

CBSE Class 12 Physics Question Paper 2025 with Solution PDF-Set 55/1/3



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) Question paper is divided into **FIVE** sections – Sections **A, B, C, D** and **E**.
- (iii) In **Section A** : Question numbers **1 to 16** are Multiple Choice (MCQ) type questions. Each question carries **1** mark.
- (iv) In **Section B** : Question numbers **17 to 21** are Very Short Answer (VSA) type questions. Each question carries **2** marks.
- (v) In **Section C** : Question numbers **22 to 28** are Short Answer (SA) type questions. Each question carries **3** marks.
- (vi) In **Section D** : Question numbers **29 & 30** are Case Study-Based questions. Each question carries **4** marks.
- (vii) In **Section E** : Question numbers **31 to 33** are Long Answer (LA) type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is NOT allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ J s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg.}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg.}$$

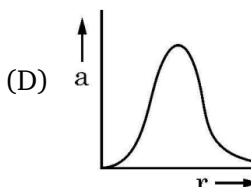
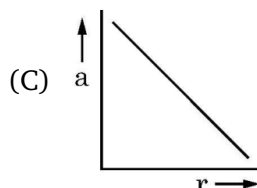
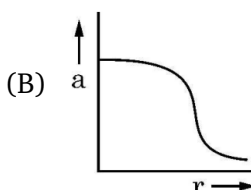
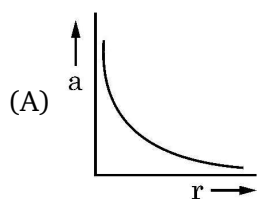
$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg.}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

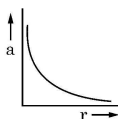
$$\text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Section A (16 marks)

1. A charge Q is fixed in position. Another charge q is brought near charge Q and released from rest. Which of the following graphs is the correct representation of the acceleration of the charge q as a function of its distance r from charge Q ? [1 mark]



Correct Answer: (A)



Solution:

Concept:

- Coulomb's law gives the magnitude of the force between point charges: $F = k|Qq|/r^2$, where $k = 1/(4\pi\epsilon_0)$ and r is their separation.
- Newton's second law gives the acceleration magnitude $a = F/m$, where m is the mass of the moving charge q .

Step 1: Express acceleration in terms of separation Since Q , q and m are fixed, the only changing factor in the force is the separation. Thus

$$a(r) = \frac{k|Qq|}{mr^2} \propto \frac{1}{r^2}.$$

Being released from rest sets the initial velocity; it does not make the initial acceleration zero. The sign of the charges determines the direction of acceleration, while the plotted positive quantity is its magnitude.

Step 2: Compare the four curves An inverse-square curve decreases steeply at small r , then flattens and approaches zero as r increases. Graph (A) has this shape. Graph (B) has a nearly constant initial portion, graph (C) shows a linear decrease, and graph (D) first increases before decreasing. None of these three follows $a \propto r^{-2}$.

Step 3: Conclusion The acceleration magnitude varies inversely as the square of the distance, as shown in graph (A). Hence, the correct option is (A).

Quick Tip to Solve: For a Coulomb-force graph, double the separation mentally: the acceleration must fall to one-quarter of its previous value.

2. Two conductors A and B of the same material have their lengths in the ratio $1 : 2$ and radii in the ratio $2 : 3$. If they are connected in parallel across a battery, the ratio $\frac{v_A}{v_B}$ of the drift velocities of electrons in them will be: [1 mark]

- (A) 2
- (B) $\frac{1}{2}$
- (C) $\frac{3}{2}$
- (D) $\frac{8}{9}$

Correct Answer: (A) 2

Solution:

Concept:

- The drift-speed magnitude is $v_d = eE\tau/m_e$, where e is the magnitude of electron charge, E the electric field, τ the relaxation time and m_e the electron mass.
- Parallel conductors have the same potential difference V ; for a uniform conductor of length l , $E = V/l$.

Step 1: Identify the dependence on length For the same material under the same conditions, τ is the same. Therefore

$$v_d = \frac{e\tau V}{m_e l} \propto \frac{1}{l}.$$

The radii affect the total currents, but do not affect the electric fields for a given common voltage and given lengths. Equivalently, in $v_d = I/(neA)$, using $I = VA/(\rho l)$ cancels the cross-sectional area A .

Step 2: Form the ratio and check the alternatives Since $l_A : l_B = 1 : 2$,

$$\frac{v_A}{v_B} = \frac{l_B}{l_A} = \frac{2}{1} = 2.$$

Option (B) reverses this inverse-length dependence. Option (C), $3/2$, uses a radius ratio that does not govern drift speed here. Option (D), $8/9$, is the current ratio I_A/I_B , which also contains the different areas; it is not the drift-speed ratio.

Step 3: Conclusion The required ratio is $v_A/v_B = 2$. Hence, the correct option is (A).

Quick Tip to Solve: For wires of the same material connected in parallel, drift speed is inversely proportional to length; the radius cancels.

3. A 1 cm segment of a wire lying along x -axis carries current of 0.5 A along $+x$ direction. A magnetic field $\vec{B} = (0.4 \text{ mT})\hat{j} + (0.6 \text{ mT})\hat{k}$ is switched on in the region. The force acting on the segment is:

[1 mark]

- (A) $(2\hat{j} + 3\hat{k}) \text{ mN}$
- (B) $(-3\hat{j} + 2\hat{k}) \mu\text{N}$
- (C) $(6\hat{j} + 4\hat{k}) \text{ mN}$
- (D) $(-4\hat{j} + 6\hat{k}) \mu\text{N}$

Correct Answer: (B) $(-3\hat{j} + 2\hat{k}) \mu\text{N}$

Solution:

Concept:

- The magnetic force on a straight current-carrying segment is $\vec{F} = I\vec{l} \times \vec{B}$, where $\vec{l}$ points along the conventional current.
- The unit-vector products are $\hat{i} \times \hat{j} = \hat{k}$ and $\hat{i} \times \hat{k} = -\hat{j}$. Use SI units before substituting.

Step 1: Convert the length and field to SI units The segment carries current along $+x$, so

$$\vec{l} = 10^{-2}\hat{i} \text{ m}, \quad \vec{B} = (0.4\hat{j} + 0.6\hat{k})10^{-3} \text{ T}.$$

The current is $I = 0.5 \text{ A}$. The centimetre and millitesla conversions together supply a factor of 10^{-5} in the force calculation.

Step 2: Evaluate the vector product Apply the cross-product directions separately to the two field components:

$$\begin{aligned}\vec{F} &= 0.5 \times 10^{-2} \times 10^{-3}(0.4\hat{k} - 0.6\hat{j}) \text{ N} \\ &= (-3\hat{j} + 2\hat{k}) \times 10^{-6} \text{ N}.\end{aligned}$$

Options (A) and (C) give a positive y -component and millinewton units, both inconsistent with this result. Option (D) has the right component signs but the wrong component magnitudes.

Step 3: Conclusion The force is $\vec{F} = (-3\hat{j} + 2\hat{k}) \mu\text{N}$. Hence, the correct option is (B).

Quick Tip to Solve: With current along $+x$, a $+z$ magnetic-field component produces force along $-y$; check that sign before comparing options.

4. The ratio of the number of turns of the primary to the secondary coils in an ideal transformer is $20 : 1$. If 240 V ac is applied from a source to the primary coil of transformer and a $6.0\ \Omega$ resistor is connected across the output terminals, then current drawn by the transformer from the source will be:

[1 mark]

- (A) 4.0 A
- (B) 3.8 A
- (C) 0.97 A
- (D) 0.10 A

Correct Answer: (D) 0.10 A

Solution:

Concept:

- For an ideal transformer, the voltage ratio equals the turns ratio: $V_s/V_p = N_s/N_p$, where subscripts p and s denote primary and secondary.
- An ideal transformer conserves power, so $V_p I_p = V_s I_s$. For the resistive load, $I_s = V_s/R$; the given ac quantities are used consistently as rms values.

Step 1: Calculate the secondary voltage and current The turns ratio is $N_p : N_s = 20 : 1$, so this is a step-down transformer. With $V_p = 240\text{ V}$,

$$V_s = 240 \times \frac{1}{20} = 12\text{ V}, \quad I_s = \frac{12}{6.0} = 2.0\text{ A}.$$

This current flows through the secondary load; the question asks for the current drawn from the primary source.

Step 2: Apply conservation of power The load consumes $P_s = 12 \times 2.0 = 24\text{ W}$. Hence the primary current must satisfy

$$I_p = \frac{V_s I_s}{V_p} = \frac{24}{240} = 0.10\text{ A}.$$

The values in options (A), (B) and (C) would require input powers of 960 W , 912 W and 232.8 W , respectively. These contradict the 24 W output of this ideal transformer.

Step 3: Conclusion The current drawn from the source is $I_p = 0.10\text{ A}$. Hence, the correct option is (D).

Quick Tip to Solve: First find the secondary voltage, then the load current; divide that current by the turns ratio to obtain the primary current.

5. You are required to design an air-filled solenoid of inductance 0.016 H having a length 0.81 m and radius 0.02 m. The number of turns in the solenoid should be: [1 mark]

(A) 2592

(B) 2866

(C) 2976

(D) 3140

Correct Answer: (B) 2866

Solution:

Concept:

- The self-inductance of a long air-filled solenoid is $L = \mu_0 N^2 A / l$, where N is its total number of turns, A its cross-sectional area and l its length.
- For a circular cross section, $A = \pi r^2$; air has permeability approximately $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$.

Step 1: Rearrange the inductance formula Since the total number of turns enters quadratically, solve for it by taking the positive square root:

$$N = \sqrt{\frac{Ll}{\mu_0 \pi r^2}}.$$

Here $L = 0.016 \text{ H}$, $l = 0.81 \text{ m}$ and $r = 0.02 \text{ m}$. The given radius must be squared to calculate the area.

Step 2: Substitute and compare the choices Substitution in SI units gives

$$\begin{aligned} N &= \sqrt{\frac{0.016 \times 0.81}{(4\pi \times 10^{-7})\pi(0.02)^2}} \\ &= \frac{9000}{\pi} \approx 2866 \quad (\pi \approx 3.14). \end{aligned}$$

Using a more precise value of π gives about 2865 turns; the listed value 2866 corresponds to the usual 3.14 approximation. The other choices, 2592, 2976 and 3140, are substantially farther from the required number and give a different inductance because $L \propto N^2$.

Step 3: Conclusion The required design uses approximately 2866 turns. Hence, the correct option is (B).

Quick Tip to Solve: For a solenoid, use the area πr^2 , then take a square root when solving for the total turns; do not confuse N with turns per metre.

6. A voltage $v = v_0 \sin \omega t$ applied to a circuit drives a current $i = i_0 \sin(\omega t + \phi)$ in the circuit. The average power consumed in the circuit over a cycle is: [1 mark]

(A) Zero

(B) $i_0 v_0 \cos \phi$

(C) $\frac{i_0 v_0}{2}$

(D) $\frac{i_0 v_0}{2} \cos \phi$

Correct Answer: (D) $\frac{i_0 v_0}{2} \cos \phi$

Solution:

Concept:

- Instantaneous electrical power is $p(t) = v(t)i(t)$. Average power is the mean of this product over one complete cycle.
- For sinusoidal voltage and current with phase difference ϕ , the power factor is $\cos \phi$, and $V_{\text{rms}} = v_0/\sqrt{2}$, $I_{\text{rms}} = i_0/\sqrt{2}$.

Step 1: Multiply voltage and current Substituting the given time dependence,

$$p(t) = v_0 i_0 \sin \omega t \sin(\omega t + \phi).$$

Use $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$ to obtain

$$p(t) = \frac{v_0 i_0}{2} [\cos \phi - \cos(2\omega t + \phi)].$$

Step 2: Average over a complete cycle The oscillating cosine term has zero average, while $\cos \phi$ is constant. Therefore

$$P_{\text{av}} = \frac{v_0 i_0}{2} \cos \phi.$$

Option (A) holds only for special phase differences with $\cos \phi = 0$. Option (B) omits the factor $1/2$ needed when using peak values. Option (C) omits the power factor and applies only when voltage and current are in phase. The sign of the stated phase lead does not change the cosine factor.

Step 3: Conclusion The average power is $P_{\text{av}} = (i_0 v_0 / 2) \cos \phi$. Hence, the correct option is (D).

Quick Tip to Solve: With peak voltage and peak current, include both factors: one-half for cycle averaging and $\cos \phi$ for the phase difference.

7. X-rays are more harmful to human beings than ultraviolet radiations because X-rays:

[1 mark]

- (A) have frequency lower than that of ultraviolet radiations.
- (B) have wavelength smaller than that of ultraviolet radiations.
- (C) move faster than ultraviolet radiations in air.
- (D) are mechanical waves but ultraviolet radiations are electromagnetic waves.

Correct Answer: (B) have wavelength smaller than that of ultraviolet radiations.

Solution:

Concept:

- Electromagnetic radiation obeys $c = \nu\lambda$ in vacuum, where ν is frequency, λ is wavelength and c is the speed of light.
- The energy of one photon is $E = h\nu = hc/\lambda$, where h is Planck's constant. Shorter wavelength therefore means higher photon energy.

Step 1: Compare photon energies X-rays occupy the shorter-wavelength, higher-frequency region of the electromagnetic spectrum compared with ultraviolet radiation. Consequently, each X-ray photon has greater energy. This higher energy can cause ionisation and damage biological molecules; it is the relevant physical basis for the comparison made in the question.

Step 2: Examine all four statements Statement (B) correctly identifies the smaller wavelength of X-rays. Statement (A) reverses the frequency order: X-rays have higher, not lower, frequency. Statement (C) incorrectly attributes the harmfulness to faster propagation; both travel at approximately the speed of light in air. Statement (D) is false because X-rays and ultraviolet radiation are both electromagnetic waves, and neither is a mechanical wave.

Step 3: Conclusion X-rays have a smaller wavelength and therefore higher photon energy than ultraviolet radiation. Hence, the correct option is (B).

Quick Tip to Solve: To compare radiation energies, use $E = hc/\lambda$: the shorter-wavelength radiation has the higher photon energy.

8. A point source is placed at the bottom of a tank containing a transparent liquid (refractive index n) to a depth H . The area of the surface of the liquid through which light from the source can emerge out is: [1 mark]

- (A) $\frac{\pi H^2}{(n-1)}$
 (B) $\frac{\pi H^2}{(n^2-1)}$
 (C) $\frac{\pi H^2}{\sqrt{n^2-1}}$
 (D) $\frac{\pi H^2}{(n^2+1)}$

Correct Answer: (B) $\frac{\pi H^2}{(n^2-1)}$

Solution:

Concept:

- At a liquid-air boundary, the critical angle C satisfies $\sin C = 1/n$, where $n > 1$ is the liquid's refractive index relative to air.
- Rays incident at angles below C emerge; rays beyond it undergo total internal reflection. The limiting cone cuts a circular area πr^2 on the surface.

Step 1: Find the radius of the escape circle The source is a vertical distance H below the surface. For the limiting ray, the right triangle between the source, the point directly above it and the surface intersection gives $r = H \tan C$. Rotating this ray about the vertical gives the circular boundary.

$$\tan^2 C = \frac{\sin^2 C}{1 - \sin^2 C} = \frac{1/n^2}{1 - 1/n^2} = \frac{1}{n^2 - 1}.$$

Step 2: Calculate the area and compare the denominators Squaring the radius is essential when finding an area:

$$A = \pi r^2 = \pi H^2 \tan^2 C = \frac{\pi H^2}{n^2 - 1}.$$

Option (A) incorrectly replaces n^2 by n . Option (C) retains a square root that belongs in the radius, whereas the area contains the squared radius. Option (D) has a plus sign, inconsistent with $\cos^2 C = 1 - \sin^2 C$.

Step 3: Conclusion The surface area through which light emerges is $\boxed{\pi H^2 / (n^2 - 1)}$. Hence, the correct option is (B).

Quick Tip to Solve: First find $r = H / \sqrt{n^2 - 1}$; then square the entire radius in $A = \pi r^2$.

9. In a photoelectric experiment with a material of work function 2.1 eV, the stopping potential is found to be 2.5 V. The maximum kinetic energy of ejected photoelectrons is: [1 mark]

- (A) 0.4 eV
- (B) 2.1 eV
- (C) 2.5 eV
- (D) 4.6 eV

Correct Answer: (C) 2.5 eV

Solution:

Concept:

- Stopping potential V_s is the retarding potential that just stops the most energetic photoelectrons; therefore $K_{\max} = eV_s$, where e is the magnitude of the electron charge.
- Einstein's equation is $h\nu = \phi + K_{\max}$, where ϕ is the work function. It is the photon energy, not the measured maximum kinetic energy, that includes ϕ .

Step 1: Convert the stopping potential into kinetic energy An electron loses 1 eV of kinetic energy on crossing a retarding potential difference of 1 V. Thus the given stopping potential directly gives

$$K_{\max} = e(2.5 \text{ V}) = 2.5 \text{ eV} = 4.0 \times 10^{-19} \text{ J}.$$

Step 2: Distinguish kinetic energy from photon energy The work function 2.1 eV has already been spent before the electron emerges. Adding it to $K_{\max}$ gives the incident photon energy 4.6 eV, which is option (D), not the quantity asked. Option (B) is just the work function; option (A), 0.4 eV, wrongly subtracts the work function from energy already measured using the stopping potential.

Step 3: Conclusion The maximum kinetic energy is 2.5 eV. Hence, the correct option is (C).

Quick Tip to Solve: When stopping potential is given in volts, its numerical value directly gives $K_{\max}$ in electron-volts.

10. When a p - n junction diode is forward biased: [1 mark]

- (A) the barrier height and the depletion layer width both increase.
- (B) the barrier height increases and the depletion layer width decreases.
- (C) the barrier height and the depletion layer width both decrease.
- (D) the barrier height decreases and the depletion layer width increases.

Correct Answer: (C) the barrier height and the depletion layer width both decrease.

Solution:

Concept:

- Forward bias connects the p side of a junction to the positive terminal and the n side to the negative terminal of the external supply.
- The applied electric field opposes the built-in junction field. For forward voltage V_F , the effective barrier potential is reduced from V_{bi} to approximately $V_{bi} - V_F$.

Step 1: Determine the change in barrier height The external potential opposes the potential barrier that normally prevents majority carriers from crossing the junction. The energy barrier $e(V_{bi} - V_F)$ is therefore smaller than the unbiased value eV_{bi} . Options (A) and (B), which predict an increase in

barrier height, cannot describe forward bias.

Step 2: Determine the change in depletion width Majority carriers are driven towards the junction: holes from the p region and electrons from the n region. Their arrival reduces the extent of the uncovered ionised donor and acceptor regions. Consequently, the depletion layer becomes narrower. Option (D) wrongly predicts a wider depletion layer; a lower barrier and a narrower layer occur together in forward bias.

Step 3: Conclusion Both the barrier height and the depletion layer width decrease. Hence, the correct option is (C).

Quick Tip to Solve: In forward bias, the supply opposes the junction barrier: remember both barrier height and depletion width decrease together.

11. Let λ_e , λ_p and λ_d be the wavelengths associated with an electron, a proton and a deuteron, all moving with the same speed. Then the correct relation between them is: [1 mark]

(A) $\lambda_d > \lambda_p > \lambda_e$

(B) $\lambda_e > \lambda_p > \lambda_d$

(C) $\lambda_p > \lambda_e > \lambda_d$

(D) $\lambda_e = \lambda_p = \lambda_d$

Correct Answer: (B) $\lambda_e > \lambda_p > \lambda_d$

Solution:

Concept:

- The de Broglie wavelength of a particle is $\lambda = h/p$, where h is Planck's constant and p is its momentum.
- At a common non-relativistic speed v , $p = mv$ and hence $\lambda = h/(mv)$. Wavelength is inversely proportional to mass when speed is fixed.

Step 1: Compare the particle masses The electron is much lighter than a proton. A deuteron contains a proton and a neutron, so its mass is approximately twice the proton mass. Thus

$$m_e < m_p < m_d.$$

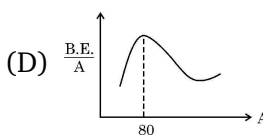
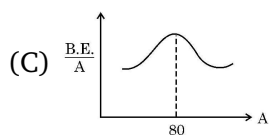
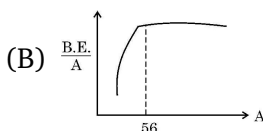
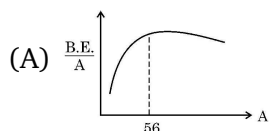
With all speeds equal, the corresponding momenta have this same ordering.

Step 2: Reverse the momentum ordering for wavelength Since $\lambda = h/p$, a larger momentum gives a smaller wavelength. Thus the electron has the longest wavelength and the deuteron the shortest. Option (A) reverses this inverse relationship, and option (C) incorrectly places the proton above the much lighter electron. Option (D) would require equal momenta, not merely equal speeds. The same mass ordering also applies relativistically because the common speed gives a common Lorentz factor.

Step 3: Conclusion The required relation is $\lambda_e > \lambda_p > \lambda_d$. Hence, the correct option is (B).

Quick Tip to Solve: For equal speeds, order the masses first and reverse that order to get the de Broglie wavelengths.

12. Which of the following figures correctly represent the shape of curve of binding energy per nucleon as a function of mass number? [1 mark]



Correct Answer: (A)

Solution:

Concept:

- Binding energy per nucleon is B/A , where B is a nucleus's total binding energy and A is its mass number. Larger B/A means greater stability against separation into individual nucleons.
- The average curve rises rapidly for light nuclei, reaches a broad maximum near the iron region at $A \simeq 56$, and then decreases gradually towards heavy nuclei.

Step 1: Identify the maximum and the heavy-nucleus trend The binding energy per nucleon has a maximum of about 8.8 MeV near $A = 56$. It remains relatively high for medium-mass nuclei but falls gradually to about 7.6 MeV for very heavy nuclei. The short range and saturation of nuclear forces explain the broad high region; increasing proton-proton repulsion contributes to the decline at large A .

Step 2: Compare all four printed curves Graph (A) best shows the rapid initial rise, a rounded maximum close to the labelled $A = 56$, and the subsequent gradual decline. Graph (B) continues rising farther beyond the marked 56 and is almost level towards the heavy-nucleus end, making it less representative of the maximum and decline. Graphs (C) and (D) instead place a pronounced peak at $A = 80$, followed by a dip and renewed rise. Neither reproduces the standard trend.

Step 3: Conclusion Graph (A) represents the rise to a maximum near the iron region and the gradual fall for heavier nuclei. Hence, the correct option is (A).

Quick Tip to Solve: Check both the maximum near $A = 56$ and the downward trend for heavy nuclei; a nearly flat top alone is insufficient.

13. Assertion (A): We cannot form a p - n junction diode by taking a slab of a p -type semiconductor and physically joining it to another slab of a n -type semiconductor.

Reason (R): In a p -type semiconductor $n_e \gg n_h$ while in a n -type semiconductor $n_h \gg n_e$. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- A practical p - n junction is formed within a continuous semiconductor crystal by suitable doping of adjacent regions. Merely pressing together two separate slabs does not create the required crystal junction.
- Holes are majority carriers in a p -type semiconductor, while electrons are majority carriers in an n -type semiconductor. Here n_h and n_e denote hole and electron concentrations.

Step 1: Judge the assertion Assertion (A) is true. Physically joining two independently prepared slabs leaves an interface with discontinuity, surface defects and possible contamination rather than the required continuous crystalline structure. A diode junction is produced by creating differently doped regions within a semiconductor crystal.

Step 2: Judge the printed reason Reason (R) is false because its inequalities are reversed. In a p -type semiconductor the correct relation is $n_h \gg n_e$, whereas in an n -type semiconductor it is $n_e \gg n_h$. The printed reason instead assigns the greater concentration to the minority carriers in both cases.

Step 3: Test whether the reason explains the assertion A false statement cannot explain the true assertion. Moreover, the limitation of physically joining slabs concerns the crystal interface; the mere existence of different majority carriers is not the cause of this limitation. Options (A) and (B) wrongly treat the reason as true, and option (D) wrongly treats the assertion as false.

Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Read the carrier subscripts carefully: p -type means holes dominate, so its majority-carrier inequality must begin with n_h .

14. Assertion (A): The potential energy of an electron revolving in any stationary orbit in a hydrogen atom is positive.

Reason (R): The total energy of a charged particle is always positive.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and Reason (R) is also false.

Solution:

Concept:

- With zero electrostatic potential energy at infinite separation, an electron and proton at distance r have attractive potential energy $U = -ke^2/r$, where $k = 1/(4\pi\epsilon_0)$.
- In a stationary Bohr orbit, $K = ke^2/(2r) > 0$, but the total energy is $E = K + U = -ke^2/(2r) < 0$. Negative total energy denotes a bound state.

Step 1: Judge the assertion using attractive potential energy The electron and proton carry opposite charges, so their interaction potential energy is negative with the usual zero at infinite separation. Every finite-radius stationary orbit therefore has $U < 0$. Assertion (A), which calls this potential energy positive, is false.

Step 2: Test the universal claim in the reason Reason (R) is also false. A bound electron in hydrogen provides a counterexample: its orbital energy is

$$E_n = -\frac{13.6}{n^2} \text{ eV}, \quad n = 1, 2, 3, \dots$$

The positive kinetic energy is smaller in magnitude than the negative potential energy, making the total negative. Charge by itself does not determine the sign of mechanical energy.

Step 3: Check the explanatory relation Since both statements are false, the reason cannot explain the assertion. Options (A) and (B) incorrectly accept both statements, while option (C) incorrectly accepts the assertion. The sign error comes from confusing positive kinetic energy with potential or total orbital energy.

Step 4: Conclusion The assertion is false and the reason is also false. Hence, the correct option is (D).

Quick Tip to Solve: For a bound hydrogen electron, use $U = -2K$ and $E = -K$: only kinetic energy is positive.

15. Assertion (A): It is difficult to move a magnet into a coil of large number of turns when the circuit of the coil is closed.

Reason (R): The direction of induced current in a coil with its circuit closed, due to motion of a magnet, is such that it opposes the cause. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Concept:

- Faraday's law gives the induced emf as $\varepsilon = -N d\Phi/dt$, where N is the number of turns and Φ is magnetic flux through one turn.
- Lenz's law states that the induced current produces a magnetic effect opposing the change in flux that produces it. A closed coil permits this current to flow.

Step 1: Evaluate the assertion As a magnet enters the coil, the magnetic flux linked with the turns changes. The induced current creates a magnetic field that opposes the magnet's approach. An external force must therefore do work against this magnetic resistance, so Assertion (A) is true.

Step 2: Evaluate the reason Reason (R) is the statement of Lenz's law: the induced current opposes its cause, namely the flux change due to the moving magnet. Thus Reason (R) is true. The opposition is to the change in flux, not necessarily to the existing flux itself.

Step 3: Test the explanation For an approaching north pole, the nearer face of the coil acts as a north pole and repels it. This opposing force directly explains the difficulty of pushing the magnet into the closed coil. Option (B) misses this causal link; (C) and (D) incorrectly classify a true statement as false.

Step 4: Conclusion Both statements are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: Identify the flux change first: an approaching pole induces the same pole on the near face of the closed coil, opposing approach.

16. Assertion (A): The deflection in a galvanometer is directly proportional to the current passing through it.

Reason (R): The coil of a galvanometer is suspended in a uniform radial magnetic field. [1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Concept:

- A current-carrying coil experiences magnetic torque $\tau_m = NIAB \sin \alpha$, where N is its turn count, A its area and α the angle between the coil's normal and the magnetic field.
- In a moving-coil galvanometer, the suspension provides restoring torque $\tau_r = k\theta$, where k is the torsional constant and θ is the angular deflection.

Step 1: Evaluate the assertion At equilibrium the magnetic torque balances the restoring torque. In the galvanometer's radial field the magnetic torque is $NIAB$, giving

$$k\theta = NIAB, \quad \theta = \frac{NAB}{k}I.$$

For the given instrument, N , A , B and k are constants; hence the deflection is directly proportional to current and Assertion (A) is true.

Step 2: Evaluate the reason The pole pieces and soft-iron core produce a radial magnetic field of approximately constant magnitude in the coil region. The coil plane remains parallel to the field as the coil turns, so its normal remains perpendicular to the field and $\sin \alpha = 1$. Reason (R) is true.

Step 3: Connect the radial field to proportional deflection The radial field removes the angle-dependent factor from the magnetic torque. With the torsional restoring torque proportional to deflection, this gives the linear current-deflection relation. Reason (R) therefore explains Assertion (A); (B) denies this connection, while (C) and (D) wrongly call true statements false.

Step 4: Conclusion Both statements are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: For a moving-coil galvanometer, use the radial field to set $\sin \alpha = 1$, then balance $NIAB$ against $k\theta$.

Section B (10 marks)

17. n identical cells, each of e.m.f. E and internal resistance r , are connected in series. Later on it was found out that two cells X and Y are connected in reverse polarities. Calculate the potential difference across the cell X . [2 marks]

Solution:

Concept:

- Series emfs add algebraically, while all internal resistances add positively. A reversed cell therefore subtracts its emf but still contributes internal resistance r .
- For a cell carrying current into its positive terminal, the terminal potential difference is $V_+ - V_- = E + Ir$. This is a signed relation with I positive into that terminal.

Step 1: Find the resultant emf and resistance Choose the emf direction of the $n - 2$ normally connected cells as positive. The two reversed cells oppose them, giving

$$E_{\text{net}} = (n - 2)E - 2E = (n - 4)E, \quad r_{\text{total}} = nr.$$

Notice that reversing a cell changes its emf sign, not the sign of its resistance.

Step 2: State the circuit assumption and determine the current The usual intended interpretation is that the series combination forms a closed loop with negligible external resistance. Under this assumption, Kirchhoff's loop rule gives

$$I = \frac{E_{\text{net}}}{r_{\text{total}}} = \frac{(n - 4)E}{nr}.$$

For $n > 4$, this current enters the positive terminal of each reversed cell, so cell X is being charged. For other values of n , the same expression is used as a signed current.

Step 3: Apply the terminal-voltage relation to the reversed cell Measure the potential difference from the negative to the positive terminal of X . The charging relation gives

$$V_X = E + Ir = E + \frac{n - 4}{n}E.$$

The printed question does not explicitly specify circuit closure or an external load. If an external resistance R is present, replace I by $(n - 4)E/(nr + R)$; if the series chain is open, no current flows.

Step 4: Conclusion For the intended closed loop with no external resistance, $V_X = \frac{2(n - 2)}{n}E$, measured as $V_+ - V_-$. If the series chain is open, $V_X = E$; thus the closed-loop assumption is needed for the first result.

Quick Tip to Solve: Two reversed cells reduce the series emf by $4E$, but the total internal resistance stays nr ; use $E + Ir$ for the reversed cell being charged.

18(a). In a diffraction experiment, the slit is illuminated by light of wavelength 600 nm. The first minimum of the pattern falls at $\theta = 30^\circ$. Calculate the width of the slit. [2 marks]

Solution:

Concept:

- For single-slit diffraction, dark fringes satisfy $a \sin \theta = m\lambda$, where a is the slit width, θ is measured from the central axis, λ is the wavelength and $m = 1, 2, \dots$
- At the first minimum, $m = 1$. The path difference between light from the slit edges is one wavelength, so contributions from corresponding points in the two slit halves cancel.

Step 1: Select the first-minimum condition The question concerns a diffraction minimum, so apply the single-slit condition with $m = 1$:

$$a \sin \theta = \lambda, \quad a = \frac{\lambda}{\sin \theta}.$$

Here $\theta = 30^\circ$ is not a small angle, so use its exact sine instead of replacing the sine by the angle in radians.

Step 2: Convert the wavelength and substitute Convert nanometres to metres and use $\sin 30^\circ = 1/2$:

$$\begin{aligned} \lambda &= 600 \times 10^{-9} \text{ m}, \\ a &= \frac{600 \times 10^{-9}}{1/2} \text{ m} = 1200 \times 10^{-9} \text{ m}. \end{aligned}$$

Thus the slit width is twice the wavelength, consistent with the edge-to-edge path difference being $a/2 = \lambda$ at the given angle.

Step 3: Conclusion The slit width is $a = 1.2 \times 10^{-6} \text{ m} = 1.2 \mu\text{m}$.

Quick Tip to Solve: For the first single-slit minimum at 30° , use $a = \lambda / \sin 30^\circ = 2\lambda$; no small-angle approximation is needed.

OR

18(b). In a Young's double-slit experiment, two light waves, each of intensity I_0 , interfere at a point, having a path difference $\frac{\lambda}{8}$ on the screen. Find the intensity at this point. [2 marks]

Solution:

Concept:

- A path difference Δx corresponds to phase difference $\phi = 2\pi\Delta x/\lambda$, where λ is the wavelength.
- For coherent interfering waves, $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$. With equal individual intensities $I_1 = I_2 = I_0$, this becomes $I = 2I_0(1 + \cos \phi)$.

Step 1: Convert the path difference to phase difference The printed path difference is $\Delta x = \lambda/8$. Therefore,

$$\phi = \frac{2\pi}{\lambda} \frac{\lambda}{8} = \frac{\pi}{4} = 45^\circ.$$

The wavelength cancels because the path difference is already specified as a fraction of one wavelength.

Step 2: Substitute the equal intensities Both waves have intensity I_0 , so their interference term is

$2I_0 \cos \phi$. Using $\cos 45^\circ = 1/\sqrt{2}$,

$$\begin{aligned} I &= I_0 + I_0 + 2I_0 \cos \frac{\pi}{4} \\ &= 2I_0 + \sqrt{2}I_0. \end{aligned}$$

The cosine is positive, so the result exceeds the sum $2I_0$ of separate intensities, but remains below the fully constructive maximum $4I_0$. This provides a useful check on the interference calculation.

Step 3: Conclusion The intensity at the point is $I = (2 + \sqrt{2})I_0$, approximately $3.414I_0$.

Quick Tip to Solve: For equal intensities, convert $\lambda/8$ to phase $\pi/4$ first, then use $I = 2I_0(1 + \cos \phi)$.

19. A double convex lens of glass has both faces of the same radius of curvature 17 cm. Find its focal length if it is immersed in water. The refractive indices of glass and water are 1.5 and 1.33 respectively. [2 marks]

Solution:

Concept:

- For a thin lens immersed in a medium, the lens-maker formula is $\frac{1}{f} = \left(\frac{n_g}{n_w} - 1\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, where n_g and n_w are the refractive indices of glass and water.
- With light travelling from left to right, a double convex lens has $R_1 > 0$ and $R_2 < 0$. Equal curvature magnitudes therefore add in the lens-maker formula.

Step 1: Assign the signed radii and relative refractive index The first surface has its centre of curvature to its right, while the second has its centre to its left. Hence,

$$R_1 = +17 \text{ cm}, \quad R_2 = -17 \text{ cm}, \quad \frac{n_g}{n_w} - 1 = \frac{1.50 - 1.33}{1.33} = \frac{0.17}{1.33}.$$

The surrounding medium is water, so the absolute glass index 1.5 cannot be used alone.

Step 2: Substitute in the lens-maker formula Keeping both radii in centimetres gives the focal length in centimetres:

$$\begin{aligned} \frac{1}{f} &= \frac{0.17}{1.33} \left(\frac{1}{17} - \frac{1}{-17} \right) \\ &= \frac{0.17}{1.33} \frac{2}{17} = \frac{0.02}{1.33} \text{ cm}^{-1}. \end{aligned}$$

The power is positive because glass is optically denser than water, so the lens remains converging.

Step 3: Conclusion The focal length in water is $f = +66.5 \text{ cm} = +0.665 \text{ m}$. Its positive sign identifies a converging lens.

Quick Tip to Solve: For an equiconvex lens in a medium, use $f = R/[2(n_g/n_w - 1)]$; retain the relative refractive index until the final substitution.

20. An electron in Bohr model of hydrogen atom makes a transition from energy level -1.51 eV to -3.40 eV. Calculate the change in the radius of its orbit. The radius of orbit of electron in its ground state is 0.53 Å. [2 marks]

Solution:

Concept:

- In the Bohr model of hydrogen, the energy of the n th orbit is $E_n = -13.6/n^2$ eV, where n is the principal quantum number.
- The orbital radius is $r_n = n^2 r_1$, with ground-state radius $r_1 = 0.53$ Å. The signed change is $\Delta r = r_{\text{final}} - r_{\text{initial}}$.

Step 1: Identify the initial and final orbits Use the magnitudes of the negative energies in $n^2 = 13.6/|E_n|$. For the initial and final levels,

$$n_i^2 = \frac{13.6}{1.51} \simeq 9, \quad n_f^2 = \frac{13.6}{3.40} = 4.$$

Thus the transition is from $n_i = 3$ to $n_f = 2$. The initial energy -1.51 eV is the rounded value of $-13.6/9$ eV; quantum numbers must be integers.

Step 2: Calculate the two radii and their difference The radius depends on the square of the orbit number:

$$\begin{aligned} r_i &= 9(0.53) = 4.77 \text{ Å}, \\ r_f &= 4(0.53) = 2.12 \text{ Å}, \\ \Delta r &= 2.12 - 4.77 = -2.65 \text{ Å}. \end{aligned}$$

The more negative final energy describes a more tightly bound electron, consistent with a smaller final radius. Also, $1 \text{ Å} = 10^{-10} \text{ m}$.

Step 3: Conclusion The radius decreases by $2.65 \text{ Å} = 2.65 \times 10^{-10} \text{ m}$. With final minus initial as the convention, $\Delta r = -2.65 \text{ Å}$.

Quick Tip to Solve: Identify the integer orbits from $13.6/|E|$, then use $\Delta r = r_1(n_f^2 - n_i^2)$ directly; keep the sign to distinguish contraction from expansion.

21. A p-type Si semiconductor is made by doping an average of one dopant atom per 5×10^7 silicon atoms. If the number density of silicon atoms in the specimen is 5×10^{28} atoms/m³, find the number of holes created per cubic centimetre in the specimen due to doping. Also give one example of such dopants. [2 marks]

Solution:

Concept:

- A trivalent acceptor impurity in silicon leaves one electron vacancy in the covalent bonding structure, producing one hole per ionised dopant atom.
- For the usual assumption of complete acceptor ionisation, the density of holes produced by doping equals the acceptor density. Also, $1 \text{ m}^3 = 10^6 \text{ cm}^3$.

Step 1: Find the density of acceptor atoms There is one dopant atom for every 5×10^7 silicon atoms.

Divide the silicon atom density by this number to obtain the dopant density:

$$N_A = \frac{5 \times 10^{28}}{5 \times 10^7} = 1 \times 10^{21} \text{ m}^{-3}.$$

Each acceptor produces one hole, so this is also the number of holes created per cubic metre due to doping. Intrinsically generated electron-hole pairs are not included in this requested dopant contribution.

Step 2: Convert the volume unit and choose a dopant One cubic metre contains 10^6 cubic centimetres, so the number of holes in one cubic centimetre is

$$p = \frac{10^{21}}{10^6} = 10^{15} \text{ cm}^{-3}.$$

Boron is a suitable dopant: it has three valence electrons, whereas silicon has four. Its electron deficiency acts as an acceptor and makes holes the majority charge carriers in the doped material.

Step 3: Conclusion Doping creates 1×10^{15} holes per cm^3 . One suitable p -type dopant is **boron**.

Quick Tip to Solve: First divide the silicon density by the number of silicon atoms per dopant; then divide a density in m^{-3} by 10^6 to express it in cm^{-3} .

Section C (21 marks)

22(a). Two batteries of emfs 3 V & 6 V and internal resistances 0.2Ω & 0.4Ω are connected in parallel. This combination is connected to a 4Ω resistor. Find:

- (i) the equivalent emf of the combination
- (ii) the equivalent internal resistance of the combination
- (iii) the current drawn from the combination

[3 marks]

Solution:

Concept:

- For two cells connected in parallel with like terminals joined, the equivalent emf is $E_{\text{eq}} = \frac{E_1/r_1 + E_2/r_2}{1/r_1 + 1/r_2}$. The individual emfs are weighted by their internal conductances.
- The equivalent internal resistance is $r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2}$. With an external resistor R , the total load current is $I = E_{\text{eq}}/(R + r_{\text{eq}})$.

Step 1: Find the equivalent emf Take $E_1 = 3 \text{ V}$, $r_1 = 0.2 \Omega$, $E_2 = 6 \text{ V}$ and $r_2 = 0.4 \Omega$, with like terminals connected. If the common terminal voltage is V , Kirchhoff's laws give

$$I = \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2}.$$

Comparing its rearranged form with $V = E_{\text{eq}} - I r_{\text{eq}}$ yields the stated parallel-cell formulas. For part (i),

$$E_{\text{eq}} = \frac{3(0.4) + 6(0.2)}{0.2 + 0.4} = \frac{2.4}{0.6} = 4 \text{ V}.$$

The emfs do not add in a parallel connection, and unequal internal resistances make a simple arithmetic average inappropriate.

Step 2: Combine the internal resistances The internal resistances enter the equivalent source in parallel. Thus, for part (ii),

$$r_{eq} = \frac{0.2 \times 0.4}{0.2 + 0.4} = \frac{0.08}{0.6} = \frac{2}{15} \Omega \simeq 0.133 \Omega.$$

This is smaller than either individual internal resistance, as expected for a parallel combination.

Step 3: Calculate the current through the external resistor The equivalent internal resistance is in series with the external 4Ω load. Retaining its exact value gives, for part (iii),

$$I = \frac{4}{4 + 2/15} = \frac{4}{62/15} = \frac{30}{31} \text{ A} \simeq 0.968 \text{ A}.$$

This is the net current delivered by the combination to the resistor; it need not equal the current in either individual battery branch.

Step 4: Conclusion (i) $E_{eq} = 4 \text{ V}$, (ii) $r_{eq} = 0.133 \Omega$, and (iii) $I \simeq 0.968 \text{ A}$ through the 4Ω resistor.

Quick Tip to Solve: For parallel cells, use the conductance-weighted emf and parallel internal resistance first; then include that internal resistance in series with the external load.

OR

22(b). (i) A conductor of length l is connected across an ideal cell of emf E . Keeping the cell connected, the length of the conductor is increased to $2l$ by gradually stretching it. If R and R' are initial and final values of resistance and v_d and v'_d are initial and final values of drift velocity, find the relation between (i) R' and R and (ii) v'_d and v_d .

(ii) When electrons drift in a conductor from lower to higher potential, does it mean that all the 'free electrons' of the conductor are moving in the same direction? [3 marks]

Solution:

Concept:

- A uniform conductor has resistance $R = \rho l/A$, where ρ is its resistivity, l its length and A its cross-sectional area. On stretching, its volume Al remains constant; assume the temperature and material properties remain unchanged.
- An ideal cell maintains the same potential difference E , so $I = E/R$. The drift-speed magnitude is $v_d = I/(neA)$, with electron number density n and charge magnitude e .

Step 1: (i) Relate the new area and resistance to their original values Stretching doubles the length but does not add material. Thus, conservation of volume gives

$$Al = A'(2l) \Rightarrow A' = \frac{A}{2}.$$

The length increases and the area decreases, both increasing the resistance. With the same resistivity,

$$R' = \frac{\rho(2l)}{A/2} = 4 \frac{\rho l}{A} = 4R.$$

Step 2: (i) Use the fixed cell voltage to compare the drift speeds Since the cell is ideal and remains

connected, its terminal voltage is still E . Therefore the current becomes $I' = E/R' = I/4$. The number density remains unchanged because the material and its volume are unchanged. Hence

$$\frac{v'_d}{v_d} = \frac{I'}{I} \frac{A}{A'} = \frac{1}{4} \times 2 = \frac{1}{2}.$$

Equivalently, the electric-field magnitude inside the wire falls from E/l to $E/(2l)$; the relation $v_d = e\tau\mathcal{E}/m$ then gives the same factor for unchanged relaxation time τ .

Step 3: (ii) Distinguish average drift from the motion of individual electrons No. Free electrons have rapid, random motion in many directions. The applied electric field adds a small average drift opposite to the field, that is, from lower to higher potential. Collisions continually change the individual velocities, so their instantaneous directions need not coincide. The net average velocity represents drift; it does not mean that every electron moves in the same direction at each instant.

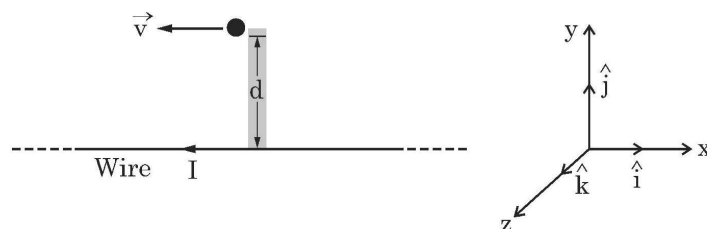
Step 4: Conclusion (i) $R' = 4R$ and $v'_d = v_d/2$. (ii) No: random electron motion persists, with only a net drift from lower to higher potential.

Quick Tip to Solve: For a stretched wire at fixed cell voltage, first use constant volume: $A \propto 1/l$, so $R \propto l^2$ and $v_d \propto 1/l$.

23. A particle of charge q is moving with a velocity $\vec{v}$ at a distance ' d ' from a long straight wire carrying a current ' I ' as shown in figure. At this instant, it is subjected to a uniform electric field $\vec{E}$ such that the particle keeps moving undeviated. In terms of unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$, find –

- the magnetic field $\vec{B}$,
- the magnetic force $\vec{F}_m$, and
- the electric field $\vec{E}$ acting on the charge.

[3 marks]



Solution:

Concept:

- The magnetic-field magnitude at perpendicular distance d from a long straight wire is $B = \mu_0 I / (2\pi d)$; the right-hand grip rule determines its direction.
- The magnetic force is $\vec{F}_m = q(\vec{v} \times \vec{B})$, and the electric force is $\vec{F}_e = q\vec{E}$. Undeviated motion requires the transverse electric and magnetic forces to cancel.

Step 1: (a) Determine the magnetic field from the printed directions In the figure, both the wire current and the particle velocity point to the left, along $-\hat{i}$. The particle lies above the wire, in the $+\hat{j}$ direction from it. By the right-hand rule, the field direction there is

$$(-\hat{i}) \times \hat{j} = -\hat{k}.$$

Thus, using the right-handed axes printed beside the wire,

$$\vec{B} = -\frac{\mu_0 I}{2\pi d} \hat{k}.$$

The field is normal to the plane containing the wire and the particle; it is not parallel to either the current or the separation vector.

Step 2: (b) Evaluate the magnetic-force cross product Write $\vec{v} = -v\hat{i}$, where v is the speed. Keeping the signs of both vectors gives

$$\begin{aligned}\vec{F}_m &= q(-v\hat{i}) \times \left(-\frac{\mu_0 I}{2\pi d} \hat{k}\right) \\ &= \frac{\mu_0 I q v}{2\pi d} (\hat{i} \times \hat{k}) = -\frac{\mu_0 I q v}{2\pi d} \hat{j}.\end{aligned}$$

Here $\hat{i} \times \hat{k} = -\hat{j}$. For a positive charge the force is toward the wire; a negative value of q reverses that force automatically.

Step 3: (c) Balance the magnetic force with the electric force For the usual crossed-field condition of undeflected motion at unchanged speed, the resultant force is zero. The electric force must therefore oppose $\vec{F}_m$:

$$\begin{aligned}q\vec{E} + q(\vec{v} \times \vec{B}) &= 0, \\ \vec{E} &= -\vec{v} \times \vec{B} = \frac{\mu_0 I v}{2\pi d} \hat{j}.\end{aligned}$$

The required electric field points upward, away from the wire. Its direction is independent of the sign of the nonzero charge, because changing that sign reverses both forces together. Its magnitude is vB , with units of $V\ m^{-1}$.

Step 4: Conclusion

$$\boxed{\vec{B} = -\frac{\mu_0 I}{2\pi d} \hat{k}}, \quad \boxed{\vec{F}_m = -\frac{\mu_0 I q v}{2\pi d} \hat{j}}, \quad \boxed{\vec{E} = \frac{\mu_0 I v}{2\pi d} \hat{j}}.$$

Quick Tip to Solve: Translate both leftward arrows into $-\hat{i}$ before taking cross products; then use $\hat{i} \times \hat{k} = -\hat{j}$ and $\vec{E} = -\vec{v} \times \vec{B}$.

24. An ac source of voltage $v = v_m \sin \omega t$ is connected to a series combination of LCR circuit. Draw the phasor diagram. Using it obtain an expression for the impedance of the circuit and the phase difference between applied voltage and the current. [3 marks]

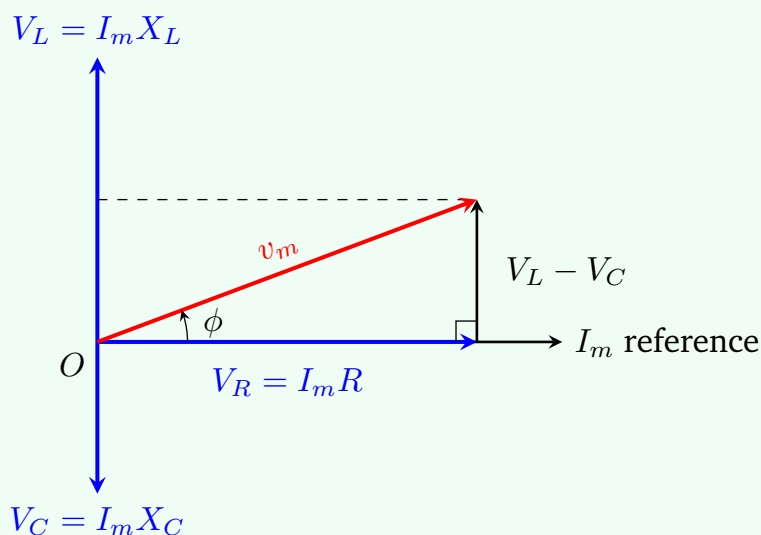
Solution:

Concept:

- In a series LCR circuit the same sinusoidal current flows through the resistance R , inductance L and capacitance C . The reactances are $X_L = \omega L$ and $X_C = 1/(\omega C)$, both in ohms.

- Taking current as the reference phasor, $V_R = I_m R$ is in phase with it, $V_L = I_m X_L$ leads it by 90° , and $V_C = I_m X_C$ lags it by 90° . Here all phasor lengths denote peak values and I_m is peak current.

Step 1: Draw and add the voltage phasors Draw I_m and V_R along the positive horizontal direction. Draw V_L vertically upward and V_C vertically downward. Their resultant is the signed vertical voltage $V_L - V_C$. The supply-voltage phasor is the vector sum of this vertical component and V_R . The diagram shows the inductive case $X_L > X_C$; ϕ is the angle by which the applied voltage leads the current.



Series LCR voltage phasors for $X_L > X_C$; the supply voltage leads the current by ϕ .

Step 2: Obtain the impedance from the right triangle The resistive and net reactive voltage components are perpendicular. Therefore, by Pythagoras,

$$\begin{aligned} v_m^2 &= V_R^2 + (V_L - V_C)^2 \\ &= I_m^2 R^2 + I_m^2 (X_L - X_C)^2. \end{aligned}$$

Defining impedance as the ratio of the supply-voltage amplitude to current amplitude gives

$$Z = \frac{v_m}{I_m} = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

Impedance is measured in Ω . The same ratio is obtained using rms voltage and rms current.

Step 3: Find the phase difference and identify its sign The opposite side of the phasor triangle is $V_L - V_C$, while the adjacent side is V_R . Hence

$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \frac{\omega L - 1/(\omega C)}{R}.$$

For $R > 0$, take $-90^\circ < \phi < 90^\circ$. Since the given voltage is $v = v_m \sin \omega t$, the current is

$$i = I_m \sin(\omega t - \phi), \quad I_m = \frac{v_m}{Z}.$$

If $X_L > X_C$, then $\phi > 0$ and the current lags the voltage. If $X_C > X_L$, then $\phi < 0$ and the current leads. At resonance, $X_L = X_C$, so $\phi = 0$, $Z = R$, and the two are in phase.

Step 4: Conclusion The required impedance and signed phase difference are

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}, \quad \phi = \tan^{-1} \left(\frac{\omega L - 1/(\omega C)}{R} \right).$$

Positive ϕ denotes voltage leading current, as shown in the phasor diagram.

Quick Tip to Solve: Put the common current along the horizontal reference. Subtract the opposing inductor and capacitor voltages before forming the right triangle for Z and $\tan \phi$.

25. (a) A parallel plate capacitor is charged by an ac source. Show that the sum of conduction current (I_c) and the displacement current (I_d) has the same value at all points of the circuit.

(b) In case (a) above, is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain. [3 marks]

Solution:

Concept:

- Conduction current is the rate of flow of charge: $I_c = dq/dt$, where $q(t)$ is the charge on a capacitor plate.
- Maxwell's displacement current is $I_d = \epsilon_0 d\Phi_E/dt$, where Φ_E is the electric flux. For a parallel plate capacitor in air, neglecting edge effects, $E = q/(\epsilon_0 A)$.
- The generalized current is $I = I_c + I_d$. Including displacement current makes current continuous even where no charges cross the dielectric gap.

Step 1: (a) Express the conduction current in the leads Let the instantaneous charge on the positive reference plate be $q(t)$, and let its area be A . The ac source makes this charge vary with time, so the signed current in the connecting wire is

$$I_c(t) = \frac{dq(t)}{dt}.$$

In the ideal conducting lead, the displacement current is negligible; therefore the total current there is $I_c + I_d = dq/dt$. The sign of this current reverses as the capacitor alternately charges and discharges.

Step 2: (a) Calculate the displacement current between the plates No conduction charge crosses the insulating gap, so $I_c = 0$ there. However, the changing plate charge produces a changing electric field and electric flux:

$$E(t) = \frac{q(t)}{\epsilon_0 A}, \quad \Phi_E(t) = E(t)A = \frac{q(t)}{\epsilon_0},$$

$$I_d(t) = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dq(t)}{dt}.$$

Thus the total current in the gap is also $I_c + I_d = dq/dt$. With a consistent direction convention,

the conduction current in a lead and the displacement current across the gap agree at every instant, including during reversal. Their sum has the same value throughout the ideal circuit.

Step 3: (b) Apply the junction rule at a capacitor plate Yes, Kirchhoff's first rule remains valid when the total current, including displacement current, is counted: conduction current entering a plate equals displacement current continuing through the gap. Thus the algebraic sum of the generalized currents at the plate is zero. If only conduction current were counted, the usual zero-sum junction statement would not apply at the charging plate: charge accumulates on it at the rate dq/dt , and no conduction current leaves through the dielectric. This is consistent with charge conservation; displacement current supplies the continuous-current description without implying that charge crosses the gap.

Step 4: Conclusion $I_c + I_d = dq/dt$ has the same instantaneous value in the wires and between the plates. Kirchhoff's junction rule is valid at each plate for the generalized current that includes displacement current.

Quick Tip to Solve: Calculate $\Phi_E = q/\epsilon_0$ first: differentiating immediately gives $I_d = dq/dt$, matching the lead current without assuming any particular ac waveform.

26. (a) Mention any three features of results of experiment on photoelectric effect which cannot be explained using the wave theory of light.

(b) In his experiment on photoelectric effect, Robert A. Millikan found the slope of the cut-off voltage versus frequency of incident light plot to be $4.12 \times 10^{-15} \text{ V s}$. Calculate the value of Planck's constant from it. [3 marks]

Solution:

Concept:

- In the classical wave picture, light transfers energy continuously, and its intensity sets the rate of energy transfer. This picture fails to explain several photoelectric observations.
- Einstein's photoelectric equation is $K_{\max} = h\nu - \phi = eV_0$, where h is Planck's constant, ν is frequency, ϕ is the work function, e is the elementary charge and V_0 is the positive magnitude of the stopping or cut-off voltage.
- A plot of V_0 against ν has slope h/e . Consequently, Planck's constant equals the slope multiplied by e .

Step 1: (a) State three observations that wave theory cannot explain **Threshold frequency:** Each emitting material has a minimum frequency ν_0 ; no photoelectrons are emitted below it, however intense the light is. Classical continuous absorption would instead allow electrons to accumulate enough energy from sufficiently intense or prolonged illumination.

Maximum kinetic energy: Above threshold, $K_{\max}$ increases linearly with light frequency and is independent of intensity at a fixed frequency. The wave picture ties transferred energy to intensity, so it cannot account for this frequency dependence together with intensity independence.

Practically instantaneous emission: Photoemission begins without a measurable accumulation delay,

even for very weak incident light of frequency above threshold. Classical absorption predicts that an electron under weak illumination should need appreciable time to collect enough energy to escape.

Step 2: (b) Relate the graph slope to Planck's constant Rearranging Einstein's equation into the straight-line form gives

$$V_0 = \frac{h}{e}\nu - \frac{\phi}{e}.$$

Hence the slope $m = \Delta V_0 / \Delta \nu = h/e$. Its supplied value is $4.12 \times 10^{-15} \text{ V s}$. The work function changes the intercept, so its value is not needed to obtain h from this slope.

Step 3: (b) Substitute the measured slope Using $e = 1.6 \times 10^{-19} \text{ C}$,

$$\begin{aligned} h &= em \\ &= (1.6 \times 10^{-19})(4.12 \times 10^{-15}) \text{ C V s} \\ &= 6.592 \times 10^{-34} \text{ J s}. \end{aligned}$$

Here $\text{C V} = \text{J}$, confirming the required unit of energy multiplied by time.

Step 4: Conclusion The unexplained observations are threshold frequency, frequency-dependent but intensity-independent maximum kinetic energy, and practically instantaneous emission. Millikan's slope gives $h \approx 6.59 \times 10^{-34} \text{ J s}$.

Quick Tip to Solve: For a stopping-voltage versus frequency graph, multiply the slope by e . If the vertical axis were maximum kinetic energy instead, the slope itself would be h .

27. (a) Draw circuit arrangement for studying V - I characteristics of a p - n junction diode.

(b) Show the shape of the characteristics of a diode.

(c) Mention two information that you can get from these characteristics.

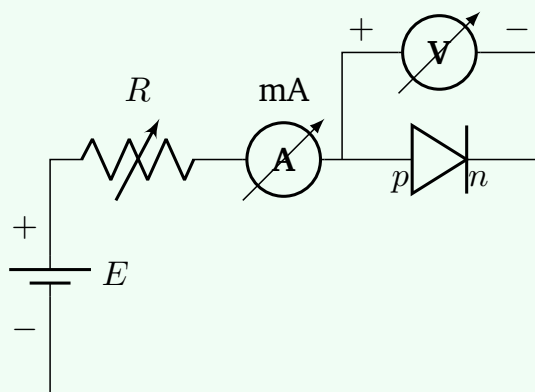
[3 marks]

Solution:

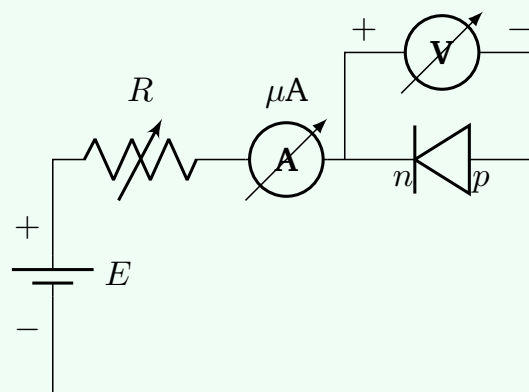
Concept:

- A diode conducts readily in forward bias (p side positive) after the barrier is sufficiently reduced. In reverse bias its small minority-carrier current remains nearly constant until breakdown.
- To measure a V - I characteristic, connect an ammeter in series and a voltmeter across the diode. At a chosen operating point the dynamic resistance is $r_d = \Delta V / \Delta I$.

Step 1: (a) Connect the measuring circuits For forward bias, connect the positive terminal of a variable dc supply to the p side through a protective variable resistor and a milliammeter. Connect the n side to the negative terminal. Connect a voltmeter in parallel with the diode, with its positive terminal on the supply-positive side. Vary the supply or the series resistor and record the voltage across the diode and the corresponding current. For reverse bias, reverse the diode connections and use a microammeter because the reverse current is much smaller. The resistor limits current and protects the diode; use suitably rated meters and supply.



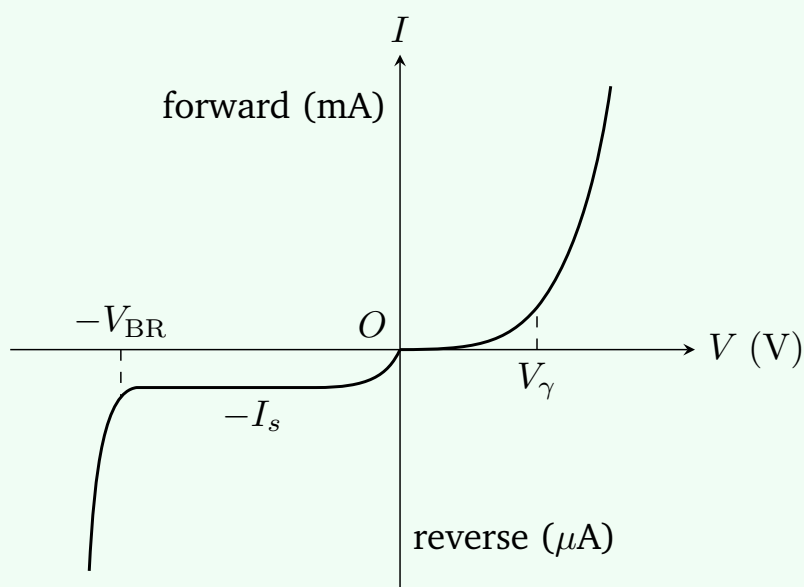
Forward bias



Reverse bias

Forward- and reverse-bias arrangements; R limits current and the voltmeter measures the diode voltage.

Step 2: (b) Plot current against diode voltage Plot forward current against positive diode voltage. Current is initially small and rises steeply beyond the knee voltage V_γ . Under reverse bias plot negative current against negative voltage: the small reverse saturation current is almost independent of voltage before breakdown. At the reverse breakdown voltage $-V_{BR}$, its magnitude increases sharply. The sketch uses different forward and reverse current scales to make the small reverse current visible.



Diode characteristic (schematic; forward and reverse current scales differ).

Step 3: (c) Read two properties from the graph First, the forward knee or cut-in voltage is read from the point where forward current starts increasing rapidly; it is typically about 0.7 V for silicon or 0.3 V for germanium. Second, the diode's dynamic resistance at an operating point follows from the inverse slope of the characteristic, $r_d = \Delta V / \Delta I$, using two nearby points. Thus the steep forward branch gives a small resistance, while the nearly flat reverse branch before breakdown gives a large resistance.

Step 4: Conclusion The circuits and characteristic show forward conduction and reverse blocking. Two quantities obtainable are the knee voltage and the dynamic resistance at a selected operating point.

Quick Tip to Solve: Put the ammeter in series and voltmeter across the diode; change from mA to microampere range when reversing the bias.

28. (a) Define 'Mass defect' and 'Binding energy' of a nucleus. Describe 'Fission process' on the basis of binding energy per nucleon.

(b) A deuteron contains a proton and a neutron and has a mass of 2.013553 u. Calculate the mass defect for it in u and its energy equivalence in MeV. ($m_p = 1.007277$ u, $m_n = 1.008665$ u, $1 \text{ u} = 931.5 \text{ MeV}/c^2$)

[3 marks]

Solution:

Concept:

- The mass defect is $\Delta m = Zm_p + (A - Z)m_n - M$, the excess of the masses of the separate free nucleons over the nuclear mass M ; Z and A are the atomic and mass numbers.
- Nuclear binding energy $B = \Delta mc^2$ is the energy needed to separate a nucleus completely into free nucleons. Binding energy per nucleon B/A measures how tightly its nucleons are bound.

Step 1: (a) Define the quantities and explain fission Mass defect is the difference between the sum of the masses of the free constituent protons and neutrons and the actual mass of the nucleus. Binding energy is the minimum energy required to separate the nucleus into those free nucleons, and is also the energy released when they combine to form it. In nuclear fission, a heavy nucleus splits into two lighter nuclei, usually with some free neutrons. Heavy nuclei have a smaller binding energy per nucleon than the medium-mass fragments formed. Consequently the total binding energy increases: the products are more tightly bound. This increase corresponds to a decrease in total rest mass, and the difference is released as energy, principally as kinetic energy of the fragments and neutrons.

Step 2: (b) Calculate the deuteron's mass defect A deuteron has one proton and one neutron, so use the given nucleon masses directly with its nuclear mass. No electron mass is involved:

$$\begin{aligned}m_p + m_n &= 1.007277 + 1.008665 = 2.015942 \text{ u}, \\ \Delta m &= (2.015942 - 2.013553) \text{ u} \\ &= 0.002389 \text{ u}.\end{aligned}$$

Step 3: Convert the missing mass into binding energy The given equivalence $1 \text{ u } c^2 = 931.5 \text{ MeV}$ converts the mass defect directly to energy:

$$B = 0.002389 \times 931.5 \text{ MeV} = 2.2253535 \text{ MeV} \simeq 2.225 \text{ MeV}.$$

This is the total binding energy of the two-nucleon deuteron; it is not the energy per nucleon. Its positive value expresses the energy that must be supplied to break it apart.

Step 4: Conclusion The deuteron's mass defect is 0.002389 u , with energy equivalent 2.225 MeV . Fission releases energy because its medium-mass products have greater total binding energy than the original heavy nucleus.

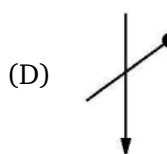
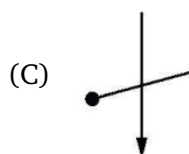
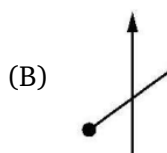
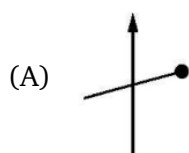
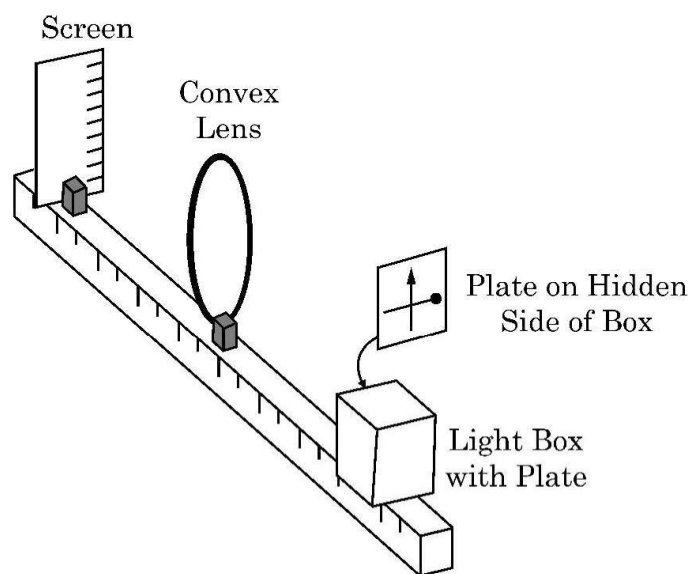
Quick Tip to Solve: First subtract the nuclear mass from the sum of free-nucleon masses; then multiply a mass defect in u by 931.5 to obtain MeV.

Section D (8 marks)

Question numbers 29 and 30 are case study based questions. Read the following paragraphs and answer the questions that follow.

29. A thin lens is a transparent optical medium bounded by two surfaces, at least one of which should be spherical. Applying the formula for image formation by a single spherical surface successively at the two surfaces of a lens, one can obtain the 'lens maker formula' and then the 'lens formula'. A lens has two foci – called 'first focal point' and 'second focal point' of the lens, one on each side. [4 marks]

29(i). (i) Consider the arrangement shown in figure. A black vertical arrow and a horizontal thick line with a ball are painted on a glass plate. It serves as the object. When the plate is illuminated, its real image is formed on the screen. Which of the following correctly represents the image formed on the screen? [1 mark]



Correct Answer: (C)

Solution:

Concept:

- A convex lens forms a real image of a real object with negative transverse magnification $m = v/u$, since $u < 0$ and $v > 0$ in the Cartesian convention.
- The transverse coordinates of each object point are multiplied by the same negative factor: the

pattern is rotated through 180° about the optical axis.

Step 1: Trace the two distinctive parts of the object The object has an upward-pointing vertical arrow and a ball at the right-hand end of the horizontal line. Real-image inversion sends the top of the arrow below the axis, so it points downward. The ball on the right goes to the left of the axis. Both changes are required; the entire pattern is inverted, not merely its vertical arrow.

Step 2: Compare every printed sketch Option (A) preserves both the upward arrow and right-hand ball. Option (B) moves the ball left but leaves the arrow upward. Option (D) turns the arrow downward but leaves the ball right. Option (C) alone has the downward arrow together with the left-hand ball, as required for the real image on the screen.

Step 3: Conclusion The real image has its arrow downward and ball to the left. Hence, the correct option is (C).

Quick Tip to Solve: Rotate both distinctive features of the object by 180 degrees; do not flip just the arrow or just the ball.

29(ii). (ii) Which of the following statements is incorrect?

[1 mark]

- (A) For a convex mirror magnification is always negative.
- (B) For all virtual images formed by a mirror magnification is positive.
- (C) For a concave lens magnification is always positive.
- (D) For real and inverted images, magnification is always negative.

Correct Answer: (A) For a convex mirror magnification is always negative.

Solution:

Concept:

- Transverse magnification $m = h_i/h_o$ is positive for an erect image and negative for an inverted image, where h_o and h_i are signed object and image heights.
- In the Cartesian convention, mirror magnification is $m = -v/u$, while lens magnification is $m = v/u$. The statements concern ordinary image formation for real objects.

Step 1: Test the convex-mirror claim A convex mirror forms a virtual, erect and diminished image of a real object. Here $u < 0$ and $v > 0$, so $m = -v/u > 0$. The image's smaller size means $0 < m < 1$; it does not make the magnification negative. Thus statement (A) has the wrong sign.

Step 2: Check the remaining statements A virtual image produced by a mirror for a real object is erect, so its magnification is positive and (B) is correct. A concave lens forms a virtual, erect image of a real object; both u and v are negative, making $v/u > 0$, so (C) is correct. For a real inverted image the image height has the opposite sign to the object height, making magnification negative; therefore (D) is also correct.

Step 3: Conclusion A convex mirror has positive magnification for a real object, so statement (A) is incorrect. Hence, the correct option is (A).

Quick Tip to Solve: Use orientation for the sign and image size for the magnitude; a diminished erect image still has positive magnification.

29(iii). (iii) A convex lens of focal length f is cut into two equal parts perpendicular to the principal axis. The focal length of each part will be: [1 mark]

- (A) f
- (B) $2f$
- (C) $\frac{f}{2}$
- (D) $\frac{f}{4}$

Correct Answer: (B) $2f$

Solution:

Concept:

- For a thin lens in air, the lens maker formula is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, where μ is the lens refractive index and R_1, R_2 are signed surface radii.
- A plane refracting face has radius ∞ , so its curvature term is zero. For the equal halves in this question, the original equally curved faces contribute equally to the lens power.

Step 1: Write the original lens power Let the magnitude of either curved surface radius be R . For light travelling left to right, the original biconvex lens has $R_1 = R$ and $R_2 = -R$. Hence

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{2(\mu - 1)}{R}.$$

Step 2: Replace the cut face by a plane surface Cutting perpendicular to the principal axis gives two identical plano-convex lenses. Each retains one curved face and has one new plane face. Each half therefore has focal length f' satisfying

$$\frac{1}{f'} = (\mu - 1) \left(\frac{1}{R} - 0 \right) = \frac{\mu - 1}{R} = \frac{1}{2f}.$$

Thus its power is halved and its focal length is doubled. Option (A) would correspond to unchanged surface curvature contributions, as in a cut along the principal axis. Options (C) and (D) would require increased power, which removing one curved-face contribution cannot produce.

Step 3: Conclusion Each plano-convex half has focal length $2f$. Hence, the correct option is (B).

Quick Tip to Solve: A perpendicular cut makes a plane face: halve the power of the equal-curvature lens, then invert to obtain twice its focal length.

29(iii). If an object in case (i) above is 20 cm from the lens and the screen is 50 cm away from the object, the focal length of the lens used is: [1 mark]

- (A) 10 cm
- (B) 12 cm
- (C) 16 cm
- (D) 20 cm

Correct Answer: (B) 12 cm

Solution:

Concept:

- For a thin lens, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, where u , v and f are the signed object distance, image distance and focal length.
- A real object and its real image lie on opposite sides of a convex lens, so their separation is the sum of the magnitudes of their distances from the lens.

Step 1: Locate the screen relative to the lens The object is 20 cm from the lens. The screen is 50 cm from the object, not from the lens. Therefore the lens-to-screen distance is $50 - 20 = 30$ cm. With light travelling from object to screen, $u = -20$ cm and $v = +30$ cm.

Step 2: Apply the lens formula

$$\frac{1}{f} = \frac{1}{30} - \frac{1}{-20} = \frac{2+3}{60} = \frac{1}{12} \text{ cm}^{-1}.$$

The positive focal length is consistent with the convex lens shown. None of 10, 16 or 20 cm satisfies this reciprocal-distance equation. Using 50 cm as v would incorrectly measure the image distance from the object.

Step 3: Conclusion The focal length is $f = +12$ cm. Hence, the correct option is (B).

Quick Tip to Solve: Subtract the object-to-lens distance from the object-to-screen separation before substituting for the image distance.

29(iv). The distance of an object from first focal point of a biconvex lens is X_1 and distance of the image from second focal point is X_2 . The focal length of the lens is: [1 mark]

- (A) $X_1 X_2$
- (B) $\sqrt{X_1 + X_2}$
- (C) $\sqrt{X_1 X_2}$
- (D) $\sqrt{\frac{X_2}{X_1}}$

Correct Answer: (C) $\sqrt{X_1 X_2}$

Solution:

Concept:

- For a real object and real image formed by a convex lens, $\frac{1}{f} = \frac{1}{U} + \frac{1}{V}$, where U and V are positive distances measured from the lens.
- Newton's lens formula is $X_1 X_2 = f^2$, where X_1 and X_2 are the object and image distances measured from their respective focal points.

Step 1: Express the lens distances using the focal distances In the real-image arrangement, the object lies beyond the first focus and the image beyond the second focus. Thus $U = f + X_1$ and $V = f + X_2$. Substituting these positive distances gives

$$\frac{1}{f} = \frac{1}{f + X_1} + \frac{1}{f + X_2}.$$

Step 2: Rearrange and compare the dimensions Multiplying by $f(f + X_1)(f + X_2)$,

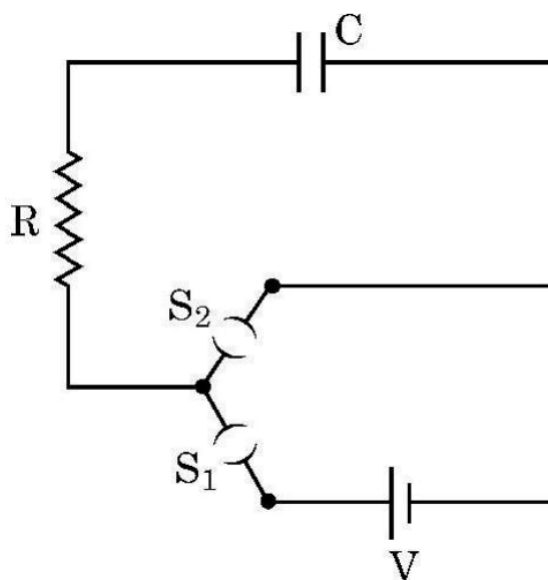
$$(f + X_1)(f + X_2) = f(2f + X_1 + X_2),$$
$$X_1 X_2 = f^2.$$

The biconvex lens has positive focal length in this arrangement. Option (A) has dimensions of length squared, (B) of the square root of length, and (D) is dimensionless. Only (C) has the required length dimension and satisfies the lens formula.

Step 3: Conclusion The focal length is $f = \sqrt{X_1 X_2}$. Hence, the correct option is (C).

Quick Tip to Solve: Distances measured from the focal points call for Newton's formula; distances measured from the lens call for the usual lens formula.

30. A circuit consisting of a capacitor C , a resistor of resistance R and an ideal battery of emf V , as shown in figure, is known as an RC series circuit. As soon as the circuit is completed by closing key S_1 (keeping S_2 open), charges begin to flow between the capacitor plates and the battery terminals. The charge on the capacitor increases and consequently the potential difference $V_c (= q/C)$ across the capacitor also increases with time. When this potential difference equals the potential difference across the battery, the capacitor is fully charged ($Q = VC$). During this process of charging, the charge q on the capacitor changes with time t as $q = Q [1 - e^{-t/RC}]$. The charging current can be obtained by differentiating it and using $\frac{d}{dx}(e^{mx}) = m e^{mx}$. Consider the case when $R = 20 \text{ k}\Omega$, $C = 500 \text{ }\mu\text{F}$ and $V = 10 \text{ V}$. [4 marks]



30(i). The final charge on the capacitor, when key S_1 is closed and S_2 is open, is: [1 mark]

- (A) $5 \text{ }\mu\text{C}$
- (B) 5 mC
- (C) 25 mC
- (D) 0.1 C

Correct Answer: (B) 5 mC

Solution:

Concept:

- A capacitor obeys $q = CV_C$, where C is its capacitance and V_C is the potential difference across its plates.
- At the end of charging through a resistor, current tends to zero, so the resistor's voltage drop tends to zero and the capacitor voltage equals the battery emf V .

Step 1: Identify the final capacitor voltage Closing S_1 with S_2 open connects the battery to the series resistor-capacitor circuit. After sufficient time, $V_C = V = 10 \text{ V}$. The resistance controls the charging rate, but not the final charge for an ideal battery.

Step 2: Calculate and convert the charge Using $C = 500 \text{ }\mu\text{F} = 500 \times 10^{-6} \text{ F}$,

$$Q = CV = (500 \times 10^{-6})(10) = 5 \times 10^{-3} \text{ C} = 5 \text{ mC}.$$

Option (A) is a thousand times too small. Options (C) and (D) would require capacitor voltages of

50 V and 200 V, respectively, so they are inconsistent with the given battery.

Step 3: Conclusion The final charge magnitude on either capacitor plate is 5 mC. Hence, the correct option is (B).

Quick Tip to Solve: For the final charge, multiply capacitance by battery voltage; the series resistance affects only how quickly the charge approaches this value.

30(ii). For sufficient time the key S_1 is closed and S_2 is open. Now S_2 is closed and S_1 is open. What is the final charge on the capacitor? [1 mark]

- (A) Zero
- (B) 5 mC
- (C) 2.5 mC
- (D) 5 μ C

Correct Answer: (A) Zero

Solution:

Concept:

- A charged capacitor connected across a resistor without a battery discharges through that resistor.
- The charge during discharge is $q(t) = Q_0 e^{-t/(RC)}$, where Q_0 is the initial charge and RC is the circuit's time constant.

Step 1: Identify the discharge path Initially, closing S_1 while keeping S_2 open charges the capacitor to $Q_0 = CV = 5$ mC. Opening S_1 disconnects the battery. Closing S_2 completes a loop containing only the charged capacitor and the resistor, through which the capacitor discharges.

Step 2: Take the long-time limit For positive R and C , the exponential in

$$q(t) = 5e^{-t/(RC)} \text{ mC}$$

tends to zero as time becomes sufficiently large. The stored electrical energy is dissipated as heat in the resistor. Option (B) is the initial charge, while the nonzero charges in (C) and (D) could occur only at intermediate discharge times, not in the final state.

Step 3: Conclusion The capacitor is fully discharged in the final limiting state, so its charge is zero. Hence, the correct option is (A).

Quick Tip to Solve: Trace the switches first: with the battery disconnected and a resistor joining the plates, the final capacitor charge tends to zero.

30(iii). The dimensional formula for RC is:

[1 mark]

- (A) $[ML^2T^{-3}A^{-2}]$
- (B) $[M^0L^0T^{-1}A^0]$
- (C) $[M^{-1}L^{-2}T^4A^2]$
- (D) $[M^0L^0TA^0]$

Correct Answer: (D) $[M^0L^0TA^0]$

Solution:

Concept:

- Resistance is $R = V/I$ and capacitance is $C = q/V$, where V , I and q denote potential difference, current and charge.
- Charge has dimensions of current multiplied by time, so $[q] = [AT]$. The product RC is the time constant of an RC circuit.

Step 1: Combine the defining relations Multiplying the dimensional relations for resistance and capacitance cancels potential difference:

$$[RC] = \left[\frac{V}{I} \frac{q}{V} \right] = \left[\frac{q}{I} \right] = \frac{[AT]}{[A]} = [T].$$

Thus the product contains no net mass, length or current dimensions.

Step 2: Check the exponential and the alternatives The exponent $-t/(RC)$ in the supplied charging law must be dimensionless. This independently requires RC to have dimensions of time. Option (A) gives the dimensions of resistance alone and (C) those of capacitance alone. Option (B) has inverse-time dimensions and would apply to $1/(RC)$, not to RC .

Step 3: Conclusion The dimensional formula of RC is $[M^0L^0T^1A^0]$. Hence, the correct option is (D).

Quick Tip to Solve: Inspect the exponential: its exponent is dimensionless, so the quantity dividing time in an RC charging law must itself be a time.

30(iv). The key S_1 is closed and S_2 is open. The value of current in the resistor after 5 seconds is:

[1 mark]

- (A) $\frac{1}{2\sqrt{e}}$ mA
- (B) $\sqrt{e}$ mA
- (C) $\frac{1}{\sqrt{e}}$ mA
- (D) $\frac{1}{2e}$ mA

Correct Answer: (A) $\frac{1}{2\sqrt{e}}$ mA

Solution:

Concept:

- Charging current is the rate of increase of capacitor charge: $I = dq/dt$.
- Differentiating $q = CV[1 - e^{-t/(RC)}]$ gives $I = (V/R)e^{-t/(RC)}$, with time constant $\tau = RC$.

Step 1: Find the initial current and time constant Convert the data to SI units: $R = 20 \times 10^3 \Omega$ and

$C = 500 \times 10^{-6} \text{ F}$. Then

$$I_0 = \frac{V}{R} = \frac{10}{20 \times 10^3} = 0.5 \text{ mA},$$
$$RC = (20 \times 10^3)(500 \times 10^{-6}) = 10 \text{ s}.$$

Step 2: Substitute the elapsed time At $t = 5 \text{ s}$, the ratio $t/(RC) = 1/2$, so

$$I = 0.5e^{-1/2} \text{ mA} = \frac{1}{2\sqrt{e}} \text{ mA}.$$

Options (B) and (C) exceed the initial current 0.5 mA , whereas charging current must decrease. Option (D) uses an exponent of -1 , which would apply after 10 s , not after 5 s .

Step 3: Conclusion The current after 5 s is $\frac{1}{2\sqrt{e}} \text{ mA}$, approximately 0.303 mA . Hence, the correct option is (A).

Quick Tip to Solve: Calculate the time constant before the exponential: here five seconds is half of RC , so the current is its initial value divided by the square root of e .

OR

30(iv). The key S_1 is closed and S_2 is open. The initial value of charging current in the resistor is: [1 mark]

- (A) 5 mA
- (B) 0.5 mA
- (C) 2 mA
- (D) 1 mA

Correct Answer: (B) 0.5 mA

Solution:

Concept:

- At the start of charging, the initially uncharged capacitor has $q = 0$, hence zero capacitor voltage $V_C = q/C$.
- Kirchhoff's loop law gives $V = IR + V_C$. Therefore the initial current is $I_0 = V/R$, also obtained by setting $t = 0$ in the charging-current formula.

Step 1: Find the initial resistor voltage Closing S_1 with S_2 open begins charging. At the initial instant the full battery emf appears across the resistor because the capacitor voltage is zero. Thus the current is limited by $R = 20 \text{ k}\Omega$, rather than by a nonzero capacitor voltage.

Step 2: Apply Ohm's law and convert the unit

$$I_0 = \frac{10 \text{ V}}{20 \times 10^3 \Omega} = 5 \times 10^{-4} \text{ A} = 0.5 \text{ mA}.$$

This is the maximum charging current; it subsequently decreases as capacitor voltage rises. The values 5, 2 and 1 mA would require resistor voltages of 100, 40 and 20 V, respectively, all larger than the given emf.

Step 3: Conclusion The initial charging current is 0.5 mA. Hence, the correct option is (B).

Quick Tip to Solve: At the instant charging begins, use the full battery voltage across the resistor; dividing volts by kilo-ohms directly gives milliamperes.

Section E (15 marks)

31(a). (i) (1) What are coherent sources? Why are they necessary for observing a sustained interference pattern?

(2) Lights from two independent sources are not coherent. Explain.

(ii) Two slits 0.1 mm apart are arranged 1.20 m from a screen. Light of wavelength 600 nm from a distant source is incident on the slits.

(1) How far apart will adjacent bright interference fringes be on the screen?

(2) Find the angular width (in degree) of the first bright fringe.

[5 marks]

Solution:

Concept:

- Coherent sources emit waves of the same frequency with a constant phase difference. A fixed phase relationship keeps the positions of interference maxima and minima stationary.
- For two overlapping light waves, $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$, where I_1, I_2 are their individual intensities and ϕ is their phase difference at the observation point.
- In Young's double-slit experiment, $\beta = \lambda D/d$ is the linear fringe width and $\Delta\theta = \beta/D = \lambda/d$ is its small-angle angular width in radians; d is the slit separation and D the screen distance.

Step 1: (i)(1) Define coherence and explain a sustained pattern Two sources are coherent when they emit light of the same frequency and maintain a constant phase difference. The phase difference need not be zero. At a fixed point on the screen, the geometrical path difference is fixed. With coherent sources, the resulting phase difference at that point also remains fixed, so constructive or destructive interference persists there. Thus, bright and dark fringes remain at definite positions and a sustained pattern is observed. If the relative phase varies randomly and rapidly, a point alternates between constructive and destructive interference. During the observation time, the interference term averages to zero and the stationary fringes disappear.

Step 2: (i)(2) Explain why independent sources are incoherent In two ordinary independent light sources, atoms emit short wave trains independently and at random times. The phase of one emitted wave train has no fixed relationship with that of a wave train emitted by the other source. Consequently, even if the sources have the same nominal frequency, their relative phase fluctuates randomly instead of remaining constant. Therefore, two independent ordinary sources cannot produce a sustained interference pattern. In Young's experiment, both slits are illuminated by the same source: they act as secondary sources with a stable relative phase and can therefore produce observable fringes.

Step 3: (ii)(1) Calculate the separation of adjacent bright fringes Convert all given quantities to SI units:

$$d = 0.1 \text{ mm} = 1.0 \times 10^{-4} \text{ m}, \quad D = 1.20 \text{ m}, \quad \lambda = 600 \text{ nm} = 6.00 \times 10^{-7} \text{ m}.$$

For a point at distance y from the central maximum, the path difference is approximately dy/D . Bright fringes satisfy $dy_n/D = n\lambda$, so

$$y_n = \frac{n\lambda D}{d}, \quad \beta = y_{n+1} - y_n = \frac{\lambda D}{d}.$$

Substitution gives

$$\beta = \frac{(6.00 \times 10^{-7})(1.20)}{1.0 \times 10^{-4}} = 7.20 \times 10^{-3} \text{ m} = 7.20 \text{ mm}.$$

Thus, adjacent bright fringes are equally spaced in the small-angle approximation, which is justified here because $d \ll D$ and the first fringes are close to the central axis.

Step 4: (ii)(2) Calculate the angular width in degrees The angular width of the first bright fringe is the angle between its bounding dark fringes. These have path differences $\lambda/2$ and $3\lambda/2$, whose difference is λ . Hence the width is the same as the angular separation of adjacent bright fringes:

$$\Delta\theta = \frac{\beta}{D} = \frac{\lambda}{d} = \frac{6.00 \times 10^{-7}}{1.0 \times 10^{-4}} = 6.00 \times 10^{-3} \text{ rad}.$$

Convert radians to degrees by multiplying by $180/\pi$:

$$\Delta\theta = (6.00 \times 10^{-3}) \frac{180}{\pi} \approx 0.344^\circ.$$

Step 5: Conclusion Coherent sources have the same frequency and a constant phase difference, which keeps an interference pattern stationary. Independent ordinary sources have random relative phases and cannot sustain the pattern. The adjacent bright fringes are 7.20 mm apart, and the angular width of the first bright fringe is $6.00 \times 10^{-3} \text{ rad} \approx 0.344^\circ$.

Quick Tip to Solve: First calculate λ/d : it gives angular fringe width in radians. Multiply by D for linear width or by $180/\pi$ for degrees; do not confuse the angular width with a half-fringe interval.

OR

31(b). (i) Define a wavefront. An incident plane wave falls on a convex lens and gets refracted through it. Draw a diagram to show the incident and refracted wavefront.

(ii) A beam of light coming from a distant source is refracted by a spherical glass ball (refractive index 1.5) of radius 15 cm. Draw the ray diagram and obtain the position of the final image formed.

[5 marks]

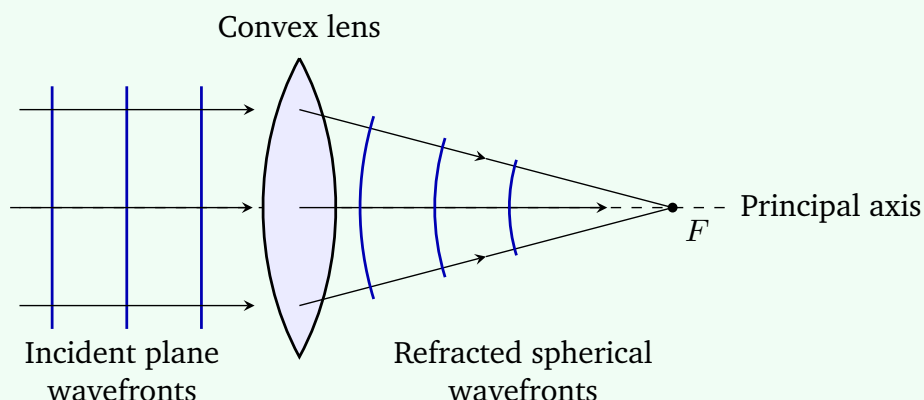
Solution:

Concept:

- A wavefront is the locus of points of a wave that vibrate in the same phase at a given instant. In an isotropic medium, light rays are normal to the wavefront.
- A convex lens converts a plane incident wavefront into a converging spherical wavefront whose centre is the principal focus on the other side of the lens.
- For paraxial refraction at a spherical surface, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, where n_1, n_2 are the refractive indices before and after refraction and u, v, R are signed distances from that surface's pole.

Step 1: (i) Define and draw the wavefronts A distant axial source supplies parallel rays, so its plane

wavefronts are perpendicular to the principal axis. A convex lens bends these rays towards its principal focus F . The emergent wavefronts are portions of spheres centred at F , with progressively smaller radii as the wave approaches the focus. Straight lines below represent incident plane wavefronts and circular arcs represent refracted spherical wavefronts; arrows show the propagation direction.



Plane wavefronts become converging spherical wavefronts after a convex lens.

Step 2: (ii) Set the sign convention for the glass ball Take light to travel from left to right through the ball in air, with all distances measured positively to the right. Let P_1 and P_2 be the front and rear poles and C the centre. The radius is $r = 15$ cm, so the separation of the two poles is $P_1P_2 = 30$ cm. At the first surface the centre lies to its right, giving $R_1 = +15$ cm; at the second it lies to its left, giving $R_2 = -15$ cm. We apply spherical refraction separately at these two surfaces, using the paraxial approximation.

Step 3: Find the image produced by the first surface For the distant source, $u_1 = -\infty$. Refraction is from air into glass, so $n_1 = 1$ and $n_2 = 1.5$. Thus

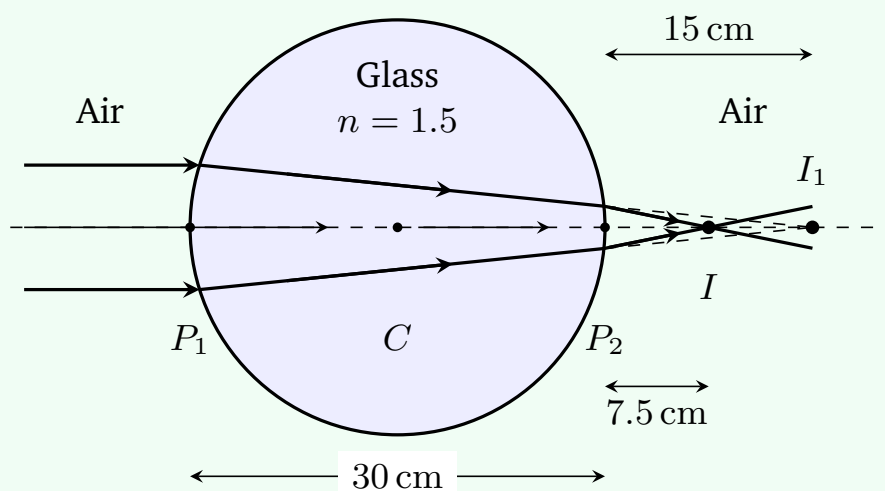
$$\begin{aligned}\frac{1.5}{v_1} - \frac{1}{-\infty} &= \frac{1.5 - 1}{15}, \\ \frac{1.5}{v_1} &= \frac{1}{30}, \\ v_1 &= +45 \text{ cm.}\end{aligned}$$

If the glass continued beyond the rear surface, these rays would meet at I_1 , 45 cm to the right of P_1 . This point is $45 - 30 = 15$ cm beyond P_2 . The rays reach the rear surface before meeting, so I_1 acts as a virtual object for that surface.

Step 4: Refract the converging rays at the rear surface For glass-to-air refraction, the incident and emergent indices are now 1.5 and 1. Since the virtual object is to the right of the rear pole, $u_2 = +15$ cm, while $R_2 = -15$ cm. Hence

$$\begin{aligned}\frac{1}{v_2} - \frac{1.5}{15} &= \frac{1 - 1.5}{-15}, \\ \frac{1}{v_2} &= \frac{1}{30} + \frac{1}{10} = \frac{2}{15}, \\ v_2 &= +7.5 \text{ cm.}\end{aligned}$$

The outgoing rays actually intersect to the right of the ball, so the final image is real. In the ray diagram, the dashed continuations of the rays inside the glass meet at I_1 ; the solid rays after the second refraction meet earlier at the final image I . The axial ray passes undeviated through both poles.



Paraxial rays form the final image at I ; dashed lines locate the first surface's image I_1 .

Step 5: Conclusion (i) A wavefront is a surface of constant phase. The convex lens changes the incident plane wavefront into a converging spherical wavefront centred at its principal focus. (ii) The glass ball forms a real image 7.5 cm beyond its rear surface, equivalently 22.5 cm from its centre on the outgoing side.

Quick Tip to Solve: After finding the first image, subtract the ball's diameter to locate the object for the rear surface. Here $u_2 = 45 - 30 = +15$ cm: it is positive because the rays are still converging towards a point beyond that surface.

32(a). (i) Two point charges $5\ \mu\text{C}$ and $-1\ \mu\text{C}$ are placed at points $(-3\ \text{cm}, 0, 0)$ and $(3\ \text{cm}, 0, 0)$ respectively. An external electric field $\vec{E} = \frac{A}{r^2} \hat{r}$, where $A = 3 \times 10^5\ \text{V m}$, is switched on in the region. Calculate the change in electrostatic energy of the system due to the electric field.

(ii) A system of two conductors is placed in air and they have net charge of $+80\ \mu\text{C}$ and $-80\ \mu\text{C}$ which causes a potential difference of 16 V between them.

(1) Find the capacitance of the system.

(2) If the air between the capacitor is replaced by a dielectric medium of dielectric constant 3, what will be the potential difference between the two conductors?

(3) If the charges on two conductors are changed to $+160\ \mu\text{C}$ and $-160\ \mu\text{C}$, will the capacitance of the system change? Give reason for your answer.

[5 marks]

Solution:

Concept:

- For a radial electrostatic field, $E_r = -dV/dr$. The potential is obtained by integrating the field, with the usual reference $V(\infty) = 0$.
- The additional potential energy of fixed charges in an external potential is $\Delta U = \sum_i q_i V_{\text{ext}}(r_i)$. Their mutual interaction energy is unchanged if their separation is unchanged.
- Capacitance is $C = Q/V$, where Q is the magnitude of the charge on either conductor and V is their potential difference. It depends on conductor geometry and the intervening dielectric.
- Completely replacing air by a dielectric of relative permittivity K gives $C' = KC$. For isolated conductors the free charge remains fixed, so $V' = V/K$.

Step 1: (i) Obtain the external potential at each charge With $E_r = A/r^2$, integrate inward from infinity:

$$V_{\text{ext}}(r) = - \int_{\infty}^r \frac{A}{r'^2} dr' = \frac{A}{r}.$$

Both points lie at the same radial distance $r = 3 \text{ cm} = 0.03 \text{ m}$ from the origin. The negative coordinate of the first charge does not make its radial distance negative. Thus the external potential at each point is

$$V_1 = V_2 = \frac{3 \times 10^5}{0.03} = 1.0 \times 10^7 \text{ V}.$$

Step 2: (i) Calculate the change in electrostatic energy The charges remain at their specified positions, so the energy of interaction between them cancels in the difference between final and initial energy. Only their interaction with the newly applied external field is added:

$$\begin{aligned} \Delta U &= q_1 V_1 + q_2 V_2 \\ &= (5 \times 10^{-6} - 1 \times 10^{-6})(1.0 \times 10^7) \text{ J} \\ &= 40 \text{ J}. \end{aligned}$$

No factor of one-half is used here: the given potential is produced by an independent external source, rather than by double-counting the pair interaction of the two charges.

Step 3: (ii)(1) Determine the capacitance in air Take the magnitude of either conductor's charge, $Q = 80 \mu\text{C}$, and divide it by the potential difference between the conductors:

$$C_{\text{air}} = \frac{80 \times 10^{-6}}{16} = 5 \times 10^{-6} \text{ F} = 5 \mu\text{F}.$$

The algebraic sum of the two conductor charges is zero, but it is not the charge used in the definition of capacitance.

Step 4: (ii)(2) Insert the dielectric at fixed charge No connection to a battery is specified. Therefore each conductor retains the stated free charge when the air is replaced by the dielectric. With $K = 3$,

$$C' = 3C_{\text{air}} = 15 \mu\text{F}, \quad V' = \frac{Q}{C'} = \frac{80}{15} \text{ V} = \frac{16}{3} \text{ V} \simeq 5.33 \text{ V}.$$

Polarisation of the dielectric reduces the electric field and hence the potential difference for the same free charge.

Step 5: (ii)(3) Explain the effect of doubling the charges Changing the charges to $+160 \mu\text{C}$ and $-160 \mu\text{C}$ doubles Q . For the same geometry and medium the potential difference also doubles, so

$$C_{\text{new}} = \frac{2Q}{2V} = \frac{Q}{V} = C.$$

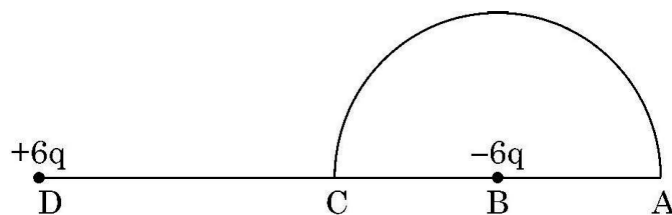
Consequently charging alone does not change the capacitance. It stays $5 \mu\text{F}$ in air, or $15 \mu\text{F}$ if the dielectric introduced in the preceding part remains present.

Step 6: Conclusion (i) The electrostatic energy increases by 40 J. (ii)(1) $C_{\text{air}} = 5 \mu\text{F}$. (2) After insertion of the dielectric, $V' = 16/3 \text{ V} \simeq 5.33 \text{ V}$. (3) Doubling the charges does not change the capacitance when geometry and medium are fixed.

Quick Tip to Solve: Equal distances from the origin mean equal external potentials, so use $(q_1 + q_2)A/r$. For the capacitor, identify whether charge or voltage is fixed before applying $C' = KC$.

32(b). (i) Consider three metal spherical shells A , B and C , each of radius R . Each shell is having a concentric metal ball of radius $R/10$. The spherical shells A , B and C are given charges $+6q$, $-4q$ and $14q$ respectively. Their inner metal balls are also given charges $-2q$, $+8q$ and $-10q$ respectively. Compare the magnitude of the electric fields due to shells A , B and C at a distance $3R$ from their centres.

(ii) A charge $-6\ \mu\text{C}$ is placed at the centre B of a semicircle of radius 5 cm , as shown in the figure. An equal and opposite charge is placed at point D at a distance of 10 cm from B . A charge $+5\ \mu\text{C}$ is moved from point C to point A along the circumference. Calculate the work done on the charge. [5 marks]



Solution:

Concept:

- Gauss's law gives the exterior field of a spherically symmetric charge distribution as $E = k|Q_{\text{enclosed}}|/r^2$, where $k = 1/(4\pi\epsilon_0)$. The enclosed charge includes both the metal shell and its concentric ball.
- Potentials add algebraically: $V = \sum_i kQ_i/r_i$. The potential due to the charge at the centre of a circle is the same at all points on that circle.
- In a quasistatic displacement of a test charge q_0 , external work is $W_{\text{ext}} = q_0(V_f - V_i) = \Delta U$. Work done by the electrostatic field has the opposite sign, $W_{\text{field}} = -\Delta U$, and does not depend on the path.

Step 1: (i) Add the charges enclosed outside each shell Draw an imaginary concentric Gaussian sphere of radius $3R$ for each system. Because $3R > R$, it encloses the shell as well as the inner ball. The algebraic totals are

$$Q_A = 6q - 2q = 4q,$$

$$Q_B = -4q + 8q = 4q,$$

$$Q_C = 14q - 10q = 4q.$$

Charge redistributes over the shell surfaces so that the field inside the metal is zero. The inner surface carries charge opposite to that of the ball; the resulting outer-surface charge is $4q$ in all three cases. Hence the different original shell charges alone do not determine the exterior field.

Step 2: (i) Apply Gauss's law at the common distance Spherical symmetry makes the field radial and constant in magnitude over each Gaussian sphere. Therefore

$$E(3R) 4\pi(3R)^2 = \frac{|4q|}{\epsilon_0}, \quad E(3R) = \frac{4k|q|}{9R^2}.$$

The enclosing radius and total charge are identical for the three systems, so their field magnitudes are equal. The ball radius $R/10$ affects the interior arrangement, but not this exterior result.

Step 3: (ii) Read the endpoint distances from the figure The points lie on a straight diameter in the

order D, C, B, A , with B at the centre of the semicircle. Thus

$$BA = BC = 0.05 \text{ m}, \quad DB = 0.10 \text{ m},$$

$$DC = DB - BC = 0.05 \text{ m}, \quad DA = DB + BA = 0.15 \text{ m}.$$

The fixed charges are $Q_B = -6 \mu\text{C}$ and $Q_D = +6 \mu\text{C}$. The moving charge is $q_0 = +5 \mu\text{C}$. Its distance from B stays constant along the arc, so the central charge contributes no potential difference between the endpoints.

Step 4: (ii) Calculate the endpoint potentials Using $k = 9 \times 10^9 \text{ N m}^2\text{C}^{-2}$, the potentials due to both fixed charges are

$$\begin{aligned} V_C &= 9 \times 10^9 \left(\frac{-6 \times 10^{-6}}{0.05} + \frac{6 \times 10^{-6}}{0.05} \right) = 0, \\ V_A &= 9 \times 10^9 \left(\frac{-6 \times 10^{-6}}{0.05} + \frac{6 \times 10^{-6}}{0.15} \right) \\ &= -7.2 \times 10^5 \text{ V}. \end{aligned}$$

The central-charge terms cancel in $V_A - V_C$. The positive charge at D gives a smaller positive potential at the more distant endpoint A , so the potential difference is negative.

Step 5: (ii) Evaluate the work with its sign convention With the usual convention for the work required to move a charge slowly, the external agent's work equals the change in electrostatic potential energy:

$$\begin{aligned} W_{\text{ext}} &= q_0(V_A - V_C) \\ &= (5 \times 10^{-6})(-7.2 \times 10^5) \text{ J} \\ &= -3.6 \text{ J}. \end{aligned}$$

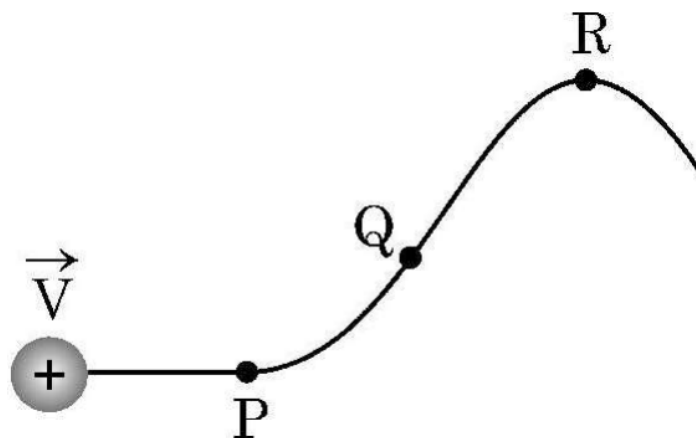
The negative sign means that the external agent removes energy during the quasistatic displacement. Equivalently, the electric field does $+3.6 \text{ J}$ of work on the charge. Stating the agent resolves the sign convention in the phrase "work done on the charge"; the curved path introduces no extra factor because the electrostatic force is conservative.

Step 6: Conclusion (i) $E_A : E_B : E_C = 1 : 1 : 1$, with each magnitude $4k|q|/(9R^2)$. (ii) The work required from an external agent is -3.6 J ; the work done by the electric field on the moving charge is $+3.6 \text{ J}$.

Quick Tip to Solve: Outside each shell, add the ball and shell charges first. Along the semicircle, cancel the central charge's equal endpoint potentials, then use $DA = 15 \text{ cm}$ and $DC = 5 \text{ cm}$ for the other charge.

33(a). (i) A proton moving with velocity $\vec{V}$ in a non-uniform magnetic field traces a path as shown in the figure. The path followed by the proton is always in the plane of the paper. What is the direction of the magnetic field in the region near points P , Q and R ? What can you say about relative magnitude of magnetic fields at these points?

(ii) A current carrying circular loop of area A produces a magnetic field B at its centre. Show that the magnetic moment of the loop is $\frac{2BA}{\mu_0} \sqrt{\frac{A}{\pi}}$. [5 marks]



Solution:

Concept:

- The magnetic force on a proton is $\vec{F} = e(\vec{v} \times \vec{B})$, where $e > 0$. Its direction is towards the local centre of curvature of the path.
- Magnetic force does no work, so the speed v stays constant. For a field normal to the plane of motion, the local radius of curvature r obeys $evB = m_p v^2/r$, hence $B = m_p v/(er)$.
- For a single circular loop of radius a , current I and area $A = \pi a^2$, the central field is $B = \mu_0 I/(2a)$ and the magnetic moment has magnitude $\mathcal{M} = IA$.

Step 1: (i) Determine the force and field near P The proton enters from the left and moves towards the right. Near P , the path starts bending upwards, so the magnetic force is upwards, towards the inside of this bend. Apply the right-hand rule to the positive charge: a rightward velocity crossed with a field directed into the paper gives an upward force. Therefore the field near P is into the plane of the paper. Reversing the field would bend the proton downwards instead.

Step 2: (i) Read the straight portion near Q and the bend near R Near Q , the sketch shows the locally straight part between the two oppositely curved portions. Here the local curvature is zero, so the transverse magnetic force is zero. In the perpendicular-field interpretation of the diagram, $B_Q = 0$, and a zero field has no direction. Near R , the proton is again moving towards the right but the path bends downwards, so the force is downwards. For a positive charge this requires the magnetic field to point out of the paper. Thus the field reverses its sense between the upward and downward bends.

Step 3: (i) Compare the field magnitudes from the curvature Equating the magnetic force to the local centripetal force gives

$$evB = \frac{m_p v^2}{r}, \quad B = \frac{m_p v}{er}.$$

Although the direction of velocity changes, its magnitude does not: the force remains perpendicular to the instantaneous velocity. Hence the same factor $m_p v/e$ applies everywhere and the field magnitude is inversely proportional to the local radius of curvature. The bend near R is sharper than near

P , so $r_R < r_P$. Near the straight portion Q , r_Q is infinite. The indicated ordering is therefore $B_R > B_P > B_Q = 0$. The drawing fixes the qualitative order, not a numerical ratio of the fields.

Step 4: (ii) Express the current in terms of the central field Write the loop radius in terms of its given area and solve the central-field formula for its current:

$$a = \sqrt{\frac{A}{\pi}}, \quad B = \frac{\mu_0 I}{2a} \Rightarrow I = \frac{2Ba}{\mu_0}.$$

Here B is the magnitude of the field produced by this loop at its centre, and μ_0 is the permeability of free space.

Step 5: (ii) Substitute the current into the magnetic moment The moment of the current loop equals current times area; its direction is normal to the loop by the right-hand rule. Substituting first for the current and then for the radius gives

$$\begin{aligned} \mathcal{M} &= IA = \frac{2Ba}{\mu_0} A \\ &= \frac{2BA}{\mu_0} \sqrt{\frac{A}{\pi}}. \end{aligned}$$

The result has units $A \, m^2$, as required for a magnetic moment.

Step 6: Conclusion (i) Near P , the magnetic field is into the paper; near Q , it is zero; near R , it is out of the paper. The magnitudes satisfy $B_R > B_P > B_Q = 0$. (ii) The loop's magnetic moment is

$$\mathcal{M} = \frac{2BA}{\mu_0} \sqrt{\frac{A}{\pi}}.$$

Quick Tip to Solve: For a positive charge, first point the force towards the inside of each bend, then use $\vec{v} \times \vec{B}$. Compare sharpness of the bends, not the slope of the path, to compare magnetic fields.

OR

33(b). (i) Derive an expression for the torque acting on a rectangular current loop suspended in a uniform magnetic field.

(ii) A charged particle is moving in a circular path with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. It is made to pass through a sheet of lead and as a consequence, it loses one half of its kinetic energy without change in its direction. How will (1) the radius of its path (2) its time period of revolution change? [5 marks]

Solution:

Concept:

- A straight conductor carrying current I in a uniform magnetic field experiences force $\vec{F} = I(\vec{l} \times \vec{B})$, where $\vec{l}$ points along the current and has magnitude equal to the conductor's length.
- Two equal and opposite parallel forces form a couple of torque $\tau = Fd$, where d is their perpendicular separation. The loop's magnetic moment is $\vec{\mu} = IA\hat{n}$, with $\hat{n}$ set by the right-hand rule.
- For nonrelativistic circular motion perpendicular to a uniform field, $r = mv/(|q|B)$, $K = mv^2/2$, and $T = 2\pi r/v$. Here m , q , v , r and T denote mass, charge, speed, radius and period.

Step 1: (i) Specify the loop geometry and its orientation Consider a rectangular loop with sides a and b , area $A = ab$, and current I . Let θ be the angle between its positive unit normal $\hat{n}$ and the uniform magnetic field $\vec{B}$. Choose the two sides of length a perpendicular to the field; the loop can be turned about an axis parallel to these sides to vary θ . The direction of the normal is found by curling the right-hand fingers in the direction of the current.

Step 2: (i) Calculate the couple acting on opposite sides The currents on the opposite sides of length a are opposite. Each side experiences a force of magnitude $F = IaB$; these forces are equal, opposite and perpendicular to the field. Their perpendicular separation is the projection of the side b along the field, namely $d = b \sin \theta$. Thus their net force is zero, but their torque is

$$\tau = Fd = (IaB)(b \sin \theta) = IAB \sin \theta.$$

The forces on the other two sides are also equal and opposite, but their lines of action coincide; therefore they contribute no couple. This accounts for all four sides and explains why a uniform field can rotate the loop while its net magnetic force is zero.

Step 3: (i) Write the vector torque and the expression for several turns The torque tends to align the magnetic moment with the external field. Since $\vec{\mu} = IA\hat{n}$, the vector form for one turn is

$$\vec{\tau} = \vec{\mu} \times \vec{B}.$$

For N identical turns, the torque adds over the turns, giving $\tau = N IAB \sin \theta$ and $\vec{\mu} = N I A \hat{n}$. Notice that the angle is measured from the normal, not from the plane of the loop: torque is zero when the normal is parallel to the field, and maximum when the plane of the loop is parallel to the field.

Step 4: (ii) Relate the reduced kinetic energy to the new speed and radius Let the original speed and radius be v and r , and the values after crossing the lead sheet be v' and r' . The particle retains half of its initial kinetic energy, while its mass and charge are unchanged:

$$\frac{1}{2}mv'^2 = \frac{1}{2} \left(\frac{1}{2}mv^2 \right) \Rightarrow v' = \frac{v}{\sqrt{2}}.$$

Its direction is unchanged at the sheet, so its velocity remains perpendicular to $\vec{B}$. Equating $|q|vB$ with mv^2/r before and after crossing gives

$$\frac{r'}{r} = \frac{v'}{v} = \frac{1}{\sqrt{2}}.$$

Hence the new circle has a smaller radius. It begins tangentially from the crossing point; the direction need not change abruptly for the subsequent curvature to change.

Step 5: (ii) Calculate the time period after the loss of energy The period of revolution is the circumference divided by speed. Substitute the magnetic-orbit radius:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \frac{mv}{|q|B} = \frac{2\pi m}{|q|B}.$$

Since m , $|q|$ and B are unchanged, $T' = T$. Equivalently, the new circumference and speed both decrease by the same factor $1/\sqrt{2}$, so their ratio does not change. This is the usual nonrelativistic result used for this problem.

Step 6: Conclusion (i) For one rectangular loop, $\tau = IAB \sin \theta$ and $\vec{\tau} = \vec{\mu} \times \vec{B}$; for N turns, $\tau = N IAB \sin \theta$. (ii) After losing half its kinetic energy, the particle has $r' = r/\sqrt{2}$, while its period

remains $T' = T = 2\pi m/(|q|B)$.

Quick Tip to Solve: Measure the torque angle from the loop's normal. For magnetic orbits at fixed mass, charge and field, use $r \propto \sqrt{K}$ and $T = 2\pi m/(|q|B)$ to compare the states directly.