

CBSE Class 12 Physics Question Paper 2025 with Solution PDF-Set 55/1/2

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) Question paper is divided into **FIVE** sections – Sections **A, B, C, D** and **E**.
- (iii) In **Section A** : Question numbers **1** to **16** are Multiple Choice (MCQ) type questions. Each question carries **1** mark.
- (iv) In **Section B** : Question numbers **17** to **21** are Very Short Answer (VSA) type questions. Each question carries **2** marks.
- (v) In **Section C** : Question numbers **22** to **28** are Short Answer (SA) type questions. Each question carries **3** marks.
- (vi) In **Section D** : Question numbers **29** & **30** are Case Study-Based questions. Each question carries **4** marks.
- (vii) In **Section E** : Question numbers **31** to **33** are Long Answer (LA) type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is NOT allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg.}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg.}$$

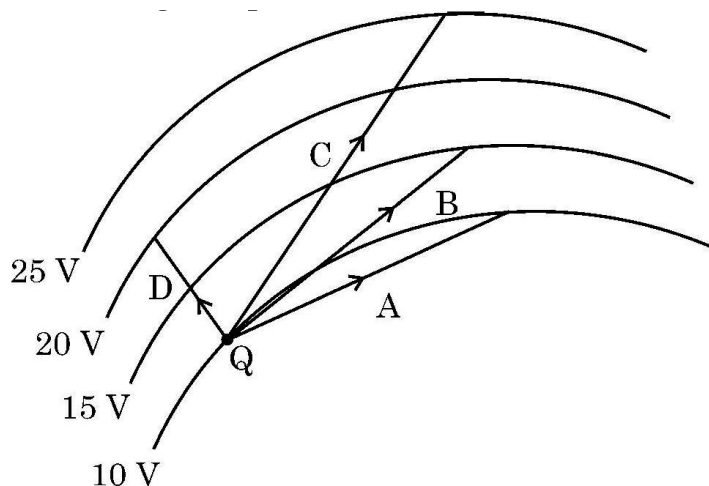
$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg.}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. In the figure curved lines represent equipotential surfaces. A charge Q is moved along different paths A , B , C and D . The work done on the charge will be maximum along the path: [1 mark]



- (A) A
- (B) B
- (C) C
- (D) D

Correct Answer: (C) C

Solution:

Concept:

- The work required to move a charge Q quasistatically between potentials V_i and V_f is $W = Q(V_f - V_i)$.
- Electrostatic work depends on the endpoint potentials, not the length or shape of the path. Every point on one equipotential has the same potential.

Step 1: Read the potentials at the endpoints The starting point Q lies on the 10 V equipotential. The endpoints of paths A , B , C and D lie on the 10 V, 15 V, 25 V and 20 V equipotentials, respectively.

Step 2: Compare the potential rises The potential changes along the four paths are

$$\Delta V_A = 0, \quad \Delta V_B = 5 \text{ V}, \quad \Delta V_C = 15 \text{ V}, \quad \Delta V_D = 10 \text{ V}.$$

For the same positive test charge, the required work increases with this potential rise. Path A needs no net work, and B and D have smaller potential rises than C ; their shape or length cannot increase the work.

Step 3: Conclusion The work done on the charge is maximum along path C , which has the greatest potential rise, 15 V. Hence, the correct option is (C).

Quick Tip to Solve: Label only the starting and ending equipotentials of each path; curved or longer paths do not change electrostatic work.

2. The resistance of a wire of length L and radius r is R . Which one of the following would provide a wire of the same material of resistance $\frac{R}{2}$? [1 mark]

- (A) Using a wire of same radius and twice the length
- (B) Using a wire of same radius and half length
- (C) Using a wire of same length and twice the radius
- (D) Using a wire of same length and half the radius

Correct Answer: (B) Using a wire of same radius and half length

Solution:

Concept:

- The resistance of a uniform wire is $R = \rho L/A = \rho L/(\pi r^2)$, where ρ is resistivity, L is length and r is radius.
- At the same temperature, wires of the same material have the same resistivity. Thus R varies directly with length and inversely with the square of radius.

Step 1: Compare the two choices with unchanged radius When the radius stays r , the cross-sectional area is unchanged. Therefore

$$R_A = \frac{\rho(2L)}{\pi r^2} = 2R, \quad R_B = \frac{\rho(L/2)}{\pi r^2} = \frac{R}{2}.$$

Doubling the length increases the resistance, whereas halving the length gives the required reduction.

Step 2: Check the choices that change the radius With length fixed, doubling the radius makes the area four times as large, while halving the radius reduces the area to one-quarter:

$$R_C = \frac{\rho L}{\pi(2r)^2} = \frac{R}{4}, \quad R_D = \frac{\rho L}{\pi(r/2)^2} = 4R.$$

Neither radius change produces $R/2$; confusing radius with area would give an incorrect choice.

Step 3: Conclusion A wire of the same radius and half the length has resistance $R/2$. Hence, the correct option is (B).

Quick Tip to Solve: Use $R'/R = (L'/L)(r/r')^2$; always square the radius ratio before comparing wire resistances.

3. A 1 cm segment of a wire lying along x -axis carries current of 0.5 A along $+x$ direction. A magnetic field $\vec{B} = (0.4 \text{ mT})\hat{j} + (0.6 \text{ mT})\hat{k}$ is switched on in the region. The force acting on the segment is: [1 mark]

- (A) $(2\hat{j} + 3\hat{k}) \text{ mN}$
(B) $(-3\hat{j} + 2\hat{k}) \mu\text{N}$
(C) $(6\hat{j} + 4\hat{k}) \text{ mN}$
(D) $(-4\hat{j} + 6\hat{k}) \mu\text{N}$

Correct Answer: (B) $(-3\hat{j} + 2\hat{k}) \mu\text{N}$

Solution:

Concept:

- A straight current-carrying segment in a magnetic field experiences $\vec{F} = I\vec{\ell} \times \vec{B}$, where $\vec{\ell}$ points along the current and has magnitude equal to the segment length.
- The unit-vector products are $\hat{i} \times \hat{j} = \hat{k}$ and $\hat{i} \times \hat{k} = -\hat{j}$. Use SI units before evaluating the force.

Step 1: Convert the length and magnetic field The current is $I = 0.5 \text{ A}$ along $+x$. Hence

$$\vec{\ell} = 10^{-2}\hat{i} \text{ m}, \quad \vec{B} = (0.4\hat{j} + 0.6\hat{k}) \times 10^{-3} \text{ T}.$$

The length conversion contributes 10^{-2} , and the millitesla conversion contributes 10^{-3} .

Step 2: Evaluate the cross product and check the components Substitution gives

$$\begin{aligned}\vec{F} &= 0.5 \times 10^{-2} \times 10^{-3}(0.4\hat{k} - 0.6\hat{j}) \text{ N} \\ &= (-3\hat{j} + 2\hat{k}) \times 10^{-6} \text{ N}.\end{aligned}$$

The $\hat{k}$ field produces a negative $\hat{j}$ force, so the positive $\hat{j}$ components in (A) and (C) are incorrect; those options also use millinewtons. Option (D) has incorrect component magnitudes.

Step 3: Conclusion The force is $\vec{F} = (-3\hat{j} + 2\hat{k}) \mu\text{N}$. Hence, the correct option is (B).

Quick Tip to Solve: For current along $+x$, write $F_y = -I\ell B_z$ and $F_z = I\ell B_y$ to avoid swapping the field components or signs.

4. A circular coil of diameter 15 mm having 300 turns is placed in a magnetic field of 30 mT such that the plane of the coil is perpendicular to the direction of magnetic field. The magnetic field is reduced uniformly to zero in 20 ms and again increased uniformly to 30 mT in 40 ms. If the emfs induced in the two time intervals are e_1 and e_2 respectively, then the value of e_1/e_2 is: [1 mark]

- (A) $\frac{1}{2}$
- (B) 1
- (C) 2
- (D) 4

Correct Answer: (C) 2

Solution:

Concept:

- Faraday's law gives the magnitude of induced emf as $|e| = N|\Delta\Phi|/\Delta t$, where N is the number of turns and Φ is the magnetic flux through one turn.
- The field is normal to the coil, so $\Phi = AB$, with A the area of one turn. For equal field changes in the same coil, the emf magnitude is inversely proportional to the time interval.

Step 1: Express the two emf magnitudes In each interval the magnitude of the field change is 30 mT = 0.030 T. The turn count and coil area are unchanged, giving

$$|e_1| = \frac{NA(0.030)}{20 \times 10^{-3}}, \quad |e_2| = \frac{NA(0.030)}{40 \times 10^{-3}}.$$

The options use positive values, so the required comparison is of emf magnitudes. The actual polarities are opposite because the field decreases first and increases next.

Step 2: Cancel the common factors Taking the ratio cancels N , A and the field change:

$$\frac{|e_1|}{|e_2|} = \frac{40 \times 10^{-3}}{20 \times 10^{-3}} = 2.$$

Option (A) reverses this ratio. Option (B) ignores the different rates of field change, and (D) incorrectly squares the time ratio. The coil diameter does not affect this comparison.

Step 3: Conclusion The first induced emf has twice the magnitude of the second, so $e_1/e_2 = 2$ for the magnitudes intended here. Hence, the correct option is (C).

Quick Tip to Solve: For the same coil and equal flux changes, compare the time intervals in reverse order: half the time gives twice the emf magnitude.

5. You are required to design an air-filled solenoid of inductance 0.016 H having a length 0.81 m and radius 0.02 m. The number of turns in the solenoid should be: [1 mark]

- (A) 2592
- (B) 2866
- (C) 2976
- (D) 3140

Correct Answer: (B) 2866

Solution:

Concept:

- An air-filled long solenoid has self-inductance $L = \mu_0 N^2 A / \ell$, where N is its total number of turns, A its cross-sectional area and ℓ its length.
- For radius r , $A = \pi r^2$. Air has relative permeability approximately unity, so use $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$.

Step 1: Rearrange the inductance formula With $L = 0.016 \text{ H}$, $\ell = 0.81 \text{ m}$ and $r = 0.02 \text{ m}$, the required total turn count satisfies

$$N = \sqrt{\frac{L\ell}{\mu_0 \pi r^2}} = \sqrt{\frac{0.016 \times 0.81}{(4\pi \times 10^{-7})\pi(0.02)^2}}.$$

The square root is essential because inductance is proportional to N^2 , not to N .

Step 2: Evaluate and compare the choices Using $\pi \simeq 3.14$,

$$N \simeq \sqrt{8.215 \times 10^6} \simeq 2866.$$

As a check, turn counts 2592, 2976 and 3140 would give inductances approximately 0.0131 H, 0.0172 H and 0.0192 H, respectively. Only the choice near 2866 reproduces 0.016 H.

Step 3: Conclusion The solenoid should have approximately 2866 turns. Hence, the correct option is (B).

Quick Tip to Solve: For a solenoid, solve for the total turns using $N = \sqrt{L\ell/(\mu_0 \pi r^2)}$; do not confuse N with turns per unit length.

6. A voltage $v = v_0 \sin \omega t$ applied to a circuit drives a current $i = i_0 \sin(\omega t + \phi)$ in the circuit. The average power consumed in the circuit over a cycle is: [1 mark]

- (A) Zero
 (B) $i_0 v_0 \cos \phi$
 (C) $\frac{i_0 v_0}{2}$
 (D) $\frac{i_0 v_0}{2} \cos \phi$

Correct Answer: (D) $\frac{i_0 v_0}{2} \cos \phi$

Solution:

Concept:

- Instantaneous electric power is $p(t) = v(t)i(t)$; average power is the mean of this product over one complete cycle.
- For sinusoidal voltage and current, $V_{\text{rms}} = v_0/\sqrt{2}$ and $I_{\text{rms}} = i_0/\sqrt{2}$, where v_0 and i_0 are peak values. The phase factor is $\cos \phi$.

Step 1: Write the instantaneous power Multiply the given waveforms and use $2 \sin a \sin b = \cos(a - b) - \cos(a + b)$:

$$\begin{aligned} p(t) &= v_0 i_0 \sin \omega t \sin(\omega t + \phi) \\ &= \frac{v_0 i_0}{2} [\cos \phi - \cos(2\omega t + \phi)] \end{aligned}$$

The second term oscillates at twice the applied frequency, while the first is independent of time.

Step 2: Average over the cycle and assess the alternatives Over a full cycle, $\langle \cos(2\omega t + \phi) \rangle = 0$. Therefore only the constant term contributes to the mean. Option (A) applies only when $\cos \phi = 0$; option (B) omits the factor $1/2$ required for peak amplitudes; option (C) assumes $\cos \phi = 1$. None is the general expression for an arbitrary phase difference.

Step 3: Conclusion The average power is $P_{\text{av}} = \frac{v_0 i_0}{2} \cos \phi = V_{\text{rms}} I_{\text{rms}} \cos \phi$. Hence, the correct option is (D).

Quick Tip to Solve: First identify whether voltage and current are peak or rms values: peak amplitudes need the factor $1/2$ in the average-power formula.

7. Which one of the following correctly represents the change in wave characteristics (all in vacuum) from microwaves to X-rays in electromagnetic spectrum? [1 mark]

- (A) Speed: Remains same; Wavelength: Decreases; Frequency: Remains same
 (B) Speed: Remains same; Wavelength: Decreases; Frequency: Increases
 (C) Speed: Increases; Wavelength: Increases; Frequency: Decreases
 (D) Speed: Remains same; Wavelength: Increases; Frequency: Remains same

Correct Answer: (B) Speed: Remains same; Wavelength: Decreases; Frequency: Increases

Solution:

Concept:

- All electromagnetic waves travel in vacuum at the same speed, $c = 3.0 \times 10^8 \text{ m s}^{-1}$, regardless of their position in the spectrum.

- Wave speed, frequency ν and wavelength λ obey $c = \nu\lambda$. At fixed speed, wavelength and frequency vary inversely.

Step 1: Compare the spectral regions Microwaves lie toward the long-wavelength, low-frequency end of the electromagnetic spectrum. X-rays have much shorter wavelengths. Thus, on moving from microwaves to X-rays, λ decreases. Since both waves are in vacuum, their speed remains c .

Step 2: Infer the frequency change and check the options Rearranging the wave relation gives

$$\nu = \frac{c}{\lambda}.$$

With the numerator fixed, a decreasing wavelength requires an increasing frequency. Options (A) and (D) incorrectly keep frequency unchanged while changing wavelength at constant speed. Option (C) incorrectly changes the vacuum speed and reverses the actual wavelength and frequency trends.

Step 3: Conclusion From microwaves to X-rays, speed remains the same, wavelength decreases and frequency increases. Hence, the correct option is (B).

Quick Tip to Solve: In vacuum, keep c fixed and use $\nu\lambda = c$: a shift toward shorter wavelengths must be accompanied by higher frequencies.

8. The speed of light in two media '1' and '2' are v_1 and $v_2 (> v_1)$ respectively. For a ray of light to undergo total internal reflection at the interface of these two media, it must be incident from:

[1 mark]

- (A) medium '1' and at an angle greater than $\sin^{-1}\left(\frac{v_1}{v_2}\right)$
- (B) medium '1' and at an angle greater than $\cos^{-1}\left(\frac{v_1}{v_2}\right)$
- (C) medium '2' and at an angle greater than $\sin^{-1}\left(\frac{v_1}{v_2}\right)$
- (D) medium '2' and at an angle greater than $\cos^{-1}\left(\frac{v_1}{v_2}\right)$

Correct Answer: (A) medium '1' and at an angle greater than $\sin^{-1}\left(\frac{v_1}{v_2}\right)$

Solution:

Concept:

- Refractive index is $n = c/v$, where v is the speed of light in the medium and c is its speed in vacuum. A lower speed means an optically denser medium.
- Total internal reflection requires incidence from the denser medium and an incidence angle i , measured from the normal, greater than the critical angle i_c .

Step 1: Identify the incident medium Since $v_2 > v_1$, the indices satisfy $n_1 > n_2$. Therefore, the ray must travel from medium 1 towards medium 2. Incidence from medium 2, as proposed in options (C) and (D), cannot produce total internal reflection.

Step 2: Find the critical angle At the critical angle the refracted ray grazes the interface, so Snell's law gives

$$n_1 \sin i_c = n_2 \sin 90^\circ, \quad \sin i_c = \frac{n_2}{n_1} = \frac{v_1}{v_2}.$$

Thus the required threshold uses inverse sine. The inverse cosine in option (B) would give the complementary angle, not the critical angle measured from the normal. At $i = i_c$ the ray only grazes the boundary; total internal reflection needs $i > i_c$.

Step 3: Conclusion The ray must be incident from medium 1 with $i > \sin^{-1}(v_1/v_2)$. Hence, the correct option is (A).

Quick Tip to Solve: Choose the slower medium first, then take inverse sine of the smaller speed divided by the larger speed.

9. A source produces monochromatic light of frequency 5.0×10^{14} Hz and the power emitted is 3.31 mW. The number of photons emitted per second by the source, on an average, is: [1 mark]

- (A) 10^{16}
- (B) 10^{24}
- (C) 10^{10}
- (D) 10^{20}

Correct Answer: (A) 10^{16}

Solution:

Concept:

- A photon of frequency ν has energy $E_\gamma = h\nu$, where $h = 6.63 \times 10^{-34}$ J s is Planck's constant.
- Power P is energy emitted per second. For monochromatic light, the average number of photons emitted per second is $\dot{N} = P/(h\nu)$.

Step 1: Convert the power and calculate photon energy The prefix milli means 10^{-3} , so $P = 3.31 \times 10^{-3}$ W. Each photon carries

$$E_\gamma = (6.63 \times 10^{-34})(5.0 \times 10^{14}) = 3.315 \times 10^{-19} \text{ J.}$$

Step 2: Divide the emitted energy rate by photon energy

$$\dot{N} = \frac{3.31 \times 10^{-3}}{3.315 \times 10^{-19}} = 9.98 \times 10^{15} \text{ s}^{-1} \simeq 1.0 \times 10^{16} \text{ s}^{-1}.$$

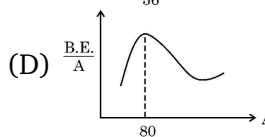
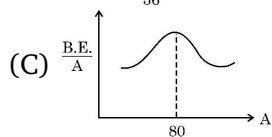
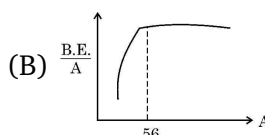
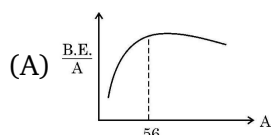
The units are $(\text{J s}^{-1})/\text{J} = \text{s}^{-1}$. The powers of ten alone give $10^{-3}/10^{-19} = 10^{16}$; options (B), (C) and (D), with exponents 24, 10 and 20, therefore do not match the energy balance.

Step 3: Conclusion The source emits approximately 10^{16} photons per second. Hence, the correct option is (A).

Quick Tip to Solve: Convert milliwatts to watts before using $P/(h\nu)$, then subtract the exponents to check the photon rate.

10. Which of the following figures correctly represent the shape of curve of binding energy per nucleon as a function of mass number?

[1 mark]



Correct Answer: (A)

Solution:

Concept:

- Binding energy per nucleon is B/A , where B is the energy required to separate a nucleus into its nucleons and A is the mass number. It measures the average binding of a nucleon.
- The B/A curve rises rapidly for light nuclei, reaches its highest values in the iron region near $A = 56$, and remains nearly constant across a broad middle-mass range before decreasing slowly for heavy nuclei.

Step 1: Identify the rise, maximum and fall Binding energy per nucleon increases rapidly for light nuclei and reaches about 8.8 MeV near $A = 56$. It then decreases smoothly, reaching about 7.6 MeV for very heavy nuclei. Nuclear-force saturation limits the average attractive binding, while increasing electrostatic repulsion between protons helps explain the decrease for heavy nuclei.

Step 2: Compare all four printed curves Curve (A) shows the rapid rise, a broad maximum near the indicated mass number 56, and the subsequent smooth decline. Curve (B) remains almost level after 56 and does not represent the fall towards heavy nuclei as well as (A). Curves (C) and (D) place a pronounced peak near 80 and show a later upturn, instead of the standard maximum near the iron region followed by a gradual decrease.

Step 3: Conclusion Curve (A) best represents the rapid initial rise, maximum near $A = 56$ and gradual decrease for heavy nuclei. Hence, the correct option is (A).

Quick Tip to Solve: Check all three features together: a rapid initial rise, a maximum near 56, and a gradual fall towards very heavy nuclei.

11. When a p - n junction diode is forward biased:

[1 mark]

- (A) the barrier height and the depletion layer width both increase.
- (B) the barrier height increases and the depletion layer width decreases.
- (C) the barrier height and the depletion layer width both decrease.
- (D) the barrier height decreases and the depletion layer width increases.

Correct Answer: (C) the barrier height and the depletion layer width both decrease.

Solution:

Concept:

- The depletion region of a p - n junction contains fixed ionised donors and acceptors. Their electric field creates a built-in potential barrier that opposes further diffusion of majority carriers.
- Forward bias connects the p -side to the positive terminal and the n -side to the negative terminal of a supply. The applied voltage opposes the junction's built-in potential.

Step 1: Determine the change in barrier height If the built-in barrier potential is V_0 and the applied forward voltage is V_F , the effective barrier is reduced approximately to $V_0 - V_F$. Majority carriers can therefore cross the junction more easily. Options (A) and (B), which require an increased barrier height, are inconsistent with forward bias.

Step 2: Determine the change in depletion width The forward voltage pushes majority carriers towards the junction. Carriers neutralise fixed ions near the depletion-region boundaries, causing these boundaries to move closer together. The depletion layer therefore becomes narrower. Option (D) incorrectly combines a lower barrier with a wider depletion layer; increasing the width is associated with reverse bias instead.

Step 3: Conclusion Both the barrier height and the depletion layer width decrease under forward bias. Hence, the correct option is (C).

Quick Tip to Solve: Forward bias assists majority carriers across the junction: remember a lower barrier together with a narrower depletion region.

12. Let λ_e , λ_p and λ_d be the wavelengths associated with an electron, a proton and a deuteron, all moving with the same speed. Then the correct relation between them is:

[1 mark]

- (A) $\lambda_d > \lambda_p > \lambda_e$
- (B) $\lambda_e > \lambda_p > \lambda_d$
- (C) $\lambda_p > \lambda_e > \lambda_d$
- (D) $\lambda_e = \lambda_p = \lambda_d$

Correct Answer: (B) $\lambda_e > \lambda_p > \lambda_d$

Solution:

Concept:

- The de Broglie wavelength of a particle is $\lambda = h/p$, where h is Planck's constant and p is momentum. At nonrelativistic speed v , $p = mv$.
- At the same speed, wavelength is inversely proportional to mass. The masses satisfy $m_e < m_p < m_d$, since a deuteron contains a proton and a neutron while an electron is much lighter than either nucleon.

Step 1: Compare the momenta at the common speed The common factor v gives

$$m_e v < m_p v < m_d v, \quad p_e < p_p < p_d.$$

Equal speeds do not imply equal momenta because the three particles have very different masses. Thus option (D), which assumes equal wavelengths, cannot follow from the given condition.

Step 2: Reverse the ordering for wavelengths Taking reciprocals in $\lambda = h/p$ reverses the momentum inequality:

$$\frac{h}{p_e} > \frac{h}{p_p} > \frac{h}{p_d}.$$

The electron therefore has the longest wavelength and the deuteron the shortest. Option (A) reverses the required order, while option (C) incorrectly gives the heavier proton a longer wavelength than the electron.

Step 3: Conclusion The correct relation is $\lambda_e > \lambda_p > \lambda_d$. Hence, the correct option is (B).

Quick Tip to Solve: When speeds are equal, list the particles from lightest to heaviest and reverse that mass order for their wavelengths.

13. Assertion (A): The potential energy of an electron revolving in any stationary orbit in a hydrogen atom is positive.

Reason (R): The total energy of a charged particle is always positive.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and Reason (R) is also false.

Solution:

Concept:

- With potential energy taken as zero at infinite separation, the electron-proton electrostatic potential energy is $U = -ke^2/r$, where $k = 1/(4\pi\epsilon_0)$, e is the elementary charge and r is their separation.
- In a Bohr orbit, the kinetic energy is $K = ke^2/(2r) > 0$, but the total energy is $E = K + U = -ke^2/(2r) < 0$. Negative total energy signifies a bound electron.

Step 1: Evaluate the assertion The electron and proton have opposite charges, so their interaction is attractive. Its potential energy in any stationary orbit is

$$U = -\frac{ke^2}{r} < 0.$$

Thus the assertion that the electron's potential energy is positive is false; it confuses potential energy with the necessarily positive kinetic energy of the revolving electron.

Step 2: Evaluate the reason Total energy includes both kinetic and potential energy. A hydrogen

electron is a counterexample to the word “always”: in the orbit with principal quantum number n ,

$$E_n = -\frac{13.6}{n^2} \text{ eV} < 0.$$

Its negative potential energy has twice the magnitude of its positive kinetic energy. Therefore, the total energy of a charged particle need not be positive, and the reason is false.

Step 3: Check the explanatory link The reason cannot explain the assertion because neither statement is true for a bound hydrogen electron. Options (A) and (B) require both statements to be true, while option (C) requires a true assertion; all three conflict with the energy signs just established.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: For a hydrogen orbit, use $U = -2K$ and $E = -K$: only the kinetic energy is positive under the usual zero-at-infinity convention.

14. Assertion (A): We cannot form a p-n junction diode by taking a slab of a p-type semiconductor and physically joining it to another slab of a n-type semiconductor.

Reason (R): In a p-type semiconductor $\eta_e \gg \eta_h$ while in a n-type semiconductor $\eta_h \gg \eta_e$. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- A p-n junction is formed by creating adjoining p-type and n-type regions within a continuous semiconductor crystal. Diffusion and recombination near their boundary establish a depletion region and a built-in electric field.
- Holes are majority carriers in p-type material, whereas electrons are majority carriers in n-type material. Using the printed notation, η_h and η_e represent hole and electron number densities respectively.

Step 1: Evaluate the assertion Simply pressing two separately prepared slabs together does not form the continuous crystal junction required for the usual p-n diode. The physical interface has surface defects and discontinuities; intimate crystal continuity is not established by mechanical contact. A junction is instead made by suitable doping of different regions of one crystal. The assertion is therefore true.

Step 2: Evaluate the printed reason In p-type material, acceptor doping makes holes the majority carriers, so $\eta_h \gg \eta_e$. In n-type material, donor doping makes electrons the majority carriers, so $\eta_e \gg \eta_h$. The printed reason states the reverse inequality in both cases. It is therefore false.

Step 3: Check whether the reason explains the assertion A false statement about carrier populations cannot explain why mechanically joining two slabs fails to create the required junction. Options (A) and (B) both require a true reason, and option (D) incorrectly declares the assertion false. The applicable code is the one for a true assertion and a false reason.

Step 4: Conclusion The assertion is true, but the printed reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Read both carrier subscripts carefully: p-type means holes dominate and n-type means electrons dominate, regardless of how the reason is worded.

15. Assertion (A): The deflection in a galvanometer is directly proportional to the current passing through it.

Reason (R): The coil of a galvanometer is suspended in a uniform radial magnetic field. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Concept:

- A current-carrying coil experiences torque $\tau_m = NIAB \sin \alpha$, where N is its number of turns, A its area, I the current and α the angle between its normal and the magnetic field $\vec{B}$.
- The suspension provides restoring torque $\tau_r = C\theta$, where C is the torsional constant and θ is the angular deflection.

Step 1: Evaluate Assertion (A) In a moving-coil galvanometer, equilibrium requires the magnetic torque to balance the restoring torque. Its fixed current sensitivity makes θ directly proportional to I . Thus Assertion (A) is true.

Step 2: Evaluate Reason (R) The pole pieces and soft-iron core provide a radial field of constant magnitude over the working region. The field remains parallel to the plane of the coil as it turns, so $\alpha = 90^\circ$ and $\tau_m = NIAB$. Thus Reason (R) is true.

Step 3: Connect the field to the deflection Equating torques in the radial field gives

$$C\theta = NIAB \Rightarrow \theta = \frac{NAB}{C}I.$$

The factor NAB/C is constant: the radial field removes any deflection-dependent sine factor. Hence R explains A. Option (B) misses this explanation, while (C) and (D) incorrectly reject a true statement.

Step 4: Conclusion Both A and R are true, and R correctly explains the proportionality of deflection to current. Hence, the correct option is (A).

Quick Tip to Solve: For a galvanometer, use the radial field to set the magnetic torque to $NIAB$, then balance it against $C\theta$.

16. Assertion (A): It is difficult to move a magnet into a coil of large number of turns when the circuit of the coil is closed.

Reason (R): The direction of induced current in a coil with its circuit closed, due to motion of a magnet, is such that it opposes the cause. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Concept:

- Faraday's law gives induced emf $\varepsilon = -N d\Phi_B/dt$, where N is the number of turns and Φ_B is the magnetic flux through each turn.
- Lenz's law states that the induced current produces a magnetic effect opposing the change of flux that causes it; a closed circuit allows this current to flow.

Step 1: Evaluate Assertion (A) Moving the magnet into the closed coil changes its flux linkage and induces a current. The resulting magnetic force resists the magnet's entry, so an external force must do work to push it in. Thus Assertion (A) is true.

Step 2: Evaluate Reason (R) The induced current flows in the direction specified by Lenz's law. For example, when the north pole approaches, the nearby face of the coil acts as a north pole and repels it. This opposes the flux increase caused by the approach. Thus Reason (R) is true.

Step 3: Relate the opposition to the required work The opposing magnetic force is precisely why entry is difficult. The external work supplies the energy dissipated in the closed circuit, consistent with conservation of energy. Therefore R explains A; option (B) denies this direct connection, and (C) and (D) wrongly label true statements as false.

Step 4: Conclusion Both A and R are true, and R correctly explains A through Lenz's law. Hence, the correct option is (A).

Quick Tip to Solve: For a magnet approaching a closed coil, make the nearer coil face the same pole as the approaching pole: the resulting repulsion opposes entry.

Section B (10 marks)

17. Show that $\vec{E} = \rho \vec{j}$ leads to Ohm's law. Write a condition in which the Ohm's law is not valid for a material. [2 marks]

Solution:

Concept:

- The microscopic relation is $\vec{E} = \rho \vec{j}$, where $\vec{E}$ is the electric field, $\vec{j}$ is the current density and ρ is the resistivity of the material.
- In a uniform conductor of length l and cross-sectional area A , the field magnitude is $E = V/l$ and

the current density is $j = I/A$, with V the potential difference and I the current.

Step 1: Convert the microscopic relation to circuit quantities For a uniform wire carrying steady current, the electric field and current density are parallel. Using magnitudes, substitute $E = V/l$ and $j = I/A$ into $E = \rho j$:

$$\frac{V}{l} = \rho \frac{I}{A} \Rightarrow V = \left(\frac{\rho l}{A} \right) I.$$

The coefficient $\rho l/A$ is the wire's resistance R . At fixed temperature and other physical conditions, ρ , l and A are constant, so R does not depend on the applied voltage.

Step 2: State a condition for failure of Ohm's law The proportionality requires constant resistance. If the material's temperature changes appreciably as the current rises, its resistivity generally changes, so the ratio V/I is no longer constant. For example, strong heating of a metallic filament makes its voltage-current relation nonlinear. Merely writing $E = \rho j$ with a variable ρ does not establish Ohm's law.

Step 3: Conclusion $V = RI$, with $R = \rho l/A$ constant, is Ohm's law. It is not valid as a linear relation when current-induced temperature changes make the resistance vary.

Quick Tip to Solve: Replace E by V/l and j by I/A ; explicitly keep ρ constant before claiming $V \propto I$.

18(a). In a diffraction experiment, the slit is illuminated by light of wavelength 600 nm. The first minimum of the pattern falls at $\theta = 30^\circ$. Calculate the width of the slit. [2 marks]

Solution:

Concept:

- In single-slit diffraction, dark fringes occur at $a \sin \theta = m\lambda$, where a is the slit width, θ is measured from the central direction, λ is the wavelength and $m = 1, 2, \dots$
- The first minimum corresponds to $m = 1$. Its condition is therefore $a \sin \theta = \lambda$; use the exact sine when the angle is not small.

Step 1: Identify the first-minimum condition Divide the slit into two equal halves. For the first dark fringe, light from corresponding points in the two halves has a path difference of $\lambda/2$ and cancels pairwise. Thus $(a/2) \sin \theta = \lambda/2$, giving

$$a \sin \theta = \lambda \Rightarrow a = \frac{\lambda}{\sin \theta}.$$

This is a diffraction minimum, so the single-slit condition is used.

Step 2: Substitute the wavelength and angle Convert the given wavelength into SI units: $\lambda = 600 \times 10^{-9}$ m. Since $\theta = 30^\circ$, use $\sin 30^\circ = 1/2$:

$$a = \frac{600 \times 10^{-9}}{1/2} \text{ m} = 1200 \times 10^{-9} \text{ m} = 1.2 \times 10^{-6} \text{ m}.$$

The result is twice the wavelength, consistent with $\sin \theta = 1/2$. The approximation $\sin \theta \simeq \theta$ is unnecessary and would reduce accuracy at this angle.

Step 3: Conclusion The slit width is $a = 1.2 \times 10^{-6} \text{ m} = 1.2 \mu\text{m}$.

Quick Tip to Solve: At the first minimum use $a = \lambda / \sin \theta$. For 30° , the width is immediately 2λ .

OR

18(b). In a Young's double-slit experiment, two light waves, each of intensity I_0 , interfere at a point, having a path difference $\frac{\lambda}{8}$ on the screen. Find the intensity at this point. [2 marks]

Solution:

Concept:

- For two coherent waves with intensities I_1 and I_2 , the resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$, where ϕ is their phase difference.
- A path difference Δ produces phase difference $\phi = 2\pi\Delta/\lambda$, with λ the wavelength. Here each wave has intensity I_0 .

Step 1: Convert path difference to phase difference The given path difference is $\Delta = \lambda/8$. One wavelength corresponds to a full phase cycle of 2π , so

$$\phi = \frac{2\pi}{\lambda} \left(\frac{\lambda}{8} \right) = \frac{\pi}{4} = 45^\circ.$$

Hence the waves are neither exactly in phase nor exactly out of phase. Their interference is partially constructive because $\cos \phi$ is positive.

Step 2: Substitute the equal wave intensities Using $I_1 = I_2 = I_0$, include the interference term as well as the individual intensities:

$$\begin{aligned} I &= I_0 + I_0 + 2\sqrt{I_0 I_0} \cos \frac{\pi}{4} \\ &= 2I_0 + 2I_0 \left(\frac{1}{\sqrt{2}} \right) \\ &= (2 + \sqrt{2})I_0. \end{aligned}$$

This is approximately $3.414I_0$, between the incoherent sum $2I_0$ and the fully constructive maximum $4I_0$, as expected for the positive interference term.

Step 3: Conclusion The intensity at the given point is $I = (2 + \sqrt{2})I_0$.

Quick Tip to Solve: Convert the path fraction into phase first: $\lambda/8$ means $\pi/4$. For equal beams, use $I = 2I_0(1 + \cos \phi)$.

19. A spherical convex surface of radius of curvature R separates glass (refractive index 1.5) from air. Light from a point source placed in air at distance $R/2$ from the surface falls on it. Find the position and nature of the image formed. [2 marks]

Solution:

Concept:

- For paraxial refraction at a spherical surface, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, where n_1 and n_2 are the incident and refracting medium indices, and u, v are distances from the pole.
- In the Cartesian sign convention, distances measured along the incident light are positive. A negative image distance places the image on the incident side and corresponds here to a virtual image.

Step 1: Assign the refractive indices and signed distances Take the incident light to travel from left to right, from air into glass. The source is to the left of the pole, while the centre of curvature of the convex glass surface is to the right. Hence,

$$n_1 = 1, \quad n_2 = 1.5, \quad u = -\frac{R}{2}, \quad R > 0.$$

The refractive indices must be assigned in the order in which the light encounters the media.

Step 2: Substitute in the spherical refraction formula Using these signed quantities gives

$$\begin{aligned} \frac{1.5}{v} - \frac{1}{-R/2} &= \frac{1.5 - 1}{R}, \\ \frac{1.5}{v} + \frac{2}{R} &= \frac{0.5}{R}, \\ \frac{1.5}{v} &= -\frac{1.5}{R}, \\ v &= -R. \end{aligned}$$

Thus the refracted rays in glass diverge; their backward extensions meet on the air side of the surface.

Step 3: Conclusion The image is virtual, at a distance R from the surface in air, on the same side as the source. Its signed image distance is $v = -R$.

Quick Tip to Solve: For light entering a convex glass surface from air, use $u < 0$ and $R > 0$ before substituting; do not give both distances the same sign.

20. The energy of an electron in an orbit of Bohr hydrogen atom is -3.4 eV. Find its angular momentum. [2 marks]

Solution:

Concept:

- In the Bohr model of hydrogen, the energy of the electron in the n th orbit is $E_n = -\frac{13.6}{n^2}$ eV, where n is a positive integer called the principal quantum number.
- Bohr's angular-momentum quantisation condition is $L = m_e v r = n \frac{h}{2\pi} = n\hbar$, where h is Planck's constant and $\hbar = h/(2\pi)$.

Step 1: Identify the orbit from its energy Equate the given energy to the Bohr energy formula for

hydrogen:

$$-\frac{13.6}{n^2} \text{ eV} = -3.4 \text{ eV}.$$

Both sides are already in electron volts, so no conversion to joules is needed to find n . Cancelling the negative signs and rearranging,

$$n^2 = \frac{13.6}{3.4} = 4, \quad n = 2.$$

Choose the positive root because the orbit number must be a positive integer.

Step 2: Apply angular-momentum quantisation Substitute the orbit number into Bohr's condition.

Using $h = 6.63 \times 10^{-34} \text{ J s}$,

$$\begin{aligned} L &= \frac{2h}{2\pi} = \frac{h}{\pi}, \\ &= \frac{6.63 \times 10^{-34}}{\pi} \text{ J s} \\ &\simeq 2.11 \times 10^{-34} \text{ J s}. \end{aligned}$$

The negative orbital energy describes a bound electron; it does not make the magnitude of its angular momentum negative.

Step 3: Conclusion The electron is in the second Bohr orbit and has angular-momentum magnitude

$$L = 2\hbar = h/\pi \simeq 2.11 \times 10^{-34} \text{ J s}.$$

Quick Tip to Solve: First calculate $n = \sqrt{13.6/|E_n|}$ with the energy in eV, then multiply n by $\hbar$; energy and angular momentum depend differently on n .

21. A p -type Si semiconductor is made by doping an average of one dopant atom per 5×10^7 silicon atoms. If the number density of silicon atoms in the specimen is $5 \times 10^{28} \text{ atoms/m}^3$, find the number of holes created per cubic centimetre in the specimen due to doping. Also give one example of such dopants. [2 marks]

Solution:

Concept:

- A trivalent acceptor dopant has one fewer valence electron than silicon. Each active acceptor atom creates one hole, so the hole density due to doping equals the acceptor atom density.
- The dopant number density is the silicon atom density divided by the number of silicon atoms per dopant. Since $1 \text{ m}^3 = 10^6 \text{ cm}^3$, a density in m^{-3} is divided by 10^6 to express it in cm^{-3} .

Step 1: Calculate the acceptor density The specimen contains one dopant atom for every 5×10^7 silicon atoms. With silicon atom density $N_{\text{Si}} = 5 \times 10^{28} \text{ m}^{-3}$, the acceptor density is

$$N_A = \frac{5 \times 10^{28}}{5 \times 10^7} = 10^{21} \text{ m}^{-3}.$$

Counting one hole per dopant gives the same number density for the holes created by doping.

Step 2: Convert the density and identify a dopant One cubic metre contains one million cubic centimetres. Therefore, the number of doping-created holes in each cubic centimetre is

$$p_{\text{doping}} = \frac{10^{21}}{10^6} = 10^{15} \text{ cm}^{-3}.$$

Boron is a suitable example of a dopant. Its three valence electrons leave an electron deficiency in the four-bond silicon structure, producing an acceptor level and a mobile hole. A pentavalent dopant would instead produce an n -type specimen.

Step 3: Conclusion The specimen has 10^{15} holes per cubic centimetre due to doping. One suitable p -type dopant is **boron**.

Quick Tip to Solve: Divide the silicon atom density by the dilution ratio first, then divide by 10^6 to change holes per cubic metre to holes per cubic centimetre.

Section C (21 marks)

22(a). Two batteries of emfs 3 V & 6 V and internal resistances $0.2\ \Omega$ & $0.4\ \Omega$ are connected in parallel. This combination is connected to a $4\ \Omega$ resistor. Find:

- the equivalent emf of the combination
- the equivalent internal resistance of the combination
- the current drawn from the combination

[3 marks]

Solution:

Concept:

- For two cells connected in parallel with like terminals joined, the equivalent emf is $E_{eq} = \frac{E_1/r_1 + E_2/r_2}{1/r_1 + 1/r_2}$, where E_1, E_2 are their emfs and r_1, r_2 their internal resistances.
- Their equivalent internal resistance is $r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$. With external resistance R , the current supplied to the load is $I = \frac{E_{eq}}{R + r_{eq}}$.

Step 1: Find the equivalent emf for part (i) With like terminals joined, the equivalent emf is the common terminal voltage when no external current is drawn. The algebraic sum of the branch currents then vanishes:

$$\frac{E_1 - E_{eq}}{r_1} + \frac{E_2 - E_{eq}}{r_2} = 0.$$

Rearranging gives the internal-conductance-weighted emf. With $E_1 = 3\text{ V}$, $r_1 = 0.2\ \Omega$, $E_2 = 6\text{ V}$ and $r_2 = 0.4\ \Omega$,

$$\begin{aligned} E_{eq} &= \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} \\ &= \frac{3(0.4) + 6(0.2)}{0.2 + 0.4} \text{ V} \\ &= \frac{2.4}{0.6} \text{ V} = 4 \text{ V}. \end{aligned}$$

Step 2: Find the equivalent internal resistance for part (ii) For the equivalent resistance, the two

internal resistances combine in parallel. Thus,

$$\begin{aligned} r_{\text{eq}} &= \frac{r_1 r_2}{r_1 + r_2} \\ &= \frac{0.2 \times 0.4}{0.2 + 0.4} \Omega = \frac{2}{15} \Omega \simeq 0.133 \Omega. \end{aligned}$$

Step 3: Calculate the load current for part (iii) Add the external 4Ω resistance to the equivalent internal resistance and apply Ohm's law:

$$\begin{aligned} I &= \frac{E_{\text{eq}}}{R + r_{\text{eq}}} \\ &= \frac{4}{4 + 2/15} \text{ A} = \frac{30}{31} \text{ A} \simeq 0.968 \text{ A}. \end{aligned}$$

Step 4: Conclusion (i) $E_{\text{eq}} = 4 \text{ V}$; (ii) $r_{\text{eq}} = 2/15 \Omega \simeq 0.133 \Omega$; (iii) $I = 30/31 \text{ A} \simeq 0.968 \text{ A}$.

Quick Tip to Solve: For unequal cells in parallel, weight each emf by $1/r$. Combine the internal resistances in parallel, then add the load resistance before calculating the current.

OR

22(b). (i) A conductor of length l is connected across an ideal cell of emf E . Keeping the cell connected, the length of the conductor is increased to $2l$ by gradually stretching it. If R and R' are initial and final values of resistance and v_d and v'_d are initial and final values of drift velocity, find the relation between (i) R' and R and (ii) v'_d and v_d .

(ii) When electrons drift in a conductor from lower to higher potential, does it mean that all the 'free electrons' of the conductor are moving in the same direction? [3 marks]

Solution:

Concept:

- Resistance is $R = \rho l/A$, where ρ is resistivity and A is cross-sectional area. Stretching conserves volume Al ; take the temperature and resistivity as unchanged.
- An ideal cell maintains potential difference E , so $I = E/R$. Drift speed obeys $I = neAv_d$, where n is free-electron number density and e is the charge magnitude.
- Electron drift is the small average velocity superposed on random thermal motion; it is not the instantaneous velocity of every electron.

Step 1: (i) Find the new area and resistance The conductor is stretched to $l' = 2l$. Conservation of volume gives

$$A'l' = Al \quad \Rightarrow \quad A' = \frac{A}{2}.$$

Using the unchanged resistivity, compare the two resistances:

$$\frac{R'}{R} = \frac{\rho(2l)/(A/2)}{\rho l/A} = 4.$$

Step 2: (i) Compare the drift speeds at constant emf The connected ideal cell maintains E , so

$$\frac{I'}{I} = \frac{E/R'}{E/R} = \frac{1}{4}.$$

The carrier number density n is unchanged, giving

$$\frac{v'_d}{v_d} = \frac{I'}{I} \frac{A}{A'} = \frac{1}{4} \times 2 = \frac{1}{2}.$$

Step 3: (ii) Distinguish net drift from individual motion No. Free electrons have random thermal velocities in all directions and undergo frequent collisions. The applied electric field adds a small net drift opposite to the field, from lower to higher potential. Individual electrons still move in different directions at any instant; random contributions cancel on averaging, leaving this drift component.

Step 4: Conclusion The required relations are $R' = 4R$ and $v'_d = v_d/2$. The electrons do not all move in the same direction: drift describes their small net average velocity amid random thermal motion.

Quick Tip to Solve: For stretching at constant volume, first set $A \propto 1/l$. With a fixed ideal cell, $R \propto l^2$ but $v_d \propto 1/l$; do not confuse the current ratio with the drift-speed ratio.

23. (a) Define magnetic moment of a current-carrying coil. Write its SI unit.

(b) A coil of 60 turns and area $1.5 \times 10^{-3} \text{ m}^2$ carrying 2 A current lies in a vertical plane. It experiences a torque of 0.12 Nm when placed in a uniform horizontal magnetic field. The torque acting on the coil changes to 0.05 Nm after the coil is rotated about its diameter by 90° , in the magnetic field. Find the magnitude of the magnetic field. [3 marks]

Solution:

Concept:

- The magnetic dipole moment of an N -turn plane coil carrying current I is $\mathbf{m} = NI\mathbf{A}$, where $\mathbf{A} = A\hat{n}$ is its vector area. Its direction is the right-hand-rule normal to the coil.
- In a uniform magnetic field $\mathbf{B}$, the torque is $\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$, of magnitude $\tau = mB \sin \theta$, where θ is the angle between the coil normal and the field.

Step 1: (a) Define the magnetic moment and its unit The magnetic moment of a current-carrying coil is the product of its number of turns, current and vector area:

$$\mathbf{m} = NI\mathbf{A}\hat{n}, \quad m = NIA.$$

Curl the fingers of the right hand in the current direction; the extended thumb gives $\hat{n}$. Its SI unit is A m^2 , equivalently J T^{-1} .

Step 2: (b) Calculate the moment of the given coil The printed question gives the area unit as m^3 , which is dimensionally a volume. For the intended numerical calculation, interpret the stated area as $A = 1.5 \times 10^{-3} \text{ m}^2$. Then

$$m = NIA = 60 \times 2 \times 1.5 \times 10^{-3} = 0.18 \text{ A m}^2.$$

Rotation changes the direction of this moment, while its magnitude remains unchanged.

Step 3: (b) Eliminate the unknown initial orientation Let the initial angle between the coil normal and the horizontal field be θ . For the intended rotation about the vertical diameter, the normal remains horizontal and turns through 90° . The two torque magnitudes therefore have sine and cosine factors:

$$\tau_1 = mB |\sin \theta| = 0.12 \text{ N m}, \quad \tau_2 = mB |\cos \theta| = 0.05 \text{ N m}.$$

Squaring and adding removes the unknown angle because $\sin^2 \theta + \cos^2 \theta = 1$:

$$\begin{aligned} \tau_1^2 + \tau_2^2 &= m^2 B^2, \\ mB &= \sqrt{(0.12)^2 + (0.05)^2} = 0.13 \text{ N m}, \\ B &= \frac{0.13}{0.18} = 0.722 \dots \text{ T}. \end{aligned}$$

Neither torque alone is the maximum torque; their quadrature sum gives mB .

Step 4: Conclusion (a) $m = NIA$, with SI unit $\boxed{\text{A m}^2}$. (b) Using the intended area unit m^2 and quarter-turn orientation, the field magnitude is $\boxed{B \simeq 0.72 \text{ T}}$.

Quick Tip to Solve: When a coil turns through 90° in the relevant plane, square and add its two torques: $B = \sqrt{\tau_1^2 + \tau_2^2} / (NIA)$. The unknown initial angle then cancels.

24. Consider two long co-axial solenoids S_1 and S_2 , each of length l ($\gg r_2$) and of radius r_1 and r_2 ($r_2 > r_1$). The number of turns per unit length are n_1 and n_2 respectively. Derive an expression for mutual inductance M_{12} of solenoid S_1 with respect to solenoid S_2 . Show that $M_{21} = M_{12}$. [3 marks]

Solution:

Concept:

- For a long air-core solenoid with n turns per unit length and current I , the interior magnetic field is $B = \mu_0 n I$. The field outside is negligible when end effects are neglected.
- Mutual inductance is the flux linkage in one coil per unit current in the other: $M_{12} = N_1 \Phi_{12} / I_2$, where Φ_{12} is the flux through one turn of S_1 due to I_2 in S_2 .
- The common cross-sectional area linked by both coaxial solenoids is the smaller area $A_1 = \pi r_1^2$. Their total turn numbers are $N_1 = n_1 l$ and $N_2 = n_2 l$.

Step 1: Find the flux in the inner solenoid due to the outer one Pass current I_2 through the outer solenoid S_2 . Its uniform interior field is $B_2 = \mu_0 n_2 I_2$, directed along the common axis. Each turn of the inner solenoid S_1 encloses area A_1 , perpendicular to this field. Hence the flux per inner turn and

the total flux linkage are

$$\Phi_{12} = B_2 A_1 = \mu_0 n_2 I_2 \pi r_1^2,$$
$$N_1 \Phi_{12} = \mu_0 n_1 n_2 l \pi r_1^2 I_2.$$

Dividing by the current producing this flux gives

$$M_{12} = \frac{N_1 \Phi_{12}}{I_2} = \mu_0 n_1 n_2 l \pi r_1^2.$$

Step 2: Reverse the roles of the two solenoids Now pass current I_1 through the inner solenoid S_1 . Its field is $B_1 = \mu_0 n_1 I_1$ within radius r_1 , and is negligible outside. Although an outer turn encloses πr_2^2 , only the inner area πr_1^2 contributes appreciably to its flux. Therefore,

$$\Phi_{21} = B_1 \pi r_1^2 = \mu_0 n_1 I_1 \pi r_1^2,$$
$$N_2 \Phi_{21} = \mu_0 n_1 n_2 l \pi r_1^2 I_1,$$
$$M_{21} = \frac{N_2 \Phi_{21}}{I_1} = \mu_0 n_1 n_2 l \pi r_1^2.$$

These expressions use the long-solenoid approximation $l \gg r_2$ and consistent positive flux directions for the two windings.

Step 3: Conclusion The mutual inductances are equal:

$$M_{21} = M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l.$$

Their SI unit is the henry (H). The smaller cross-sectional area determines the linked flux in either direction.

Quick Tip to Solve: Use the inner area πr_1^2 in both calculations, and multiply the flux per turn by the number of turns of the receiving solenoid before dividing by the source current.

25. (a) A parallel plate capacitor is charged by an ac source. Show that the sum of conduction current (I_c) and the displacement current (I_d) has the same value at all points of the circuit.

(b) In case (a) above, is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain. [3 marks]

Solution:

Concept:

- Conduction current is due to the motion of free charges: $I_c = dq/dt$, where q is the charge delivered to a capacitor plate.
- Maxwell's displacement current is $I_d = \epsilon_0 d\Phi_E/dt$, where Φ_E is the electric flux. The total current is $I = I_c + I_d$.
- Kirchhoff's junction rule expresses current continuity. At a capacitor plate, displacement current must be included when applying this rule to the total current.

Step 1: (a) Relate the flux between the plates to charge Let the ideal parallel plate capacitor have plate area A , separation d , and instantaneous plate charges $+q(t)$ and $-q(t)$. Neglect edge effects and

take the gap to be vacuum or air. The electric field and flux through the gap are

$$E = \frac{q(t)}{\varepsilon_0 A}, \quad \Phi_E = EA = \frac{q(t)}{\varepsilon_0}.$$

The changing ac voltage makes $q(t)$, and hence this electric flux, vary with time.

Step 2: (a) Compare the total current in a wire and in the gap In the connecting wire, the ideal-conductor approximation gives conduction current $I_c = dq/dt$ and negligible displacement current. In the insulating gap no charge crosses from one plate to the other, so $I_c = 0$, but

$$I_d = \varepsilon_0 \frac{d}{dt} \left(\frac{q(t)}{\varepsilon_0} \right) = \frac{dq}{dt}.$$

Thus, at each instant, the wire carries total current $dq/dt + 0$, while the gap carries total current $0 + dq/dt$. These are the same, including their direction during each half-cycle. For example, if $V(t) = V_m \sin \omega t$, then $q(t) = CV(t)$ gives $I = \omega CV_m \cos \omega t$ in both regions. The two contributions are evaluated at the same cross-section; they must not be added from different regions to give $2I$.

Step 3: (b) Apply the junction rule at the plates Yes, the junction rule holds at each plate when the current means the total conduction plus displacement current. At the first plate, conduction current in the lead equals displacement current in the gap; at the second plate, displacement current equals conduction current in its lead. Hence the algebraic sum of total currents at each plate is zero. If only conduction current is counted, it is not zero at a charging plate: charge is accumulating there, with net conduction inflow dq/dt . Including displacement current restores continuity without implying that charges physically cross the insulating gap.

Step 4: Conclusion $I_c + I_d = dq/dt$ has the same instantaneous value throughout the circuit. Kirchhoff's first rule is valid at both plates for this total current, although a conduction-current-only balance omits the changing electric field in the gap.

Quick Tip to Solve: Compare two cross-sections: use $I_c = dq/dt$ in the wire and $I_d = \varepsilon_0 d(EA)/dt$ in the gap. Both give the same current.

26. (a) Draw a plot of frequency ν of incident radiations as a function of stopping potential V_0 for a given photo emissive material. What information can be obtained from the value of the intercept on the stopping potential axis?

(b) Calculate: (i) the momentum and (ii) de Broglie wavelength, of an electron with kinetic energy of 80 eV. [3 marks]

Solution:

Concept:

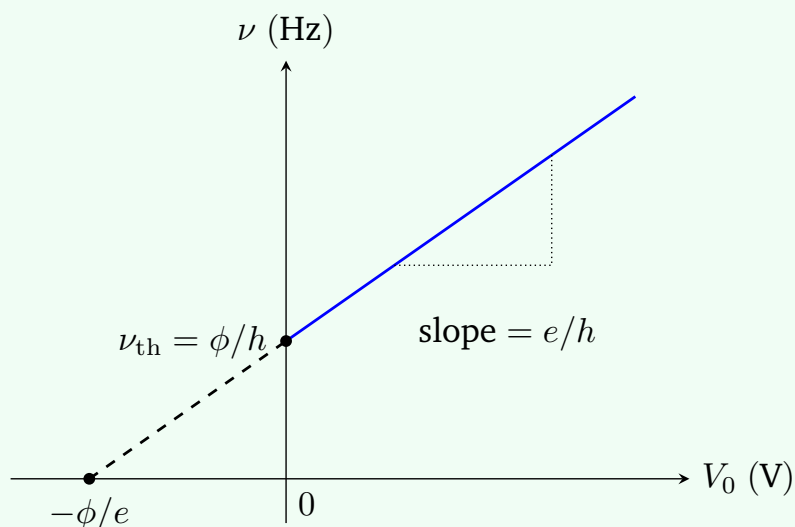
- Einstein's photoelectric equation is $eV_0 = h\nu - \phi$, where V_0 is the stopping potential, e the magnitude of electron charge, h Planck's constant, and ϕ the work function.
- For a non-relativistic electron of mass m_e and kinetic energy K , the momentum magnitude is

$p = \sqrt{2m_e K}$ and its de Broglie wavelength is $\lambda = h/p$.

Step 1: (a) Plot frequency against stopping potential Rearrange Einstein's equation to put the requested dependent variable ν on the left:

$$\nu = \frac{e}{h} V_0 + \frac{\phi}{h}.$$

Therefore, take V_0 on the horizontal axis and ν on the vertical axis. The graph is a straight line of positive slope e/h . It meets the frequency axis at the threshold frequency $\nu_{th} = \phi/h$. The solid portion shows the photoelectric relation for $V_0 \geq 0$; extend the line backwards to locate its stopping-potential-axis intercept.



Frequency as a function of stopping potential; the dashed extension locates the negative intercept.

Step 2: (a) Interpret the horizontal-axis intercept At the intercept on the stopping potential axis, set $\nu = 0$ in the equation of the extrapolated line:

$$0 = \frac{e}{h} V_{0,int} + \frac{\phi}{h}, \quad V_{0,int} = -\frac{\phi}{e}.$$

Thus the negative intercept determines the work function: $\phi = e|V_{0,int}|$. If its magnitude is a volts, the work function is numerically a electron-volts. The negative intercept is obtained by extrapolation; it does not describe photoemission at zero frequency.

Step 3: (b)(i) Calculate the momentum Convert the kinetic energy to joules before using the mechanical relation. With $e = 1.6 \times 10^{-19}$ C,

$$K = 80(1.6 \times 10^{-19}) = 1.28 \times 10^{-17} \text{ J}.$$

This energy is much smaller than the electron rest energy, so the non-relativistic expression applies.

Substituting $m_e = 9.1 \times 10^{-31} \text{ kg}$,

$$\begin{aligned} p &= \sqrt{2m_e K} \\ &= \sqrt{2(9.1 \times 10^{-31})(1.28 \times 10^{-17})} \\ &= \sqrt{2.3296 \times 10^{-47}} \\ &\simeq 4.83 \times 10^{-24} \text{ kg m s}^{-1}. \end{aligned}$$

Step 4: (b)(ii) Calculate the de Broglie wavelength Use the momentum just obtained and the given constant $h = 6.63 \times 10^{-34} \text{ J s}$:

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{4.83 \times 10^{-24}} \simeq 1.37 \times 10^{-10} \text{ m}.$$

This is approximately 0.137 nm, an atomic-scale wavelength.

Step 5: Conclusion The ν -versus- V_0 graph has slope e/h and horizontal intercept $-\phi/e$, whose magnitude determines the work function. The electron has $p \simeq 4.83 \times 10^{-24} \text{ kg m s}^{-1}$ and $\lambda \simeq 1.37 \times 10^{-10} \text{ m}$.

Quick Tip to Solve: Read the axis order carefully: plotting frequency against stopping potential gives slope e/h , and the negative horizontal intercept gives $-\phi/e$.

27. (a) Draw circuit arrangement for studying V - I characteristics of a p - n junction diode.

(b) Show the shape of the characteristics of a diode.

(c) Mention two information that you can get from these characteristics.

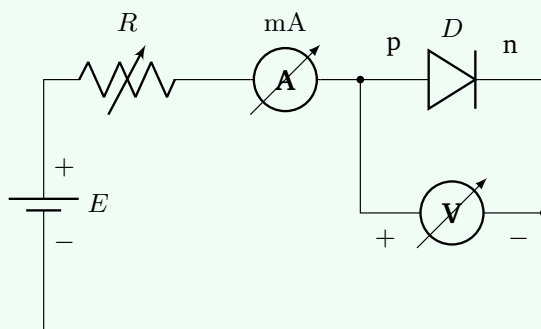
[3 marks]

Solution:

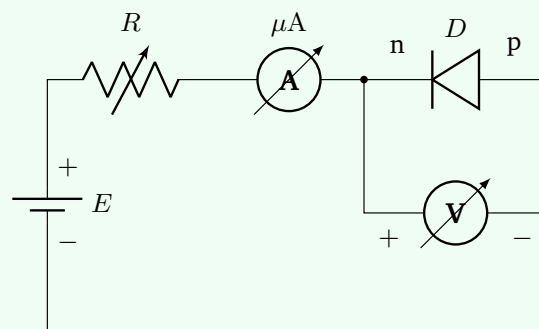
Concept:

- A diode is forward biased when its p-side is connected to the positive terminal and its n-side to the negative terminal; reversing these connections gives reverse bias.
- An ammeter measures the diode current in series, while a voltmeter connected in parallel measures the potential difference across the diode. A variable resistance limits and adjusts the current.
- The characteristic is nonlinear: current rises rapidly beyond the forward knee voltage, whereas reverse current remains small until breakdown.

Step 1: (a) Connect the circuits and record readings The arrangements below use a DC source E , variable resistance R , and a voltmeter across the diode. For forward bias, connect the p-side towards the positive terminal and use a milliammeter in series. Vary R to change the applied diode voltage and record pairs of voltage and current readings. For reverse bias, interchange the diode connections and use a microammeter, because the reverse current is much smaller. Record reverse voltage and current magnitudes in the same way; plot both as negative when using the p-to-n sign convention. Keep the current limited; an ordinary diode should not be driven into damaging breakdown during the measurement.



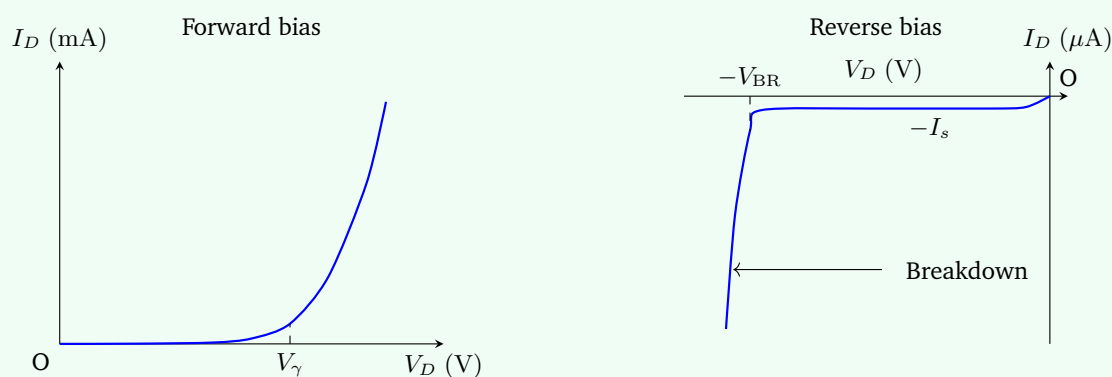
Forward bias



Reverse bias

Forward and reverse bias arrangements; R is the current-limiting variable resistance.

Step 2: (b) Plot the two characteristics Take the signed diode voltage $V_D = V_p - V_n$ on the horizontal axis and current positive from p to n. In forward bias, little current flows initially, followed by a rapid increase beyond the knee voltage V_γ . In reverse bias, a small, nearly constant reverse saturation current I_s flows. Its magnitude increases sharply if the reverse voltage reaches the breakdown value V_{BR} . The two plots use different current scales to make both behaviours visible.



Schematic characteristics: forward current is in mA and reverse current in μA ; voltage scales differ.

Step 3: (c) Read two useful quantities from the graph First, the forward knee or cut-in voltage V_γ is obtained from the bend where the forward current starts increasing rapidly. It is typically about 0.7 V for silicon and 0.3 V for germanium. Second, the small-signal or dynamic resistance at an operating point is found from the reciprocal slope of the local characteristic:

$$r_d = \frac{\Delta V_D}{\Delta I_D}.$$

Choose two nearby points around the operating point, express current in amperes, and take their voltage difference divided by their current difference. The resistance is therefore measured in ohms.

Step 4: Conclusion The circuits measure the diode voltage in parallel and current in series. The plotted characteristics show strong forward conduction and a very small reverse current before breakdown; they provide the cut-in voltage and dynamic resistance.

Quick Tip to Solve: Put the voltmeter across the diode and the current meter in series. Use mA for forward bias and μA for reverse bias; convert to amperes before calculating resistance.

28. (a) Define 'Mass defect' and 'Binding energy' of a nucleus. Describe 'Fission process' on the basis of binding energy per nucleon.

(b) A deuteron contains a proton and a neutron and has a mass of 2.013553 u. Calculate the mass defect for it in u and its energy equivalence in MeV. ($m_p = 1.007277$ u, $m_n = 1.008665$ u, $1 \text{ u} = 931.5 \text{ MeV}/c^2$)

[3 marks]

Solution:

Concept:

- The mass defect of a nucleus is $\Delta m = Zm_p + (A - Z)m_n - M$, where Z is its proton number, A its nucleon number, and M its nuclear mass.
- Nuclear binding energy is $B = \Delta mc^2$. It is the energy needed to separate the nucleus completely into its free protons and neutrons; the same energy is released when it forms from them.
- Binding energy per nucleon, B/A , measures average binding. A nuclear reaction releases energy when the products have greater total binding energy than the reactants.

Step 1: (a) Define the mass defect and explain fission The mass of a bound nucleus is smaller than the sum of the masses of its constituent free nucleons. This positive difference is its mass defect; its energy equivalent is the binding energy. In fission, a heavy nucleus splits into two medium-mass nuclei, generally with a few neutrons emitted. Medium-mass nuclei lie higher on the binding-energy-per-nucleon curve than very heavy nuclei. Their nucleons are therefore more tightly bound. The increase in total binding energy appears mainly as kinetic energy of the fragments and emitted neutrons, with some radiation.

Step 2: (b) Calculate the deuteron's mass defect A deuteron contains one proton and one neutron, so use the given nuclear masses directly; no electron mass correction is needed.

$$\begin{aligned}m_p + m_n &= 1.007277 + 1.008665 = 2.015942 \text{ u}, \\ \Delta m &= (2.015942 - 2.013553) \text{ u} \\ &= 0.002389 \text{ u}.\end{aligned}$$

Step 3: Convert the missing mass to energy Since $1 \text{ u } c^2 = 931.5 \text{ MeV}$, multiply the mass defect in atomic mass units by the supplied conversion factor:

$$\begin{aligned}B &= \Delta mc^2 \\ &= 0.002389 \times 931.5 \text{ MeV} \\ &= 2.2253535 \text{ MeV} \simeq 2.225 \text{ MeV}.\end{aligned}$$

This is the energy required to dissociate one deuteron into a free proton and neutron.

Step 4: Conclusion The deuteron's mass defect is 0.002389 u, with energy equivalent 2.225 MeV. Fission releases energy because the medium-mass products have greater total binding energy than the original heavy nucleus.

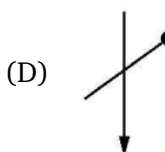
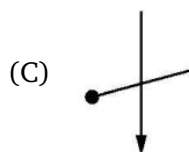
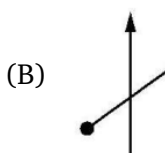
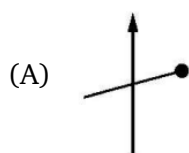
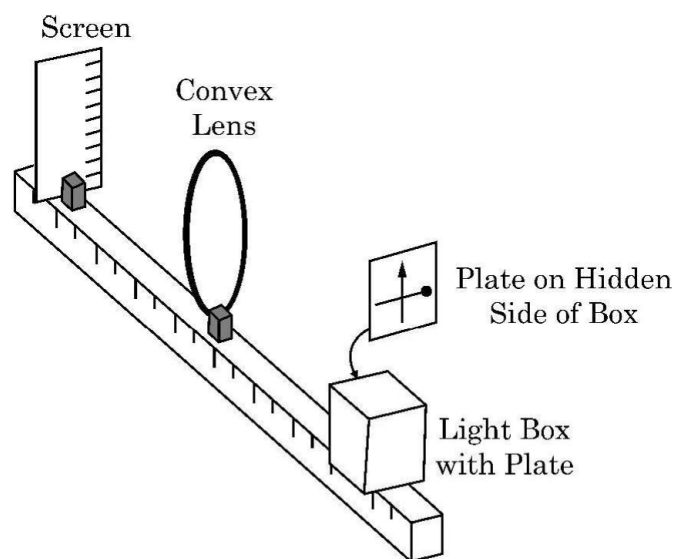
Quick Tip to Solve: Add the masses of the free nucleons first, subtract the nuclear mass, and only then multiply by 931.5 to obtain the binding energy in MeV.

Section D (8 marks)

Question numbers 29 and 30 are case study based questions. Read the following paragraphs and answer the questions that follow.

29. A thin lens is a transparent optical medium bounded by two surfaces, at least one of which should be spherical. Applying the formula for image formation by a single spherical surface successively at the two surfaces of a lens, one can obtain the 'lens maker formula' and then the 'lens formula'. A lens has two foci – called 'first focal point' and 'second focal point' of the lens, one on each side. [4 marks]

29(i). (i) Consider the arrangement shown in figure. A black vertical arrow and a horizontal thick line with a ball are painted on a glass plate. It serves as the object. When the plate is illuminated, its real image is formed on the screen. Which of the following correctly represents the image formed on the screen? [1 mark]



Correct Answer: (C)

Solution:

Concept:

- A convex lens forms a real image of an object outside its focal length with negative transverse magnification, $m = v/u < 0$, since $u < 0$ and $v > 0$.
- Inversion occurs in both transverse directions: an object point above the axis images below it, and

a point on one lateral side images on the opposite side.

Step 1: Reverse both transverse coordinates The real image can be received on a screen. The upward black arrow must therefore point downward in the image. At the same time, the ball at the right end of the horizontal line moves to its left end. This is a half-turn of the pattern in the image plane, with its size set by the magnification.

Step 2: Check the four printed patterns Option (A) retains both the upward arrow and the ball on the right. Option (B) reverses the ball's position but leaves the arrow upward. Option (D) reverses the arrow but leaves the ball on the right. Only option (C) combines a downward arrow with the ball on the left, as required by real-image inversion.

Step 3: Conclusion The screen image has the arrow pointing downward and the ball on the left. Hence, the correct option is (C).

Quick Tip to Solve: For this asymmetric object, check both the arrow tip and the ball: real-image inversion reverses both positions about the optical axis.

29(ii). (ii) Which of the following statements is incorrect?

[1 mark]

- (A) For a convex mirror magnification is always negative.
- (B) For all virtual images formed by a mirror magnification is positive.
- (C) For a concave lens magnification is always positive.
- (D) For real and inverted images, magnification is always negative.

Correct Answer: (A) For a convex mirror magnification is always negative.

Solution:

Concept:

- Transverse magnification is $m = h_i/h_o$, where h_i and h_o are signed image and object heights. An erect image has $m > 0$; an inverted image has $m < 0$.
- For a mirror, $m = -v/u$; for a thin lens, $m = v/u$. Use the usual real-object situation and Cartesian sign convention in these statements.

Step 1: Check the convex-mirror statement A convex mirror produces a virtual, erect and diminished image of a real object. Thus its magnification lies between zero and one. In the mirror formula, $u < 0$ and $v > 0$, so $-v/u > 0$. The claim in (A) that this magnification is negative is incorrect.

Step 2: Verify the remaining alternatives For a real object, a virtual mirror image is erect, so (B) correctly gives positive magnification. A concave lens also forms an erect virtual image of a real object, with $u < 0$ and $v < 0$, hence $v/u > 0$; (C) is correct. Real, inverted images have image and object heights of opposite signs, making their magnification negative; (D) is correct.

Step 3: Conclusion A convex mirror has positive magnification for a real object, so statement (A) is incorrect. Hence, the correct option is (A).

Quick Tip to Solve: Decide whether an image is erect or inverted before using a formula: erect means positive magnification, even if the image is diminished.

29(iii). (iii) A convex lens of focal length f is cut into two equal parts perpendicular to the principal axis. The focal length of each part will be: [1 mark]

- (A) f
- (B) $2f$
- (C) $\frac{f}{2}$
- (D) $\frac{f}{4}$

Correct Answer: (B) $2f$

Solution:

Concept:

- For a thin lens in air, $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is the refractive index and R_1, R_2 are the signed surface radii.
- Cutting the convex lens into two equal halves perpendicular to its axis gives two identical plano-convex lenses. A plane surface has zero curvature, $1/R_{\text{plane}} = 0$.

Step 1: Write the power of the complete lens For the symmetric convex lens implied by the two equal halves, let the magnitudes of the two curved-surface radii be R . With light travelling from left to right, $R_1 = +R$ and $R_2 = -R$. Thus

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{2(\mu - 1)}{R}.$$

Step 2: Compare the power after cutting Each half retains one curved surface and gains one plane face. Its power is therefore

$$\frac{1}{f_{\text{half}}} = (\mu - 1) \left(\frac{1}{R} - 0 \right) = \frac{1}{2f}.$$

Option (A) would apply to the familiar cut along the principal axis, which retains both original curvatures. Options (C) and (D) would increase the power, whereas this perpendicular cut halves it.

Step 3: Conclusion Each part has focal length $2f$. Hence, the correct option is (B).

Quick Tip to Solve: A perpendicular cut makes a plane face: half the original power means twice the focal length, not half the focal length.

29(iii). If an object in case (i) above is 20 cm from the lens and the screen is 50 cm away from the object, the focal length of the lens used is [1 mark]

- (A) 10 cm
- (B) 12 cm
- (C) 16 cm
- (D) 20 cm

Correct Answer: (B) 12 cm

Solution:

Concept:

- A real image is formed on a screen on the opposite side of a convex lens from the real object.
- The thin-lens formula is $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, where u and v are signed distances measured from the lens and f is its focal length.

Step 1: Locate the object and screen relative to the lens The object-to-screen separation includes the object-to-lens and lens-to-screen distances. Since the object is 20 cm from the lens, the screen is $50 - 20 = 30$ cm beyond it. With light travelling from the object towards the lens, the Cartesian signs are $u = -20$ cm and $v = +30$ cm. The given 50 cm must not be substituted as v .

Step 2: Substitute the signed distances in the lens formula

$$\frac{1}{f} = \frac{1}{30} - \frac{1}{-20} = \frac{1}{30} + \frac{1}{20} = \frac{5}{60} = \frac{1}{12} \text{ cm}^{-1}.$$

The options 10 cm, 16 cm and 20 cm have different reciprocals and therefore cannot reproduce these object and image distances. The positive reciprocal is also consistent with a converging lens.

Step 3: Conclusion The focal length is $f = +12$ cm. Hence, the correct option is (B).

Quick Tip to Solve: Subtract the object-to-lens distance from the object-to-screen separation before using the lens formula.

29(iv). The distance of an object from first focal point of a biconvex lens is X_1 and distance of the image from second focal point is X_2 . The focal length of the lens is [1 mark]

- (A) $X_1 X_2$
- (B) $\sqrt{X_1 + X_2}$
- (C) $\sqrt{X_1 X_2}$
- (D) $\sqrt{\frac{X_2}{X_1}}$

Correct Answer: (C) $\sqrt{X_1 X_2}$

Solution:

Concept:

- In the real-object, real-image arrangement, the positive object and image distances from the lens are $s = f + X_1$ and $s' = f + X_2$; X_1 and X_2 are measured from the respective focal points.

- The lens formula for these positive distances is $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$. It leads to Newton's lens relation $X_1 X_2 = f^2$.

Step 1: Express the lens formula in focal-point distances Substitute $s = f + X_1$ and $s' = f + X_2$, rather than treating the given distances as measured from the optical centre:

$$\begin{aligned}\frac{1}{f} &= \frac{1}{f + X_1} + \frac{1}{f + X_2}, \\ (f + X_1)(f + X_2) &= f(2f + X_1 + X_2), \\ f^2 + f(X_1 + X_2) + X_1 X_2 &= 2f^2 + f(X_1 + X_2).\end{aligned}$$

Cancelling the common terms gives $X_1 X_2 = f^2$.

Step 2: Select the positive focal length and check dimensions A converging biconvex lens in this arrangement has positive focal length, so take the positive square root. Both X_1 and X_2 have dimensions of length. Thus option (A) has dimensions of length squared, option (B) has dimensions of the square root of length, and option (D) is dimensionless. Only option (C) has dimensions of length and satisfies the derived relation.

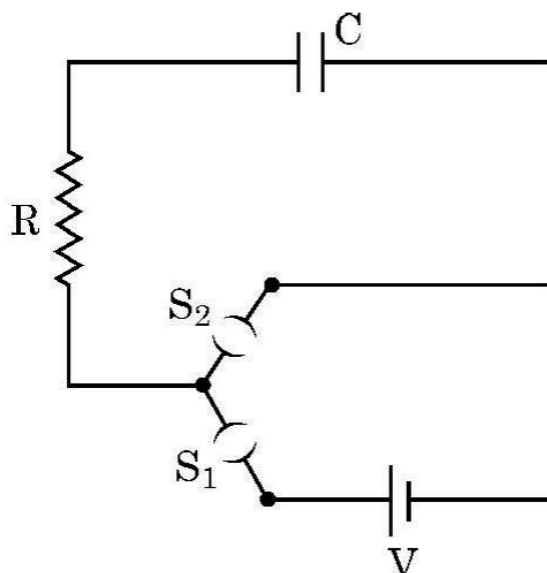
Step 3: Conclusion The focal length is $f = \sqrt{X_1 X_2}$. Hence, the correct option is (C).

Quick Tip to Solve: Distances measured from the focal points signal Newton's relation: multiply them to obtain f^2 .

30. A circuit consisting of a capacitor C , a resistor of resistance R and an ideal battery of emf V , as shown in figure is known as RC series circuit. As soon as the circuit is completed by closing key S_1 (keeping S_2 open) charges begin to flow between the capacitor plates and the battery terminals. The charge on the capacitor increases and consequently the potential difference $V_c (= q/C)$ across the capacitor also increases with time. When this potential difference equals the potential difference across the battery, the capacitor is fully charged ($Q = VC$). During this process of charging, the charge q on the capacitor changes with time t as

$$q = Q \left[1 - e^{-t/RC} \right].$$

The charging current can be obtained by differentiating it and using $\frac{d}{dx}(e^{mx}) = me^{mx}$. Consider the case when $R = 20 \text{ k}\Omega$, $C = 500 \text{ }\mu\text{F}$ and $V = 10 \text{ V}$. [4 marks]



30(i). The final charge on the capacitor, when key S_1 is closed and S_2 is open, is

[1 mark]

- (A) $5 \mu\text{C}$
- (B) 5 mC
- (C) 25 mC
- (D) 0.1 C

Correct Answer: (B) 5 mC

Solution:

Concept:

- For a capacitor, $q = CV_c$, where C is its capacitance and V_c is the potential difference across its plates.
- After a long charging time, the current through the series resistor tends to zero. Its voltage drop therefore vanishes, and the capacitor voltage approaches the battery emf V , giving $Q = CV$.

Step 1: Identify the final capacitor voltage With S_1 closed and S_2 open, the battery charges the capacitor through R . In the final steady state, $IR = 0$, so Kirchhoff's loop rule gives $V_c = 10 \text{ V}$. The resistance controls the charging rate but does not change this final voltage.

Step 2: Convert the capacitance and calculate the charge

$$Q = CV = (500 \times 10^{-6} \text{ F})(10 \text{ V}) = 5.0 \times 10^{-3} \text{ C}.$$

Since $1 \text{ mC} = 10^{-3} \text{ C}$, this is a millicoulomb value. Option (A), $5 \mu\text{C}$, is smaller by a factor of 1000, while 25 mC and 0.1 C do not equal the product CV .

Step 3: Conclusion The final charge on the capacitor is $Q = 5 \text{ mC}$. Hence, the correct option is (B).

Quick Tip to Solve: Multiplying capacitance in microfarads by voltage gives charge in microcoulombs:
 $500 \times 10 = 5000 \mu\text{C}$.

30(ii). For sufficient time the key S_1 is closed and S_2 is open. Now key S_2 is closed and S_1 is open. What is the final charge on the capacitor? [1 mark]

- (A) Zero
- (B) 5 mC
- (C) 2.5 mC
- (D) 5 μC

Correct Answer: (A) Zero

Solution:

Concept:

- A charged capacitor discharges through a resistor when the battery is disconnected and a closed conducting path remains between its plates.
- During discharge, $q(t) = Q_0 e^{-t/(RC)}$, where Q_0 is the initial charge, R is the discharge resistance and C is the capacitance.

Step 1: Identify the circuit after the keys are switched The initial long charging period gives $Q_0 = CV = 5 \text{ mC}$. Opening S_1 removes the battery from the active loop. Closing S_2 completes a loop containing only the resistor and capacitor. The capacitor therefore sends a transient discharge current through the resistor, and its stored energy is dissipated as heat.

Step 2: Take the long-time limit of the discharge Here $RC = (20 \times 10^3)(500 \times 10^{-6}) = 10 \text{ s}$. Hence

$$q(t) = (5 \text{ mC})e^{-t/(10 \text{ s})}, \quad \lim_{t \rightarrow \infty} q(t) = 0.$$

The final charge refers to this long-time limit. Option (B) is the initial charge before discharge, and the nonzero values in (C) and (D) could occur only during discharge, not in the final state. The charge does not instantly disappear when the keys are switched.

Step 3: Conclusion The capacitor becomes fully discharged, so its final charge is zero. Hence, the correct option is (A).

Quick Tip to Solve: After switching, trace the closed loop: a capacitor-resistor loop without the battery ultimately leaves the capacitor uncharged.

30(iii). The dimensional formula for RC is

[1 mark]

- (A) $[ML^2T^{-3}A^{-2}]$
- (B) $[M^0L^0T^{-1}A^0]$
- (C) $[M^{-1}L^{-2}T^4A^2]$
- (D) $[M^0L^0TA^0]$

Correct Answer: (D) $[M^0L^0TA^0]$

Solution:

Concept:

- Resistance satisfies $R = V/I$, while capacitance satisfies $C = q/V$. Here V , I and q denote potential difference, current and charge.
- The exponent of an exponential function must be dimensionless. Thus the charging factor $e^{-t/(RC)}$ requires RC to have the same dimensions as time t .

Step 1: Find the dimensions of the product directly Use the defining relations so that the potential-difference dimensions cancel:

$$[RC] = \frac{[V]}{[I]} \frac{[q]}{[V]} = \frac{[q]}{[I]} = \frac{AT}{A} = T.$$

Equivalently, $\Omega \cdot F = (V/A)(C/V) = s$, where the upright C denotes the coulomb unit.

Step 2: Compare the four dimensional formulae Mass, length and electric current cancel, leaving time to the first power. Option (A) is the dimensional formula of resistance alone, and option (C) is that of capacitance alone. Option (B) represents inverse time and would apply to $1/(RC)$. The time dimension also ensures that $t/(RC)$ in the given charging exponential is dimensionless.

Step 3: Conclusion The dimensional formula of RC is $[M^0 L^0 T A^0]$. Hence, the correct option is (D).

Quick Tip to Solve: Read the exponent $-t/(RC)$: its denominator must have dimensions of time, so RC is a time constant.

30(iv). The key S_1 is closed and S_2 is open. The value of current in the resistor after 5 seconds, is [1 mark]

- (A) $\frac{1}{2\sqrt{e}}$ mA
 (B) $\sqrt{e}$ mA
 (C) $\frac{1}{\sqrt{e}}$ mA
 (D) $\frac{1}{2e}$ mA

Correct Answer: (A) $\frac{1}{2\sqrt{e}}$ mA

Solution:

Concept:

- Charging current is the rate of increase of capacitor charge: $i = dq/dt$. The given charge is $q = Q[1 - e^{-t/(RC)}]$, with $Q = CV$.
- The time constant is $\tau = RC$; it sets the rate at which the charging current decays from its initial value V/R .

Step 1: Differentiate the charge expression The minus sign outside the exponential cancels the minus sign from its derivative:

$$i(t) = \frac{dq}{dt} = \frac{Q}{RC} e^{-t/(RC)} = \frac{V}{R} e^{-t/(RC)}.$$

Thus the charging current decreases with time, even though the stored charge increases.

Step 2: Evaluate the current at the stated time Convert $R = 20 \times 10^3 \Omega$ and $C = 500 \times 10^{-6} F$. Then

$$\begin{aligned} RC &= (20 \times 10^3)(500 \times 10^{-6}) = 10 \text{ s}, \\ \frac{V}{R} &= \frac{10}{20 \times 10^3} \text{ A} = 0.5 \text{ mA}, \\ i(5 \text{ s}) &= (0.5 \text{ mA})e^{-5/10} = \frac{1}{2\sqrt{e}} \text{ mA}. \end{aligned}$$

Options (B) and (C) exceed the initial current 0.5 mA, which a decaying current cannot do. Option

(D) uses the factor e^{-1} , corresponding to 10 s, whereas the stated time is only half a time constant.

Step 3: Conclusion The current after 5 s is $\frac{1}{2\sqrt{e}}$ mA, approximately 0.303 mA. Hence, the correct option is (A).

Quick Tip to Solve: Calculate V/R and RC first; at half a time constant, divide the initial charging current by $\sqrt{e}$.

OR

30(iv). The key S_1 is closed and S_2 is open. The initial value of charging current in the resistor, is [1 mark]

- (A) 5 mA
- (B) 0.5 mA
- (C) 2 mA
- (D) 1 mA

Correct Answer: (B) 0.5 mA

Solution:

Concept:

- An initially uncharged capacitor has $V_c = q/C = 0$ at the instant charging begins. The full battery emf is therefore initially across the series resistor.
- Ohm's law gives $i_0 = V/R$. Equivalently, the charging-current expression $i(t) = (V/R)e^{-t/(RC)}$ has $e^0 = 1$ at $t = 0$.

Step 1: Apply the loop rule at the start of charging The given charging law implies $q(0) = Q(1-1) = 0$, so the capacitor is initially uncharged. With S_1 closed and S_2 open, the loop equation is $V = iR + V_c$. At the initial instant, $V_c = 0$, leaving $i_0R = V$. This is the largest charging current; it subsequently decreases as the capacitor voltage builds up.

Step 2: Substitute the battery emf and resistance

$$i_0 = \frac{10 \text{ V}}{20 \times 10^3 \Omega} = 5 \times 10^{-4} \text{ A} = 0.5 \text{ mA}.$$

The values 5 mA, 2 mA and 1 mA would require initial resistor voltage drops of 100 V, 40 V and 20 V, respectively. Each exceeds the given 10 V, so none satisfies the initial loop equation.

Step 3: Conclusion The initial charging current is $i_0 = 0.5$ mA. Hence, the correct option is (B).

Quick Tip to Solve: For an initially uncharged capacitor, calculate the starting current using only V/R ; capacitance controls how quickly it changes.

Section E (15 marks)

31(a). (i) (1) What are coherent sources? Why are they necessary for observing a sustained interference pattern?

(2) Lights from two independent sources are not coherent. Explain.

(ii) Two slits 0.1 mm apart are arranged 1.20 m from a screen. Light of wavelength 600 nm from a distant source is incident on the slits.

(1) How far apart will adjacent bright interference fringes be on the screen?

(2) Find the angular width (in degree) of the first bright fringe.

[5 marks]

Solution:

Concept:

- Coherent sources emit waves of the same frequency with a phase difference that remains constant with time. Their superposition can produce a stationary interference pattern.
- For two waves of intensities I_1 and I_2 , the resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$, where ϕ is their phase difference at the observation point.
- In Young's double-slit experiment, the fringe width is $\beta = \lambda D/d$, where λ is the wavelength, D the slit-to-screen distance and d the slit separation. For small angles, the angular fringe width is $\Delta\theta = \beta/D = \lambda/d$ in radians.

Step 1: (i)(1) Define coherence and explain its necessity Two sources are coherent when their emitted waves have the same frequency and maintain a constant phase difference. At any fixed point on the screen, a fixed path difference then gives a phase difference that does not change with time. Thus the positions of constructive interference (bright fringes) and destructive interference (dark fringes) remain fixed, producing a sustained pattern.

If the relative phase varies randomly, the bright and dark positions fluctuate rapidly. Over the observation time, the average of the interference term vanishes, since $\langle \cos \phi \rangle = 0$. The observed intensity becomes $I_1 + I_2$, and distinct stationary fringes are lost. This explains why coherence is necessary.

Step 2: (i)(2) Explain why independent sources are incoherent In two independent ordinary light sources, different atoms emit separate wave trains at random times with unrelated initial phases. Their emission processes are independent, so the phase difference between the two beams changes unpredictably and rapidly. Even if both sources emit light of the same nominal frequency, they do not maintain a constant phase difference and are therefore not coherent.

In Young's experiment, illuminating both slits with one source creates two secondary sources with a stable relative phase. A common distant source supplies approximately plane wavefronts to the two slits.

Step 3: (ii)(1) Calculate the separation of adjacent bright fringes First express all lengths in SI units:

$$d = 0.1 \text{ mm} = 1.0 \times 10^{-4} \text{ m},$$

$$D = 1.20 \text{ m},$$

$$\lambda = 600 \text{ nm} = 6.00 \times 10^{-7} \text{ m}.$$

For a point a distance y from the central maximum, the path difference is approximately dy/D . The condition for the n th bright fringe is $dy_n/D = n\lambda$, giving $y_n = n\lambda D/d$. Hence the separation of

adjacent bright fringes is

$$\begin{aligned}\beta &= y_{n+1} - y_n = \frac{\lambda D}{d} \\ &= \frac{(6.00 \times 10^{-7})(1.20)}{1.0 \times 10^{-4}} \text{ m} \\ &= 7.20 \times 10^{-3} \text{ m} = 7.20 \text{ mm}.\end{aligned}$$

Step 4: (ii)(2) Find the angular width in degrees The first bright fringe is bounded by the neighbouring dark fringes, whose path differences are $\lambda/2$ and $3\lambda/2$. Their separation is one fringe width β . Thus, using the small-angle approximation,

$$\begin{aligned}\Delta\theta &= \frac{\beta}{D} = \frac{\lambda}{d} \\ &= \frac{6.00 \times 10^{-7}}{1.0 \times 10^{-4}} \text{ rad} \\ &= 6.00 \times 10^{-3} \text{ rad}.\end{aligned}$$

Convert radians to degrees by multiplying by $180/\pi$:

$$\Delta\theta = 6.00 \times 10^{-3} \times \frac{180}{\pi}^\circ \approx 0.344^\circ.$$

The small value of this angle also confirms that the paraxial fringe-width formula is appropriate here.

Step 5: Conclusion Coherent sources have the same frequency and a constant phase difference, keeping the interference fringes stationary. Independent ordinary sources have randomly varying relative phases and cannot produce a sustained pattern. The adjacent bright fringes are **7.20 mm** apart, and the angular width of the first bright fringe is **0.344°** (approximately 0.34°).

Quick Tip to Solve: Convert both slit separation and wavelength to metres first. Use $\lambda D/d$ for linear fringe width and λ/d for angular width; multiply the latter by $180/\pi$ only when degrees are requested.

OR

31(b). (i) Define a wavefront. An incident plane wave falls on a convex lens and gets refracted through it. Draw a diagram to show the incident and refracted wavefront.

(ii) A beam of light coming from a distant source is refracted by a spherical glass ball (refractive index 1.5) of radius 15 cm. Draw the ray diagram and obtain the position of the final image formed.

[5 marks]

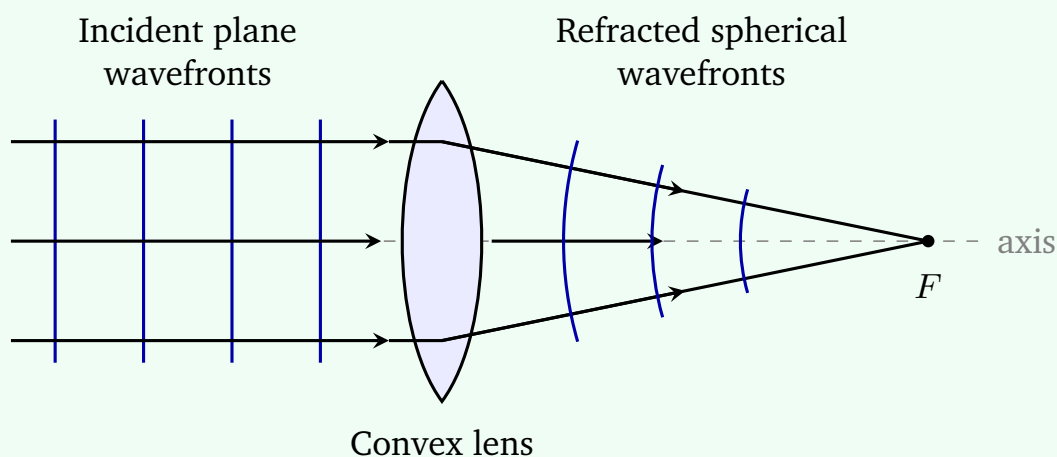
Solution:

Concept:

- A wavefront is the locus of points of a wave that are in the same phase at a given instant. In an isotropic medium, light rays are perpendicular to the wavefront.
- A convex lens converts a plane wavefront incident parallel to its principal axis into a converging spherical wavefront whose centre is the principal focus on the other side.
- For paraxial refraction at a spherical surface, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, where n_1, n_2 are the incident and refracted medium indices, and u, v, R are signed distances measured from that surface's pole.

Step 1: (i) Define and draw the wavefronts A wavefront joins points oscillating in the same phase

at one instant. For a distant source the rays reaching the convex lens are parallel, so the incident wavefronts are parallel planes, shown edge-on as straight lines perpendicular to the principal axis. The lens makes the emergent rays converge at its principal focus F . Consequently, the refracted wavefronts are portions of spheres centred at F , shown as circular arcs concave towards F . The ray arrows point towards F and are normal to these arcs. After passing through F , the wave would diverge; the diagram shows the converging region before the focus.



Plane wavefronts become converging spherical wavefronts; the rays are normal to the wavefronts.

Step 2: (ii) Set the sign convention for the glass ball Let light travel from left to right through the glass ball in air. Denote the first and second poles by P_1 and P_2 , and the centre by C . The separation of the poles is the diameter,

$$P_1P_2 = 2R = 30 \text{ cm.}$$

Take rightward distances as positive, with the origin at the pole of the surface currently being considered. At the first surface, the centre lies to the right, so $R_1 = +15 \text{ cm}$. At the second surface it lies to the left, so $R_2 = -15 \text{ cm}$. The distant source provides parallel paraxial rays, and hence $u_1 = -\infty$.

Step 3: Find the image produced by the first surface alone At P_1 , light enters glass from air, so $n_1 = 1$ and $n_2 = 1.5$. Using all distances in centimetres,

$$\begin{aligned} \frac{1.5}{v_1} - \frac{1}{-\infty} &= \frac{1.5 - 1}{15}, \\ \frac{1.5}{v_1} &= \frac{1}{30}, \\ v_1 &= +45 \text{ cm.} \end{aligned}$$

Thus the rays inside the glass are directed towards a point I_1 , located 45 cm to the right of P_1 . They encounter the second surface before reaching that point. The dashed lines in the ray diagram show where the internal rays would meet if their directions remained unchanged.

Step 4: Use the intermediate image as the second surface's object Measured from P_2 , the point I_1 lies at

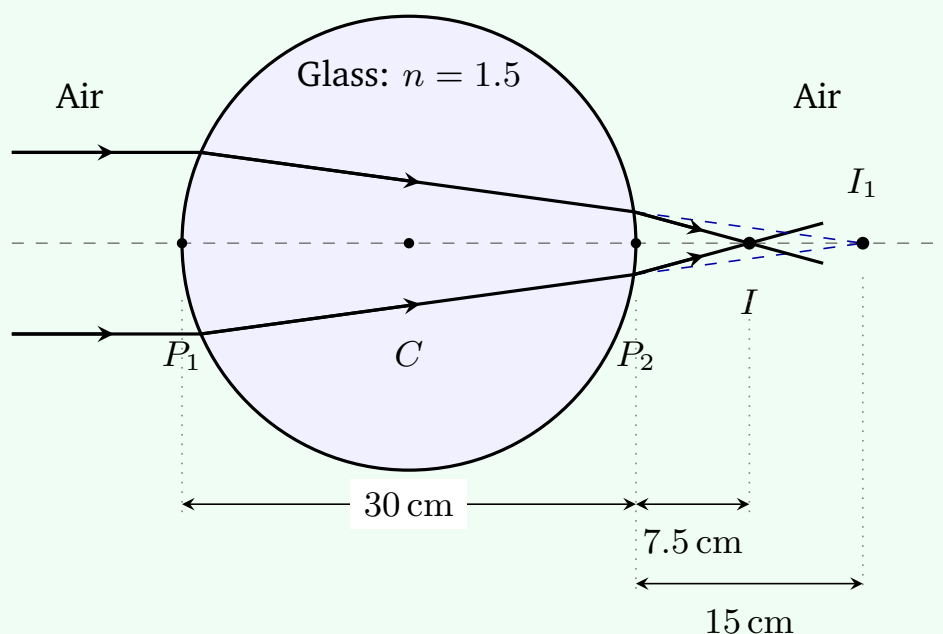
$$u_2 = P_1I_1 - P_1P_2 = 45 - 30 = +15 \text{ cm.}$$

This is a virtual object for the second surface: the incident rays are already converging towards a point on its outgoing side. Therefore u_2 is positive. At this surface light passes from glass to air, so the incident index is 1.5, the refracted index is 1, and $R_2 = -15 \text{ cm}$.

Step 5: Calculate the final image position Apply the spherical refraction formula separately at P_2 :

$$\begin{aligned}\frac{1}{v_2} - \frac{1.5}{15} &= \frac{1 - 1.5}{-15}, \\ \frac{1}{v_2} - \frac{1}{10} &= \frac{1}{30}, \\ \frac{1}{v_2} &= \frac{1}{30} + \frac{1}{10} = \frac{2}{15}, \\ v_2 &= +7.5 \text{ cm.}\end{aligned}$$

The positive image distance means that the emergent rays actually meet in air to the right of P_2 . Relative to the centre of the ball, the image coordinate is $15 + 7.5 = 22.5$ cm on the outgoing side. These positions use the paraxial approximation underlying the spherical-surface formula.



Paraxial ray diagram (schematic). Solid rays meet at the final image I ; dashed extensions of the internal rays meet at I_1 .

Step 6: Conclusion (i) A wavefront is a surface of constant phase. The convex lens changes the incident plane wavefront into a converging spherical wavefront centred at its principal focus. (ii) The glass ball forms a real final image 7.5 cm beyond its second surface, or 22.5 cm from its centre on the outgoing side.

Quick Tip to Solve: For a glass ball, refract at each surface separately. Subtract the diameter from the first image distance to obtain u_2 ; a convergence point beyond the second surface gives a positive virtual-object distance.

32(a). (i) Two point charges $5\ \mu\text{C}$ and $-1\ \mu\text{C}$ are placed at points $(-3\ \text{cm}, 0, 0)$ and $(3\ \text{cm}, 0, 0)$ respectively. An external electric field $\vec{E} = \frac{A}{r^2} \hat{r}$, where $A = 3 \times 10^5\ \text{V m}$, is switched on in the region. Calculate the change in electrostatic energy of the system due to the electric field.

(ii) A system of two conductors is placed in air and they have net charge of $+80\ \mu\text{C}$ and $-80\ \mu\text{C}$ which causes a potential difference of $16\ \text{V}$ between them.

(1) Find the capacitance of the system.

(2) If the air between the capacitor is replaced by a dielectric medium of dielectric constant 3, what will be the potential difference between the two conductors?

(3) If the charges on two conductors are changed to $+160\ \mu\text{C}$ and $-160\ \mu\text{C}$, will the capacitance of the system change? Give reason for your answer. [5 marks]

Solution:

Concept:

- Electric potential and radial electric field are related by $E_r = -dV/dr$. With $V(\infty) = 0$, the field $\vec{E} = (A/r^2)\hat{r}$ has potential $V(r) = A/r$, where r is distance from the origin.
- For fixed point charges q_i , the added interaction energy in an externally imposed potential is $\Delta U = \sum_i q_i V_{\text{ext}}(r_i)$. The mutual energy between the fixed charges does not change.
- Capacitance is $C = Q/V$, where Q is the magnitude of charge on either conductor and V is their potential difference. It depends on geometry and the intervening medium.
- Filling a capacitor with a dielectric of relative permittivity K gives $C' = KC$. For an isolated capacitor, Q remains fixed and $V' = V/K$.

Step 1: (i) Find the external potential at both charges The coordinates have opposite signs, but both charges are at the same radial distance from the origin: $r_1 = r_2 = 3\ \text{cm} = 0.03\ \text{m}$. The potential is a scalar and depends on this positive distance. Integrating the radial field from infinity gives

$$V(r) = - \int_{\infty}^r \frac{A}{r'^2} dr' = \frac{A}{r}.$$

Hence the external potential at either charge is

$$V_1 = V_2 = \frac{3 \times 10^5}{0.03} = 1.0 \times 10^7\ \text{V}.$$

Step 2: (i) Calculate the energy added by the external field The separation of the two charges remains $6\ \text{cm}$, so their mutual potential energy is unchanged. Only their interaction with the newly applied field contributes to the requested change:

$$\begin{aligned} \Delta U &= q_1 V_1 + q_2 V_2 \\ &= (5 \times 10^{-6})(10^7) + (-1 \times 10^{-6})(10^7) \\ &= 50 - 10 = +40\ \text{J}. \end{aligned}$$

The negative charge contributes negative energy; the positive charge contributes a larger positive energy. There is no factor of $1/2$ when calculating the interaction of these charges with the specified

external potential.

Step 3: (ii)(1) Calculate the capacitance in air Use the magnitude of the charge on one conductor, $Q = 80 \mu\text{C}$, rather than the zero algebraic sum of the two charges:

$$C = \frac{Q}{V} = \frac{80 \times 10^{-6}}{16} = 5.0 \times 10^{-6} \text{ F} = 5.0 \mu\text{F}.$$

Step 4: (ii)(2) Find the potential difference after inserting the dielectric No battery connection is specified, so the two conductors retain their given charges. For $K = 3$, the new capacitance and potential difference are

$$C' = KC = 3(5) = 15 \mu\text{F},$$
$$V' = \frac{Q}{C'} = \frac{80 \mu\text{C}}{15 \mu\text{F}} = \frac{16}{3} \text{ V} \approx 5.33 \text{ V}.$$

The increased capacitance reduces the potential difference for the same stored charge.

Step 5: (ii)(3) Explain the effect of doubling the charges Doubling the charges to $\pm 160 \mu\text{C}$ doubles the potential difference for a fixed geometry and medium. Therefore the ratio Q/V , and hence the capacitance, remains unchanged. It is $5 \mu\text{F}$ in air or $15 \mu\text{F}$ if the dielectric remains present; changing charge alone cannot change capacitance.

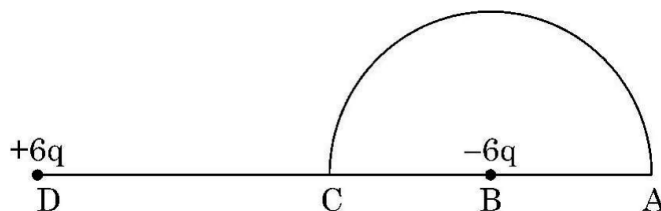
Step 6: Conclusion (i) The electrostatic energy increases by 40 J . (ii)(1) $C = 5 \mu\text{F}$; (2) $V' = 16/3 \text{ V} \approx 5.33 \text{ V}$; (3) the capacitance does not change when the charges are doubled, provided the geometry and medium stay fixed.

Quick Tip to Solve: Equal distances from the origin give equal external potentials even on opposite sides. For dielectric insertion, decide first whether charge is fixed; here Q stays fixed, so divide the original voltage by K .

OR

32(b). (i) Consider three metal spherical shells A , B and C , each of radius R . Each shell is having a concentric metal ball of radius $R/10$. The spherical shells A , B and C are given charges $+6q$, $-4q$ and $14q$ respectively. Their inner metal balls are also given charges $-2q$, $+8q$ and $-10q$ respectively. Compare the magnitude of the electric fields due to shells A , B and C at a distance $3R$ from their centres.

(ii) A charge $-6 \mu\text{C}$ is placed at the centre B of a semicircle of radius 5 cm , as shown in the figure. An equal and opposite charge is placed at point D at a distance of 10 cm from B . A charge $+5 \mu\text{C}$ is moved from point C to point A along the circumference. Calculate the work done on the charge. [5 marks]



Solution:

Concept:

- By Gauss's law, the external field of a spherically symmetric charge distribution is $E = k|Q_{\text{enclosed}}|/r^2$, where $k = 1/(4\pi\epsilon_0)$, Q_{enclosed} is its total charge and r is distance from its centre.

- The electric field within conducting material is zero. A concentric charged ball induces an equal and opposite charge on the inner surface of the surrounding shell; the remaining charge appears on its outer surface.
- Electric potential is a scalar: $V = k \sum_i Q_i/r_i$. The potential energy change of a moving charge q_0 is $\Delta U = q_0(V_f - V_i)$.
- Electrostatic work is path independent. The external work for slow transfer is $W_{\text{ext}} = \Delta U$, while the electric field does work $W_{\text{field}} = -\Delta U$.

Step 1: (i) Find the total enclosed charge for each arrangement At $r = 3R$, a Gaussian surface encloses both the metal shell and its inner ball. The total charges are

$$\begin{aligned}Q_A &= 6q - 2q = 4q, \\Q_B &= -4q + 8q = 4q, \\Q_C &= 14q - 10q = 4q.\end{aligned}$$

Equivalently, the charges induced on the inner surfaces are $+2q, -8q, +10q$, respectively. Subtracting these from the specified shell charges leaves $+4q$ on every outer surface. Thus the differently charged inner balls do not produce different external fields in these three arrangements.

Step 2: (i) Apply Gauss's law at the common distance The concentric spherical geometry ensures spherical symmetry. For each arrangement,

$$E(4\pi r^2) = \frac{4q}{\epsilon_0}, \quad E_A = E_B = E_C = \frac{4kq}{(3R)^2} = \frac{4kq}{9R^2}.$$

Here q denotes the positive charge magnitude. Therefore the ratio of the field magnitudes is $E_A : E_B : E_C = 1 : 1 : 1$. The ball radius $R/10$ does not enter the external-field result because all its charge is enclosed.

Step 3: (ii) Read the initial and final distances from the figure The fixed charges are $Q_B = -6 \mu\text{C}$ and $Q_D = +6 \mu\text{C}$. The moving charge is $q_0 = +5 \mu\text{C}$, initially at C and finally at A . Since B is the centre of the semicircle,

$$BC = BA = 0.05 \text{ m}, \quad BD = 0.10 \text{ m}.$$

The collinear order is D, C, B, A , so

$$DC = BD - BC = 0.05 \text{ m}, \quad DA = DB + BA = 0.15 \text{ m}.$$

The contribution from the negative charge at B is the same at both endpoints because their distances from B are equal.

Step 4: (ii) Calculate the potentials at the endpoints Taking zero potential at infinity and using $k = 9 \times 10^9 \text{ N m}^2\text{C}^{-2}$, add the potentials of the two fixed charges:

$$V_C = 9 \times 10^9 \left(\frac{-6 \times 10^{-6}}{0.05} + \frac{6 \times 10^{-6}}{0.05} \right) = 0.$$

At A , the positive charge is farther away, so its positive contribution is smaller:

$$\begin{aligned}V_A &= 9 \times 10^9 \left(\frac{-6 \times 10^{-6}}{0.05} + \frac{6 \times 10^{-6}}{0.15} \right) \\&= -7.2 \times 10^5 \text{ V}.\end{aligned}$$

Step 5: (ii) Use the potential difference to obtain the work The work required from an external agent for a slow transfer equals the change in potential energy:

$$\begin{aligned} W_{\text{ext}} &= \Delta U = q_0(V_A - V_C) \\ &= (5 \times 10^{-6})(-7.2 \times 10^5 - 0) \\ &= -3.6 \text{ J.} \end{aligned}$$

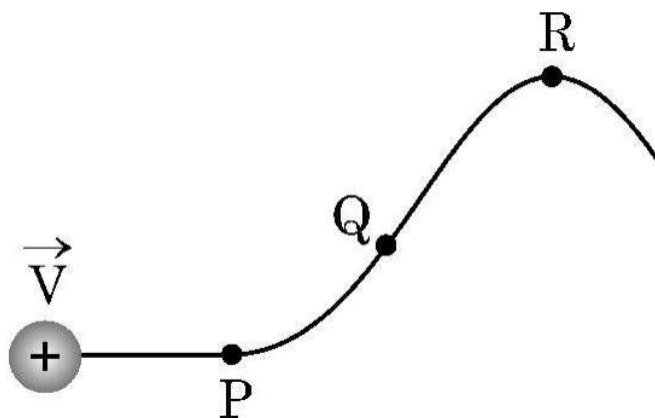
The negative sign means that the external agent removes energy while transferring the positive charge to a lower potential. The work done by the electric field itself is $+3.6 \text{ J}$. Stating the agency resolves the sign convention in the phrase “work done on the charge.” The semicircular path does not alter either result because the electrostatic field is conservative.

Step 6: Conclusion (i) $E_A : E_B : E_C = 1 : 1 : 1$, with each field magnitude $4kq/(9R^2)$. (ii) The required external work is -3.6 J ; equivalently, the electric field does $+3.6 \text{ J}$ of work on the charge.

Quick Tip to Solve: Outside a shell, add the shell and ball charges before applying Gauss’s law. Along the semicircle, the central charge contributes no potential difference; only the changing distance from D matters.

33(a). (i) A proton moving with velocity $\vec{V}$ in a non-uniform magnetic field traces a path as shown in the figure. The path followed by the proton is always in the plane of the paper. What is the direction of the magnetic field in the region near points P , Q and R ? What can you say about relative magnitude of magnetic fields at these points?

(ii) A current carrying circular loop of area A produces a magnetic field B at its centre. Show that the magnetic moment of the loop is $\frac{2BA}{\mu_0} \sqrt{\frac{A}{\pi}}$. [5 marks]



Solution:

Concept:

- A proton has positive charge e , so its magnetic force is $\vec{F} = e(\vec{v} \times \vec{B})$. The force is perpendicular to its velocity and bends the path towards the local centre of curvature.
- A magnetic field does no work, so the proton’s speed v remains constant. For a transverse field, $evB = m_p v^2/r$, giving $B = m_p v/(er)$, where m_p is the proton mass and r the local radius of curvature.
- For a circular loop of radius a , current I and area A , the central field is $B = \mu_0 I/(2a)$, while the magnitude of its magnetic moment is $M = IA$.

Step 1: (i) Read the bending direction at P and R Follow the proton from the left towards P , then through Q to R . Near P , its velocity is towards the right and its path starts bending upwards. The magnetic force must therefore point upwards. For a positive charge, the right-hand rule for $\vec{v} \times \vec{B}$ gives a magnetic field directed **into the plane of the paper** near P . At the crest R , the proton again moves towards the right, but the path bends downwards. The force is now downwards, requiring the field to point **out of the plane of the paper** near R . Reversing the curvature reverses the transverse field direction; the sign of the proton's charge remains positive throughout.

Step 2: (i) Identify the field at Q and compare magnitudes The central portion near Q is shown locally straight, at the change between upward and downward bending. Its curvature is zero, so there is no transverse magnetic force there. Thus the field represented in the diagram is zero at Q , and no field direction can be assigned there. For the curved portions, equate the magnetic force to the local centripetal force:

$$evB = \frac{m_p v^2}{r} \implies B = \frac{m_p v}{er}.$$

The speed is unchanged because $\vec{F} \cdot \vec{v} = 0$. Hence the field magnitude is inversely proportional to the local radius of curvature. The bend near R is sharper than near P , so $r_R < r_P$ and $B_R > B_P$, while $B_Q = 0$. The sketch supports this ordering, but gives no numerical ratio of the nonzero fields.

Step 3: (ii) Express the loop radius and current in terms of A and B Let the circular loop have radius a . Its area gives

$$A = \pi a^2 \implies a = \sqrt{\frac{A}{\pi}}.$$

From the magnetic field at the centre of a circular current loop,

$$B = \frac{\mu_0 I}{2a} \implies I = \frac{2aB}{\mu_0}.$$

Here B is the field produced by this loop itself, and μ_0 is the permeability of free space.

Step 4: (ii) Substitute into the magnetic moment The magnetic moment equals current times enclosed area. Substituting first for the current and then for the radius gives

$$\begin{aligned} M &= IA \\ &= \frac{2aB}{\mu_0} A \\ &= \frac{2BA}{\mu_0} \sqrt{\frac{A}{\pi}}. \end{aligned}$$

Its vector is normal to the loop, with direction given by curling the right-hand fingers in the current direction. The derived expression is its magnitude, measured in A m^2 .

Step 5: Conclusion (i) The field is into the paper near P , zero at Q , and out of the paper near R . Its magnitudes satisfy $B_R > B_P > B_Q = 0$. (ii) The loop's magnetic moment is $M = \frac{2BA}{\mu_0} \sqrt{\frac{A}{\pi}}$.

Quick Tip to Solve: For a positive particle, use the direction of bending to find the field direction, then compare curvature for field strength. For the loop, eliminate its radius using $a = \sqrt{A/\pi}$.

33(b). (i) Derive an expression for the torque acting on a rectangular current loop suspended in a uniform magnetic field.

(ii) A charged particle is moving in a circular path with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. It is made to pass through a sheet of lead and as a consequence, it loses one half of its kinetic energy without change in its direction. How will (1) the radius of its path (2) its time period of revolution change? [5 marks]

Solution:

Concept:

- A straight conductor carrying current I in a uniform magnetic field $\vec{B}$ experiences force $\vec{F} = I(\vec{L} \times \vec{B})$, where $\vec{L}$ points along the current.
- A force couple has torque equal to either force times the perpendicular separation of their lines of action. The magnetic moment of a plane loop is $\vec{\mu} = IA\hat{n}$, with $\hat{n}$ fixed by the right-hand rule.
- For a nonrelativistic particle of mass m and charge magnitude $|q|$ moving perpendicular to $\vec{B}$, $r = mv/(|q|B)$, $K = mv^2/2$, and $T = 2\pi r/v$.

Step 1: (i) Choose the loop geometry and evaluate the forces Consider a single rectangular loop of sides l and b , area $A = lb$, carrying current I . Let θ be the angle between its area normal $\hat{n}$ and the uniform field $\vec{B}$. Choose the two sides of length l perpendicular to the field; the field lies in the plane perpendicular to those sides. The currents in these two opposite sides have opposite directions. Each side experiences a magnetic force of magnitude

$$F = IlB \sin 90^\circ = IlB.$$

These equal and opposite forces form a couple. The forces on the other two sides, of length b , are also equal and opposite, but act along the same line through their midpoints, so they give no net torque. The net magnetic force on the complete loop is zero in the uniform field.

Step 2: (i) Obtain the couple arm and torque The perpendicular separation of the force lines on the l -sides is $b \sin \theta$. It vanishes when the loop's normal is parallel to $\vec{B}$, and equals b when the normal is perpendicular to $\vec{B}$. Therefore,

$$\begin{aligned}\tau &= F(b \sin \theta) \\ &= (IlB)b \sin \theta \\ &= IAB \sin \theta.\end{aligned}$$

Since $\mu = IA$, the vector result is

$$\vec{\tau} = \vec{\mu} \times \vec{B}.$$

The torque tends to align the magnetic moment with the field. For a coil of N identical turns, each turn contributes the same torque, giving $\tau = NIAB \sin \theta$ and $\vec{\mu} = NIA\hat{n}$. The angle used here is with the normal, not with the plane of the loop.

Step 3: (ii) Determine the speed after the energy loss Let v be the initial speed and v' the speed just after the lead sheet. The particle loses half its kinetic energy, so the remaining energy is $K' = K/2$. With its mass and charge unchanged,

$$\frac{1}{2}m(v')^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right) \implies v' = \frac{v}{\sqrt{2}}.$$

The initial circular motion means the velocity is perpendicular to the field. Because the sheet does not change its direction, it remains perpendicular after emerging, so the subsequent path is again circular in the same uniform field.

Step 4: (ii)(1) Compare the radii of the two circular paths The magnetic force supplies the centripetal force outside the sheet:

$$|q|vB = \frac{mv^2}{r} \implies r = \frac{mv}{|q|B}.$$

The field, mass and charge are the same before and after crossing. Hence

$$\frac{r'}{r} = \frac{v'}{v} = \frac{1}{\sqrt{2}}.$$

Thus the new circular orbit has a smaller radius; the radius scales with the square root of the kinetic energy, rather than with the kinetic energy itself.

Step 5: (ii)(2) Compare the periods of revolution Substitute the radius into the distance-per-revolution expression:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \frac{mv}{|q|B} = \frac{2\pi m}{|q|B}.$$

The speed cancels, so the period is independent of kinetic energy in this nonrelativistic treatment. Equivalently, the circumference and the speed both fall by the same factor $1/\sqrt{2}$, leaving the revolution time unchanged.

Step 6: Conclusion (i) A rectangular current loop experiences $\vec{\tau} = \vec{\mu} \times \vec{B}$, with magnitude $\tau = IAB \sin \theta$ for one turn, or $NIAB \sin \theta$ for N turns. (ii) After the energy loss, $r' = r/\sqrt{2}$, while $T' = T = 2\pi m/(|q|B)$.

Quick Tip to Solve: Measure the torque angle from the loop's normal. For the particle, first use $v \propto \sqrt{K}$: the radius follows v , while $T = 2\pi m/(|q|B)$ stays fixed.