnet is rotated is : [1 mark]

- (A) 30°
- (B) 45°
- (C) 60°
- (D) 75°

Correct Answer: (C) 60°

Solution:

Concept:

- The torque magnitude on a bar magnet in a uniform magnetic field is $\tau = MB \sin \theta$, where M is magnetic dipole moment and θ is its angle with the field.
- Torque is maximum when the magnet is perpendicular to the field. The angle turned is the change in orientation, not the final angle with the field.

Step 1: Relate the final torque to the initial torque Initially $\theta_i = 90^\circ$, so $\tau_i = MB$. For the final torque to be one-half of this value,

$$MB \sin \theta_f = \frac{MB}{2} \Rightarrow \sin \theta_f = \frac{1}{2}.$$

Rotating towards the field gives $\theta_f = 30^\circ$. Rotating towards the opposite direction gives 150° ; either is the same smallest rotation from the initial position.

Step 2: Calculate the change in orientation The angle turned from the initial perpendicular position is

$$\alpha = 90^\circ - 30^\circ = 60^\circ.$$

Option (A) confuses the final angle with the angle turned. For rotations of 45° and 75° , the torque fractions are $\cos 45^\circ$ and $\cos 75^\circ$, respectively; neither is $1/2$.

Step 3: Conclusion The bar magnet is rotated through 60° . Hence, the correct option is (C).

Quick Tip to Solve: Starting from maximum torque, use $\tau/\tau_{\max} = \cos \alpha$, where α is the rotation from the perpendicular position.

5. Which of the following substances has magnetic permeability less than that of free space ? [1 mark]

- (A) Sodium
- (B) Iron
- (C) Aluminium
- (D) Copper

Correct Answer: (D) Copper

Solution:

Concept:

- Magnetic permeability μ relates to susceptibility χ_m by $\mu = \mu_0(1 + \chi_m)$, where μ_0 is the permeability of free space.
- A diamagnetic substance has $\chi_m < 0$, so its relative permeability $\mu_r = \mu/\mu_0$ is slightly less than unity. Its induced magnetisation opposes the applied field.

Step 1: Identify the required magnetic behaviour The condition $\mu < \mu_0$ means $1 + \chi_m < 1$, or $\chi_m < 0$. Thus the question asks for a diamagnetic substance, rather than one whose magnetisation reinforces the applied magnetic field.

Step 2: Classify each listed substance Sodium and aluminium are paramagnetic and have a small positive susceptibility, so their permeabilities are slightly greater than μ_0 . Iron is ferromagnetic and has a much larger positive magnetic response. Copper is diamagnetic and has negative susceptibility. It alone satisfies the required inequality among the four options.

Step 3: Conclusion Copper has magnetic permeability less than that of free space. Hence, the correct option is (D).

Quick Tip to Solve: Convert $\mu < \mu_0$ to $\chi_m < 0$, then select the diamagnetic material: here it is copper.

6. The magnetic flux linked with a closed coil (in Wb) varies with time t (in s) as $\phi = 5t^2 + 4t - 2$. If the resistance of the circuit is $14\ \Omega$, the magnitude of induced current in the coil at $t = 1\text{ s}$ will be : [1 mark]

- (A) 0.5 A
- (B) 1.0 A
- (C) 1.5 A
- (D) 2.0 A

Correct Answer: (B) 1.0 A

Solution:

Concept:

- Faraday's law gives the induced emf as $\varepsilon = -d\phi/dt$, where ϕ is the magnetic flux linkage and t is time. The negative sign expresses Lenz's law.
- For a circuit of resistance R , the induced current magnitude is $I = |\varepsilon|/R$. Use the rate of change of flux, rather than the instantaneous flux itself.

Step 1: Differentiate the given flux The constant term does not contribute to induction. Differentiating the complete polynomial before substituting the time gives

$$\frac{d\phi}{dt} = \frac{d}{dt}(5t^2 + 4t - 2) = 10t + 4.$$

At $t = 1\text{ s}$, the rate of change is 14 Wb s^{-1} , so the emf magnitude is 14 V.

Step 2: Apply Ohm's law to the circuit Divide the emf magnitude by the specified resistance:

$$I = \frac{|\varepsilon|}{R} = \frac{14}{14} = 1.0\text{ A}.$$

The options 0.5 A, 1.5 A and 2.0 A would require emf magnitudes of 7 V, 21 V and 28 V, respectively; none matches the differentiated flux. The sign of the emf sets direction and does not make the requested magnitude negative.

Step 3: Conclusion The magnitude of the induced current at $t = 1\text{ s}$ is 1.0 A. Hence, the correct option is (B).

Quick Tip to Solve: Differentiate $5t^2 + 4t - 2$ first, insert $t = 1$, and divide the resulting emf magnitude by $14\ \Omega$.

7. In an electromagnetic wave travelling in free space, the amplitude of magnetic field is $6.0 \times 10^{-4} \text{ T}$. The amplitude of its electric field is : [1 mark]

- (A) $2 \times 10^4 \text{ V m}^{-1}$
- (B) $1.5 \times 10^{12} \text{ V m}^{-1}$
- (C) $1.8 \times 10^5 \text{ V m}^{-1}$
- (D) $0.3 \times 10^4 \text{ V m}^{-1}$

Correct Answer: (C) $1.8 \times 10^5 \text{ V m}^{-1}$

Solution:

Concept:

- For an electromagnetic wave in free space, the electric and magnetic field amplitudes obey $E_0/B_0 = c$, where $c = 3.0 \times 10^8 \text{ m s}^{-1}$ is the speed of light.
- Both E_0 and B_0 denote amplitudes, so they can be compared directly. No root-mean-square conversion factor is needed.

Step 1: Express the electric amplitude in terms of the magnetic amplitude Rearrange the free-space wave relation to obtain $E_0 = cB_0$. The magnetic field amplitude is already in SI units, $B_0 = 6.0 \times 10^{-4} \text{ T}$, and hence needs no further conversion.

Step 2: Multiply the coefficient and the power of ten Substitute the values, multiplying the coefficients and adding the exponents:

$$E_0 = (3.0 \times 10^8)(6.0 \times 10^{-4}) = 18 \times 10^4 = 1.8 \times 10^5 \text{ V m}^{-1}.$$

The unit is correct because $\text{T m s}^{-1} = \text{V m}^{-1}$. The coefficients in options (A) and (D) do not equal the required product. The enormous value in (B) is also inconsistent with multiplying by the small magnetic amplitude; dividing by B_0 would use the wrong rearrangement of the wave relation.

Step 3: Conclusion The electric field amplitude is $1.8 \times 10^5 \text{ V m}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $E_0 = cB_0$: multiply 3 by 6 and add exponents $8 + (-4) = 4$, then normalise 18×10^4 .

8. A long straight wire is held vertically and carries a steady current in upward direction. The shape of magnetic field lines produced by the current-carrying wire are : [1 mark]

- (A) horizontal straight lines directed radially out from the wire.
- (B) straight lines parallel to the current-carrying wire.
- (C) concentric horizontal circles around the wire.
- (D) coaxial helixes around the wire.

Correct Answer: (C) concentric horizontal circles around the wire.

Solution:

Concept:

- The magnetic field of a long straight current-carrying wire forms closed circles centred on the wire, in planes perpendicular to its length.
- The right-hand thumb rule gives the direction: point the right thumb along the current; the curled fingers point along the magnetic field.

Step 1: Relate the field plane to the wire The wire is vertical, so every plane perpendicular to it is horizontal. Cylindrical symmetry about the wire means that the field is tangential to a circle at each fixed distance from the wire. Thus the circles share the point where the wire passes through each horizontal plane as their centre.

Step 2: Check the alternatives For upward current the field runs anticlockwise when viewed from above. Radial straight lines in (A) point in the wrong direction, while (B) would place the field along the current. The helical curves in (D) would require a component along the wire, which this wire's field does not have.

Step 3: Conclusion The magnetic field lines are concentric horizontal circles around the wire. Hence, the correct option is (C).

Quick Tip to Solve: First draw a plane perpendicular to the wire; the field circles lie in that plane, while the thumb rule fixes their sense.

9. The magnification produced by a spherical mirror is -2.0 . The mirror used and the nature of the image formed will be :

[1 mark]

- (A) Convex and virtual
- (B) Concave and real
- (C) Concave and virtual
- (D) Convex and real

Correct Answer: (B) Concave and real

Solution:

Concept:

- Mirror magnification is $m = h_i/h_o = -v/u$, where h_i, h_o are the image and object heights and v, u are their signed distances from the mirror.
- For a real object, negative magnification denotes an inverted real image; positive magnification denotes an erect virtual image.

Step 1: Interpret the sign and magnitude Here $m = -2.0$. Its negative sign tells us that the image is inverted, and its magnitude 2.0 tells us that the image is twice the object's height. In the usual real-object arrangement for a spherical mirror, this is a real enlarged image.

Step 2: Identify the mirror A concave mirror can form this image when the real object is between its focus and centre of curvature. A convex mirror gives an erect, virtual, diminished image for a real object, so both convex-mirror alternatives fail. A concave virtual image is erect and has positive magnification, so (C) fails as well.

Step 3: Conclusion The mirror is concave and the image is real, inverted and enlarged. Hence, the correct option is (B).

Quick Tip to Solve: Read the sign of magnification before its size: a negative value identifies an inverted image, while its magnitude gives enlargement.

10. Choose the correct statement :

[1 mark]

- (A) Photons of light show diffraction whereas electrons do not show diffraction.
- (B) Electrons have momentum whereas photons do not have momentum.
- (C) Photons of light and electrons both exhibit dual nature.
- (D) All electromagnetic radiations do not have photons.

Correct Answer: (C) Photons of light and electrons both exhibit dual nature.

Solution:

Concept:

- Wave-particle duality means that radiation and matter show wave properties in some experiments and particle properties in others.
- A photon has energy $E = h\nu$ and momentum $p = h/\lambda$. A moving electron has de Broglie wavelength $\lambda = h/p$, where h is Planck's constant.

Step 1: Establish the dual behaviour Light shows interference and diffraction, which reveal wave behaviour, and the photoelectric effect reveals its transfer of energy in photons. Electrons possess particle properties such as charge and momentum, and electron diffraction demonstrates their wave behaviour.

Step 2: Test the other statements Option (A) is false because electrons also diffract. Option (B) is false because photons carry nonzero momentum even though their rest mass is zero. Option (D) is false because the photon description applies throughout the electromagnetic spectrum, not only to visible light.

Step 3: Conclusion Both photons of light and electrons exhibit dual nature. Hence, the correct option is (C).

Quick Tip to Solve: Use electron diffraction to reject (A), and $p = h/\lambda$ to reject the claim that a photon has no momentum.

11. A beam of red light and a beam of blue light have equal intensities. Which of the following statements is true ?

[1 mark]

- (A) The blue beam has more number of photons than the red beam.
- (B) The red beam has more number of photons than the blue beam.
- (C) Wavelength of red light is lesser than wavelength of blue light.
- (D) The blue light beam has lesser energy per photon than that in the red light beam.

Correct Answer: (B) The red beam has more number of photons than the blue beam.

Solution:

Concept:

- A photon's energy is $E = h\nu = hc/\lambda$, where ν is frequency, λ is wavelength, h is Planck's constant and c is the speed of light.
- Intensity is energy crossing unit area per unit time. If N photons cross area A in time t , then $I = NE/(At)$.

Step 1: Compare individual photon energies Red light has a longer wavelength and lower frequency than blue light. Therefore $E_{\text{red}} < E_{\text{blue}}$. Options (C) and (D) reverse these wavelength and energy

comparisons and are false.

Step 2: Compare photon numbers for equal intensities Compare the photons crossing equal areas in equal times. Equal intensities mean equal total energies, so fewer of the more energetic blue photons are needed. Quantitatively,

$$\frac{N_{\text{red}}}{N_{\text{blue}}} = \frac{E_{\text{blue}}}{E_{\text{red}}} = \frac{\lambda_{\text{red}}}{\lambda_{\text{blue}}} > 1.$$

Thus option (A), which assigns more photons to the blue beam, reverses the correct comparison.

Step 3: Conclusion The red beam has more photons per equal area and equal time than the blue beam. Hence, the correct option is (B).

Quick Tip to Solve: At fixed intensity, photon number flux is proportional to wavelength: lower-energy red photons must arrive in greater numbers.

12. Which of the following is an electrical conductor at room temperature?

[1 mark]

- (A) Sn
- (B) Mica
- (C) Si
- (D) C

Correct Answer: (A) Sn

Solution:

Concept:

- Metals conduct electricity because they possess mobile electrons. In band language, a partly filled band or overlapping bands provide readily accessible states for conduction.
- Silicon is a semiconductor and mica is an insulator. A material's classification concerns its available mobile charge carriers at the stated temperature.

Step 1: Identify the metallic conductor Sn is tin, which is a metal and an electrical conductor at room temperature. Its mobile conduction electrons produce a current under an applied electric field, making it the standard conductor among the listed choices.

Step 2: Compare the remaining materials Mica has very few mobile charge carriers and is insulating. Silicon has a small energy gap and belongs to the semiconductor class, so it is not classified as a metallic conductor. Carbon requires an allotrope qualification: diamond is insulating whereas graphite conducts. The option simply states C; in the standard classification intended here, tin is the definite metallic-conductor choice.

Step 3: Conclusion Tin, Sn, is the intended electrical conductor at room temperature. Hence, the correct option is (A).

Quick Tip to Solve: Recognise Sn as tin, a metal; do not mistake the similar-looking symbol Si, silicon, for it.

13. Assertion (A): In double slit experiment if one slit is closed, diffraction pattern due to the other slit will appear on the screen.

Reason (R): For interference, at least two waves are required.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- A slit of finite width produces diffraction because wavelets from different points across the same slit superpose with direction-dependent phase differences.
- Interference requires superposition of at least two waves; a stable two-slit interference pattern requires coherent waves from the two slits.

Step 1: Evaluate the assertion Closing one slit removes the interference between the two slit beams. Light still passes through the remaining slit of finite width, and wavelets from different parts of that slit produce its single-slit diffraction pattern on the screen. Assertion (A) is therefore true.

Step 2: Evaluate the reason Interference is the redistribution of intensity due to superposition of waves. At least two waves are needed for it, so Reason (R) is true. Consequently the codes declaring the reason false, (C) and (D), do not apply.

Step 3: Check the explanatory link The reason explains why the usual two-slit interference contribution disappears after one slit is closed. It does not explain why the surviving slit produces diffraction: that requires superposition of secondary wavelets from different points across its finite width. Thus the reason is not the correct explanation of the assertion, which rules out (A).

Step 4: Conclusion Both statements are true, but the reason does not correctly explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Distinguish loss of interference between two slits from diffraction by one finite slit; the latter involves wavelets within that slit.

14. Assertion (A): For monochromatic incident radiation, the emitted photoelectrons from a given metal have speed ranging from zero to a certain maximum value.

Reason (R): Each metal has a definite work function.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = \frac{1}{2}m_e v_{\max}^2$, where ν is the incident frequency, ϕ the work function and m_e the electron mass.
- The work function is the minimum energy needed to remove an electron from the metal surface; it determines the maximum kinetic energy, not the kinetic energy of every emitted electron.

Step 1: Evaluate the assertion For incident radiation above the threshold frequency, emitted electrons have a range of kinetic energies, from nearly zero up to $K_{\max}$. Electrons can originate at different depths and lose different amounts of energy before escaping. Thus monochromatic light does not make all photoelectrons equally fast, and Assertion (A) is true.

Step 2: Evaluate the reason For a given metal and surface condition, the work function has a definite value. It is the minimum energy required for an electron to escape from that surface, so Reason (R) is also true. Options (C) and (D), which state that the reason is false, are excluded.

Step 3: Check the explanatory link A definite work function together with the fixed incident frequency gives the upper limit $K_{\max} = h\nu - \phi$. It does not explain the spread of energies below this limit. That spread arises from the differing circumstances and energy losses of escaping electrons. Therefore the reason does not correctly explain the assertion, excluding (A).

Step 4: Conclusion Both statements are true, but the reason does not correctly explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Einstein's equation predicts the maximum photoelectron speed; differing losses before escape explain the speeds below that maximum.

15. Assertion (A): n-type semiconductor is not negatively charged.

Reason (R): Neutral pentavalent impurity atom doped in intrinsic semiconductor (neutral) donates its fifth unpaired electron to the crystal lattice and becomes a positive donor. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- An n-type semiconductor has electrons as majority carriers, but majority-carrier type does not specify the net charge of the crystal.
- A neutral donor atom releases one conduction electron and becomes a fixed positive donor ion; these charges balance.

Step 1: Evaluate the assertion Doping a neutral intrinsic semiconductor with neutral pentavalent atoms does not add net charge. Although the mobile electron concentration rises, the specimen remains electrically neutral. Thus Assertion (A) is true.

Step 2: Evaluate the reason Four valence electrons of a pentavalent dopant participate in covalent bonds with neighbouring semiconductor atoms. The weakly bound fifth electron becomes available for conduction, leaving a positively charged donor ion at its lattice site. Thus Reason (R) is true.

Step 3: Check the explanatory connection Each released electron of charge $-e$ is balanced by a donor ion of charge $+e$. The reason therefore explains why the increased population of negative mobile carriers does not make the crystal negatively charged. Option (B) incorrectly denies this connection, while (C) and (D) incorrectly declare true statements false.

Step 4: Conclusion Both Assertion (A) and Reason (R) are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: When checking neutrality after doping, count fixed ion charges as well as mobile electron and hole charges.

16. Assertion (A): A series LCR circuit behaves as a pure resistive circuit at resonance.

Reason (R): At resonance, $X_L = X_C$ gives $\omega = \frac{1}{\sqrt{LC}}$. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- In a series LCR circuit, the net reactance is $X_L - X_C$, with $X_L = \omega L$ and $X_C = 1/(\omega C)$.
- The impedance magnitude is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and the phase angle obeys $\tan \phi = (X_L - X_C)/R$.

Step 1: Evaluate the assertion At resonance, inductive and capacitive reactances are equal. Their voltage contributions are opposite in phase and cancel in the total applied voltage. Consequently $Z = R$ and $\phi = 0$: the applied voltage and current are in phase. Assertion (A) is true.

Step 2: Evaluate the reason Equating the two reactances gives the resonance angular frequency:

$$\omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}.$$

Hence Reason (R) is true; L and C are the inductance and capacitance.

Step 3: Link the reason to the circuit behaviour The equality $X_L = X_C$ stated in the reason directly makes the net reactance zero, explaining the purely resistive response. Option (B) overlooks this cancellation. Options (C) and (D) cannot apply because the reason and assertion are both true.

Step 4: Conclusion Both Assertion (A) and Reason (R) are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: At series resonance, set the difference of reactances to zero; the individual reactances need not vanish.

Section B (10 marks)

17. In an intrinsic semiconductor, carrier's concentration is $5 \times 10^8 \text{ m}^{-3}$. On doping with impurity atoms, the hole concentration becomes $8 \times 10^{12} \text{ m}^{-3}$.

(a) Identify (i) the type of dopant and (ii) the extrinsic semiconductor so formed.

(b) Calculate the electron concentration in the extrinsic semiconductor.

[2 marks]

Solution:

Concept:

- A trivalent acceptor impurity creates holes in a tetravalent semiconductor, forming a p-type semiconductor with holes as majority carriers.
- At thermal equilibrium and unchanged temperature, the mass-action law is $np = n_i^2$, where n , p and n_i are the electron, hole and intrinsic carrier concentrations.

Step 1: Identify the dopant and semiconductor in part (a) Initially the intrinsic electron and hole concentrations are equal: $n = p = n_i = 5 \times 10^8 \text{ m}^{-3}$. Doping raises the hole concentration to $8 \times 10^{12} \text{ m}^{-3}$, much greater than n_i . Thus holes become the majority carriers. The dopant is trivalent (an acceptor, such as boron), and the resulting extrinsic semiconductor is p-type.

Step 2: Apply the mass-action law for part (b) The product of the two equilibrium carrier concentrations stays equal to n_i^2 at the same temperature. Therefore the minority electron concentration is

$$\begin{aligned} n &= \frac{n_i^2}{p} = \frac{(5 \times 10^8)^2}{8 \times 10^{12}} \text{ m}^{-3} \\ &= \frac{25 \times 10^{16}}{8 \times 10^{12}} \text{ m}^{-3} = 3.125 \times 10^4 \text{ m}^{-3}. \end{aligned}$$

This is smaller than the intrinsic electron concentration, as expected when acceptor doping increases holes.

Step 3: Conclusion (a) The dopant is trivalent (acceptor) and the extrinsic semiconductor is p-type. (b) The electron concentration is $3.125 \times 10^4 \text{ m}^{-3}$.

Quick Tip to Solve: Square the intrinsic concentration before dividing by the majority-carrier concentration; do not assume electrons and holes stay equal after doping.

18. In Young's double slit experiment, the screen is moved 30 cm towards the slits. As a consequence, the fringe width of the pattern changes by 0.09 mm. If the slits separation used is 2 mm, calculate the wavelength of light used in the experiment.

[2 marks]

Solution:

Concept:

- In Young's double-slit experiment, the fringe width is $\beta = \lambda D/d$, where λ is wavelength, D is slit-to-screen distance and d is slit separation.
- For fixed λ and d , fringe width is proportional to D . Moving the screen towards the slits decreases the fringe width.

Step 1: Relate the two changes Let the original screen distance be D , and let $s = 0.30 \text{ m}$ be the distance through which it moves towards the slits. The new distance is $D - s$. Subtracting the new

fringe width from the original width eliminates the unknown D :

$$\begin{aligned}\beta_1 - \beta_2 &= \frac{\lambda D}{d} - \frac{\lambda(D-s)}{d} \\ &= \frac{\lambda s}{d}.\end{aligned}$$

The given 0.09 mm is the magnitude of this decrease, so $\lambda = d(\beta_1 - \beta_2)/s$.

Step 2: Convert to SI units and substitute Use $d = 2 \times 10^{-3}$ m, $\beta_1 - \beta_2 = 9 \times 10^{-5}$ m, and $s = 0.30$ m:

$$\lambda = \frac{(2 \times 10^{-3})(9 \times 10^{-5})}{0.30} \text{ m} = 6.0 \times 10^{-7} \text{ m}.$$

The ratio of the two lengths $(\beta_1 - \beta_2)/s$ is dimensionless, leaving wavelength in metres.

Step 3: Conclusion The wavelength of the light is $\lambda = 6.0 \times 10^{-7}$ m = 600 nm.

Quick Tip to Solve: Subtract the two fringe-width equations to remove the unknown original screen distance, then use consistent units for both changes.

19. The two surfaces of a biconvex lens are of radius of curvature ' R ' each. Obtain the condition under which its focal length ' f ' be equal to ' R '. If one of the two surfaces of this lens is made plane, what will be the new focal length of the lens ? [2 marks]

Solution:

Concept:

- For a thin lens in air, the lens-maker's formula is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is the refractive index of the lens material relative to air.
- With light travelling from left to right, a symmetric biconvex lens has $R_1 = +R$ and $R_2 = -R$. A plane surface has infinite radius and contributes zero to the curvature term.

Step 1: Find the condition for the original lens Substituting the signed radii into the lens-maker's formula gives

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{2(\mu - 1)}{R}.$$

The specified condition $f = R$ then requires

$$\frac{1}{R} = \frac{2(\mu - 1)}{R} \Rightarrow \mu = \frac{3}{2} = 1.5.$$

Thus it is the refractive index relative to the surrounding air that must have this value.

Step 2: Replace one curved surface by a plane surface The modified lens is plano-convex, with the same material and remaining curved radius R . Taking $R_1 = R$ and $R_2 = \infty$, its focal length f' satisfies

$$\frac{1}{f'} = (\mu - 1) \left(\frac{1}{R} - 0 \right) = \frac{1}{2R}.$$

The curvature contribution, and hence the optical power, has been halved. Reversing this thin plano-convex lens would give the same focal length.

Step 3: Conclusion The required refractive index is $\mu = 1.5$. After one surface is made plane, the new focal length is $f' = 2R = 2f$.

Quick Tip to Solve: Use opposite signs for the two biconvex radii; flattening either surface removes half the power of a symmetric thin lens.

20. In Bohr's model of hydrogen atom, find the percentage change in the radius of its orbit when an electron makes a transition from $n = 3$ state to $n = 2$ state. [2 marks]

Solution:

Concept:

- In Bohr's model, the radius of the n th allowed orbit of hydrogen is $r_n = a_0 n^2$, where a_0 is the radius of its first orbit and n is the principal quantum number.
- Percentage change is measured relative to the initial radius: $\frac{r_f - r_i}{r_i} \times 100\%$. A negative result means that the orbit becomes smaller.

Step 1: Express the initial and final radii The electron starts in the $n = 3$ state and ends in the $n = 2$ state. Therefore,

$$r_i = r_3 = 9a_0, \quad r_f = r_2 = 4a_0.$$

The radius depends on the square of the quantum number, so its ratio is $r_f/r_i = 4/9$, not $2/3$. The common constant a_0 cancels, and its numerical value is unnecessary.

Step 2: Calculate the fractional change Subtract the initial radius from the final radius and divide by the initial value:

$$\begin{aligned} \frac{\Delta r}{r_i} \times 100\% &= \frac{4a_0 - 9a_0}{9a_0} \times 100\% \\ &= -\frac{5}{9} \times 100\% = -55.56\%. \end{aligned}$$

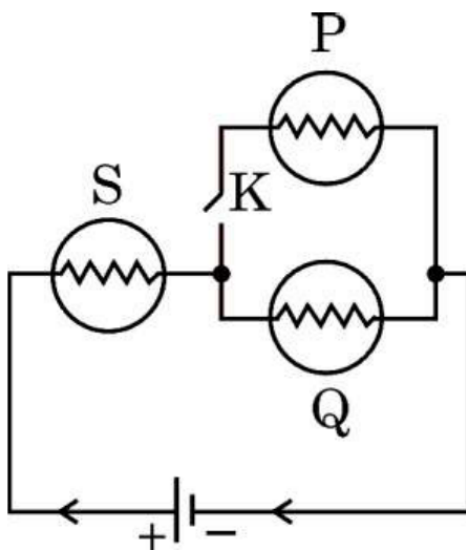
The minus sign is consistent with a transition to a lower orbit. Using the final radius in the denominator would describe a different comparison and would not give the requested change from the original orbit.

Step 3: Conclusion The percentage change in orbital radius is -55.56% ; hence the radius **decreases by approximately 55.6%**.

Quick Tip to Solve: For a Bohr-orbit transition, use $100[(n_f/n_i)^2 - 1]\%$, always keeping the initial quantum number in the denominator.

21(a). In the given figure, three identical bulbs P , Q and S are connected to a battery.
 (i) Compare the brightness of bulbs P and Q with that of bulb S when key K is closed.
 (ii) Compare the brightness of the bulbs S and Q when the key K is opened.
 Justify your answer in both cases.

[2 marks]



Solution:

Concept:

- The brightness of an identical bulb is compared through the electrical power it dissipates. For bulb resistance R and current I , this power is $\mathcal{P} = I^2 R$.
- Equal resistances connected in parallel carry equal currents. In a series path, the same current passes through every element; at a junction the incoming current equals the sum of the outgoing currents.

Step 1: Compare the bulbs with the key closed When K is closed, P and Q form two identical parallel branches, and their combination is in series with S . Let each parallel branch carry current i . Then the current through S is $i + i = 2i$. Thus,

$$\mathcal{P}_P = \mathcal{P}_Q = i^2 R, \quad \mathcal{P}_S = (2i)^2 R = 4i^2 R.$$

Bulbs P and Q therefore glow equally brightly, while S is brighter than either one. The dissipated-power ratio is $\mathcal{P}_P : \mathcal{P}_Q : \mathcal{P}_S = 1 : 1 : 4$, using the common resistance of the identical bulbs.

Step 2: Trace the conducting path with the key open Opening K breaks only the branch containing P , so P carries no current and does not glow. Bulbs S and Q still form a complete series circuit with the battery. They carry the same current I' , and their resistances are equal:

$$\mathcal{P}_S = I'^2 R = \mathcal{P}_Q.$$

Consequently, S and Q have equal brightness when the key is open. Opening this key does not break

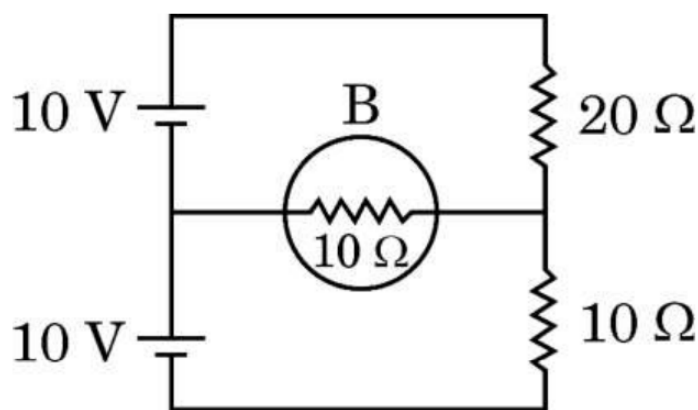
the entire battery circuit.

Step 3: Conclusion With K closed, P and Q are equally bright and each is dimmer than S . With K open, S and Q are equally bright, while P is off.

Quick Tip to Solve: Redraw the conducting branches for each key position: S carries the sum of the two branch currents when the key is closed, but only the current through Q when it is open.

OR

21(b). Two cells of emf 10 V each, two resistors of $20\ \Omega$ and $10\ \Omega$ and a bulb B of $10\ \Omega$ resistance are connected together as shown in the figure. Find the current that flows through the bulb. [2 marks]



Solution:

Concept:

- Kirchhoff's junction rule states that the algebraic sum of currents at a junction is zero. A resistor of resistance R between potentials V_1 and V_2 carries current $(V_1 - V_2)/R$ from the first node towards the second.
- Across an ideal cell, potential rises by its emf on moving from the short plate (negative terminal) to the long plate (positive terminal). The two 10 V cells shown have their positive terminals upwards.

Step 1: Assign the node potentials Take the bottom connecting wire as the zero-potential reference. Moving upwards through the lower cell gives 10 V at the cells' midpoint, which is the left end of the bulb. Moving upwards through the upper cell gives 20 V at the top wire. Let V be the potential at the right end of the bulb, where the two resistors meet. The bulb has resistance $10\ \Omega$.

Step 2: Apply the junction rule at the resistor midpoint Express each branch current as flowing out of the junction of potential V . The three branches lead to the top wire, bottom wire and cells' midpoint, respectively:

$$\frac{V - 20\text{ V}}{20\ \Omega} + \frac{V}{10\ \Omega} + \frac{V - 10\text{ V}}{10\ \Omega} = 0.$$

Multiplying by $20\ \Omega$ and simplifying gives

$$V - 20\text{ V} + 2V + 2(V - 10\text{ V}) = 0, \quad V = 8\text{ V}.$$

The bulb's left terminal is therefore at the higher potential. Its current magnitude is

$$I_B = \frac{10\text{ V} - 8\text{ V}}{10\ \Omega} = 0.20\text{ A}.$$

As a check, the upper resistor supplies 0.60 A to the junction and the lower resistor takes 0.80 A; the remaining 0.20 A enters through the bulb, satisfying charge conservation.

Step 3: Conclusion The current through bulb *B* is 0.20 A, flowing from left to right, from the cells' midpoint towards the junction of the two resistors.

Quick Tip to Solve: Set the bottom wire to zero and use the cell polarities to fix the left-side potentials; one junction equation then determines the current through the connecting bulb.

Section C (21 marks)

22. (a) What are majority and minority charge carriers of p-type and n-type semiconductors ?

(b) Explain briefly the formation of diffusion current and drift current in a p-n junction diode.

[3 marks]

Solution:

Concept:

- Majority carriers are the more numerous mobile charge carriers in a doped semiconductor; minority carriers are the less numerous mobile carriers. Electrons carry negative charge, while holes act as positive carriers.
- Diffusion is carrier motion from higher to lower concentration. Drift is carrier motion under an electric field. Conventional current is along hole motion and opposite to electron motion.

Step 1: Identify the charge carriers in each semiconductor (a) In a **p-type semiconductor**, acceptor doping produces a large concentration of holes, so **holes are majority carriers** and **electrons are minority carriers**. In an **n-type semiconductor**, donor doping supplies many free electrons, so **electrons are majority carriers** and **holes are minority carriers**. Thermal generation provides minority carriers in both materials. These statements describe mobile carriers; the ionised dopant atoms remain fixed in the crystal lattice.

Step 2: Explain the formation of diffusion current (b) When the p-side and n-side form a junction, their different carrier concentrations cause majority electrons to diffuse from *n* to *p*, while majority holes diffuse from *p* to *n*. Both motions produce conventional current from *p* to *n*; this is the **diffusion current**. Near the junction, electrons and holes recombine. Their removal leaves immobile positive donor ions on the n-side and negative acceptor ions on the p-side, forming the depletion region.

Step 3: Explain the formation of drift current The exposed ions produce a built-in electric field directed from the positive n-side towards the negative p-side. This field opposes the continued diffusion of majority carriers. It also sweeps thermally generated minority electrons from *p* to *n* and minority holes from *n* to *p*. Both motions give conventional current from *n* to *p*, called the **drift current**. Thus drift current is driven by the junction electric field, whereas diffusion current is driven by the concentration difference. In an unbiased junction at thermal equilibrium, the two currents are equal in magnitude and opposite in direction, giving zero net current.

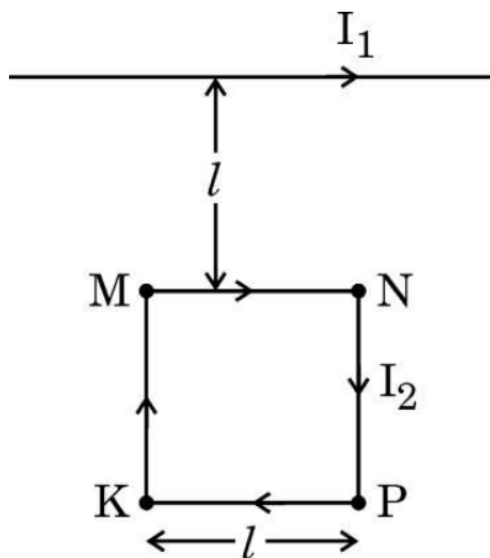
Step 4: Conclusion P-type material has majority holes and minority electrons; n-type material has majority electrons and minority holes. Diffusion current flows conventionally from *p* to *n*, while the

built-in field produces drift current from n to p . They balance at thermal equilibrium.

Quick Tip to Solve: First write the actual motion of electrons and holes; reverse only the electron direction to obtain conventional current. Diffusion follows concentration differences, while drift follows the junction electric field.

23. (a) In which cases does a charged particle not experience a force in a magnetic field ?

(b) A square loop $MNPK$ of side ' l ' carrying a current ' I_2 ' is kept close to a long straight wire in the same plane and the wire carries a steady current I_1 , as shown in the figure. Obtain the magnitude of magnetic force exerted by the wire on the loop. [3 marks]



Solution:

Concept:

- The magnetic force on a charge q moving with velocity $\vec{v}$ is $\vec{F} = q\vec{v} \times \vec{B}$, of magnitude $|q|vB \sin \theta$, where θ is the angle between velocity and magnetic field.
- A long straight wire carrying current I_1 produces $B(r) = \mu_0 I_1 / (2\pi r)$ at perpendicular distance r . A perpendicular wire segment of length l , carrying current I_2 , experiences force $I_2 l B$.

Step 1: (a) Identify when the magnetic force vanishes In a nonzero magnetic field, a charged particle experiences no magnetic force if it is at rest, because $v = 0$. A moving charged particle also experiences no magnetic force when it travels parallel or antiparallel to the field: $\theta = 0^\circ$ or 180° , so $\sin \theta = 0$. Thus it is the velocity component perpendicular to the magnetic field that determines the magnetic force.

Step 2: (b) Find the forces on the parallel sides In the printed figure, side MN is a distance l below the straight wire, whereas side KP is a distance $2l$ below it. Each side has length l . The currents in the straight wire and MN have the same direction, so MN is attracted towards the wire. The current in PK is opposite to I_1 , so that side is repelled away from the wire. Their force magnitudes are

$$F_{MN} = I_2 l \frac{\mu_0 I_1}{2\pi l} = \frac{\mu_0 I_1 I_2}{2\pi},$$
$$F_{PK} = I_2 l \frac{\mu_0 I_1}{2\pi(2l)} = \frac{\mu_0 I_1 I_2}{4\pi}.$$

Step 3: Cancel the forces on the other two sides The magnetic field below the straight wire points into the page. At each distance r from the wire, equal elements of sides KM and NP experience equal magnetic fields. Their currents have opposite directions, giving equal and opposite horizontal forces. These cancel pairwise. The two remaining vertical forces oppose each other, and their difference is

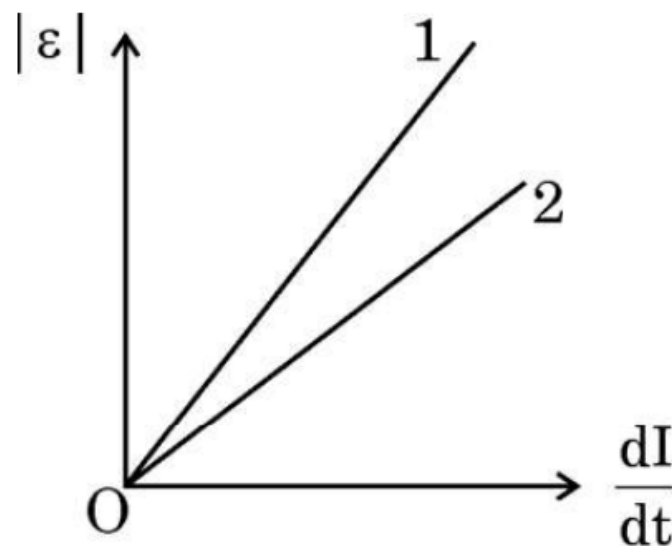
$$F_{\text{net}} = \frac{\mu_0 I_1 I_2}{2\pi} - \frac{\mu_0 I_1 I_2}{4\pi}.$$

Step 4: Conclusion (a) There is no magnetic force for a stationary charge or a charge moving parallel or antiparallel to the field. (b) The loop experiences a net force of magnitude $\frac{\mu_0 I_1 I_2}{4\pi}$, directed towards the straight wire because the nearer parallel side is attracted more strongly.

Quick Tip to Solve: Read both distances from the figure: the near side is at l , the far side at $2l$. Cancel the transverse sides and subtract the two parallel-side forces.

24. (a) Obtain an expression for the self-inductance of a long solenoid of length ' l ', cross-sectional area ' A ' and having ' N ' turns.

(b) The figure shows the plot of magnitude of induced emf (ε) versus the rate of change of current in two coils '1' and '2'. Which coil has greater value of self-inductance and why? [3 marks]



Solution:

Concept:

- For a long air-core solenoid, $B = \mu_0 n I$, where $n = N/l$ is the number of turns per unit length, I is current and μ_0 is the permeability of free space.
- Self-inductance L relates flux linkage to current by $N\Phi = LI$. The induced emf obeys $\varepsilon = -L dI/dt$; its magnitude is $|\varepsilon| = L|dI/dt|$.

Step 1: (a) Calculate the flux through one turn Let a current I flow in the N turns of the solenoid. For a long solenoid, end effects are negligible and the magnetic field is approximately uniform inside and directed along its axis. The plane of each turn is perpendicular to this field. With no core material

specified, take the interior to be air or vacuum. Hence

$$B = \mu_0 \frac{N}{l} I, \quad \Phi = BA = \frac{\mu_0 N I A}{l}.$$

Here A is the cross-sectional area enclosed by each turn; Φ is the magnetic flux through one turn.

Step 2: Relate the total flux linkage to current All turns link approximately the same flux, so the total flux linkage is $N\Phi$, rather than merely the flux through one turn. Substituting the expression above in the definition gives

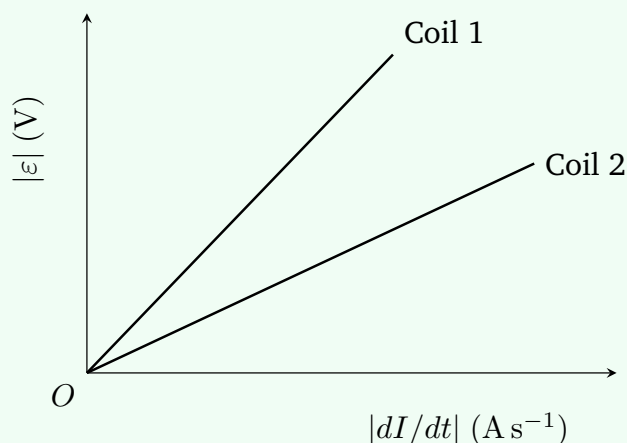
$$LI = N\Phi = N \left(\frac{\mu_0 N I A}{l} \right), \quad L = \frac{N\Phi}{I} = \frac{\mu_0 N^2 A}{l}.$$

The SI unit of L is henry. For a uniformly filled linear magnetic core, the same derivation uses its permeability μ in place of μ_0 .

Step 3: (b) Compare the slopes of the two lines The graph plots induced-emf magnitude vertically against the positive rate of change of current horizontally. From $|\varepsilon| = L|dI/dt|$, the gradient of each straight line is

$$\text{slope} = \frac{\Delta|\varepsilon|}{\Delta|dI/dt|} = L.$$

In the printed graph, line 1 is steeper than line 2. At the same rate of current change, coil 1 therefore produces a larger induced-emf magnitude. This directly compares the coefficients of self-induction without needing numerical axis values.



The steeper line for coil 1 shows $L_1 > L_2$.

Step 4: Conclusion (a) The self-inductance of the long air-core solenoid is $L = \mu_0 N^2 A / l$. (b) $L_1 > L_2$: coil 1 has greater self-inductance because its emf versus current-change-rate graph has the greater slope.

Quick Tip to Solve: Multiply the flux per turn by N before dividing by current. On an induced-emf versus current-change-rate graph, the steeper line represents the larger inductance.

25. Explain the following observations using Einstein's photoelectric equation :

(a) Photoelectric emission does not occur from a surface when the frequency of the light incident on it is less than a certain minimum value.

(b) It is the frequency, and not the intensity of the incident light which affects the maximum kinetic energy of the photoelectrons.

(c) The cut-off voltage (V_0) versus frequency (ν) of the incident light curve is a straight line with a slope $\frac{h}{e}$.

[3 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $h\nu = \phi + K_{\max}$, where $h\nu$ is the energy of one incident photon, ϕ is the metal's work function and $K_{\max}$ is the maximum photoelectron kinetic energy.
- The threshold frequency is $\nu_0 = \phi/h$. The positive magnitude of the cut-off or stopping voltage satisfies $eV_0 = K_{\max}$, where e is the magnitude of the electron charge.

Step 1: (a) Apply the minimum energy condition In the photoelectric process, an electron absorbs the energy of a single photon. It first needs energy ϕ to escape from the metal surface. For emission to occur, the remaining kinetic energy must be nonnegative:

$$K_{\max} = h\nu - \phi \geq 0 \quad \Rightarrow \quad \nu \geq \frac{\phi}{h} = \nu_0.$$

If $\nu < \nu_0$, an individual photon supplies insufficient energy to release an electron. Increasing the number of such photons does not increase the energy of any one photon, so increasing intensity cannot compensate for a frequency below the threshold in the usual photoelectric effect.

Step 2: (b) Separate photon energy from photon number For a fixed metal, ϕ is constant. Einstein's equation gives

$$K_{\max} = h\nu - \phi.$$

Increasing frequency raises the energy supplied by each photon and therefore raises the maximum electron kinetic energy. Intensity at a fixed frequency changes the number of photons arriving per unit area per unit time, while each still has energy $h\nu$. Above threshold, greater intensity increases the number of emitted electrons and the saturation photocurrent; it does not change $K_{\max}$.

Step 3: (c) Express the stopping voltage as a function of frequency At the cut-off voltage, even the most energetic photoelectrons are just stopped by the opposing electric field. Equating their maximum kinetic energy to the electric work gives

$$\begin{aligned} eV_0 &= h\nu - \phi, \\ V_0 &= \frac{h}{e}\nu - \frac{\phi}{e} = \frac{h}{e}(\nu - \nu_0). \end{aligned}$$

This has the straight-line form $y = mx + c$, with slope h/e . The line meets the frequency axis at ν_0 ; its extrapolated voltage intercept is $-\phi/e$. The physical stopping-voltage relation applies where photoemission occurs, $\nu \geq \nu_0$.

Step 4: Conclusion (a) No emission occurs below $\nu_0 = \phi/h$ because each photon lacks the escape energy. (b) $K_{\max}$ depends on frequency through $h\nu - \phi$, whereas intensity controls photon number. (c) $V_0 = (h/e)\nu - \phi/e$ is a straight line of slope $\boxed{h/e}$.

Quick Tip to Solve: Start all three explanations from $h\nu = \phi + K_{\max}$; for the voltage graph, replace $K_{\max}$ by eV_0 and compare with $y = mx + c$.

26. (a) Write the conditions under which two light waves originating from two coherent sources can interfere each other (i) constructively, and (ii) destructively, in terms of wavelength. Can these be applied for two lights originating from two sodium lamps? Give reason.

(b) Monochromatic light of green colour is used in Young's double slit experiment and an interference pattern is observed on a screen. If the green light is replaced by red monochromatic light of the same intensity, how will the fringe width of interference pattern be affected? Justify your answer. [3 marks]

Solution:

Concept:

- Coherent waves have the same frequency and a constant phase difference. Interference depends on the total phase difference at the point of observation.
- A path difference Δ introduces a phase difference $2\pi\Delta/\lambda$, where λ is the wavelength.
- In Young's double-slit experiment, fringe width is $\beta = \lambda D/d$, where D is the slit-to-screen distance and d is the slit separation.

Step 1: State the constructive and destructive conditions (a) For waves emitted in phase, constructive interference occurs when their path difference is an integral multiple of the wavelength:

$$\Delta = n\lambda, \quad n = 0, \pm 1, \pm 2, \dots$$

Their crests and troughs arrive together, so the resultant intensity is maximum. Destructive interference occurs for an odd multiple of half the wavelength:

$$\Delta = \left(n + \frac{1}{2}\right)\lambda.$$

A crest of one wave then meets a trough of the other, giving minimum intensity; the minimum is zero when their amplitudes are equal. More generally, the total phase difference, including any fixed initial phase difference, must be $2n\pi$ for a maximum and $(2n + 1)\pi$ for a minimum.

Step 2: Check two independent sodium lamps Two independent sodium lamps are not mutually coherent, even though they emit light of approximately the same characteristic wavelength. Emission from the atoms in the two lamps occurs independently, so their relative phase changes randomly. The interference contribution averages to zero over the observation time. Thus the stated conditions cannot produce a stable observable interference pattern using two independent sodium lamps.

Step 3: Compare red and green fringe widths (b) The geometrical quantities D and d remain unchanged when the light is replaced. Therefore,

$$\frac{\beta_{\text{red}}}{\beta_{\text{green}}} = \frac{\lambda_{\text{red}}}{\lambda_{\text{green}}} > 1.$$

Red light has a longer wavelength than green light, so the fringes become farther apart. Keeping the

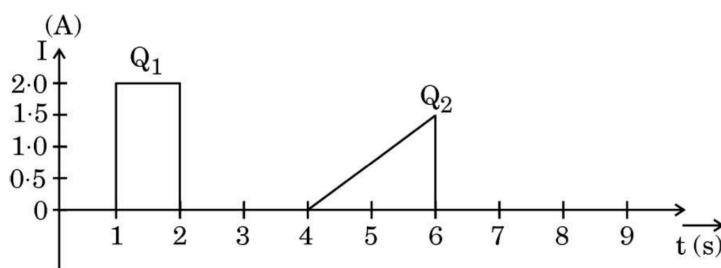
incident intensity the same does not keep the fringe width fixed, because intensity is absent from the expression for β .

Step 4: Conclusion For in-phase coherent sources, maxima occur at $\Delta = n\lambda$ and minima at $\Delta = (n + \frac{1}{2})\lambda$. Independent sodium lamps do not give sustained interference. Replacing green light by red light increases the fringe width.

Quick Tip to Solve: Separate the two requirements: coherence keeps the pattern steady, while $\beta = \lambda D/d$ sets its spacing. A longer wavelength gives wider fringes.

27(a). (i) Derive an expression for the resistivity of a conductor in terms of number density of free electrons and relaxation time.

(ii) The figure shows the plot of current through a cross-section of wire over two different time intervals. Compare the charges (Q_1 and Q_2) that pass through the cross-section during these time intervals. [3 marks]



Solution:

Concept:

- In the free-electron model, the drift-speed magnitude is $v_d = e\mathcal{E}\tau/m_e$, where $\mathcal{E}$ is electric field, e is the magnitude of electron charge, m_e is electron mass, and τ is relaxation time.
- Current density is $J = nev_d$, where n is the number of free electrons per unit volume. The microscopic form of Ohm's law is $\mathcal{E} = \rho J$.
- The charge passing through a cross-section is $Q = \int I dt$, equal to the area under the current-time graph.

Step 1: Obtain the mean drift speed (i) An electron in an electric field experiences force $\vec{F} = -e\vec{\mathcal{E}}$, hence acceleration $\vec{a} = -e\vec{\mathcal{E}}/m_e$. Random thermal velocities have zero mean, while collisions interrupt the directed motion. If the mean relaxation time is τ , the mean drift velocity is

$$\vec{v}_d = -\frac{e\tau}{m_e}\vec{\mathcal{E}}.$$

The electron drift is opposite to the field; conventional current is along the field. Using magnitudes gives $v_d = e\mathcal{E}\tau/m_e$.

Step 2: Relate current density to resistivity For a conductor of cross-sectional area A , electrons in a length $v_d dt$ cross a section in time dt . Their charge magnitude is $dQ = neAv_d dt$, so $I = neAv_d$. Therefore,

$$J = \frac{I}{A} = nev_d = \frac{ne^2\tau}{m_e}\mathcal{E}.$$

Comparing with $J = \mathcal{E}/\rho$ yields

$$\rho = \frac{m_e}{ne^2\tau}.$$

Thus a larger free-electron density or a longer relaxation time gives a smaller resistivity. Its SI unit is $\Omega \text{ m}$.

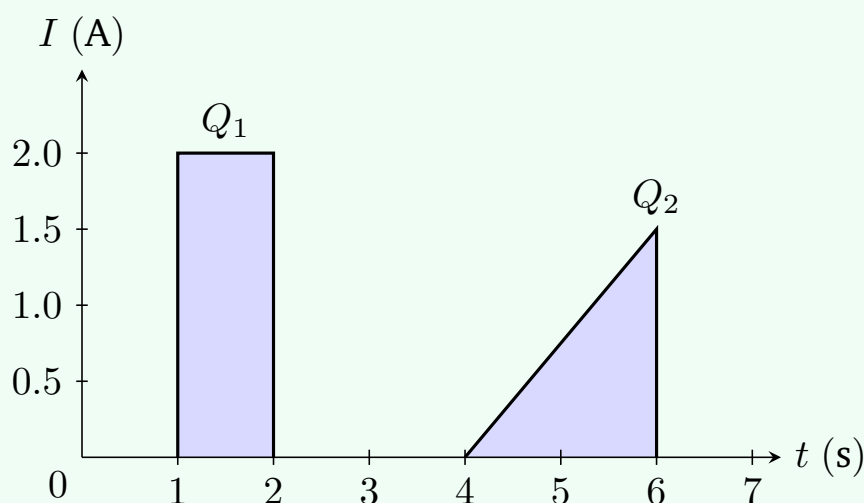
Step 3: Calculate the two areas under the graph (ii) The first pulse is a rectangle of height 2.0 A, extending from 1 s to 2 s. The second pulse is a triangle extending from 4 s to 6 s, with maximum current 1.5 A. Hence,

$$Q_1 = 2.0(2 - 1) = 2.0 \text{ C},$$

$$Q_2 = \frac{1}{2}(6 - 4)(1.5) = 1.5 \text{ C},$$

$$\frac{Q_1}{Q_2} = \frac{2.0}{1.5} = \frac{4}{3}.$$

The instantaneous jumps shown by the vertical graph segments have zero time width and do not add any area.



The shaded rectangle and triangle represent Q_1 and Q_2 , respectively.

Step 4: Conclusion The resistivity is $\rho = m_e / (ne^2\tau)$. The charges are $Q_1 = 2.0 \text{ C}$ and $Q_2 = 1.5 \text{ C}$, so $Q_1 : Q_2 = 4 : 3$.

Quick Tip to Solve: Compare areas, not peak currents: use a rectangle for the first pulse and half the base times height for the second pulse.

27(b). (i) A battery of emf E and internal resistance r is connected to a variable external resistance R .

(I) Obtain the expression for current I in the circuit and the value of maximum current the battery can supply.

(II) Obtain the terminal voltage V across the battery and its maximum possible value.

(ii) The above battery sends a current I_1 when $R = R_1$ and a current I_2 when $R = R_2$. Obtain the internal resistance of the battery in terms of I_1 , I_2 , R_1 and R_2 . [3 marks]

Solution:

Concept:

- A battery of emf E and internal resistance r supplying current I obeys the loop equation $E = IR + Ir$, where R is the external resistance.
- During discharge, the terminal voltage is $V = E - Ir = IR$. The internal voltage drop Ir reduces the voltage available to the external circuit.
- The same battery has the same emf and internal resistance in both measurements, allowing E to be eliminated to obtain r .

Step 1: Determine the current and its maximum (i)(I) The external resistance and the internal resistance are in series. Applying the loop rule gives

$$E - Ir - IR = 0 \Rightarrow I = \frac{E}{R + r}.$$

For fixed E and positive r , current becomes larger as the non-negative external resistance becomes smaller. Its maximum value occurs at $R = 0$, the short-circuit condition:

$$I_{\max} = \frac{E}{r}.$$

This limit refers to the given battery model, in which the emf and internal resistance remain constant.

Step 2: Determine the terminal voltage and its maximum (i)(II) The potential difference across the battery terminals equals the potential difference across the external resistance. Substituting the current gives

$$V = E - Ir = \frac{ER}{R + r}.$$

As R increases, the current and the internal voltage loss Ir decrease, so V increases. The maximum possible terminal voltage is obtained on open circuit, when $I = 0$, corresponding to $R \rightarrow \infty$:

$$V_{\max} = E.$$

For any finite positive load resistance with $r > 0$, the terminal voltage is less than the emf. At short circuit it becomes zero, despite the current being maximum.

Step 3: Eliminate the emf between the two measurements (ii) Applying the current expression separately to the two external resistances gives

$$E = I_1(R_1 + r) = I_2(R_2 + r).$$

Expanding and collecting the terms containing r ,

$$\begin{aligned}I_1 R_1 + I_1 r &= I_2 R_2 + I_2 r, \\(I_1 - I_2)r &= I_2 R_2 - I_1 R_1, \\r &= \frac{I_2 R_2 - I_1 R_1}{I_1 - I_2}.\end{aligned}$$

The two measurements must have different currents for this expression to determine r . The numerator has units of voltage and the denominator has units of current, giving resistance in ohms.

Step 4: Conclusion $I = E/(R+r)$, with $I_{\max} = E/r$ at short circuit; $V = ER/(R+r)$, with $V_{\max} = E$ on open circuit. The internal resistance is

$$r = \frac{I_2 R_2 - I_1 R_1}{I_1 - I_2}.$$

Quick Tip to Solve: Write $E = I(R+r)$ for each load and equate the two expressions; keep the subtraction order consistent in the numerator and denominator.

28. (a) State any three characteristics of electromagnetic waves.

(b) Briefly explain how and where the displacement current exists during the charging of a capacitor.

[3 marks]

Solution:

Concept:

- An electromagnetic wave consists of oscillating electric and magnetic fields that propagate together and transport energy.
- Maxwell's displacement current is $I_d = \epsilon_0 d\Phi_E/dt$, where Φ_E is the electric flux through a surface and ϵ_0 is the permittivity of free space.

Step 1: State three characteristics of electromagnetic waves (a) First, electromagnetic waves are transverse: their electric field $\vec{E}$ and magnetic field $\vec{B}$ are perpendicular to each other and to the direction of propagation. Second, they do not require a material medium and travel in vacuum with speed

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m s}^{-1}.$$

Third, the electric and magnetic fields oscillate in the same phase: they reach maxima and vanish simultaneously at a given point, with amplitudes related by $E_0 = cB_0$ in vacuum.

Step 2: Locate the displacement current during charging (b) While a capacitor is charging, conduction current in its connecting wires deposits opposite charges on its plates. The increasing plate charge produces a time-varying electric field, and hence a changing electric flux, in the gap between the plates. This changing field gives rise to displacement current through the gap even though free charge does not cross the insulating space from one plate to the other.

Step 3: Relate the displacement current to the charging current For an ideal parallel-plate capacitor with vacuum or air between the plates, neglect fringing and let A be the plate area and Q the charge

on the positive plate. Then

$$E = \frac{Q}{\varepsilon_0 A}, \quad \Phi_E = EA = \frac{Q}{\varepsilon_0},$$
$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} = I.$$

Thus the changing field supplies the current term required for continuity across the capacitor. Once charging ends and the electric field becomes constant, the displacement current vanishes.

Step 4: Conclusion Electromagnetic waves are transverse, propagate in vacuum at c , and have electric and magnetic fields in phase. During capacitor charging, displacement current exists in the changing electric field between the plates, with $I_d = \varepsilon_0 d\Phi_E/dt = I$ for the ideal air or vacuum capacitor.

Quick Tip to Solve: For a charging capacitor, use $\Phi_E = Q/\varepsilon_0$: differentiating immediately shows why the gap's displacement current equals the current in the wire.

Section D (8 marks)

Questions number 29 and 30 are Case Study-based questions. Read the following paragraphs and answer the questions that follow.

29. A hydrogen atom consists of an electron revolving in a circular orbit of radius r with certain velocity v around a proton located at the nucleus of the atom. The electrostatic force of attraction between the revolving electron and the proton provides the requisite centripetal force to keep it in the orbit. According to Bohr's model, an electron can revolve only in certain stable orbits. The angular momentum of the electron in these orbits is some integral multiple of $\frac{h}{2\pi}$, where h is the Planck's constant. Further, when an electron makes a transition from one orbit of higher energy, to that of lower energy, a photon is emitted having energy equal to the difference between energies of the initial and final states. Assuming the mass and charge of an electron as m and e respectively, answer the following questions. [4 marks]

29(i). (i) The expression for the speed of electron v in terms of radius of the orbit (r) and physical constant $\left(K = \frac{1}{4\pi\epsilon_0}\right)$ is : [1 mark]

- (A) $\frac{Ke^2}{mr}$
(B) $\frac{Ke^2}{mr^2}$
(C) $\sqrt{\frac{Ke^2}{mr}}$
(D) $\sqrt{\frac{Ke^2}{mr^2}}$

Correct Answer: (C) $\sqrt{\frac{Ke^2}{mr}}$

Solution:

Concept:

- The electrostatic attraction between the proton and electron is $F = Ke^2/r^2$, where $K = 1/(4\pi\epsilon_0)$, e is the charge magnitude and r is the orbit radius.
- Circular motion of an electron of mass m and speed v requires inward centripetal force mv^2/r .

Step 1: Equate the electrostatic and centripetal forces The attraction towards the proton is the force that maintains the circular orbit. Therefore

$$\frac{mv^2}{r} = \frac{Ke^2}{r^2} \Rightarrow v^2 = \frac{Ke^2}{mr}.$$

Speed is a nonnegative scalar, so we must take the positive square root of this expression.

Step 2: Check the powers and the printed options One power of r cancels when the force equation is multiplied by r ; the denominator of v^2 therefore contains r , not r^2 . Option (A) gives v^2 , rather than v . Option (B) gives the magnitude of the centripetal acceleration, and option (D) is its square

root; neither has the dimensions of speed. Only option (C) has the correct square root and radius dependence.

Step 3: Conclusion The electron speed is $v = \sqrt{Ke^2/(mr)}$. Hence, the correct option is (C).

Quick Tip to Solve: First solve the force equation for v^2 ; cancel one radius factor before taking the square root to avoid both common distractors.

29(ii). (ii) The total energy of the atom in terms of r and physical constant K is :

[1 mark]

- (A) $\frac{Ke^2}{r}$
(B) $-\frac{Ke^2}{2r}$
(C) $\frac{Ke^2}{2r}$
(D) $\frac{3Ke^2}{2r}$

Correct Answer: (B) $-\frac{Ke^2}{2r}$

Solution:

Concept:

- With the electrostatic potential energy taken as zero at infinite separation, a proton-electron pair has potential energy $U = -Ke^2/r$.
- The total orbital energy is $E = T + U$, where $T = mv^2/2$ is the electron's kinetic energy; the proton is treated as stationary in this model.

Step 1: Obtain the kinetic energy from the orbit condition Electrostatic attraction supplies the required centripetal force. Hence

$$\frac{mv^2}{r} = \frac{Ke^2}{r^2} \Rightarrow T = \frac{1}{2}mv^2 = \frac{Ke^2}{2r}.$$

The kinetic energy is positive and equals half the magnitude of the negative potential energy.

Step 2: Add kinetic and potential energy with their signs The attraction makes U negative, so the total energy is

$$E = \frac{Ke^2}{2r} - \frac{Ke^2}{r} = -\frac{Ke^2}{2r}.$$

This negative sign identifies a bound atom. Option (A) is the magnitude of the potential energy, option (C) is the kinetic energy alone, and option (D) adds the two energy magnitudes as though the potential energy were positive. None is the total energy of the bound state.

Step 3: Conclusion The atom's total energy is $E = -Ke^2/(2r)$. Hence, the correct option is (B).

Quick Tip to Solve: In a Bohr orbit remember $U = -2T$, so $E = -T$; this fixes both the minus sign and the factor of two at once.

29(iii). (iii) A photon of wavelength 500 nm is emitted when an electron makes a transition from one state to the other state in an atom. The change in the total energy of the electron and change in its kinetic energy in eV as per Bohr's model, respectively will be : [1 mark]

(A) 2.48, -2.48

(B) -1.24, 1.24

(C) -2.48, 2.48

(D) 1.24, -1.24

Correct Answer: (C) -2.48, 2.48

Solution:

Concept:

- The energy carried by a photon of wavelength λ is $E_\gamma = hc/\lambda$, where h is Planck's constant and c is the speed of light.
- In a Bohr orbit, the total electronic energy and kinetic energy obey $E = -T$. Changes mean final minus initial, so $\Delta T = -\Delta E$.

Step 1: Calculate the positive energy of the emitted photon Convert the wavelength to metres and then the photon energy from joules to electron volts:

$$\begin{aligned}\lambda &= 500 \times 10^{-9} \text{ m}, \\ E_\gamma &= \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{500 \times 10^{-9}} = 3.978 \times 10^{-19} \text{ J}, \\ E_\gamma &= \frac{3.978 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} \simeq 2.48 \text{ eV}.\end{aligned}$$

This is the energy leaving the atom, not the signed change of its electronic energy.

Step 2: Assign the signs of both electronic energy changes Emission lowers the total energy: $\Delta E = -E_\gamma$. Since $E = -T$ for each stationary orbit, its kinetic energy increases by the same magnitude, $\Delta T = +E_\gamma$. Option (A) reverses the emission signs. Options (B) and (D) use half the photon energy, although the photon carries the entire difference in total energy; (D) also reverses the signs.

Step 3: Conclusion The changes in total and kinetic energy are respectively -2.48 eV and +2.48 eV. Hence, the correct option is (C).

Quick Tip to Solve: For emission, write $\Delta E = -hc/\lambda$ first; then use $\Delta T = -\Delta E$ to get the kinetic-energy sign without another calculation.

29(iv)(a). (iv) (a) In Bohr's model of hydrogen atom, the frequency of revolution of electron in its n^{th} orbit is proportional to :

[1 mark]

- (A) n
- (B) $\frac{1}{n}$
- (C) $\frac{1}{n^2}$
- (D) $\frac{1}{n^3}$

Correct Answer: (D) $\frac{1}{n^3}$

Solution:

Concept:

- In the hydrogen atom, the radius of the n^{th} Bohr orbit obeys $r_n \propto n^2$, while the electron's orbital speed obeys $v_n \propto n^{-1}$.
- The frequency of revolution is the number of complete orbits per second: $f_n = v_n/(2\pi r_n)$. It is distinct from the frequency of a photon emitted during a transition.

Step 1: Obtain the speed and radius dependences The Coulomb force supplies the centripetal force, so $mv_n^2/r_n = Ke^2/r_n^2$. Bohr's quantisation condition is $mv_nr_n = n\hbar$, with $\hbar = h/(2\pi)$. Combining the two equations gives

$$v_n = \frac{Ke^2}{n\hbar}, \quad r_n = \frac{n^2\hbar^2}{mKe^2}.$$

Thus the electron slows down as n^{-1} , and the distance travelled in one revolution grows as n^2 .

Step 2: Divide speed by the orbital circumference Using the circumference $2\pi r_n$,

$$f_n = \frac{v_n}{2\pi r_n} \propto \frac{n^{-1}}{n^2} = n^{-3}.$$

Option (B) gives only the speed dependence and ignores the increasing circumference. Option (C) corresponds to the inverse radius alone and ignores the changing speed. Option (A) predicts increasing frequency even though the electron travels more slowly along a larger orbit.

Step 3: Conclusion The frequency of revolution is proportional to $1/n^3$. Hence, the correct option is (D).

Quick Tip to Solve: For orbital frequency, divide $v_n \propto n^{-1}$ by $r_n \propto n^2$; subtract the powers to obtain n^{-3} .

29(iv)(b). (b) An electron makes a transition from -3.4 eV state to the ground state in hydrogen atom. Its radius of orbit changes by : (radius of orbit of electron in ground state = $0.53 \text{ \AA}$) [1 mark]

- (A) $0.53 \text{ \AA}$
- (B) $1.06 \text{ \AA}$
- (C) $1.59 \text{ \AA}$
- (D) $2.12 \text{ \AA}$

Correct Answer: (C) $1.59 \text{ \AA}$

Solution:

Concept:

- The hydrogen energy levels obey $E_n = -13.6/n^2$ eV, with $n = 1$ representing the ground state.
- Bohr orbit radii obey $r_n = n^2 r_1$, where the given ground-state radius is $r_1 = 0.53 \text{ \AA}$. A transition to a lower level reduces the orbit radius.

Step 1: Identify the initial energy level Equate the given initial energy to the hydrogen energy formula:

$$-\frac{13.6}{n_i^2} = -3.4 \Rightarrow n_i^2 = 4 \Rightarrow n_i = 2.$$

The final state is the ground state, so $n_f = 1$. The energy sign identifies a bound state; it does not make the radius negative.

Step 2: Calculate both radii and their difference The initial and final radii are

$$r_i = 4(0.53) = 2.12 \text{ \AA}, \quad r_f = 0.53 \text{ \AA}.$$

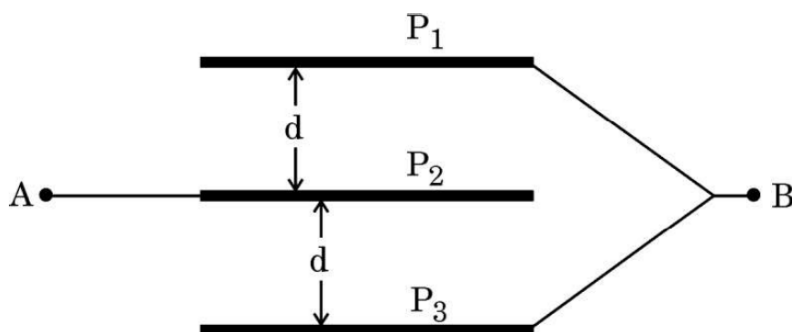
Their signed change is $r_f - r_i = 0.53 - 2.12 = -1.59 \text{ \AA}$, so the radius decreases and its change in magnitude is $1.59 \text{ \AA}$. Option (A) is only the final radius, and option (D) is only the initial radius. Option (B) follows from incorrectly taking radius proportional to n instead of n^2 ; it is not the radius difference.

Step 3: Conclusion The orbital radius decreases by $1.59 \text{ \AA}$. Hence, the correct option is (C).

Quick Tip to Solve: First find $n_i^2 = 13.6/3.4 = 4$; the radius decrease to the ground state is then $(4 - 1)r_1$.

30. A parallel plate capacitor consists of two conducting plates kept generally parallel to each other at a distance. When the capacitor is charged, the charge resides on the inner surfaces of the plates and an electric field is set up between them. Thus, electrostatic energy is stored in the capacitor. The figure shows three large square metallic plates, each of side ' L ' held parallel and equidistant from each other. The space between P_1 and P_2 and P_2 and P_3 is completely filled with mica sheets of dielectric constant ' K '. The plate P_2 is connected to point A and other plates P_1 and P_3 are connected to point B . Point A is maintained at a positive potential with respect to point B and the potential difference between A and B is V .

[4 marks]



30(i). (i) The capacitance of the system between A and B will be :

[1 mark]

- (A) $\frac{\epsilon_0 K L^2}{d}$
 (B) $\frac{\epsilon_0 K L^2}{2d}$
 (C) $\frac{2\epsilon_0 K L^2}{d}$
 (D) $\frac{2\epsilon_0 K d}{L^2}$

Correct Answer: (C) $\frac{2\epsilon_0 K L^2}{d}$

Solution:

Concept:

- A dielectric-filled parallel plate capacitor has capacitance $C_0 = K\epsilon_0 S/d$, where S is the plate area, d the gap and K the dielectric constant.
- Capacitors connected across the same two terminals are in parallel, and their capacitances add.

Step 1: Identify the two capacitors The gaps P_1P_2 and P_2P_3 each form a capacitor of plate area $S = L^2$, separation d and dielectric constant K . Both have one plate at terminal A and one at terminal B , so each experiences the full voltage V .

$$C_0 = \frac{K\epsilon_0 L^2}{d}.$$

Step 2: Add the parallel capacitances As the outer plates are joined together, the two capacitors are in parallel, even though the plates are geometrically stacked.

$$C_{AB} = C_0 + C_0 = \frac{2K\epsilon_0 L^2}{d}.$$

Option (A) counts only one gap. Option (B) treats the pair as a series combination. Option (D) reverses the dependence on area and separation and does not have the dimensions of capacitance.

Step 3: Conclusion The equivalent capacitance is $C_{AB} = \frac{2\varepsilon_0 K L^2}{d}$. Hence, the correct option is (C).

Quick Tip to Solve: Trace the terminals: each gap has one plate at A and one at B, so add the two identical capacitances.

30(ii). (ii) The charge on plate P_1 is :

[1 mark]

- (A) $\frac{\varepsilon_0 V K L^2}{2d}$
(B) $\frac{\varepsilon_0 V K L^2}{d}$
(C) $\frac{2\varepsilon_0 V K L^2}{d}$
(D) $\frac{\varepsilon_0 V K L^2}{4d}$

Correct Answer: (B) $\frac{\varepsilon_0 V K L^2}{d}$

Solution:

Concept:

- For one dielectric-filled gap, $C_0 = K\varepsilon_0 L^2/d$, with plate area L^2 , separation d and dielectric constant K .
- The magnitude of charge on each plate of a capacitor is $Q = C_0 V$; the lower-potential plate carries negative charge.

Step 1: Select the gap adjoining the specified plate The printed question asks for the charge on P_1 . This outer plate forms one capacitor with P_2 ; their potential difference is V . The second gap is adjoining P_3 , so its charge must not be added to that on P_1 .

Step 2: Calculate the charge and identify its sign Substituting the capacitance of this single gap gives

$$|Q_1| = C_0 V = \frac{\varepsilon_0 V K L^2}{d}.$$

Since P_1 is at terminal B, below the potential of P_2 at A, its signed charge is $Q_1 = -\varepsilon_0 V K L^2/d$. The positive quantities in the options specify magnitudes. Option (C) instead gives the total positive charge on the two faces of P_2 ; options (A) and (D) incorrectly divide the single-gap charge.

Step 3: Conclusion The magnitude of charge on P_1 is $\frac{\varepsilon_0 V K L^2}{d}$, and the charge is negative. Hence, the correct option is (B).

Quick Tip to Solve: An outer plate bounds one active gap; only the middle plate carries the sum of the charges from both gaps.

30(iii). (iii) The electric field in the region between P_1 and P_2 is :

[1 mark]

- (A) $\frac{V}{d}$
- (B) $\frac{2V}{d}$
- (C) $\frac{V}{2d}$
- (D) $\frac{d}{V}$

Correct Answer: (A) $\frac{V}{d}$

Solution:

Concept:

- Between large parallel plates, neglecting edge effects, the electric field is uniform and its magnitude is $E = |\Delta V|/d$, where d is their separation.
- The field points from higher to lower potential; a dielectric changes the charge needed to maintain a fixed voltage but does not change $E = V/d$.

Step 1: Identify the voltage across the specified gap Plate P_2 is connected to A , and plate P_1 is connected to B . Therefore the potential difference across the gap P_1P_2 is the full applied voltage V . The gap width shown in the figure is d , not the outer-plate separation $2d$.

Step 2: Use the uniform-field relation Applying the potential-gradient relation to that gap gives

$$E = \frac{V_A - V_B}{d} = \frac{V}{d}.$$

The direction is from P_2 towards P_1 . Option (B) doubles the field without changing either voltage or gap; option (C) wrongly uses $2d$ as the gap. Option (D) is the reciprocal of the required ratio and has the wrong dimensions.

Step 3: Conclusion The electric field magnitude between P_1 and P_2 is V/d , directed from P_2 to P_1 . Hence, the correct option is (A).

Quick Tip to Solve: For each dielectric-filled gap held at a fixed voltage, use that gap's own separation in $E = V/d$.

30(iv). (iv) (a) The separation between the plates of same area (L^2) of a parallel plate air capacitor having capacitance equal to that of this system, will be : [1 mark]

- (A) $\frac{d}{K}$
- (B) $\frac{2d}{K}$
- (C) $\frac{d}{2K}$
- (D) $\frac{d}{4K}$

Correct Answer: (C) $\frac{d}{2K}$

Solution:

Concept:

- A parallel plate air capacitor of area S and separation x has capacitance $C_{\text{air}} = \varepsilon_0 S/x$, treating the relative permittivity of air as unity.
- The two identical dielectric-filled gaps of the given system act in parallel, giving $C_{AB} = 2K\varepsilon_0 L^2/d$.

Step 1: Equate the required capacitances Let the separation of the equivalent air capacitor be x . Its plate area remains L^2 ; only its separation is to be determined. Requiring its capacitance to equal that of both gaps together gives

$$\frac{\varepsilon_0 L^2}{x} = \frac{2K\varepsilon_0 L^2}{d}.$$

Step 2: Cancel the common factors and compare options Cancel $\varepsilon_0 L^2$ from both sides and solve for the separation:

$$\frac{1}{x} = \frac{2K}{d}, \quad x = \frac{d}{2K}.$$

Option (A) accounts for just one dielectric-filled gap. Option (B) gives one quarter of the required system capacitance, while option (D) gives twice the required capacitance. The factor of two in the system capacitance must reduce the equivalent separation by two.

Step 3: Conclusion The required plate separation is $x = \frac{d}{2K}$. Hence, the correct option is (C).

Quick Tip to Solve: At fixed plate area, multiply capacitance by a factor only by dividing the air gap by that same factor.

30(iv). (b) If the source of potential difference applied between A and B is removed, and then A and B are connected by a conducting wire, the net charge on the system will be : [1 mark]

- (A) $\frac{\epsilon_0 V K L^2}{4d}$
 (B) $\frac{\epsilon_0 V K L^2}{2d}$
 (C) $\frac{\epsilon_0 V K L^2}{d}$
 (D) Zero

Correct Answer: (D) Zero

Solution:

Concept:

- Charging an initially neutral capacitor separates equal and opposite charges; the total charge of the complete capacitor system remains zero.
- After removal of the source, joining the terminals by a conducting wire allows charge to redistribute until all connected plates have the same potential.

Step 1: Add the initial charges on all three plates Write the capacitance of each dielectric-filled gap as $C_0 = K\epsilon_0 L^2/d$. Plate P_2 , connected to the positive terminal, carries $+2C_0V$, whereas each outer plate carries $-C_0V$. Therefore the algebraic total is

$$Q_{\text{net}} = 2C_0V - C_0V - C_0V = 0.$$

Step 2: Apply charge conservation during discharge Removing the voltage source isolates the capacitor from that source. Connecting A and B then lets opposite charges neutralise through the wire. The final voltage across both gaps becomes zero, so the stored capacitor charge disappears, while the total charge remains its initial value of zero. All three nonzero alternatives represent fractions of an individual plate's initial charge magnitude; none is the algebraic net charge of the system.

Step 3: Conclusion The net charge on the system is zero after the terminals are joined. Hence, the correct option is (D).

Quick Tip to Solve: For the net charge of a capacitor system, add signed charges on all plates before considering the discharge process.

Section E (15 marks)

31(a). (i) An object is placed 30 cm from a thin convex lens of focal length 10 cm. The lens forms a sharp image on a screen. If a thin concave lens is placed in contact with the convex lens, the sharp image on the screen is formed when the screen is moved by 45 cm from its initial position. Calculate the focal length of the concave lens.

(ii) Calculate the angle of minimum deviation of an equilateral prism. The refractive index of the prism is $\sqrt{3}$. Calculate the angle of incidence for this case of minimum deviation also. [5 marks]

Solution:

Concept:

- With the Cartesian sign convention, a thin lens obeys $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$. Here u and v are the signed object and image distances, and f is the focal length.
- For two thin lenses in contact, powers add: $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$, where F is the equivalent focal length. A concave lens has negative focal length.
- At minimum deviation, a prism has a symmetric ray path: $i = e$, $r_1 = r_2 = A/2$, and $\delta_m = 2i - A$. For a prism in air, $\mu = \sin i / \sin(A/2)$.

Step 1: (i) Locate the original real image Take incident light to travel from left to right. The real object lies to the left, so $u = -30$ cm, while the convex lens has $f_1 = +10$ cm. Applying the lens formula gives

$$\frac{1}{10} = \frac{1}{v_1} - \frac{1}{-30},$$

$$\frac{1}{v_1} = \frac{1}{10} - \frac{1}{30} = \frac{1}{15}.$$

Thus the screen initially lies 15 cm to the right of the lens.

Step 2: Determine the focal length of the combination Adding a concave lens reduces the convergence of the rays, so a real image forms farther from the lenses. The screen therefore moves away from them, giving $v_2 = 15 + 45 = 60$ cm. A movement towards the lenses would place it at -30 cm, which would correspond to a virtual image and could not give the stated sharp image on a screen. The object distance remains unchanged because the lenses are in contact. Hence

$$\frac{1}{F} = \frac{1}{60} - \frac{1}{-30} = \frac{1}{60} + \frac{1}{30} = \frac{1}{20}, \quad F = 20 \text{ cm.}$$

Step 3: Obtain the concave lens focal length Substitute the known convex lens and equivalent focal lengths into the contact-lens relation:

$$\frac{1}{20} = \frac{1}{10} + \frac{1}{f_2},$$

$$\frac{1}{f_2} = \frac{1}{20} - \frac{1}{10} = -\frac{1}{20},$$

$$f_2 = -20 \text{ cm.}$$

The negative sign is consistent with the added lens being diverging; the combination still has a positive focal length.

Step 4: (ii) Apply the minimum-deviation condition An equilateral prism has prism angle $A = 60^\circ$. At minimum deviation the internal refraction angles are equal, so $r_1 = r_2 = 30^\circ$. Snell's law at entry

then gives

$$\sin i = \mu \sin r_1 = \sqrt{3} \sin 30^\circ = \frac{\sqrt{3}}{2}.$$

The physical angle of incidence lies between 0° and 90° , giving $i = 60^\circ$. Since $e = i$, the minimum deviation is

$$\delta_m = i + e - A = 2(60^\circ) - 60^\circ = 60^\circ.$$

Step 5: Conclusion (i) The concave lens has focal length $f_2 = -20 \text{ cm}$. (ii) The prism's minimum deviation is $\delta_m = 60^\circ$, with angle of incidence $i = 60^\circ$.

Quick Tip to Solve: First find the original image distance; the screen displacement is not the new image distance. For minimum deviation in an equilateral prism, immediately set both internal refraction angles to 30° .

OR

31(b). (i) A physics teacher wants to demonstrate interference with the help of double slit experiment using a laser beam of 633 nm wavelength. Since the hall is large enough, interference pattern is formed on the wall 5.0 m from the slits. For clear and comfortable view by all the students they want the fringe width 5 mm .

(I) Find the slit separation for obtaining the desired interference pattern.

(II) How far will the first minimum be from the central maximum ?

(ii) A parallel beam of light of wavelength 650 nm passes through a slit of width 0.6 mm . The diffraction pattern is obtained on a screen kept 60 cm away from the slit. Find the distance between first order minima on both sides of the central maximum. [5 marks]

Solution:

Concept:

- In Young's double-slit experiment, the fringe width is $\beta = \lambda D/d$, where λ is the wavelength, D the screen distance, and d the slit separation. This uses the small-angle approximation.
- Destructive interference occurs when the path difference is $(n + \frac{1}{2})\lambda$. The first dark fringe on either side of the central bright fringe therefore lies at $y = \pm\beta/2$.
- Single-slit diffraction minima satisfy $a \sin \theta = m\lambda$, where a is the slit width and $m = \pm 1, \pm 2, \dots$. For small angles, $y_m \simeq m\lambda D/a$, and the central maximum extends between the two first minima.

Step 1: (i)(I) Express the interference data in SI units For the laser double-slit experiment, the given quantities are

$$\lambda = 633 \times 10^{-9} \text{ m}, \quad D = 5.0 \text{ m}, \quad \beta = 5 \times 10^{-3} \text{ m}.$$

The desired fringe width means the separation between consecutive bright fringes, or between consecutive dark fringes. Rearranging $\beta = \lambda D/d$ gives the slit separation required for that spacing.

Step 2: Calculate the required slit separation Substitute the converted quantities:

$$\begin{aligned} d &= \frac{\lambda D}{\beta} = \frac{(633 \times 10^{-9})(5.0)}{5 \times 10^{-3}} \text{ m} \\ &= 6.33 \times 10^{-4} \text{ m} = 0.633 \text{ mm}. \end{aligned}$$

This separation is much smaller than the wall distance, as required for the usual Young's experiment

geometry. Increasing the separation beyond this value would reduce the fringe width.

Step 3: (i)(II) Locate the first interference minimum At the central maximum, the two waves have zero path difference. The nearest minimum occurs when their path difference first reaches $\lambda/2$, so it lies halfway between the central maximum and the first neighbouring bright fringe:

$$|y_{\text{first dark}}| = \frac{\lambda D}{2d} = \frac{\beta}{2} = \frac{5}{2} \text{ mm} = 2.5 \text{ mm}.$$

There is one such minimum on each side of the centre, at signed positions $y = \pm 2.5 \text{ mm}$. The requested distance is measured from the central maximum to either one of these minima.

Step 4: (ii) Find the two first diffraction minima This is a separate single-slit experiment. Use its own wavelength, slit width and screen distance:

$$\lambda = 650 \times 10^{-9} \text{ m}, \quad a = 0.6 \times 10^{-3} \text{ m}, \quad D = 0.60 \text{ m}.$$

At the first minima, $m = \pm 1$. Here $\lambda/a \simeq 1.08 \times 10^{-3}$, so the angles are small and $\tan \theta \simeq \sin \theta$. Each first minimum is at distance

$$|y_1| = \frac{\lambda D}{a} = \frac{(650 \times 10^{-9})(0.60)}{0.6 \times 10^{-3}} \text{ m} = 6.50 \times 10^{-4} \text{ m} = 0.65 \text{ mm}.$$

The two minima lie on opposite sides of the centre. Their separation, equal to the width of the central diffraction maximum, is therefore

$$y_{+1} - y_{-1} = 2|y_1| = 2(0.65) \text{ mm} = 1.30 \text{ mm}.$$

Step 5: Conclusion (i)(I) Required slit separation: $d = 0.633 \text{ mm}$. (i)(II) Distance of the first interference minimum from the centre: 2.5 mm on either side. (ii) Separation between the first diffraction minima: 1.30 mm .

Quick Tip to Solve: The first double-slit dark fringe lies at $\beta/2$, but a single-slit central maximum has width $2\lambda D/a$. Check whether the question asks for a distance from the centre or a separation across both sides.

32(a). (i) Two point charges $+q$ and $-q$ are held at $(a, 0)$ and $(-a, 0)$ in x - y plane. Obtain an expression for the net electric field due to the charges at a point $(0, y)$. Hence, find electric field at a far off point ($y \gg a$).

(ii) Three point charges of -2 nC , -1 nC , and $+5 \text{ nC}$ are kept at the vertices A , B and C of an equilateral triangle of side 0.2 m . Find the total amount of work done in shifting the charges from A to A_1 , B to B_1 and C to C_1 . Here A_1 , B_1 and C_1 are the midpoints of sides AB , BC and CA , respectively. [5 marks]

Solution:

Concept:

- Coulomb's law and superposition give the net electric field: $\vec{E} = \sum_i k q_i \vec{R}_i / R_i^3$, where $k = 1/(4\pi\epsilon_0)$ and $\vec{R}_i$ points from charge q_i to the observation point.
- The electric dipole moment points from the negative charge to the positive charge. Here $\vec{p} = 2qa \hat{i}$; the point $(0, y)$ lies on the perpendicular bisector, or equatorial line, of the dipole.
- The electrostatic potential energy of three charges is $U = k(q_A q_B / r_{AB} + q_B q_C / r_{BC} + q_C q_A / r_{CA})$.

Each distinct pair is counted once, with the algebraic signs of its charges.

- When the charges are shifted slowly with no change in kinetic energy, the work done by an external agent is $W_{\text{ext}} = U_f - U_i$. Work done by the electric field has the opposite sign.

Step 1: (i) Write the two electric-field vectors Let $P = (0, y)$. The distances of P from $+q$ at $(a, 0)$ and $-q$ at $(-a, 0)$ are equal: $R = \sqrt{a^2 + y^2}$. The displacement vectors from the two charges to P are respectively $-a\hat{i} + y\hat{j}$ and $a\hat{i} + y\hat{j}$. Thus,

$$\begin{aligned}\vec{E}_+ &= \frac{kq}{(a^2 + y^2)^{3/2}}(-a\hat{i} + y\hat{j}), \\ \vec{E}_- &= \frac{-kq}{(a^2 + y^2)^{3/2}}(a\hat{i} + y\hat{j}).\end{aligned}$$

The field of the positive charge points away from it, while that of the negative charge points towards it. Consequently the two vertical components cancel and the horizontal components both point along the negative x -axis.

Step 2: Add the components and take the far-field limit On adding the vectors, the exact field at P is

$$\vec{E}(0, y) = -\frac{2kqa}{(a^2 + y^2)^{3/2}}\hat{i} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{(a^2 + y^2)^{3/2}}.$$

For the specified far-off point, $y \gg a$, the term a^2 is negligible compared with y^2 . Therefore,

$$\vec{E}_{\text{far}} \simeq -\frac{2qa}{4\pi\epsilon_0 y^3}\hat{i} = -\frac{\vec{p}}{4\pi\epsilon_0 y^3}.$$

The inverse-cube dependence is characteristic of the field of a dipole far from it. The field on this equatorial line is opposite to $\vec{p}$.

Step 3: (ii) Calculate the initial potential energy Initially every separation is $L = 0.2$ m. With $q_A = -2 \times 10^{-9}$ C, $q_B = -1 \times 10^{-9}$ C, and $q_C = 5 \times 10^{-9}$ C, the sum of the pair products is

$$\begin{aligned}S &= q_A q_B + q_B q_C + q_C q_A \\ &= [(-2)(-1) + (-1)(5) + (5)(-2)] \times 10^{-18} \text{ C}^2 \\ &= -13 \times 10^{-18} \text{ C}^2.\end{aligned}$$

Using $k = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$,

$$U_i = \frac{kS}{L} = \frac{(9 \times 10^9)(-13 \times 10^{-18})}{0.2} = -5.85 \times 10^{-7} \text{ J}.$$

Step 4: Calculate the final energy and the work done The midpoints A_1, B_1, C_1 form the medial equilateral triangle. By the midpoint theorem, each new pair separation equals half the original side, so $A_1 B_1 = B_1 C_1 = C_1 A_1 = 0.1$ m. The charges are unchanged. Hence every pair-energy term doubles:

$$U_f = \frac{kS}{L/2} = 2U_i = -1.17 \times 10^{-6} \text{ J}.$$

The total work by the external agent depends only on these initial and final configurations, irrespective

of the order in which the charges are shifted:

$$\begin{aligned} W_{\text{ext}} &= U_f - U_i \\ &= -1.17 \times 10^{-6} - (-5.85 \times 10^{-7}) \\ &= -5.85 \times 10^{-7} \text{ J.} \end{aligned}$$

The negative sign means that electrostatic potential energy is released during the shift. Equivalently, the electric field does positive work of $5.85 \times 10^{-7} \text{ J}$.

Step 5: Conclusion (i) $\vec{E} = -\frac{2qa}{4\pi\epsilon_0(a^2 + y^2)^{3/2}}\hat{i}$, and for $y \gg a$, $\vec{E} \simeq -\frac{\vec{p}}{4\pi\epsilon_0 y^3}$. Both point along the negative x -axis. (ii) The work done in shifting the charges by an external agent is $-5.85 \times 10^{-7} \text{ J}$; the electric field does $+5.85 \times 10^{-7} \text{ J}$.

Quick Tip to Solve: On a dipole's perpendicular bisector, cancel the transverse components before adding the fields. Moving all three charges to the side midpoints halves every pair distance, so the final potential energy is twice the initial energy.

OR

32(b). (i) Show that Gauss's theorem is consistent with Coulomb's law. Using it, derive an expression for the electric field due to a uniformly charged thin spherical shell of radius r at a point at a distance y from the centre of the shell such that (I) $y > r$, and (II) $y < r$.

(ii) A point charge of $+2 \text{ nC}$ is kept at the origin of a three-dimensional coordinate system. Find the type and magnitude of the charge which should be kept at $(0, 0, -6 \text{ m})$ so that the potential due to the system becomes zero at $(0, 0, 2 \text{ m})$. [5 marks]

Solution:

Concept:

- Gauss's theorem states $\oint_S \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the net charge enclosed by a closed surface S , and $d\vec{A}$ is directed outwards.
- For a point charge or a uniformly charged spherical shell, spherical symmetry makes the field radial and its magnitude constant over a concentric spherical Gaussian surface.
- Coulomb's law gives the radial field of a point charge: $\vec{E} = Q\hat{R}/(4\pi\epsilon_0 R^2)$, where R is the distance from the charge. A test charge q_0 experiences $\vec{F} = q_0\vec{E}$.
- Electric potential is a scalar: $V = k \sum_i q_i/d_i$, with $k = 1/(4\pi\epsilon_0)$, d_i the positive distance from each source charge, and zero potential chosen at infinity.

Step 1: (i) Check Gauss's theorem using Coulomb's field Put a point charge Q at the centre of a spherical surface of radius R . Coulomb's law gives $\vec{E} = Q\hat{R}/(4\pi\epsilon_0 R^2)$. The outward area vector is radial, so the signed outward field component is constant. Its total electric flux is

$$\begin{aligned} \Phi &= \oint_S \vec{E} \cdot d\vec{A} \\ &= \frac{Q}{4\pi\epsilon_0 R^2} \oint_S dA \\ &= \frac{Q}{4\pi\epsilon_0 R^2} (4\pi R^2) = \frac{Q}{\epsilon_0}. \end{aligned}$$

This is precisely Gauss's theorem. Conversely, spherical symmetry and Gauss's theorem give $E_R(4\pi R^2) = Q/\epsilon_0$, where E_R is the signed radial component. Hence

$$\vec{F} = q_0 \vec{E} = \frac{Qq_0}{4\pi\epsilon_0 R^2} \hat{\mathbf{R}},$$

recovering Coulomb's inverse-square force law, including attraction or repulsion through the signs of the charges.

Step 2: Derive the field outside the spherical shell Let the uniformly charged thin shell have radius r , total charge Q , and surface charge density $\sigma = Q/(4\pi r^2)$. For a point at distance $y > r$, choose a concentric Gaussian sphere of radius y . Spherical symmetry makes its field radial, with the same signed component E_y everywhere on this surface. The whole shell lies inside, so $Q_{\text{enc}} = Q$. Therefore,

$$E_y(4\pi y^2) = \frac{Q}{\epsilon_0},$$

$$\vec{E}(y) = \frac{Q}{4\pi\epsilon_0 y^2} \hat{\mathbf{y}} = \frac{\sigma r^2}{\epsilon_0 y^2} \hat{\mathbf{y}}, \quad y > r.$$

Here $\hat{\mathbf{y}}$ denotes the unit vector from the centre to the observation point. Thus the outside field is the same as if the entire shell charge were concentrated at its centre; it is outward for $Q > 0$ and inward for $Q < 0$.

Step 3: Derive the field inside the shell For $0 < y < r$, take a concentric Gaussian sphere entirely inside the shell. All the charge lies on the shell at radius r , so this Gaussian surface encloses no charge. Using the same spherical symmetry,

$$E_y(4\pi y^2) = \frac{0}{\epsilon_0} = 0 \quad \Rightarrow \quad \vec{E} = \vec{0}.$$

At the centre, symmetry likewise requires zero field. Therefore the field vanishes everywhere inside the shell. This inference uses both zero enclosed charge and spherical symmetry: zero net flux alone does not generally mean zero field at every point.

Step 4: (ii) Use the actual distances to the zero-potential point Let $P = (0, 0, 2 \text{ m})$, and let the unknown charge q be at $(0, 0, -6 \text{ m})$. The distance of the source charge $Q_1 = +2 \text{ nC}$ at the origin from P is

$$d_1 = 2 \text{ m}.$$

The other distance is

$$d_2 = |2 - (-6)| = 8 \text{ m}.$$

Applying scalar superposition,

$$0 = V(P) = k \left(\frac{2 \times 10^{-9}}{2} + \frac{q}{8} \right),$$

$$\frac{q}{8} = -1 \times 10^{-9},$$

$$q = -8 \times 10^{-9} \text{ C} = -8 \text{ nC}.$$

A negative charge is required because its potential must cancel the positive potential of the charge at the origin. It is four times farther from P , so its magnitude must be four times larger; substituting gives potentials of equal magnitude and opposite sign.

Step 5: Conclusion (i) Coulomb's law gives flux Q/ϵ_0 , in agreement with Gauss's theorem. For a shell

of total charge Q ,

$$\vec{E} = \begin{cases} \frac{Q}{4\pi\epsilon_0 y^2} \hat{y}, & y > r, \\ \vec{0}, & y < r. \end{cases}$$

(ii) The required charge is negative with magnitude 8 nC , that is,

$$q = -8 \text{ nC}.$$

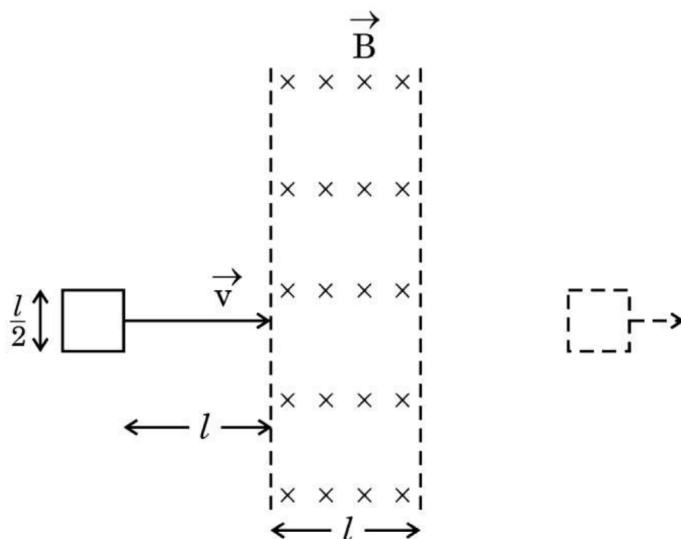
Quick Tip to Solve: In a spherical Gauss-law calculation, identify the enclosed charge before substituting. For zero potential, use signed charges but positive source-to-observation distances: the second distance here is 8 m, not 6 m.

33(a). (i) State Lenz's law and explain how this law is a consequence of conservation of energy principle.

(ii) A square shaped loop of side $\frac{\ell}{2}$ is initially lying outside a region of uniform magnetic field $\vec{B}$ as shown in the figure. The loop is moved towards right with a constant velocity $\vec{v}$ till it goes out of the region of magnetic field.

(I) What will be the directions of induced current when the loop enters the field and when it leaves the field ?

(II) Draw the plots showing the variation of magnetic flux ϕ linked with the loop with time t and variation of induced emf E with time t . Mark the relevant values of E , ϕ and t on the graphs. [5 marks]



Solution:

Concept:

- Lenz's law: the induced current flows in a direction such that its magnetic effect opposes the change in magnetic flux that produces it.
- Faraday's law for one turn is $E = -d\phi/dt$, where $\phi = BA_{\text{in}}$ and A_{in} is the loop area within the uniform field.
- With loop side $a = \ell/2$ and speed v , the area changes at rate av during entry and exit. Thus

$$E_0 = Bav = B\ell v/2 \text{ and } \phi_0 = Ba^2 = B\ell^2/4.$$

Step 1: (i) Relate Lenz's law to conservation of energy When a loop is moved into or out of a magnetic field, the induced current produces a magnetic force opposing this motion. An external agent must therefore do work to keep the speed constant. This mechanical work supplies the electrical energy, which is dissipated as heat if the loop has resistance. For resistance R , during entry or exit, $I = E_0/R$ and the opposing magnetic force has magnitude $F = B Ia$. Hence

$$Fv = (Bav)I = E_0 I = I^2 R.$$

If the induced effect aided the change instead, the loop would accelerate while also producing electrical energy without an external energy supply. That would violate conservation of energy. The opposing direction expressed by Lenz's law prevents this, and is represented by the minus sign in Faraday's law.

Step 2: (ii)(I) Determine the current directions The crosses in the printed figure show that the applied field points into the page. During entry, the inward flux increases, so the induced field must point out of the page. By the right-hand rule, the induced current is anticlockwise as viewed on the page. During exit, the inward flux decreases, so the induced field must point into the page to oppose this decrease. The current is then clockwise. When the loop is wholly outside or wholly inside the uniform field, its flux is constant and no current is induced.

Step 3: (ii)(II) Fix the time origin and boundary times Take $t = 0$ at the initial position shown. The leading edge is initially a distance ℓ from the near boundary, and the field region has width ℓ . The four transition times are therefore

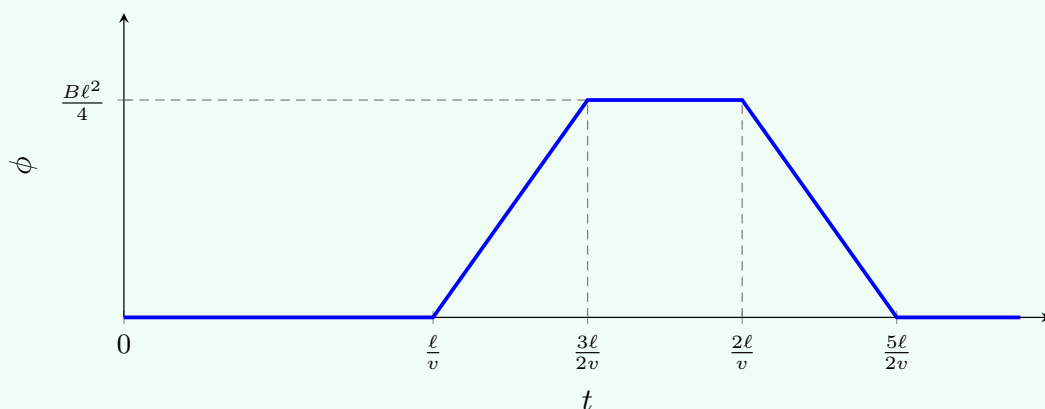
$$\begin{aligned} t_1 &= \frac{\ell}{v} && \text{(entry begins),} \\ t_2 &= \frac{\ell + \ell/2}{v} = \frac{3\ell}{2v} && \text{(fully inside),} \\ t_3 &= \frac{\ell + \ell}{v} = \frac{2\ell}{v} && \text{(exit begins),} \\ t_4 &= \frac{\ell + \ell + \ell/2}{v} = \frac{5\ell}{2v} && \text{(fully outside).} \end{aligned}$$

Entry, full immersion and exit each last $\ell/(2v)$. The initial field-free travel lasts ℓ/v .

Step 4: Plot the magnetic flux Choose the positive area normal into the page. While entering, the overlap width is $v(t - t_1)$; while leaving it is $v(t_4 - t)$. Thus the signed flux is positive and obeys

$$\phi(t) = \begin{cases} 0, & 0 \leq t \leq t_1, \\ \frac{B\ell v}{2}(t - t_1), & t_1 \leq t \leq t_2, \\ \frac{B\ell^2}{4}, & t_2 \leq t \leq t_3, \\ \frac{B\ell v}{2}(t_4 - t), & t_3 \leq t \leq t_4, \\ 0, & t \geq t_4. \end{cases}$$

The plot rises linearly, stays at $\phi_0 = B\ell^2/4$, and falls linearly to zero, with no jumps in flux.

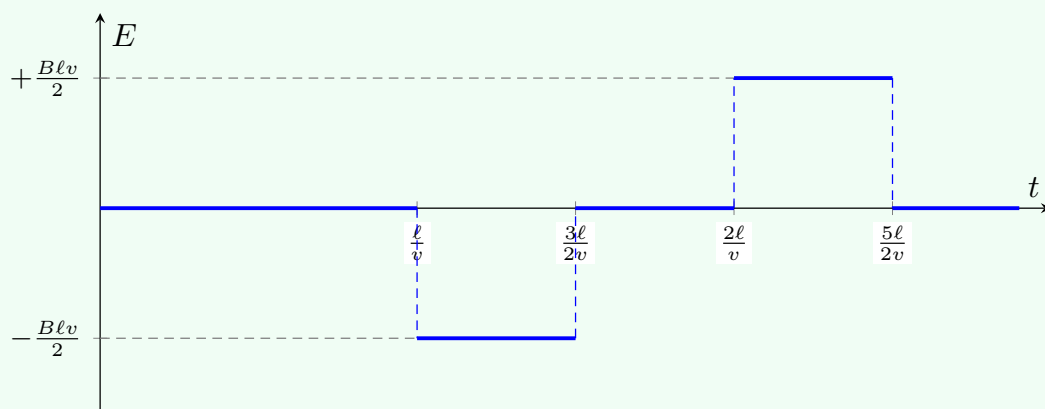


Flux versus time, with the positive normal into the page and $t = 0$ at the initial position.

Step 5: Plot the signed induced emf With the chosen inward normal, clockwise circulation is positive. The emf equals the negative slope of the flux graph, so

$$E(t) = \begin{cases} 0, & 0 < t < t_1, \\ -\frac{B\ell v}{2}, & t_1 < t < t_2, \\ 0, & t_2 < t < t_3, \\ +\frac{B\ell v}{2}, & t_3 < t < t_4, \\ 0, & t > t_4. \end{cases}$$

The negative entry pulse corresponds to anticlockwise current; the positive exit pulse corresponds to clockwise current. The abrupt changes shown at the transition times idealise the sharp boundaries of the uniform field. Reversing the sign convention reverses both signed plots while preserving the physical current directions.



Signed induced emf versus time; clockwise circulation is positive.

Step 6: Conclusion Lenz's law expresses conservation of energy: induced effects oppose the flux change. The current is anticlockwise on entry and clockwise on exit. With time measured from the printed initial position, the transitions occur at ℓ/v , $3\ell/(2v)$, $2\ell/v$, $5\ell/(2v)$. The maximum flux is $B\ell^2/4$, and the nonzero emf magnitude is $B\ell v/2$.

Quick Tip to Solve: Track the leading and trailing edges separately. Include the initial gap ℓ , then obtain the emf graph by taking the negative slope of each segment of the flux graph.

- 33(b).** (i) Differentiate between peak and rms values of alternating current. How are they related ?
 (ii) A current element X is connected across an ac source of emf $V = V_0 \sin 2\pi\nu t$. It is found that the voltage leads the current in phase by $\frac{\pi}{2}$ radian. If element X was replaced by element Y , the voltage lags behind the current in phase by $\frac{\pi}{2}$ radian.
 (I) Identify elements X and Y by drawing phasor diagrams.
 (II) Obtain the condition of resonance when both elements X and Y are connected in series to the source and obtain expression for resonant frequency. What is the impedance value in this case ?

[5 marks]

Solution:**Concept:**

- The peak current I_0 is the maximum magnitude of the instantaneous alternating current. The rms current $I_{\text{rms}} = \sqrt{\langle i^2 \rangle}$ is its effective heating value, where the average is taken over one complete cycle.
- For an ideal inductor, $v = L di/dt$; for an ideal capacitor, $i = C dv/dt$. Their reactances are $X_L = \omega L$ and $X_C = 1/(\omega C)$, where L is inductance, C is capacitance and $\omega = 2\pi\nu$.
- In a series circuit the same current flows through both elements. Their voltage phasors oppose one another, so the net reactance is $X_L - X_C$. Resonance occurs when this net reactance vanishes.

Step 1: (i) Distinguish peak current from rms current The peak value I_0 specifies the largest instantaneous magnitude reached by the current during a cycle. The rms value specifies the steady direct current that would produce the same average heating power in a given resistor as the alternating current. Thus, peak value measures the amplitude, while rms value measures the effective heating strength.

For a sinusoidal current $i = I_0 \sin \omega t$, with period T , square the current and average over one cycle:

$$I_{\text{rms}}^2 = \frac{1}{T} \int_0^T i^2 dt = \frac{I_0^2}{T} \int_0^T \sin^2 \omega t dt$$

$$= \frac{I_0^2}{2}.$$

Here the average of $\sin^2 \omega t$ over a full cycle is $1/2$. Taking the positive square root gives

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \simeq 0.707 I_0, \quad I_0 = \sqrt{2} I_{\text{rms}}.$$

Although the average sinusoidal current itself is zero, its mean square and hence its heating effect are nonzero.

Step 2: (ii)(I) Identify the elements using their phase relations In element X , the voltage leads the current by $\pi/2$. This is the property of a pure inductor. Using the given voltage $v = V_0 \sin \omega t$ in $v = L di/dt$, the sinusoidal current is

$$i_X = \frac{V_0}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right).$$

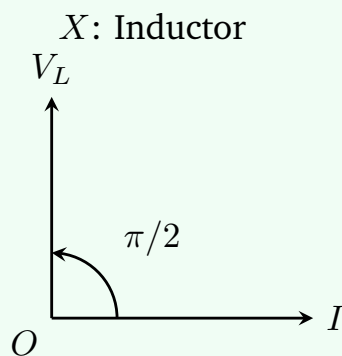
Thus X is an inductor of inductance L .

In element Y , the voltage lags behind the current by $\pi/2$. This is the property of a pure capacitor,

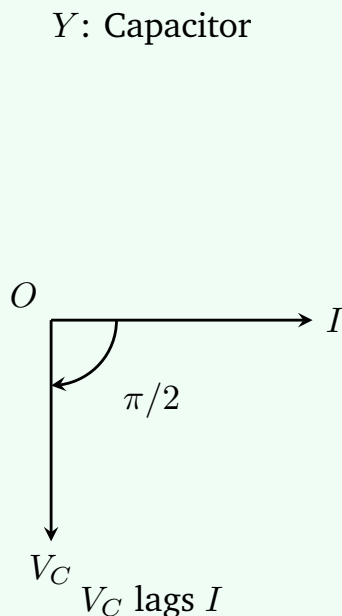
because

$$\begin{aligned} i_Y &= C \frac{dv}{dt} = \omega C V_0 \cos \omega t \\ &= \omega C V_0 \sin \left(\omega t + \frac{\pi}{2} \right). \end{aligned}$$

Thus Y is a capacitor of capacitance C . In each phasor diagram below, take the current I along the positive horizontal direction and measure phase counterclockwise. The inductor voltage V_L is ahead by $\pi/2$, whereas the capacitor voltage V_C is behind by $\pi/2$.



V_L leads I



Current-reference phasors: voltage leads for $X = L$ and lags for $Y = C$.

Step 3: (ii)(II) Derive the resonance condition and frequency Connect L and C in series. Since their voltage phasors point in opposite directions, the magnitude of the net reactive voltage is $I|X_L - X_C|$. At resonance the inductive and capacitive reactances are equal, so the two voltage phasors cancel:

$$\begin{aligned} X_L &= X_C, \\ \omega_0 L &= \frac{1}{\omega_0 C}, \\ \omega_0^2 &= \frac{1}{LC}. \end{aligned}$$

Consequently, the positive resonant angular frequency and ordinary frequency are

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad \nu_0 = \frac{1}{2\pi\sqrt{LC}}.$$

With L in henries and C in farads, ν_0 is in hertz.

Step 4: State the impedance at resonance The question connects only the two ideal reactive elements in series. Their impedance magnitude is therefore

$$Z = \left| \omega L - \frac{1}{\omega C} \right|.$$

At $\omega = \omega_0$, the reactances cancel and the ideal series LC impedance is $Z = 0$. If a practical circuit has

total series resistance R , its impedance is instead

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2},$$

which reduces to its minimum value R at resonance. Thus the zero value refers to the ideal LC combination specified here; any actual series resistance limits the resonant current.

Step 5: Conclusion I_0 is the maximum current magnitude and I_{rms} is the equivalent dc heating current; for a sinusoid, $I_{\text{rms}} = I_0/\sqrt{2}$. Element X is an inductor and Y is a capacitor. Series resonance requires $X_L = X_C$, giving $\nu_0 = 1/(2\pi\sqrt{LC})$. The ideal LC impedance at resonance is $0\ \Omega$; with series resistance R , it is R .

Quick Tip to Solve: Keep current as the horizontal reference: inductor voltage points upward and capacitor voltage downward. Equal opposite reactive voltages give resonance; retain any actual series resistance when stating the impedance.