

CBSE Class 12 Physics Question Paper 2025 with Solution PDF-Set 55/2/2



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – Sections **A, B, C, D** and **E**.
- (iii) In **Section A** : Question numbers **1** to **16** are Multiple Choice type questions. Each question carries **1** mark.
- (iv) In **Section B** : Question numbers **17** to **21** are Very Short Answer type questions. Each question carries **2** marks.
- (v) In **Section C** : Question numbers **22** to **28** are Short Answer type questions. Each question carries **3** marks.
- (vi) In **Section D** : Question numbers **29** & **30** are case study-based questions. Each question carries **4** marks.
- (vii) In **Section E** : Question numbers **31** to **33** are Long Answer type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is **not** allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg.}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg.}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg.}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. Two identical point charges are placed at the two vertices A and B of an equilateral triangle of side l . The magnitude of the electric field at the third vertex P is E . If a hollow conducting sphere of radius $(l/4)$ is placed at P, the magnitude of the electric field at point P now becomes [1 mark]

- (A) $> E$
- (B) E
- (C) $\frac{E}{2}$
- (D) zero

Correct Answer: (D) zero

Solution:

Concept:

- In electrostatic equilibrium a closed conducting shell shields its empty cavity from external electric fields.
- Free charges on the conductor redistribute until its surface is equipotential; with no charge inside the cavity, the electric field throughout the cavity is zero.

Step 1: Locate the charges relative to the shell The point P is enclosed by the hollow conducting sphere. The source charges at A and B are each a distance l from P , greater than the shell radius $l/4$, so both charges remain outside the sphere. There is no charge in its cavity.

Step 2: Apply electrostatic shielding The external charges induce a surface-charge distribution on the conductor. Its field cancels the original field inside the empty cavity. Thus the earlier value E does not persist at P . Neither an increased field, the unchanged field, nor half the field is possible there; options (A), (B) and (C) disregard shielding.

Step 3: Conclusion The electric field at P is 0 . Hence, the correct option is (D).

Quick Tip to Solve: First check whether the observation point and all source charges lie inside or outside the conducting shell; an empty closed cavity has zero electrostatic field.

2. A battery of e.m.f. 12 V and internal resistance $0.5\ \Omega$ is connected to a $9.5\ \Omega$ resistor through a key. The ratio of potential difference between the two terminals of the battery, when the key is open to that when the key is closed, is [1 mark]

- (A) 1.05
- (B) 1
- (C) 0.95
- (D) 1.1

Correct Answer: (A) 1.05

Solution:

Concept:

- The emf $\mathcal{E}$ is the terminal potential difference when no current flows through the battery.
- For a discharging cell, $V = \mathcal{E} - Ir$ and $I = \mathcal{E}/(R + r)$, where r is internal resistance and R is external resistance.

Step 1: Find the two terminal voltages With the key open, $I = 0$, so $V_{\text{open}} = 12 \text{ V}$. With the key closed, the total resistance is $9.5 + 0.5 = 10 \Omega$. Therefore

$$I = \frac{12}{10} = 1.2 \text{ A}, \quad V_{\text{closed}} = 12 - (1.2)(0.5) = 11.4 \text{ V}.$$

Step 2: Form the ratio in the requested order The question asks for open-key voltage divided by closed-key voltage:

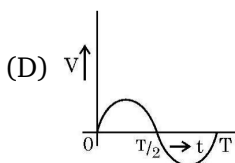
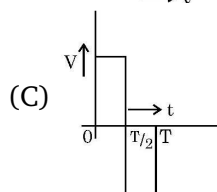
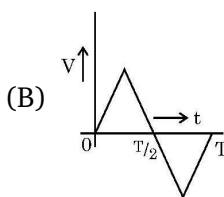
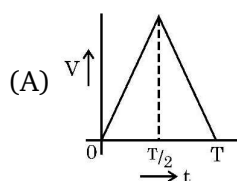
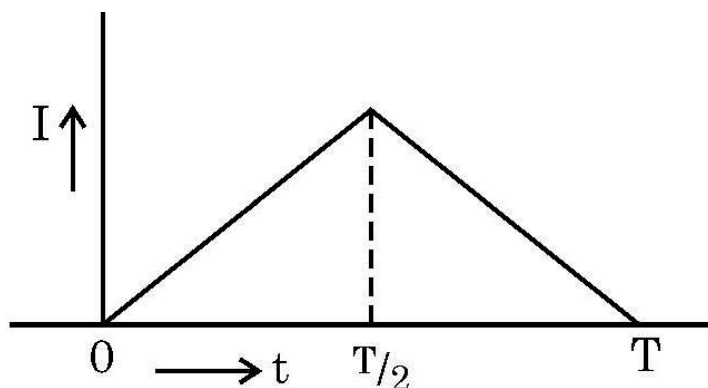
$$\frac{V_{\text{open}}}{V_{\text{closed}}} = \frac{12}{11.4} = 1.0526 \dots \simeq 1.05.$$

The value 0.95 reverses the ratio. A ratio of 1 neglects internal resistance, while 1.1 is inconsistent with the given resistance values.

Step 3: Conclusion The required ratio is 1.05. Hence, the correct option is (A).

Quick Tip to Solve: Cancel the emf before calculating: $V_{\text{open}}/V_{\text{closed}} = (R + r)/R$, which must exceed one for a discharging battery.

3. The alternating current I in an inductor is observed to vary with time t as shown in the graph for a cycle. Which one of the following graphs is the correct representation of wave form of voltage V with time t ? [1 mark]



Correct Answer: (C)

Solution:**Concept:**

- For an ideal inductor, terminal voltage is $V = L \, dI/dt$, where L is its inductance.
- A straight current-time segment gives constant voltage; its slope determines the sign.

Step 1: Read the two current slopes For peak current I_0 , the rising slope is $2I_0/T$ during $0 < t < T/2$. The falling slope is $-2I_0/T$ during $T/2 < t < T$.

Step 2: Match the resulting voltage waveform Multiplying the slopes by L gives

$$V = \begin{cases} 2LI_0/T, & 0 < t < T/2, \\ -2LI_0/T, & T/2 < t < T. \end{cases}$$

Only (C) has these constant levels; (A), (B) and (D) show varying voltage. Here V is terminal voltage; $-L \, dI/dt$ describes the opposing back emf.

Step 3: Conclusion The voltage is the rectangular waveform in (C). Hence, the correct option is (C).

Quick Tip to Solve: Replace each straight current-time segment by a constant voltage level: rising current gives positive voltage and falling current gives negative voltage.

4. A diamagnetic substance is brought, one by one, near the north pole and the south pole of a bar magnet. It is [1 mark]

- (A) repelled by north pole and attracted by south pole.
- (B) attracted by north pole and repelled by south pole.
- (C) attracted by north pole as well as by south pole.
- (D) repelled by north pole as well as by south pole.

Correct Answer: (D) repelled by north pole as well as by south pole.

Solution:**Concept:**

- A diamagnetic substance has negative magnetic susceptibility, $\chi_m < 0$, and develops magnetisation opposite to the applied magnetic field.
- In a non-uniform field, a diamagnetic substance tends to move from a stronger-field region to a weaker-field region.

Step 1: Consider the north pole Near a north pole the magnetic field is stronger close to the pole than farther away. The induced magnetic response opposes the applied field, so the diamagnetic sample tends to move away from the stronger-field region. It is therefore repelled by the north pole.

Step 2: Reverse the pole and compare the options Near the south pole the field direction reverses, and the induced magnetisation reverses with it. The tendency to move towards weaker field is unchanged, so the sample is also repelled by the south pole. Options (A) and (B) incorrectly make the force switch

from repulsion to attraction when the pole changes. Option (C) describes attraction to stronger fields, unlike diamagnetism.

Step 3: Conclusion The substance is repelled by both the north pole and the south pole. Hence, the correct option is (D).

Quick Tip to Solve: For a diamagnetic sample near either magnet pole, identify the weaker-field region: its motion is away from the pole, regardless of field direction.

5. Two long solenoids of radii r_1 and r_2 ($r_2 > r_1$) and number of turns per unit length n_1 and n_2 respectively are co-axially wrapped one over the other. The ratio of self-inductance of inner solenoid to their mutual inductance is [1 mark]

- (A) $\frac{n_1}{n_2}$
(B) $\frac{n_2}{n_1}$
(C) $\frac{n_1 r_1^2}{n_2 r_2^2}$
(D) $\frac{n_2 r_1^2}{n_1 r_2^2}$

Correct Answer: (A) $\frac{n_1}{n_2}$

Solution:

Concept:

- For a long solenoid of length ℓ , turn density n , and cross-sectional area A , the self-inductance is $L = \mu_0 n^2 A \ell$.
- For two co-axial long solenoids sharing length ℓ , the mutual inductance is $M = \mu_0 n_1 n_2 A_{\text{common}} \ell$; only the common flux area is used.

Step 1: Write the inductances using the inner area Since $r_2 > r_1$, the inner solenoid has area $A_1 = \pi r_1^2$. Its self-inductance is

$$L_1 = \mu_0 n_1^2 \pi r_1^2 \ell.$$

The magnetic field produced by a long inner solenoid is essentially confined to this area, so the flux linking the outer turns also uses πr_1^2 , giving

$$M = \mu_0 n_1 n_2 \pi r_1^2 \ell.$$

Step 2: Cancel the common factors Taking the requested ratio gives

$$\frac{L_1}{M} = \frac{\mu_0 n_1^2 \pi r_1^2 \ell}{\mu_0 n_1 n_2 \pi r_1^2 \ell} = \frac{n_1}{n_2}.$$

Option (B) is the reciprocal. Options (C) and (D) incorrectly retain a radius ratio: the same inner area

occurs in both inductances and cancels.

Step 3: Conclusion The ratio is $L_1/M = n_1/n_2$. Hence, the correct option is (A).

Quick Tip to Solve: For co-axial long solenoids, use the smaller cross-sectional area in mutual inductance; it cancels when compared with the inner solenoid self-inductance.

6. A 1 cm straight segment of a conductor carrying 1 A current in x direction lies symmetrically at origin of Cartesian coordinate system. The magnetic field due to this segment at point (1 m, 1 m, 0) is [1 mark]

- (A) $1.0 \times 10^{-9} \hat{k}$ T
(B) $-1.0 \times 10^{-9} \hat{k}$ T
(C) $\frac{5.0}{\sqrt{2}} \times 10^{-10} \hat{k}$ T
(D) $-\frac{5.0}{\sqrt{2}} \times 10^{-10} \hat{k}$ T

Correct Answer: (C) $\frac{5.0}{\sqrt{2}} \times 10^{-10} \hat{k}$ T

Solution:

Concept:

- For a short current element, the Biot-Savart law gives $\Delta \vec{B} = (\mu_0/4\pi)I(\Delta \vec{\ell} \times \vec{r})/r^3$, where $\vec{r}$ points from the element to the observation point.
- The direction follows the cross product, with $\hat{i} \times \hat{j} = \hat{k}$, and $\mu_0/(4\pi) = 10^{-7} \text{ T m A}^{-1}$.

Step 1: Convert the length and establish the geometry The segment is only 0.01 m long, much shorter than the distance to the observation point, so it is treated as a short current element centred at the origin. The vectors and distance are

$$\Delta \vec{\ell} = 0.01 \hat{i} \text{ m}, \quad \vec{r} = (\hat{i} + \hat{j}) \text{ m}, \quad r = \sqrt{2} \text{ m}.$$

Thus $\Delta \vec{\ell} \times \vec{r} = 0.01 \hat{k} \text{ m}^2$.

Step 2: Substitute in the vector law Using $I = 1 \text{ A}$,

$$\Delta \vec{B} \simeq 10^{-7} \frac{(1)(0.01)\hat{k}}{(\sqrt{2})^3} = \frac{5.0}{\sqrt{2}} \times 10^{-10} \hat{k} \text{ T}.$$

The direction is positive z , ruling out (B) and (D). Option (A) misses the geometric factor from the oblique observation direction and its distance $r = \sqrt{2} \text{ m}$.

Step 3: Conclusion The field is $\frac{5.0}{\sqrt{2}} \times 10^{-10} \hat{k} \text{ T}$. Hence, the correct option is (C).

Quick Tip to Solve: Convert centimetres to metres first, then compute $\Delta \vec{\ell} \times \vec{r}$; using r^3 with this vector form avoids losing the angle factor.

7. A coil of an ac generator, having 100 turns and area 0.1 m^2 each, rotates at half a rotation per second in a magnetic field of 0.02 T . The maximum emf generated in the coil is [1 mark]

- (A) 0.31 V
- (B) 0.20 V
- (C) 0.63 V
- (D) 0.10 V

Correct Answer: (C) 0.63 V

Solution:

Concept:

- For a coil rotating uniformly in a uniform magnetic field, the flux linkage is $N\Phi = NBA \cos \omega t$.
- Faraday's law gives peak emf $\mathcal{E}_0 = NBA\omega$, where N is the number of turns, A is each turn's area, B is field strength, and $\omega = 2\pi f$.

Step 1: Convert the rotation rate to angular speed Half a rotation per second means $f = 0.5 \text{ Hz}$. The angular speed must be expressed in radians per second:

$$\omega = 2\pi f = 2\pi(0.5) = \pi \text{ rad s}^{-1}.$$

The factor 2π converts revolutions into radians.

Step 2: Calculate the maximum emf Substitute $N = 100$, $A = 0.1 \text{ m}^2$, and $B = 0.02 \text{ T}$:

$$\mathcal{E}_0 = 100 \times 0.02 \times 0.1 \times \pi = 0.2\pi \text{ V} \simeq 0.63 \text{ V}.$$

Option (D), 0.10 V , follows from using f instead of ω . Option (B) omits the angular-speed factor, and option (A) is approximately half the peak value.

Step 3: Conclusion The maximum emf is 0.63 V . Hence, the correct option is (C).

Quick Tip to Solve: Write $\omega = 2\pi f$ before substituting in $NBA\omega$; half a revolution per second corresponds to π , not 0.5 , radians per second.

8. Atomic spectral emission lines of hydrogen atom are incident on a zinc surface. The lines which can emit photoelectrons from the surface are members of [1 mark]

- (A) Balmer series
- (B) Paschen series
- (C) Lyman series
- (D) Neither Balmer, nor Paschen nor Lyman series

Correct Answer: (C) Lyman series

Solution:

Concept:

- Photoelectric emission requires a single photon's energy $h\nu$ to be at least the metal's work function ϕ : $h\nu \geq \phi$.
- For hydrogen, a transition from n_i to n_f emits energy $13.6(1/n_f^2 - 1/n_i^2) \text{ eV}$. The Lyman, Balmer and Paschen series end at $n_f = 1, 2, 3$, respectively.

Step 1: Compare the photon energies with zinc's threshold Zinc requires ultraviolet radiation for photoemission; its work function is approximately 4.3 eV. The most energetic Balmer photon has energy approaching $13.6/4 = 3.4$ eV, and the Paschen limit is only $13.6/9 \approx 1.51$ eV. Thus neither of these series supplies enough energy, excluding (A) and (B).

Step 2: Test the Lyman series Even the least energetic Lyman line, from $n_i = 2$ to $n_f = 1$, has energy

$$E = 13.6 \left(1 - \frac{1}{4} \right) = 10.2 \text{ eV}.$$

This exceeds zinc's work function. The remaining Lyman lines have still higher photon energies, so the claim in (D) that none of the three series works is false.

Step 3: Conclusion The photoemitting lines belong to the Lyman series. Hence, the correct option is (C).

Quick Tip to Solve: Compare a series' limiting photon energy with the work function; increasing intensity cannot compensate for photons below threshold.

9. The focal length of a concave mirror in air is f . When the mirror is immersed in a liquid of refractive index $\frac{5}{3}$, its focal length will become [1 mark]

- (A) $\frac{5}{3}f$
- (B) $\frac{3}{5}f$
- (C) $\frac{2}{3}f$
- (D) f

Correct Answer: (D) f

Solution:

Concept:

- A spherical mirror's paraxial focal length is $f = R/2$, where R is its radius of curvature, with the appropriate sign convention.
- Reflection obeys $i = r$, where i and r are the angles with the normal. This law does not depend on the refractive index of the surrounding medium.

Step 1: Identify what sets the focus Rays parallel to the principal axis are brought to the focus by reflection at the concave surface. The mirror's curvature and the law of reflection determine their meeting point. Immersing the mirror changes the speed of light in its surroundings but does not change its radius of curvature or the reflected ray angles.

Step 2: Apply the mirror relation after immersion The radius R remains unchanged, so

$$f_{\text{liquid}} = \frac{R}{2} = f_{\text{air}} = f.$$

Options (A), (B) and (C) incorrectly multiply the focal length by a factor involving the liquid's refractive index. Such a refractive-index dependence is relevant to lenses, whose action is refraction, and not to this reflecting surface.

Step 3: Conclusion The focal length remains f . Hence, the correct option is (D).

Quick Tip to Solve: For a mirror immersed in a uniform medium, keep $f = R/2$; do not apply the refractive-index scaling used for lenses.

10. Which one of the following statements is correct? Electric field due to static charges is [1 mark]

- (A) conservative and field lines do not form closed loops.
- (B) conservative and field lines form closed loops.
- (C) non-conservative and field lines do not form closed loops.
- (D) non-conservative and field lines form closed loops.

Correct Answer: (A) conservative and field lines do not form closed loops.

Solution:

Concept:

- An electrostatic field is conservative: the work done around any closed path is zero, so $\oint \vec{E} \cdot d\vec{l} = 0$.
- Electric field lines point in the direction of $\vec{E}$, from higher to lower potential. They originate on positive charges and terminate on negative charges, or extend to infinity.

Step 1: Check the conservative property Static charges produce an electric field with a single-valued electrostatic potential V , related by $\vec{E} = -\vec{\nabla}V$. Work between two points therefore depends only on the endpoints. Options (C) and (D), both of which call this field non-conservative, are incorrect.

Step 2: Check whether a field line can close Along a field line followed in its arrow direction, potential decreases. A closed field line would return to its starting point after a net potential decrease, contradicting the single-valued potential. Equivalently, $\vec{E} \cdot d\vec{l}$ would be positive all around such a line, contrary to the zero closed-loop integral. Thus the closed-loop statement in (B) is also false.

Step 3: Conclusion The electrostatic field is conservative and its field lines do not form closed loops. Hence, the correct option is (A).

Quick Tip to Solve: Pair static electric fields with zero work around a closed path and field lines that never close on themselves.

11. When the resistance measured between p and n ends of a p-n junction diode is high, it can act as a/an [1 mark]

- (A) resistor
- (B) inductor
- (C) capacitor
- (D) switch

Correct Answer: (D) switch

Solution:

Concept:

- A p-n junction conducts readily in forward bias but offers a very high resistance in reverse bias, below breakdown.
- A switch is described by two limiting states: low resistance permits current (closed), while high resistance blocks current (open).

Step 1: Interpret the high resistance High resistance between the p and n ends means that only a negligible current flows for the applied voltage. This is the blocking state of the diode, normally associated with reverse bias. In the ideal circuit model, its current is zero and the conducting path is effectively interrupted.

Step 2: Match the state to a circuit function An open switch also interrupts a path by presenting a very large resistance, so the diode can act as a switch in its OFF state. A resistor is not the intended two-state device, and an inductor is identified by magnetic energy storage rather than a blocking resistance. A junction does possess capacitance, but charge storage is a separate property; the stated high-resistance measurement identifies its open-switch behaviour.

Step 3: Conclusion The diode acts as an open switch. Hence, the correct option is (D).

Quick Tip to Solve: Translate a diode's high resistance into OFF or open switch, and its low forward resistance into ON or closed switch.

12. The energy of an electron in a hydrogen atom in ground state is -13.6 eV . Its energy in an orbit corresponding to quantum number n is -0.544 eV . The value of n is [1 mark]

- (A) 2
- (B) 3
- (C) 4
- (D) 5

Correct Answer: (D) 5

Solution:

Concept:

- In the Bohr model of hydrogen, the energy of the n th stationary orbit is $E_n = E_1/n^2$, where $E_1 = -13.6\text{ eV}$ and n is a positive integer.
- Negative energy denotes a bound electron. As n increases, the energy approaches zero from below; its magnitude decreases as $1/n^2$.

Step 1: Substitute the given orbital energy Equating the Bohr expression to the given value gives

$$-0.544 = \frac{-13.6}{n^2}, \quad n^2 = \frac{13.6}{0.544} = 25.$$

Both energies are in electron volts, so their ratio is dimensionless. The two negative signs cancel; using opposite signs would incorrectly make n^2 negative.

Step 2: Select the allowed quantum number A principal quantum number must be positive, so the root to test is $\sqrt{25}$. The alternatives $n = 2, 3, 4$ give energies -3.4 eV, approximately -1.51 eV, and -0.85 eV, respectively, none of which equals the specified energy. Substituting $n = 5$ reproduces $-13.6/25 = -0.544$ eV.

Step 3: Conclusion The required quantum number is $n = 5$. Hence, the correct option is (D).

Quick Tip to Solve: Divide the magnitude of the ground-state energy by the given energy magnitude, then take the positive square root to obtain n .

13. Assertion (A): Out of Infrared and radio waves, the radio waves show more diffraction effect.

Reason (R): Radio waves have greater frequency than infrared waves.

[1 mark]

- (A) If both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) If both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(C) If Assertion (A) is true but Reason (R) is false.
(D) If both Assertion (A) and Reason (R) are false.

Correct Answer: (C) If Assertion (A) is true but Reason (R) is false.

Solution:

Concept:

- Diffraction is the spreading of a wave around an obstacle or through an aperture. For a fixed aperture size a , a longer wavelength λ gives stronger spreading; the characteristic angular scale is proportional to λ/a .
- Electromagnetic waves in vacuum obey $c = \nu\lambda$, where c is their speed and ν their frequency. A longer wavelength corresponds to a lower frequency.

Step 1: Evaluate the assertion Radio waves have much longer wavelengths than infrared waves. For the same obstacle or aperture, radio waves therefore bend and spread more noticeably. The assertion that radio waves show more diffraction is true.

Step 2: Evaluate the reason Since $\nu = c/\lambda$, the longer wavelength of radio waves means their frequency is smaller than that of infrared waves. The reason states the opposite frequency ordering and is therefore false.

Step 3: Test the explanatory link The increased diffraction follows from radio waves' greater wavelength, not a greater frequency. A false reason cannot explain the true assertion. Options (A)

and (B) require both statements to be true, while (D) requires the assertion also to be false; each is excluded by the separate checks.

Step 4: Conclusion The assertion is true but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: For the same obstacle, choose the longer wavelength for stronger diffraction, then use $\nu\lambda = c$ to check the frequency ordering.

14. Assertion (A): In an ideal step-down transformer, the electrical energy is not lost.

Reason (R): In a step-down transformer, voltage decreases but the current increases. [1 mark]

- (A) If both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) If both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(C) If Assertion (A) is true but Reason (R) is false.
(D) If both Assertion (A) and Reason (R) are false.

Correct Answer: (B) If both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

Solution:

Concept:

- An ideal transformer has no resistive, magnetic or flux-leakage losses. Thus its output power equals its input power and its efficiency is unity.
- A step-down transformer reduces the voltage and increases the current. For an ideal transformer, $V_s/V_p = N_s/N_p$ and $I_s/I_p = N_p/N_s$, where p, s denote primary and secondary and N is the number of turns.

Step 1: Evaluate the assertion The word ideal excludes copper heating, eddy-current, hysteresis and leakage losses. Hence all the supplied electrical energy is transferred to the secondary circuit, and the assertion is true. The absence of losses is an assumption of the ideal-transformer model.

Step 2: Evaluate the reason A step-down transformer has fewer turns on its secondary, so its output voltage is lower and its output current is greater. This familiar voltage-current behaviour makes the reason true in the stated context.

Step 3: Test whether the reason establishes losslessness The printed reason states only the directions of change; it does not state that the voltage and current change by exactly reciprocal factors. A lower voltage and a larger current can still give an output power smaller than the input power in a lossy transformer. Exact power equality follows from ideality, and cannot be deduced from those qualitative trends alone. Therefore the reason does not explain the assertion. Option (A) incorrectly asserts this explanatory link; (C) and (D) incorrectly classify a true statement as false.

Step 4: Conclusion Both statements are true, but the reason is not the correct explanation of the assertion. Hence, the correct option is (B).

Quick Tip to Solve: For transformer Assertion-Reason questions, distinguish qualitative voltage-current changes from the exact power equality that requires ideality.

15. Assertion (A): In Bohr model of hydrogen atom, the angular momentum of an electron in n^{th} orbit is proportional to the square root of its orbit radius r_n .

Reason (R): According to Bohr model, electron can jump to its nearest orbits only.

[1 mark]

- (A) If both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) If both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(C) If Assertion (A) is true but Reason (R) is false.
(D) If both Assertion (A) and Reason (R) are false.

Correct Answer: (C) If Assertion (A) is true but Reason (R) is false.

Solution:

Concept:

- In the Bohr model, the electron's orbital angular momentum is $L_n = m_e v_n r_n = n\hbar$, where n is the principal quantum number and $\hbar = h/(2\pi)$.
- For hydrogen, the radius is $r_n = a_0 n^2$, where a_0 is the radius of the first orbit. An allowed transition exchanges energy $h\nu = |E_f - E_i|$.

Step 1: Test the assertion Eliminate n between the radius and angular-momentum formulae:

$$n = \sqrt{\frac{r_n}{a_0}}, \quad L_n = \frac{\hbar}{\sqrt{a_0}} \sqrt{r_n}.$$

Since a_0 is constant for hydrogen, L_n is proportional to $\sqrt{r_n}$. Assertion (A) is true.

Step 2: Test the reason An electron is not restricted to transitions between neighbouring Bohr orbits. For example, a direct transition from $n = 3$ to $n = 1$ emits a photon whose energy equals that difference. Hence Reason (R), which permits only the nearest orbits, is false.

Step 3: Assess the explanation and codes The radius dependence follows from the two Bohr formulae, not from an adjacency restriction on transitions. Codes (A) and (B) require a true reason, while code (D) requires a false assertion; each is inconsistent with the statements just tested.

Step 4: Conclusion The assertion is true but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Combine $L_n \propto n$ with $r_n \propto n^2$; separately test the word “only” in any proposed transition restriction.

16. Assertion (A): In a semiconductor diode the thickness of depletion layer is not fixed.

Reason (R): Thickness of depletion layer in a semiconductor device depends upon many factors such as biasing of the semiconductor. [1 mark]

- (A) If both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) If both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
(C) If Assertion (A) is true but Reason (R) is false.
(D) If both Assertion (A) and Reason (R) are false.

Correct Answer: (A) If both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Concept:

- A p-n junction has a depletion region containing immobile ionised donors and acceptors, with very few mobile charge carriers.
- An external bias changes the junction's potential barrier and therefore the width of its depletion region: forward bias narrows it, while reverse bias widens it.

Step 1: Test the assertion The depletion-layer thickness is not a rigid geometrical property of a diode. Its boundaries shift as mobile carriers enter or leave the junction region. In particular, changing the applied bias changes the region's thickness. Thus Assertion (A) is true.

Step 2: Test the reason Forward bias lowers the potential barrier and pushes majority carriers towards the junction, reducing the depleted region. Reverse bias raises the barrier and pulls majority carriers away, widening that region. Doping also affects the width. Therefore the dependence on factors such as biasing stated in Reason (R) is true.

Step 3: Connect the reason to the assertion Dependence on a variable bias directly explains why the thickness is not fixed. Hence the code for two true statements without an explanatory relation is unsuitable. The remaining codes also fail because they require the reason, or both statements, to be false.

Step 4: Conclusion Both statements are true, and the reason correctly explains the assertion. Hence, the correct option is (A).

Quick Tip to Solve: Remember the depletion-width test: forward bias narrows the region, reverse bias widens it; this immediately establishes its variable thickness.

Section B (10 marks)

17. The threshold voltage of a silicon diode is 0.7 V. It is operated at this point by connecting the diode in series with a battery of V volt and a resistor of $1000\ \Omega$. Find the value of V when the current drawn is 15 mA. [2 marks]

Solution:

Concept:

- At the stated operating point, the forward-biased silicon diode has a voltage drop $V_D = 0.7\text{ V}$.
- The resistor and diode carry the same series current I . Ohm's law gives the resistor drop $V_R = IR$,

and Kirchhoff's loop rule gives the battery voltage $V = V_R + V_D$.

Step 1: Calculate the voltage across the resistor Convert the current to amperes before using the resistance in ohms:

$$I = 15 \text{ mA} = 15 \times 10^{-3} \text{ A}, \quad R = 1000 \Omega.$$

The potential difference needed across the resistor is therefore

$$V_R = IR = (15 \times 10^{-3})(1000) = 15 \text{ V}.$$

This is only the resistor's share of the battery voltage; the diode also has a nonzero forward drop.

Step 2: Apply the voltage balance around the series loop With the battery polarity chosen to forward-bias the diode, the battery rise balances both component drops:

$$V - IR - V_D = 0, \quad V = 15 + 0.7 = 15.7 \text{ V}.$$

As a check, the resulting resistor current is $(15.7 - 0.7)/1000 = 0.015 \text{ A}$, equal to the prescribed value. The threshold voltage must thus be added to, rather than subtracted from, the resistor drop when finding the supply voltage.

Step 3: Conclusion The required battery voltage is $V = 15.7 \text{ V}$.

Quick Tip to Solve: For a series diode and resistor, first find IR , then add the stated forward diode drop: here $15 \text{ mA} \times 1 \text{ k}\Omega = 15 \text{ V}$.

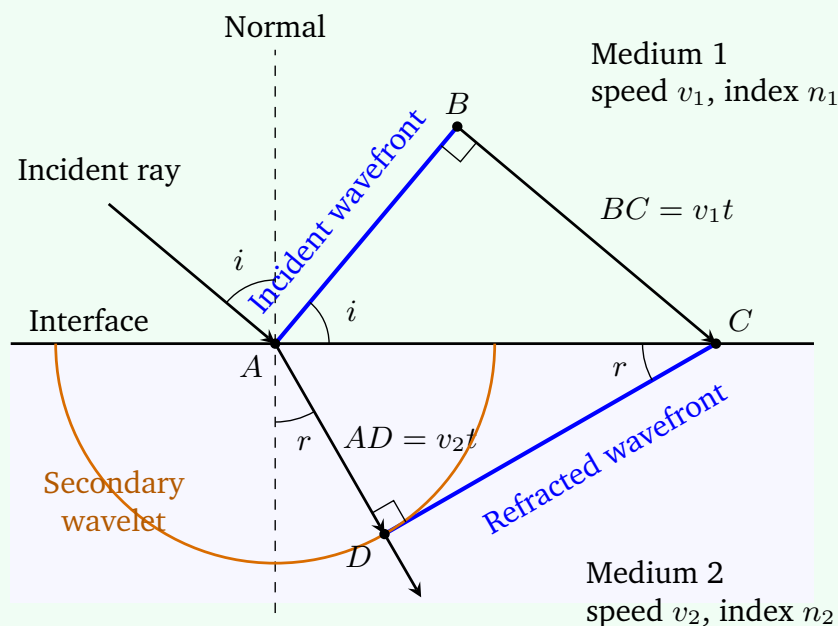
18. Show the refraction of light wave at a plane interface using Huygens' principle and prove Snell's law. [2 marks]

Solution:

Concept:

- Each point on a wavefront emits secondary wavelets; their common forward tangent gives the new wavefront. Rays are perpendicular to wavefronts.
- Let v_1, v_2 be the speeds in media of refractive indices n_1, n_2 , respectively. Since $n = c/v$, their speed ratio is $v_1/v_2 = n_2/n_1$.

Step 1: Construct the refracted wavefront Let the incident wavefront AB reach the interface first at A . In time t , point B reaches C , so $BC = v_1 t$. Meanwhile, the wavelet from A travels into medium 2 with radius $AD = v_2 t$. Its tangent CD is the refracted wavefront; the perpendicular AD is a refracted ray. The sketch shows $v_2 < v_1$.



Huygens' construction at a plane interface; the example shows $v_2 < v_1$.

Step 2: Use the two right triangles to prove Snell's law Since rays are normal to wavefronts, the angles between the interface and AB , CD equal i , r , respectively. Triangles ABC and ADC are right-angled at B and D . Hence

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}, \quad \sin r = \frac{AD}{AC} = \frac{v_2 t}{AC}.$$

Dividing cancels the common time and interface segment:

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{n_2}{n_1}.$$

Step 3: Conclusion Huygens' construction gives the refracted wavefront CD and establishes Snell's law, $n_1 \sin i = n_2 \sin r$. Light entering the slower medium bends towards the normal.

Quick Tip to Solve: Use the same time t for $BC = v_1 t$ and $AD = v_2 t$; both right triangles share AC , so division immediately produces Snell's law.

19. Two convex lenses A and B, each of focal length 10.0 cm, are mounted on an optical bench at 50.0 cm and 70.0 cm respectively. An object is mounted at 20.0 cm. Find the nature and position of the final image formed by the combination. [2 marks]

Solution:

Concept:

- For each thin lens, the Cartesian sign convention gives $1/f = 1/v - 1/u$. Distances u and v are measured from that lens, positive along the incident light; both convex lenses have $f = +10.0$ cm.
- The image formed by the first lens serves as the object for the second. Transverse magnification is $m = v/u$, and the total magnification of the combination is $m_A m_B$.

Step 1: Locate the image made by lens A Light travels from the object at the 20.0 cm mark towards

lens A at the 50.0 cm mark. Its object distance is

$$u_A = 20.0 - 50.0 = -30.0 \text{ cm.}$$

Therefore

$$\frac{1}{v_A} = \frac{1}{10.0} + \frac{1}{-30.0} = \frac{1}{15.0}, \quad v_A = +15.0 \text{ cm.}$$

This real intermediate image is at the $50.0 + 15.0 = 65.0$ cm bench mark, to the left of lens B.

Step 2: Use that image as the object for lens B Lens B is at 70.0 cm, so its object distance must be recalculated from B:

$$u_B = 65.0 - 70.0 = -5.0 \text{ cm.}$$

Applying the lens formula again,

$$\frac{1}{v_B} = \frac{1}{10.0} + \frac{1}{-5.0} = -\frac{1}{10.0}, \quad v_B = -10.0 \text{ cm.}$$

The negative image distance means that the rays emerging from B appear to diverge from a point 10.0 cm to its left. Its bench coordinate is $70.0 - 10.0 = 60.0$ cm.

Step 3: Check the image orientation and size The individual and total magnifications are

$$m_A = \frac{15.0}{-30.0} = -\frac{1}{2}, \quad m_B = \frac{-10.0}{-5.0} = 2, \quad m = m_A m_B = -1.$$

Thus lens B's virtual image is erect relative to its intermediate object, but inverted relative to the original object. Its size equals the original object's size.

Step 4: Conclusion The final image is **virtual, inverted relative to the original object, and of the same size**, at the 60.0 cm bench mark, between A and B and 10.0 cm to the left of B.

Quick Tip to Solve: Convert the first image distance to a bench coordinate before finding u_B . Here the intermediate image at 65.0 cm lies only 5.0 cm before B, inside its focal length.

20. Radiations of two frequencies are incident on a metal surface of work function 2.0 eV one by one. The energies of their photons are 2.5 eV and 4.5 eV respectively. Find the ratio of the maximum speed of the electrons emitted in the two cases. [2 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $K_{\max} = E_\gamma - \phi$, where E_γ is the photon energy and ϕ is the metal's work function.
- For an emitted electron of mass m_e , $K_{\max} = \frac{1}{2}m_e v_{\max}^2$. Therefore maximum speed is proportional to the square root of the excess photon energy.

Step 1: Subtract the work function in each case Both photon energies exceed the work function 2.0 eV, so both radiations eject electrons. The energy needed to escape from the metal must be subtracted before comparing their kinetic energies:

$$K_1 = 2.5 - 2.0 = 0.5 \text{ eV,}$$

$$K_2 = 4.5 - 2.0 = 2.5 \text{ eV.}$$

Step 2: Compare the maximum speeds The electron mass is the same in both cases. Dividing the kinetic-energy equations eliminates this mass and the factor $1/2$:

$$\frac{v_1}{v_2} = \sqrt{\frac{K_1}{K_2}} = \sqrt{\frac{0.5}{2.5}} = \frac{1}{\sqrt{5}}.$$

Conversion from electronvolts to joules is unnecessary for this ratio because the same conversion factor would occur in both the numerator and denominator.

Step 3: Conclusion In the order of photon energies 2.5 eV and 4.5 eV, the ratio of maximum electron speeds is $v_1 : v_2 = 1 : \sqrt{5}$.

Quick Tip to Solve: Subtract the common work function first, then take the square root of the kinetic-energy ratio; the photon-energy ratio alone is incorrect.

21(a). Two wires of the same material and the same radius have their lengths in the ratio 2 : 3. They are connected in parallel to a battery which supplies a current of 15 A. Find the current through the wires. [2 marks]

Solution:

Concept:

- A uniform wire has resistance $R = \rho l/A$, where ρ is resistivity, l is length and A is cross-sectional area.
- Parallel branches have the same potential difference, so their currents are inversely proportional to their resistances. The total supplied current equals the sum of the branch currents.

Step 1: Relate the two resistances Label the shorter wire 1 and the longer wire 2. Their common material gives the same resistivity, and their equal radii give the same cross-sectional area. Thus resistance is proportional to length:

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} = \frac{2}{3}.$$

We can consequently write $R_1 = 2k$ and $R_2 = 3k$, where k is a common positive resistance factor.

Step 2: Divide the total current between the branches By Ohm's law, the common branch voltage gives $I_1 R_1 = I_2 R_2$. Therefore

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{3}{2}, \quad I_1 + I_2 = 15 \text{ A}.$$

The five equal current shares each represent $15/5 = 3$ A, giving

$$I_1 = \frac{3}{5}(15) = 9 \text{ A}, \quad I_2 = \frac{2}{5}(15) = 6 \text{ A}.$$

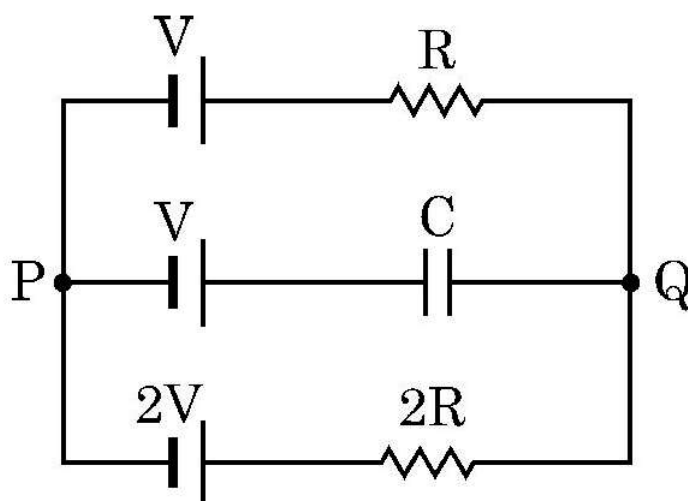
The larger current correctly flows through the shorter wire because that branch has the smaller resistance.

Step 3: Conclusion The wire of relative length 2 carries 9 A , and the wire of relative length 3 carries 6 A .

Quick Tip to Solve: For equal material and radius in parallel, invert the length ratio to obtain the current ratio before dividing the total current.

OR

21(b). In the circuit three ideal cells of e.m.f. V , V and $2V$ are connected to a resistor of resistance R , a capacitor of capacitance C and another resistor of resistance $2R$ as shown in figure. In the steady state find (i) the potential difference between P and Q and (ii) potential difference across capacitor C . [2 marks]



Solution:

Concept:

- In a steady direct-current state an ideal capacitor carries no current. Thus the middle branch is open for current, but its cell still establishes a potential rise.
- Kirchhoff's loop rule gives zero net potential change around a closed loop. On crossing a cell from its short plate to its long plate, the potential rises by its emf.

Step 1: Find the current in the outer loop In the printed circuit, the positive plate of each cell is on the right. The lower $2V$ cell drives current I from P towards Q through $2R$, returning from Q towards P through the upper R . The two outer emfs oppose each other around this loop, while the resistances add:

$$2V - V = I(2R + R), \quad I = \frac{V}{3R}.$$

Step 2: Determine the potential difference between P and Q Following the lower branch from P to Q gives a rise $2V$ across the cell and a drop $2IR$ across the resistor. Denoting node potentials by U_P and U_Q ,

$$U_Q - U_P = 2V - 2IR = 2V - \frac{2V}{3} = \frac{4V}{3}.$$

Thus Q is at the higher potential. As a check, following the upper branch from P to Q gives $V + IR = 4V/3$, since that traversal is opposite to its current.

Step 3: Find the voltage across the capacitor Let X be the capacitor's left plate, directly after the

middle cell. This cell raises its potential to $U_X = U_P + V$. The capacitor's right plate is connected to Q, hence

$$U_Q - U_X = (U_Q - U_P) - V = \frac{4V}{3} - V = \frac{V}{3}.$$

No resistor drop occurs in the middle branch; capacitance controls the stored charge, not this steady-state voltage.

Step 4: Conclusion (i) The potential difference is $\boxed{4V/3}$, with Q higher than P; explicitly $U_P - U_Q = -4V/3$. (ii) The capacitor voltage is $\boxed{V/3}$, with its Q-side plate positive.

Quick Tip to Solve: Open only the capacitor branch for the current calculation; retain its cell's voltage when comparing the two capacitor-plate potentials.

Section C (21 marks)

22. (a) Define resistivity of a conductor. Discuss its dependence on temperature of the conductor and draw a plot of resistivity of copper as a function of temperature.

(b) (i) "A low voltage battery from which high current is required must have low internal resistance." Justify.

(ii) "A high voltage battery must have a large internal resistance." Justify.

[3 marks]

Solution:

Concept:

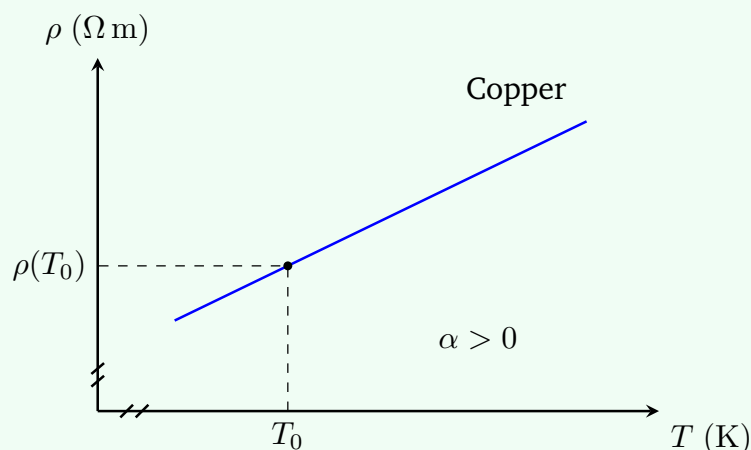
- Resistivity ρ characterises the material at a specified temperature: $\rho = RA/l$, where R , A and l are the conductor's resistance, cross-sectional area and length.
- Over a moderate temperature interval, a metal obeys $\rho(T) = \rho(T_0)[1 + \alpha(T - T_0)]$, where α is its temperature coefficient of resistivity.
- A battery of emf $\mathcal{E}$ and internal resistance r , connected to load resistance R_L , delivers $I = \mathcal{E}/(R_L + r)$. Its internal voltage drop is Ir .

Step 1: Define resistivity and describe its temperature dependence (a) Resistivity is the resistance of a conductor of unit length and unit cross-sectional area. Its SI unit is $\Omega \text{ m}$. It depends on the material and temperature, but not on the dimensions of a uniform specimen.

For copper and ordinary metals, increasing temperature increases lattice vibrations and electron scattering, reducing the mean time between collisions. Hence their resistivity increases, with $\alpha > 0$. Around ordinary temperatures the graph is approximately a straight line of positive slope:

$$\rho(T) = \rho(T_0) + \alpha\rho(T_0)(T - T_0).$$

This linear relation is an approximation over a limited interval; it should not be extrapolated to absolute zero. Semiconductor resistivity generally decreases with increasing temperature because additional charge carriers are produced.



Copper: approximate increase of resistivity over a moderate temperature interval.

Step 2: Explain the low-voltage battery requirement (b)(i) From $I = \mathcal{E}/(R_L + r)$, obtaining a large current with a small emf requires small total circuit resistance. The load resistance must be small and the battery's internal resistance must also be low. Otherwise the internal drop Ir consumes a substantial fraction of its already small emf and limits the useful current. Low r also reduces the internal heating I^2r at the required current.

Step 3: Explain the high-voltage battery requirement (b)(ii) For the high-voltage, small-current application intended in this statement, a large internal resistance limits the current if a low-resistance path is connected. In particular, the short-circuit current is

$$I_{\text{sc}} = \frac{\mathcal{E}}{r}.$$

A large r keeps this current within a tolerable value, reducing the risk of damage from an excessive current. Thus the stated requirement is a current-limiting design consideration: high emf by itself does not universally require large internal resistance. A high-voltage source designed to supply large useful currents instead requires low internal resistance and separate protection.

Step 4: Conclusion Resistivity is $\rho = RA/l$, measured in $\Omega \text{ m}$; copper's resistivity increases approximately linearly with temperature near ordinary temperatures. Low-voltage, high-current batteries need low internal resistance, while large internal resistance in the intended high-voltage, small-current battery limits excessive current.

Quick Tip to Solve: Use $I = \mathcal{E}/(R_L + r)$ for both battery statements: small r permits high current; large r limits current for a high emf.

23. (a) When a parallel beam of light enters water surface obliquely at some angle, what is the effect on the width of the beam?
- (b) With the help of a ray diagram, show that a straw appears bent when it is partly dipped in water and explain it.
- (c) Explain the transmission of optical signal through an optical fibre by a diagram. [3 marks]

Solution:

Concept:

- Snell's law is $n_1 \sin i = n_2 \sin r$, where i and r are measured from the normal. Light entering water from air bends toward the normal.
- Light emerging from water into air bends away from the normal. The backward extensions of the refracted rays locate a raised virtual image of an underwater point.
- Total internal reflection occurs from a denser to a rarer medium when the incidence angle exceeds the critical angle i_c , with $\sin i_c = n_2/n_1$ for $n_1 > n_2$.

Step 1: Compare the beam widths in part (a) Measure each beam width perpendicular to its own direction of propagation. Let the two bounding rays meet the water surface at points separated by distance d . Their widths in air and water are respectively

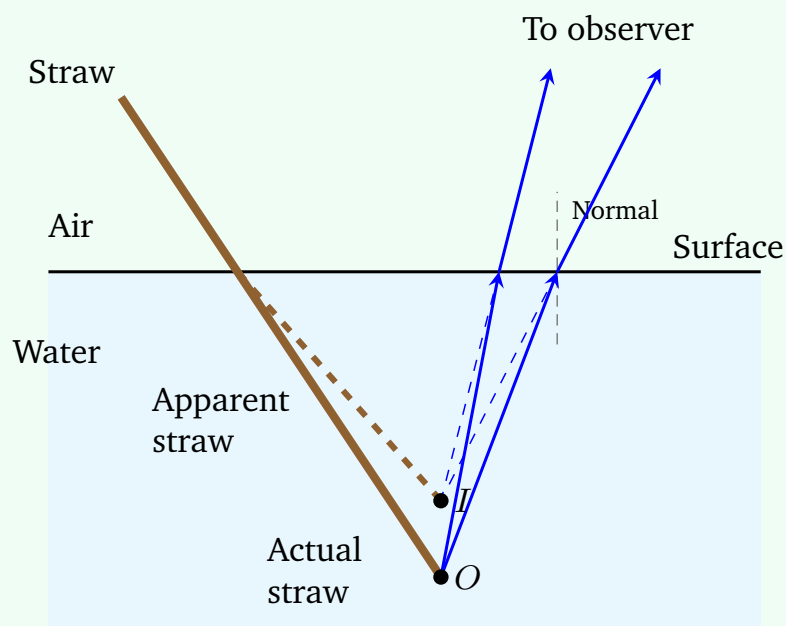
$$w_{\text{air}} = d \cos i, \quad w_{\text{water}} = d \cos r.$$

Since water has the greater refractive index, Snell's law gives $r < i$ for oblique incidence. Hence $\cos r > \cos i$, and

$$\frac{w_{\text{water}}}{w_{\text{air}}} = \frac{\cos r}{\cos i} > 1.$$

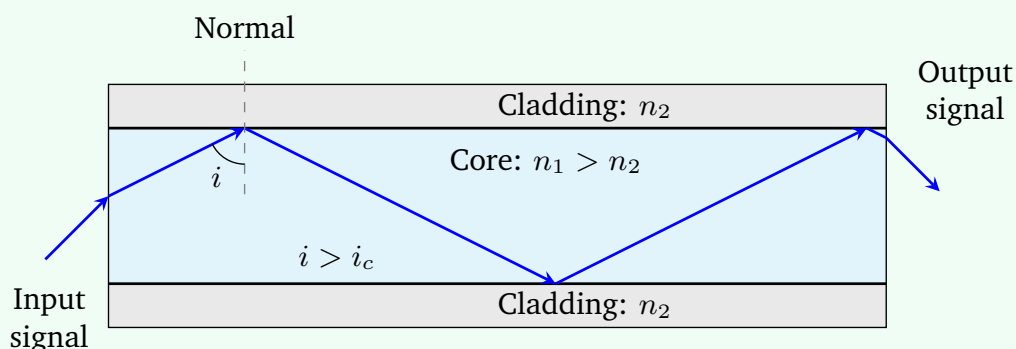
The parallel beam therefore becomes wider on entering water; its rays remain mutually parallel at a plane surface.

Step 2: Locate the apparent submerged end in part (b) In the ray diagram, O is the submerged end of the straight straw. Rays from O travel through water and refract away from the normal on entering air. An observer above the water traces the emergent rays backward as straight lines; their dashed extensions meet at a virtual point I , above O . The underwater portion therefore appears raised, whereas the portion in air is seen at its actual position. The two portions appear to meet at a bend at the water surface. For viewing close to the normal, the apparent depth is approximately the real depth divided by the refractive index of water relative to air.



Refraction raises the apparent tip from O to I ; the dashed rays are backward extensions (schematic).

Step 3: Trace an optical signal through the fibre in part (c) An optical fibre has a transparent core of refractive index n_1 , surrounded by cladding of lower refractive index n_2 . A light signal entering within the fibre's acceptance range reaches the core–cladding boundary with incidence angle $i > i_c$, measured from the normal. It is then totally internally reflected. Repeated total internal reflections guide the signal along the core, as the zigzag ray shows, allowing it to emerge from the distant end with little loss. The cladding maintains the required refractive-index difference and helps confine the light.



Repeated total internal reflection confines the light to the core, with $n_1 > n_2$ and $i > i_c$.

Step 4: Conclusion (a) The beam width increases. (b) Refraction raises the apparent underwater portion, so the straw looks bent at the surface. (c) Repeated total internal reflection at the denser-core/rarer-cladding boundary transmits the optical signal.

Quick Tip to Solve: Compare beam widths perpendicular to the rays using $d \cos \theta$; for the straw extend only the emergent rays backward, and for a fibre measure incidence from the boundary normal.

24. Differentiate between the peak value and root mean square value of an alternating current. Derive the expression for the root mean square value of alternating current, in terms of its peak value. [3 marks]

Solution:

Concept:

- For a sinusoidal alternating current, $i(t) = I_0 \sin \omega t$, where I_0 is its peak value and $\omega = 2\pi/T$ is the angular frequency for period T .
- The root mean square current is $I_{\text{rms}} = \sqrt{\langle i^2 \rangle}$. It equals the steady direct current that produces the same heat in the same resistor over the same time.

Step 1: Distinguish the peak and effective values The peak current I_0 is the maximum magnitude of the instantaneous current during a cycle; the positive and negative extrema are $+I_0$ and $-I_0$. The rms current measures the heating effect over a complete cycle, rather than the instantaneous maximum. Although the arithmetic mean of a sinusoidal current over a full cycle is zero, its rms value is nonzero because the heating rate $i^2 R$ is positive for both directions of current.

Step 2: Average the heat produced over one period Let the alternating current pass through a resistor of constant resistance R . The heat generated in one period is

$$H_{\text{ac}} = \int_0^T i^2 R dt = I_0^2 R \int_0^T \sin^2 \omega t dt.$$

Using $\sin^2 \omega t = (1 - \cos 2\omega t)/2$ and $\omega T = 2\pi$,

$$\begin{aligned} \int_0^T \sin^2 \omega t dt &= \left[\frac{t}{2} - \frac{\sin 2\omega t}{4\omega} \right]_0^T \\ &= \frac{T}{2}. \end{aligned}$$

Thus $H_{\text{ac}} = I_0^2 R T / 2$; the oscillating cosine term contributes zero over a complete cycle.

Step 3: Equate this heat to that of a steady current By definition, a steady current I_{rms} must produce the same heat in resistance R during time T . Therefore,

$$I_{\text{rms}}^2 R T = \frac{I_0^2 R T}{2} \implies I_{\text{rms}}^2 = \frac{I_0^2}{2}.$$

The rms value is a nonnegative magnitude, so we choose the positive square root. This factor applies to sinusoidal alternating current; other waveforms require their own mean-square calculation.

Step 4: Conclusion The peak value is the maximum instantaneous magnitude, whereas the rms value is the equivalent dc heating value. For sinusoidal current,

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \simeq 0.707 I_0.$$

Quick Tip to Solve: For rms current, square first, average over one cycle, and then take the square root; averaging the current itself gives zero.

25. (a) How is an electromagnetic wave produced?

(b) An electromagnetic wave is travelling in vertically upward direction. At an instant, its electric field vector points in west direction. In which direction does the magnetic field vector point at that instant?

(c) Estimate the ratio of shortest wave length of radio waves to the longest wave length of gamma waves. [3 marks]

Solution:

Concept:

- An accelerated charge emits electromagnetic radiation. The emitted wave contains time-varying electric and magnetic fields, each perpendicular to the direction of propagation and to the other field.
- The direction of energy propagation is along $\vec{E} \times \vec{B}$. Approximate wavelength limits in the electromagnetic spectrum allow an order-of-magnitude estimate of wavelength ratios.

Step 1: Explain the production of the wave in part (a) An electromagnetic wave is produced by an accelerating electric charge, for example a charge oscillating in a transmitting antenna. Its changing electric field is associated with a changing magnetic field, and the changing magnetic field is associated with a changing electric field. These coupled disturbances propagate away from the source as electromagnetic radiation and carry energy. A material medium is not necessary for their propagation; in vacuum their speed is c .

Step 2: Apply the cross-product rule in part (b) Choose east, north and vertically upward as the positive x , y and z directions, respectively. The given electric field points west, so $\vec{E} = -E\hat{x}$. For propagation upward, the vector $\vec{E} \times \vec{B}$ must point along $+\hat{z}$. Taking the magnetic field toward the south gives

$$(-E\hat{x}) \times (-B\hat{y}) = EB\hat{z}.$$

This satisfies the required propagation direction. A northward magnetic field would instead make the cross product point downward, so its sign would be incorrect.

Step 3: Use the approximate spectrum limits in part (c) In the usual school-level classification, radio waves extend down to wavelengths of order 10^{-1} m, while gamma rays extend up to wavelengths of order 10^{-12} m. Using these approximate boundary values,

$$\frac{\lambda_{\text{radio, shortest}}}{\lambda_{\text{gamma, longest}}} \simeq \frac{10^{-1} \text{ m}}{10^{-12} \text{ m}} = 10^{11}.$$

The metres cancel, so the ratio is dimensionless. Spectrum boundaries are approximate, as is appropriate for the requested estimate.

Step 4: Conclusion (a) Accelerating charges produce electromagnetic waves. (b) The magnetic field points **south**. (c) The required wavelength ratio is approximately 10^{11} .

Quick Tip to Solve: Use east, north and up as a right-handed set: west crossed with south points up. For the wavelength ratio, subtract the powers of ten.

26(a). In a region of a uniform electric field $\vec{E}$, a negatively charged particle is moving with a constant velocity $\vec{v} = -v_o\hat{i}$ near a long straight conductor coinciding with XX' axis and carrying current I towards $-X$ axis. The particle remains at a distance d from the conductor.

(i) Draw diagram showing direction of electric and magnetic fields.

(ii) What are the various forces acting on the charged particle?

(iii) Find the value of v_o in terms of E , d and I .

[3 marks]

Solution:

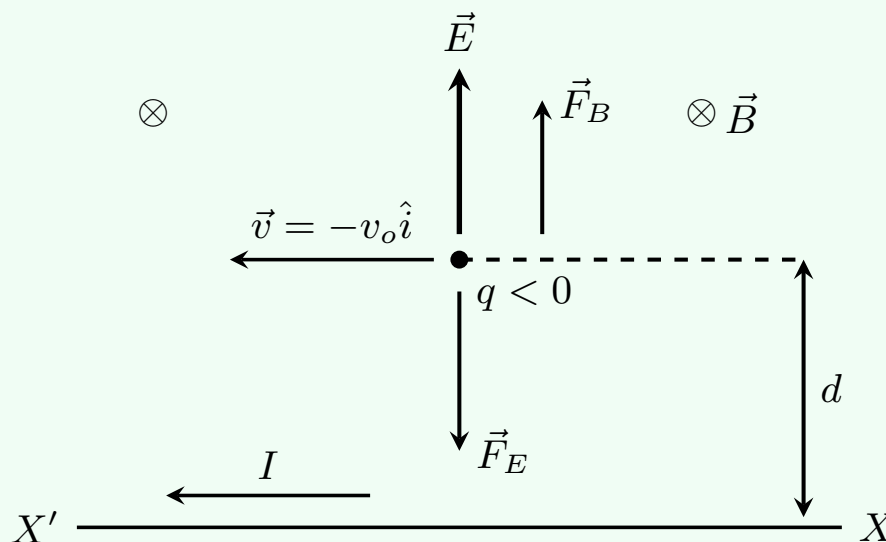
Concept:

- At perpendicular distance d from a long straight wire carrying current I , the magnetic field has magnitude $B = \mu_0 I / (2\pi d)$. Its direction follows the right-hand thumb rule.
- A particle of charge q in electric and magnetic fields experiences $\vec{F}_E = q\vec{E}$ and $\vec{F}_B = q(\vec{v} \times \vec{B})$. For a negative charge, each force is opposite to the corresponding field or cross-product direction.
- Constant velocity requires zero resultant force: $q(\vec{E} + \vec{v} \times \vec{B}) = 0$. Here the velocity is perpendicular to the magnetic field, so the magnetic-force magnitude is $|q|v_oB$.

Step 1: (i) Choose the particle's position and find the field directions Take the particle to be above the wire, at $y = d$, with $+\hat{k}$ directed out of the page. Both the current and the particle velocity point along $-\hat{i}$. The right-hand thumb rule gives $\vec{B} = -B\hat{k}$, into the page at the particle. Hence

$$\vec{v} \times \vec{B} = (-v_o\hat{i}) \times (-B\hat{k}) = -v_oB\hat{j}.$$

Since $q < 0$, the magnetic force points upward, away from the wire. The balancing electric force must point downward, towards the wire. The electric field therefore points upward, $\vec{E} = E\hat{j}$, because the charge is negative. The sketch shows these directions for a particle above the wire; the field and force directions reverse vertically if the particle is placed below it.



Field and force directions for the negative particle at $y = d$; crosses denote magnetic field into the page.

Step 2: (ii) State the two opposing electromagnetic forces Write $q = -|q|$. The electric force attracts the particle towards the wire, while the magnetic force pushes it away from the wire in the chosen

geometry:

$$\vec{F}_E = -|q|E\hat{j},$$
$$\vec{F}_B = +|q|v_o \frac{\mu_0 I}{2\pi d} \hat{j}.$$

These forces are equal and opposite because the particle has no acceleration. Their balance keeps the particle at the fixed separation d while it travels parallel to the wire.

Step 3: (iii) Apply the condition for constant velocity Equate the magnitudes of the electric and magnetic forces and substitute the field of the wire:

$$|q|E = |q|v_o B,$$
$$E = v_o \frac{\mu_0 I}{2\pi d},$$
$$v_o = \frac{2\pi d E}{\mu_0 I}.$$

The charge magnitude cancels, so the required speed depends only on the fields, or equivalently on E , I and d . The ratio E/B has SI units of speed.

Step 4: Conclusion For a particle above the wire, $\vec{B}$ is into the page and $\vec{E}$ is upward. The magnetic force is away from the wire and the electric force is towards it. Their balance gives $v_o = 2\pi d E / (\mu_0 I)$, with $\vec{v} = -v_o \hat{i}$.

Quick Tip to Solve: First determine $\vec{v} \times \vec{B}$, then reverse it for the negative charge; use $E = v_o B$ only after confirming that the forces oppose each other.

OR

26(b). Two infinitely long conductors kept along XX' and YY' axes are carrying current I_1 and I_2 along $-X$ axis and $-Y$ axis respectively. Find the magnitude and direction of the net magnetic field produced at point $P(X, Y)$. [3 marks]

Solution:

Concept:

- An infinitely long straight conductor carrying current I produces magnetic field $B = \mu_0 I / (2\pi r)$ at perpendicular distance r . The right-hand thumb rule gives its direction.
- Magnetic fields obey vector superposition: $\vec{B}_{\text{net}} = \vec{B}_1 + \vec{B}_2$. Both fields here are perpendicular to the XY plane, so their signed components can be added along $\hat{k}$, the outward normal to that plane.

Step 1: Find the field due to the conductor along the X-axis First take $P(X, Y)$ in the first quadrant, so $X > 0$ and $Y > 0$. Its perpendicular distance from the conductor along XX' is Y . The current I_1 points along $-\hat{i}$. Curling the fingers of the right hand about that direction shows that the field at a point above the X-axis points into the page, along $-\hat{k}$. Therefore

$$\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi Y} \hat{k}.$$

Step 2: Find the field due to the conductor along the Y-axis The perpendicular distance from the conductor along YY' to P is X . The current I_2 points along $-\hat{j}$. At a point to the right of this conductor,

the right-hand thumb rule gives a field out of the page, along $+\hat{k}$, since $(-\hat{j}) \times \hat{i} = +\hat{k}$. Thus

$$\vec{B}_2 = +\frac{\mu_0 I_2}{2\pi X} \hat{k}.$$

The two contributions oppose each other in the first quadrant; adding their magnitudes would therefore give the wrong net field there.

Step 3: Superpose the fields and identify the direction Adding the signed field components gives

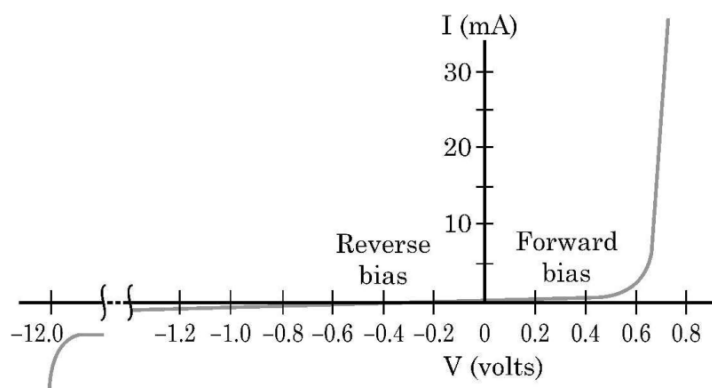
$$\begin{aligned}\vec{B}_{\text{net}} &= \frac{\mu_0}{2\pi} \left(\frac{I_2}{X} - \frac{I_1}{Y} \right) \hat{k}, \\ B_{\text{net}} &= \frac{\mu_0}{2\pi} \left| \frac{I_2}{X} - \frac{I_1}{Y} \right|.\end{aligned}$$

A positive coefficient of $\hat{k}$ means out of the page, while a negative coefficient means into the page. If the coefficient vanishes, the resultant field is zero and has no direction. Because the question gives coordinates without restricting the quadrant, the same vector formula also applies for signed nonzero X, Y : the sign of each coordinate automatically reverses the relevant field across its wire. A point on either ideal line conductor is excluded from this expression.

Step 4: Conclusion $B_{\text{net}} = \frac{\mu_0}{2\pi} \left| \frac{I_2}{X} - \frac{I_1}{Y} \right|$. The field is out of the XY plane when $I_2/X - I_1/Y > 0$, into it when this quantity is negative, and zero when it is zero.

Quick Tip to Solve: Pair each wire with the other coordinate for its distance: the X -axis wire uses Y , and the Y -axis wire uses X . Determine each direction before adding.

27. (a) What are majority and minority charge carriers in an extrinsic semiconductor?
- (b) A p-n junction is forward biased. Describe the movement of the charge carriers which produce current in it.
- (c) The graph shows the variation of current with voltage for a p-n junction diode. Estimate the dynamic resistance of diode at $V = -0.6$ volt. [3 marks]



Solution:

Concept:

- An extrinsic semiconductor is doped so that one type of mobile carrier has a much higher concentration than the other. The more numerous carriers are majority carriers; the less numerous carriers are minority carriers.
- Forward bias reduces the potential barrier and narrows the depletion region of a p-n junction. Majority carriers can then diffuse across the junction and produce an appreciable current.
- The dynamic or small-signal resistance at an operating point is $r_d = dV/dI$, estimated as $\Delta V/\Delta I$ along the local tangent to the I -versus- V curve. It is the reciprocal of the local slope, rather than the ratio V/I .

Step 1: (a) Identify the majority and minority charge carriers In an n-type semiconductor, donor impurities supply extra electrons: electrons are the majority carriers and holes are the minority carriers. In a p-type semiconductor, acceptor impurities create holes: holes are the majority carriers and electrons are the minority carriers. Minority carriers are still present because thermal generation creates electron-hole pairs; doping changes the relative concentrations, rather than removing one carrier type completely.

Step 2: (b) Describe the charge motion under forward bias Forward bias connects the p-side to the positive terminal and the n-side to the negative terminal of the source. The applied voltage opposes the built-in junction field, lowering the barrier. Majority holes from the p-region move towards and across the junction into the n-region, while majority electrons from the n-region move towards and across it into the p-region. After crossing, these injected carriers recombine with majority carriers in the opposite region; the source continuously supplies carriers to maintain the current. Hole motion from p to n and electron motion from n to p both give conventional current across the junction from p to n.

Step 3: (c) Read the local slope at the stated negative voltage The operating point $V = -0.6$ V lies on the reverse-bias part of the characteristic, well before breakdown. The plotted reverse current is approximately constant in the neighbourhood of this point: the tangent to the graph is essentially

horizontal. A small change in reverse voltage therefore causes negligible change in current:

$$\frac{\Delta I}{\Delta V} \approx 0, \quad r_d = \frac{\Delta V}{\Delta I} \rightarrow \infty.$$

Thus the graph supports an effectively infinite, or very large, dynamic resistance at this point. It does not resolve a finite numerical value; using the steep forward-bias branch near +0.6 V would answer a different operating-point question.

Step 4: Conclusion (a) Electrons and holes are the majority and minority carriers, respectively, in n-type material; their roles reverse in p-type material. (b) Forward current arises from holes crossing p to n and electrons crossing n to p. (c) At -0.6 V , $r_d \approx \infty$ in the idealized graph, meaning a very large resistance.

Quick Tip to Solve: Keep the minus sign in -0.6 V : use the reverse-bias tangent. A nearly horizontal I -versus- V curve means a very large dynamic resistance.

28. (a) Show the variation of binding energy per nucleon with mass number. Write the significance of the binding energy curve.

(b) Two nuclei with lower binding energy per nucleon form a nuclei with more binding energy per nucleon.

(i) What type of nuclear reaction is it?

(ii) Whether the total mass of nuclei increases, decreases or remains unchanged?

(iii) Does the process require energy or produce energy?

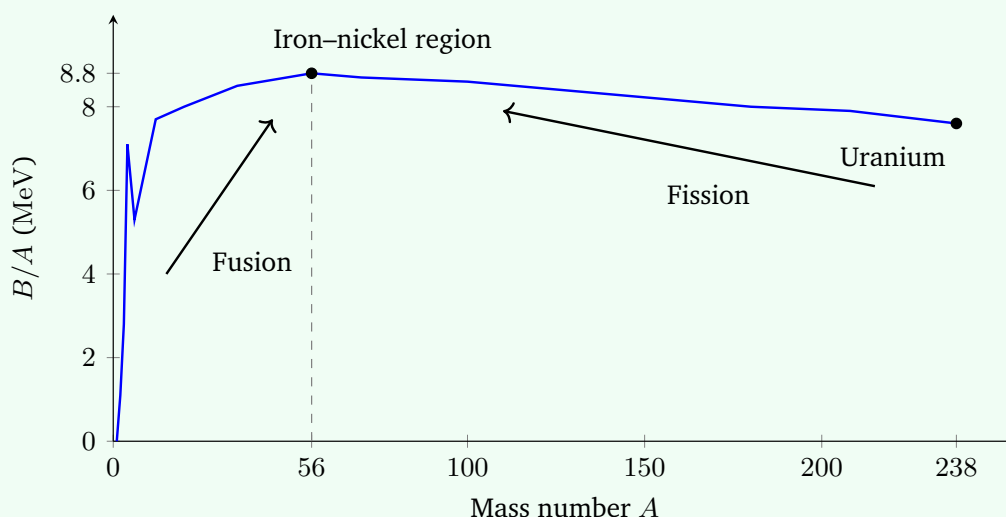
[3 marks]

Solution:

Concept:

- Binding energy B is the energy needed to separate a nucleus completely into its constituent nucleons. For mass number A , the binding energy per nucleon is B/A ; a larger value generally indicates greater nuclear stability.
- Nuclear binding energy comes from the mass defect: $B = \Delta mc^2$, where Δm is the difference between the sum of free-nucleon masses and the bound nucleus's mass, and c is the speed of light.

Step 1: (a) Draw and interpret the binding-energy curve The curve rises rapidly, with fluctuations for light nuclei, and reaches a broad maximum of about 8.8 MeV per nucleon near $A \simeq 56$, in the iron–nickel region. It then falls slowly towards about 7.6 MeV for very heavy nuclei such as uranium. The sketch shows the overall trend; the light-nucleus fluctuations are schematic. Nuclei near the maximum are especially stable. Energy is released when a nuclear reaction moves its products towards a larger binding energy per nucleon: fusion can do this for light nuclei, while fission can do this for very heavy nuclei. The nearly flat middle region also reflects the short-range, saturating nature of nuclear forces.



Schematic binding energy per nucleon versus mass number.

Step 2: (b)(i) Identify the reaction Two lighter nuclei combine to form a more tightly bound nucleus. Combining nuclei in this way is nuclear fusion. Here the product has a larger binding energy per nucleon than the initial nuclei, so its total binding energy is larger, since the total number of nucleons is conserved.

Step 3: (b)(ii)–(iii) Relate the mass change to energy release Let M_i be the sum of the initial nuclear rest masses, and M_f the final nuclear rest mass. Because more energy is bound in the product, its mass defect is larger, giving

$$Q = B_f - (B_1 + B_2) = (M_i - M_f)c^2 > 0.$$

Thus $M_f < M_i$: the total nuclear rest mass decreases, and the difference appears as released energy. This is a net energy-producing process, even though initial energy is generally needed to overcome the electrostatic repulsion between the nuclei. Total mass-energy remains conserved.

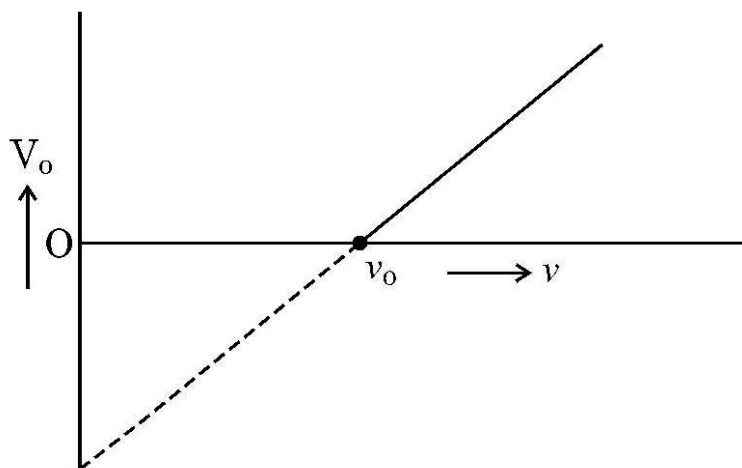
Step 4: Conclusion The binding-energy curve peaks near the iron–nickel region, showing the greater stability of intermediate-mass nuclei and explaining energy release in light-nucleus fusion and heavy-nucleus fission. In (b), the reaction is fusion: the total nuclear rest mass decreases and energy is produced.

Quick Tip to Solve: On a binding-energy-per-nucleon graph, moving upwards means more binding: the nuclear rest mass falls and the released energy equals the increase in total binding energy.

Section D (8 marks)

Question numbers 29 and 30 are case study based questions. Read the following paragraphs and answer the questions that follow.

29. When a photon of suitable frequency is incident on a metal surface, photoelectron is emitted from it. If the frequency is below a threshold frequency ν_0 for the surface, no photoelectron is emitted. For a photon of frequency ν ($\nu > \nu_0$), the kinetic energy of the emitted photoelectrons is $h(\nu - \nu_0)$. The photocurrent can be stopped by applying a potential V_0 called 'stopping potential' on the anode. Thus maximum kinetic energy of photoelectrons $K_m = eV_0 = h(\nu - \nu_0)$. The experimental graph between V_0 and ν for a metal is shown in figure. This is a straight line of slope m . [4 marks]



29(i). The straight line graphs obtained for two metals

[1 mark]

- (A) coincide each other.
- (B) are parallel to each other.
- (C) are not parallel to each other and cross at a point on ν -axis.
- (D) are not parallel to each other and do not cross at a point on ν -axis.

Correct Answer: (B) are parallel to each other.

Solution:

Concept:

- Einstein's equation is $eV_0 = h\nu - \phi$, where V_0 is stopping potential, ν incident frequency, ϕ work function, h Planck's constant and e electron-charge magnitude.
- A straight line $y = mx + b$ has slope m . Equal slopes give parallel lines; the intercept depends on the constant term b .

Step 1: Put the equation in straight-line form Divide the photoelectric equation by e :

$$V_0 = \frac{h}{e}\nu - \frac{\phi}{e}.$$

Therefore the slope of the stopping-potential versus frequency graph is h/e . Both h and e are universal constants, independent of the metal.

Step 2: Compare the two metals Different work functions change the intercept $-\phi/e$ and threshold frequency ϕ/h , but not the slope. The graphs are parallel. Coincidence additionally requires equal work functions, which is not generally true for different metals. Options (C) and (D) incorrectly

require unequal slopes.

Step 3: Conclusion The straight-line graphs for the two metals have the same slope h/e and are parallel to each other. Hence, the correct option is (B).

Quick Tip to Solve: For a stopping-potential versus frequency graph, read the slope as h/e ; changing the metal changes the threshold, not this slope.

29(ii). The value of Planck's constant for this metal is

[1 mark]

- (A) $\frac{e}{m}$
- (B) $\frac{1}{me}$
- (C) me
- (D) $\frac{m}{e}$

Correct Answer: (C) me

Solution:

Concept:

- The photoelectric equation is $eV_0 = h(\nu - \nu_0)$, where e is the magnitude of the electron charge, V_0 the stopping potential, h Planck's constant and ν_0 the threshold frequency.
- In a graph of V_0 against ν , the slope $m = \Delta V_0 / \Delta \nu$ is the coefficient of ν in the straight-line equation.

Step 1: Identify the slope Rewrite Einstein's equation with the vertical-axis quantity isolated:

$$V_0 = \frac{h}{e}\nu - \frac{h\nu_0}{e}.$$

Comparing with a straight line of slope m gives $m = h/e$. The metal changes ν_0 , and hence the intercept, but not the slope.

Step 2: Rearrange and check the options Multiplying the slope relation by e gives $h = me$. As a dimensional check, m has units Vs, so me has units CVs = Js, appropriate for Planck's constant. The expressions e/m , $1/(me)$ and m/e all contradict $m = h/e$; the last also fails to convert voltage to energy.

Step 3: Conclusion Planck's constant is $h = me$. Hence, the correct option is (C).

Quick Tip to Solve: First check which quantity is on the vertical axis: for V_0 versus ν , multiply the slope by e to obtain h .

29(iii). The intercepts on ν -axis and V_0 -axis of the graph are respectively:

[1 mark]

- (A) $\nu_0, \frac{h\nu_0}{e}$
(B) $\nu_0, h\nu_0$
(C) $\frac{h\nu_0}{e}, \nu_0$
(D) $h\nu_0, \nu_0$

Correct Answer: (A) $\nu_0, \frac{h\nu_0}{e}$

Solution:

Concept:

- The graph obeys $V_0 = (h/e)\nu - h\nu_0/e$, with ν_0 the threshold frequency, h Planck's constant and e electron-charge magnitude.
- At a horizontal-axis intercept the vertical coordinate is zero, and vice versa. An intercept's length is positive even when its coordinate is negative.

Step 1: Find the frequency-axis intercept Set $V_0 = 0$. Einstein's equation then gives $0 = h(\nu - \nu_0)$, so the line meets the frequency axis at $\nu = \nu_0$. This is the minimum frequency needed for photoemission.

Step 2: Find the vertical intercept and its magnitude Extrapolating the line to $\nu = 0$ gives

$$V_0 = -\frac{h\nu_0}{e}.$$

The graph therefore cuts the negative vertical axis. The printed choices use its magnitude, $h\nu_0/e$, making (A) the intended pair of intercept lengths. If signed coordinates were requested, no option would give the negative sign. Option (B) gives energy rather than voltage; (C) and (D) also interchange the frequency and voltage quantities.

Step 3: Conclusion The intercept magnitudes are ν_0 and $h\nu_0/e$; the signed vertical intercept is $-h\nu_0/e$. Hence, the correct option is (A), interpreting the printed intercepts as lengths.

Quick Tip to Solve: Set one axis variable to zero at a time; preserve the negative sign before comparing an intercept coordinate with its magnitude.

29(iii). When the wavelength of a photon is doubled, how many times its wave number and frequency become, respectively? [1 mark]

- (A) $2, \frac{1}{2}$
 (B) $\frac{1}{2}, \frac{1}{2}$
 (C) $\frac{1}{2}, 2$
 (D) $2, 2$

Correct Answer: (B) $\frac{1}{2}, \frac{1}{2}$

Solution:

Concept:

- Spectroscopic wave number is $\bar{\nu} = 1/\lambda$, where λ is wavelength. If angular wave number $k = 2\pi/\lambda$ is used instead, it has the same inverse dependence on wavelength.
- For a photon travelling in vacuum, $\nu = c/\lambda$, where ν is frequency and c the speed of light.

Step 1: Compare the wave numbers Write the new wavelength as $\lambda' = 2\lambda$. Then $\bar{\nu}' = 1/(2\lambda) = \bar{\nu}/2$. Thus the wave-number ratio is $\bar{\nu}'/\bar{\nu} = 1/2$; it is unchanged by choosing the angular-wave-number convention.

Step 2: Compare the frequencies and test the choices The speed of light stays constant, so $\nu' = c/(2\lambda) = \nu/2$. Both quantities are halved. Option (A) incorrectly doubles the wave number, (C) incorrectly doubles the frequency, and (D) incorrectly doubles both: each conflicts with the inverse dependence on wavelength.

Step 3: Conclusion The wave number and frequency become $1/2$ and $1/2$ times their original values, respectively. Hence, the correct option is (B).

Quick Tip to Solve: At fixed wave speed, doubling wavelength halves both frequency and wave number because each contains $1/\lambda$.

29(iv). The momentum of a photon is 5.0×10^{-29} kg m/s. Ignoring relativistic effects (if any), the wavelength of the photon is [1 mark]

- (A) $1.33 \mu\text{m}$
 (B) $3.3 \mu\text{m}$
 (C) $16.6 \mu\text{m}$
 (D) $13.3 \mu\text{m}$

Correct Answer: (D) $13.3 \mu\text{m}$

Solution:

Concept:

- A photon obeys $p = h/\lambda$, where p is momentum, h Planck's constant and λ wavelength. Thus $\lambda = h/p$; no electron-mass or nonrelativistic kinetic-energy formula is needed.
- Use $h = 6.63 \times 10^{-34}$ J s and $1 \mu\text{m} = 10^{-6}$ m to express the wavelength in the units of the choices.

Step 1: Substitute the photon momentum With $p = 5.0 \times 10^{-29} \text{ kg m/s}$,

$$\lambda = \frac{6.63 \times 10^{-34}}{5.0 \times 10^{-29}} = 1.326 \times 10^{-5} \text{ m.}$$

The powers subtract to give $-34 - (-29) = -5$; dividing Js by momentum leaves metres.

Step 2: Convert to micrometres and compare Since $10^{-5} \text{ m} = 10 \mu\text{m}$, the result is $13.26 \mu\text{m}$, matching the listed $13.3 \mu\text{m}$. Option (A), $1.33 \mu\text{m}$, loses a factor of ten in this conversion. The values in (B) and (C) also do not equal h/p .

Step 3: Conclusion The photon wavelength is approximately $13.3 \mu\text{m}$, as listed. Hence, the correct option is (D).

Quick Tip to Solve: Keep the wavelength in metres until the last line; $1.326 \times 10^{-5} \text{ m}$ is $13.26 \mu\text{m}$, not $1.326 \mu\text{m}$.

30. A parallel plate capacitor has two parallel plates which are separated by an insulating medium like air, mica, etc. When the plates are connected to the terminals of a battery, they get equal and opposite charges and an electric field is set up in between them. This electric field between the two plates depends upon the potential difference applied, the separation of the plates and nature of the medium between the plates. [4 marks]

30(i). The electric field between the plates of a parallel plate capacitor is E . Now the separation between the plates is doubled and simultaneously the applied potential difference between the plates is reduced to half of its initial value. The new value of the electric field between the plates will be: [1 mark]

- (A) E
- (B) $2E$
- (C) $\frac{E}{4}$
- (D) $\frac{E}{2}$

Correct Answer: (C) $\frac{E}{4}$

Solution:

Concept:

- The approximately uniform field between parallel capacitor plates has magnitude $E = V/d$, where V is their potential difference and d their separation; edge effects are neglected.
- A ratio comparison gives $E'/E = (V'/V)/(d'/d)$, so changes in the voltage and separation must both be included.

Step 1: Substitute both changes The new separation is $d' = 2d$, while the new potential difference is $V' = V/2$. Since the voltage is explicitly changed, the charge on the plates need not remain constant.

$$E' = \frac{V'}{d'} = \frac{V/2}{2d} = \frac{V}{4d}.$$

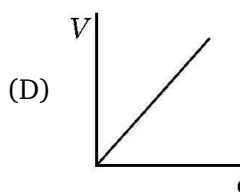
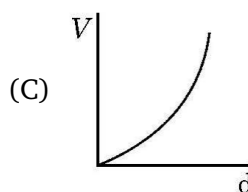
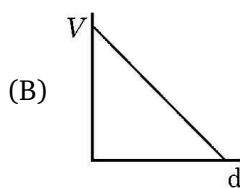
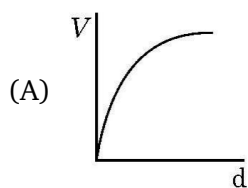
Step 2: Compare with the original field Each change reduces the field by a factor of two. Together they reduce it by a factor of four. Option (D) accounts for only one change; option (A) would require the voltage and separation to change by the same factor, and option (B) incorrectly predicts an

increase.

Step 3: Conclusion The new field is $E' = E/4$. Hence, the correct option is (C).

Quick Tip to Solve: For simultaneous capacitor changes, form $E'/E = (V'/V)(d/d')$ before substituting either factor.

30(ii). A constant electric field is to be maintained between the two plates of a capacitor whose separation d changes with time. Which of the graphs correctly depict the potential difference (V) to be applied between the plates as a function of separation between the plates (d) to maintain the constant electric field? [1 mark]



Correct Answer: (D)

Solution:

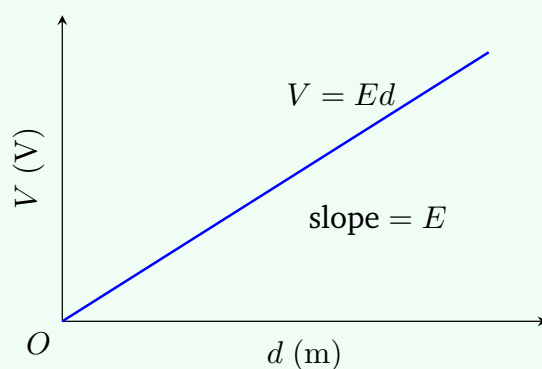
Concept:

- For parallel plates, the electric field magnitude is $E = V/d$, with potential difference V across plate separation d .
- Holding E constant requires $V = Ed$: the graph of V against d is a straight line through the origin with constant positive slope E .

Step 1: Express voltage in terms of separation Rearrange the field relation with E fixed:

$$V(d) = Ed, \quad \frac{V}{d} = E = \text{constant}.$$

Thus, doubling the separation requires doubling the applied voltage. The voltage must increase in direct proportion to separation, regardless of how fast the separation changes with time.



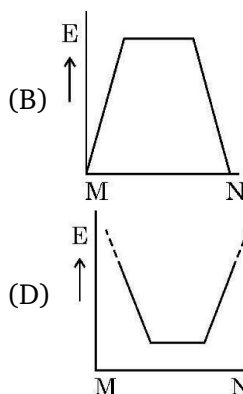
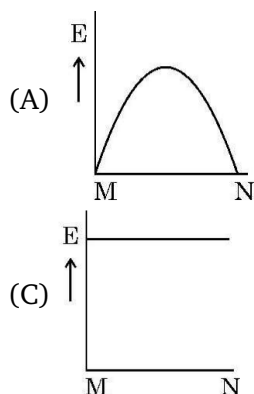
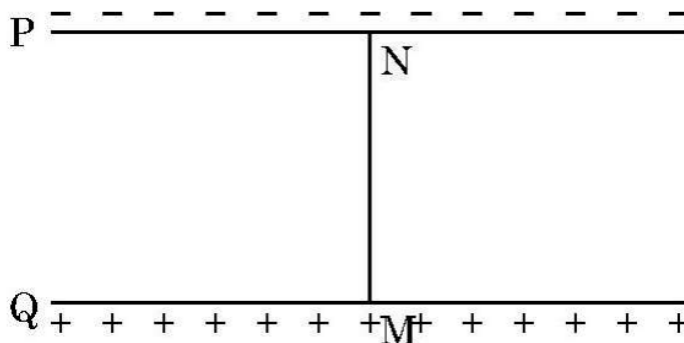
Applied potential difference for a fixed electric field

Step 2: Identify the straight line through the origin Graph (D) has the required constant positive slope and zero intercept. Graphs (A) and (C) are curved, so their ratio V/d changes with separation. Graph (B) decreases as separation increases and also has a nonzero voltage intercept, contrary to $V = Ed$.

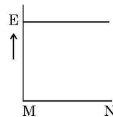
Step 3: Conclusion The required graph is the rising straight line through the origin, with slope E . Hence, the correct option is (D).

Quick Tip to Solve: For a constant capacitor field, remember $V = Ed$: voltage tracks separation proportionally, giving a line through the origin.

30(iii). In the above figure P, Q are the two parallel plates of a capacitor. Plate Q is at positive potential with respect to plate P. MN is an imaginary line drawn perpendicular to the plates. Which of the graphs shows correctly the variations of the magnitude of electric field strength E along the line MN? [1 mark]



Correct Answer: (C)



Solution:

Concept:

- In the ideal parallel-plate approximation, equal and opposite surface charge densities produce a uniform electric field throughout the space between the plates, away from their edges.
- The field points from the positive plate to the negative plate. Its magnitude between plates is $E = \sigma/\epsilon$, where σ is the free surface charge density and ϵ is the permittivity of the medium.

Step 1: Locate the line segment in the capacitor gap The diagram places M on the positive plate Q and N on plate P . The segment MN lies between the plates, perpendicular to them; it does not extend outside the capacitor. The field is directed from Q towards P , along M to N .

Step 2: Compare the field magnitude at different positions As a point moves along the gap from M to N , the magnitude remains constant. The required plot is therefore horizontal and nonzero, as in graph (C). Graphs (A), (B) and (D) all show a magnitude changing with position and thus do not represent the ideal field along this segment. The graph concerns the gap, not the zero field inside the conducting material.

Step 3: Conclusion The field magnitude is uniform along MN , so the horizontal graph is correct. Hence, the correct option is (C).

Quick Tip to Solve: First check whether the plotted path lies inside the plate gap or extends outside it; MN here lies wholly in the uniform-field gap.

30(iv). Three parallel plates are placed above each other with equal displacement $\vec{d}$ between neighbouring plates. The electric field between the first pair of the plates is $\vec{E}_1$ and the electric field between the second pair of the plates is $\vec{E}_2$. The potential difference between the third and the first plate is – [1 mark]

- (A) $(\vec{E}_1 + \vec{E}_2) \cdot \vec{d}$
- (B) $(\vec{E}_1 - \vec{E}_2) \cdot \vec{d}$
- (C) $(\vec{E}_2 - \vec{E}_1) \cdot \vec{d}$
- (D) $\frac{d(E_1 + E_2)}{2}$

Correct Answer: (A) $(\vec{E}_1 + \vec{E}_2) \cdot \vec{d}$

Solution:

Concept:

- Electrostatic potential changes according to $V_b - V_a = - \int_a^b \vec{E} \cdot d\vec{r}$. The potential drop in the direction of a displacement is $\vec{E} \cdot \vec{d}$ for a uniform field.
- Potential differences add across successive gaps. Contributions from both fields must be added algebraically; the dot products already account for their directions.

Step 1: State the displacement and voltage convention Let the potentials of the successive plates be V_1, V_2, V_3 , and take $\vec{d}$ from plate 1 to plate 2 and from plate 2 to plate 3. The potential drops across

the two gaps are

$$V_1 - V_2 = \vec{E}_1 \cdot \vec{d}, \quad V_2 - V_3 = \vec{E}_2 \cdot \vec{d}.$$

Step 2: Add the two contributions and check the sign Adding cancels the intermediate potential:

$$V_1 - V_3 = (\vec{E}_1 + \vec{E}_2) \cdot \vec{d}.$$

This potential-drop convention gives option (A). Options (B) and (C) incorrectly subtract one gap contribution, while (D) introduces an unjustified factor of one-half. The printed word “between” does not specify a subtraction order: if the required signed quantity is instead $V_3 - V_1$ for the same $\vec{d}$, the answer is the negative of option (A). Reversing the displacement reverses the sign as well.

Step 3: Conclusion The intended potential difference, taken as the total drop, is $(\vec{E}_1 + \vec{E}_2) \cdot \vec{d}$. Hence, the correct option is (A), with the potential-drop convention stated above.

Quick Tip to Solve: Write each gap’s signed potential change before adding; reversing the voltage order or the displacement changes the overall sign.

OR

30(iv). A material of dielectric constant K is filled in a parallel plate capacitor of capacitance C . The new value of its capacitance becomes [1 mark]

- (A) C
- (B) $\frac{C}{K}$
- (C) CK
- (D) $C \left(1 + \frac{1}{K}\right)$

Correct Answer: (C) CK

Solution:

Concept:

- For an air-filled parallel-plate capacitor, $C = \epsilon_0 A/d$, where A is the overlapping plate area and d is their separation; air is treated as vacuum to the usual approximation.
- A dielectric of relative permittivity K has absolute permittivity $\epsilon = K\epsilon_0$. Completely filling the gap changes the permittivity while leaving the plate geometry unchanged.

Step 1: Replace the permittivity in the capacitance formula With the dielectric filling the entire space between the plates, the new capacitance is

$$C' = \frac{\epsilon A}{d} = \frac{K\epsilon_0 A}{d}.$$

This relation depends on the material and geometry, not on whether a battery remains connected.

Step 2: Compare the new capacitance with the initial value Dividing the new expression by $C = \epsilon_0 A/d$

gives $C'/C = K$. For a usual dielectric, $K > 1$, so capacitance increases. Option (A) would ignore the dielectric, option (B) predicts a decrease, and option (D) is not the result for a fully filled gap; even at $K = 1$, option (D) would incorrectly double the capacitance.

Step 3: Conclusion The new capacitance is $C' = KC$. Hence, the correct option is (C).

Quick Tip to Solve: When a dielectric completely fills an unchanged air capacitor, replace ϵ_0 by $K\epsilon_0$, multiplying capacitance by K .

Section E (15 marks)

31(a). (i) A thin pencil of length $f/4$ is placed coinciding with the principal axis of a mirror of focal length f . The image of the pencil is real and enlarged, just touches the pencil. Calculate the magnification produced by the mirror.

(ii) A ray of light is incident on a refracting face AB of a prism ABC at an angle of 45° . The ray emerges from face AC and the angle of deviation is 15° . The angle of prism is 30° . Show that the emergent ray is normal to the face AC from which it emerges out. Find the refraction index of the material of the prism. [5 marks]

Solution:

Concept:

- A concave mirror gives a real, enlarged image of an object placed between its focus and centre of curvature. Write $f > 0$ for the magnitude of its focal length. For real axial object and image distances $U, V > 0$, the mirror formula is $1/f = 1/U + 1/V$.
- An axial pencil must be imaged endpoint by endpoint. Its length magnification is $M = L_{\text{image}}/L_{\text{pencil}}$, where both lengths are positive; the transverse formula $-v/u$ does not directly give the length ratio of a finite axial pencil.
- For a prism, $\delta = i + e - A$ and $r_1 + r_2 = A$, where i, e are the external angles to the face normals, r_1, r_2 are the internal angles, A is the prism angle and δ is the deviation. In air, Snell's law gives $\mu = \sin i / \sin r_1$.

Step 1: (i) Locate the touching end of the pencil Since the image is real and enlarged, the mirror is concave and the pencil lies on the focus side of the centre of curvature C . Its image lies on the opposite side of C , farther from the mirror. The axial imaging relation $V = fU/(U - f)$ decreases with U , so the end of the pencil farther from the mirror forms the nearer end of the image. For the two lengths to just touch without overlapping, that far end must coincide with its own image. Thus its object and image distances are equal. Applying the mirror formula gives

$$\frac{1}{f} = \frac{1}{U_C} + \frac{1}{U_C} \Rightarrow U_C = V_C = 2f.$$

Therefore the touching point is C . The pencil's nearer end P is at

$$U_P = 2f - \frac{f}{4} = \frac{7f}{4}.$$

Step 2: (i) Image the nearer end and compare the lengths Apply the mirror formula to end P , using

positive distance magnitudes for this real object and real image:

$$\begin{aligned}\frac{1}{V_P} &= \frac{1}{f} - \frac{1}{7f/4} \\ &= \frac{1}{f} - \frac{4}{7f} = \frac{3}{7f}, \quad V_P = \frac{7f}{3}.\end{aligned}$$

The image extends from $2f$ to $7f/3$, so its length and the corresponding magnification are

$$L_{\text{image}} = \frac{7f}{3} - 2f = \frac{f}{3}, \quad M = \frac{f/3}{f/4} = \frac{4}{3}.$$

This is greater than unity, as required by the printed condition. The endpoints exchange their near-to-far order in the image. Hence the signed longitudinal ratio would be negative if directed endpoint separations were used, while the requested enlargement is the positive length ratio.

Step 3: (ii) Determine the angle of emergence Substitute the given incidence $i = 45^\circ$, prism angle $A = 30^\circ$, and deviation $\delta = 15^\circ$ into the prism deviation relation:

$$\begin{aligned}e &= \delta + A - i \\ &= 15^\circ + 30^\circ - 45^\circ = 0^\circ.\end{aligned}$$

The angle of emergence is measured from the normal to AC. Therefore $e = 0^\circ$ establishes that the emergent ray is normal to face AC; it does not mean that the ray travels along that face.

Step 4: (ii) Apply Snell's law at the two faces At the second face, Snell's law reads $\mu \sin r_2 = \sin e$. Because $e = 0^\circ$, the internal angle is also $r_2 = 0^\circ$. The internal angles obey

$$r_1 + r_2 = A \quad \Rightarrow \quad r_1 = 30^\circ.$$

Now use refraction from air into the prism at AB:

$$\mu = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2} \simeq 1.414.$$

No minimum-deviation condition is needed here: the supplied incidence and deviation already determine the emergence angle, and the normal exit fixes the first internal angle.

Step 5: Conclusion (i) The pencil's image has length $f/3$, giving an enlargement (length magnification) of $\boxed{4/3}$. (ii) $\boxed{e = 0^\circ}$, so the emergent ray is normal to AC, and the prism material has refractive index $\boxed{\mu = \sqrt{2} \simeq 1.414}$.

Quick Tip to Solve: For an axial pencil, image both endpoints and subtract their distances. For the prism, calculate $e = \delta + A - i$ first; a normal exit immediately gives $r_1 = A$.

31(b). (i) Light consisting of two wavelengths 600 nm and 480 nm is used to obtain interference fringes in a double slit experiment. The screen is placed 1.0 m away from slits which are 1.0 nm apart.

(1) Calculate the distance of the third bright fringe on the screen from the central maximum for wavelength 600 nm.

(2) Find the least distance from the central maximum where the bright fringes due to both the wavelengths coincide.

(ii) (1) Draw the variation of intensity with angle of diffraction in single slit diffraction pattern. Write the expression for value of angle corresponding to zero intensity locations.

(2) In what way diffraction of light waves differs from diffraction of sound waves?

[5 marks]

Solution:

Concept:

- In Young's double-slit experiment, constructive interference requires path difference $d \sin \theta = m\lambda$, where d is slit separation, λ is wavelength and m is an integer. The screen coordinate is $y = D \tan \theta$; only for small angles does $y_m \simeq m\lambda D/d$.
- Coincident bright fringes of wavelengths λ_1 and λ_2 require $m_1\lambda_1 = m_2\lambda_2$. The smallest positive integer pair gives the first noncentral coincidence, provided that such interference orders physically exist.
- For a single slit of width a , $I(\theta) = I_0(\sin \beta/\beta)^2$, with $\beta = \pi a \sin \theta/\lambda$. Diffraction is appreciable when the aperture or obstacle size is comparable with the wavelength.

Step 1: (i)(1) Check the printed slit separation and the third order The official paper prints $d = 1.0 \text{ nm} = 1.0 \times 10^{-9} \text{ m}$, while $D = 1.0 \text{ m}$ and $\lambda_1 = 600 \text{ nm}$. With these printed data, a third-order bright fringe would require

$$\sin \theta_3 = \frac{3\lambda_1}{d} = \frac{3 \times 600 \text{ nm}}{1.0 \text{ nm}} = 1800.$$

Since $|\sin \theta| \leq 1$, this order cannot occur. More generally, the path difference between waves from the slits never exceeds d , so no nonzero bright-fringe order exists for either wavelength when d is smaller than both wavelengths.

A purely formal substitution into the usual small-angle formula gives

$$y_3^{\text{formal}} = \frac{3(600 \times 10^{-9})(1.0)}{1.0 \times 10^{-9}} = 1800 \text{ m}.$$

This is not a physically valid screen position: the formula assumes $y \ll D$, which the result grossly violates. The printed data therefore do not define a physical third bright fringe.

Step 2: (i)(2) Find the common order and test its existence For a common bright fringe, the two path differences must be equal:

$$\begin{aligned} m_1(600) &= m_2(480), \\ 5m_1 &= 4m_2. \end{aligned}$$

The smallest positive integer solution is $m_1 = 4$, $m_2 = 5$, corresponding to path difference 2400 nm. Formally, the small-angle position would be

$$y^{\text{formal}} = \frac{4(600 \times 10^{-9})(1.0)}{1.0 \times 10^{-9}} = 2400 \text{ m}.$$

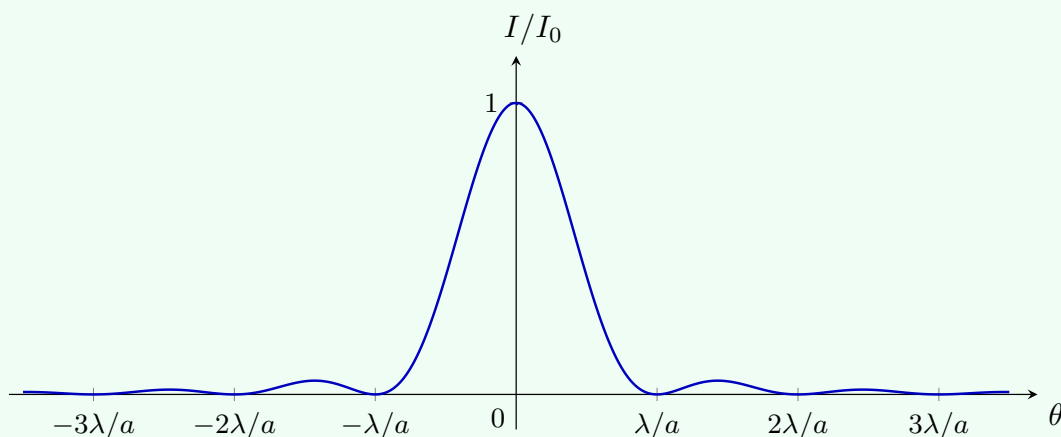
However, the necessary path difference is 2400 times the printed slit separation, so this common order is impossible. Only the central maxima coincide, at $y = 0$; there is no nonzero distance at which bright fringes coincide for the printed data. If a slit separation of 1.0 mm was intended instead, the familiar small-angle answers would be 1.8 mm and 2.4 mm, respectively. Those conditional values require changing the printed unit and are not answers for $d = 1.0$ nm.

Step 3: (ii)(1) Plot single-slit intensity and locate the minima The intensity is greatest at $\theta = 0$, where the limiting value of $\sin \beta / \beta$ is one. On either side is a sequence of much weaker secondary maxima, separated by zeros. The central maximum extends between the first minima and has twice the angular width of a secondary maximum in the small-angle approximation.

Zero intensity occurs when $\sin \beta = 0$ but $\beta \neq 0$. Thus

$$\begin{aligned}\beta &= m\pi, & m &= \pm 1, \pm 2, \dots, \\ a \sin \theta_m &= m\lambda, \\ \theta_m &= \sin^{-1} \left(\frac{m\lambda}{a} \right).\end{aligned}$$

Only orders with $|m\lambda/a| \leq 1$ occur. For small diffraction angles, $\theta_m \simeq m\lambda/a$ in radians. The point $m = 0$ is the central maximum, not a minimum.



Single-slit intensity distribution; horizontal positions use the small-angle approximation. The central peak is twice as wide as each secondary lobe.

Step 4: (ii)(2) Compare diffraction of light and sound Light and sound both obey the same general diffraction principle: waves spread around obstacles and through apertures. The practical difference follows from their wavelengths. Visible light has a wavelength of order 10^{-7} m, much smaller than everyday openings and obstacles, so appreciable spreading usually requires a very narrow slit or a very small obstacle. This is why ordinary light rays appear to travel approximately straight and form relatively sharp shadows.

Audible sound has much longer wavelengths, often comparable with doors and ordinary obstacles. It therefore diffracts appreciably in everyday situations; sound can be heard around a corner even when its source cannot be seen. The difference is in the relative sizes of wavelength and obstacle, rather than a different diffraction law.

Step 5: Conclusion For the printed 1.0 nm slit separation, the third bright fringe and a noncentral common bright fringe do not exist; only the central coincidence is at $y = 0$. The values 1800 m and 2400 m are formal, invalid small-angle substitutions. Single-slit minima satisfy $a \sin \theta_m = m\lambda$, $m = \pm 1, \pm 2, \dots$. Sound shows appreciable everyday diffraction because its wavelength is much larger

than that of visible light.

Quick Tip to Solve: Before using $y_m = m\lambda D/d$, check that the requested order can satisfy $m\lambda \leq d$. For fringe coincidence here, reduce $600 : 480$ to $5 : 4$ to obtain orders 4 and 5.

32(a). (i) A small conducting sphere A of radius r charged to a potential V , is enclosed by a spherical conducting shell B of radius R . If A and B are connected by a thin wire, calculate the final potential on sphere A and shell B.

(ii) Write two characteristics of equipotential surfaces. A uniform electric field of 50 N C^{-1} is set up in a region along $+x$ axis. If the potential at the origin $(0, 0)$ is 220 V , find the potential at a point $(4 \text{ m}, 3 \text{ m})$. [5 marks]

Solution:

Concept:

- The capacitance of an isolated conducting sphere of radius r is $C = 4\pi\epsilon_0 r$. Its charge at potential V , measured relative to infinity, is $Q = CV$.
- Conductors connected by a wire have the same potential in electrostatic equilibrium. With no isolated charge left in their cavity, the net excess charge lies on the outermost conducting surface.
- On an equipotential surface, $dV = 0$. The electric field satisfies $\vec{E} = -\nabla V$, so $\Delta V = -\int \vec{E} \cdot d\vec{l}$.

Step 1: (i) Express the initial charge on sphere A The small sphere is first charged to the given potential V . Taking the enclosing shell as initially uncharged, as implied by the question, the total charge available in the isolated arrangement is the charge initially placed on A. For the sphere before enclosure,

$$V = \frac{Q}{4\pi\epsilon_0 r} \Rightarrow Q = 4\pi\epsilon_0 r V.$$

Enclosing it with the neutral shell induces opposite charges on the shell's inner and outer surfaces, but does not change this total charge.

Step 2: (i) Use charge conservation after connecting A and B Charge flows through the wire until A and B form one equipotential conductor. Any initial charge on A moves to the exterior of the connected conductor; at equilibrium there is no field in the cavity and no net charge remains on A or the inner surface of B. Thus the conserved charge Q resides on the outer spherical surface of radius R . Neglecting the thin wire's effect on the spherical geometry, the potential of this surface and every point inside it is

$$V_f = \frac{Q}{4\pi\epsilon_0 R} = \frac{4\pi\epsilon_0 r V}{4\pi\epsilon_0 R} = \frac{r}{R} V.$$

The wire ensures that sphere A shares the same potential as shell B. Since $R > r$, the final potential has a smaller magnitude than the original potential of the isolated sphere.

Step 3: (ii) State two characteristics of equipotential surfaces First, no work is done by the electric field in moving a test charge between any two points on an equipotential surface. Both points have the same potential, so for a test charge q , $W = q(V_1 - V_2) = 0$.

Second, the electric field is perpendicular to an equipotential surface wherever the field is nonzero. A tangential field component would do work during displacement along the surface and would produce a potential difference, contradicting $dV = 0$.

Step 4: (ii) Calculate the potential change along the field Here the uniform field is $\vec{E} = 50\hat{i} \text{ N C}^{-1}$, and the displacement from the origin is $4\hat{i} + 3\hat{j}$ metres. Only the displacement along $+x$ contributes

to the dot product. Therefore,

$$\begin{aligned} V(4, 3) - V(0, 0) &= -\vec{E} \cdot \Delta \vec{r} \\ &= -50 \times 4 = -200 \text{ V}, \\ V(4, 3) &= 220 - 200 = 20 \text{ V}. \end{aligned}$$

The 3 m displacement along y changes no potential because $E_y = 0$. The negative sign of the potential change expresses the decrease of potential in the direction of the electric field.

Step 5: Conclusion (i) The final potentials are $V_A = V_B = (r/R)V$. (ii) Motion along an equipotential surface requires no electrostatic work, and the electric field is normal to it. The requested potential is $V(4 \text{ m}, 3 \text{ m}) = 20 \text{ V}$.

Quick Tip to Solve: For connected nested conductors, conserve total charge and use the outer radius for the final potential. In a uniform field, use only displacement parallel to the field in $\Delta V = -E\Delta x$.

OR

32(b). (i) What is difference between an open surface and a closed surface? Draw elementary surface vector $d\vec{S}$ for a spherical surface S .

(ii) Define electric flux through a surface. Give the significance of a Gaussian surface. A charge outside a Gaussian surface does not contribute to total electric flux through the surface. Why?

(iii) A small spherical shell S_1 has point charges $q_1 = -3 \mu\text{C}$, $q_2 = -2 \mu\text{C}$ and $q_3 = 9 \mu\text{C}$ inside it. This shell is enclosed by another big spherical shell S_2 . A point charge Q is placed in between the two surfaces S_1 and S_2 . If the electric flux through the surface S_2 is four times the flux through surface S_1 , find charge Q . [5 marks]

Solution:

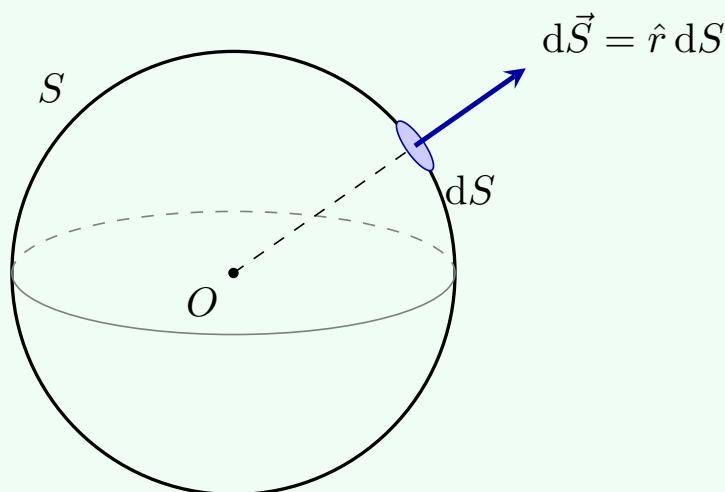
Concept:

- The elementary area vector is $d\vec{S} = \hat{n} dS$, where dS is the area of a small surface element and $\hat{n}$ is its unit normal. For a closed surface, the positive normal points outwards.
- Electric flux measures the electric field passing normally through a surface: $\Phi_E = \int_S \vec{E} \cdot d\vec{S}$, where $\vec{E}$ is the electric field. Its SI unit is $\text{N m}^2 \text{C}^{-1}$.
- Gauss's law states $\oint_S \vec{E} \cdot d\vec{S} = q_{\text{enc}}/\epsilon_0$, where q_{enc} is the algebraic sum of charges inside the closed surface and ϵ_0 is the permittivity of free space.

Step 1: (i) Distinguish the surfaces and show the area vector An open surface has an edge or boundary and does not completely enclose a volume, for example a plane disc or a hemispherical surface without its base. Either of its two normal directions may be chosen consistently as positive. A closed surface has no edge and completely encloses a volume, for example the complete surface of a sphere. Its area vectors are conventionally taken along the outward normals. For a spherical surface S , the normal at every point is radial. Therefore, at a small patch of area dS ,

$$d\vec{S} = \hat{r} dS,$$

where $\hat{r}$ points from the centre O towards the patch and then outwards. The arrow in the diagram starts at the patch and is perpendicular to the local tangent plane.



Spherical surface

The area vector at a small patch of a spherical surface points radially outwards.

Step 2: (ii) Define electric flux and explain a Gaussian surface Electric flux through a surface is the surface integral of the normal component of the electric field. If θ is the angle between the field and the chosen area vector, the contribution of a small element is

$$d\Phi_E = \vec{E} \cdot d\vec{S} = E \cos \theta dS.$$

Integrating gives the total flux. For a uniform field over a plane area S , this reduces to $\Phi_E = ES \cos \theta$. A Gaussian surface is an imaginary closed surface chosen to apply Gauss's law. A suitable choice, using the symmetry of a charge distribution, makes the flux integral simple and allows the electric field to be calculated from the enclosed charge. It need not be a physical conducting surface.

Step 3: (ii) Explain why an external charge gives zero net flux Consider the field of the external charge alone. That charge is not enclosed, so its enclosed charge is zero. Gauss's law gives

$$\oint_S \vec{E}_{\text{external}} \cdot d\vec{S} = 0.$$

In the field-line picture, any line that enters the closed surface also leaves it; its negative inward flux is balanced by its positive outward flux. The external charge can produce a nonzero field and nonzero flux through individual patches, but its contributions cancel over the whole closed surface.

Step 4: (iii) Calculate the charge enclosed by each shell The three charges inside S_1 have the algebraic sum

$$q_{\text{in}} = q_1 + q_2 + q_3 = (-3 - 2 + 9) \mu\text{C} = 4 \mu\text{C}.$$

Thus $\Phi_1 = q_{\text{in}}/\epsilon_0$. The charge Q lies outside S_1 , so it does not change this net flux. The larger shell S_2 encloses all three charges as well as Q , giving

$$\Phi_2 = \frac{q_{\text{in}} + Q}{\epsilon_0}.$$

Step 5: (iii) Apply the stated flux ratio Substitute $\Phi_2 = 4\Phi_1$ into the expressions from Gauss's law:

$$\begin{aligned}\frac{q_{\text{in}} + Q}{\epsilon_0} &= 4 \frac{q_{\text{in}}}{\epsilon_0}, \\ q_{\text{in}} + Q &= 4q_{\text{in}}, \\ Q &= 3q_{\text{in}} = 3(4 \mu\text{C}) = +12 \mu\text{C}.\end{aligned}$$

As a check, the total enclosed charge increases from $4 \mu\text{C}$ to $16 \mu\text{C}$, giving exactly the required factor of four in the outward flux. The radii of the shells do not enter the net-flux calculation.

Step 6: Conclusion An open surface has a boundary, whereas a closed surface completely encloses a volume. On a sphere, $d\vec{S} = \hat{r} dS$ is radially outward. A Gaussian surface relates net flux to enclosed charge; an external charge contributes zero net flux. The required charge between the shells is $Q = +12 \mu\text{C}$.

Quick Tip to Solve: Add the enclosed charges with their signs before applying Gauss's law. If the outer flux is k times the inner flux, the extra enclosed charge is $(k - 1)q_{\text{in}}$.

33(a). (i) What is the source of force acting on a current-carrying conductor placed in a magnetic field? Obtain the expression for force acting between two long straight parallel conductors carrying steady currents and hence define 'ampere'.

(ii) A point charge q is moving with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. Find the work done by the magnetic force on the charge.

(iii) Explain the necessary conditions in which the trajectory of a charged particle is helical in a uniform magnetic field. [5 marks]

Solution:

Concept:

- A carrier of charge q , moving with drift velocity $\vec{v}_d$, experiences magnetic force $q\vec{v}_d \times \vec{B}$; the conductor experiences $\vec{F} = I\vec{\ell} \times \vec{B}$.
- An infinitely long straight wire carrying current I produces $B = \mu_0 I / (2\pi d)$ at perpendicular distance d in vacuum.
- Magnetic force is perpendicular to velocity, so its power $\vec{F} \cdot \vec{v}$ is zero. Resolve velocity into components parallel and perpendicular to the field to determine the trajectory.

Step 1: (i) Identify the microscopic source of force Current consists of moving charge carriers. The external magnetic field exerts a Lorentz force on these carriers, and their interaction with the lattice transmits the force to the conductor. For carrier number density n , cross-sectional area A , and length ℓ , the segment contains $nA\ell$ carriers. Using the signed carrier charge and $\vec{J} = nq\vec{v}_d$, the total force is

$$\vec{F} = nA\ell q \vec{v}_d \times \vec{B} = I\vec{\ell} \times \vec{B},$$

where $\vec{\ell}$ points along conventional current. Thus moving charges in the conductor are responsible for its magnetic force.

Step 2: (i) Derive the force between parallel conductors Consider two long straight parallel wires in vacuum, separated by distance d , carrying steady currents I_1 and I_2 . Neglect end effects. The

magnetic field produced by wire 1 at wire 2 is

$$B_1 = \frac{\mu_0 I_1}{2\pi d}.$$

This field is perpendicular to wire 2, so a length ℓ of that wire experiences

$$F_{21} = I_2 \ell B_1 = \frac{\mu_0 I_1 I_2 \ell}{2\pi d}, \quad \frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}.$$

Interchanging the wires gives an equal force magnitude on wire 1. The right-hand rule for the magnetic field followed by $I\vec{\ell} \times \vec{B}$ gives attraction for currents in the same direction and repulsion for currents in opposite directions.

Step 3: (i) State the force-based definition of ampere In the force-based definition requested here, one ampere is that constant current which, when maintained in each of two straight, parallel conductors of infinite length and negligible circular cross-section, placed one metre apart in vacuum, produces a force of 2×10^{-7} N per metre between them. Indeed, using the textbook value $\mu_0 = 4\pi \times 10^{-7}$ N A⁻², with $I_1 = I_2 = 1$ A and $d = 1$ m,

$$\frac{F}{\ell} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7} \text{ N m}^{-1}.$$

Step 4: (ii) Evaluate the magnetic work The force on the point charge is $\vec{F}_B = q\vec{v} \times \vec{B}$, while its infinitesimal displacement is $d\vec{r} = \vec{v} dt$. Therefore

$$dW = \vec{F}_B \cdot d\vec{r} = q(\vec{v} \times \vec{B}) \cdot \vec{v} dt = 0.$$

Integrating over any part of the path gives $W_B = 0$. The field can change the direction of velocity, but cannot change the particle's speed or kinetic energy by magnetic work.

Step 5: (iii) Resolve the motion to obtain a helix For a charged particle acted upon only by a uniform, steady magnetic field, both $v_{\parallel}$ and $v_{\perp}$ must be nonzero. Thus its initial velocity must be oblique to the field, neither parallel nor perpendicular. The parallel component experiences no magnetic force and remains constant; the perpendicular component undergoes uniform circular motion. For particle mass m ,

$$r = \frac{mv_{\perp}}{|q|B}, \quad T = \frac{2\pi m}{|q|B}, \quad p = |v_{\parallel}|T.$$

Their superposition is a helix with axis parallel to $\vec{B}$ and pitch p . If $v_{\parallel} = 0$, the path reduces to a circle; if $v_{\perp} = 0$, it is a straight line. The sign of the charge determines the sense of rotation.

Step 6: Conclusion The conductor's force originates in magnetic forces on its moving carriers. Parallel wires exert $F/\ell = \mu_0 I_1 I_2 / (2\pi d)$, giving the stated force-based ampere definition. Magnetic work is zero. A helix requires nonzero velocity components both along and perpendicular to the uniform field.

Quick Tip to Solve: For a particle in a magnetic field, first split its velocity into parallel and perpendicular components: the former sets the pitch, the latter the radius, and neither changes its speed.

33(b). (i) A current carrying loop can be considered as a magnetic dipole placed along its axis. Explain.

(ii) Obtain the relation for magnetic dipole moment $\vec{M}$ of current carrying coil. Give the direction of $\vec{M}$.

(iii) A current carrying coil is placed in an external uniform magnetic field. The coil is free to turn in the magnetic field. What is the net force acting on the coil? Obtain the orientation of the coil in stable equilibrium. Show that in this orientation the flux of the total field (field produced by the loop + external field) through the coil is maximum. [5 marks]

Solution:

Concept:

- The magnetic field of a small current loop has the same dipole form as that of a bar magnet at distances large compared with the loop's size.
- A plane coil of N turns, current I , and area A per turn has moment $\vec{M} = NIA\hat{n}$, with $\hat{n}$ fixed by the right-hand rule.
- In a uniform external field $\vec{B}$, a magnetic dipole has torque $\vec{\tau} = \vec{M} \times \vec{B}$ and potential energy $U = -\vec{M} \cdot \vec{B}$. Stable equilibrium minimises U .

Step 1: (i) Compare the loop with a magnetic dipole The field lines of a current loop emerge from one face and enter the other, just as for a bar magnet. Viewed from a face, anticlockwise current makes that face act as a north pole, while clockwise current makes it act as a south pole. The loop axis is therefore its magnetic axis, and its moment points along the axis from its effective south face towards its effective north face. For a circular loop of radius R carrying current I , the field at axial distance x is

$$B_{\text{loop}} = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}}.$$

At $x \gg R$, this becomes $B_{\text{loop}} = \mu_0 IR^2 / (2x^3)$. Its inverse-cube dependence and direction are those of an axial magnetic dipole. Thus a small loop behaves as a magnetic dipole situated at its centre and directed along its axis; the point-dipole field comparison applies at large distances.

Step 2: (ii) Obtain the dipole moment and its direction A magnetic dipole of moment magnitude M produces the far axial field

$$B_{\text{dipole}} = \frac{\mu_0}{4\pi} \frac{2M}{x^3}.$$

Equating this with the loop's far field gives

$$\frac{\mu_0 IR^2}{2x^3} = \frac{\mu_0 M}{2\pi x^3} \Rightarrow M = I\pi R^2 = IA.$$

The moments of N identical coplanar turns add, giving $\vec{M} = NIA\hat{n}$, in A m^2 . The area rule applies to any plane loop. For example, take a rectangular turn with sides a, b , with the sides of length a perpendicular to $\vec{B}$. The forces IaB on two opposite sides form a couple with arm $b \sin \theta$, giving $\tau = IabB \sin \theta = IAB \sin \theta$, the dipole torque with $M = IA$. Curl the fingers of the right hand in the current's direction; the extended thumb gives $\hat{n}$ and hence $\vec{M}$, perpendicular to the plane of the coil.

Step 3: (iii) Calculate the net force in a uniform field A current element experiences $d\vec{F} = I d\vec{\ell} \times \vec{B}$.

Since the external field is constant around a closed turn,

$$\vec{F}_{\text{net}} = NI \oint d\vec{\ell} \times \vec{B} = NI \left(\oint d\vec{\ell} \right) \times \vec{B} = \vec{0}.$$

The displacement integral around the closed loop vanishes. Forces on different portions can still produce a torque, even though the resultant force is zero.

Step 4: (iii) Identify the stable orientation Let θ be the angle between $\vec{M}$ and the external field. The torque magnitude is $MB \sin \theta$, so the equilibrium orientations are $\theta = 0$ and $\theta = \pi$. To distinguish their stability, use

$$U = -MB \cos \theta, \quad \frac{d^2U}{d\theta^2} = MB \cos \theta.$$

At $\theta = 0$, $U = -MB$ is a minimum and a small angular displacement produces a restoring torque. Hence the stable orientation has $\vec{M}$ parallel to $\vec{B}$, with the coil's plane perpendicular to the field. The antiparallel orientation has maximum energy and is unstable.

Step 5: (iii) Maximise the flux of the total field Take positive area normal along $\vec{M}$. For a rigid coil carrying steady current, its own field and coil rotate together, so its self-flux Φ_{self} through a turn is independent of its orientation in the external field. By superposition, the total flux per turn is

$$\Phi_{\text{total}}(\theta) = \Phi_{\text{self}} + BA \cos \theta.$$

As $\cos \theta \leq 1$, the maximum occurs at $\theta = 0$, giving $\Phi_{\text{total,max}} = \Phi_{\text{self}} + BA$. This is exactly the stable orientation found from minimum potential energy: the external field threads the coil in the direction of its own field. For identical turns, the total flux linkage is $N\Phi_{\text{total}}$ and is maximal at the same orientation.

Step 6: Conclusion A current loop behaves as an axial magnetic dipole with $\vec{M} = NIA\hat{n}$, directed by the right-hand rule. Its net force in a uniform field is zero. It is stable when $\vec{M} \parallel \vec{B}$; then its plane is perpendicular to the field and its total flux is maximum.

Quick Tip to Solve: Measure the angle from the coil's normal, not its plane: both minimum energy $-MB \cos \theta$ and maximum external flux $BA \cos \theta$ occur when that normal follows the field.