

CBSE Class 12 Physics Question Paper 2025 with Solution PDF-Set 55/7/1



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – **Sections A, B, C, D** and **E**.
- (iii) In **Section A** – Questions no. **1 to 16** are Multiple Choice type questions. Each question carries **1** mark.
- (iv) In **Section B** – Questions no. **17 to 21** are Very Short Answer type questions. Each question carries **2** marks.
- (v) In **Section C** – Questions no. **22 to 28** are Short Answer type questions. Each question carries **3** marks.
- (vi) In **Section D** – Questions no. **29** and **30** are case study-based questions. Each question carries **4** marks.
- (vii) In **Section E** – Questions no. **31 to 33** are Long Answer type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is not allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. Two horizontal plates, separated by 1 cm, are arranged one above the other. A particle of mass 5 mg and charge 2 nC is released in air between the plates. The potential difference that should be applied to the plates so that the particle remains suspended between them, is: [1 mark]

- (A) 250 V
- (B) 200 V
- (C) 100 V
- (D) 50 V

Correct Answer: (A) 250 V

Solution:

Concept:

- A suspended particle is in equilibrium: the upward electric force balances its downward weight, so $qE = mg$.
- Between parallel plates, $E = V/d$, where V is the potential difference and d is their separation.

Step 1: Convert the data and choose the field direction The mass is $m = 5 \times 10^{-6}$ kg, the charge is $q = 2 \times 10^{-9}$ C, and the separation is $d = 10^{-2}$ m. Because the charge is positive, the field must point upward; hence the lower plate is at higher potential.

Step 2: Balance the two forces Substituting $E = V/d$ into the equilibrium condition gives, using $g \simeq 10 \text{ m s}^{-2}$,

$$V = \frac{mgd}{q} = \frac{(5 \times 10^{-6})(10)(10^{-2})}{2 \times 10^{-9}} = 250 \text{ V}.$$

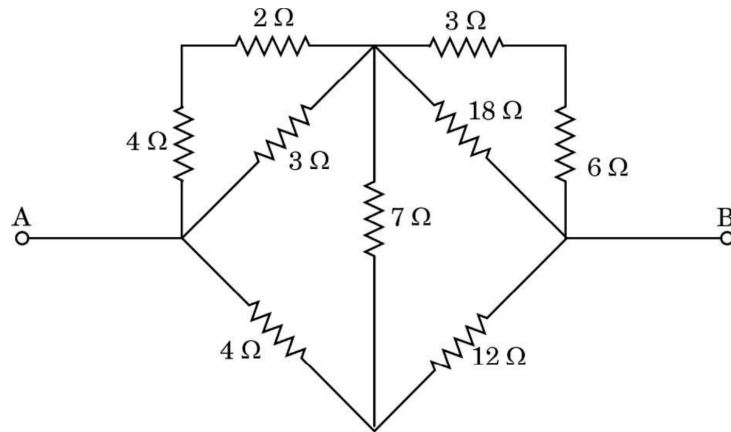
The alternatives 200 V, 100 V, and 50 V give electric forces smaller than the weight and cannot suspend the particle. Using $g = 9.8 \text{ m s}^{-2}$ gives 245 V, consistent with the rounded listed value.

Step 3: Conclusion The required potential difference is 250 V, with the lower plate positive relative to the upper plate. Hence, the correct option is (A).

Quick Tip to Solve: Use $V = mgd/q$ directly, but convert milligrams to 10^{-6} kg, not 10^{-3} kg.

2. The effective resistance between points A and B in the given circuit is:

[1 mark]



- (A) $6\ \Omega$
- (B) $\frac{8}{3}\ \Omega$
- (C) $\frac{16}{3}\ \Omega$
- (D) $2\ \Omega$

Correct Answer: (C) $\frac{16}{3}\ \Omega$

Solution:

Concept:

- Series resistances add; two parallel resistances have equivalent resistance $R_1 R_2 / (R_1 + R_2)$.
- A Wheatstone bridge is balanced when the ratios of its two arms are equal. Its bridge resistor then carries no current.

Step 1: Reduce the upper arms Call the upper and lower diamond junctions C and D . Between A and C , the outer $4\ \Omega$ and $2\ \Omega$ resistors are in series and parallel to $3\ \Omega$. Similarly, between C and B , the outer $3\ \Omega$ and $6\ \Omega$ are in series and parallel to $18\ \Omega$:

$$R_{AC} = \frac{6 \times 3}{6 + 3} = 2\ \Omega, \quad R_{CB} = \frac{9 \times 18}{9 + 18} = 6\ \Omega.$$

Step 2: Check balance and combine the paths The lower arms are $R_{AD} = 4\ \Omega$ and $R_{DB} = 12\ \Omega$. Since $R_{AC}/R_{CB} = 2/6 = 4/12 = R_{AD}/R_{DB}$, junctions C and D are equipotential. The $7\ \Omega$ resistor carries no current. The remaining paths have resistances $2 + 6 = 8\ \Omega$ and $4 + 12 = 16\ \Omega$, in parallel:

$$R_{AB} = \frac{8 \times 16}{8 + 16} = \frac{16}{3}\ \Omega.$$

The value $2\ \Omega$ is only one reduced arm, and $6\ \Omega$ is another. Neither is the whole network; $8/3\ \Omega$ also disagrees with the parallel combination.

Step 3: Conclusion The effective resistance is $\frac{16}{3}\ \Omega$. Hence, the correct option is (C).

Quick Tip to Solve: Reduce the outer series branches first; then test bridge balance before including the central $7\ \Omega$ resistor.

3. A rectangular coil of area A is kept in a uniform magnetic field $\vec{B}$ such that the plane of the coil makes an angle α with $\vec{B}$. The magnetic flux linked with the coil is: [1 mark]

- (A) $BA \sin \alpha$
- (B) $BA \cos \alpha$
- (C) BA
- (D) zero

Correct Answer: (A) $BA \sin \alpha$

Solution:

Concept:

- Magnetic flux through a plane surface is $\Phi = \vec{B} \cdot \vec{A} = BA \cos \theta$, where θ is the angle between the field and the surface normal.
- The area vector is perpendicular to the plane, so an angle α with the plane corresponds to $\theta = 90^\circ - \alpha$ with the normal.

Step 1: Identify the angle required by the flux formula The question gives the angle with the plane of the coil, not with its normal. Therefore the component of the magnetic field passing perpendicular to the coil is $B \cos(90^\circ - \alpha) = B \sin \alpha$.

Step 2: Multiply the normal field by the area For uniform field over the rectangular coil,

$$\Phi = BA \cos(90^\circ - \alpha) = BA \sin \alpha.$$

The expression $BA \cos \alpha$ uses the wrong reference direction for the given angle. The value BA applies only when the plane is perpendicular to the field, and zero applies only when it is parallel to the field; neither is the general answer.

Step 3: Conclusion The magnetic flux linked with the coil is $BA \sin \alpha$. Hence, the correct option is (A).

Quick Tip to Solve: Angle with the plane means use sine; angle with the normal means use cosine. Check that a parallel field gives zero flux.

4. An alternating current is given by $I = I_0 \cos(100\pi)t$. The least time the current takes to decrease from its maximum value to zero will be:

[1 mark]

- (A) $\left(\frac{1}{200}\right)$ s
(B) $\left(\frac{1}{150}\right)$ s
(C) $\left(\frac{1}{100}\right)$ s
(D) $\left(\frac{1}{50}\right)$ s

Correct Answer: (A) $\left(\frac{1}{200}\right)$ s

Solution:

Concept:

- A sinusoidal current $I = I_0 \cos \omega t$ has angular frequency ω and period $T = 2\pi/\omega$.
- From a positive maximum to the next zero, the phase advances by $\pi/2$, corresponding to one quarter of a cycle.

Step 1: Identify the starting phase Here $\omega = 100\pi \text{ rad s}^{-1}$. At $t = 0$, $\cos 0 = 1$, so the current is at its positive maximum I_0 . The first subsequent zero occurs when the cosine argument reaches $\pi/2$.

Step 2: Calculate the first zero time Set $100\pi t = \pi/2$:

$$t = \frac{\pi/2}{100\pi} = \frac{1}{200} \text{ s.}$$

For comparison, $T = 1/50 \text{ s}$, so $1/100 \text{ s}$ is half a cycle and gives the negative maximum; $1/50 \text{ s}$ returns to the positive maximum. The value $1/150 \text{ s}$ gives phase $2\pi/3$, whose cosine is $-1/2$, not zero.

Step 3: Conclusion The least time is $\frac{1}{200} \text{ s} = 5 \times 10^{-3} \text{ s}$. Hence, the correct option is (A).

Quick Tip to Solve: For any sinusoidal current, maximum to the next zero takes $T/4$; here obtain T from $2\pi/\omega$.

5. A capacitor and an inductor are connected in series across an ac source of voltage of variable frequency. The frequency is increased continuously. The nature of the circuit before and after the resonance will be: [1 mark]

- (A) inductive only
- (B) capacitive only
- (C) capacitive and inductive respectively
- (D) inductive and capacitive respectively

Correct Answer: (C) capacitive and inductive respectively

Solution:

Concept:

- The inductive reactance is $X_L = 2\pi fL$, while the capacitive reactance is $X_C = 1/(2\pi fC)$, where f is frequency.
- The net series reactance is $X = X_L - X_C$. Positive reactance is inductive and negative reactance is capacitive; resonance occurs at $f_0 = 1/(2\pi\sqrt{LC})$.

Step 1: Compare the reactances below resonance At low frequency, X_L is small while X_C is large. More precisely, $X_L/X_C = (f/f_0)^2 < 1$ for $f < f_0$. Thus the capacitive reactance dominates and the circuit behaves capacitively before resonance.

Step 2: Follow the change above resonance As frequency increases, X_L increases and X_C decreases. They are equal at resonance. For $f > f_0$, their ratio exceeds unity, so $X_L > X_C$ and the circuit becomes inductive. It cannot remain only inductive or only capacitive throughout this change, and the opposite ordering would incorrectly reverse the frequency dependences of the two reactances.

Step 3: Conclusion Before resonance the circuit is capacitive, and after resonance it is inductive. Hence, the correct option is (C).

Quick Tip to Solve: As frequency rises, X_L rises and X_C falls: the sequence across series resonance is capacitive, then inductive.

6. A metal rod of length 50 cm is held vertically and moved with a velocity of 10 m/s towards east. The horizontal component of the Earth's magnetic field at the place is 0.4 G. The emf induced across the ends of the rod is: [1 mark]

- (A) 0.1 mV
- (B) 0.2 mV
- (C) 0.8 mV
- (D) 1.6 mV

Correct Answer: (B) 0.2 mV

Solution:

Concept:

- The motional emf along a rod is $\mathcal{E} = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$. For mutually perpendicular rod, velocity and field, its magnitude is Blv .
- The horizontal terrestrial field points north. Eastward motion therefore makes $\vec{v} \times \vec{B}_H$ vertical, along the rod. The conversion is $1 \text{ G} = 10^{-4} \text{ T}$.

Step 1: Identify the effective field and convert units The rod has length $l = 50 \text{ cm} = 0.50 \text{ m}$, speed $v = 10 \text{ m s}^{-1}$, and horizontal field $B_H = 0.4 \times 10^{-4} = 4 \times 10^{-5} \text{ T}$. The horizontal field and eastward motion are perpendicular; their magnetic force acts along the vertical rod. Any vertical field component produces a horizontal force and contributes no emf between its ends.

Step 2: Calculate the potential difference Using the perpendicular geometry,

$$\mathcal{E} = B_H l v = (4 \times 10^{-5})(0.50)(10) = 2 \times 10^{-4} \text{ V} = 0.2 \text{ mV}.$$

The upper end becomes positive because $\vec{v} \times \vec{B}_H$ points upward. The alternatives 0.1 mV, 0.8 mV, and 1.6 mV do not follow from the given field, length and speed; no extra geometric factor is required here.

Step 3: Conclusion The induced emf is 0.2 mV. Hence, the correct option is (B).

Quick Tip to Solve: Convert gauss to tesla before using Blv , and retain only the component of $\vec{v} \times \vec{B}$ along the rod.

7. The dimensions of 'self-inductance' are:

[1 mark]

- (A) $[MLT^{-2}A^{-2}]$
- (B) $[ML^2T^{-1}A^{-1}]$
- (C) $[ML^{-1}T^{-2}A^{-2}]$
- (D) $[ML^2T^{-2}A^{-2}]$

Correct Answer: (D) $[ML^2T^{-2}A^{-2}]$

Solution:

Concept:

- The energy stored in an inductor is $U = \frac{1}{2}L_{\text{ind}}I^2$, where L_{ind} is its self-inductance and I is current.
- Energy has dimensions $[ML^2T^{-2}]$, and current has dimension $[A]$. Here the L inside dimension brackets denotes length, not inductance.

Step 1: Express inductance in terms of energy and current Rearrange the energy relation as $L_{\text{ind}} = 2U/I^2$. The numerical factor 2 is dimensionless, so dividing the dimensions of energy by those of current squared gives

$$[L_{\text{ind}}] = \frac{[ML^2T^{-2}]}{[A^2]} = [ML^2T^{-2}A^{-2}].$$

Step 2: Check the powers in the options Option (A) has only the first power of length, and option (C) has inverse length, so neither can give energy on multiplication by current squared. Option (B) has $T^{-1}A^{-1}$, so both its time and current powers are incorrect for self-inductance. Only option (D) has all four required powers. The same result follows from $\mathcal{E} = L_{\text{ind}} dI/dt$.

Step 3: Conclusion The dimensions of self-inductance are $[ML^2T^{-2}A^{-2}]$. Hence, the correct option is (D).

Quick Tip to Solve: Recall $U = \frac{1}{2}LI^2$: self-inductance has the dimensions of energy divided by current squared.

8. The frequency of a photon of energy 1.326 eV is:

[1 mark]

- (A) 1.18×10^{14} Hz
- (B) 3.20×10^{14} Hz
- (C) 4.20×10^{15} Hz
- (D) 4.80×10^{15} Hz

Correct Answer: (B) 3.20×10^{14} Hz

Solution:

Concept:

- A photon's energy and frequency are related by $E = h\nu$, where h is Planck's constant and ν is frequency.
- Convert electronvolts to joules before using $h = 6.63 \times 10^{-34}$ J s: $1 \text{ eV} = 1.6 \times 10^{-19}$ J.

Step 1: Convert the photon energy The given energy in SI units is

$$E = 1.326 \times 1.6 \times 10^{-19} = 2.1216 \times 10^{-19} \text{ J.}$$

Step 2: Calculate and compare the frequency Rearranging Planck's relation gives

$$\nu = \frac{E}{h} = \frac{2.1216 \times 10^{-19}}{6.63 \times 10^{-34}} = 3.20 \times 10^{14} \text{ Hz.}$$

Option (A) has the wrong coefficient. Options (C) and (D) are over an order of magnitude too large; each would correspond to a much higher photon energy.

Step 3: Conclusion The frequency is 3.20×10^{14} Hz. Hence, the correct option is (B).

Quick Tip to Solve: For energy in electronvolts, multiply its numerical value by approximately 2.414×10^{14} to obtain frequency in hertz.

9. Germanium crystal is doped at room temperature with a minute quantity of boron. The charge carriers in the doped semiconductors will be:

[1 mark]

- (A) electrons only
- (B) holes only
- (C) holes and few electrons
- (D) electrons and few holes

Correct Answer: (C) holes and few electrons

Solution:

Concept:

- Germanium is tetravalent, whereas boron is trivalent. A boron impurity acts as an acceptor and produces a p-type semiconductor.
- In p-type material, holes are majority carriers and electrons are minority carriers; thermal generation produces some electron-hole pairs at room temperature.

Step 1: Identify the impurity's effect A boron atom replacing a germanium atom has only three valence electrons for four covalent bonds. The resulting electron deficiency supplies a mobile hole, so doping increases the hole concentration.

Step 2: Include the minority carriers At room temperature, thermal excitation also leaves a small population of conduction electrons. Thus, neither electrons only nor holes only describes the doped crystal. Electrons with few holes would describe n-type material produced by a pentavalent donor, not boron doping.

Step 3: Conclusion The carriers are holes and a few electrons. Hence, the correct option is (C).

Quick Tip to Solve: A group-III dopant in germanium gives p-type material: choose majority holes but retain a few thermally generated electrons at room temperature.

10. Out of the four options given, in which transition will the emitted photon have the maximum wavelength? [1 mark]

- (A) $n = 4$ to $n = 3$
- (B) $n = 3$ to $n = 2$
- (C) $n = 2$ to $n = 1$
- (D) $n = 3$ to $n = 1$

Correct Answer: (A) $n = 4$ to $n = 3$

Solution:

Concept:

- For hydrogen energy levels, $E_n = -13.6/n^2$ eV, where n is the principal quantum number.
- Emission from n_i to $n_f < n_i$ gives $\Delta E = 13.6(1/n_f^2 - 1/n_i^2)$ eV. Since $\lambda = hc/\Delta E$, the smallest energy gap gives the largest wavelength.

Step 1: Compare the four energy gaps Evaluate the positive energy released in each downward transition:

$$4 \rightarrow 3 : 13.6(1/9 - 1/16) = 0.661 \text{ eV},$$

$$3 \rightarrow 2 : 13.6(1/4 - 1/9) = 1.89 \text{ eV},$$

$$2 \rightarrow 1 : 13.6(1 - 1/4) = 10.2 \text{ eV},$$

$$3 \rightarrow 1 : 13.6(1 - 1/9) = 12.09 \text{ eV}.$$

Step 2: Use the inverse energy-wavelength relation The smallest gap is between the adjacent upper levels $n = 4$ and $n = 3$. Each of the other three transitions releases more energy and therefore produces a shorter wavelength, excluding options (B), (C) and (D).

Step 3: Conclusion The transition $n = 4$ to $n = 3$ emits the photon of maximum wavelength. Hence, the correct option is (A).

Quick Tip to Solve: For a hydrogen spectrum, compare $1/n_f^2 - 1/n_i^2$; the smallest positive difference gives the longest emission wavelength.

11. A p-n junction diode is forward biased. As a result,

[1 mark]

- (A) both the potential barrier height and the width of depletion layer decrease.
- (B) both the potential barrier height and the width of depletion layer increase.
- (C) the potential barrier height decreases and the width of depletion layer increases.
- (D) the potential barrier height increases and the width of depletion layer decreases.

Correct Answer: (A) both the potential barrier height and the width of depletion layer decrease.

Solution:

Concept:

- A p-n junction has a depletion region containing fixed ionised donors and acceptors; their electric field creates the junction's potential barrier.
- Forward bias connects the p side to the positive terminal and the n side to the negative terminal, applying a voltage that opposes the built-in barrier.

Step 1: Determine the change in barrier height The forward voltage V_F reduces the effective barrier from V_{bi} to approximately $V_{bi} - V_F$. Majority carriers can consequently cross the junction more easily. A rising barrier, as in options (B) and (D), is inconsistent with forward bias.

Step 2: Determine the change in depletion width Holes from the p side and electrons from the n side are pushed towards the junction. This reduces the region depleted of mobile carriers, so its width decreases. Option (C) has the correct barrier change but the wrong width change; option (B) also incorrectly predicts a wider layer.

Step 3: Conclusion Both the potential barrier height and the width of the depletion layer decrease. Hence, the correct option is (A).

Quick Tip to Solve: Forward bias opposes the junction's built-in field: pair a lower barrier with a narrower depletion layer.

12. Isotones are the nuclides having:

[1 mark]

- (A) same mass numbers
- (B) same atomic numbers
- (C) same neutron number, but different atomic number
- (D) different neutron number, and different mass number

Correct Answer: (C) same neutron number, but different atomic number

Solution:

Concept:

- For a nuclide ${}_Z^AX$, A is its mass number, Z is its atomic number and its neutron number is $N = A - Z$.
- Isotones have equal neutron numbers but different atomic numbers. Isotopes instead share Z , while isobars share A .

Step 1: Apply the isotone condition For two isotones, the defining conditions are

$$A_1 - Z_1 = A_2 - Z_2, \quad Z_1 \neq Z_2.$$

For example, ${}^{14}_6\text{C}$ and ${}^{15}_7\text{N}$ each have eight neutrons, while their atomic numbers differ.

Step 2: Distinguish the other nuclear relationships The equal mass numbers in option (A) define isobars, and the equal atomic numbers in option (B) define isotopes. Option (D) explicitly requires different neutron numbers, contradicting the essential isotone condition. Option (C) states both required features.

Step 3: Conclusion Isotones have the same neutron number but different atomic numbers. Hence, the correct option is (C).

Quick Tip to Solve: Calculate $A - Z$ for each nuclide: equal results with different Z identify isotones immediately.

13. Assertion (A): A charged particle is moving with velocity v in x - y plane, making an angle θ ($0 < \theta < \frac{\pi}{2}$) with x -axis. If a uniform magnetic field $\vec{B}$ is applied in the region, along y -axis, the particle will move in a helical path with its axis parallel to x -axis.

Reason (R): The direction of the magnetic force acting on a charged particle moving in a magnetic field is along the velocity of the particle.

Select the correct answer:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (D) Both Assertion (A) and Reason (R) are false.

Solution:

Concept:

- The magnetic Lorentz force is $\vec{F} = q(\vec{v} \times \vec{B})$, perpendicular to both the velocity and the magnetic field.
- A nonzero velocity component parallel to a uniform magnetic field remains constant, while the perpendicular component produces circular motion. Their combination is a helix with axis parallel to the field.

Step 1: Evaluate the assertion Here $\vec{B}$ is along the y -axis. Since the initial velocity makes angle θ with the x -axis,

$$v_{\parallel} = v \sin \theta, \quad v_{\perp} = v \cos \theta.$$

Both are nonzero for $0 < \theta < \pi/2$. The particle therefore follows a helix whose axis is parallel to the y -axis. The assertion specifies the x -axis and is false.

Step 2: Evaluate the reason The cross product in $\vec{F} = q(\vec{v} \times \vec{B})$ makes the magnetic force perpendicular to the instantaneous velocity, not along it. Consequently, $\vec{F} \cdot \vec{v} = 0$, and the magnetic field changes direction of motion without doing work. The reason is false.

Step 3: Check the explanatory relation A false claim about the force direction cannot explain the false assertion about the helix axis. Options (A), (B) and (C) all require a true assertion, so none applies. The printed option (D) correctly combines the two false statements.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: The axis of a magnetic helix follows $\vec{B}$, regardless of the initial velocity's transverse direction; magnetic force is always perpendicular to velocity.

14. Assertion (A): A ray of light is incident normally on the face of a prism. The emergent ray will graze along the opposite face of the prism when the critical angle at glass-air interface is equal to the angle of the prism.

Reason (R): The refractive index of a prism depends on angle of the prism.

Select the correct answer:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- Inside a prism, the refraction angles measured from the two face normals satisfy $r_1 + r_2 = A_p$, where A_p is the prism angle.
- For a glass-air boundary, light incident at the critical angle C emerges at 90° to the normal, grazing the face. The refractive index is a property of the material for the specified wavelength and conditions.

Step 1: Evaluate the assertion Normal incidence on the first face gives $i_1 = 0$ and hence $r_1 = 0$ by Snell's law. Therefore, the incidence angle on the second face is

$$r_2 = A_p - r_1 = A_p.$$

If the critical angle equals the prism angle, then $r_2 = C$. The emergent ray has refraction angle 90° and grazes the opposite face. Thus, the assertion is true.

Step 2: Evaluate the reason Changing a prism's angle changes its geometry and the resulting deviation, but does not change the material's refractive index at a given wavelength and temperature. Although μ can be measured using the prism angle and minimum deviation together, it does not intrinsically depend on the prism angle. The reason is false.

Step 3: Check the explanatory relation Grazing emergence follows from the prism-angle relation and the definition of critical angle, not from the claimed dependence of refractive index on shape. Options (A) and (B) require a true reason; option (D) requires a false assertion. All three are excluded.

Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: With normal entry into a prism, set $r_1 = 0$, so the second-face incidence equals the prism angle; compare that angle directly with the critical angle.

15. Assertion (A): EM waves do not require a medium for their propagation.

Reason (R): EM waves are transverse waves.

Select the correct answer:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- An electromagnetic wave consists of time-varying electric and magnetic fields and can propagate through vacuum.
- In a plane electromagnetic wave, $\vec{E}$ and $\vec{B}$ are perpendicular to each other and to the propagation direction; this is the transverse nature of the wave.

Step 1: Judge the assertion The assertion is true. Electromagnetic waves require no material particles to carry the disturbance. Changing electric and magnetic fields sustain propagation in vacuum, with speed $c = 1/\sqrt{\mu_0\epsilon_0}$.

Step 2: Judge the reason The reason is also true: both field vectors oscillate perpendicular to the direction of travel, so electromagnetic waves are transverse.

Step 3: Test the explanatory link Transverse character specifies the direction of oscillation; it does not establish whether a medium is required. For example, a transverse wave on a stretched string does require a material medium. Therefore, R does not explain A. Option (A) incorrectly makes an explanatory connection, while (C) and (D) incorrectly declare a true statement false.

Step 4: Conclusion Both A and R are true, but R is not the correct explanation of A. Hence, the correct option is (B).

Quick Tip to Solve: Separate wave geometry from the need for a medium: a transverse string wave is a quick counterexample to the proposed explanation.

16. Assertion (A): The minimum negative potential applied to the anode in a photoelectric experiment at which photoelectric current becomes zero, is called cut-off voltage.

Reason (R): The threshold frequency for a metal is the minimum frequency of incident radiation below which emission of photoelectrons does not take place.

Select the correct answer:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- The stopping or cut-off potential V_0 is the magnitude of the minimum retarding potential that stops even the fastest photoelectrons: $eV_0 = K_{\max}$, where e is the elementary charge.
- The threshold frequency ν_0 is set by the metal's work function ϕ : $h\nu_0 = \phi$. Emission requires $\nu \geq \nu_0$.

Step 1: Judge the assertion The assertion is true. Making the collecting anode negative relative to the emitting cathode retards the emitted electrons. At the least retarding voltage sufficient to stop the most energetic electrons, the photocurrent becomes zero. This defines the cut-off voltage.

Step 2: Judge the reason The reason is true. Below ν_0 , each incident photon has energy $h\nu < \phi$, so a photon cannot supply enough energy to release an electron from the metal.

Step 3: Test whether the reason explains the assertion The reason states the condition for emission at the metal surface; it does not explain how a retarding voltage prevents already emitted electrons from reaching the anode. Although Einstein's equation relates the quantities through $eV_0 = h(\nu - \nu_0)$, the stated definition of threshold frequency is not the explanation of cut-off voltage. Thus (A) supplies a false causal link, and (C) and (D) wrongly reject true statements.

Step 4: Conclusion Both A and R are true, but R is not the correct explanation of A. Hence, the correct option is (B).

Quick Tip to Solve: Threshold frequency controls whether electrons leave the metal; stopping potential controls whether emitted electrons reach the collector.

Section B (10 marks)

17. A cell of emf E and internal resistance r is connected across a resistor of variable resistance R . Show graphically the variation of

(a) the terminal voltage across the cell,

(b) the current supplied by the cell,

with R as it is increased from 0 to the maximum value.

[2 marks]

Solution:

Concept:

- A cell of emf E and internal resistance $r > 0$, supplying an external resistance R , drives current $I = E/(R + r)$.
- The terminal voltage is $V = E - Ir = IR$; the internal voltage drop Ir is lost inside the cell.

Step 1: Obtain the terminal voltage curve Substituting the current into the terminal voltage gives

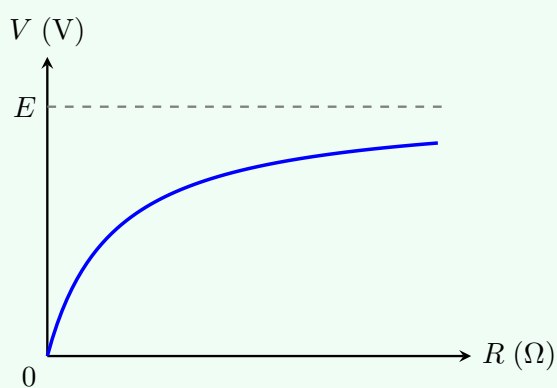
$$V = \frac{ER}{R + r}.$$

At $R = 0$, the terminals are short-circuited and $V = 0$. As R increases, the current and internal voltage drop fall, so V rises. The increase becomes progressively slower. For very large R , V approaches E from below; the dashed line is its horizontal asymptote. At $R = r$, $V = E/2$.

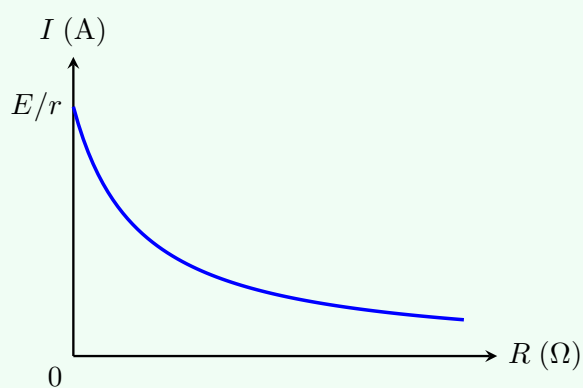
Step 2: Obtain the current curve Applying the circuit equation $E = I(R + r)$,

$$I = \frac{E}{R + r}.$$

The current starts from its largest value E/r at $R = 0$. It decreases rapidly at first and then more slowly as R increases, approaching zero for very large R . At $R = r$, $I = E/(2r)$. The two curves below show these shapes; for a finite maximum resistance, terminate each curve at that value of R .



(a) Terminal voltage



(b) Current

Terminal voltage and supplied current as the external resistance increases.

Step 3: Conclusion The V -versus- R graph rises from zero towards E , while the I -versus- R graph falls from E/r towards zero. Both variations are nonlinear.

Quick Tip to Solve: Fix the intercepts using short circuit $R = 0$, then use the open-circuit limit $R \rightarrow \infty$ to place the two asymptotes.

18(a). Using the mirror equation and the formula of magnification, deduce that “the virtual image produced by a convex mirror is always diminished in size and is located between the pole and the focus.”

[2 marks]

Solution:

Concept:

- With the Cartesian sign convention, the mirror formula is $1/f = 1/v + 1/u$, where u , v and f are object distance, image distance and focal length measured from the pole.
- The linear magnification is $m = h_i/h_o = -v/u$. Positive magnification means an erect image; $0 < m < 1$ means an erect, diminished image.

Step 1: Apply the mirror formula with correct signs For a real object in front of the convex mirror, put $u = -a$, where $a > 0$. The principal focus is behind the mirror, so $f > 0$. Then

$$\begin{aligned}\frac{1}{f} &= \frac{1}{v} - \frac{1}{a}, \\ \frac{1}{v} &= \frac{1}{f} + \frac{1}{a}, \\ v &= \frac{af}{a+f}.\end{aligned}$$

Since $a, f > 0$, the image distance is positive and $v/f = a/(a+f) < 1$. Thus $0 < v < f$: the virtual image is behind the mirror, between its pole and focus, for every finite real-object distance.

Step 2: Evaluate the magnification Using the same sign convention,

$$m = -\frac{v}{u} = \frac{v}{a} = \frac{f}{a+f}.$$

The denominator exceeds the positive numerator, giving $0 < m < 1$. The positive sign establishes that the image is erect, and the magnitude less than unity establishes that its height is smaller than the object's. As the object recedes to infinity, the image tends to the focus and its size tends to zero.

Step 3: Conclusion A convex mirror forms an erect, virtual and diminished image of a real object, with $0 < v < f$ and $0 < m < 1$. For an object at infinity, the limiting image is at the focus.

Quick Tip to Solve: Write $u = -a$ and $f > 0$ first; the fractions $v/f = a/(a+f)$ and $m = f/(a+f)$ immediately prove the two required inequalities.

OR

18(b). A convex lens of focal length 10 cm, a concave lens of focal length 15 cm and a third lens of unknown focal length are placed coaxially in contact. If the focal length of the combination is +12 cm, find the nature and focal length of the third lens, if all lenses are thin. Will the answer change if the lenses were thick?

[2 marks]

Solution:

Concept:

- For thin lenses in contact, powers add: $1/F = 1/f_1 + 1/f_2 + 1/f_3$, where F is the equivalent focal length and f_i are the individual focal lengths in the same units.
- A converging convex lens has positive focal length and a diverging concave lens has negative focal

length. The contact-power formula assumes negligible lens thickness.

Step 1: Insert signed focal lengths The first convex lens has $f_1 = +10$ cm, the concave lens has $f_2 = -15$ cm, and the combination has $F = +12$ cm. Keeping every focal length in centimetres,

$$\frac{1}{12} = \frac{1}{10} - \frac{1}{15} + \frac{1}{f_3}.$$

The negative sign for the concave lens is essential: its power partly cancels that of the first convex lens.

Step 2: Calculate the third lens's focal length and nature Rearranging gives

$$\begin{aligned}\frac{1}{f_3} &= \frac{1}{12} - \frac{1}{10} + \frac{1}{15} \\ &= \frac{5 - 6 + 4}{60} = \frac{1}{20} \text{ cm}^{-1}.\end{aligned}$$

Hence $f_3 = +20$ cm. Its positive focal length means that the third lens is converging, namely a convex lens under the usual glass-in-air convention.

Step 3: Examine the thick-lens case For thick lenses, the effective principal planes generally do not coincide even when the lens surfaces touch. Thickness and separations between these planes affect the equivalent power, so simply adding the individual powers is no longer valid. The inferred focal length may therefore change; $+20$ cm is not guaranteed. The information supplied is insufficient to calculate the thick-lens result.

Step 4: Conclusion For thin lenses, the third lens is convex with focal length $+20$ cm. For thick lenses the result generally changes and requires additional optical and geometrical data.

Quick Tip to Solve: Assign the concave lens a negative focal length before subtracting known powers; use the simple sum only for thin lenses in contact.

19. Write two differences in the patterns of double-slit interference experiment and single-slit diffraction experiment.

Light waves from two pinholes illuminated by two sodium lamps do not produce interference patterns. Explain why. [2 marks]

Solution:

Concept:

- Interference is the superposition of waves from two coherent sources; a sustained pattern needs a constant relative phase.
- Single-slit diffraction is the superposition of secondary wavelets from different parts of one slit. Its minima satisfy $a \sin \theta = n\lambda$, where a is the slit width.

Step 1: Compare the widths of the bright fringes In the usual double-slit interference pattern, bright fringes have equal widths and are equally spaced, with fringe width $\beta = \lambda D/d$; D is the screen distance and d is the slit separation. In single-slit diffraction, the central bright maximum has width $2\lambda D/a$, approximately twice the width of a secondary bright maximum.

Step 2: Compare the brightness of the maxima For equal-intensity, narrow coherent slits, the interference maxima have equal intensities in the ideal interference pattern. In single-slit diffraction

the central maximum is the brightest and the secondary maxima become successively weaker away from the centre. A real double-slit pattern can also have a diffraction envelope; the comparison here is with the ideal interference pattern.

Step 3: Explain the result for two sodium lamps Separate sodium lamps emit light independently. Even though their characteristic wavelengths are nearly the same, their relative phase changes rapidly and randomly. Thus the two illuminated pinholes are not coherent sources. The bright and dark interference positions fluctuate too rapidly, so the time-averaged intensity has no stable interference fringes.

Step 4: Conclusion Interference fringes are equally wide with equal ideal maximum intensities; diffraction has a central maximum twice as wide and progressively weaker secondary maxima. Two independent sodium lamps lack a constant phase difference and hence give no sustained interference pattern.

Quick Tip to Solve: Compare width and intensity separately; matching wavelengths alone does not make two independent lamps coherent.

20. Draw energy band diagrams of n-type and p-type semiconductors at temperature $T > 0$ K. Show the donor/acceptor energy levels with the order of difference of their energies from the bands.

[2 marks]

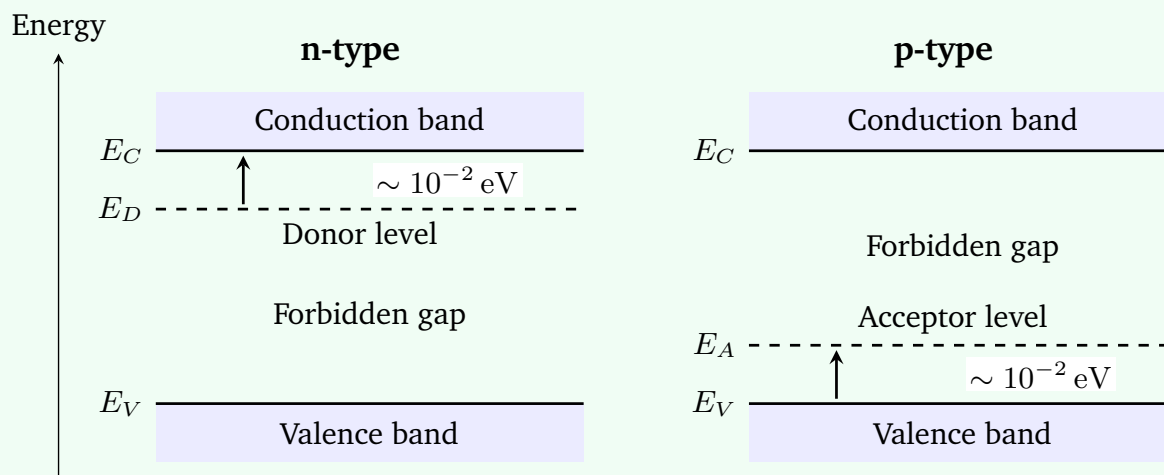
Solution:

Concept:

- Doping introduces allowed impurity levels in the forbidden energy gap between the valence-band edge E_V and conduction-band edge E_C .
- A donor supplies an electron to the conduction band; an acceptor takes an electron from the valence band and leaves a mobile hole there.

Step 1: Place the donor level in the n-type semiconductor Pentavalent impurities produce a donor level E_D just below E_C . Its separation is of order 10^{-2} eV, much smaller than the main forbidden gap of order 1 eV. At nonzero temperature, thermal excitation can lift donor electrons into the conduction band, leaving positively ionised donor atoms. Electrons are the majority carriers.

Step 2: Place the acceptor level in the p-type semiconductor Trivalent impurities produce an acceptor level E_A just above E_V , again separated by an energy of order 10^{-2} eV. A valence electron can gain this small thermal energy and occupy the acceptor level. The impurity becomes negatively ionised and a mobile hole remains in the valence band. Holes are the majority carriers. The arrows in the diagrams show electron excitation; occupation depends on temperature.



Energy bands at $T > 0\text{ K}$; arrows show electron excitation. Separations are schematic.

Step 3: Conclusion The n-type donor level lies just below the conduction band, with $E_C - E_D \sim 10^{-2}\text{ eV}$. The p-type acceptor level lies just above the valence band, with $E_A - E_V \sim 10^{-2}\text{ eV}$.

Quick Tip to Solve: Remember the destination of the electron: donor to conduction band, but valence band to acceptor level.

21. Briefly explain how energy is produced in stars, giving two examples of the nuclear reactions involved. [2 marks]

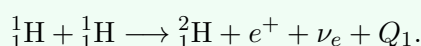
Solution:

Concept:

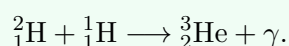
- Nuclear fusion combines light nuclei into more tightly bound nuclei. The decrease in total rest mass releases energy according to $Q = \Delta mc^2$, where c is the speed of light.
- Very high temperature and pressure in stellar cores permit thermonuclear reactions; hydrogen burning converts hydrogen into helium through a sequence of reactions.

Step 1: Explain the source of stellar energy At the high core temperature, light nuclei have large kinetic energies and can approach closely enough for fusion, aided by quantum tunnelling through the electrostatic barrier. The products have greater binding energy per nucleon and lower total rest mass than the reactants. The released energy heats the stellar material and ultimately emerges as radiation. Thus fusion, rather than an ordinary chemical reaction, sustains the energy output of stars.

Step 2: Give two reactions in the proton-proton chain A first reaction forms deuterium from two protons, emitting a positron and an electron neutrino:



The deuterium nucleus then captures another proton to form helium-3 and emit a gamma photon:



Here Q_1 denotes released energy in the first reaction; in the second, the gamma photon and recoil carry the released energy. Both equations conserve charge and nucleon number. Further reactions complete the conversion to helium-4.

Step 3: Conclusion Stars produce energy by thermonuclear fusion of light nuclei. The two proton-proton-chain reactions above release the mass-defect energy as particle kinetic energy and radiation.

Quick Tip to Solve: In each fusion equation, check nucleon number and charge separately; include the positron and neutrino when two protons form deuterium.

Section C (21 marks)

22. Three cells A , B and C of emfs 2 V , 3 V and 5 V respectively are connected in parallel to each other. Their internal resistances are $5\ \Omega$, $5\ \Omega$ and $1\ \Omega$ respectively. Calculate the currents flowing through the cells A , B and C . [3 marks]

Solution:

Concept:

- Cells connected in parallel with like terminals joined have the same terminal potential difference V . For cell j , take I_j positive when it supplies current out of its positive terminal, so $I_j = (E_j - V)/r_j$.
- Kirchhoff's junction law expresses charge conservation. With no external branch, the algebraic sum of currents supplied by the three cells is zero: $I_A + I_B + I_C = 0$.

Step 1: Express each current using the common terminal voltage Let V be the potential of the common positive-terminal junction relative to the common negative-terminal junction. The given emfs are $E_A = 2\text{ V}$, $E_B = 3\text{ V}$, and $E_C = 5\text{ V}$, with internal resistances $r_A = r_B = 5\ \Omega$ and $r_C = 1\ \Omega$. Therefore, with V in volts,

$$I_A = \frac{2 - V}{5}, \quad I_B = \frac{3 - V}{5}, \quad I_C = 5 - V,$$

where the numerical current values are in amperes. A negative value will mean that the cell absorbs current at its positive terminal.

Step 2: Apply the junction law to find the voltage No external load is specified, so current supplied by a cell must return through the other cells. Hence

$$\begin{aligned} \frac{2 - V}{5} + \frac{3 - V}{5} + (5 - V) &= 0, \\ (2 - V) + (3 - V) + 5(5 - V) &= 0, \\ 30 - 7V &= 0. \end{aligned}$$

The common terminal voltage is $V = 30/7\text{ V}$. It lies below E_C , but above E_A and E_B , so C supplies energy while A and B absorb it.

Step 3: Calculate the branch currents and interpret their signs Substitute the common terminal voltage into the three branch equations:

$$\begin{aligned} I_A &= \frac{2 - 30/7}{5} = -\frac{16}{35}\text{ A}, \\ I_B &= \frac{3 - 30/7}{5} = -\frac{9}{35}\text{ A}, \\ I_C &= 5 - \frac{30}{7} = \frac{5}{7}\text{ A}. \end{aligned}$$

As a check, $-16/35 - 9/35 + 25/35 = 0$. Inside A and B, current flows from their positive terminals to their negative terminals; inside C, it flows from its negative terminal to its positive terminal.

Step 4: Conclusion The currents are 0.457 A through A and 0.257 A through B, both entering their positive terminals (charging direction), and 0.714 A through C, leaving its positive terminal (discharging direction).

Quick Tip to Solve: First find the shared terminal voltage; comparing it with each emf immediately gives the direction of that cell's current.

23(a). (i) Write Biot-Savart's law in vector form.

(ii) Two identical circular coils A and B, each of radius R , carrying currents I and $\sqrt{3}I$ respectively, are placed concentrically in XY and YZ planes respectively. Find the magnitude and direction of the net magnetic field at their common centre. [3 marks]

Solution:

Concept:

- Biot-Savart's law gives the field due to a current element; its direction is perpendicular to both the current element and the displacement towards the observation point.
- A circular loop of radius R , carrying current I , produces a central field $B = \mu_0 I / (2R)$, normal to its plane, with sense given by the right-hand rule.
- Magnetic fields superpose as vectors. Perpendicular fields combine by the Pythagorean theorem, not by adding their magnitudes.

Step 1: State the vector law for part (i) For a current element $I d\vec{l}$, let $\vec{r} = r\hat{r}$ point from the element to the observation point. In free space,

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{r}}{r^3}.$$

Here $d\vec{l}$ points along conventional current and μ_0 is the permeability of free space. The cross product fixes the direction as well as the sine factor in the magnitude.

Step 2: Find the two central fields in part (ii) Coil A lies in the XY plane, so its field is along the Z axis. Coil B lies in the YZ plane, so its field is along the X axis. Thus the fields are perpendicular and their magnitudes are

$$B_A = \frac{\mu_0 I}{2R}, \quad B_B = \frac{\sqrt{3}\mu_0 I}{2R}.$$

The problem does not specify the circulation senses. For fields along positive Z and positive X , respectively,

$$\vec{B}_{\text{net}} = \frac{\mu_0 I}{2R} (\sqrt{3}\hat{i} + \hat{k}).$$

Reversing either current reverses only its corresponding component.

Step 3: Combine the perpendicular components The resultant lies in the XZ plane. Its magnitude and its angle θ with the field of coil A follow from

$$B_{\text{net}} = \sqrt{B_A^2 + B_B^2} = \frac{\mu_0 I}{2R} \sqrt{1+3},$$

$$\tan \theta = \frac{B_B}{B_A} = \sqrt{3}.$$

The direction is towards the field of coil B from that of coil A ; the actual positive or negative axis senses follow from the currents by the right-hand rule.

Step 4: Conclusion Biot-Savart's law is $d\vec{B} = (\mu_0 I / 4\pi r^2) d\vec{l} \times \hat{r}$. The net field is $B_{\text{net}} = \mu_0 I / R$, at 60° to $\vec{B}_A$ and 30° to $\vec{B}_B$, in the XZ plane. If each coil has N turns, all field magnitudes acquire the same factor N ; the angles are unchanged.

Quick Tip to Solve: First draw the normals to the coil planes: these are the field directions. Their ratio $1 : \sqrt{3}$ immediately gives the resultant's angles.

OR

23(b). (i) A rectangular loop of sides l and b carries a current I clockwise. Write the magnetic moment $\vec{m}$ of the loop and show its direction in a diagram.

(ii) The loop is placed in a uniform magnetic field $\vec{B}$ and is free to rotate about an axis which is perpendicular to $\vec{B}$. Prove that the loop experiences no net force, but a torque $\vec{\tau} = \vec{m} \times \vec{B}$. [3 marks]

Solution:

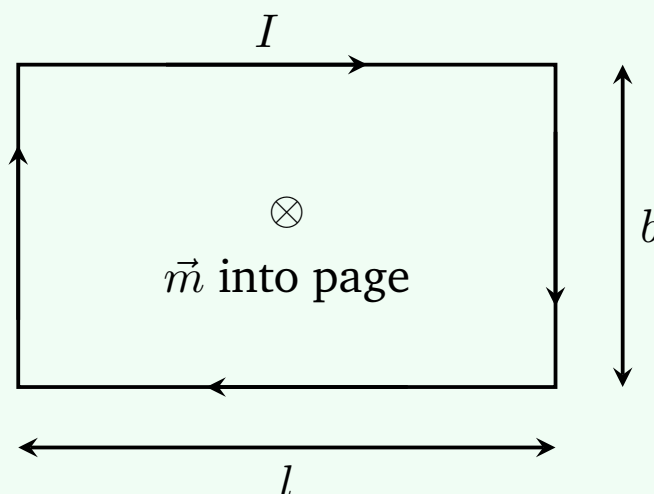
Concept:

- A plane current loop has magnetic moment $\vec{m} = IA\hat{n}$, where A is its area and $\hat{n}$ is the unit normal chosen by the right-hand rule.
- The magnetic force on a straight side with length vector $\vec{L}$ is $\vec{F} = I\vec{L} \times \vec{B}$. Equal, opposite forces can form a couple even when their resultant force is zero.

Step 1: Find and show the magnetic moment in part (i) The rectangular loop has area $A = lb$. Curl the fingers of the right hand along its clockwise current: the thumb points into the page. Thus, if $\hat{k}$ points out of the page,

$$\vec{m} = Ilb\hat{n} = -Ilb\hat{k}.$$

The magnitude is Ilb , in Am^2 . The cross at the centre of the diagram represents a vector directed away from the viewer, into the page.



Clockwise current produces a magnetic moment into the page.

Step 2: Add the forces in the uniform field for part (ii) The length vectors of opposite sides are equal

and opposite. Since the same uniform $\vec{B}$ acts on each side, their magnetic forces are also equal and opposite. More compactly, summing around the closed loop gives

$$\vec{F}_{\text{net}} = I \oint d\vec{l} \times \vec{B} = I \left(\oint d\vec{l} \right) \times \vec{B} = \vec{0}.$$

The integral of displacement around a closed path is zero. Uniformity is essential: it permits taking $\vec{B}$ outside the integral.

Step 3: Calculate the couple and its direction Let θ be the angle between the loop normal $\hat{n}$ and $\vec{B}$. To evaluate the turning couple, take the two length- l sides perpendicular to $\vec{B}$. Each then experiences force $F = IlB$, in opposite senses. Their perpendicular separation along the field direction is $b \sin \theta$, so

$$\tau = F(b \sin \theta) = IlbB \sin \theta = mB \sin \theta.$$

The forces on the other two sides have the same line of action and hence add no couple. Applying the right-hand rule to the turning sense gives the direction $\hat{n} \times \vec{B}$. Hence the vector relation is

$$\vec{\tau} = Ilb \hat{n} \times \vec{B} = \vec{m} \times \vec{B}.$$

This torque tends to align $\vec{m}$ with $\vec{B}$; it is zero when they are parallel or antiparallel. The freely rotating axis perpendicular to the field permits this turning motion.

Step 4: Conclusion The clockwise loop has $\vec{m} = Ilb\hat{n}$, directed into the page. In a uniform magnetic field its net force is zero, but it experiences $\vec{\tau} = \vec{m} \times \vec{B}$, of magnitude $IlbB \sin \theta$.

Quick Tip to Solve: Use the loop normal, not its plane, to define θ . Clockwise current means a cross for the magnetic moment; a zero resultant force need not mean zero torque.

24. (a) State Faraday's law of electromagnetic induction and explain the role of negative sign in its expression.

(b) Explain, with an example, that Lenz's law is consistent with the law of conservation of energy.

[3 marks]

Solution:

Concept:

- Magnetic flux through a loop is $\Phi_B = \int \vec{B} \cdot d\vec{A}$. A change in magnetic flux linkage induces an emf, whether the change is caused by the field, the loop area or its orientation.
- Faraday's law sets the magnitude of induced emf; Lenz's law fixes its sense so that the resulting current opposes the change responsible for induction.

Step 1: State Faraday's law for part (a) Whenever the magnetic flux linked with a circuit changes, an emf is induced in it. Its magnitude equals the rate of change of flux linkage. For a coil of N turns, with the same flux Φ_B through each turn,

$$\varepsilon = -\frac{d(N\Phi_B)}{dt} = -N\frac{d\Phi_B}{dt}.$$

Here ε is measured in volts and Φ_B in webers. An induced current flows if the circuit is closed; the emf can exist even when the circuit is open.

Step 2: Interpret the negative sign Choose the positive loop direction and positive area normal consistently by the right-hand rule. The minus sign expresses Lenz's law: the induced current produces a magnetic effect opposing the *change* in external flux. Thus an increasing flux is opposed, whereas a decreasing flux is supported. It does not mean that the induced field must always oppose the external field, nor that every measured emf is numerically negative.

Step 3: Use an approaching magnet for part (b) Move the north pole of a bar magnet towards a closed conducting coil. The flux through the coil increases. The induced current makes the near face of the coil a north pole, which repels the approaching magnet and opposes its motion. An external agent must therefore do work to keep pushing the magnet towards the coil. For a resistive coil, this supplied mechanical energy ultimately appears as Joule heat, with instantaneous dissipation $P = I^2 R$; energy can also be temporarily stored in the magnetic field.

Step 4: Check energy conservation by reversing the argument If the induced current instead aided the approach, the coil would attract the approaching north pole, speeding the magnet up while also generating electrical energy. The increased speed would increase induction further, producing energy without the required external work or loss of another energy store. Lenz's opposing sense prevents this. On withdrawing the magnet, the near face becomes south and attracts it, again opposing the motion, so external work is still required.

Step 5: Conclusion Faraday's law is $\mathcal{E} = -N d\Phi_B/dt$. Its negative sign represents opposition to the change in flux. The work required to move a magnet against the induced magnetic force supplies the electrical energy, so Lenz's law is consistent with conservation of energy.

Quick Tip to Solve: For a moving magnet, first decide whether it approaches or recedes. Make the coil repel an approaching pole and attract a receding pole; both oppose the motion causing induction.

25. (a) Differentiate between 'conduction current' and 'displacement current', giving one similarity and one dissimilarity between them.

(b) Explain the existence of electromagnetic waves in free space, using the concept of displacement current.

[3 marks]

Solution:

Concept:

- Conduction current is the rate of transport of electric charge through a surface: $I_c = dq/dt$.
- Maxwell's displacement current in vacuum is $I_d = \epsilon_0 d\Phi_E/dt$, where Φ_E is electric flux through the surface and ϵ_0 is the permittivity of free space.
- A changing electric field produces a magnetic field through the displacement-current term, while a changing magnetic field produces an electric field by Faraday's law.

Step 1: (a) Compare the physical origins of the currents Conduction current arises from the actual motion of charged particles, for example the drift of free electrons in a metallic wire. Displacement current arises from a time-varying electric flux and does not require transport of charges through the region. Thus it can exist in the vacuum between the plates of a charging capacitor, where no conduction current crosses the gap. This is the required dissimilarity.

Step 2: (a) State their common magnetic effect Both currents produce a magnetic field. They enter

the generalized Ampere law in the same additive way:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(I_c + I_d).$$

Here μ_0 is the permeability of free space. For an ideal charging capacitor the displacement current between the plates equals the conduction current in the connecting wires, ensuring a consistent magnetic field for different spanning surfaces.

Step 3: (b) Explain propagation in free space Free space contains no material charge carriers, so $I_c = 0$. Nevertheless, a time-varying electric field gives a nonzero I_d , and the generalized Ampere law therefore permits a magnetic field there. If that magnetic field varies with time, Faraday's law produces an electric field:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}.$$

The coupled, time-varying electric and magnetic fields propagate through space as an electromagnetic wave. No material medium is needed. The two fields are perpendicular to each other and to the direction of propagation, and their speed in vacuum is $c = 1/\sqrt{\mu_0\epsilon_0}$.

Step 4: Conclusion Conduction and displacement currents both produce magnetic fields, but only conduction current requires charge transport. Displacement current allows the coupled electric and magnetic fields of an electromagnetic wave to propagate in free space.

Quick Tip to Solve: Use the charging-capacitor gap to remember the distinction: charge crosses the wire, but only changing electric flux crosses the vacuum gap.

26. (a) Define 'work function' of a metal. How can its value be determined from a graph between stopping potential and frequency of the incident radiation?

(b) The work function of a metal is 2.4 eV. A stopping potential of 0.6 V is required to reduce the photocurrent to zero, in a photoelectric experiment. Calculate the wavelength of light used. [3 marks]

Solution:

Concept:

- The work function ϕ is the minimum energy required to remove an electron from the surface of a metal, leaving it just outside the surface with zero kinetic energy.
- Einstein's photoelectric equation is $h\nu = \phi + K_{\max}$, with $K_{\max} = eV_s$. Here h is Planck's constant, ν is frequency, e is the elementary charge and V_s is the magnitude of the stopping potential.
- The photon energy is $h\nu = hc/\lambda$, where c is the speed of light and λ is its wavelength in vacuum.

Step 1: (a) Relate the stopping-potential graph to the work function Rearrange Einstein's equation into the straight-line form for a plot of V_s on the vertical axis against ν on the horizontal axis:

$$V_s = \frac{h}{e}\nu - \frac{\phi}{e}.$$

The line has slope h/e . Its intercept on the frequency axis, where $V_s = 0$, is the threshold frequency ν_0 . Read ν_0 from the graph and calculate $\phi = h\nu_0$. Equivalently, if the line is extrapolated to the vertical axis and its intercept is b , then $b = -\phi/e$, so $\phi = e|b|$. The negative intercept is an extrapolation; photoemission does not occur below the threshold frequency.

Step 2: (b) Find the incident photon energy The maximum photoelectron kinetic energy equals

the electrical energy needed to stop it. A stopping potential of 0.6 V corresponds to $K_{\max} = 0.6 \text{ eV}$. Therefore,

$$E_{\gamma} = \phi + K_{\max} = 2.4 + 0.6 = 3.0 \text{ eV},$$

$$E_{\gamma} = 3.0(1.6 \times 10^{-19}) = 4.8 \times 10^{-19} \text{ J}.$$

The incident energy includes both the energy spent escaping the metal and the kinetic energy retained by the fastest emitted electron.

Step 3: Convert photon energy into wavelength Using $h = 6.63 \times 10^{-34} \text{ J s}$ and $c = 3.0 \times 10^8 \text{ m s}^{-1}$,

$$\lambda = \frac{hc}{E_{\gamma}} = \frac{(6.63 \times 10^{-34})(3.0 \times 10^8)}{4.8 \times 10^{-19}} = 4.14 \times 10^{-7} \text{ m}.$$

Since $1 \text{ nm} = 10^{-9} \text{ m}$, this is approximately 414 nm.

Step 4: Conclusion The work function is obtained from the frequency-axis intercept as $\phi = h\nu_0$, or from the extrapolated vertical intercept as $\phi = e|b|$. The incident wavelength is $\lambda \approx 4.14 \times 10^{-7} \text{ m} = 414 \text{ nm}$.

Quick Tip to Solve: Add the work function in eV to the numerical stopping potential in volts, then use $\lambda(\text{nm}) \approx 1240/E_{\gamma}(\text{eV})$.

27. Write the mathematical forms of three postulates of Bohr's theory of the hydrogen atom. Using them prove that, for an electron revolving in the n^{th} orbit,

(a) the radius of the orbit is proportional to n^2 , and

(b) the total energy of the atom is proportional to $\left(\frac{1}{n^2}\right)$.

[3 marks]

Solution:

Concept:

- In Bohr's hydrogen model, an electron of mass m and charge magnitude e moves in a stationary circular orbit around the proton; write $k = 1/(4\pi\epsilon_0)$.
- Angular momentum is quantized as $mv_nr_n = n\hbar$, where $\hbar = h/(2\pi)$ and $n = 1, 2, 3, \dots$
- The electrostatic potential energy is $U = -ke^2/r$, taking zero energy at infinite separation. The total energy is $E = K + U$.

Step 1: State the three postulates mathematically First, an electron can revolve in certain stable, stationary orbits without radiating energy. In such an orbit $dE_n/dt = 0$ and $P_{\text{rad}} = 0$; the electrostatic force supplies its centripetal force:

$$\frac{mv_n^2}{r_n} = \frac{ke^2}{r_n^2}.$$

Second, only orbits with quantized angular momentum are allowed:

$$mv_nr_n = \frac{n\hbar}{2\pi} = n\hbar.$$

Third, radiation is emitted or absorbed only in a transition between two stationary states. Its frequency obeys

$$h\nu = |E_i - E_f|.$$

For emission $E_i > E_f$; for absorption the atom moves to the higher-energy state.

Step 2: (a) Eliminate the speed to obtain the radius From angular-momentum quantization, $v_n = n\hbar/(mr_n)$. Substitute this into the circular-orbit force equation:

$$\frac{n^2\hbar^2}{mr_n^3} = \frac{ke^2}{r_n^2},$$

$$r_n = \frac{n^2\hbar^2}{mke^2} = \frac{\varepsilon_0\hbar^2}{\pi me^2} n^2.$$

Every factor multiplying n^2 is constant for hydrogen; this establishes the required dependence of the orbit radius on the principal quantum number.

Step 3: (b) Combine kinetic and potential energies The force equation gives $mv_n^2 = ke^2/r_n$, so the kinetic energy is $K_n = ke^2/(2r_n)$. Adding the attractive electrostatic potential energy gives

$$E_n = \frac{ke^2}{2r_n} - \frac{ke^2}{r_n} = -\frac{ke^2}{2r_n},$$

$$E_n = -\frac{mk^2e^4}{2\hbar^2} \frac{1}{n^2} = -\frac{me^4}{8\varepsilon_0^2\hbar^2} \frac{1}{n^2}.$$

Thus the coefficient of $1/n^2$ is a negative constant. The negative sign means the electron is bound; energy must be supplied to separate it from the proton. As n increases the energy approaches zero from below, while its magnitude decreases.

Step 4: Conclusion $r_n \propto n^2$ and $E_n = -C/n^2$, where $C = me^4/(8\varepsilon_0^2\hbar^2) > 0$. Hence the printed proportionality $E_n \propto 1/n^2$ holds with a negative proportionality constant; for hydrogen, $E_n = -13.6/n^2$ eV.

Quick Tip to Solve: Eliminate speed with $mvr = n\hbar$ to get the radius first; then use $E = -ke^2/(2r)$ to obtain the energy dependence without further speed algebra.

28. Explain the process of formation of ‘depletion layer’ and ‘potential barrier’ in a p-n junction region of a diode, with the help of a suitable diagram. Which feature of junction diode makes it suitable for its use as a rectifier? [3 marks]

Solution:

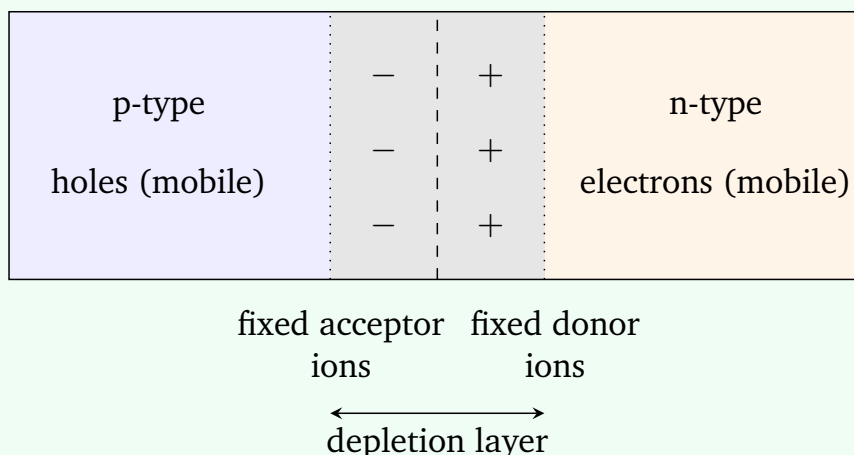
Concept:

- Diffusion carries majority carriers from a region of high carrier concentration to a region of low carrier concentration across a newly formed p-n junction.
- The uncovered, immobile ions form a space-charge region whose electric field opposes further majority-carrier diffusion. Its potential difference is the built-in potential barrier.

Step 1: Explain formation of the depletion layer On joining p-type and n-type semiconductor regions, electrons diffuse from the n-side to the p-side, while holes diffuse from the p-side to the n-side because of their concentration gradients. Electrons and holes recombine near the junction. The n-side close to the junction is consequently left with positively charged, immobile donor ions; the p-side is left with negatively charged, immobile acceptor ions. This narrow region becomes depleted of mobile majority carriers and is called the depletion layer. It contains fixed ions, so depletion does not mean absence of all charge.

Potential barrier: $V_n - V_p > 0$

$\vec{E}$ (n to p)
←



Unbiased p-n junction: fixed ions and electric field in the depletion region (schematic).

Step 2: Explain the potential barrier and equilibrium The fixed positive and negative ion layers establish an electric field directed from the n-side towards the p-side. Thus the n-side is at higher electric potential than the p-side. The field opposes the diffusion of majority carriers: its force on electrons is towards the n-side, and its force on holes is towards the p-side. The resulting potential difference across the depletion region is the potential barrier. As the barrier builds up, diffusion decreases. At thermal equilibrium, the drift current due to the junction field balances the diffusion current, giving zero net current without an external bias.

Step 3: State the property used for rectification A junction diode offers low resistance under forward bias, which reduces the barrier, and very high resistance under reverse bias, which increases it, before breakdown. It therefore conducts appreciable current in one direction and only a very small reverse current in the other. With an alternating input this asymmetric conduction allows one polarity to pass, producing a unidirectional, pulsating output.

Step 4: Conclusion Diffusion and recombination produce the depletion layer of fixed ions, whose electric field establishes the potential barrier. The diode's low forward resistance and high reverse resistance make it suitable as a rectifier.

Quick Tip to Solve: In the junction sketch place negative fixed ions on the p-side and positive fixed ions on the n-side; draw the barrier field from n to p.

Section D (8 marks)

Questions number 29 and 30 are case study-based questions. Read the following paragraphs and answer the questions that follow.

29. In a metallic conductor, an electron, moving due to thermal motion, suffers collisions with the heavy fixed ions but after collision, it will emerge out with the same speed but in random directions. If we consider all the electrons, their average velocity will be zero. When an electric field is applied, electrons move with an average velocity, known as drift velocity (v_d). The average time between successive collisions is known as relaxation time (τ). The magnitude of drift velocity per unit electric field is called mobility (μ). An expression for current through the conductor can be obtained in terms of drift velocity, number of electrons per unit volume (n), electronic charge ($-e$), and the cross-sectional area (A) of the conductor. This expression leads to an expression between current density ($\vec{J}$) and the electric field ($\vec{E}$). Hence, an expression for resistivity (ρ) of a metal is obtained. This expression helps us to understand increase in resistivity of a metal with increase in its temperature, in terms of change in the relaxation time (τ) and change in the number density of electrons (n). [4 marks]

29(i). (i) Consider two cylindrical conductors A and B , made of the same metal connected in series to a battery. The length and the radius of B are twice that of A . If μ_A and μ_B are the mobility of electrons in A and B respectively, then $\frac{\mu_A}{\mu_B}$ is: [1 mark]

- (A) $\frac{1}{2}$
- (B) $\frac{1}{4}$
- (C) 2
- (D) 1

Correct Answer: (D) 1

Solution:

Concept:

- Electron mobility is $\mu = v_d/E = e\tau/m_e$, where v_d is drift speed, E is electric field, e is the magnitude of electronic charge, τ is relaxation time and m_e is electron mass.
- For the same metal at the same temperature, mobility is a material property; it does not depend on a conductor's length or radius.

Step 1: Identify what determines mobility Both wires are made of the same metal, so under the usual common-temperature assumption their electron relaxation times are equal. Hence $e\tau/m_e$ is the same for both wires. Doubling the length and radius changes resistance and current density but does not directly change mobility.

Step 2: Check the geometric distractors In series the wires carry the same current. The area of B is four times that of A, so both its drift speed and electric field are one-fourth those in A. Their ratio v_d/E is therefore unchanged. Options (A), (B) and (C) incorrectly assign a geometric factor to mobility; in particular, $1/4$ describes the ratio of drift speeds B to A, not the requested mobility ratio.

Step 3: Conclusion $\mu_A/\mu_B = 1$. Hence, the correct option is (D).

Quick Tip to Solve: Write $\mu = e\tau/m_e$ before using the given dimensions: length and radius do not appear in mobility.

29(ii). (ii) A wire of length 0.5 m and cross-sectional area $1.0 \times 10^{-7} \text{ m}^2$ is connected to a battery of 2 V that maintains a current of 1.5 A in it. The conductivity of the material of the wire (in $\Omega^{-1} \text{ m}^{-1}$) is: [1 mark]

- (A) 2.5×10^4
- (B) 3.0×10^5
- (C) 3.75×10^6
- (D) 5.0×10^7

Correct Answer: (C) 3.75×10^6

Solution:

Concept:

- Ohm's law gives $R = V/I$, where V is potential difference and I is current.
- A uniform wire has $R = \rho L/A$, so its conductivity is $\sigma = 1/\rho = L/(RA) = IL/(VA)$; L and A are length and cross-sectional area.

Step 1: Relate conductivity to the measured quantities The voltage and current give $R = 2/1.5 = 4/3 \Omega$. Conductivity is the reciprocal of resistivity, rather than the reciprocal of resistance alone. Include the wire's length and area using $\sigma = L/(RA)$.

Step 2: Substitute in SI units and compare the options All data are already in SI units. Therefore,

$$\sigma = \frac{1.5 \times 0.5}{2 \times 1.0 \times 10^{-7}} = \frac{0.75}{2.0 \times 10^{-7}} = 3.75 \times 10^6 \Omega^{-1} \text{ m}^{-1}.$$

Only option (C) has both the correct numerical coefficient and power of ten. Options (A) and (B) are too small and (D) is too large; none follows from the supplied voltage, current and dimensions.

Step 3: Conclusion The conductivity is $3.75 \times 10^6 \Omega^{-1} \text{ m}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $\sigma = IL/(VA)$ directly and handle the reciprocal of 10^{-7} carefully before normalising the coefficient.

29(iii). (iii) The temperature coefficient of resistance of nichrome is $1.70 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$. In order to increase resistance of a nichrome wire by 8.5%, the temperature of the wire should be increased by: [1 mark]

- (A) 250°C
- (B) 500°C
- (C) 850°C
- (D) 1000°C

Correct Answer: (B) 500°C

Solution:

Concept:

- With temperature coefficient α , the linear resistance law is $R = R_0(1 + \alpha\Delta T)$, where R_0 is the initial resistance and ΔT is the temperature rise.
- The fractional resistance increase is $\Delta R/R_0 = \alpha\Delta T$. A percentage must be divided by 100 before

substituting in this relation.

Step 1: Express the stated rise as a fraction An increase of 8.5% means $\Delta R/R_0 = 8.5/100 = 0.085$. The given coefficient is $1.70 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$. The original resistance is not needed because it cancels in the fractional increase.

Step 2: Calculate the required temperature change Applying the stated linear law,

$$\Delta T = \frac{0.085}{1.70 \times 10^{-4}} = 500 \text{ }^\circ\text{C}.$$

Check the alternatives through $100\alpha\Delta T$: temperature rises of 250, 850 and $1000 \text{ }^\circ\text{C}$ give increases of 4.25%, 14.45% and 17%, respectively. Thus (A), (C) and (D) fail the required percentage.

Step 3: Conclusion The temperature must increase by $500 \text{ }^\circ\text{C}$. Hence, the correct option is (B).

Quick Tip to Solve: Divide the percentage by 100 first; $\Delta T = (\text{percentage increase})/(100\alpha)$ gives the rise, not the final temperature.

29(iv)(a). Consider the contribution of the following two factors I and II in resistivity of a metal:

I. Relaxation time of electrons

II. Number of electrons per unit volume

The resistivity of a metal increases with increase in its temperature because:

[1 mark]

- (A) I decreases and II increases.
- (B) I increases and II is almost constant.
- (C) Both I and II increase.
- (D) I decreases and II is almost constant.

Correct Answer: (D) I decreases and II is almost constant.

Solution:

Concept:

- In the free-electron model a metal has resistivity $\rho = m_e/(ne^2\tau)$, where m_e is electron mass, n is free-electron number density, e is electronic charge magnitude and τ is relaxation time.
- Raising the temperature of a metal increases lattice vibrations and electron scattering, while its free-electron number density remains approximately constant.

Step 1: Determine the changes in the two factors Stronger thermal vibrations of lattice ions cause more frequent scattering of conduction electrons. The average interval between successive collisions, τ , therefore decreases. A metal already has many free electrons, so heating does not produce a large change in their number density n ; factor II remains almost constant.

Step 2: Connect these changes to resistivity With n almost fixed, ρ is inversely proportional to τ , so the decrease of factor I increases resistivity. Option (A) incorrectly requires a significant increase in II. Options (B) and (C) incorrectly state that the relaxation time increases; increasing τ alone would lower resistivity.

Step 3: Conclusion Factor I decreases and factor II remains almost constant. Hence, the correct option is (D).

Quick Tip to Solve: For a heated metal, remember the chain: stronger lattice vibrations, shorter relaxation time, greater resistivity.

OR

29(iv)(b). A steady current flows in a copper wire of non-uniform cross-section. Consider the following three physical quantities:

I. Electric field

II. Current density

III. Drift speed

Then at the different points along the wire:

[1 mark]

(A) II and III change, but I is constant.

(B) I and II change, but III is constant.

(C) I and III change, but II is constant.

(D) All I, II and III change.

Correct Answer: (D) All I, II and III change.

Solution:

Concept:

- Charge conservation makes the steady current I identical through all cross-sections of a wire, but the current density is $J = I/A$, where A is the local cross-sectional area.
- In the same material, $E = \rho J$ and $v_d = J/(ne)$, where ρ is resistivity, n is free-electron number density and e is the magnitude of electron charge.

Step 1: Distinguish current from current density Steady flow prevents charge from continually accumulating within the wire, so the same current passes through every section. Since the area varies along the wire, $J = I/A$ varies: a narrower section has a greater current density. Constant current does not imply constant current density, eliminating option (C).

Step 2: Determine electric field and drift speed For a copper wire under the usual uniform-material and common-temperature assumption, ρ and n are constant. Thus

$$E = \frac{\rho I}{A}, \quad v_d = \frac{I}{neA}.$$

Both electric field and drift speed vary inversely with the local area, just as current density does. Option (A) wrongly keeps the electric field constant, and option (B) wrongly keeps the drift speed constant. All three quantities are larger in a narrower section.

Step 3: Conclusion Electric field, current density and drift speed all change along the non-uniform wire. Hence, the correct option is (D).

Quick Tip to Solve: Keep I constant in a steady wire, then write J , E and v_d in terms of the local area to test each option.

30. When light travels from an optically denser medium to an optically rarer medium, at the interface it is partly reflected back into the same medium and partly refracted to the second medium. The angle of incidence corresponding to an angle of refraction 90° is called the critical angle (i_c) for the given pair of media. This angle is related to the refractive index of medium 1 with respect to medium 2. Refraction of light through a prism involves refraction at two plane interfaces. A relation for the refractive index of the material of the prism can be obtained in terms of the refracting angle of the prism and the angle of minimum deviation. For a thin prism, this relation reduces to a simple equation. Laws of refraction are also valid for refraction of light at a spherical interface. When an object is placed in front of a spherical surface separating two media, its image is formed. A relation between object and image distance, in terms of refractive indices of two media and the radius of curvature of the spherical surface can be obtained. Using this relation for two surfaces of a lens, 'lens maker formula' is obtained. [4 marks]

30(i). (i) A small bulb is placed at the bottom of a tank containing a transparent liquid (refractive index n) to a depth H . The radius of the circular area of the surface of liquid, through which light from the bulb can emerge out, is R . Then $\left(\frac{R}{H}\right)$ is: [1 mark]

(A) $\frac{1}{\sqrt{n^2 - 1}}$

(B) $\sqrt{n^2 - 1}$

(C) $\frac{1}{\sqrt{n^2 + 1}}$

(D) $\sqrt{n^2 + 1}$

Correct Answer: (A) $\frac{1}{\sqrt{n^2 - 1}}$

Solution:

Concept:

- For light going from a liquid of refractive index n into air, the critical angle i_c satisfies $n \sin i_c = 1$.
- The limiting ray makes a right triangle with height H and horizontal side R , so $R/H = \tan i_c$.

Step 1: Identify the limiting ray At the escape circle's boundary, incidence is at i_c ; beyond it, total internal reflection occurs. The right triangle gives $R = H \tan i_c$.

Step 2: Express the tangent in terms of refractive index Using $\sin i_c = 1/n$,

$$\frac{R}{H} = \frac{\sin i_c}{\sqrt{1 - \sin^2 i_c}} = \frac{1/n}{\sqrt{1 - 1/n^2}} = \frac{1}{\sqrt{n^2 - 1}}.$$

Option (B) inverts the ratio; (C) and (D) violate $\cos^2 i_c = 1 - \sin^2 i_c$.

Step 3: Conclusion $R/H = 1/\sqrt{n^2 - 1}$. Hence, the correct option is (A).

Quick Tip to Solve: For an escape circle, use $R/H = \tan i_c$, converting the critical-angle sine to a tangent.

30(ii)(a). A parallel beam of light is incident on a face of a prism with refracting angle 60° . The angle of minimum deviation is found to be 30° . The refractive index of the material of the prism is close to: [1 mark]

- (A) 1.3
- (B) 1.4
- (C) 1.5
- (D) 1.6

Correct Answer: (B) 1.4

Solution:

Concept:

- At minimum deviation through a prism in air, the light path is symmetric: $i = e$ and $r_1 = r_2 = A/2$, where A is its refracting angle.
- The prism's refractive index is $n = \sin[(A + D_m)/2] / \sin(A/2)$, where D_m is the minimum deviation.

Step 1: Find the angles at minimum deviation With $A = 60^\circ$ and $D_m = 30^\circ$, the refraction angle at the first face is $r = A/2 = 30^\circ$. From $D_m = 2i - A$, the incidence angle is $i = (60^\circ + 30^\circ)/2 = 45^\circ$.

Step 2: Apply Snell's law and compare the options The surrounding medium is air, so

$$n = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2} \simeq 1.414.$$

Among 1.3, 1.4, 1.5, 1.6, the closest value is 1.4. The other three values are farther from 1.414. The thin-prism approximation is unsuitable for a refracting angle of 60° .

Step 3: Conclusion The refractive index is approximately 1.4. Hence, the correct option is (B).

Quick Tip to Solve: At minimum deviation, halve $A + D_m$ for the incidence angle and halve A for the refraction angle before taking their sine ratio.

OR

30(ii)(b). The angle of minimum deviation for a ray of light incident on a thin prism, made of crown glass ($n = 1.52$) is D_m . If the prism was made of dense flint glass ($n = 1.62$) instead of crown glass, the angle of minimum deviation will: [1 mark]

- (A) decrease by 4%
- (B) increase by 4%
- (C) decrease by 19%
- (D) increase by 19%

Correct Answer: (D) increase by 19%

Solution:

Concept:

- For a thin prism in air, the minimum deviation is approximately $D_m = (n - 1)A$, where n is the refractive index and A the small refracting angle.
- For prisms of unchanged geometry, deviation is proportional to $n - 1$; percentage change is measured relative to the original deviation.

Step 1: Compare the two deviations Let D_c and D_f be the deviations for crown and dense flint glass, respectively. The prism angle stays the same, hence

$$D_c = (1.52 - 1)A = 0.52A, \quad D_f = (1.62 - 1)A = 0.62A.$$

The increase in refractive index increases the deviation, ruling out both decrease options (A) and (C).

Step 2: Calculate the percentage increase Relative to the crown-glass deviation,

$$\frac{D_f - D_c}{D_c} \times 100\% = \frac{0.62 - 0.52}{0.52} \times 100\% \simeq 19.23\%.$$

Thus the increase is close to 19%, whereas the 4% increase in option (B) does not satisfy the thin-prism relation. Comparing the indices directly would use the wrong denominator.

Step 3: Conclusion The minimum deviation increases by approximately 19%. Hence, the correct option is (D).

Quick Tip to Solve: For a fixed thin prism, calculate the percentage change in $n - 1$, using the original $n - 1$ as the denominator.

30(iii). (iii) An object is placed in front of a convex spherical glass surface ($n = 1.5$ and radius of curvature R) at a distance of $4R$ from it. As the object is moved slowly close to the surface, the image formed is: [1 mark]

- (A) always real
- (B) always virtual
- (C) first real and then virtual
- (D) first virtual and then real

Correct Answer: (C) first real and then virtual

Solution:

Concept:

- Refraction at a spherical surface obeys $n_2/v - n_1/u = (n_2 - n_1)/R$, with distances measured from the pole using the Cartesian sign convention.
- For an object in air to the left of a convex glass surface, $u = -d$, $n_1 = 1$, $n_2 = 1.5$ and $R > 0$. A positive image distance is real; a negative one is virtual.

Step 1: Determine the initial image Here d is the positive object distance. Substitution gives

$$\frac{1.5}{v} + \frac{1}{d} = \frac{0.5}{R}, \quad v = \frac{3Rd}{d - 2R}.$$

Initially $d = 4R$, so $v = 6R > 0$: the image is real and lies inside the glass. This excludes options (B) and (D).

Step 2: Follow the image as the object approaches For $d > 2R$, the denominator $d - 2R$ is positive and the image remains real. At $d = 2R$, the refracted rays are parallel and the image is at infinity. For $0 < d < 2R$, the denominator becomes negative, giving a virtual image on the object's side. Therefore it cannot remain always real as stated in option (A).

Step 3: Conclusion The image is first real and then virtual as the object moves closer. Hence, the

correct option is (C).

Quick Tip to Solve: For this convex air-to-glass surface, track the sign of $d - 2R$ in $v = 3Rd/(d - 2R)$; the sign changes when the object passes $d = 2R$.

30(iv). (iv) A double-convex lens, made of glass of refractive index 1.5, has focal length 10 cm. The radius of curvature of its each face, is: [1 mark]

- (A) 10 cm
- (B) 15 cm
- (C) 20 cm
- (D) 40 cm

Correct Answer: (A) 10 cm

Solution:

Concept:

- For a thin lens in air, the lens maker's formula is $1/f = (n - 1)(1/R_1 - 1/R_2)$, where f is focal length and R_1, R_2 are signed surface radii.
- The common radius asked for implies equally curved convex faces. With light travelling left to right, $R_1 = +R$ and $R_2 = -R$, where R is the positive magnitude.

Step 1: Substitute the surface signs The first surface has its centre of curvature to the right; the second has its centre to the left. Thus their radius magnitudes are equal but their signs are opposite:

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{2(n - 1)}{R}.$$

Step 2: Calculate the common radius With $n = 1.5$ and $f = 10$ cm,

$$R = 2(n - 1)f = 2(0.5)(10 \text{ cm}) = 10 \text{ cm}.$$

Here $2(n - 1) = 1$, so the radius magnitude equals the focal length. Radii of 15, 20, or 40 cm would give those respective focal lengths, not the specified 10 cm; options (B), (C) and (D) therefore fail.

Step 3: Conclusion Each face has radius of curvature of magnitude 10 cm. Hence, the correct option is (A).

Quick Tip to Solve: For an equiconvex lens in air, use $R = 2(n - 1)f$; at $n = 1.5$, this simplifies immediately to $R = f$.

Section E (15 marks)

31(a). (i) A parallel plate capacitor with plate area A and plate separation d has a capacitance C_0 . A slab of dielectric constant K having area A and thickness $\left(\frac{d}{4}\right)$ is inserted in the capacitor, parallel to the plates. Find the new value of its capacitance.

(ii) You are provided with a large number of $1\ \mu\text{F}$ identical capacitors and a power supply of $1200\ \text{V}$. The dielectric medium used in each capacitor can withstand up to $200\ \text{V}$ only. Find the minimum number of capacitors and their arrangement, required to build a capacitor system of equivalent capacitance of $2\ \mu\text{F}$ for use with this supply. [5 marks]

Solution:

Concept:

- For parallel plates of area A separated by air through distance d , $C_0 = \epsilon_0 A/d$, neglecting edge effects. Dielectric layers perpendicular to the electric field act as capacitors in series.
- A dielectric slab of thickness t and relative permittivity K reduces that layer's electric field by a factor K . The effective air separation is $d - t + t/K$.
- Identical capacitors in series share the applied voltage equally; their reciprocal capacitances add. Parallel branches have equal voltage and their capacitances add.

Step 1: (i) Add the potential drops across the two media Let the plate charge be Q , so that the surface charge density is Q/A . The total air thickness is $d - t$, even if the slab does not touch either plate. The electric fields in air and dielectric are respectively $Q/(\epsilon_0 A)$ and $Q/(K\epsilon_0 A)$. Thus,

$$V = \frac{Q}{\epsilon_0 A}(d - t) + \frac{Q}{K\epsilon_0 A}t, \quad C = \frac{Q}{V} = \frac{\epsilon_0 A}{d - t + t/K}.$$

Step 2: Substitute the printed slab thickness The slab occupies $t = d/4$, leaving an air gap of total thickness $3d/4$. Therefore,

$$C = \frac{\epsilon_0 A}{3d/4 + d/(4K)} = \frac{4K\epsilon_0 A}{(3K + 1)d} = \frac{4K}{3K + 1}C_0.$$

For $K = 1$, this gives $C = C_0$, as expected. Even for a very large K , the remaining air gap limits the capacitance to $4C_0/3$.

Step 3: (ii) Determine the number of capacitors in each series branch Let n identical capacitors be connected in series across the supply. Each carries the same charge and has the same capacitance, so its potential difference is $1200/n$ volts. The dielectric condition requires

$$\frac{1200}{n} \leq 200 \Rightarrow n \geq 6.$$

Choose six capacitors in each branch. Each then operates at $200\ \text{V}$, and the branch capacitance is $C_s = (1/6)\ \mu\text{F}$.

Step 4: Combine sufficient branches in parallel If m such branches are connected in parallel, the equivalent capacitance is

$$C_{\text{eq}} = mC_s = \frac{m}{6}\ \mu\text{F}.$$

Setting this equal to $2\ \mu\text{F}$ gives $m = 12$. The arrangement therefore contains 12 parallel branches, each with 6 capacitors in series, using $N = 12 \times 6 = 72$ capacitors.

Step 5: Verify that this number is the minimum A capacitor of $1\ \mu\text{F}$, operated at no more than 200 V , can store at most $\frac{1}{2}CV^2 = 0.020\text{ J}$. The required system stores

$$U = \frac{1}{2}(2 \times 10^{-6})(1200)^2 = 1.44\text{ J}.$$

Since capacitor energies add, any valid arrangement must have $N(0.020) \geq 1.44$, or $N \geq 72$. The proposed arrangement reaches this lower bound with every capacitor at its allowed voltage.

Step 6: Conclusion (i) The new capacitance is $C = \frac{4K}{3K+1}C_0$. (ii) The minimum is 72 capacitors: connect 12 branches in parallel, with 6 identical $1\ \mu\text{F}$ capacitors in series per branch. The equivalent capacitance is $2\ \mu\text{F}$, and each capacitor has 200 V across it.

Quick Tip to Solve: For a partially filled gap, replace only the dielectric thickness t by t/K . For the capacitor bank, first meet the voltage rating using series connections, then recover the required capacitance using parallel branches.

OR

31(b). (i) An electric dipole of dipole moment $\vec{p}$ consists of point charges q and $-q$, separated by $2a$. Derive an expression for electric potential in terms of its dipole moment at a point at a distance x ($\gg a$) from its centre and lying (I) along its axis, and (II) along its bisector line.

(ii) An electric dipole of dipole moment $\vec{p} = (0.8\hat{i} + 0.6\hat{j}) \times 10^{-29}\text{ C m}$ is placed in an electric field $\vec{E} = 1.0 \times 10^7\hat{k}\text{ V/m}$. Calculate the magnitude of the torque acting on it and the angle it makes with the x -axis, at this instant. [5 marks]

Solution:

Concept:

- Electric potential obeys scalar superposition: $V = (4\pi\epsilon_0)^{-1} \sum_i q_i/r_i$, taking zero potential at infinity. The sign of each charge must be retained.
- A dipole consists of charges $-q$ and $+q$ separated by $2a$. Its dipole moment has magnitude $p = 2aq$ and points from the negative charge towards the positive charge.
- A dipole in a uniform electric field experiences torque $\vec{\tau} = \vec{p} \times \vec{E}$, with magnitude $pE \sin \theta$, where θ is the angle between $\vec{p}$ and $\vec{E}$.

Step 1: (i)(I) Write the exact potential on the dipole axis Choose the dipole centre as origin and consider an axial point a distance $x > a$ away on the side of $+q$. Its distances from $+q$ and $-q$ are $x - a$ and $x + a$, respectively. Adding their scalar potentials gives

$$\begin{aligned} V_{\text{axial}} &= \frac{1}{4\pi\epsilon_0} \left(\frac{q}{x-a} - \frac{q}{x+a} \right) \\ &= \frac{1}{4\pi\epsilon_0} \frac{2aq}{x^2 - a^2} = \frac{1}{4\pi\epsilon_0} \frac{p}{x^2 - a^2}. \end{aligned}$$

Step 2: Use the far-field condition Because $x \gg a$, the ratio a^2/x^2 is negligible compared with unity. Hence

$$V_{\text{axial}} \simeq \frac{1}{4\pi\epsilon_0} \frac{p}{x^2}.$$

This is positive on the side towards which $\vec{p}$ points. At the equally distant axial point on the negative-

charge side, the potential has the same magnitude and a negative sign. More generally, the far-field expression is $V = p \cos \theta / (4\pi\epsilon_0 x^2)$, with θ measured from $\vec{p}$.

Step 3: (i)(II) Evaluate the potential on the perpendicular bisector A point on the perpendicular bisector, at distance x from the centre, is equally far from both charges. Each distance is $r = \sqrt{x^2 + a^2}$. Therefore,

$$V_{\text{bisector}} = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{\sqrt{x^2 + a^2}} - \frac{q}{\sqrt{x^2 + a^2}} \right) = 0.$$

This cancellation is exact, even without the far-field approximation. Zero potential here does not imply zero electric field; the two scalar potentials cancel while the electric-field vectors need not.

Step 4: (ii) Compute the torque vector and its magnitude The given dipole moment lies in the xy -plane and the field points along $+z$, so their mutual angle is 90° . Using $\hat{i} \times \hat{k} = -\hat{j}$ and $\hat{j} \times \hat{k} = \hat{i}$,

$$\begin{aligned} \vec{\tau} &= (0.8\hat{i} + 0.6\hat{j}) \times \hat{k} 10^{-22} \\ &= (0.6\hat{i} - 0.8\hat{j}) \times 10^{-22} \text{ N m}, \\ |\vec{\tau}| &= \sqrt{0.6^2 + 0.8^2} \times 10^{-22} = 1.0 \times 10^{-22} \text{ N m}. \end{aligned}$$

The unit follows from $(\text{C m})(\text{V/m}) = \text{C V} = \text{N m}$.

Step 5: Determine the torque direction relative to the positive x-axis The torque has a positive x -component and a negative y -component, so it lies in the fourth quadrant of the xy -plane. If α is its angle with the positive x -axis,

$$\cos \alpha = \frac{\tau_x}{|\vec{\tau}|} = 0.6, \quad \alpha = \cos^{-1}(0.6) = 53.13^\circ.$$

Thus the torque points 53.13° below the positive x -axis, towards negative y . Its directed azimuth is -53.13° , or equivalently 306.87° ; the ordinary angle between the vectors is 53.13° .

Step 6: Conclusion (i) For $x \gg a$, $V_{\text{axial}} = \frac{p}{4\pi\epsilon_0 x^2}$ on the positive-charge side (negative on the opposite side), while $V_{\text{bisector}} = 0$. (ii) $|\vec{\tau}| = 1.0 \times 10^{-22} \text{ N m}$, directed 53.13° below the positive x -axis in the xy -plane.

Quick Tip to Solve: Add potentials as signed scalars. For torque, calculate $\vec{p} \times \vec{E}$ first and inspect the component signs before assigning an angle: here the negative y -component fixes the fourth quadrant.

32(a). (i) With the help of a labelled diagram, explain the principle of working of a moving coil galvanometer. Write the purpose of using (i) radial magnetic field, and (ii) soft iron core, in it.

(ii) Define current sensitivity of a galvanometer. "Increasing the current sensitivity may not necessarily increase the voltage sensitivity." Give reason. [5 marks]

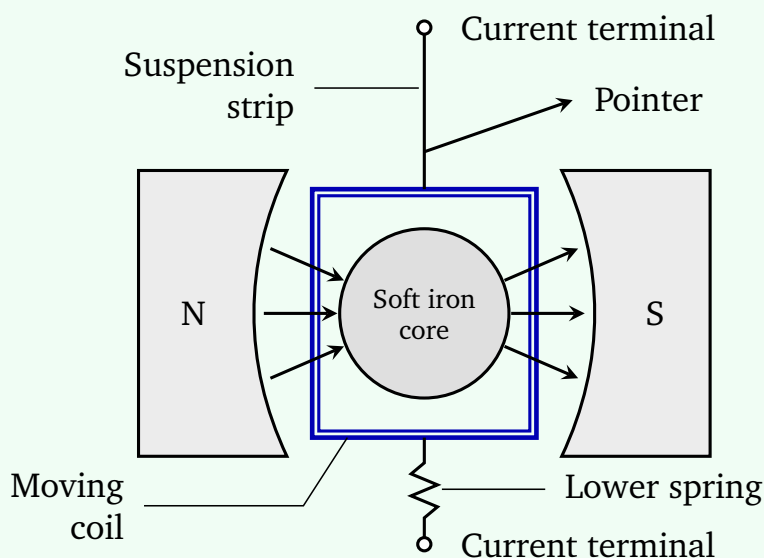
Solution:

Concept:

- A coil of N turns, area A , carrying current I in magnetic field B , experiences torque $\tau_m = NIAB \sin \phi$, where ϕ is the angle between its normal and the field.
- In a radial field, the field lies in the plane of the coil at every allowed position, so $\phi = 90^\circ$ and $\tau_m = NIAB$. A suspension or spring supplies restoring torque $\tau_r = k\theta$, where k is the torsional constant.

- Current sensitivity is deflection per unit current, $S_I = \theta/I$; voltage sensitivity is deflection per unit potential difference, $S_V = \theta/V$.

Step 1: (i) Describe the construction and the magnetic torque A light rectangular coil of many turns of fine insulated wire is suspended between the concave pole pieces of a strong permanent magnet. A cylindrical soft iron core is fixed inside the coil without touching it. A fine conducting suspension strip and a lower spring carry current into and out of the coil and provide the restoring torque. A pointer attached to the coil, or a mirror arrangement, measures its angular deflection. When current flows, the magnetic forces on opposite vertical sides of the coil form a couple. The coil therefore rotates; reversing the current reverses its deflection. This turning effect of a magnetic field on a current-carrying coil is the working principle of the moving coil galvanometer.



Concave N and S pole pieces give a radial field in the gap.

Moving coil galvanometer: labelled schematic of the coil, pole pieces and suspension.

Step 2: Obtain the equilibrium deflection Because the field is radial, the magnetic torque is $NIAB$, independent of the instantaneous deflection. As the coil turns, the suspension develops an opposing torque $k\theta$. At the final steady position these torques balance:

$$NIAB = k\theta \Rightarrow \theta = \frac{NAB}{k}I.$$

Hence deflection is directly proportional to current. A calibrated uniform scale can therefore measure small currents; N , A , B , and k are fixed for a given instrument.

Step 3: State the purposes of the radial field and soft iron core **Radial magnetic field:** It keeps the coil's plane parallel to the local field, so its normal remains perpendicular to the field. Thus $\sin \phi = 1$ throughout the useful angular range, giving maximum magnetic torque for a given current and a linear current-deflection relation.

Soft iron core: Its high magnetic permeability strengthens the field in the air gap and, together with the concave pole pieces, makes the field radial. The stronger field increases the torque produced by a small current and therefore increases sensitivity.

Step 4: (ii) Relate current sensitivity to voltage sensitivity The current sensitivity of a galvanometer

is the angular deflection produced per unit current through it:

$$S_I = \frac{\theta}{I} = \frac{NAB}{k}.$$

Its SI unit is rad A^{-1} . Let G be the coil resistance and V the potential difference across it. Using $I = V/G$,

$$S_V = \frac{\theta}{V} = \frac{NAB}{kG} = \frac{S_I}{G}.$$

Increasing the number of turns increases S_I , but also increases the length of wire and hence G . For otherwise similar turns of the same wire, both N and G increase in the same proportion, so N/G , and therefore S_V , need not increase. An increase in current sensitivity alone is therefore insufficient to guarantee an increase in voltage sensitivity; the accompanying resistance change must also be considered.

Step 5: Conclusion The galvanometer works by magnetic torque balanced by restoring torque, giving $\theta = NIAB/k$. The radial field ensures linear deflection and the soft iron core strengthens and shapes the field. Since $S_V = S_I/G$, greater current sensitivity need not mean greater voltage sensitivity.

Quick Tip to Solve: Write $S_V = S_I/G$ before judging a sensitivity change: more turns increase both magnetic torque and coil resistance.

OR

32(b). (i) (I) Write Ampere's circuital law in mathematical form and explain the terms used.

(II) As the current carrying solenoid is made longer, the magnetic field produced outside it approaches zero. Why?

(III) A flexible loop of irregular shape carrying current when located in an external magnetic field, changes to a circular shape. Give reason.

(ii) A galvanometer of resistance G is converted into a voltmeter to measure up to V volts, by connecting a resistance R_1 in series with the coil. If R_1 is replaced by R_2 , then it can only measure up to $\frac{V}{2}$ volt. Find the value of the resistance R_3 (in terms of R_1 and R_2) needed to convert it into a voltmeter that can read up to $2V$.

[5 marks]

Solution:

Concept:

- Ampere's circuital law relates the circulation of a steady magnetic field around a closed path to the net current threading that path: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$.
- A current element experiences $d\vec{F} = I d\vec{l} \times \vec{B}$. A planar loop has magnetic moment $\vec{m} = I\vec{A}$, where $\vec{A}$ is its vector area, and magnetic potential energy $U = -\vec{m} \cdot \vec{B}$.
- A galvanometer becomes a voltmeter by adding a large series resistance. At its fixed full-scale current I_g , the voltage range is $V_{\text{range}} = I_g(G + R)$, where G is the coil resistance and R the added resistance.

Step 1: (i)(I) State Ampere's circuital law and define its terms For steady currents in free space, the line integral of the magnetic field around any closed path is μ_0 times the algebraic sum of the currents enclosed by it:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \sum_k I_k.$$

Here $\vec{B}$ is the magnetic field at a point of the path C , $d\vec{l}$ is its directed infinitesimal tangent element, and μ_0 is the permeability of free space. The closed-integral sign means that the integration covers the entire loop. The enclosed current is the net current through a surface bounded by C ; its positive direction follows the right-hand rule relative to the chosen circulation direction.

Step 2: (i)(II) Explain the weak external field of a long solenoid Outside a long solenoid, the magnetic fields due to its different turns largely cancel, whereas inside they reinforce. The external return field spreads over a large region and is appreciable mainly near the ends. As the length becomes much greater than the radius, end effects at points near the central portion become negligible, and the field there approaches that of an ideal infinitely long solenoid: zero outside. For a finite solenoid the external field is small rather than exactly zero; the limiting statement applies away from its ends.

Step 3: (i)(III) Explain why the flexible loop becomes circular In the stable orientation of the loop in a uniform external field, its magnetic moment is parallel to $\vec{B}$, so the field is normal to the loop. The magnetic force $I d\vec{l} \times \vec{B}$ on each element acts in the plane, perpendicular to that element, and tends to expand the loop. Equivalently, in this orientation $U = -IAB$, so at fixed current and field the loop lowers its magnetic potential energy by increasing its enclosed area A . The wire length, and hence its perimeter, is fixed. A circle encloses the maximum area for a given perimeter; thus the freely flexible loop tends to acquire a circular shape.

Step 4: (ii) Use the two given voltage ranges The same galvanometer is used in both conversions, so G and I_g remain unchanged. At full-scale deflection,

$$V = I_g(G + R_1),$$

$$\frac{V}{2} = I_g(G + R_2).$$

Multiplying the second relation by two and comparing it with the first eliminates both V and I_g :

$$G + R_1 = 2(G + R_2),$$

$$G = R_1 - 2R_2.$$

This is the coil resistance expressed in terms of the two given series resistances.

Step 5: (ii) Calculate the resistance for the doubled range To measure up to $2V$, the full-scale current must still be I_g . Therefore,

$$2V = I_g(G + R_3),$$

$$G + R_3 = 2(G + R_1),$$

$$R_3 = G + 2R_1,$$

$$= (R_1 - 2R_2) + 2R_1.$$

It is the total voltmeter resistance, including the galvanometer coil, that doubles when the range doubles; doubling only the added resistance would give an incorrect range.

Step 6: Conclusion Ampere's law is $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$. The external field of a long solenoid approaches zero away from its ends through cancellation of the fields of its turns. The flexible loop becomes circular to maximise its area for a fixed perimeter in its stable orientation. The required series resistance is $R_3 = 3R_1 - 2R_2$.

Quick Tip to Solve: Write each voltmeter range as $I_g(G + R)$; compare the ranges to eliminate I_g , and retain the coil resistance G until the last substitution.

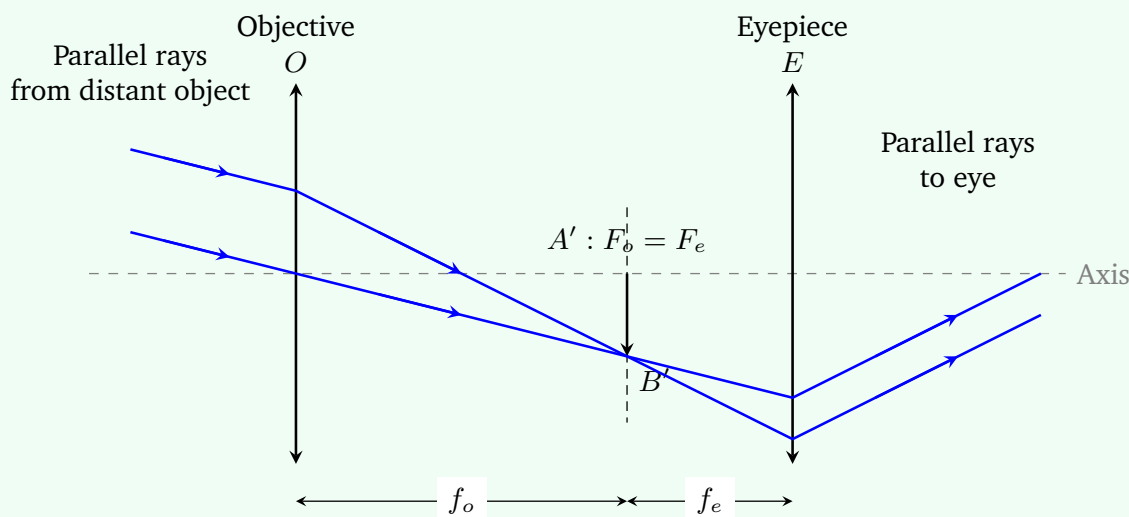
- 33(a).** (i) Explain with the help of a labelled ray diagram the formation of final image by an astronomical telescope at infinity. Write the expression for its magnifying power.
- (ii) The total magnification produced by a compound microscope is 20. The magnification produced by the eyepiece is 5. When the microscope is focussed on a certain object, the distance between the objective and eyepiece is observed to be 14 cm. Calculate the focal lengths of the objective and the eyepiece. (Given that the least distance of distinct vision = 25 cm) [5 marks]

Solution:

Concept:

- A telescope in normal adjustment forms its final image at infinity. Its convex objective forms a real image in the common focal plane of the objective and eyepiece.
- Angular magnifying power is $M = \beta/\alpha$, where α and β are the small angles subtended at the unaided eye and through the telescope. For normal adjustment, $M = -f_o/f_e$.
- A compound microscope has magnification magnitude $|m| = |m_o|m_e$. For its eyepiece, $m_e = 1 + D/f_e$ when the final image is at the near point D .
- With Cartesian signs, a thin lens obeys $1/f = 1/v - 1/u$, and its transverse magnification is $m = v/u$. Lens separation is the sum of the objective image distance and the magnitude of the eyepiece object distance.

Step 1: (i) Trace the image formation in the telescope The objective has a large aperture and a long focal length f_o , whereas the eyepiece has a short focal length f_e . Parallel rays from a distant object enter the objective and converge to form a real, inverted, diminished intermediate image $A'B'$ at its rear focal plane. Adjust the eyepiece so that this image also lies in its front focal plane. Each pencil of rays from B' then emerges parallel after refraction through the eyepiece, placing the final virtual image at infinity. The image remains inverted relative to the distant object, and the separation of the two lenses is $f_o + f_e$. The diagram shows rays from one off-axis object point; it is schematic and not to scale.



Astronomical telescope in normal adjustment: the intermediate image lies in the common focal plane, and emergent rays are parallel.

Step 2: (i) Obtain the angular magnifying power Let the magnitude of the intermediate image height be h . For small angles, $|\alpha| \simeq h/f_o$ and $|\beta| \simeq h/f_e$. Therefore,

$$|M| = \frac{|\beta|}{|\alpha|} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}.$$

Including the inversion gives the signed magnifying power $M = -f_o/f_e$. A longer-focal-length objective and a shorter-focal-length eyepiece provide greater angular magnification.

Step 3: (ii) Find the microscope eyepiece focal length Use the usual near-point viewing condition for the microscope, taking its final virtual image at the given least distance of distinct vision, $D = 25$ cm. Then

$$5 = 1 + \frac{25}{f_e} \Rightarrow f_e = 6.25 \text{ cm}.$$

The eyepiece image distance is $v_e = -25$ cm. Its lens equation gives

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{6.25} = -\frac{1}{5} \text{ cm}^{-1},$$

so $u_e = -5$ cm. The intermediate image lies 5 cm to the left of the eyepiece.

Step 4: (ii) Use the lens separation to find the objective focal length The objective magnification has magnitude $|m_o| = 20/5 = 4$. Since the intermediate image is real and inverted, $m_o = -4$. With objective-eyepiece separation 14 cm,

$$v_o = 14 - 5 = 9 \text{ cm}, \quad u_o = \frac{v_o}{m_o} = -\frac{9}{4} = -2.25 \text{ cm}.$$

Apply the objective lens equation:

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{9} + \frac{4}{9} = \frac{5}{9} \text{ cm}^{-1}, \quad f_o = 1.8 \text{ cm}.$$

The printed question does not explicitly state the microscope's final-image position. The values above use near-point viewing. If instead its final image is at infinity, $m_e = D/f_e$ gives $f_e = 5$ cm; then $|u_e| = f_e = 5$ cm, so the same objective value $f_o = 1.8$ cm follows.

Step 5: Conclusion (i) In normal adjustment the final telescope image is at infinity and inverted, with $M = -f_o/f_e$ and lens separation $f_o + f_e$. (ii) For the microscope with final image at the near point, $f_o = 1.8 \text{ cm}$, $f_e = 6.25 \text{ cm}$. If infinity viewing is intended instead, $f_e = 5 \text{ cm}$, while f_o remains 1.8 cm .

Quick Tip to Solve: Use the microscope's actual lens separation to get $v_o = 14 - |u_e|$; do not substitute 14 cm directly for v_o . State the eyepiece viewing condition before choosing its magnification formula.

OR

33(b). (i) Two coherent light waves, each of intensity I_0 superpose each other and produce interference pattern on a screen. Obtain the expression for the resultant intensity at a point where the phase difference between the waves is ϕ . Write its maximum and minimum possible values.

(ii) In a single slit diffraction experiment, the aperture of the slit is 3 mm and the separation between the slit and the screen is 1.5 m . A monochromatic light of wavelength 600 nm is normally incident on the slit. Calculate the distance of (I) first order minimum, and (II) second order maximum, from the centre of the screen. [5 marks]

Solution:

Concept:

- Coherent waves have the same frequency and a constant phase difference. Their displacements add, and intensity is proportional to the square of the resultant amplitude.
- For amplitudes A_1, A_2 and phase difference ϕ , the resultant amplitude obeys $A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos \phi$.
- Single-slit minima satisfy $a \sin \theta = n\lambda$, where a is slit width and $n = 1, 2, \dots$. For a distant screen, the displacement is $y \simeq D\theta \simeq D \sin \theta$, with screen distance D .
- Secondary diffraction maxima lie approximately midway between successive minima: $a \sin \theta \simeq (n + \frac{1}{2})\lambda$, with $n = 1$ for the first secondary maximum. The central maximum has order zero.

Step 1: (i) Add the coherent wave displacements Write the two displacements at the point as $u_1 = A_0 \cos \omega t$ and $u_2 = A_0 \cos(\omega t + \phi)$. Equal intensities imply equal amplitudes A_0 . Applying the cosine addition identity gives

$$\begin{aligned} u &= u_1 + u_2 \\ &= 2A_0 \cos(\phi/2) \cos(\omega t + \phi/2). \end{aligned}$$

Thus the squared resultant amplitude is $4A_0^2 \cos^2(\phi/2)$. If $I_0 = kA_0^2$, where k is the common proportionality constant, the resultant intensity is

$$I = k [4A_0^2 \cos^2(\phi/2)] = 4I_0 \cos^2(\phi/2) = 2I_0(1 + \cos \phi).$$

The additional term $2I_0 \cos \phi$ is the interference term; adding the individual intensities alone would omit its phase dependence.

Step 2: Find the maximum and minimum intensities Constructive interference occurs when $\phi = 2m\pi$, for any integer m . Then the amplitudes reinforce and $\cos^2(\phi/2) = 1$, giving $I_{\max} = 4I_0$. Destructive interference occurs when $\phi = (2m+1)\pi$, for which $\cos^2(\phi/2) = 0$, giving $I_{\min} = 0$. Complete darkness is possible here because the two interfering amplitudes are equal.

Step 3: (ii)(I) Locate the first minimum Convert all the supplied quantities to SI units:

$$a = 3 \times 10^{-3} \text{ m}, \quad D = 1.5 \text{ m}, \quad \lambda = 600 \times 10^{-9} \text{ m}.$$

At the first minimum, the path difference across the slit is λ . Each ray from one half of the slit cancels a corresponding ray from the other half, whose path difference is $\lambda/2$. Hence $a \sin \theta_1 = \lambda$. Since $\lambda/a = 2 \times 10^{-4}$, the small-angle approximation is well justified:

$$y_{1,\min} = \frac{D\lambda}{a} = \frac{1.5 \times 600 \times 10^{-9}}{3 \times 10^{-3}} = 3.0 \times 10^{-4} \text{ m} = 0.30 \text{ mm}.$$

Step 4: (ii)(II) Locate the second secondary maximum The first secondary maximum lies between the first and second minima; the second secondary maximum lies between the second and third minima. Using the usual approximate midpoint condition for secondary maxima,

$$a \sin \theta_{2,\max} \simeq \frac{5\lambda}{2}.$$

Therefore its distance from the centre is

$$y_{2,\max} \simeq \frac{5D\lambda}{2a} = \frac{5}{2}(0.30 \text{ mm}) = 0.75 \text{ mm}.$$

Unlike the minima condition, the midpoint condition is an approximation; it is the standard school-level estimate for the secondary maxima. The pattern is symmetric, so each calculated distance occurs on both sides of the central maximum.

Step 5: Conclusion (i) $I = 4I_0 \cos^2(\phi/2) = 2I_0(1 + \cos \phi)$, with $I_{\max} = 4I_0$ and $I_{\min} = 0$. (ii) The first minimum is 0.30 mm from the centre, and the second secondary maximum is approximately 0.75 mm from the centre, on either side.

Quick Tip to Solve: Calculate $D\lambda/a$ once: it is the first-minimum distance. Multiply it by $5/2$ for the approximate second secondary maximum; do not count the central maximum as the first order.