

CBSE Class 12 Physics Question Paper 2025 with Solution PDF-Set 55/1/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. All questions are **compulsory**.
- (ii) Question paper is divided into **FIVE** sections – Sections **A, B, C, D** and **E**.
- (iii) In **Section A** : Question numbers **1** to **16** are Multiple Choice (MCQ) type questions. Each question carries **1** mark.
- (iv) In **Section B** : Question numbers **17** to **21** are Very Short Answer (VSA) type questions. Each question carries **2** marks.
- (v) In **Section C** : Question numbers **22** to **28** are Short Answer (SA) type questions. Each question carries **3** marks.
- (vi) In **Section D** : Question numbers **29** & **30** are Case Study-Based questions. Each question carries **4** marks.
- (vii) In **Section E** : Question numbers **31** to **33** are Long Answer (LA) type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is NOT allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg.}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg.}$$

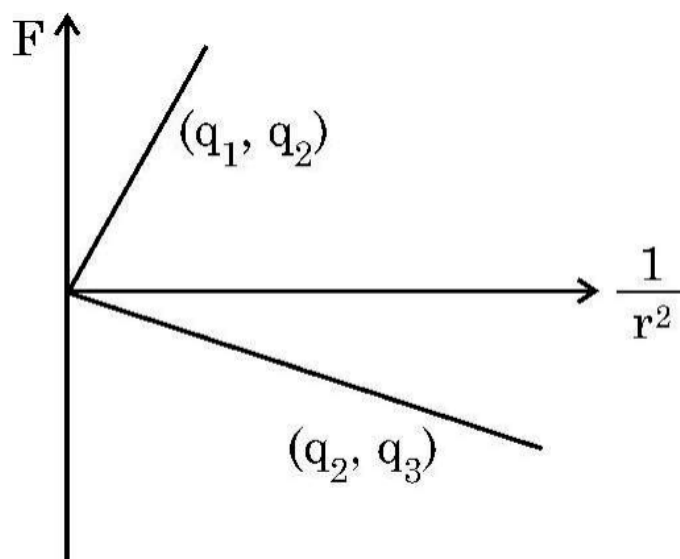
$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg.}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. Figure shows variation of Coulomb force (F) acting between two point charges with $\frac{1}{r^2}$, r being the separation between the two charges (q_1, q_2) and (q_2, q_3). If q_2 is positive and least in magnitude, then the magnitudes of q_1, q_2 and q_3 are such that: [1 mark]



- (A) $q_2 < q_3 < q_1$
- (B) $q_3 < q_1 < q_2$
- (C) $q_1 < q_2 < q_3$
- (D) $q_2 < q_1 < q_3$

Correct Answer: (A) $q_2 < q_3 < q_1$

Solution:

Concept:

- Coulomb's law gives the signed force $F = kq_iq_j/r^2$, where $k = 1/(4\pi\epsilon_0)$ and r is the separation.
- In a graph of F against $1/r^2$, the magnitude of the slope is $k|q_iq_j|$. Compare absolute slopes to compare charge magnitudes.

Step 1: Compare the slopes of the two lines The line for (q_1, q_2) is steeper in magnitude than the line for (q_2, q_3) . Thus $k|q_1q_2| > k|q_2q_3|$. Cancelling the common positive factor $k|q_2|$ gives $|q_1| > |q_3|$. The negative slope of the second line indicates attraction, not a negative charge magnitude.

Step 2: Use the given smallest magnitude The question states that q_2 has the least magnitude. Combining this information with the slope comparison places $|q_2|$ first, $|q_3|$ next and $|q_1|$ last. Options (B) and (C) contradict the stated smallest magnitude; option (D) reverses the slope comparison.

Step 3: Conclusion The magnitudes obey $|q_2| < |q_3| < |q_1|$, written in the options as $q_2 < q_3 < q_1$. Hence, the correct option is (A).

Quick Tip to Solve: For two force-versus- $1/r^2$ lines sharing one charge, cancel that charge and compare the absolute slopes.

2. Two wires P and Q are made of the same material. The wire Q has twice the diameter and half the length as that of wire P . If the resistance of wire P is R , the resistance of the wire Q will be: [1 mark]

- (A) R
- (B) $\frac{R}{2}$
- (C) $\frac{R}{8}$
- (D) $2R$

Correct Answer: (C) $\frac{R}{8}$

Solution:

Concept:

- A uniform wire has resistance $R = \rho L/A$, where ρ is resistivity, L is length and A is cross-sectional area.
- For a circular wire of diameter d , $A = \pi d^2/4$. Wires of the same material at the same temperature have the same resistivity.

Step 1: Find the changes in length and area Let wire P have length L and diameter d . Wire Q has length $L/2$ and diameter $2d$. Since area depends on the square of the diameter, $A_Q = 4A_P$, not $2A_P$.

Step 2: Form the resistance ratio The common resistivity cancels in the ratio:

$$\frac{R_Q}{R_P} = \frac{L_Q}{L_P} \frac{A_P}{A_Q} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}.$$

Option (B) accounts only for the halved length and ignores the increased area. Option (A) would require the length and area factors to cancel, while option (D) incorrectly predicts an increase even though both changes reduce resistance.

Step 3: Conclusion The resistance of wire Q is $R_Q = R/8$. Hence, the correct option is (C).

Quick Tip to Solve: Compare wire resistances using $R \propto L/d^2$: halving length and doubling diameter gives $(1/2)/2^2 = 1/8$.

3. A 1 cm segment of a wire lying along x -axis carries current of 0.5 A along $+x$ direction. A magnetic field $\vec{B} = (0.4 \text{ mT})\hat{j} + (0.6 \text{ mT})\hat{k}$ is switched on in the region. The force acting on the segment is: [1 mark]

- (A) $(2\hat{j} + 3\hat{k}) \text{ mN}$
- (B) $(-3\hat{j} + 2\hat{k}) \mu\text{N}$
- (C) $(6\hat{j} + 4\hat{k}) \text{ mN}$
- (D) $(-4\hat{j} + 6\hat{k}) \mu\text{N}$

Correct Answer: (B) $(-3\hat{j} + 2\hat{k}) \mu\text{N}$

Solution:

Concept:

- The magnetic force on a straight current element is $\vec{F} = I\vec{\ell} \times \vec{B}$, where $\vec{\ell}$ points along the conventional current.
- Use $\hat{i} \times \hat{j} = \hat{k}$ and $\hat{i} \times \hat{k} = -\hat{j}$; also $1 \text{ cm} = 10^{-2} \text{ m}$ and $1 \text{ mT} = 10^{-3} \text{ T}$.

Step 1: Convert the data and form the cross product Here $I = 0.5 \text{ A}$, $\vec{\ell} = 10^{-2}\hat{i} \text{ m}$, and $\vec{B} = (0.4\hat{j} + 0.6\hat{k})10^{-3} \text{ T}$. Therefore,

$$\vec{F} = 0.5 \times 10^{-2} \times 10^{-3}(0.4\hat{k} - 0.6\hat{j}) \text{ N}.$$

Step 2: Evaluate and check directions The components are $F_y = -3 \times 10^{-6} \text{ N}$ and $F_z = 2 \times 10^{-6} \text{ N}$. The field along $+z$ produces force along $-y$, while the field along $+y$ produces force along $+z$. Options (A) and (C) have the wrong sign of the y -component and use millinewtons rather than micronewtons. Option (D) has incorrect component magnitudes.

Step 3: Conclusion The force is $\vec{F} = (-3\hat{j} + 2\hat{k}) \mu\text{N}$. Hence, the correct option is (B).

Quick Tip to Solve: With current along $+x$, write $F_y = -I\ell B_z$ and $F_z = I\ell B_y$ before substituting the values.

4. A coil has 100 turns, each of area 0.05 m^2 and total resistance 1.5Ω . It is inserted at an instant in a magnetic field of 90 mT , with its axis parallel to the field. The charge induced in the coil at that instant is: [1 mark]

- (A) 3.0 mC
- (B) 0.30 C
- (C) 0.45 C
- (D) 1.5 C

Correct Answer: (B) 0.30 C

Solution:

Concept:

- Faraday's law gives $\mathcal{E} = -N d\Phi/dt$, where N is the number of turns and $\Phi = BA \cos \theta$ is flux through one turn.
- Using $I = \mathcal{E}/R$ and integrating over the insertion gives the magnitude of induced charge $Q = N|\Delta\Phi|/R$, independent of the insertion time.

Step 1: Calculate the change in flux linkage Initially the coil is outside the field, so its initial flux is zero. The coil axis is normal to its plane; an axis parallel to the field means $\theta = 0$. With $B = 90 \times 10^{-3} \text{ T}$, the change in flux linkage is

$$N\Delta\Phi = NBA = 100 \times 90 \times 10^{-3} \times 0.05 = 0.45 \text{ Wb-turn}.$$

Step 2: Divide by the resistance The charge passing around the coil during insertion is

$$Q = \frac{0.45}{1.5} = 0.30 \text{ C}.$$

Option (C) uses the numerical flux linkage without dividing by resistance. Option (A) results from omitting the factor of 100 turns. Option (D) does not follow from the flux-change relation. An

instantaneous charge is understood here as the total charge induced during the insertion.

Step 3: Conclusion The induced charge has magnitude 0.30 C. Hence, the correct option is (B).

Quick Tip to Solve: For total induced charge, use $Q = N\Delta\Phi/R$ directly; the coil axis is the area-normal direction.

5. You are required to design an air-filled solenoid of inductance 0.016 H having a length 0.81 m and radius 0.02 m. The number of turns in the solenoid should be: [1 mark]

- (A) 2592
- (B) 2866
- (C) 2976
- (D) 3140

Correct Answer: (B) 2866

Solution:

Concept:

- The self-inductance of a long air-filled solenoid is $L = \mu_0 N^2 A / \ell$, where N is its number of turns, ℓ its length and A its cross-sectional area.
- For a solenoid of radius r , $A = \pi r^2$; air has permeability approximately $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$.

Step 1: Rearrange the inductance formula The radius is 0.02 m, so $A = \pi(0.02)^2 = 4\pi \times 10^{-4} \text{ m}^2$. Solving for the positive number of turns gives

$$N = \sqrt{\frac{L\ell}{\mu_0 A}}.$$

The square root is essential because inductance is proportional to the square of the total number of turns.

Step 2: Substitute and compare the options Substituting the stated inductance and length,

$$N = \sqrt{\frac{0.016 \times 0.81}{(4\pi \times 10^{-7})(4\pi \times 10^{-4})}} = \frac{9000}{\pi} \approx 2865 \approx 2866.$$

Using $\pi = 3.14$ gives approximately 2866 turns. The other proposed counts, 2592, 2976 and 3140, give approximately 0.0131, 0.0173 and 0.0192 H, respectively, rather than the required inductance.

Step 3: Conclusion The required number of turns is approximately 2866. Hence, the correct option is (B).

Quick Tip to Solve: First compute $A = \pi r^2$, then use $N = \sqrt{L\ell/(\mu_0 A)}$; do not substitute radius directly for area.

6. A voltage $v = v_0 \sin \omega t$ applied to a circuit drives a current $i = i_0 \sin(\omega t + \phi)$ in the circuit. The average power consumed in the circuit over a cycle is: [1 mark]

(A) Zero

(B) $i_0 v_0 \cos \phi$

(C) $\frac{i_0 v_0}{2}$

(D) $\frac{i_0 v_0}{2} \cos \phi$

Correct Answer: (D) $\frac{i_0 v_0}{2} \cos \phi$

Solution:

Concept:

- Instantaneous electric power is $p(t) = v(t)i(t)$. Its average over one complete cycle is the average power consumed.
- For sinusoidal voltage and current, $V_{\text{rms}} = v_0/\sqrt{2}$ and $I_{\text{rms}} = i_0/\sqrt{2}$; the phase factor is $\cos \phi$.

Step 1: Multiply voltage and current Put $x = \omega t$. Using $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$, the instantaneous power becomes

$$p = \frac{v_0 i_0}{2} [\cos \phi - \cos(2\omega t + \phi)].$$

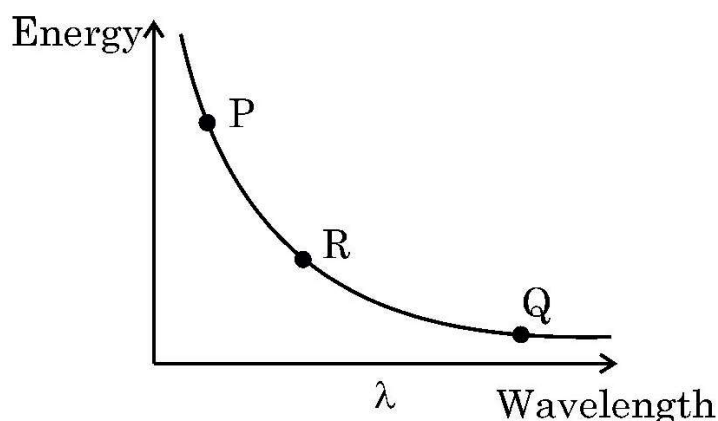
The first term is constant; the second oscillates at twice the supply frequency.

Step 2: Average over a complete cycle The cosine term $\cos(2\omega t + \phi)$ completes two full oscillations during one supply cycle, so its average is zero. Only the constant term contributes. Option (B) misses the factor $1/2$ because the given amplitudes are peak values. Option (C) applies only when $\cos \phi = 1$, and option (A) only when the power factor is zero; neither is the general expression.

Step 3: Conclusion The average power is $P_{\text{av}} = \frac{v_0 i_0}{2} \cos \phi = V_{\text{rms}} I_{\text{rms}} \cos \phi$. Hence, the correct option is (D).

Quick Tip to Solve: If voltage and current are given as peak amplitudes, include the factor $1/2$ as well as the power factor $\cos \phi$.

7. The given diagram exhibits the relationship between the wavelength of the electromagnetic waves and the energy of photon associated with them. The three points P , Q and R marked on the diagram may correspond respectively to: [1 mark]



- (A) X-rays, microwaves, UV radiation
- (B) X-rays, UV radiation, microwaves
- (C) UV radiation, microwaves, X-rays
- (D) Microwaves, UV radiation, X-rays

Correct Answer: (A) X-rays, microwaves, UV radiation

Solution:

Concept:

- A photon's energy is $E = h\nu = hc/\lambda$, where h is Planck's constant, c the speed of light and λ the wavelength.
- Photon energy decreases as wavelength increases. Among the named bands, X-rays have shorter wavelengths than ultraviolet radiation, which has shorter wavelengths than microwaves.

Step 1: Read the point labels carefully Point P lies at the smallest wavelength and highest energy. Point R occupies the intermediate position, while point Q lies at the greatest wavelength and lowest energy. Thus $\lambda_P < \lambda_R < \lambda_Q$ and $E_P > E_R > E_Q$. Notice that the points along the horizontal axis are ordered P, R, Q , although the question asks for P, Q, R .

Step 2: Match the electromagnetic bands The shortest-wavelength point P corresponds to X-rays; the longest-wavelength point Q corresponds to microwaves; the intermediate point R corresponds to UV radiation. Option (B) interchanges Q and R . Option (C) wrongly assigns X-rays to the intermediate point, and option (D) assigns microwaves to the highest-energy point.

Step 3: Conclusion Points P, Q, R correspond respectively to X-rays, microwaves and UV radiation. Hence, the correct option is (A).

Quick Tip to Solve: Rank the points by their plotted wavelength first, then report the bands in the requested order P, Q, R .

8. A beaker is filled with water (refractive index $\frac{4}{3}$) upto a height H . A coin is placed at its bottom. The depth of the coin, when viewed along the near normal direction, will be: [1 mark]

- (A) $\frac{H}{4}$
(B) $\frac{3H}{4}$
(C) H
(D) $\frac{4H}{3}$

Correct Answer: (B) $\frac{3H}{4}$

Solution:

Concept:

- For near-normal viewing from air into water, refractive index is $\mu = \text{real depth}/\text{apparent depth}$.
- Refraction makes the submerged coin appear raised; apparent depth is measured downward from the water surface.

Step 1: Calculate the apparent depth The coin is on the bottom, so its real depth below the surface is H . Using the refractive index of water relative to air,

$$d_{\text{app}} = \frac{H}{\mu} = \frac{H}{4/3} = \frac{3H}{4}.$$

Division by the refractive index is appropriate because the observer is in the rarer medium.

Step 2: Distinguish depth from upward displacement The apparent upward shift is $H - 3H/4 = H/4$, which is option (A), but the question asks for depth, not shift. Option (C) neglects refraction. Option (D) would put the apparent coin deeper than its actual position and results from reversing the depth ratio.

Step 3: Conclusion The apparent depth of the coin is $3H/4$. Hence, the correct option is (B).

Quick Tip to Solve: For a coin viewed from air, divide the water depth by its refractive index; subtract from the real depth only when asked for the apparent shift.

9. The stopping potential V_0 measured in a photoelectric experiment for a metal surface is plotted against frequency ν of the incident radiation. Let m be the slope of the straight line so obtained. Then the value of charge of an electron is given by (h is the Planck's constant.) [1 mark]

- (A) mh
- (B) $\frac{m}{h}$
- (C) $\frac{h}{m}$
- (D) $\frac{1}{mh}$

Correct Answer: (C) $\frac{h}{m}$

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi$, where ϕ is the work function and h is Planck's constant.
- At stopping potential V_0 , the maximum kinetic energy is $K_{\max} = eV_0$, with e the magnitude of the electron charge.

Step 1: Express stopping potential as a linear function Combining the energy relations and dividing by e gives

$$eV_0 = h\nu - \phi, \quad V_0 = \frac{h}{e}\nu - \frac{\phi}{e}.$$

Thus the slope of a graph with V_0 on the vertical axis and ν on the horizontal axis is the coefficient of ν . The work function changes the intercept, not this slope.

Step 2: Rearrange the measured slope Comparing with the straight-line equation gives

$$m = \frac{h}{e} \Rightarrow e = \frac{h}{m}.$$

Option (B), m/h , is the reciprocal of the charge magnitude. Multiplying the slope by h , as in (A), or taking $1/(mh)$, as in (D), does not follow from the slope equation. The positive quantity requested is the electron's charge magnitude; its signed charge is $-e$.

Step 3: Conclusion The magnitude of the electron charge is $e = h/m$. Hence, the correct option is (C).

Quick Tip to Solve: Check which quantity is on each graph axis: the slope of stopping potential against frequency is h/e , whereas the reverse plot has slope e/h .

10. Let λ_e , λ_p and λ_d be the wavelengths associated with an electron, a proton and a deuteron, all moving with the same speed. Then the correct relation between them is: [1 mark]

(A) $\lambda_d > \lambda_p > \lambda_e$

(B) $\lambda_e > \lambda_p > \lambda_d$

(C) $\lambda_p > \lambda_e > \lambda_d$

(D) $\lambda_e = \lambda_p = \lambda_d$

Correct Answer: (B) $\lambda_e > \lambda_p > \lambda_d$

Solution:

Concept:

- The de Broglie wavelength of a particle is $\lambda = h/p$, where h is Planck's constant and p is the particle's momentum.
- For a particle of mass m and speed v , $p = mv$ in the non-relativistic treatment. Thus at the same speed, wavelength varies inversely with mass.

Step 1: Compare the three particle masses An electron is much lighter than a proton. A deuteron is the nucleus of deuterium and contains one proton and one neutron, so it is heavier than a proton. Therefore,

$$m_e < m_p < m_d.$$

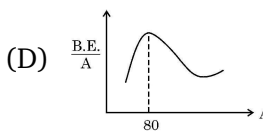
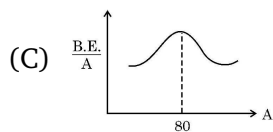
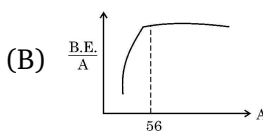
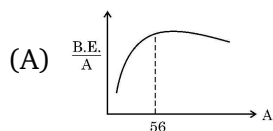
Since their speeds are equal, their momenta increase in this same order.

Step 2: Reverse the mass order for wavelengths The numerator h and speed v are common to all three particles. Hence $\lambda = h/(mv)$ gives the largest wavelength to the lightest particle and the smallest wavelength to the heaviest particle. Option (A) reverses this inverse relation; (C) incorrectly places the proton ahead of the lighter electron; (D) would require equal momenta, not merely equal speeds.

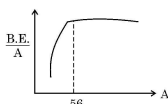
Step 3: Conclusion The wavelength order is $\lambda_e > \lambda_p > \lambda_d$. Hence, the correct option is (B).

Quick Tip to Solve: At equal speed, reverse the mass order to obtain the de Broglie wavelength order: electron first, proton next, deuteron last.

11. Which of the following figures correctly represent the shape of curve of binding energy per nucleon as a function of mass number? [1 mark]



Correct Answer: (B)



Solution:

Concept:

- Binding energy per nucleon is B/A , where B is the total binding energy and A is the mass number. It measures the average energy required to remove a nucleon.
- The empirical B/A curve rises rapidly for light nuclei, reaches about 8.8 MeV near the iron region $A \approx 56$, and varies only slowly for medium and heavy nuclei.

Step 1: Identify the characteristic shape Light nuclei show a steep overall increase of binding energy per nucleon with mass number. Near $A = 56$, the curve reaches its high, nearly flat region. Beyond this region, it decreases gently toward very heavy nuclei. The saturation of the short-range nuclear force accounts for the broad near-constant region, while increasing Coulomb repulsion lowers the value for heavy nuclei.

Step 2: Compare the printed graphs Graph (B) shows the rapid initial rise followed by the broad, almost level region and a small decline. Graph (A) continues rising more noticeably beyond the marked 56, giving a rounded hump and a more pronounced fall instead of the characteristic broad plateau. Graphs (C) and (D) put a prominent maximum near $A = 80$, followed by a deep dip and an upturn, which do not represent the observed trend.

Step 3: Conclusion Graph (B) best represents binding energy per nucleon as a function of mass number. Hence, the correct option is (B).

Quick Tip to Solve: Select the curve that rises rapidly to the iron region near $A = 56$, then stays nearly flat with a gentle decline toward heavy nuclei.

12. When a p - n junction diode is forward biased:

[1 mark]

- (A) the barrier height and the depletion layer width both increase.
- (B) the barrier height increases and the depletion layer width decreases.
- (C) the barrier height and the depletion layer width both decrease.
- (D) the barrier height decreases and the depletion layer width increases.

Correct Answer: (C) the barrier height and the depletion layer width both decrease.

Solution:

Concept:

- The depletion layer of a p - n junction contains fixed ionised donor and acceptor atoms. Its electric field creates a potential barrier to majority-carrier diffusion.
- In forward bias, the p side is connected to the positive terminal and the n side to the negative terminal, so the applied field opposes the junction's built-in field.

Step 1: Determine the effect on the barrier The forward voltage reduces the net potential difference across the depletion region. In the simple junction model, the effective barrier changes from V_{bi} to approximately $V_{bi} - V_F$, where V_F is the applied forward voltage. Majority carriers can therefore cross the junction more easily. The barrier height decreases.

Step 2: Determine the effect on the depletion width The external supply drives electrons on the n side and holes on the p side toward the junction. This reduces the exposed space-charge region, so the depletion layer becomes narrower. Both effects are reductions. Option (A) describes the reverse-bias tendency, while (B) and (D) incorrectly predict opposite changes in barrier height and depletion width.

Step 3: Conclusion In forward bias, the barrier height and depletion layer width both decrease. Hence, the correct option is (C).

Quick Tip to Solve: Remember that forward bias opposes the built-in field: a lower barrier accompanies a thinner depletion layer, allowing a large forward current.

13. Assertion (A): It is difficult to move a magnet into a coil of large number of turns when the circuit of the coil is closed.

Reason (R): The direction of induced current in a coil with its circuit closed, due to motion of a magnet, is such that it opposes the cause. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Concept:

- Faraday's law gives $\mathcal{E} = -N d\Phi/dt$, where N is the number of turns and Φ is the magnetic flux through one turn.
- Lenz's law states that the induced current produces a magnetic effect opposing the change in flux that causes it. A closed conducting circuit allows this current to flow.

Step 1: Evaluate the assertion Moving a magnet into the coil changes the flux linked with its turns and induces an emf. Because the coil circuit is closed, an induced current flows and produces a magnetic field opposing the magnet's approach. An external force must do work against this magnetic opposition, so insertion is difficult. Assertion (A) is true.

Step 2: Evaluate the reason The stated direction of induced current is precisely Lenz's law: its magnetic effect opposes the cause of induction, namely the flux change due to the moving magnet. The opposition is to the change, not necessarily to the original magnetic field itself. Reason (R) is true.

Step 3: Test the explanatory connection As a pole approaches, the nearby face of the coil acts as a like pole and repels it. This opposition explains the extra mechanical effort needed to push the magnet in. Thus the reason directly explains the assertion; option (B) misses that causal link, and (C) and (D) wrongly classify true statements as false.

Step 4: Conclusion Both statements are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: For an approaching magnet and a closed coil, make the near coil face the same pole: the resulting repulsion is Lenz's-law opposition to insertion.

14. Assertion (A): The deflection in a galvanometer is directly proportional to the current passing through it.

Reason (R): The coil of a galvanometer is suspended in a uniform radial magnetic field. [1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Concept:

- A moving-coil galvanometer in a radial magnetic field has magnetic torque $\tau_m = NBA_c I$, where N is the number of turns, B the field strength, A_c the coil area and I the current.
- The suspension provides restoring torque $\tau_r = k\theta$, where k is the torsional constant and θ is the angular deflection.

Step 1: Evaluate the assertion At equilibrium, magnetic torque and restoring torque balance:

$$NBA_c I = k\theta \Rightarrow \theta = \frac{NBA_c}{k} I.$$

The instrument quantities N , B , A_c and k are fixed, so deflection is directly proportional to current. Assertion (A) is true.

Step 2: Evaluate the reason The coil operates in a uniform radial magnetic field produced using shaped pole pieces and a soft-iron core. The field remains parallel to the plane of the coil as it turns, so the angle between its area normal and the field is always 90° . Reason (R) is true.

Step 3: Connect radial geometry to proportional deflection The radial field keeps the sine factor in $\tau_m = NBA_c I \sin \alpha$ equal to one, making the magnetic torque independent of deflection angle. With the suspension's restoring torque proportional to θ , this gives the linear current-deflection relation. Therefore the reason explains the assertion. Option (B) denies this link, whereas (C) and (D) incorrectly call a true statement false.

Step 4: Conclusion Both statements are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: In a radial-field galvanometer, write $NBA_c I = k\theta$ immediately: the radial field removes the angle-dependent sine factor from magnetic torque.

15. Assertion (A): We cannot form a p - n junction diode by taking a slab of a p -type semiconductor and physically joining it to another slab of a n -type semiconductor.

Reason (R): In a p -type semiconductor $n_e \gg n_h$, while in a n -type semiconductor $n_h \gg n_e$. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- A practical p - n junction is formed by creating adjacent p -type and n -type regions within a single semiconductor crystal.
- Holes are the majority carriers in a p -type semiconductor, whereas electrons are the majority carriers in an n -type semiconductor. Here n_h and n_e denote hole and electron number densities.

Step 1: Evaluate the assertion Simply pressing two separately prepared slabs together does not establish the continuous crystal lattice needed for a normal junction. The surfaces contain defects and impurities, and the physical contact is not equivalent to adjoining doped regions in one crystal. Thus the assertion is true.

Step 2: Evaluate the printed reason The correct carrier-density relations are $n_h \gg n_e$ for p -type material and $n_e \gg n_h$ for n -type material. The printed reason states the reverse relations in both cases, so it is false.

Step 3: Test the explanation and the answer codes A false reason cannot explain the true assertion. Options (A) and (B) require both statements to be true, and option (D) requires both to be false; none matches these truth values.

Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Read the subscripts in the reason carefully: p -type means holes dominate, and n -type means electrons dominate.

16. Assertion (A): The potential energy of an electron revolving in any stationary orbit in a hydrogen atom is positive.

Reason (R): The total energy of a charged particle is always positive.

[1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and Reason (R) is also false.

Solution:

Concept:

- With potential energy zero at infinite separation, the electron-proton interaction energy at distance r is $U = -ke^2/r$, where $k = 1/(4\pi\epsilon_0)$ and e is the elementary charge.
- In a Bohr stationary orbit of hydrogen, $K = ke^2/(2r)$ and $E = K + U = -ke^2/(2r)$. Negative total energy signifies a bound electron.

Step 1: Evaluate the assertion The electron and proton have opposite charges, so their electrostatic potential energy is negative at every finite separation when its zero is at infinity. Therefore an electron in any stationary orbit has $U < 0$, and the assertion that this energy is positive is false.

Step 2: Evaluate the reason The total energy is the sum of kinetic and potential energies. Although kinetic energy is non-negative, potential energy can be negative. For a bound hydrogen electron, $U = -2K$, giving $E = -K < 0$. This is a direct counterexample to the claim that the total energy of a charged particle is always positive; the reason is false.

Step 3: Test the explanation and the answer codes Since both statements are false, the reason does not provide a valid explanation. Options (A), (B) and (C) all require a true assertion, so they are excluded.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: For a bound hydrogen electron, remember $U = -2K$ and $E = -K$; positive kinetic energy does not imply positive total energy.

Section B (10 marks)

17. A battery of emf E and internal resistance r is connected to a rheostat. When a current of 2 A is drawn from the battery, the potential difference across the rheostat is 5 V. The potential difference becomes 4 V when a current of 4 A is drawn from the battery. Calculate the value of E and r . [2 marks]

Solution:

Concept:

- A cell delivering current I has terminal potential difference $V = E - Ir$, where E is its emf and r is its internal resistance.
- The rheostat is connected across the battery terminals, so its potential difference is V . The same E and r apply to both current settings.

Step 1: Write the two terminal-voltage equations At the first setting, $I_1 = 2$ A and $V_1 = 5$ V; at the

second, $I_2 = 4\text{ A}$ and $V_2 = 4\text{ V}$. The internal voltage drop must be added to the terminal voltage to recover the emf:

$$E = 5\text{ V} + (2\text{ A})r,$$

$$E = 4\text{ V} + (4\text{ A})r.$$

Step 2: Eliminate the emf and determine the resistance Equating these expressions eliminates the unknown emf. The fall in terminal voltage equals the increase in current multiplied by the internal resistance:

$$r = \frac{V_1 - V_2}{I_2 - I_1} = \frac{5 - 4\text{ V}}{4 - 2\text{ A}} = 0.5\ \Omega.$$

The positive resistance agrees with the observed decrease in terminal voltage as more current is drawn.

Step 3: Substitute back to find the emf Use the first setting to obtain

$$E = 5\text{ V} + (2\text{ A})(0.5\ \Omega) = 6\text{ V}.$$

The second setting gives $6 - (4)(0.5) = 4\text{ V}$, confirming the other observation.

Step 4: Conclusion The battery emf is $E = 6\text{ V}$, and its internal resistance is $r = 0.5\ \Omega$.

Quick Tip to Solve: Find r from the voltage drop divided by the current increase, then use $E = V + Ir$ at either setting.

18(a). In a diffraction experiment, the slit is illuminated by light of wavelength 600 nm . The first minimum of the pattern falls at $\theta = 30^\circ$. Calculate the width of the slit. [2 marks]

Solution:

Concept:

- In single-slit diffraction, dark fringes satisfy $a \sin \theta = m\lambda$, where a is the slit width, θ is measured from the central axis, λ is the wavelength, and $m = 1, 2, \dots$
- The first minimum corresponds to $m = 1$, so $a = \lambda / \sin \theta$. The exact sine is needed here because 30° is not a small angle.

Step 1: Identify the first-minimum condition At the first minimum, the path difference between rays from opposite edges of the slit is one wavelength. The slit can then be divided into two equal halves: corresponding rays from the two halves differ in path by $\lambda/2$ and cancel. Hence,

$$a \sin 30^\circ = \lambda.$$

This is a diffraction minimum, so the single-slit condition is used rather than the double-slit interference condition.

Step 2: Convert the wavelength and calculate the width Convert the given wavelength to SI units: $600\text{ nm} = 600 \times 10^{-9}\text{ m}$. Using $\sin 30^\circ = 1/2$,

$$\begin{aligned} a &= \frac{600 \times 10^{-9}}{1/2}\text{ m} \\ &= 1200 \times 10^{-9}\text{ m} \\ &= 1.2 \times 10^{-6}\text{ m}. \end{aligned}$$

Thus the width is twice the wavelength, as expected directly from the sine value.

Step 3: Conclusion The slit width is $a = 1.2 \times 10^{-6} \text{ m} = 1.2 \mu\text{m}$.

Quick Tip to Solve: For the first single-slit minimum, use $a \sin \theta = \lambda$; at 30° , the slit width is simply 2λ .

OR

18(b). In a Young's double-slit experiment, two light waves, each of intensity I_0 , interfere at a point, having a path difference $\frac{\lambda}{8}$ on the screen. Find the intensity at this point. [2 marks]

Solution:

Concept:

- A path difference Δx between coherent waves of wavelength λ produces phase difference $\phi = 2\pi \Delta x / \lambda$.
- The resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$. For the equal individual intensities $I_1 = I_2 = I_0$, this becomes $I = 2I_0(1 + \cos \phi)$.

Step 1: Convert the printed path difference to phase The given path difference is $\Delta x = \lambda/8$. One full wavelength corresponds to a phase change of 2π , so one-eighth of a wavelength corresponds to

$$\phi = \frac{2\pi}{\lambda} \frac{\lambda}{8} = \frac{\pi}{4}.$$

The waves therefore arrive with a phase difference of 45° , neither exactly in phase nor exactly in opposition.

Step 2: Add the intensities with the interference term Substitute the equal intensities and $\cos(\pi/4) = 1/\sqrt{2}$ into the interference formula:

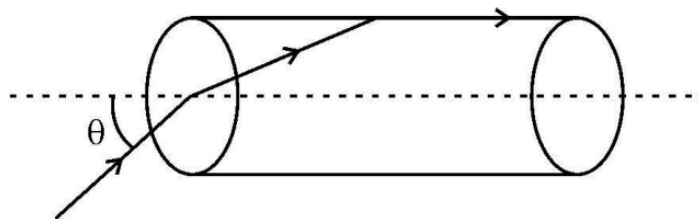
$$\begin{aligned} I &= I_0 + I_0 + 2\sqrt{I_0 I_0} \cos \frac{\pi}{4} \\ &= 2I_0 + \sqrt{2} I_0 \\ &= (2 + \sqrt{2}) I_0. \end{aligned}$$

The positive interference term makes the intensity greater than $2I_0$. It remains below the maximum $4I_0$, consistently with the nonzero phase difference.

Step 3: Conclusion The intensity at the given point is $I = (2 + \sqrt{2}) I_0$, approximately $3.414 I_0$.

Quick Tip to Solve: Convert path difference to phase before substituting: $\lambda/8$ gives $\pi/4$, and equal beams give $I = 2I_0(1 + \cos \phi)$.

19. A transparent solid cylindrical rod (refractive index $\frac{2}{\sqrt{3}}$) is kept in air. A ray of light incident on its face travels along the surface of the rod, as shown in figure. Calculate the angle θ . [2 marks]



Solution:

Concept:

- For refraction from the rod into air, the critical angle C satisfies $\sin C = 1/\mu$, where μ is the refractive index of the rod relative to air.
- Snell's law at the plane entrance face is $\sin \theta = \mu \sin r$, where r is measured from the rod's axis, the normal to this face.

Step 1: Use grazing emergence at the curved surface The ray emerging along the surface makes 90° with the outward normal. Therefore the angle of incidence inside the rod at this surface equals the critical angle. Using the printed value $\mu = 2/\sqrt{3}$,

$$\sin C = \frac{1}{2/\sqrt{3}} = \frac{\sqrt{3}}{2}, \quad C = 60^\circ.$$

The normal to the curved surface in the shown axial plane is perpendicular to the rod's axis. Hence the refracted ray's angle to the axis is

$$r = 90^\circ - C = 30^\circ.$$

Step 2: Apply Snell's law at the entrance face The incident ray enters from air, of refractive index approximately unity, through the flat end face. The marked angle θ is measured from its normal, so

$$1 \sin \theta = \frac{2}{\sqrt{3}} \sin 30^\circ = \frac{1}{\sqrt{3}}.$$

The physical incidence angle lies between 0° and 90° ; choose the acute inverse-sine value.

Step 3: Conclusion The required angle is $\theta = \sin^{-1}(1/\sqrt{3}) \approx 35.3^\circ$.

Quick Tip to Solve: For a ray that emerges along the rod's side, first set the side incidence to the critical angle; then use its complement at the flat entrance face.

20. Prove that, in Bohr model of hydrogen atom, the time period of revolution of an electron in n^{th} orbit is proportional to n^3 . [2 marks]

Solution:

Concept:

- In Bohr's hydrogen model, electrostatic attraction provides the centripetal force: $m_e v_n^2 / r_n = e^2 / (4\pi\epsilon_0 r_n^2)$, where r_n and v_n are the radius and speed in orbit n .
- Angular momentum is quantised: $m_e v_n r_n = nh / (2\pi)$. The revolution period is distance divided by speed, $T_n = 2\pi r_n / v_n$.

Step 1: Obtain the dependence of radius and speed on orbit number From angular-momentum quantisation,

$$v_n = \frac{nh}{2\pi m_e r_n}.$$

Insert this expression in the centripetal-force equation and cancel common factors:

$$\frac{n^2 h^2}{4\pi^2 m_e r_n^3} = \frac{e^2}{4\pi\epsilon_0 r_n^2} \Rightarrow r_n = \frac{\epsilon_0 h^2}{\pi m_e e^2} n^2.$$

Substituting this radius back into the speed expression gives

$$v_n = \frac{e^2}{2\epsilon_0 h} \frac{1}{n}.$$

Thus the orbit circumference grows as n^2 , while the electron's orbital speed decreases as n^{-1} .

Step 2: Combine radius and speed to find the period For one complete revolution the electron travels the circumference $2\pi r_n$. Therefore

$$\begin{aligned} T_n &= \frac{2\pi r_n}{v_n} \\ &= 2\pi \left(\frac{\epsilon_0 h^2 n^2}{\pi m_e e^2} \right) \left(\frac{2\epsilon_0 h n}{e^2} \right) \\ &= \frac{4\epsilon_0^2 h^3}{m_e e^4} n^3. \end{aligned}$$

Here ϵ_0 , h , m_e and e are constants independent of the orbit number.

Step 3: Conclusion In the Bohr model of hydrogen, $T_n \propto n^3$, as required.

Quick Tip to Solve: Recall $r_n \propto n^2$ and $v_n \propto n^{-1}$; in $T_n = 2\pi r_n / v_n$, subtract the exponents to obtain $2 - (-1) = 3$.

21. A *p*-type Si semiconductor is made by doping an average of one dopant atom per 5×10^7 silicon atoms. If the number density of silicon atoms in the specimen is 5×10^{28} atoms/m³, find the number of holes created per cubic centimetre in the specimen due to doping. Also give one example of such dopants. [2 marks]

Solution:

Concept:

- A trivalent acceptor impurity produces *p*-type silicon. In the usual complete-ionisation approximation, each dopant atom creates one hole, so the doping-created hole density equals the acceptor density.
- Number density is the number of particles per unit volume. Since $1 \text{ m}^3 = 10^6 \text{ cm}^3$, a density in m^{-3} is divided by 10^6 to express it in cm^{-3} .

Step 1: Calculate the acceptor density Let $N_{\text{Si}} = 5 \times 10^{28} \text{ m}^{-3}$ be the silicon atom density. One dopant atom is added for every 5×10^7 silicon atoms. Hence the acceptor density is

$$N_A = \frac{N_{\text{Si}}}{5 \times 10^7} = \frac{5 \times 10^{28}}{5 \times 10^7} = 10^{21} \text{ m}^{-3}.$$

With one hole supplied per acceptor, the number of holes created by doping per cubic metre is also 10^{21} .

Step 2: Convert the density and identify a suitable dopant A cubic centimetre occupies 10^{-6} m^3 , so the number of doping-created holes in that volume is

$$N_h = (10^{21} \text{ m}^{-3})(10^{-6} \text{ m}^3) = 10^{15}.$$

Boron is a suitable dopant. Its three valence electrons form only three complete covalent bonds with neighbouring silicon atoms; the electron deficiency associated with the fourth bond gives rise to a hole. Thus it acts as an acceptor, making holes the majority charge carriers.

Step 3: Conclusion The specimen has 10^{15} doping-created holes per cm^3 . One suitable trivalent dopant is boron.

Quick Tip to Solve: First divide the silicon density by the number of silicon atoms per dopant; then divide by 10^6 to convert holes per cubic metre into holes per cubic centimetre.

Section C (21 marks)

22(a). Two batteries of emfs 3 V & 6 V and internal resistances 0.2Ω & 0.4Ω are connected in parallel. This combination is connected to a 4Ω resistor. Find:

- the equivalent emf of the combination
- the equivalent internal resistance of the combination
- the current drawn from the combination

[3 marks]

Solution:

Concept:

- For two batteries connected in parallel with like terminals joined, the equivalent emf is $E_{\text{eq}} = (E_1/r_1 + E_2/r_2)/(1/r_1 + 1/r_2)$, where E_1, E_2 and r_1, r_2 are their emfs and internal resistances.

- Their equivalent internal resistance is $r_{eq} = r_1 r_2 / (r_1 + r_2)$. The net current through an external resistance R is $I = E_{eq} / (R + r_{eq})$.

Step 1: Find the equivalent emf of the parallel combination Here $E_1 = 3\text{ V}$, $r_1 = 0.2\ \Omega$, $E_2 = 6\text{ V}$, and $r_2 = 0.4\ \Omega$. At a common terminal voltage V , the net current supplied by the combination is

$$I = \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2}.$$

Comparing this with $I = (E_{eq} - V) / r_{eq}$ gives the equivalent-source formula. For part (i),

$$E_{eq} = \frac{3/0.2 + 6/0.4}{1/0.2 + 1/0.4} = \frac{15 + 15}{5 + 2.5} = 4\text{ V}.$$

The emfs are weighted by the reciprocals of their internal resistances; they are not simply added or averaged arithmetically.

Step 2: Combine the internal resistances in parallel For part (ii), replace the ideal emf sources by zero-emf sources to obtain the resistance seen at the terminals. The internal resistances are then in parallel:

$$r_{eq} = \frac{0.2 \times 0.4}{0.2 + 0.4} = \frac{0.08}{0.6} = \frac{2}{15}\ \Omega \approx 0.133\ \Omega.$$

This is smaller than either individual internal resistance, as expected for parallel resistors.

Step 3: Calculate the current through the external resistor For part (iii), the external resistance $R = 4\ \Omega$ is in series with the internal resistance of the equivalent battery. Using its emf in Ohm's law gives

$$I = \frac{4}{4 + 2/15} = \frac{4}{62/15} = \frac{30}{31}\text{ A} \approx 0.968\text{ A}.$$

This is the net current drawn from the whole combination by the load, rather than the current in either battery branch separately.

Step 4: Conclusion The results are (i) $E_{eq} = 4\text{ V}$, (ii) $r_{eq} \approx 0.133\ \Omega$, and (iii) $I \approx 0.968\text{ A}$.

Quick Tip to Solve: For unequal cells in parallel, first calculate the resistance-weighted equivalent emf; keep $r_{eq} = 2/15\ \Omega$ as a fraction until the final current calculation.

22(b). (i) A conductor of length l is connected across an ideal cell of emf E . Keeping the cell connected, the length of the conductor is increased to $2l$ by gradually stretching it. If R and R' are initial and final values of resistance and v_d and v'_d are initial and final values of drift velocity, find the relation between (i) R' and R and (ii) v'_d and v_d .

(ii) When electrons drift in a conductor from lower to higher potential, does it mean that all the 'free electrons' of the conductor are moving in the same direction? [3 marks]

Solution:

Concept:

- For a uniform conductor, $R = \rho l/A$, where ρ is resistivity, l is length and A is cross-sectional area. On stretching, its volume Al remains constant; take temperature and resistivity as unchanged.
- The current is $I = neAv_d$, where n is the free-electron number density, e is the magnitude of electron charge and v_d is the magnitude of drift velocity. An ideal cell keeps the potential difference equal to its emf E .
- Drift velocity is the small average velocity superposed on the random thermal motion of electrons; it is not the instantaneous velocity of every electron.

Step 1: (i) Use constant volume to find the final resistance The length changes from l to $l' = 2l$. The conductor becomes thinner because its material volume stays constant:

$$Al = A'l' = A'(2l) \Rightarrow A' = \frac{A}{2}.$$

Substituting the new length and area into the resistance formula gives

$$R' = \frac{\rho(2l)}{A/2} = 4\frac{\rho l}{A} = 4R.$$

Both the doubled length and the halved cross-sectional area increase the resistance.

Step 2: (i) Compare currents and drift velocities Because the cell remains connected and has no internal resistance, the potential difference stays E . Ohm's law therefore gives

$$I = \frac{E}{R}, \quad I' = \frac{E}{R'} = \frac{I}{4}.$$

Stretching does not change the number density n . Divide the two forms of $v_d = I/(neA)$:

$$\frac{v'_d}{v_d} = \frac{I'}{I} \frac{A}{A'} = \frac{1}{4} \times 2 = \frac{1}{2}.$$

As a check, the electric field decreases from E/l to $E/(2l)$, which halves the drift speed for unchanged relaxation time.

Step 3: (ii) Distinguish drift from random electron motion No. Free electrons continue to move randomly in all directions with large thermal velocities and suffer collisions. The applied electric field adds a small systematic average velocity opposite to the field. Since electrons have negative charge,

this net drift is from lower to higher potential. Individual electrons need not move in that direction at a given instant; only the ensemble average has the drift direction.

Step 4: Conclusion The stretched conductor has $R' = 4R$ and $v'_d = v_d/2$. All free electrons do not move in the same direction: their random motion persists, with a net drift towards higher potential.

Quick Tip to Solve: For stretching at fixed voltage, first use constant volume to change the area; resistance varies as length squared, while drift speed varies inversely with length.

23. Using Biot-Savart law, derive expression for the magnetic field ($\vec{B}$) due to a circular current carrying loop at a point on its axis and hence at its centre. [3 marks]

Solution:

Concept:

- Biot-Savart law gives the field of a current element: $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^3}$, where $\vec{r}$ points from the element to the observation point and μ_0 is the permeability of free space.
- For a circular loop, transverse field components from diametrically opposite elements cancel on its axis. The axial components add, with their direction fixed by the right-hand thumb rule.

Step 1: Define the geometry and apply Biot-Savart law Consider a circular loop of radius a , carrying current I , with centre O . Let P lie on its axis at distance x from O . Choose $\hat{n}$ along the normal given by the curled right-hand fingers following the current. Every current element is at the same distance from P :

$$r = \sqrt{a^2 + x^2}.$$

The tangent element $d\vec{\ell}$ is perpendicular both to the radius in the loop plane and to its axis. Thus it is perpendicular to the vector $\vec{r}$ joining the element to P . Biot-Savart law reduces to

$$dB = \frac{\mu_0 I d\ell}{4\pi r^2}.$$

Step 2: Resolve along the axis and integrate around the loop Let α be the angle between $d\vec{B}$ and the loop axis. Since $d\vec{B}$ is perpendicular to $\vec{r}$, the right-triangle geometry gives $\cos \alpha = a/r$. Opposite elements produce equal and opposite transverse components, leaving the axial contribution

$$dB_{\text{axis}} = dB \cos \alpha = \frac{\mu_0 I a d\ell}{4\pi r^3}.$$

The factors I , a and r are constant over the loop, while $\oint d\ell = 2\pi a$. Therefore

$$\begin{aligned} B_{\text{axis}} &= \frac{\mu_0 I a}{4\pi (a^2 + x^2)^{3/2}} \oint d\ell \\ &= \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}. \end{aligned}$$

The field points along $\hat{n}$; reversing the current reverses this direction.

Step 3: Set the axial distance to zero for the centre At the centre, $x = 0$, so the general result becomes

$$B_O = \frac{\mu_0 I a^2}{2(a^2)^{3/2}} = \frac{\mu_0 I}{2a}.$$

All elementary fields are perpendicular to the loop plane and have the same sense at the centre. If the current appears anticlockwise to an observer, the field points towards that observer; for clockwise current it points away.

Step 4: Conclusion For one circular loop,

$$\vec{B}(x) = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}} \hat{n}, \quad \vec{B}_O = \frac{\mu_0 I}{2a} \hat{n}.$$

Here $\hat{n}$ is the normal fixed by the right-hand rule, and the field is measured in tesla.

Quick Tip to Solve: On the axis, integrate only the axial component: the extra projection factor is a/r . Check the result by putting $x = 0$ to recover $\mu_0 I / (2a)$.

- 24. (a)** Show that the energy required to build up the current I in a coil of inductance L is $\frac{1}{2}LI^2$.
(b) Considering the case of magnetic field produced by air-filled current carrying solenoid, show that the magnetic energy density of a magnetic field B is $\frac{B^2}{2\mu_0}$. [3 marks]

Solution:

Concept:

- A coil of constant inductance L opposes a changing current through the back emf $\varepsilon_{\text{ind}} = -L di/dt$. Work done against this emf is stored as magnetic field energy.
- For a long air-filled solenoid, $B = \mu_0 nI$ and $L = \mu_0 n^2 A\ell$, where $n = N/\ell$ is the turns per unit length, A is the cross-sectional area and ℓ is its length. End effects are neglected.
- Magnetic energy density is the stored magnetic energy divided by the volume occupied by the uniform field: $u_B = U/(A\ell)$, in J m^{-3} .

Step 1: (a) Calculate the work against the induced emf Let the instantaneous current be i , increasing from zero towards I . Lenz's law makes the induced emf oppose this increase. The external source must supply the opposing voltage $L di/dt$. In time dt , the charge passing through the coil is $dq = i dt$, so the work stored in the magnetic field is

$$dU = \left(L \frac{di}{dt} \right) dq = \left(L \frac{di}{dt} \right) i dt = Li di.$$

This is the work against self-induction; any Joule heat produced by coil resistance is a separate energy expenditure.

Step 2: Integrate as the current builds up For a constant inductance and an initially unenergised coil, integrate from $i = 0$ to the final current I :

$$U = \int_0^I Li di = L \left[\frac{i^2}{2} \right]_0^I = \frac{1}{2} LI^2.$$

The energy is positive despite the negative sign of the induced emf, because the source does work against that emf. It can be returned to the circuit when the magnetic field decreases.

Step 3: (b) Express the solenoid energy per unit volume The nearly uniform field inside a long

solenoid links flux $\Phi = BA$ with each turn. Its total flux linkage gives

$$LI = N\Phi = NBA, \quad L = \frac{N(\mu_0 nI)A}{I} = \mu_0 n^2 A\ell.$$

Substituting this inductance into the energy formula and dividing by the field volume $A\ell$, we obtain

$$u_B = \frac{U}{A\ell} = \frac{\frac{1}{2}\mu_0 n^2 A\ell I^2}{A\ell} = \frac{1}{2}\mu_0 n^2 I^2.$$

Finally, $B = \mu_0 nI$ implies $nI = B/\mu_0$, so

$$u_B = \frac{1}{2}\mu_0 \left(\frac{B}{\mu_0}\right)^2 = \frac{B^2}{2\mu_0}.$$

Step 4: Conclusion The work stored magnetically in the coil is $U = \frac{1}{2}LI^2$, measured in joules. For an air-filled solenoid, the magnetic energy density is $u_B = B^2/(2\mu_0)$, measured in J m^{-3} .

Quick Tip to Solve: Derive the factor $1/2$ by integrating $Li \, di$; for energy density, divide by $A\ell$ before replacing nI with B/μ_0 .

25. (a) A parallel plate capacitor is charged by an ac source. Show that the sum of conduction current (I_c) and the displacement current (I_d) has the same value at all points of the circuit.

(b) In case (a) above, is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain. [3 marks]

Solution:

Concept:

- Conduction current is the rate of flow of charge through a conductor: $I_c = dQ/dt$, where Q is the charge delivered to a capacitor plate.
- Maxwell's displacement current is $I_d = \epsilon_0 d\Phi_E/dt$, where Φ_E is the electric flux between the plates. It is associated with a changing electric field.
- For an ideal parallel-plate capacitor in air or vacuum, neglecting fringing, $E = Q/(\epsilon_0 A)$, with A the plate area. The total current is $I_c + I_d$.

Step 1: (a) Calculate the displacement current between the plates The ac source changes the instantaneous plate charge $Q(t)$. The electric field in the gap is uniform in the ideal approximation, so the electric flux through the plate area is

$$\Phi_E = EA = \frac{Q}{\epsilon_0 A} A = \frac{Q}{\epsilon_0}.$$

Consequently,

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{d}{dt} \left(\frac{Q}{\epsilon_0} \right) = \frac{dQ}{dt} = I_c \text{ (in the connecting wire).}$$

The equality holds with consistent current directions, including when the ac current reverses.

Step 2: Compare the total current at different circuit sections In an ideal conducting lead, the current is conduction current and the displacement contribution is negligible. In the insulating gap, no

charge crosses from one plate to the other, so conduction current is zero and the current is entirely displacement current. Thus, at the same instant,

$$(I_c + I_d)_{\text{wire}} = \frac{dQ}{dt} + 0,$$

$$(I_c + I_d)_{\text{gap}} = 0 + \frac{dQ}{dt}.$$

Thus the instantaneous total current is spatially continuous, although it varies with time.

Step 3: (b) Apply the junction rule at each capacitor plate Kirchhoff's first rule remains valid at each plate when both kinds of current are included. The conduction current in each lead matches the displacement current in the gap, so the algebraic sum of total currents at either plate is zero. For conduction current alone, the imbalance equals the rate of charge accumulation on the plate, so the usual zero-sum statement does not apply. Displacement current restores continuity; it does not mean electrons cross the insulating gap.

Step 4: Conclusion $I_c + I_d = dQ/dt$ has the same value at all circuit sections at a given instant. Kirchhoff's junction rule is valid at both plates for this total current; conduction current alone requires allowance for charge accumulation.

Quick Tip to Solve: First use $\Phi_E = Q/\epsilon_0$. Then compare a section through the wire with one through the gap; do not add the wire current and gap current as if they occurred at the same section.

26. Answer the following giving reason:

- All the photo electrons do not eject with the same kinetic energy when monochromatic light is incident on a metal surface.
- The saturation current in case (a) is different for different intensity.
- If one goes on increasing the wavelength of light incident on a metal surface, keeping its intensity constant, emission of photo electrons stops at a certain wavelength for this metal. [3 marks]

Solution:

Concept:

- A photon of frequency ν and wavelength λ has energy $E = h\nu = hc/\lambda$, where h is Planck's constant and c is the speed of light.
- Einstein's photoelectric equation is $K_{\text{max}} = h\nu - \phi$, where ϕ is the metal's work function, the minimum energy required to liberate an electron.
- Saturation current measures the total number of emitted photoelectrons collected per second. The threshold wavelength is $\lambda_0 = hc/\phi$.

Step 1: (a) Explain the spread of photoelectron energies Monochromatic light supplies photons of the same energy $h\nu$, but electrons in the metal have different initial energies and occupy different depths. Electrons travelling to the surface can lose different amounts of energy in collisions. Hence their kinetic energies on escape differ. Electrons requiring the least escape energy and suffering no additional loss emerge with the maximum kinetic energy

$$K_{\text{max}} = h\nu - \phi.$$

Other emitted electrons have less kinetic energy; this equation specifies only the maximum.

Step 2: (b) Relate saturation current to intensity At fixed frequency above threshold, greater intensity supplies more photons per second to the same area, so more photoelectrons are emitted. At saturation all emitted electrons are collected. For emission rate $\dot{N}_e$,

$$I_s = e\dot{N}_e.$$

Thus saturation current increases with intensity, while maximum kinetic energy is unchanged.

Step 3: (c) Apply the threshold condition as wavelength increases Increasing wavelength reduces the energy of each photon because $E = hc/\lambda$. Photoemission requires the energy of a single photon to be at least the work function:

$$\frac{hc}{\lambda} \geq \phi \implies \lambda \leq \lambda_0 = \frac{hc}{\phi}.$$

At threshold $K_{\max} = 0$; beyond λ_0 , photoemission stops. Constant intensity cannot compensate for insufficient energy per photon.

Step 4: Conclusion Photoelectron energies differ because electrons have different initial energies and energy losses. Saturation current increases with intensity at fixed frequency. Emission ceases beyond the metal's threshold wavelength $\lambda_0 = hc/\phi$.

Quick Tip to Solve: Separate photon energy from photon number: frequency controls $K_{\max}$ and the threshold, while intensity at fixed frequency controls the saturation current.

27. (a) Define 'Mass defect' and 'Binding energy' of a nucleus. Describe 'Fission process' on the basis of binding energy per nucleon.

(b) A deuteron contains a proton and a neutron and has a mass of 2.013553 u. Calculate the mass defect for it in u and its energy equivalence in MeV. ($m_p = 1.007277$ u, $m_n = 1.008665$ u, $1 \text{ u} = 931.5 \text{ MeV}/c^2$)

[3 marks]

Solution:

Concept:

- The mass defect of a nucleus is $\Delta m = Zm_p + (A - Z)m_n - M$, where Z and A are its atomic and mass numbers, and M is its nuclear mass.
- Nuclear binding energy is $B = \Delta mc^2$. Greater binding energy per nucleon, B/A , generally signifies greater nuclear stability.
- For the given masses, $1 \text{ u } c^2 = 931.5 \text{ MeV}$; this converts a mass defect in atomic mass units directly into its energy equivalent.

Step 1: (a) Define mass defect and binding energy Mass defect is the difference between the sum of the rest masses of the free protons and neutrons that constitute a nucleus and the actual rest mass of that nucleus. The nucleus has the smaller mass. Binding energy is the minimum energy required to separate the nucleus completely into its constituent free nucleons. It is also the energy released when those nucleons assemble into the nucleus. The mass decrease and released energy are related by $B = \Delta mc^2$.

Step 2: Explain fission using binding energy per nucleon In nuclear fission a heavy nucleus splits into two lighter nuclei, usually with emission of neutrons. Heavy nuclei have lower binding energy per nucleon than typical medium-mass fission fragments. The products therefore have greater total binding energy than the original nucleus, even after the emitted neutrons are included in the nucleon

count. The increase in total binding energy appears as released energy, mainly kinetic energy of the fragments and neutrons:

$$Q = B_{\text{products}} - B_{\text{initial}} > 0.$$

Thus the products are more tightly bound per nucleon, and their combined rest mass is smaller.

Step 3: (b) Subtract the deuteron mass from the free-nucleon masses A deuteron consists of one proton and one neutron. Use their nuclear masses as given; electron masses are not involved:

$$\begin{aligned} m_p + m_n &= (1.007277 + 1.008665) \text{ u} = 2.015942 \text{ u}, \\ \Delta m &= (2.015942 - 2.013553) \text{ u} = 0.002389 \text{ u}. \end{aligned}$$

Step 4: Convert the mass defect into energy Multiplying by the given energy equivalent of one atomic mass unit gives

$$\begin{aligned} B &= \Delta mc^2 = 0.002389 \times 931.5 \text{ MeV} \\ &= 2.2253535 \text{ MeV} \simeq 2.225 \text{ MeV}. \end{aligned}$$

This is the total binding energy of the deuteron, not the binding energy per nucleon.

Step 5: Conclusion Fission releases energy because its products have greater total binding energy. The deuteron's mass defect is 0.002389 u, with energy equivalent 2.225 MeV (approximately 2.23 MeV).

Quick Tip to Solve: For a deuteron, add exactly one proton mass and one neutron mass, subtract its nuclear mass, and multiply by 931.5 to obtain the binding energy in MeV.

28. (a) Draw circuit arrangement for studying V - I characteristics of a p - n junction diode.

(b) Show the shape of the characteristics of a diode.

(c) Mention two information that you can get from these characteristics.

[3 marks]

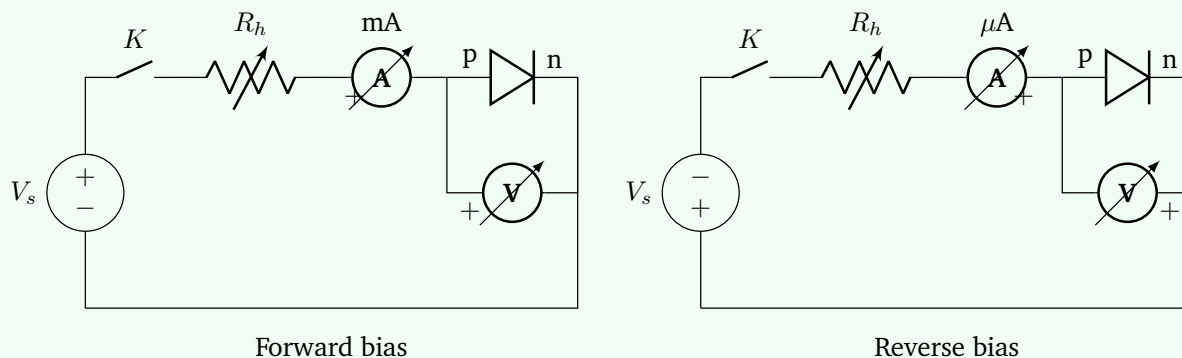
Solution:

Concept:

- A junction diode conducts appreciably when its p-side (anode) is positive relative to its n-side (cathode). In reverse bias only a small reverse saturation current flows until breakdown.
- A voltmeter measures the voltage across the diode in parallel; an ammeter measures the diode current in series. The characteristic is the graph of diode current I against diode voltage V .

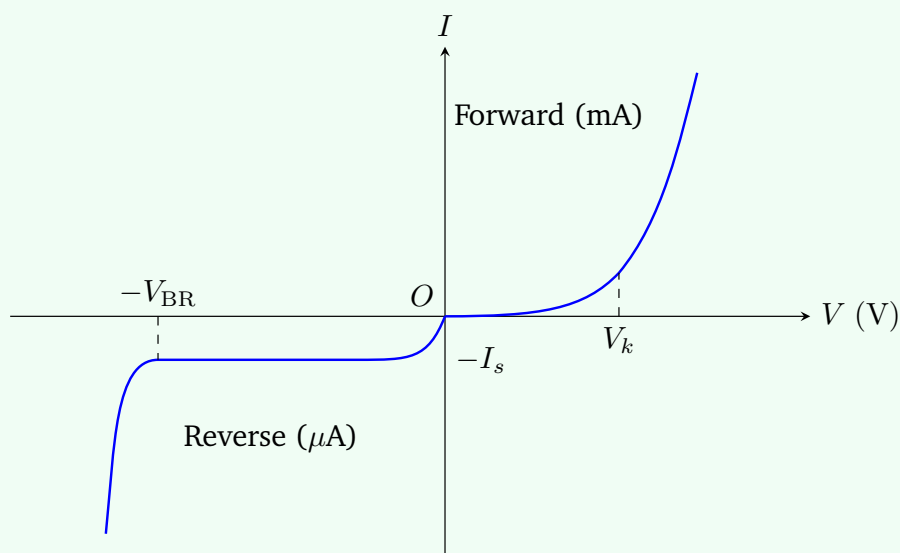
Step 1: (a) Connect the circuits for both biases Connect a variable dc supply, key K , rheostat R_h , current meter and diode in series, with a voltmeter across the diode. For forward bias connect the positive supply terminal to the p-side through the meter and rheostat, and the negative terminal to the n-side. Use a milliammeter, with its positive terminal towards the positive supply. Increase the voltage gradually and record the corresponding diode voltage and current. For reverse bias reverse the supply polarity, use a microammeter to resolve the much smaller current, and reverse the meter connections so that both meters read positive magnitudes. Keep the applied reverse voltage below the

diode's safe rating. The rheostat limits current, especially near the steep forward region.



Forward and reverse bias circuits; meter polarity signs show the positive terminals.

Step 2: (b) Plot the characteristic Plot forward voltage and current in the first quadrant. The current remains small initially, then rises rapidly beyond the knee voltage V_k . With the signed convention $V = V_p - V_n$, reverse voltage and current are negative, so the reverse branch is in the third quadrant. The small reverse saturation current $-I_s$ is nearly constant until the reverse breakdown voltage $-V_{BR}$, after which the current magnitude rises sharply. The sketch is schematic; forward and reverse current scales are generally different because their magnitudes differ greatly.



Schematic diode characteristic, with separate magnitudes of current in forward and reverse bias.

Step 3: (c) Read two useful quantities First, locate the knee (cut-in) voltage from the bend of the forward curve: it indicates when appreciable forward conduction begins, typically about 0.7 V for silicon or 0.3 V for germanium. Second, find the dynamic resistance at a chosen operating point from the local slope:

$$r_d = \frac{\Delta V}{\Delta I}.$$

Choose two nearby points around that operating point; the inverse slope of the I -versus- V curve gives r_d . The steep conducting forward branch therefore has low dynamic resistance. These readings

characterize the diode rather than the external series resistor.

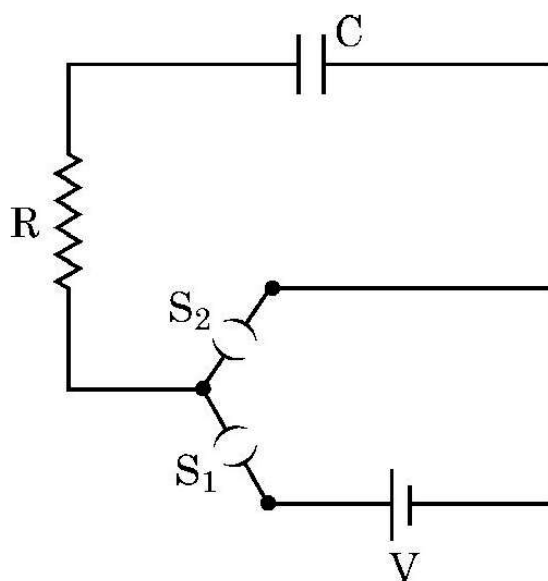
Step 4: Conclusion The series current meter and parallel voltmeter give the diode's forward and reverse V - I curves. The curves reveal the knee voltage and the dynamic resistance at a selected operating point.

Quick Tip to Solve: Put the voltmeter across only the diode, and use a microammeter for reverse bias; read resistance as inverse slope, since current is on the vertical axis.

Section D (8 marks)

Question numbers 29 and 30 are case study based questions. Read the following paragraphs and answer the questions that follow.

29. Case Study (Q29): A circuit consisting of a capacitor C , a resistor of resistance R and an ideal battery of emf V , as shown in figure, is known as an RC series circuit. As soon as the circuit is completed by closing key S_1 (keeping S_2 open), charges begin to flow between the capacitor plates and the battery terminals. The charge on the capacitor increases and consequently the potential difference $V_c (= q/C)$ across the capacitor also increases with time. When this potential difference equals the potential difference across the battery, the capacitor is fully charged ($Q = VC$). During this process of charging, the charge q on the capacitor changes with time t as $q = Q [1 - e^{-t/RC}]$. The charging current can be obtained by differentiating it and using $\frac{d}{dx}(e^{mx}) = m e^{mx}$. Consider the case when $R = 20 \text{ k}\Omega$, $C = 500 \text{ }\mu\text{F}$ and $V = 10 \text{ V}$. [4 marks]



29(i). (i) The final charge on the capacitor, when key S_1 is closed and S_2 is open, is: [1 mark]

- (A) $5 \text{ }\mu\text{C}$
- (B) 5 mC
- (C) 25 mC
- (D) 0.1 C

Correct Answer: (B) 5 mC

Solution:

Concept:

- At full charge the capacitor voltage equals the ideal battery emf, so $Q = CV$, where C is capacitance and V is the applied voltage.
- The series resistor sets the charging rate, not the final charge; $1 \text{ }\mu\text{F} = 10^{-6} \text{ F}$ and $1 \text{ mC} = 10^{-3} \text{ C}$.

Step 1: Apply the final steady state condition Closing S_1 with S_2 open connects the battery to the capacitor through R . After a sufficiently long time the charging current tends to zero. The resistor then has zero voltage drop, leaving the full 10 V across the capacitor.

Step 2: Calculate and compare the options Convert the capacitance before multiplying:

$$Q = (500 \times 10^{-6} \text{ F})(10 \text{ V}) = 5 \times 10^{-3} \text{ C} = 5 \text{ mC}.$$

Option (A) is a thousand times too small because of a prefix error. Options (C) and (D) exceed CV and would require a larger battery voltage; none describes the given steady state.

Step 3: Conclusion The final charge is 5 mC. Hence, the correct option is (B).

Quick Tip to Solve: Multiply capacitance in microfarads by volts to get microcoulombs: $500 \times 10 = 5000 \mu\text{C} = 5 \text{ mC}$.

29(ii). (ii) For sufficient time the key S_1 is closed and S_2 is open. Now S_2 is closed and S_1 is open. What is the final charge on the capacitor? [1 mark]

- (A) Zero
- (B) 5 mC
- (C) 2.5 mC
- (D) $5 \mu\text{C}$

Correct Answer: (A) Zero

Solution:

Concept:

- When a charged capacitor is connected across a resistor without a battery, it discharges: $q(t) = Qe^{-t/(RC)}$, where Q is its initial charge.
- At the final steady state no current flows through the resistor; its potential difference and the capacitor's potential difference both tend to zero.

Step 1: Identify the new circuit Initially the closed S_1 charges the capacitor to $Q = CV = 5 \text{ mC}$. Opening S_1 removes the battery. Closing S_2 completes a loop containing only the capacitor and resistor, so the stored charge can flow through the resistor.

Step 2: Take the long time limit Here $RC = (20 \times 10^3)(500 \times 10^{-6}) = 10 \text{ s}$. Thus

$$\lim_{t \rightarrow \infty} q(t) = Q \lim_{t \rightarrow \infty} e^{-t/(10 \text{ s})} = 0.$$

The 5 mC in (B) is the charge just before discharge begins. The smaller nonzero values in (C) and (D) can occur at intermediate times, but neither is the final charge after complete discharge.

Step 3: Conclusion The final charge on the capacitor is zero. Hence, the correct option is (A).

Quick Tip to Solve: Trace the loop after changing the switches: a capacitor left across a resistor with no battery eventually discharges completely.

29(iii). (iii) The dimensional formula for RC is:

[1 mark]

- (A) $[ML^2T^{-3}A^{-2}]$
- (B) $[M^0L^0T^{-1}A^0]$
- (C) $[M^{-1}L^{-2}T^4A^2]$
- (D) $[M^0L^0TA^0]$

Correct Answer: (D) $[M^0L^0TA^0]$

Solution:

Concept:

- Resistance is potential difference divided by current: $R = V/I$. Capacitance is charge divided by potential difference: $C = q/V$.
- The exponent of an exponential must be dimensionless. Therefore the ratio $t/(RC)$ in the charging law requires RC to have the dimensions of time.

Step 1: Multiply the defining dimensions Eliminate voltage from the product of the definitions:

$$[RC] = \left[\frac{V}{I} \frac{q}{V} \right] = \left[\frac{q}{I} \right] = \frac{[AT]}{[A]} = [T].$$

Here the symbol A in a dimensional formula denotes the electric current dimension, and charge has the dimension current multiplied by time.

Step 2: Check each proposed formula Option (A), $[ML^2T^{-3}A^{-2}]$, is the dimension of resistance alone. Option (C), $[M^{-1}L^{-2}T^4A^2]$, is the dimension of capacitance alone. Their product cancels mass, length and current, leaving one power of time. Option (B) instead represents inverse time and would make $t/(RC)$ dimensional.

Step 3: Conclusion The dimensional formula of RC is $[M^0L^0TA^0]$. Hence, the correct option is (D).

Quick Tip to Solve: Read the exponent first: whenever the law contains $e^{-t/\tau}$, the quantity τ must have units of time.

29(iv). (iv) The key S_1 is closed and S_2 is open. The value of current in the resistor after 5 seconds is:

[1 mark]

- (A) $\frac{1}{2\sqrt{e}}$ mA
- (B) $\sqrt{e}$ mA
- (C) $\frac{1}{\sqrt{e}}$ mA
- (D) $\frac{1}{2e}$ mA

Correct Answer: (A) $\frac{1}{2\sqrt{e}}$ mA

Solution:

Concept:

- Charging current is the time derivative of charge: $i = dq/dt$. For $q = Q[1 - e^{-t/(RC)}]$, this gives $i = (Q/RC)e^{-t/(RC)}$.
- Since $Q = CV$, the initial current is V/R and the time constant is $\tau = RC$; therefore $i =$

$$(V/R)e^{-t/\tau}.$$

Step 1: Find the current scale and time constant Convert the given resistance and capacitance to SI units:

$$\tau = (20 \times 10^3)(500 \times 10^{-6}) = 10 \text{ s},$$

$$i_0 = \frac{10}{20 \times 10^3} \text{ A} = 0.5 \text{ mA}.$$

Differentiating the negative exponential in the charge expression produces a positive charging current, which then decays with time.

Step 2: Substitute the elapsed time At $t = 5 \text{ s}$, the exponent is $-5/10 = -1/2$. Hence

$$i = (0.5 \text{ mA})e^{-1/2} = \frac{1}{2\sqrt{e}} \text{ mA}.$$

Options (B) and (C) exceed the initial 0.5 mA, whereas charging current must decrease. Option (D) is i_0/e , which is reached at $t = \tau = 10 \text{ s}$, not at half a time constant.

Step 3: Conclusion After 5 s, the current is $\frac{1}{2\sqrt{e}} \text{ mA}$, approximately 0.303 mA. Hence, the correct option is (A).

Quick Tip to Solve: Compute t/RC before using the exponential: half a time constant reduces the initial current by $\sqrt{e}$, not by e .

OR

29(iv). (iv) The key S_1 is closed and S_2 is open. The initial value of charging current in the resistor is: [1 mark]

- (A) 5 mA
- (B) 0.5 mA
- (C) 2 mA
- (D) 1 mA

Correct Answer: (B) 0.5 mA

Solution:

Concept:

- At the start of charging an initially uncharged capacitor has $q = 0$, so its voltage $V_C = q/C$ is zero.
- Kirchhoff's voltage law gives $V = iR + V_C$; hence the initial charging current is $i_0 = V/R$. Equivalently, use $i = (V/R)e^{-t/(RC)}$ at $t = 0$.

Step 1: Identify the initial voltage across the resistor Closing S_1 while S_2 remains open starts the charging process. The capacitor cannot acquire a finite charge instantaneously through the finite resistance, so initially the entire 10 V battery emf appears across the resistor.

Step 2: Apply Ohm's law and check the options With $R = 20 \text{ k}\Omega = 20 \times 10^3 \Omega$,

$$i_0 = \frac{10}{20 \times 10^3} \text{ A} = 5 \times 10^{-4} \text{ A} = 0.5 \text{ mA}.$$

At $t = 0$ the exponential factor is exactly one, so no time-dependent reduction is needed. Options (A),

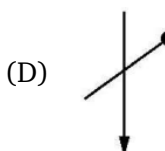
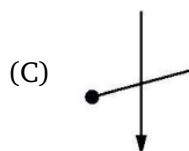
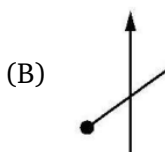
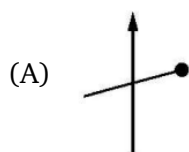
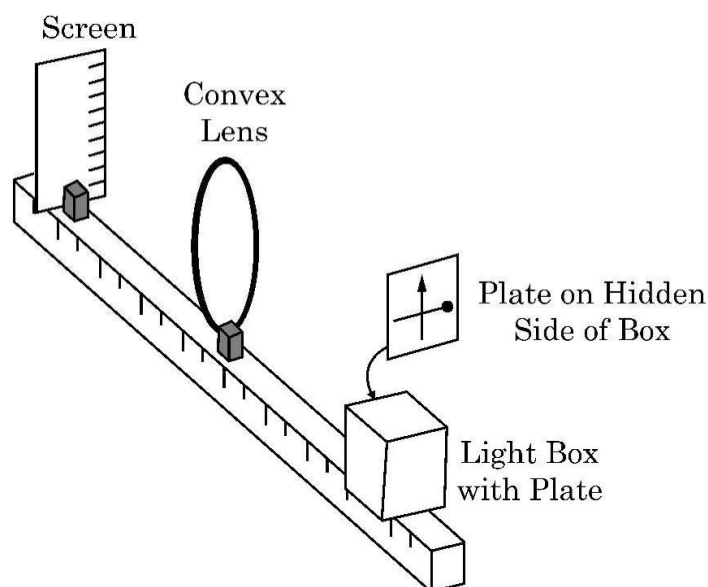
(C) and (D) would require resistor voltage drops of 100 V, 40 V and 20 V, respectively, all greater than the available battery emf.

Step 3: Conclusion The initial charging current is 0.5 mA. Hence, the correct option is (B).

Quick Tip to Solve: For the instant charging starts, set the initially uncharged capacitor voltage to zero; volts divided by kilohms directly gives milliamperes.

30. A thin lens is a transparent optical medium bounded by two surfaces, at least one of which should be spherical. Applying the formula for image formation by a single spherical surface successively at the two surfaces of a lens, one can obtain the 'lens maker formula' and then the 'lens formula'. A lens has two foci – called 'first focal point' and 'second focal point' of the lens, one on each side. [4 marks]

30(i). (i) Consider the arrangement shown in figure. A black vertical arrow and a horizontal thick line with a ball are painted on a glass plate. It serves as the object. When the plate is illuminated, its real image is formed on the screen. Which of the following correctly represents the image formed on the screen? [1 mark]



Correct Answer: (C)

Solution:

Concept:

- A convex lens forms a real image of a real object when the object lies beyond its principal focus. Its transverse magnification $m = v/u$ is negative, since $u < 0$ and $v > 0$.
- Negative magnification reverses both transverse coordinates: $x' = mx$ and $y' = my$. Thus the pattern undergoes a half-turn about the optical axis in the image plane.

Step 1: Track the arrow and the ball In the object pattern the arrow points upward and the ball is on the right-hand end of the thick horizontal line. Real image formation places points above the axis below it, and points to the right on the left. Consequently the image arrow must point downward and the ball must lie on the left-hand end.

Step 2: Compare both features in the options Option (A) retains the upward arrow and the ball on the right. Option (B) moves the ball left but retains the upward arrow. Option (D) reverses the arrow but leaves the ball on the right. Only option (C) simultaneously has the downward arrow and the ball on the left, as required by negative transverse magnification.

Step 3: Conclusion The real image reverses both transverse directions, giving a downward arrow and a ball on the left. Hence, the correct option is (C).

Quick Tip to Solve: Rotate the object pattern through 180° in its plane; check both the arrowhead and the ball before choosing an image.

30(ii). (ii) Which of the following statements is incorrect?

[1 mark]

- (A) For a convex mirror magnification is always negative.
- (B) For all virtual images formed by a mirror magnification is positive.
- (C) For a concave lens magnification is always positive.
- (D) For real and inverted images, magnification is always negative.

Correct Answer: (A) For a convex mirror magnification is always negative.

Solution:

Concept:

- Transverse magnification is $m = h_i/h_o$, where h_i and h_o are the signed image and object heights. An erect image has $m > 0$; an inverted image has $m < 0$.
- For a real object, a convex mirror and a concave lens form virtual, erect, diminished images. Their magnifications are positive and smaller than unity.

Step 1: Test the convex-mirror statement A convex mirror gives an erect image of a real object, so its magnification is positive. Equivalently, the mirror formula for magnification is $m = -v/u$: here $u < 0$ and the virtual image lies behind the mirror, so $v > 0$, giving $m > 0$. Thus statement (A), which claims that it is always negative, is incorrect.

Step 2: Check the remaining statements Under the usual real-object conditions of these statements, a

virtual mirror image is erect, so (B) is correct. A concave lens also forms a virtual erect image and has positive magnification, so (C) is correct. An inverted image has height of the opposite sign to the object, which makes its magnification negative; hence (D) is correct as well.

Step 3: Conclusion The incorrect statement is that a convex mirror always has negative magnification; its magnification is positive for a real object. Hence, the correct option is (A).

Quick Tip to Solve: First associate erect with positive and inverted with negative; the word “incorrect” then singles out the convex-mirror claim.

30(iii). (iii) A convex lens of focal length f is cut into two equal parts perpendicular to the principal axis. The focal length of each part will be: [1 mark]

- (A) f
- (B) $2f$
- (C) $\frac{f}{2}$
- (D) $\frac{f}{4}$

Correct Answer: (B) $2f$

Solution:

Concept:

- The lens maker formula in air is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is the refractive index and R_1, R_2 are the signed surface radii.
- Cutting the symmetric convex lens into equal halves perpendicular to the principal axis gives two identical plano-convex lenses. Each retains one curved surface and acquires a plane surface with infinite radius.

Step 1: Write the original lens power For the equal-curvature biconvex lens intended by the equal-halves construction, take $R_1 = R$ and $R_2 = -R$. Its two curved surfaces contribute equal positive powers, giving

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{2(\mu - 1)}{R}.$$

Step 2: Compare a cut half with the original For either plano-convex half, the plane surface contributes zero to the lens maker formula. If its focal length is f' , then

$$\frac{1}{f'} = \frac{\mu - 1}{R} = \frac{1}{2f}.$$

Option (A) would apply to cutting parallel to the principal axis, which preserves the original surface curvatures. Options (C) and (D) imply greater power, whereas removal of one curved surface reduces the power by half.

Step 3: Conclusion Each equal plano-convex part has focal length $f' = 2f$. Hence, the correct option is (B).

Quick Tip to Solve: A perpendicular central cut removes one curved surface from each half: halve the power and double the focal length.

30(iii). (iii) If an object in case (i) above is 20 cm from the lens and the screen is 50 cm away from the object, the focal length of the lens used is: [1 mark]

- (A) 10 cm
- (B) 12 cm
- (C) 16 cm
- (D) 20 cm

Correct Answer: (B) 12 cm

Solution:

Concept:

- A sharp image on a screen is real. The lens lies between this real image and the object, so their separation equals the sum of the object-to-lens and lens-to-screen distances.
- With the Cartesian sign convention, the thin-lens formula is $1/f = 1/v - 1/u$. For light travelling from the object towards the lens, a real object has $u < 0$ and a real image has $v > 0$.

Step 1: Find the lens-to-screen distance The stated 50 cm is the distance from the object to the screen, not from the lens to the screen. Since the object is 20 cm from the lens, the image distance is

$$v = 50 - 20 = 30 \text{ cm}, \quad u = -20 \text{ cm}.$$

Step 2: Substitute into the lens formula Using the signed distances,

$$\begin{aligned} \frac{1}{f} &= \frac{1}{30} - \frac{1}{-20} \\ &= \frac{1}{30} + \frac{1}{20} = \frac{5}{60} = \frac{1}{12} \text{ cm}^{-1}. \end{aligned}$$

Thus the required reciprocal focal length equals the reciprocal of 12 cm. The values 10 cm, 16 cm and 20 cm in the other options do not satisfy these conjugate distances.

Step 3: Conclusion The convex lens has focal length $f = +12$ cm. Hence, the correct option is (B).

Quick Tip to Solve: Subtract 20 cm from the total separation first; then use $f = ab/(a + b)$ with positive distances $a = 20$ and $b = 30$.

30(iv). (iv) The distance of an object from first focal point of a biconvex lens is X_1 and distance of the image from second focal point is X_2 . The focal length of the lens is: [1 mark]

- (A) $X_1 X_2$
(B) $\sqrt{X_1 + X_2}$
(C) $\sqrt{X_1 X_2}$
(D) $\sqrt{\frac{X_2}{X_1}}$

Correct Answer: (C) $\sqrt{X_1 X_2}$

Solution:

Concept:

- Newton's form of the thin-lens equation relates distances from the focal points: $X_1 X_2 = f^2$, where f is the focal length.
- For a real object beyond the first focus and its real image beyond the second focus, the positive distances from the lens are $a = X_1 + f$ and $b = X_2 + f$, respectively.

Step 1: Express the lens equation using focal distances Using positive object and real-image distances, the lens equation is $1/f = 1/a + 1/b$. Substituting the distances measured from the focal points gives

$$\frac{1}{f} = \frac{1}{X_1 + f} + \frac{1}{X_2 + f} = \frac{X_1 + X_2 + 2f}{(X_1 + f)(X_2 + f)}.$$

Step 2: Simplify and check the dimensions Cross-multiplication and expansion yield

$$\begin{aligned}(X_1 + f)(X_2 + f) &= f(X_1 + X_2 + 2f), \\ X_1 X_2 + f(X_1 + X_2) + f^2 &= f(X_1 + X_2) + 2f^2, \\ X_1 X_2 &= f^2.\end{aligned}$$

Option (A) has dimensions of length squared and option (B) has dimensions of the square root of length. Option (D), as printed, also fails the dimensional requirement for a focal length. The square root of the product, option (C), has dimensions of length and satisfies the derived relation.

Step 3: Conclusion Taking the positive root for the converging biconvex lens gives $f = \sqrt{X_1 X_2}$. Hence, the correct option is (C).

Quick Tip to Solve: When both distances are measured from focal points rather than the optical centre, apply Newton's relation $X_1 X_2 = f^2$.

Section E (15 marks)

31(a). (i) Two point charges $5\ \mu\text{C}$ and $-1\ \mu\text{C}$ are placed at points $(-3\ \text{cm}, 0, 0)$ and $(3\ \text{cm}, 0, 0)$ respectively. An external electric field $\vec{E} = \frac{A}{r^2} \hat{r}$, where $A = 3 \times 10^5\ \text{V m}$, is switched on in the region. Calculate the change in electrostatic energy of the system due to the electric field.

(ii) A system of two conductors is placed in air and they have net charge of $+80\ \mu\text{C}$ and $-80\ \mu\text{C}$ which causes a potential difference of $16\ \text{V}$ between them.

(1) Find the capacitance of the system.

(2) If the air between the capacitor is replaced by a dielectric medium of dielectric constant 3, what will be the potential difference between the two conductors?

(3) If the charges on two conductors are changed to $+160\ \mu\text{C}$ and $-160\ \mu\text{C}$, will the capacitance of the system change? Give reason for your answer. [5 marks]

Solution:

Concept:

- For a radial electric field, $E_r = -dV/dr$. Taking $V(\infty) = 0$, the field $\vec{E} = (A/r^2)\hat{r}$ has potential $V(r) = A/r$.
- The interaction energy of charges with an externally imposed potential is $U_{\text{ext}} = \sum_i q_i V_{\text{ext}}(\vec{r}_i)$. There is no factor $1/2$ in this sum because the external field is not produced by these charges.
- The capacitance of two conductors is $C = Q/\Delta V$, where Q is the magnitude of the charge on either conductor. It depends on the geometry and the intervening medium.
- Completely replacing air with a dielectric of relative permittivity K gives $C' = KC$. For fixed free charge, the potential difference becomes $\Delta V' = \Delta V/K$.

Step 1: (i) Find the external potential at the charges Both points have the same distance from the origin: $r_1 = r_2 = 3\ \text{cm} = 0.03\ \text{m}$. Although their position vectors point in opposite directions, potential is a scalar and depends only on radial distance here. Integrating the given radial field gives

$$V(r) - V(\infty) = - \int_{\infty}^r \frac{A}{r'^2} dr' = \frac{A}{r}.$$

Hence the external potential is the same at both charge locations:

$$V_1 = V_2 = \frac{3 \times 10^5}{0.03} = 1.0 \times 10^7\ \text{V}.$$

Step 2: (i) Calculate the change in electrostatic energy The two charges remain at their stated positions, so their mutual interaction energy does not change. The increase is therefore the sum of their interaction energies with the field that has been switched on:

$$\begin{aligned} \Delta U &= q_1 V_1 + q_2 V_2 \\ &= (5 \times 10^{-6})(10^7) + (-1 \times 10^{-6})(10^7) \\ &= 50 - 10 = +40\ \text{J}. \end{aligned}$$

The negative charge contributes negatively to the external-field energy; its sign must be retained rather than adding the charge magnitudes.

Step 3: (ii)(1) Calculate the capacitance in air Use the magnitude of the charge on one conductor, $Q = 80\ \mu\text{C}$, and the potential difference between them. The equal and opposite charges do not mean

that the capacitance is zero, nor should their magnitudes be added:

$$C = \frac{Q}{\Delta V} = \frac{80 \times 10^{-6}}{16} = 5 \times 10^{-6} \text{ F} = 5 \mu\text{F}.$$

Step 4: (ii)(2) Insert the dielectric at fixed charge No battery connection is stated, so the charges on the two conductors remain unchanged. With the space between them filled by a dielectric of constant $K = 3$, the capacitance increases to $C' = 3C = 15 \mu\text{F}$. Thus

$$\Delta V' = \frac{Q}{C'} = \frac{80 \mu\text{C}}{15 \mu\text{F}} = \frac{16}{3} \text{ V} \approx 5.33 \text{ V}.$$

Polarization reduces the electric field and therefore the potential difference for the same free charge.

Step 5: (ii)(3) Examine the effect of doubling the charges Changing the charges to $+160 \mu\text{C}$ and $-160 \mu\text{C}$ doubles Q . With the geometry and medium fixed, the potential difference also doubles, leaving their ratio unchanged:

$$C_{\text{new}} = \frac{2Q}{2\Delta V} = C.$$

The capacitance is therefore independent of the amount of charge: it remains $5 \mu\text{F}$ in air, or $15 \mu\text{F}$ if the dielectric in part (2) is retained.

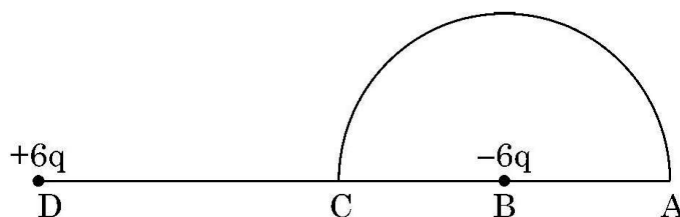
Step 6: Conclusion (i) The electrostatic energy increases by $[40 \text{ J}]$. (ii)(1) $[C = 5 \mu\text{F}]$ in air; (2) $[\Delta V' = 16/3 \text{ V} \approx 5.33 \text{ V}]$; (3) doubling the charges does not change the capacitance for a fixed geometry and medium.

Quick Tip to Solve: Equal radial distances give equal external potentials, so use $(q_1 + q_2)A/r$. For the capacitor, first decide whether charge or voltage stays fixed before applying the dielectric factor.

OR

31(b). (i) Consider three metal spherical shells A , B and C , each of radius R . Each shell is having a concentric metal ball of radius $R/10$. The spherical shells A , B and C are given charges $+6q$, $-4q$ and $14q$ respectively. Their inner metal balls are also given charges $-2q$, $+8q$ and $-10q$ respectively. Compare the magnitude of the electric fields due to shells A , B and C at a distance $3R$ from their centres.

(ii) A charge $-6 \mu\text{C}$ is placed at the centre B of a semicircle of radius 5 cm , as shown in the figure. An equal and opposite charge is placed at point D at a distance of 10 cm from B . A charge $+5 \mu\text{C}$ is moved from point C to point A along the circumference. Calculate the work done on the charge. [5 marks]



Solution:

Concept:

- For a spherically symmetric charge distribution, Gauss's law gives the external field $E = k|Q_{\text{enc}}|/r^2$, where $k = 1/(4\pi\epsilon_0)$, Q_{enc} is the total enclosed charge, and r is the distance from its centre.
- A central charge inside a conducting shell induces an equal and opposite charge on the inner surface. The outer-surface charge is the sum of the shell's net charge and the central charge.
- Electric potential obeys scalar superposition: $V = k\sum_i Q_i/r_i$. In an electrostatic field, work depends only on the initial and final positions, not on the path followed.
- For a charge q_0 moved slowly from C to A , external work is $W_{\text{ext}} = \Delta U = q_0(V_A - V_C)$. Work done by the electric field is $W_{\text{field}} = -\Delta U$.

Step 1: (i) Add the shell and inner-ball charges At a distance $3R$, a Gaussian sphere encloses both the shell and its inner metal ball. Their charges must therefore be added algebraically:

$$Q_A = +6q - 2q = +4q,$$

$$Q_B = -4q + 8q = +4q,$$

$$Q_C = +14q - 10q = +4q.$$

For example, the $-2q$ ball in system A induces $+2q$ on the shell's inner surface, leaving $+4q$ on its outer surface. Similarly, the outer-surface charges of B and C are $+4q$. The ball radius $R/10$ does not affect the field outside the concentric arrangement.

Step 2: (i) Apply Gauss's law at the common distance Spherical symmetry makes the field constant in magnitude and radial on a Gaussian sphere of radius $r = 3R$. Therefore

$$E(4\pi r^2) = \frac{Q_{\text{enc}}}{\epsilon_0}, \quad E = \frac{k|Q_{\text{enc}}|}{r^2}.$$

All three arrangements have equal net enclosed charge and are observed at the same radius:

$$E_A = E_B = E_C = \frac{4kq}{9R^2}, \quad E_A : E_B : E_C = 1 : 1 : 1.$$

Here q denotes a positive charge magnitude; the exterior fields point radially outward.

Step 3: (ii) Read the endpoint distances from the figure The order of the points on the horizontal line is D, C, B, A , with B the centre of the semicircle. Hence

$$BC = BA = 0.05 \text{ m}, \quad BD = 0.10 \text{ m},$$

and the distances from the positive charge at D are

$$DC = BD - BC = 0.05 \text{ m}, \quad DA = DB + BA = 0.15 \text{ m}.$$

The moving charge stays at a fixed distance from the charge at B , so that charge contributes equally to the potentials at C and A . Its contribution cancels from the potential difference.

Step 4: (ii) Calculate the potential difference and work With $Q_B = -6 \times 10^{-6} \text{ C}$ and $Q_D = +6 \times 10^{-6} \text{ C}$, scalar addition gives

$$V_C = k \left(\frac{-6 \times 10^{-6}}{0.05} + \frac{6 \times 10^{-6}}{0.05} \right) = 0,$$

$$V_A = 9 \times 10^9 \left(\frac{-6 \times 10^{-6}}{0.05} + \frac{6 \times 10^{-6}}{0.15} \right) \\ = -7.2 \times 10^5 \text{ V.}$$

Therefore, for $q_0 = +5 \times 10^{-6} \text{ C}$, the work required by an external agent in moving the charge slowly is

$$W_{\text{ext}} = q_0(V_A - V_C) = (5 \times 10^{-6})(-7.2 \times 10^5) = -3.6 \text{ J.}$$

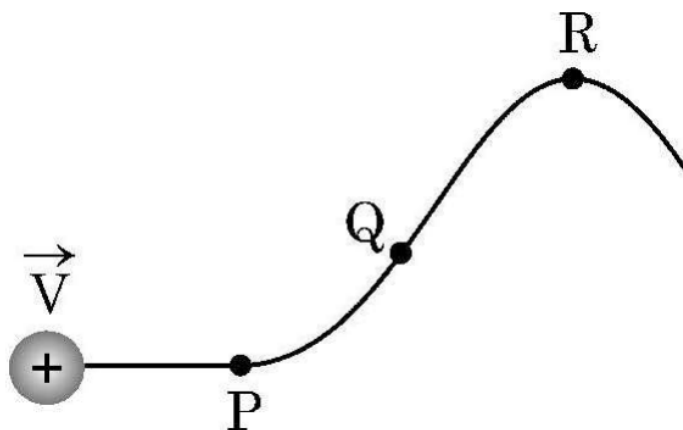
The negative sign means that the electric potential energy decreases. The electric field itself does $+3.6 \text{ J}$ of work on the charge; an external agent maintaining negligible change in kinetic energy removes that energy. The semicircular path does not alter either result.

Step 5: Conclusion (i) $E_A : E_B : E_C = 1 : 1 : 1$. (ii) The external work required is -3.6 J ; equivalently, the work done by the electric field on the charge is $+3.6 \text{ J}$.

Quick Tip to Solve: Outside a concentric shell and ball, add all enclosed charges. For motion along a circle about a fixed charge, cancel that charge's equal endpoint potentials before evaluating the other source's contribution.

32(a). (i) A proton moving with velocity $\vec{V}$ in a non-uniform magnetic field traces a path as shown in the figure. The path followed by the proton is always in the plane of the paper. What is the direction of the magnetic field in the region near points P , Q and R ? What can you say about relative magnitude of magnetic fields at these points?

(ii) A current carrying circular loop of area A produces a magnetic field B at its centre. Show that the magnetic moment of the loop is $\frac{2BA}{\mu_0} \sqrt{\frac{A}{\pi}}$. [5 marks]



Solution:

Concept:

- A proton has positive charge e , so its magnetic force is $\vec{F} = e(\vec{v} \times \vec{B})$. The force points towards the local centre of curvature of its path.
- A magnetic field does no work. Thus the proton's speed v is constant, and the transverse field obeys $evB = mv^2/r$, where m is the proton mass and r the local radius of curvature.
- For a circular current loop of radius a , current I and area $A = \pi a^2$, the central field is $B = \mu_0 I / (2a)$ and the magnetic moment has magnitude $\mu = IA$.

Step 1: (i) Use the bend at P to find the field direction Follow the proton from the left towards P , Q

and R . Near P , its velocity is towards the right and the path bends upwards, so the magnetic force is upwards. For a positive charge, the right-hand rule applied to $\vec{v} \times \vec{B}$ gives a field directed **into the plane of the paper** at P . A field out of the paper would bend this right-moving proton downwards instead.

Step 2: (i) Identify the straight region and the reverse bend Near Q , the drawn path is locally straight: this is the transition between the two opposite curvatures. Its local radius of curvature is effectively infinite, so the transverse magnetic force and field there are zero. At R , the velocity is towards the right while the path bends downwards. The required downward force on a positive charge therefore corresponds to a field **out of the plane of the paper**. As usual in this path diagram, these directions refer to the field perpendicular to the paper; any component parallel to the instantaneous velocity produces no magnetic force and is not determined by the path.

Step 3: (i) Compare the field strengths using curvature Equate the magnetic force to the local centripetal force:

$$evB = \frac{mv^2}{r} \implies B = \frac{mv}{er}.$$

Since m , e and v are unchanged, stronger curvature means a stronger field. The bend at R is sharper than at P , so $r_R < r_P$ and $B_R > B_P$. At the locally straight point Q , $r_Q \rightarrow \infty$, giving $B_Q = 0$. Thus the relative transverse field strengths are $B_R > B_P > B_Q = 0$.

Step 4: (ii) Eliminate the current and loop radius Write the field at the centre of a circular loop and solve for its current:

$$B = \frac{\mu_0 I}{2a} \implies I = \frac{2aB}{\mu_0}.$$

The area gives $a = \sqrt{A/\pi}$. Substituting both relations into the definition of magnetic moment yields

$$\begin{aligned} \mu &= IA = \frac{2aBA}{\mu_0} \\ &= \frac{2BA}{\mu_0} \sqrt{\frac{A}{\pi}}. \end{aligned}$$

The moment vector is normal to the loop, with its direction given by curling the right-hand fingers along the current. The expression above is its magnitude; its SI unit is A m^2 .

Step 5: Conclusion (i) The field is into the paper at P , zero at the straight region near Q , and out of the paper at R , with $B_R > B_P > B_Q = 0$. (ii) The loop's magnetic moment is $\mu = \frac{2BA}{\mu_0} \sqrt{\frac{A}{\pi}}$.

Quick Tip to Solve: First follow the positive particle's velocity and identify which way its path bends; then use $B \propto 1/r$. For the loop, eliminate I using the central field before replacing the radius by $\sqrt{A/\pi}$.

32(b). (i) Derive an expression for the torque acting on a rectangular current loop suspended in a uniform magnetic field.

(ii) A charged particle is moving in a circular path with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. It is made to pass through a sheet of lead and as a consequence, it loses one half of its kinetic energy without change in its direction. How will (1) the radius of its path (2) its time period of revolution change? [5 marks]

Solution:

Concept:

- A straight conductor carrying current I in a uniform field experiences $\vec{F} = I(\vec{\ell} \times \vec{B})$, where $\vec{\ell}$ is directed along the current.
- A pair of equal, opposite, parallel forces with different lines of action forms a couple. Its torque is one force multiplied by the perpendicular distance between the lines of action.
- The magnetic moment of a loop is $\vec{\mu} = IA\hat{n}$, with area A and right-hand normal $\hat{n}$. For a non-relativistic charged particle, $K = mv^2/2$ and $|q|vB = mv^2/r$.

Step 1: (i) Specify the loop geometry and its orientation Consider a single rectangular loop with sides a and b , area $A = ab$, carrying steady current I . Let θ be the angle between its normal $\hat{n}$ and the uniform field $\vec{B}$. Choose the pair of sides of length a perpendicular to $\vec{B}$; the loop can turn about an axis parallel to these sides. The forces on these two opposite sides have equal magnitude $F = IaB$ and opposite directions because their currents are opposite.

Step 2: (i) Calculate the torque of the force couple The separation of the two sides is b , but the distance perpendicular to their force directions is $b \sin \theta$. Their resultant force is zero, while their combined turning effect is

$$\begin{aligned}\tau &= F(b \sin \theta) \\ &= (IaB)b \sin \theta = IAB \sin \theta.\end{aligned}$$

The forces on the other pair of sides are also equal and opposite. Their resultant lines of action are collinear along the direction parallel to the a -sides through the loop's centre, so this pair produces no net torque. Hence the torque just obtained is the total torque on the rectangular loop.

Step 3: (i) Express the result using magnetic moment With $\vec{\mu} = IA\hat{n}$, the torque is

$$\vec{\tau} = \vec{\mu} \times \vec{B}, \quad |\vec{\tau}| = \mu B \sin \theta.$$

Its direction is given by the right-hand rule, and it tends to align the magnetic moment with the field. The torque vanishes when the normal is parallel or antiparallel to the field, and is maximum when the normal is perpendicular to it. For a coil of N identical turns, each turn contributes equally, giving $\tau = NIAB \sin \theta$ and $\mu = NIA$.

Step 4: (ii) Relate the reduced kinetic energy to the new speed Let the speed before crossing the lead

sheet be v , and afterwards be v' . The particle retains half its initial kinetic energy, so

$$\frac{1}{2}mv'^2 = \frac{1}{2} \left(\frac{1}{2}mv^2 \right) \implies v' = \frac{v}{\sqrt{2}}.$$

The particle's mass and charge remain the same. Since its direction does not change on crossing the sheet, it remains perpendicular to the field and starts a smaller circular orbit tangent to its original direction of motion.

Step 5: (ii)(1) Compare the radii of the circular paths The magnetic force supplies the centripetal force on each side of the sheet:

$$r = \frac{mv}{|q|B}, \quad r' = \frac{mv'}{|q|B}.$$

The field, charge and mass are unchanged, so

$$\frac{r'}{r} = \frac{v'}{v} = \frac{1}{\sqrt{2}}.$$

Thus the radius is reduced by a factor $1/\sqrt{2}$, rather than by a factor $1/2$; radius depends on speed, while kinetic energy depends on speed squared.

Step 6: (ii)(2) Compare the time periods of revolution The period equals circumference divided by speed. Substituting the radius expression,

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \frac{mv}{|q|B} = \frac{2\pi m}{|q|B}.$$

The speed cancels. After crossing the sheet, both the circumference and speed decrease in the same ratio, so $T' = 2\pi m/(|q|B) = T$. This is the standard non-relativistic result for circular motion in a uniform magnetic field.

Step 7: Conclusion (i) A rectangular loop experiences $\vec{\tau} = \vec{\mu} \times \vec{B}$, with magnitude $IAB \sin \theta$ for one turn or $NIAB \sin \theta$ for N turns. (ii) The new radius is $r' = r/\sqrt{2}$, while the time period is unchanged: $T' = T$.

Quick Tip to Solve: Measure θ from the loop's normal when using $IAB \sin \theta$. For the particle, use $r \propto \sqrt{K}$ and $T = 2\pi m/(|q|B)$ to compare the two orbits directly.

33(a). (i) (1) What are coherent sources? Why are they necessary for observing a sustained interference pattern?

(2) Lights from two independent sources are not coherent. Explain.

(ii) Two slits 0.1 mm apart are arranged 1.20 m from a screen. Light of wavelength 600 nm from a distant source is incident on the slits.

(1) How far apart will adjacent bright interference fringes be on the screen?

(2) Find the angular width (in degree) of the first bright fringe.

[5 marks]

Solution:

Concept:

- Coherent sources emit waves of the same frequency with a constant phase difference. This makes the positions of constructive and destructive interference stable.
- For two overlapping waves of intensities I_1 and I_2 , the resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$, where ϕ is their phase difference at the observation point.
- In Young's double-slit experiment, the path difference at a small angle θ is $d \sin \theta \simeq dy/D$, where d is slit separation, D the screen distance and y the transverse position on the screen.
- Adjacent bright fringes have separation $\beta = \lambda D/d$, and a bright fringe has angular width $\Delta\theta \simeq \beta/D = \lambda/d$, in radians. Here λ is the wavelength.

Step 1: (i)(1) Define coherence and explain its necessity Two sources are coherent when they have the same frequency and maintain a fixed phase difference with time. Their phase difference need not be zero. At a given screen position, the geometrical path difference then produces a fixed additional phase difference, so the total ϕ remains steady.

Consequently, points where the waves reinforce remain bright and points where they cancel remain dark. The interference term $2\sqrt{I_1 I_2} \cos \phi$ does not average away during observation, and the pattern is sustained. If the relative phase fluctuates rapidly and randomly, the bright and dark positions keep shifting; an observer or detector averages these variations and cannot resolve a stationary fringe pattern.

Step 2: (i)(2) Explain independent-source incoherence In two independent ordinary light sources, atoms emit light in separate, short wave trains. The emission events occur independently, and the initial phases of successive wave trains are random. Thus, even if both sources have the same nominal colour or frequency, their relative phase changes unpredictably with time.

Their waves do not maintain the fixed phase relationship needed for sustained interference. Over the observation time, the interference term averages to zero and the recorded intensity is approximately $I_1 + I_2$. In Young's experiment, illuminating both slits with one source provides two related portions of the same wavefront and hence coherent secondary sources.

Step 3: (ii)(1) Derive and calculate the bright-fringe separation A bright fringe occurs when the path difference is an integral multiple of the wavelength. For the m th bright fringe, the small-angle relation gives

$$\frac{dy_m}{D} = m\lambda \quad \Rightarrow \quad y_m = \frac{m\lambda D}{d}.$$

Subtracting the positions of successive bright fringes gives

$$\beta = y_{m+1} - y_m = \frac{\lambda D}{d}.$$

Convert the supplied quantities into SI units:

$$d = 0.1 \text{ mm} = 1.0 \times 10^{-4} \text{ m}, \quad \lambda = 600 \text{ nm} = 6.0 \times 10^{-7} \text{ m}.$$

With $D = 1.20 \text{ m}$, substitution gives

$$\beta = \frac{(6.0 \times 10^{-7})(1.20)}{1.0 \times 10^{-4}} = 7.2 \times 10^{-3} \text{ m}.$$

Step 4: (ii)(2) Obtain the angular width of the first bright fringe The first bright fringe lies between the neighbouring dark fringes whose path differences are $\lambda/2$ and $3\lambda/2$. At small angles their directions satisfy

$$\theta_- \simeq \frac{\lambda}{2d}, \quad \theta_+ \simeq \frac{3\lambda}{2d}.$$

Therefore its angular width, measured between these minima, is

$$\Delta\theta = \theta_+ - \theta_- \simeq \frac{\lambda}{d} = \frac{6.0 \times 10^{-7}}{1.0 \times 10^{-4}} = 6.0 \times 10^{-3} \text{ rad}.$$

Since $\pi \text{ rad} = 180^\circ$, the conversion to degrees is

$$\Delta\theta = (6.0 \times 10^{-3}) \frac{180^\circ}{\pi} \simeq 0.344^\circ.$$

The angles are much smaller than one radian, so the small-angle approximation is justified. The screen distance cancels in β/D , as expected for an angular width.

Step 5: Conclusion Coherent sources have the same frequency and a constant phase difference, giving a sustained interference pattern. Independent ordinary sources have randomly varying relative phases and do not give a sustained pattern. The adjacent bright fringes are 7.2 mm apart, and the angular width of the first bright fringe is 0.344° , approximately.

Quick Tip to Solve: Convert both slit separation and wavelength to metres first. Use $\lambda D/d$ for separation on the screen, but λ/d for angular width; multiply the latter by $180/\pi$ to obtain degrees.

OR

33(b). (i) Define a wavefront. An incident plane wave falls on a convex lens and gets refracted through it. Draw a diagram to show the incident and refracted wavefront.

(ii) A beam of light coming from a distant source is refracted by a spherical glass ball (refractive index 1.5) of radius 15 cm. Draw the ray diagram and obtain the position of the final image formed.

[5 marks]

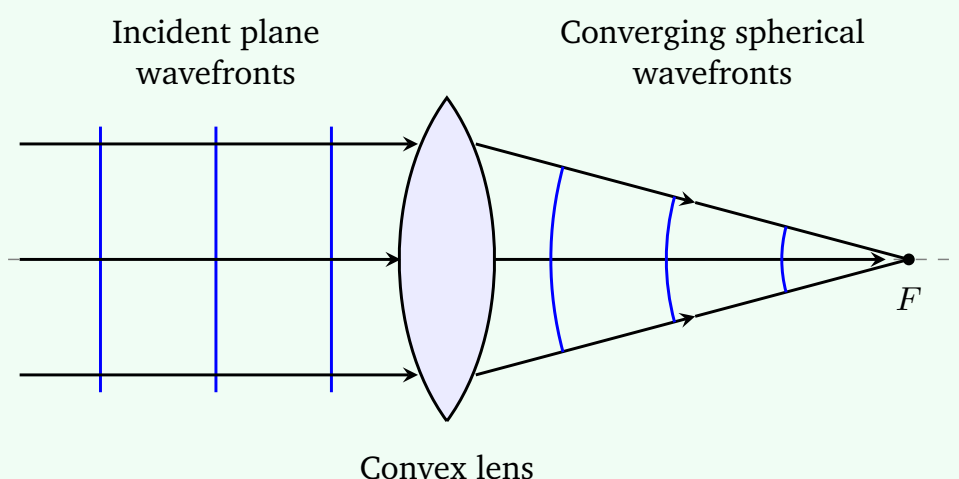
Solution:

Concept:

- A wavefront is the locus of all points of a wave that vibrate in the same phase at a given instant. Rays are normal to the wavefront in an isotropic medium.
- A convex lens transforms an incident plane wavefront into a converging spherical wavefront whose centre is at the principal focus, for paraxial light in air.

- For refraction at a spherical surface, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, where n_1, n_2 are the incident and refracted media's refractive indices. Object distance u , image distance v , and radius R are measured from that surface's pole, with the direction of incident light positive.

Step 1: (i) Define and draw the wavefronts Since every point on a wavefront has the same phase, a plane wavefront corresponds to a parallel incident beam. The convex lens is thicker near its principal axis and introduces a greater optical delay there than near its edges. The emergent wavefront is therefore curved and converges towards the principal focus F . In the diagram, the straight vertical lines are incident plane wavefronts and the circular arcs are successive converging spherical wavefronts. Their normals, shown as rays, meet at F ; after crossing the focus the wavefronts become diverging.



Plane wavefronts become converging spherical wavefronts after a convex lens; rays are normal to the fronts.

Step 2: (ii) Find the image formed by the first surface Treat the glass ball as two successive refracting spherical surfaces, with poles P_1 and P_2 separated by the diameter 30 cm. Take light to travel from left to right, and use the paraxial approximation. At the first surface, light goes from air to glass and the centre of curvature is to the right:

$$n_1 = 1, \quad n_2 = 1.5, \quad u_1 = -\infty, \quad R_1 = +15 \text{ cm.}$$

Substituting in the spherical-refraction formula gives

$$\begin{aligned} \frac{1.5}{v_1} - \frac{1}{-\infty} &= \frac{1.5 - 1}{15}, \\ \frac{1.5}{v_1} &= \frac{1}{30}, \\ v_1 &= +45 \text{ cm.} \end{aligned}$$

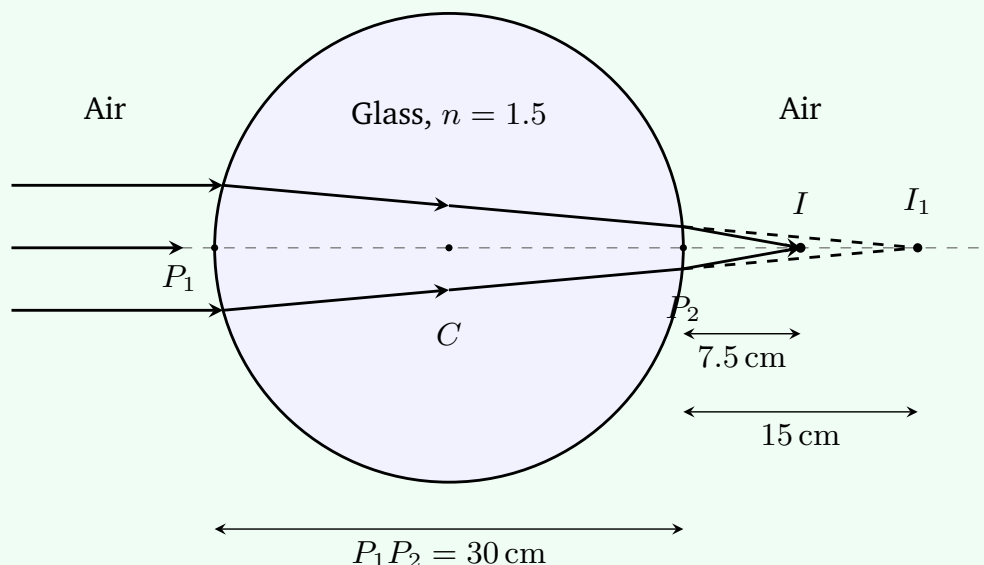
Thus, if the second surface were absent, the rays in glass would meet at I_1 , located 45 cm to the right of P_1 . The rays encounter the second surface before reaching this point.

Step 3: Locate the object for the second refraction The point towards which the rays are already converging acts as a virtual object for the second surface. Its distance from P_2 , rather than from P_1 , is required:

$$u_2 = v_1 - P_1P_2 = 45 - 30 = +15 \text{ cm.}$$

The positive sign follows because I_1 lies to the right of the second pole. At this surface light passes

from glass to air, while the centre of curvature lies to its left, so $n_1 = 1.5$, $n_2 = 1$, and $R_2 = -15$ cm. The dashed extensions in the ray diagram show the intermediate image I_1 ; the actual emergent rays meet sooner at I .



Paraxial ray diagram for the glass ball (schematic). Dashed extensions locate the first-surface image I_1 ; the actual final image is I .

Step 4: Calculate the final image distance Apply the same formula to the second surface, using its own pole as the origin:

$$\begin{aligned}\frac{1}{v_2} - \frac{1.5}{15} &= \frac{1 - 1.5}{-15}, \\ \frac{1}{v_2} - \frac{1}{10} &= \frac{1}{30}, \\ \frac{1}{v_2} &= \frac{1}{10} + \frac{1}{30} = \frac{2}{15}, \\ v_2 &= +7.5 \text{ cm}.\end{aligned}$$

A positive image distance signifies that the emergent rays actually converge on the right of the second surface. The distance from the centre of the ball is $15 + 7.5 = 22.5$ cm.

Step 5: Conclusion (i) A wavefront is a surface of constant phase; the convex lens converts a plane wavefront into a converging spherical wavefront. (ii) The final image is real and is formed 7.5 cm beyond the far surface of the glass ball, or 22.5 cm from its centre on the emergent side.

Quick Tip to Solve: For the second surface of a ball, first subtract the diameter from the first image distance. Here $45 - 30 = +15$ cm: keeping this virtual-object distance positive is essential.