

CBSE Class 12 Physics Question Paper 2025 with Solution PDF-Set 55/7/3



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – **Sections A, B, C, D** and **E**.
- (iii) In **Section A** – Questions no. **1 to 16** are Multiple Choice type questions. Each question carries **1** mark.
- (iv) In **Section B** – Questions no. **17 to 21** are Very Short Answer type questions. Each question carries **2** marks.
- (v) In **Section C** – Questions no. **22 to 28** are Short Answer type questions. Each question carries **3** marks.
- (vi) In **Section D** – Questions no. **29** and **30** are case study-based questions. Each question carries **4** marks.
- (vii) In **Section E** – Questions no. **31 to 33** are Long Answer type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is **not** allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

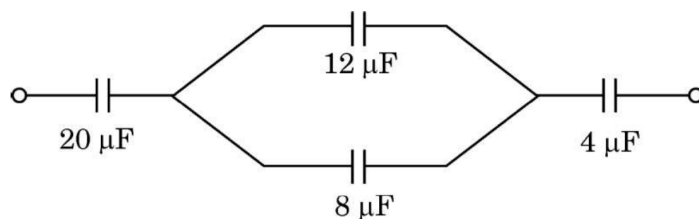
$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. In the circuit shown, the charge on the left plate of $20\ \mu\text{F}$ capacitor is $-50\ \mu\text{C}$. The charge on the right plate of the $12\ \mu\text{F}$ capacitor is: [1 mark]



- (A) $20\ \mu\text{C}$
- (B) $-20\ \mu\text{C}$
- (C) $30\ \mu\text{C}$
- (D) $-30\ \mu\text{C}$

Correct Answer: (C) $30\ \mu\text{C}$

Solution:

Concept:

- For a capacitor, the plate charges are equal and opposite and their magnitude is $Q = C|V|$, where C is capacitance and V is potential difference.
- Parallel capacitors share the same potential difference. Charge is conserved on each initially neutral isolated junction.

Step 1: Find the charge entering the parallel combination The right plate of the $20\ \mu\text{F}$ capacitor carries $+50\ \mu\text{C}$. Its connection to the left plates of the $12\ \mu\text{F}$ and $8\ \mu\text{F}$ capacitors is an isolated neutral conductor, so their left plates together carry $-50\ \mu\text{C}$. Their right plates therefore carry a total $+50\ \mu\text{C}$.

Step 2: Divide the charge according to capacitance Since the two capacitors are parallel, their charges have the ratio $12 : 8$. Thus the charge on the right plate of the $12\ \mu\text{F}$ capacitor is

$$Q_{12} = \frac{12}{12 + 8}(50) = +30\ \mu\text{C}.$$

The magnitude $20\ \mu\text{C}$ belongs to the $8\ \mu\text{F}$ capacitor, excluding (A) and (B). Option (D) has the correct magnitude but the wrong plate sign.

Step 3: Conclusion The required charge is $+30\ \mu\text{C}$. Hence, the correct option is (C).

Quick Tip to Solve: In a parallel pair, divide the total charge in the capacitance ratio; track the sign by conserving charge at the intervening junction.

2. Germanium crystal is doped at room temperature with a minute quantity of boron. The charge carriers in the doped semiconductors will be: [1 mark]

- (A) electrons only
- (B) holes only
- (C) holes and few electrons
- (D) electrons and few holes

Correct Answer: (C) holes and few electrons

Solution:

Concept:

- A trivalent impurity in tetravalent germanium acts as an acceptor and produces a p-type semiconductor.
- At room temperature, thermal excitation creates electron-hole pairs. Holes are majority carriers in p-type material and electrons are minority carriers.

Step 1: Identify the effect of boron doping Boron has three valence electrons whereas germanium has four. A boron atom replacing a germanium atom leaves an incomplete covalent bond. It can accept an electron from a neighbouring bond, leaving a mobile hole. Thus boron doping greatly increases the hole concentration.

Step 2: Include the thermally generated carriers At room temperature, some electrons are thermally excited into the conduction band. Therefore the semiconductor contains many holes and a small concentration of electrons. Option (B) wrongly removes all minority electrons. Option (A) excludes the majority holes, and option (D) describes an n-type material with electrons as majority carriers.

Step 3: Conclusion The charge carriers are holes and a few electrons. Hence, the correct option is (C).

Quick Tip to Solve: Trivalent doping means p-type: choose holes as majority carriers, but retain minority electrons whenever the temperature is nonzero.

3. A steady current I is passed through a conductor at room temperature for time t . It is observed that its temperature rises by 0.5°C . If $2I$ current is passed through the conductor (at room temperature) for the same duration, the rise in its temperature will be approximately: [1 mark]

- (A) 1.0°C
- (B) 1.5°C
- (C) 2.0°C
- (D) 4.0°C

Correct Answer: (C) 2.0°C

Solution:

Concept:

- Joule heating gives $H = I^2Rt$, where I is current, R is resistance and t is the heating duration.
- For a small temperature rise, take resistance and heat capacity mc as constant; then $mc\Delta T \approx I^2Rt$.

Step 1: Compare the heating in the two trials The same conductor starts at room temperature in both cases, and the heating time is unchanged. For these small rises, changes in resistance and heat

losses are neglected in the approximate estimate. Hence the temperature rise is proportional to the square of the current, not to the current itself.

Step 2: Apply the square-current ratio The ratio of temperature rises is

$$\frac{\Delta T_2}{\Delta T_1} = \frac{(2I)^2}{I^2} = 4, \quad \Delta T_2 = 4(0.5) = 2.0^\circ\text{C}.$$

Option (A) incorrectly uses a linear dependence on current. Options (B) and (D) imply factors of three and eight respectively, which do not follow from Joule's law.

Step 3: Conclusion The approximate temperature rise is 2.0°C . Hence, the correct option is (C).

Quick Tip to Solve: With the same conductor and heating duration, multiply the original temperature rise by the square of the current multiplier.

4. The dimensions of 'self-inductance' are:

[1 mark]

- (A) $[MLT^{-2}A^{-2}]$
- (B) $[ML^2T^{-1}A^{-1}]$
- (C) $[ML^{-1}T^{-2}A^{-2}]$
- (D) $[ML^2T^{-2}A^{-2}]$

Correct Answer: (D) $[ML^2T^{-2}A^{-2}]$

Solution:

Concept:

- The energy stored in an inductor is $U = \frac{1}{2}\mathcal{L}I^2$, where $\mathcal{L}$ denotes self-inductance and I denotes current.
- Energy has dimensions $[ML^2T^{-2}]$, and electric current has dimension $[A]$. Numerical constants have no dimensions.

Step 1: Express inductance in terms of energy and current Rearranging the magnetic-energy relation gives $\mathcal{L} = 2U/I^2$. Therefore inductance has the dimensions of energy divided by current squared; the factor of two contributes no dimension.

Step 2: Substitute the fundamental dimensions

$$[\mathcal{L}] = \frac{[U]}{[I]^2} = \frac{ML^2T^{-2}}{A^2} = ML^2T^{-2}A^{-2}.$$

Options (A) and (C) have incorrect powers of length. Option (B) has incorrect powers of time and current. Only (D) has all four powers required by the energy relation.

Step 3: Conclusion The dimensions of self-inductance are $[ML^2T^{-2}A^{-2}]$. Hence, the correct option is (D).

Quick Tip to Solve: For inductance dimensions, use stored energy divided by current squared; it avoids introducing voltage or magnetic flux as extra quantities.

5. Isotones are the nuclides having:

[1 mark]

- (A) same mass numbers
- (B) same atomic numbers
- (C) same neutron number, but different atomic number
- (D) different neutron number, and different mass number

Correct Answer: (C) same neutron number, but different atomic number

Solution:

Concept:

- For a nuclide, mass number A , atomic number Z and neutron number N obey $A = Z + N$.
- Isotones have equal neutron numbers and different atomic numbers. Isotopes have equal atomic numbers, while isobars have equal mass numbers.

Step 1: Apply the definition of isotones Two different nuclides are isotones when $N_1 = N_2$ but $Z_1 \neq Z_2$. For example, ${}^{14}_6\text{C}$ and ${}^{15}_7\text{N}$ each contain eight neutrons, although their proton numbers are six and seven respectively.

Step 2: Distinguish the neighbouring definitions Option (A), equal mass numbers, describes isobars. Option (B), equal atomic numbers, describes isotopes. Option (D) explicitly requires different neutron numbers and therefore contradicts the definition of isotones. Equal neutron numbers with unequal atomic numbers is exactly the condition stated in (C).

Step 3: Conclusion Isotones have the same neutron number but different atomic numbers. Hence, the correct option is (C).

Quick Tip to Solve: To test whether a pair is isotonic, calculate $N = A - Z$ for each nuclide and check that these values agree while the proton numbers differ.

6. A metal rod of length 50 cm is held vertically and moved with a velocity of 10 m/s towards east. The horizontal component of the Earth's magnetic field at the place is 0.4 G. The emf induced across the ends of the rod is:

[1 mark]

- (A) 0.1 mV
- (B) 0.2 mV
- (C) 0.8 mV
- (D) 1.6 mV

Correct Answer: (B) 0.2 mV

Solution:

Concept:

- A rod moving perpendicular to its length and a magnetic field develops motional emf $\varepsilon = Blv$, where B is the effective field, l is rod length and v is speed.
- Earth's horizontal magnetic field is along the north-south direction, perpendicular to the eastward velocity. Use $1 \text{ G} = 10^{-4} \text{ T}$.

Step 1: Identify the effective magnetic-field component For eastward motion in the horizontal northward field, $\vec{v} \times \vec{B}_H$ is vertical, along the rod. Thus the horizontal component causes charge separation between its ends. The vertical field component produces a horizontal magnetic force and

contributes no emf along the vertical rod.

Step 2: Convert the quantities and calculate the emf Here $l = 0.50 \text{ m}$, $v = 10 \text{ m s}^{-1}$ and $B_H = 0.4 \times 10^{-4} \text{ T}$. Therefore

$$\varepsilon = (0.4 \times 10^{-4})(0.50)(10) = 2.0 \times 10^{-4} \text{ V} = 0.2 \text{ mV}.$$

Option (A) is half this value, while (C) and (D) are four and eight times it. None follows from the given rod length, speed and field in SI units.

Step 3: Conclusion The induced emf has magnitude 0.2 mV. Hence, the correct option is (B).

Quick Tip to Solve: Convert gauss to tesla and centimetres to metres before using Blv ; only the component of $\vec{v} \times \vec{B}$ along the rod contributes.

7. The frequency of a photon of energy 1.326 eV is:

[1 mark]

(A) $1.18 \times 10^{14} \text{ Hz}$

(B) $3.20 \times 10^{14} \text{ Hz}$

(C) $4.20 \times 10^{15} \text{ Hz}$

(D) $4.80 \times 10^{15} \text{ Hz}$

Correct Answer: (B) $3.20 \times 10^{14} \text{ Hz}$

Solution:

Concept:

- Planck's relation is $E = h\nu$, where E is photon energy, h is Planck's constant and ν is frequency.
- Convert electronvolts to joules before using $h = 6.63 \times 10^{-34} \text{ J s}$: $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$.

Step 1: Convert the photon energy into SI units The given energy is an energy in electronvolts, not a potential difference. Multiplying by the joule value of one electronvolt gives

$$E = 1.326(1.6 \times 10^{-19}) = 2.1216 \times 10^{-19} \text{ J}.$$

Step 2: Divide the energy by Planck's constant From $\nu = E/h$,

$$\nu = \frac{2.1216 \times 10^{-19}}{6.63 \times 10^{-34}} = 0.32 \times 10^{15} = 3.20 \times 10^{14} \text{ Hz}.$$

Option (A) has the right order of magnitude but the wrong coefficient. Options (C) and (D) are more than ten times the calculated frequency and would correspond to much larger photon energies.

Step 3: Conclusion The photon frequency is $3.20 \times 10^{14} \text{ Hz}$. Hence, the correct option is (B).

Quick Tip to Solve: Keep the energy unit consistent with Planck's constant: convert eV to J first, then combine the powers of ten separately from the coefficients.

8. An alternating current is given by $I = I_0 \cos(100\pi)t$. The least time the current takes to decrease from its maximum value to zero will be: [1 mark]

- (A) $\left(\frac{1}{200}\right)$ s
(B) $\left(\frac{1}{150}\right)$ s
(C) $\left(\frac{1}{100}\right)$ s
(D) $\left(\frac{1}{50}\right)$ s

Correct Answer: (A) $\left(\frac{1}{200}\right)$ s

Solution:

Concept:

- For a sinusoidal current $I = I_0 \cos \omega t$, I_0 is the peak current and ω is the angular frequency.
- A cosine decreases from its positive maximum to its first zero in one quarter of a period: $\Delta t = T/4$, where $T = 2\pi/\omega$.

Step 1: Identify the first zero after a maximum At $t = 0$, the current equals I_0 . Here $\omega = 100\pi \text{ rad s}^{-1}$. Its first subsequent zero occurs when the phase increases from zero to $\pi/2$, because $\cos(\pi/2) = 0$.

Step 2: Calculate the quarter-period interval

$$100\pi \Delta t = \frac{\pi}{2} \Rightarrow \Delta t = \frac{1}{200} \text{ s.}$$

The period is $1/50$ s, so option (D) is a full cycle and (C) is a half-cycle, ending at the negative maximum. At the time in (B), the phase is $2\pi/3$, already beyond the first zero.

Step 3: Conclusion The least time is $1/200 \text{ s} = 5.0 \times 10^{-3} \text{ s}$. Hence, the correct option is (A).

Quick Tip to Solve: For a sinusoidal current, use one quarter-period for the first change from a maximum to zero.

9. A rectangular coil of area A is kept in a uniform magnetic field $\vec{B}$ such that the plane of the coil makes an angle α with $\vec{B}$. The magnetic flux linked with the coil is: [1 mark]

- (A) $BA \sin \alpha$
(B) $BA \cos \alpha$
(C) BA
(D) zero

Correct Answer: (A) $BA \sin \alpha$

Solution:

Concept:

- Magnetic flux through a plane surface in a uniform field is $\Phi = \vec{B} \cdot \vec{A} = BA \cos \theta$, where θ is measured from the surface normal.
- The area vector is perpendicular to the plane. If the field makes angle α with the plane, it makes angle $90^\circ - \alpha$ with its normal.

Step 1: Convert the given angle to the normal angle The question specifies the angle between the field and the plane of the coil. Therefore substituting α directly in the cosine formula would use the wrong angle. The required normal angle is $\theta = 90^\circ - \alpha$.

Step 2: Evaluate the flux and check the alternatives

$$\Phi = BA \cos(90^\circ - \alpha) = BA \sin \alpha.$$

Option (B) would apply if α were measured from the normal. Option (C) is valid only when the field is perpendicular to the plane; zero flux in (D) occurs only when it is parallel to the plane. Neither is the general expression.

Step 3: Conclusion The magnetic flux linked with the coil is $BA \sin \alpha$. Hence, the correct option is (A).

Quick Tip to Solve: Use sine when the angle is given with the plane, and cosine when it is given with the normal.

10. The minimum energy required to free the electron from the ground state of the hydrogen atom is: [1 mark]

- (A) 3.4 eV
- (B) less than 3.4 eV
- (C) 13.6 eV
- (D) between 13.6 eV and 3.4 eV

Correct Answer: (C) 13.6 eV

Solution:

Concept:

- The energy of the electron in the n th level of hydrogen is $E_n = -13.6/n^2$ eV, with zero energy defined for a free electron at rest infinitely far from the nucleus.
- Ionisation energy is the positive energy required to move an electron from its initial bound level to this zero-energy limit.

Step 1: Identify the initial and final energies In the ground state, $n = 1$, so $E_1 = -13.6$ eV. To just free the electron with no residual kinetic energy, its final energy must be $E_\infty = 0$. The negative initial energy indicates that the electron is bound.

Step 2: Calculate the required energy difference

$$\Delta E = E_\infty - E_1 = 0 - (-13.6) = 13.6 \text{ eV.}$$

The value 3.4 eV is the ionisation energy from $n = 2$, not from the ground state. Options (A), (B) and (D) all supply less than the ground-state binding energy and therefore cannot free this electron.

Step 3: Conclusion The minimum required energy is 13.6 eV. Hence, the correct option is (C).

Quick Tip to Solve: For hydrogen ionisation from level n , take the magnitude of the bound-state energy, $13.6/n^2$ eV.

11. A capacitor and an inductor are connected in series across an ac source of voltage of variable frequency. The frequency is increased continuously. The nature of the circuit before and after the resonance will be: [1 mark]

- (A) inductive only
- (B) capacitive only
- (C) capacitive and inductive respectively
- (D) inductive and capacitive respectively

Correct Answer: (C) capacitive and inductive respectively

Solution:

Concept:

- In a series LC circuit, inductive reactance is $X_L = \omega L$ and capacitive reactance is $X_C = 1/(\omega C)$, where $\omega = 2\pi f$.
- The circuit is capacitive when $X_C > X_L$, and inductive when $X_L > X_C$. At resonance these reactances are equal and $\omega_0 = 1/\sqrt{LC}$.

Step 1: Compare reactances below resonance At low frequency, X_C is large and X_L is small. For $\omega < \omega_0$, $\omega^2 LC < 1$, giving $\omega L < 1/(\omega C)$. The negative net reactance $X_L - X_C$ means that the circuit is capacitive before resonance.

Step 2: Compare reactances above resonance Increasing frequency raises X_L and lowers X_C . For $\omega > \omega_0$, $X_L > X_C$, so the circuit is inductive after resonance. Options (A) and (B) wrongly assign one nature on both sides; option (D) reverses the order of the two frequency regions.

Step 3: Conclusion Before and after resonance the circuit is capacitive and inductive, respectively. Hence, the correct option is (C).

Quick Tip to Solve: As frequency rises, the capacitor's reactance falls while the inductor's rises: capacitive first, inductive later.

12. A p-n junction diode is forward biased. As a result, [1 mark]

- (A) both the potential barrier height and the width of depletion layer decrease.
- (B) both the potential barrier height and the width of depletion layer increase.
- (C) the potential barrier height decreases and the width of depletion layer increases.
- (D) the potential barrier height increases and the width of depletion layer decreases.

Correct Answer: (A) both the potential barrier height and the width of depletion layer decrease.

Solution:

Concept:

- The depletion region of a p-n junction contains fixed ionised donors and acceptors, whose electric field produces the built-in potential barrier.
- Forward bias connects the p-side to the positive terminal and the n-side to the negative terminal. Its applied electric field opposes the built-in junction field.

Step 1: Determine the change in barrier height Since the applied voltage opposes the built-in potential, it reduces the effective barrier from V_0 to approximately $V_0 - V_F$, where V_F is the forward voltage. Majority carriers can therefore cross the junction more readily.

Step 2: Determine the change in depletion width The forward-bias supply pushes majority carriers towards the junction, reducing the region depleted of mobile carriers. Thus its width also decreases. Option (B) describes the reverse-bias trend, while (C) and (D) incorrectly predict opposite changes in the barrier height and depletion width.

Step 3: Conclusion Both the potential barrier height and the width of the depletion layer decrease. Hence, the correct option is (A).

Quick Tip to Solve: Remember that forward bias opposes the junction field, lowering the barrier and narrowing the depletion region together.

13. Assertion (A): EM waves do not require a medium for their propagation.

Reason (R): EM waves are transverse waves.

Select the correct answer:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- An electromagnetic wave consists of time-varying electric and magnetic fields that can propagate through vacuum without material particles.
- In a freely propagating electromagnetic wave, the electric and magnetic field vectors are perpendicular to the direction of propagation; the wave is transverse.

Step 1: Judge the assertion The assertion is true. Electromagnetic disturbances can travel in vacuum: for example, sunlight reaches Earth across space. A material medium is not needed because changing electric and magnetic fields sustain the propagating disturbance.

Step 2: Judge the reason The reason is also true. The electric and magnetic fields oscillate at right angles to the direction in which an electromagnetic wave travels, which is the meaning of transverse propagation.

Step 3: Test the explanatory link Being transverse specifies the orientation of oscillations, not whether a medium is necessary. A transverse wave on a stretched string still needs a material string. Therefore the reason does not explain the assertion. Option (A) wrongly claims an explanation, while (C) and (D) wrongly classify a true statement as false.

Step 4: Conclusion Both the assertion and the reason are true, but the reason does not correctly explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Test the explanation separately from truth: transverse string waves show that transverse does not mean medium-free.

14. Assertion (A): A charged particle is moving with velocity v in x - y plane, making an angle θ ($0 < \theta < \frac{\pi}{2}$) with x -axis. If a uniform magnetic field $\vec{B}$ is applied in the region, along y -axis, the particle will move in a helical path with its axis parallel to x -axis.

Reason (R): The direction of the magnetic force acting on a charged particle moving in a magnetic field is along the velocity of the particle.

Select the correct answer:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (D) Both Assertion (A) and Reason (R) are false.

Solution:

Concept:

- The magnetic force on a charge is $\vec{F} = q\vec{v} \times \vec{B}$. It is perpendicular to both the velocity and the magnetic field, so it does no work.
- A velocity component parallel to a uniform magnetic field remains unchanged, while the perpendicular component produces circular motion. Their combination gives a helix along the field.

Step 1: Judge the assertion using velocity components Here $\vec{B}$ is along the y -axis. The parallel component is $v_y = v \sin \theta$, and the perpendicular component is $v_x = v \cos \theta$. Both are nonzero for the given angle range. The particle therefore follows a helix whose axis is parallel to the y -axis, not the x -axis. The assertion is false.

Step 2: Judge the reason using the force law The cross product $q\vec{v} \times \vec{B}$ makes the magnetic force perpendicular to the instantaneous velocity. In particular, $\vec{F} \cdot \vec{v} = 0$; the force changes direction of motion rather than speed. Thus the claim that the force acts along the velocity is false.

Step 3: Test the explanation and select the code Since the reason is false, it cannot explain the assertion. The true magnetic-force law yields a helix about the field direction. Options (A) and (B) require two true statements, while (C) requires a true assertion; none matches the independent checks above.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: For helical motion in a uniform magnetic field, align the helix axis with the field, and keep magnetic force perpendicular to velocity.

15. Assertion (A): The minimum negative potential applied to the anode in a photoelectric experiment at which photoelectric current becomes zero, is called cut-off voltage.

Reason (R): The threshold frequency for a metal is the minimum frequency of incident radiation below which emission of photoelectrons does not take place.

Select the correct answer:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- The stopping or cut-off potential V_0 is the magnitude of the retarding potential needed to stop even the fastest photoelectrons: $eV_0 = K_{\max}$.
- The threshold frequency ν_0 satisfies $h\nu_0 = \phi$, where ϕ is the work function of the metal.

Step 1: Evaluate the assertion Making the anode negative relative to the emitting cathode repels the emitted electrons. At the minimum retarding potential that prevents even the most energetic electrons from reaching the anode, the photocurrent becomes zero. This is precisely the cut-off voltage, so Assertion (A) is true.

Step 2: Evaluate the reason A photon must supply at least the work function to release an electron. Thus emission cannot occur when $h\nu < \phi$, or $\nu < \nu_0$. Reason (R) correctly defines the threshold frequency and is true.

Step 3: Test the explanatory link The reason states the condition for emission to begin; it does not explain how a negative anode potential stops already emitted electrons. The assertion follows from the electrostatic retardation of photoelectrons. Hence the two true statements are not an explanation pair; options (A), (C) and (D) do not apply.

Step 4: Conclusion Both statements are true, but the reason does not correctly explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Separate the emission threshold $h\nu_0 = \phi$ from the stopping condition $eV_0 = K_{\max}$ when judging the explanation link.

16. Assertion (A): A ray of light is incident normally on the face of a prism. The emergent ray will graze along the opposite face of the prism when the critical angle at glass-air interface is equal to the angle of the prism.

Reason (R): The refractive index of a prism depends on angle of the prism.

Select the correct answer:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- For a prism of angle A , the internal refraction angles obey $r_1 + r_2 = A$.
- At a glass-air interface, grazing emergence occurs when the internal incidence equals the critical angle C , where $\sin C = 1/\mu$ and μ is the refractive index of glass relative to air.

Step 1: Evaluate the assertion Normal incidence on the first face gives $i_1 = r_1 = 0$, so the ray enters without deviation. The prism relation then gives $r_2 = A$. The emergent ray grazes the second face when its angle of emergence is 90° , requiring $r_2 = C$. Thus the stated condition $A = C$ is correct and the assertion is true.

Step 2: Evaluate the reason Refractive index is a property of the optical material for a specified wavelength and physical conditions. Changing the prism's angle changes the geometry of refraction, but does not change its material's refractive index. Therefore the reason is false.

Step 3: Check the explanation and select the code Since the reason is false, it cannot explain the true assertion. The assertion instead follows from the prism-angle relation and the critical-angle condition. The codes requiring both statements to be true or both to be false are therefore excluded.

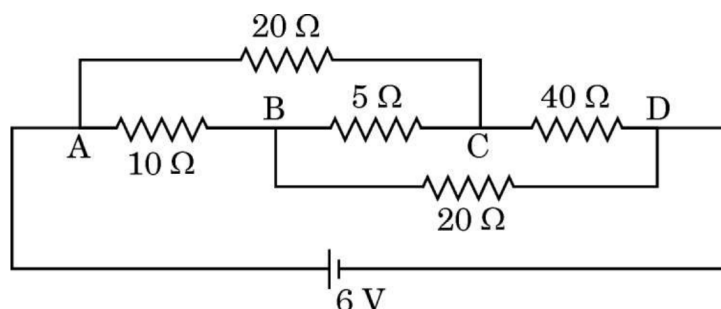
Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: For normal entry into a prism, set $r_1 = 0$ immediately; the incidence at the second face then equals the prism angle.

Section B (10 marks)

17. Calculate the value of the current passing through the battery in the given circuit diagram.

[2 marks]



Solution:

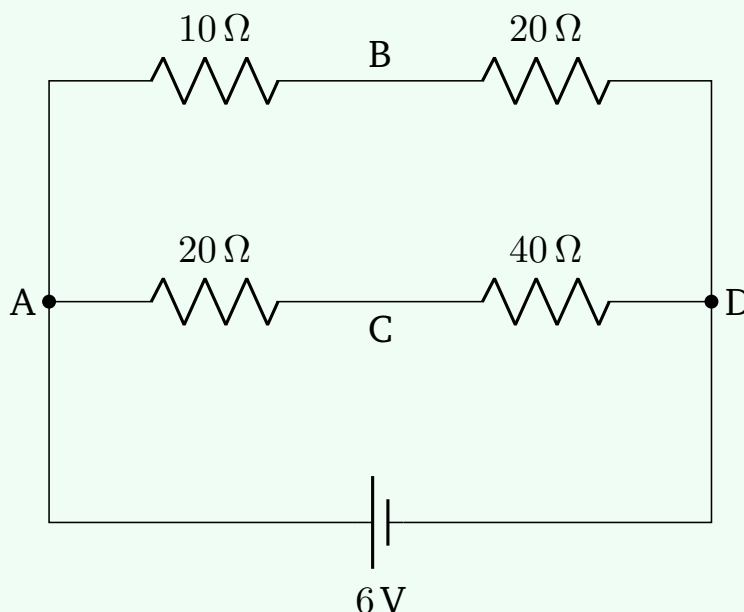
Concept:

- A Wheatstone bridge is balanced when the resistance ratios in its two branches are equal. The middle junctions are then at equal potential, so the bridge resistor carries no current.
- Series resistances add, while two parallel resistances satisfy $R_{eq} = R_1 R_2 / (R_1 + R_2)$. The battery current is $I = V / R_{eq}$.

Step 1: Identify the balanced bridge The battery is connected across A and D. One branch is A–B–D, with resistances $10\ \Omega$ and $20\ \Omega$; the other is A–C–D, with resistances $20\ \Omega$ and $40\ \Omega$. Their ratios satisfy

$$\frac{R_{AB}}{R_{BD}} = \frac{10}{20} = \frac{20}{40} = \frac{R_{AC}}{R_{CD}}.$$

Hence B and C are at the same potential and no current flows through the $5\ \Omega$ resistor between them. It may be omitted when calculating the resistance seen by the battery.



Equivalent circuit: the balanced B–C branch carries zero current.

Step 2: Reduce the two branches and apply Ohm's law The diagram shows the equivalent network after removal of the inactive bridge branch. The series combinations are $R_1 = 10 + 20 = 30\ \Omega$ and

$R_2 = 20 + 40 = 60 \Omega$, connected in parallel across the battery. Therefore,

$$R_{\text{eq}} = \frac{30 \times 60}{30 + 60} = 20 \Omega, \quad I = \frac{6}{20} = 0.30 \text{ A}.$$

As a check, the branch currents are $6/30 = 0.20 \text{ A}$ and $6/60 = 0.10 \text{ A}$, whose sum gives the battery current.

Step 3: Conclusion The current passing through the 6 V battery is $\boxed{0.30 \text{ A}}$.

Quick Tip to Solve: First compare the ratios along A–B–D and A–C–D; the balanced 5Ω bridge can be removed before combining resistors.

18. In a Young's double-slit experiment, the intensity at the central maximum in the interference pattern on the screen is I_0 . Find the intensity at a point on the screen where the path difference between the interfering waves is $\frac{\lambda}{6}$. [2 marks]

Solution:

Concept:

- A path difference Δ between coherent waves of wavelength λ produces a phase difference $\varphi = 2\pi\Delta/\lambda$.
- For equal-intensity slits in Young's experiment, $I = 4I_s \cos^2(\varphi/2)$, where I_s is the intensity from either slit alone. The central maximum has intensity $I_0 = 4I_s$.

Step 1: Convert the printed path difference to phase difference The official question gives $\Delta = \lambda/6$. A full path difference of one wavelength corresponds to a phase difference of 2π ; therefore this fraction of a wavelength gives

$$\varphi = \frac{2\pi}{\lambda} \frac{\lambda}{6} = \frac{\pi}{3}.$$

At the central maximum the path and phase differences are zero, so the waves add in phase.

Step 2: Express the intensity in terms of the central maximum Divide the equal-slit interference formula by its maximum value $4I_s$. The resulting ratio depends only on the phase difference:

$$\frac{I}{I_0} = \cos^2\left(\frac{\varphi}{2}\right) = \cos^2\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}.$$

The phase difference is less than π , so the point lies between the central bright fringe and the first dark fringe; its nonzero intensity is consistent with partial constructive interference.

Step 3: Conclusion The required intensity is $\boxed{I = \frac{3I_0}{4}}$.

Quick Tip to Solve: With central intensity supplied, use $I/I_0 = \cos^2(\pi\Delta/\lambda)$; insert the path fraction before evaluating the cosine.

19. Define the ‘distance of closest approach’. An α -particle of kinetic energy K is bombarded on a thin gold foil. Derive an expression for the ‘distance of closest approach’. [2 marks]

Solution:

Concept:

- The distance of closest approach is the minimum separation between the centres of an incident α -particle and the nucleus in a head-on encounter, where the particle momentarily comes to rest before returning.
- The repulsive electrostatic potential energy of charges q_1 and q_2 separated by r is $U = q_1 q_2 / (4\pi\epsilon_0 r)$, taking zero potential energy at infinity.

Step 1: Specify the charges and energy conditions An α -particle has charge $+2e$, and a gold nucleus has charge $+Ze$, with $Z = 79$. Consider a head-on approach to the heavy nucleus, whose recoil is neglected. Far from it, the alpha particle has kinetic energy K and negligible electrostatic potential energy. As the two positive charges repel, the particle slows down and its kinetic energy becomes electrostatic potential energy.

Step 2: Apply conservation of energy at the turning point At the closest separation r_0 , the head-on alpha particle momentarily has zero speed. Equating its initial kinetic energy to the final electrostatic potential energy gives

$$K = \frac{1}{4\pi\epsilon_0} \frac{(2e)(Ze)}{r_0}.$$

Rearranging and then using the atomic number of gold,

$$r_0 = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{K} = \frac{1}{4\pi\epsilon_0} \frac{158e^2}{K}.$$

Thus a larger initial kinetic energy allows the alpha particle to penetrate closer to the nucleus. The formula gives r_0 in metres when K is in joules and the other quantities are in SI units.

Step 3: Conclusion The distance of closest approach is the turning-point separation in a head-on encounter. For gold,

$$r_0 = \frac{158e^2}{4\pi\epsilon_0 K}.$$

Quick Tip to Solve: Use the product of charges $(2e)(79e)$, and set the final kinetic energy to zero only for the head-on turning point.

20(a). Using the mirror equation and the formula of magnification, deduce that “the virtual image produced by a convex mirror is always diminished in size and is located between the pole and the focus.” [2 marks]

Solution:

Concept:

- The mirror equation is $1/f = 1/v + 1/u$, where u , v and f are signed object distance, image distance and focal length.
- For a convex mirror and a real object, the Cartesian convention gives $f > 0$, $u < 0$; transverse magnification is $m = -v/u$.

Step 1: Express the image distance using positive quantities Put $u = -a$, where $a > 0$ is the distance of the real object in front of the mirror. Substitution into the mirror equation gives

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{a}, \quad v = \frac{af}{a+f}.$$

Since a and f are positive, $v > 0$; the image is behind the mirror and hence virtual. Moreover,

$$\frac{v}{f} = \frac{a}{a+f} < 1.$$

Thus $0 < v < f$, so the image lies strictly between the pole and focus for every finite real-object distance.

Step 2: Compare the image size with the object size The magnification follows directly from the signed distances:

$$m = -\frac{v}{u} = \frac{v}{a} = \frac{f}{a+f}.$$

This ratio is positive and smaller than unity. Its positive sign means the image is erect, while its magnitude being less than one means the image height is less than the object height. For an object at infinity the limiting image is at the focus and is point-sized.

Step 3: Conclusion For a finite real object, $0 < v < f$ and $0 < m < 1$. A convex mirror therefore produces a virtual, erect, diminished image between its pole and focus.

Quick Tip to Solve: Set $u = -a$ with $a > 0$; both required inequalities then follow immediately because $a + f$ exceeds either positive term.

OR

20(b). A convex lens of focal length 10 cm, a concave lens of focal length 15 cm and a third lens of unknown focal length are placed coaxially in contact. If the focal length of the combination is +12 cm, find the nature and focal length of the third lens, if all lenses are thin. Will the answer change if the lenses were thick? [2 marks]

Solution:

Concept:

- For coaxial thin lenses in contact in air, powers add: $1/F = 1/f_1 + 1/f_2 + 1/f_3$, where F is the combination's focal length.
- A converging lens has positive focal length and a diverging lens negative focal length. The thin-lens contact approximation neglects thickness and separation of principal planes.

Step 1: Substitute the signed focal lengths The convex lens has $f_1 = +10$ cm, the concave lens has $f_2 = -15$ cm, and the combination has $F = +12$ cm. Therefore,

$$\begin{aligned} \frac{1}{f_3} &= \frac{1}{12} - \frac{1}{10} + \frac{1}{15} \\ &= \frac{5 - 6 + 4}{60} = \frac{1}{20} \text{ cm}^{-1}. \end{aligned}$$

The positive reciprocal indicates that the third lens supplies converging power. Equivalently, the first two lenses together have power $+10 - 6.67 = +3.33$ D, while the required total is $+8.33$ D.

Step 2: Examine the effect of lens thickness For thick lenses, even lenses whose surfaces touch generally have separated principal planes. The simple addition of thin-lens powers is then not valid: the equivalent power depends also on thicknesses, refractive indices and surface geometry. Hence the inferred focal length can change; the given data alone do not determine a unique revised value for thick lenses.

Step 3: Conclusion The third thin lens is converging (convex), with $f_3 = +20$ cm. For thick lenses the answer generally changes, and further geometrical and optical data are required.

Quick Tip to Solve: Insert the minus sign for the concave lens before rearranging the power formula; use one common length unit throughout.

21. Draw energy band diagrams of n-type and p-type semiconductors at temperature $T > 0$ K. Show the donor/acceptor energy levels with the order of difference of their energies from the bands.

[2 marks]

Solution:

Concept:

- Doping introduces impurity energy levels inside the forbidden energy gap. Let E_C be the conduction-band minimum and E_V the valence-band maximum.
- Donors supply electrons from a level E_D just below E_C ; acceptors take electrons into a level E_A just above E_V , leaving holes in the valence band.

Step 1: Draw the donor level for the n-type semiconductor Pentavalent donor atoms introduce a shallow level close to the conduction band. The small separation is of order

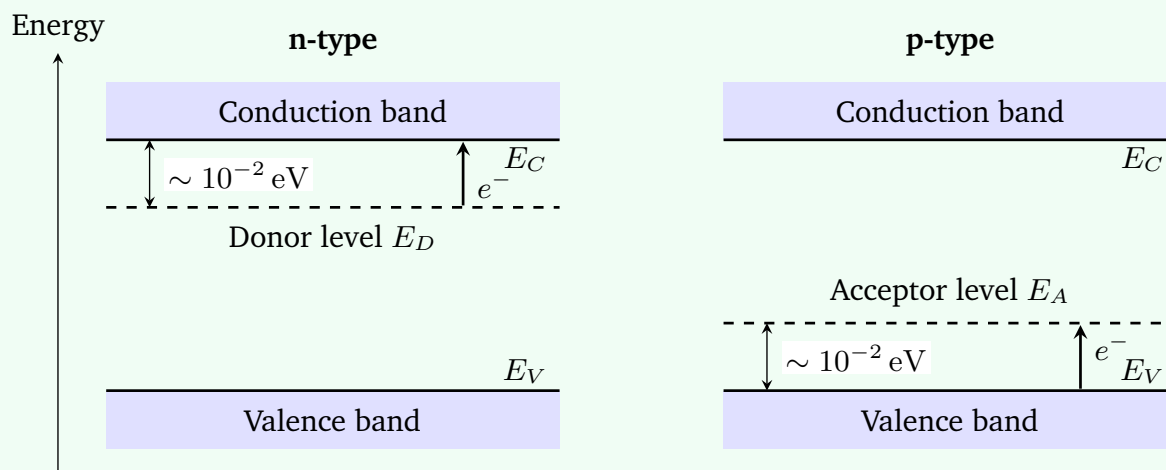
$$E_C - E_D \sim 10^{-2} \text{ eV.}$$

At a nonzero temperature, thermal excitation can transfer electrons from these donor states into the conduction band, leaving positive ionised donors. Electrons are the majority charge carriers. The diagram places the donor level inside the gap, not inside the conduction band.

Step 2: Draw the acceptor level for the p-type semiconductor Trivalent acceptors introduce a shallow level close to the valence band, with separation of order

$$E_A - E_V \sim 10^{-2} \text{ eV.}$$

A valence electron can be thermally excited into an acceptor level. The resulting mobile hole remains in the valence band, while the acceptor becomes negatively ionised. Holes are the majority carriers. The small impurity-ionisation energies are much less than the full semiconductor band gap, which is typically of order 1 eV.



Impurity energy levels for $T > 0\text{ K}$; vertical positions are schematic and not to scale.

Step 3: Conclusion The n-type donor level lies just below the conduction band; the p-type acceptor level lies just above the valence band. Each separation is of order 10^{-2} eV , as shown.

Quick Tip to Solve: Remember donor-to-conduction and valence-to-acceptor: each electron needs only a small upward energy jump, not the full band gap.

Section C (21 marks)

22. Explain the process of formation of ‘depletion layer’ and ‘potential barrier’ in a p-n junction region of a diode, with the help of a suitable diagram. Which feature of junction diode makes it suitable for its use as a rectifier? [3 marks]

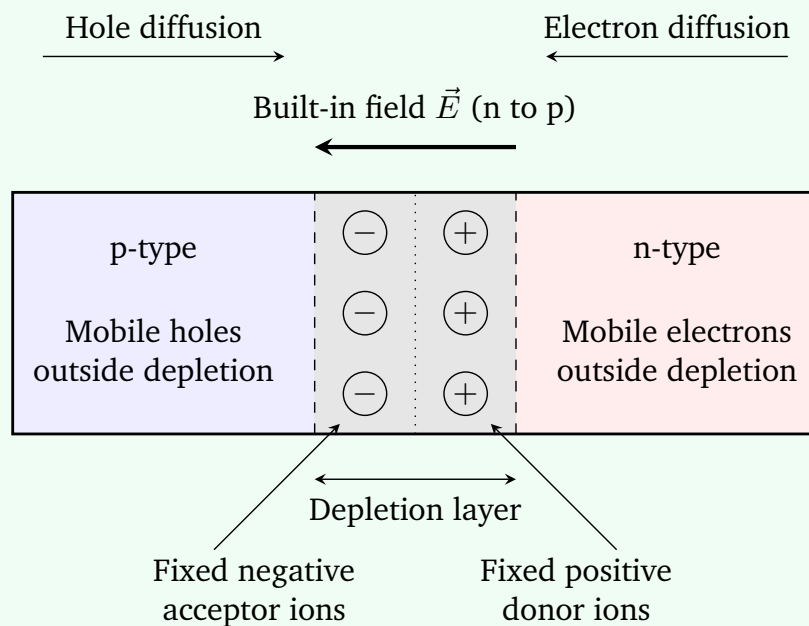
Solution:

Concept:

- A carrier concentration gradient drives diffusion: electrons move from the n-side to the p-side and holes from the p-side to the n-side.
- Uncovered, immobile ionised dopants create a space-charge region and an electric field. At thermal equilibrium, drift and diffusion currents balance.
- A diode rectifies because it conducts readily under forward bias and offers very high resistance under reverse bias below breakdown.

Step 1: Explain the formation of the depletion layer When p-type and n-type regions form a junction, their majority carriers diffuse across it because of the concentration difference. Electrons entering the p-side recombine with holes; holes entering the n-side recombine with electrons. Close to the junction this removes most mobile carriers. Fixed negative acceptor ions are left on the p-side, and fixed positive donor ions on the n-side. The resulting region is called the depletion layer because it is depleted of mobile charge carriers; it is not devoid of all charge.

Step 2: Explain the potential barrier and equilibrium The uncovered ions set up an electric field directed from the positive n-side to the negative p-side. This field opposes further diffusion: its force drives electrons toward the n-side and holes toward the p-side. The associated built-in potential is higher on the n-side than on the p-side; crossing the junction by diffusion requires overcoming an energy barrier. As the space charge builds up, the opposing drift current increases until it balances the diffusion current. With no applied bias, the net current is then zero and the depletion width stabilises.



Unbiased p-n junction: fixed ions, depleted region and built-in electric field. Diffusion directions describe the formation process.

Step 3: Connect the bias response to rectification Forward bias lowers the potential barrier and narrows the depletion layer, allowing a substantial current. Reverse bias raises the barrier and widens the layer; only a small minority-carrier leakage current flows before breakdown. This strongly asymmetric current-voltage response gives the diode low forward resistance and high reverse resistance. In an alternating input it therefore passes mainly one half-cycle, producing a unidirectional, pulsating output current.

Step 4: Conclusion Carrier diffusion and recombination leave fixed ions that form the depletion layer and its opposing potential barrier. The diode's preferential conduction under forward bias makes it suitable for rectification.

Quick Tip to Solve: Draw negative fixed ions on the p-side and positive fixed ions on the n-side; the built-in field always points from n to p.

23. An electric field $\vec{E}$ given by:

$$\vec{E} = 100 \hat{i} \text{ N/C for } x > 0$$

$$\vec{E} = -100 \hat{i} \text{ N/C for } x < 0$$

exists in a region. A right circular cylinder of length 10 cm and radius 2 cm, is placed in the region such that its axis coincides with x -axis and its two faces are at $x = -5$ cm and $x = 5$ cm. Calculate:

(a) the net outward flux through the cylinder, and

(b) the net charge inside the cylinder.

[3 marks]

Solution:

Concept:

- Electric flux through a surface is $\Phi = \int \vec{E} \cdot d\vec{A}$, where $d\vec{A}$ points along the outward normal for a closed surface.
- Gauss's law gives $\Phi_{\text{net}} = Q_{\text{enclosed}}/\epsilon_0$, where $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$.

Step 1: Identify the contributing surfaces Divide the closed cylinder into its curved surface and its two circular end faces. The electric field is parallel to the cylinder's axis, whereas the outward normal to the curved surface is perpendicular to that axis. Thus $\vec{E} \cdot d\vec{A} = 0$ on the curved surface and its flux is zero. The radius in SI units is $r = 2 \text{ cm} = 0.02 \text{ m}$, so each end has area

$$A = \pi r^2 = \pi(0.02)^2 = 4\pi \times 10^{-4} \text{ m}^2.$$

Step 2: Calculate the outward flux through both ends At $x = +5 \text{ cm}$, both the field and the outward area vector point along $+\hat{i}$, giving $\Phi_+ = 100A$. At $x = -5 \text{ cm}$, both point along $-\hat{i}$, so their dot product is again positive: $\Phi_- = (-100\hat{i}) \cdot (-A\hat{i}) = 100A$. Consequently the two fluxes add, even though the field directions are opposite:

$$\Phi_{\text{net}} = 100A + 100A = 0.08\pi \simeq 0.2513 \text{ N m}^2\text{C}^{-1}.$$

Step 3: Apply Gauss's law for the enclosed charge The total outward flux fixes the net charge inside the closed cylinder, irrespective of its detailed distribution. Substituting the flux gives

$$Q_{\text{enclosed}} = \epsilon_0 \Phi_{\text{net}} = (8.854 \times 10^{-12})(0.08\pi) \simeq 2.23 \times 10^{-12} \text{ C}.$$

The charge is positive because the net flux is outward. The cylinder length does not enter the result: the field has constant magnitude on each end face, and the curved surface contributes no flux.

Step 4: Conclusion (a) The net outward flux is $0.08\pi \simeq 0.2513 \text{ N m}^2\text{C}^{-1}$. (b) The net enclosed charge is $+2.23 \times 10^{-12} \text{ C}$.

Quick Tip to Solve: Assign an outward normal separately to each end cap; opposite fields need not cancel when the outward normals are also opposite.

24. Answer the following, giving reasons:

- The maximum kinetic energy of the photoelectrons is independent of the intensity of incident radiation.
- Photoelectric current increases with the increase in the intensity of the incident radiation.
- The stopping potential V_0 varies linearly with the frequency ν of the incident radiation for a given photosensitive surface.

[3 marks]

Solution:

Concept:

- A photon of frequency ν carries energy $h\nu$, where h is Planck's constant. In the usual photoelectric process, one electron absorbs energy from one photon.
- Einstein's photoelectric equation is $K_{\text{max}} = h\nu - \phi$, where ϕ is the surface work function. Emission requires $\nu \geq \nu_0 = \phi/h$.
- Stopping potential is related to maximum kinetic energy by $eV_0 = K_{\text{max}}$, with e the positive magnitude of the electron charge. Photocurrent measures collected charge per unit time.

Step 1: Explain why intensity does not set maximum kinetic energy (a) At a fixed frequency above threshold, increasing intensity increases the number of photons reaching the surface per second, but the energy $h\nu$ of each photon remains unchanged. An electron uses energy ϕ to escape, so the largest

possible remaining energy is

$$K_{\max} = h\nu - \phi.$$

For a given photosensitive surface, ϕ is fixed. Thus intensity changes the number of emitted electrons, not their maximum kinetic energy.

Step 2: Relate photon arrival rate to photoelectric current (b) For monochromatic radiation of fixed frequency, incident power P supplies photons at the rate $\dot{N}_\gamma = P/(h\nu)$. Increasing intensity at fixed illuminated area increases P , hence the number of photons available to eject electrons each second. With unchanged collection conditions, more photoelectrons reach the anode per second. Since

$$I_{\text{ph}} = e\dot{N}_{\text{collected}},$$

the photoelectric current increases. Saturation current is proportional to intensity because all emitted electrons are collected. Below threshold, increasing intensity cannot initiate emission in the usual photoelectric experiment.

Step 3: Obtain the frequency dependence of stopping potential (c) At the stopping potential, the retarding electric field just stops even the fastest photoelectrons. Therefore

$$eV_0 = h\nu - \phi \quad \Rightarrow \quad V_0 = \frac{h}{e}\nu - \frac{\phi}{e}.$$

This has the straight-line form $y = mx + c$, with slope h/e and extrapolated vertical intercept $-\phi/e$. It meets the frequency axis at $\nu_0 = \phi/h$, where the maximum kinetic energy becomes zero.

Step 4: Conclusion At fixed frequency, intensity controls the photon arrival rate and thus photocurrent, while $K_{\max}$ depends on photon energy. For a fixed surface, $V_0 = (h/e)\nu - \phi/e$, so stopping potential varies linearly with frequency above threshold.

Quick Tip to Solve: Separate photon energy from photon number: frequency controls $K_{\max}$ and V_0 ; intensity controls the number of photoelectrons emitted per second.

25(a). (i) Write Biot-Savart's law in vector form.

(ii) Two identical circular coils A and B, each of radius R , carrying currents I and $\sqrt{3}I$ respectively, are placed concentrically in XY and YZ planes respectively. Find the magnitude and direction of the net magnetic field at their common centre. [3 marks]

Solution:

Concept:

- Biot-Savart's law gives the magnetic field of a current element: $d\vec{B} = (\mu_0/4\pi)I(d\vec{\ell} \times \hat{r})/r^2$. The vector $\hat{r}$ points from the current element to the observation point, and $d\vec{\ell}$ follows the current.
- The field at the centre of a single circular loop of radius R is $B = \mu_0 I/(2R)$, normal to the loop's plane, with its sense fixed by the right-hand rule. Fields add vectorially by superposition.

Step 1: State Biot-Savart's law in vector form (i) For a steady current element $I d\vec{\ell}$, the field at

displacement $\vec{r} = r\hat{r}$ is

$$d\vec{B} = \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2} = \frac{\mu_0 I d\vec{\ell} \times \vec{r}}{4\pi r^3}.$$

Here μ_0 is the permeability of free space. The cross product makes $d\vec{B}$ perpendicular to both the current element and its displacement towards the observation point.

Step 2: Find the perpendicular fields of the two coils (ii) Coil A lies in the XY plane, so its field is along the Z axis. Coil B lies in the YZ plane, so its field is along the X axis. Using the circular-loop centre-field formula, their magnitudes are

$$B_A = \frac{\mu_0 I}{2R}, \quad B_B = \frac{\sqrt{3}\mu_0 I}{2R}.$$

These fields are perpendicular; their magnitudes must not be added directly.

Step 3: Add the fields and specify the resultant direction The magnitude follows from Pythagoras:

$$B_{\text{net}} = \sqrt{B_A^2 + B_B^2} = \frac{\mu_0 I}{2R} \sqrt{1 + 3} = \frac{\mu_0 I}{R}.$$

Let θ be the angle from $\vec{B}_B$ towards $\vec{B}_A$. Then

$$\tan \theta = \frac{B_A}{B_B} = \frac{1}{\sqrt{3}}, \quad \theta = 30^\circ.$$

Thus the resultant lies in the XZ plane, at 30° to coil B 's field and 60° to coil A 's field. The paper does not specify the circulation senses. If the currents produce fields along $+X$ and $+Z$, respectively, then

$$\vec{B}_{\text{net}} = \frac{\mu_0 I}{2R} (\sqrt{3}\hat{i} + \hat{k}).$$

Reversing either current reverses its corresponding component. If each coil has N turns, all field magnitudes acquire the same factor N .

Step 4: Conclusion For the stated loops, $B_{\text{net}} = \mu_0 I/R$. It lies in the XZ plane, 30° from $\vec{B}_B$ towards $\vec{B}_A$ (60° from $\vec{B}_A$); the signed axial components follow the current senses by the right-hand rule.

Quick Tip to Solve: First take the normal to each coil's plane, then combine the perpendicular fields. Measure the smaller angle from the larger field, here coil B 's field.

OR

25(b). (i) A rectangular loop of sides l and b carries a current I clockwise. Write the magnetic moment $\vec{m}$ of the loop and show its direction in a diagram.

(ii) The loop is placed in a uniform magnetic field $\vec{B}$ and is free to rotate about an axis which is perpendicular to $\vec{B}$. Prove that the loop experiences no net force, but a torque $\vec{\tau} = \vec{m} \times \vec{B}$. [3 marks]

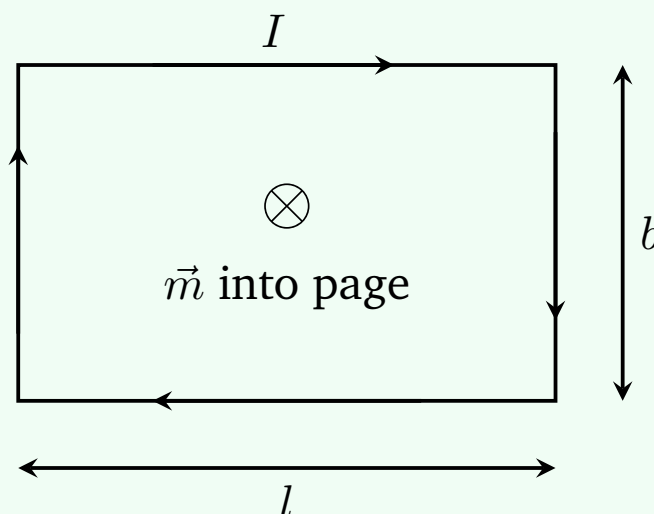
Solution:

Concept:

- A plane current loop has magnetic moment $\vec{m} = IA\hat{n}$, where A is its area and $\hat{n}$ is the right-hand-rule normal.

- In a uniform magnetic field the force on a straight side is $\vec{F} = I\vec{L} \times \vec{B}$, where $\vec{L}$ points along the current. Equal opposite forces can form a couple.

Step 1: (i) Find the magnetic moment and its direction The area of the rectangular loop is $A = lb$, so its moment has magnitude $m = Ilb$, in A m^2 . Curl the fingers of the right hand along the clockwise current as viewed in the diagram. The thumb points into the page, perpendicular to the loop. Hence $\vec{m} = Ilb\hat{n}$, with $\hat{n}$ directed into the page, represented by $\otimes$.



Clockwise current; the magnetic moment is normal to the loop and into the page.

Step 2: (ii) Add the forces on all four sides Each side has an opposite side of equal length carrying current in the opposite direction. Since the field is the same everywhere, their magnetic forces cancel pairwise. Equivalently, taking the vector sum around the closed rectangle,

$$\vec{F}_{\text{net}} = I \left(\sum_{\text{sides}} \vec{L} \right) \times \vec{B} = \vec{0}.$$

Zero resultant force therefore prevents translational acceleration due to the field, but does not require zero torque.

Step 3: Calculate the couple acting on the loop Let θ be the angle between $\vec{m}$ and $\vec{B}$. Take the rotation axis through the loop centre parallel to the sides of length l , perpendicular to $\vec{B}$. Each of these two sides experiences force $F = IlB$. Their forces are opposite and have perpendicular separation $b \sin \theta$, so their couple is

$$\tau = F(b \sin \theta) = IlbB \sin \theta = mB \sin \theta.$$

The forces on the other two sides act along their common line through the loop centre and give no resultant torque. The couple turns the loop to align its magnetic moment with the field. The right-hand rule gives its vector direction as $\vec{m} \times \vec{B}$. This expression is independent of the chosen drawing orientation and vanishes when the moment is parallel or antiparallel to the field.

Step 4: Conclusion $\vec{m} = Ilb\hat{n}$, directed into the page for the clockwise loop. In a uniform field, $\vec{F}_{\text{net}} = 0$ and $\vec{\tau} = \vec{m} \times \vec{B}$.

Quick Tip to Solve: Measure the torque angle from the normal to the loop, not from its plane; clockwise current gives a normal into the page.

26. Name the electromagnetic radiations and write their frequency range:

- (a) Used to take the photograph of the bones
- (b) Produced by hot bodies
- (c) Used in television communication systems

[3 marks]

Solution:

Concept:

- Electromagnetic waves form a continuous spectrum; the conventional frequency boundaries between named bands are approximate and may overlap.
- Frequency ν and wavelength λ are related by $\nu = c/\lambda$, where $c = 3 \times 10^8 \text{ m s}^{-1}$. Higher frequency corresponds to shorter wavelength.

Step 1: (a) Identify the radiation used for bone photographs X-rays are used for radiographs of bones. Their approximate frequency range is 10^{16} – 10^{20} Hz. Soft tissue transmits X-rays more readily than dense bone, so the different absorption produces contrast on the detector. The whole X-ray band is not used in a single medical exposure; the answer names the radiation and its broad spectral range.

Step 2: (b) Identify radiation associated with hot bodies Infrared radiation is the characteristic heat radiation emitted strongly by ordinary hot bodies. Its approximate frequency range is 10^{11} – 4×10^{14} Hz. This lies below visible light in frequency. Thermal radiation is a continuous spectrum: a sufficiently hot body also emits visible light, so saying infrared does not imply that all hot bodies emit only infrared.

Step 3: (c) Identify the waves used for television communication Radio waves, specifically the VHF and UHF bands, are used in terrestrial television communication. Television frequencies are approximately 4×10^7 – 10^9 Hz, that is, about 40 MHz to 1 GHz. These are a portion of the much broader radio spectrum, rather than the full radio-wave band. The carrier frequency is far higher than the audio and picture information frequencies impressed on it.

Step 4: Conclusion (a) X-rays: approximately 10^{16} – 10^{20} Hz. (b) Infrared: approximately 10^{11} – 4×10^{14} Hz. (c) Radio waves in the television VHF/UHF bands: approximately 4×10^7 – 10^9 Hz.

Quick Tip to Solve: Match bones with penetrating X-rays, heat with infrared, and terrestrial television with VHF/UHF radio waves; quote frequencies in hertz.

27. An ac source of voltage $V = V_0 \sin \omega t$ is connected to a circuit element X . It is observed that the current flowing through X varies as $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$.

- (a) Identify the element X and write the expression for its reactance.
- (b) Plot a graph to show the variation of reactance of the element with the frequency of the applied voltage.
- (c) Draw plots showing the variation of voltage and current with time over one cycle of applied ac.

[3 marks]

Solution:

Concept:

- For an ideal inductor of inductance L , the applied voltage is $V = L dI/dt$; the sinusoidal current lags the voltage by $\pi/2$.
- Inductive reactance is $X_L = \omega L = 2\pi f L$, where ω is angular frequency and f is frequency in hertz.

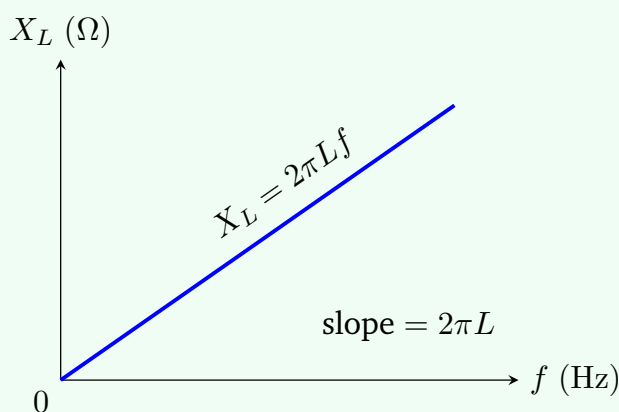
The period is $T = 1/f$.

Step 1: (a) Identify the element using its phase relation The voltage phase is ωt , whereas the current phase is $\omega t - \pi/2$. Thus current lags voltage by one quarter of a cycle, identifying X as an **ideal inductor**. A resistor would have zero phase difference, while a capacitor would make current lead voltage. Differentiate the given current:

$$\frac{dI}{dt} = \omega I_0 \cos(\omega t - \pi/2) = \omega I_0 \sin \omega t,$$
$$V = \omega L I_0 \sin \omega t.$$

Comparing with $V_0 \sin \omega t$ gives $V_0 = \omega L I_0$, and therefore $X_L = V_0/I_0 = \omega L = 2\pi f L$, measured in ohms.

Step 2: (b) Plot reactance against frequency With L constant, $X_L = 2\pi L f$ is directly proportional to f . The graph is a straight line through the origin, with positive slope $2\pi L$. Its intercept is zero because an ideal inductor has zero reactance at zero frequency. Doubling the applied frequency doubles the reactance.

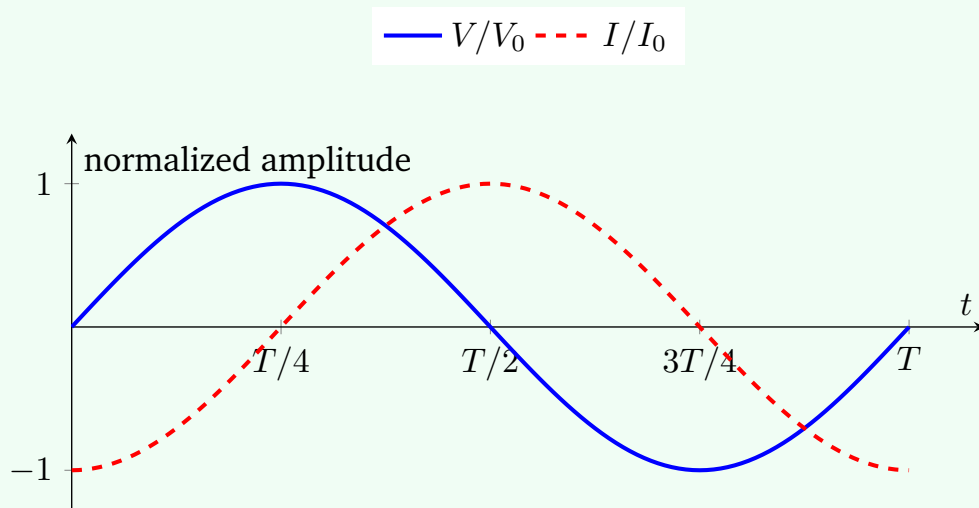


Inductive reactance versus frequency for constant inductance.

Step 3: (c) Plot voltage and current over one complete cycle Write the waveforms in terms of the period:

$$\frac{V}{V_0} = \sin \frac{2\pi t}{T}, \quad \frac{I}{I_0} = -\cos \frac{2\pi t}{T}.$$

At $t = 0$, voltage is zero and increasing, while current is $-I_0$. Voltage reaches its positive maximum at $T/4$; current reaches its positive maximum at $T/2$. Thus the current curve is shifted to the right by $T/4$. The plotted ordinates are normalized to their respective amplitudes so that voltage and current, which have different units, can be compared on the same axes.



One cycle: voltage (solid) and current (dashed), normalized to their own amplitudes.

Step 4: Conclusion X is an ideal inductor, with $X_L = \omega L = 2\pi f L$. Its reactance rises linearly with frequency, and its current lags the voltage by $\pi/2 \text{ rad}$, corresponding to a time delay of $T/4$.

Quick Tip to Solve: The minus sign inside the current phase means lag. Start the current plot at its negative maximum, not at zero, when voltage is a sine wave.

28. Write the mathematical forms of three postulates of Bohr's theory of the hydrogen atom. Using them prove that, for an electron revolving in the n^{th} orbit,

(a) the radius of the orbit is proportional to n^2 , and

(b) the total energy of the atom is proportional to $\left(\frac{1}{n^2}\right)$.

[3 marks]

Solution:

Concept:

- In the Bohr model, the electrostatic attraction supplies centripetal force: $m_e v_n^2 / r_n = k e^2 / r_n^2$, where $k = 1/(4\pi\epsilon_0)$, m_e is electron mass and e is the magnitude of its charge.
- The allowed angular momenta are integral multiples of $\hbar = h/(2\pi)$. Potential energy is measured from zero at infinite separation, so a bound hydrogen atom has negative total energy.

Step 1: State the three postulates mathematically An electron revolves in certain stationary orbits without radiating energy: in an allowed orbit, $dE_n/dt = 0$ and the radiated power is zero. Only orbits with quantised angular momentum are allowed:

$$m_e v_n r_n = n\hbar = \frac{nh}{2\pi}, \quad n = 1, 2, 3, \dots$$

Radiation is emitted or absorbed only during a transition between stationary states. Its frequency satisfies $h\nu = |E_i - E_f|$; for emission from a higher to a lower state, $h\nu = E_i - E_f$.

Step 2: Derive the radius dependence for part (a) Apply the force equation and substitute the speed

from angular-momentum quantisation:

$$m_e v_n^2 r_n = k e^2, \quad v_n = \frac{n \hbar}{m_e r_n},$$
$$\frac{n^2 \hbar^2}{m_e r_n} = k e^2, \quad r_n = \frac{\hbar^2}{m_e k e^2} n^2.$$

The coefficient contains only constants for hydrogen; increasing the orbit number therefore changes the radius by the square of that number.

Step 3: Derive the total-energy dependence for part (b) From the same force equation, kinetic energy is $K_n = \frac{1}{2} m_e v_n^2 = k e^2 / (2 r_n)$. The electron and proton attract, so their electrostatic potential energy is $U_n = -k e^2 / r_n$. Therefore,

$$E_n = K_n + U_n = -\frac{k e^2}{2 r_n}$$
$$= -\frac{m_e k^2 e^4}{2 \hbar^2} \frac{1}{n^2}.$$

Thus the proportionality coefficient in the printed statement $E_n \propto 1/n^2$ is negative. The magnitude decreases as $1/n^2$, while the signed energy approaches zero from below as n increases.

Step 4: Conclusion $r_n = a_0 n^2$, where $a_0 = \hbar^2 / (m_e k e^2)$, and $E_n = -C/n^2$, where $C = m_e k^2 e^4 / (2 \hbar^2) > 0$. Hence the radius is proportional to n^2 , and the total energy has the stated inverse-square dependence with a negative proportionality constant.

Quick Tip to Solve: Eliminate speed between Coulomb force and angular momentum to obtain the radius, then use $E = -K = -k e^2 / (2 r)$ to find the energy dependence immediately.

Section D (8 marks)

Questions number 29 and 30 are case study-based questions. Read the following paragraphs and answer the questions that follow.

29. In a metallic conductor, an electron, moving due to thermal motion, suffers collisions with the heavy fixed ions but after collision, it will emerge out with the same speed but in random directions. If we consider all the electrons, their average velocity will be zero. When an electric field is applied, electrons move with an average velocity, known as drift velocity (v_d). The average time between successive collisions is known as relaxation time (τ). The magnitude of drift velocity per unit electric field is called mobility (μ). An expression for current through the conductor can be obtained in terms of drift velocity, number of electrons per unit volume (n), electronic charge ($-e$), and the cross-sectional area (A) of the conductor. This expression leads to an expression between current density ($\vec{J}$) and the electric field ($\vec{E}$). Hence, an expression for resistivity (ρ) of a metal is obtained. This expression helps us to understand increase in resistivity of a metal with increase in its temperature, in terms of change in the relaxation time (τ) and change in the number density of electrons (n). [4 marks]

29(i)(a). Consider the contribution of the following two factors I and II in resistivity of a metal:

I. Relaxation time of electrons

II. Number of electrons per unit volume

The resistivity of a metal increases with increase in its temperature because:

[1 mark]

- (A) I decreases and II increases.
- (B) I increases and II is almost constant.
- (C) Both I and II increase.
- (D) I decreases and II is almost constant.

Correct Answer: (D) I decreases and II is almost constant.

Solution:

Concept:

- The Drude expression for resistivity is $\rho = m/(ne^2\tau)$, where m is the electron mass, n the free-electron number density and τ the relaxation time.
- Heating a metal increases lattice vibrations and the frequency of electron collisions; its free-electron number density remains approximately constant.

Step 1: Identify the temperature dependence Stronger thermal vibrations of the ions scatter conduction electrons more frequently. The mean time between collisions, factor I, therefore decreases. Unlike a semiconductor, a metal already has a large supply of conduction electrons, so heating does not appreciably increase factor II.

Step 2: Apply the resistivity relation With m , e and approximately n fixed, $\rho \propto 1/\tau$. A decrease in τ raises the resistivity. Option (A) incorrectly asserts an increase in n ; options (B) and (C) incorrectly assert an increase in τ .

Step 3: Conclusion Factor I decreases and factor II is almost constant. Hence, the correct option is (D).

Quick Tip to Solve: For a heated metal, hold n approximately fixed and track the decrease in collision time in $\rho = m/(ne^2\tau)$.

29(i)(b). A steady current flows in a copper wire of non-uniform cross-section. Consider the following three physical quantities:

I. Electric field

II. Current density

III. Drift speed

Then at the different points along the wire:

[1 mark]

(A) II and III change, but I is constant.

(B) I and II change, but III is constant.

(C) I and III change, but II is constant.

(D) All I, II and III change.

Correct Answer: (D) All I, II and III change.

Solution:

Concept:

- In steady flow through one continuous wire, charge conservation makes the current I the same through every cross-section.
- The local relations are $J = I/A$, $v_d = I/(neA)$ and $E = \rho J$, where A is the cross-sectional area, n the electron number density and ρ the resistivity.

Step 1: Compare current density and drift speed Copper has the same material parameters along the wire under the usual uniform-temperature assumption. Since A varies and the steady current is constant, both J and v_d vary inversely with A . They are larger in a narrower part of the wire. Thus quantities II and III change.

Step 2: Determine the electric field The microscopic Ohm law gives $E = \rho I/A$. The field also varies inversely with cross-sectional area, so quantity I changes. Constant current does not mean constant field. Option (A) incorrectly keeps E constant, (B) keeps v_d constant and (C) keeps J constant.

Step 3: Conclusion Electric field, current density and drift speed all change along the non-uniform wire. Hence, the correct option is (D).

Quick Tip to Solve: In a steady current through a varying cross-section, conserve I ; then J , v_d and E all scale as $1/A$.

29(ii). The temperature coefficient of resistance of nichrome is $1.70 \times 10^{-4} \text{ }^{\circ}\text{C}^{-1}$. In order to increase resistance of a nichrome wire by 8.5%, the temperature of the wire should be increased by: [1 mark]

- (A) 250°C
- (B) 500°C
- (C) 850°C
- (D) 1000°C

Correct Answer: (B) 500°C

Solution:

Concept:

- The linear temperature law is $R = R_0(1 + \alpha\Delta T)$, where R_0 is the initial resistance and α the given temperature coefficient.
- The fractional increase in resistance is $\Delta R/R_0 = \alpha\Delta T$; a percentage must be divided by 100 before substitution.

Step 1: Express the increase as a fraction The stated increase is 8.5%, so $\Delta R/R_0 = 8.5/100 = 0.085$. With $\alpha = 1.70 \times 10^{-4} \text{ }^{\circ}\text{C}^{-1}$, the initial resistance cancels from the calculation.

Step 2: Calculate the temperature rise Rearranging the linear law gives

$$\Delta T = \frac{0.085}{1.70 \times 10^{-4}} = 500^{\circ}\text{C}.$$

The alternatives 250, 850 and 1000°C would produce increases of 4.25%, 14.45% and 17%, respectively, so they do not satisfy the specified increase.

Step 3: Conclusion The required temperature rise is 500°C . Hence, the correct option is (B).

Quick Tip to Solve: Divide the percentage increase by 100α ; the result is a temperature rise, so no initial temperature is needed.

29(iii). A wire of length 0.5 m and cross-sectional area $1.0 \times 10^{-7} \text{ m}^2$ is connected to a battery of 2 V that maintains a current of 1.5 A in it. The conductivity of the material of the wire (in $\Omega^{-1} \text{ m}^{-1}$) is: [1 mark]

- (A) 2.5×10^4
- (B) 3.0×10^5
- (C) 3.75×10^6
- (D) 5.0×10^7

Correct Answer: (C) 3.75×10^6

Solution:

Concept:

- Conductivity σ relates current density and electric field through $J = \sigma E$; it is the reciprocal of resistivity.
- For a uniform wire of length L and area A , $J = I/A$ and $E = V/L$, giving $\sigma = IL/(VA)$.

Step 1: Find the field and current density The voltage across the wire is 2 V, its length is 0.5 m, and

its area is $1.0 \times 10^{-7} \text{ m}^2$. Thus

$$E = \frac{2}{0.5} = 4 \text{ V m}^{-1}, \quad J = \frac{1.5}{1.0 \times 10^{-7}} = 1.5 \times 10^7 \text{ A m}^{-2}.$$

Step 2: Calculate and check conductivity Taking the ratio yields

$$\sigma = \frac{J}{E} = \frac{1.5 \times 10^7}{4} = 3.75 \times 10^6 \Omega^{-1} \text{ m}^{-1}.$$

As a check, $R = V/I = 4/3 \Omega$ and $\sigma = L/(RA)$ give the same value. None of the other three options satisfies $I = \sigma AV/L$; confusing conductivity with resistivity would invert the required result.

Step 3: Conclusion The conductivity is $3.75 \times 10^6 \Omega^{-1} \text{ m}^{-1}$. Hence, the correct option is (C).

Quick Tip to Solve: Use $\sigma = IL/(VA)$ directly and retain the negative exponent in area before evaluating the power of ten.

29(iv). Consider two cylindrical conductors A and B , made of the same metal connected in series to a battery. The length and the radius of B are twice that of A . If μ_A and μ_B are the mobility of electrons in A and B respectively, then $\frac{\mu_A}{\mu_B}$ is: [1 mark]

- (A) $\frac{1}{2}$
- (B) $\frac{1}{4}$
- (C) 2
- (D) 1

Correct Answer: (D) 1

Solution:

Concept:

- Electron mobility is $\mu = v_d/E = e\tau/m$, where v_d is drift speed, E the electric-field magnitude and τ the relaxation time.
- For the same metal at the same temperature, mobility is a material property and does not depend on the length or radius of the specimen.

Step 1: Compare the material properties Both conductors contain electrons with the same charge and mass and have the same relaxation time under the usual common-temperature assumption. Therefore their expressions $e\tau/m$ are equal. Geometrical differences affect resistance and local field but do not change this material parameter.

Step 2: Check the effect of the larger radius The radius of conductor B is twice that of A , so its cross-sectional area is four times as large. Their series current is identical; hence $v_{d,B} = v_{d,A}/4$. Since $E = \rho I/A$, also $E_B = E_A/4$. These factors cancel in v_d/E . Option (B) confuses a drift-speed ratio with mobility, while (A) and (C) incorrectly attribute a geometrical dependence to mobility.

Step 3: Conclusion $\mu_A = \mu_B$, so $\mu_A/\mu_B = 1$. Hence, the correct option is (D).

Quick Tip to Solve: Before using length or radius ratios, check whether the requested quantity is a material property: mobility is $e\tau/m$.

30. When light travels from an optically denser medium to an optically rarer medium, at the interface it is partly reflected back into the same medium and partly refracted to the second medium. The angle of incidence corresponding to an angle of refraction 90° is called the critical angle (i_c) for the given pair of media. This angle is related to the refractive index of medium 1 with respect to medium 2. Refraction of light through a prism involves refraction at two plane interfaces. A relation for the refractive index of the material of the prism can be obtained in terms of the refracting angle of the prism and the angle of minimum deviation. For a thin prism, this relation reduces to a simple equation. Laws of refraction are also valid for refraction of light at a spherical interface. When an object is placed in front of a spherical surface separating two media, its image is formed. A relation between object and image distance, in terms of refractive indices of two media and the radius of curvature of the spherical surface can be obtained. Using this relation for two surfaces of a lens, 'lens maker formula' is obtained. [4 marks]

30(i). An object is placed in front of a convex spherical glass surface ($n = 1.5$ and radius of curvature R) at a distance of $4R$ from it. As the object is moved slowly close to the surface, the image formed is: [1 mark]

- (A) always real
- (B) always virtual
- (C) first real and then virtual
- (D) first virtual and then real

Correct Answer: (C) first real and then virtual

Solution:

Concept:

- Refraction from air into glass obeys $n_2/v - n_1/u = (n_2 - n_1)/R$, where u, v are signed object and image distances.
- With light travelling into the convex glass surface, $R > 0$, a real object has $u < 0$, and an image with $v > 0$ is real in glass.

Step 1: Apply the surface formula Write the positive object distance as x , so $u = -x$. With $n_1 = 1$ and $n_2 = 1.5$,

$$\frac{1.5}{v} + \frac{1}{x} = \frac{0.5}{R}, \quad v = \frac{3Rx}{x - 2R}.$$

Initially $x = 4R$, giving $v = 6R > 0$. Thus the initial image lies inside the glass and is real.

Step 2: Track the sign as the object approaches As x decreases from $4R$ to $2R$, the denominator stays positive and the real image moves towards infinity. At $x = 2R$, the emerging rays are parallel. For $0 < x < 2R$, $v < 0$, so the refracted rays diverge and their backward extensions give a virtual image. Consequently neither an always-real nor an always-virtual image is possible; the reverse sequence also contradicts the positive initial image distance.

Step 3: Conclusion The image is first real and then virtual as the object crosses the object-focus position $2R$. Hence, the correct option is (C).

Quick Tip to Solve: Find where the denominator of $v = 3Rx/(x - 2R)$ changes sign; this locates the real-to-virtual transition immediately.

30(ii). A double-convex lens, made of glass of refractive index 1.5, has focal length 10 cm. The radius of curvature of its each face, is: [1 mark]

- (A) 10 cm
- (B) 15 cm
- (C) 20 cm
- (D) 40 cm

Correct Answer: (A) 10 cm

Solution:

Concept:

- For a thin glass lens in air, the lens maker formula is $1/f = (n - 1)(1/R_1 - 1/R_2)$, where n is its refractive index.
- For a double-convex lens with equal curvature magnitudes R , the Cartesian signs are $R_1 = +R$ and $R_2 = -R$.

Step 1: Insert the signed radii Light entering the first convex surface meets a centre of curvature on the positive side, whereas that of the second surface lies on the negative side. Therefore,

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R} - \frac{1}{-R} \right) = \frac{2(n - 1)}{R}.$$

The contributions of both refracting surfaces add; they do not cancel.

Step 2: Substitute the refractive index and focal length Rearranging gives $R = 2(n - 1)f$. Here $n = 1.5$ and $f = 10$ cm, hence

$$R = 2(1.5 - 1)(10 \text{ cm}) = 10 \text{ cm}.$$

The alternatives 15, 20 and 40 cm do not satisfy the lens maker relation. In particular, $R = 2f$ is the spherical-mirror relation and cannot be applied to this lens.

Step 3: Conclusion The radius of curvature of each face is 10 cm. Hence, the correct option is (A).

Quick Tip to Solve: For an equiconvex glass lens with $n = 1.5$, the factor $2(n - 1)$ equals one, so each radius equals the focal length.

30(iii). A small bulb is placed at the bottom of a tank containing a transparent liquid (refractive index n) to a depth H . The radius of the circular area of the surface of liquid, through which light from the bulb can emerge out, is R . Then $\left(\frac{R}{H}\right)$ is: [1 mark]

(A) $\frac{1}{\sqrt{n^2 - 1}}$

(B) $\sqrt{n^2 - 1}$

(C) $\frac{1}{\sqrt{n^2 + 1}}$

(D) $\sqrt{n^2 + 1}$

Correct Answer: (A) $\frac{1}{\sqrt{n^2 - 1}}$

Solution:

Concept:

- Light travelling from liquid of refractive index n into air has critical angle i_c , with $\sin i_c = 1/n$.
- The limiting ray from a bulb at depth H reaches the edge of the escape circle of radius R , giving $\tan i_c = R/H$.

Step 1: Identify the limiting emerging ray A ray directed to a surface point at horizontal distance r from the point above the bulb makes angle i with the vertical normal, where $\tan i = r/H$. Rays with $i < i_c$ emerge, while those with $i > i_c$ undergo total internal reflection. Thus the boundary is set by $i = i_c$ and $r = R$.

Step 2: Convert the critical-angle relation to a tangent From $\sin i_c = 1/n$, the positive cosine is $\cos i_c = \sqrt{n^2 - 1}/n$. Hence

$$\frac{R}{H} = \tan i_c = \frac{1/n}{\sqrt{n^2 - 1}/n} = \frac{1}{\sqrt{n^2 - 1}}.$$

Option (B) gives the reciprocal ratio H/R . The plus signs in (C) and (D) are incompatible with $\cos^2 i_c = 1 - \sin^2 i_c$; they cannot describe the boundary ray.

Step 3: Conclusion The radius-to-depth ratio is $R/H = 1/\sqrt{n^2 - 1}$. Hence, the correct option is (A).

Quick Tip to Solve: At the escape-circle boundary, use $R/H = \tan i_c$, not $\sin i_c$; the depth is the adjacent side of the triangle.

30(iv)(a). A parallel beam of light is incident on a face of a prism with refracting angle 60° . The angle of minimum deviation is found to be 30° . The refractive index of the material of the prism is close to: [1 mark]

- (A) 1.3
- (B) 1.4
- (C) 1.5
- (D) 1.6

Correct Answer: (B) 1.4

Solution:

Concept:

- At minimum deviation, the path through a prism is symmetric: $i = e$ and $r_1 = r_2 = A/2$, where A is the prism angle.
- For a prism in air, its refractive index is $n = \sin[(A + D_m)/2] / \sin(A/2)$, with minimum deviation D_m .

Step 1: Determine the symmetric ray angles The deviation relation is $D_m = i + e - A = 2i - A$. With $A = 60^\circ$ and $D_m = 30^\circ$, the incidence angle is

$$i = \frac{60^\circ + 30^\circ}{2} = 45^\circ, \quad r = \frac{A}{2} = 30^\circ.$$

These are the angles to use in Snell's law at the first interface.

Step 2: Evaluate the refractive index Applying $n = \sin i / \sin r$,

$$n = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2} \simeq 1.414.$$

The nearest printed value is 1.4; the distances from 1.3, 1.5 and 1.6 are larger. Using the thin-prism approximation here would give 1.5, but a 60° prism is not a thin prism, so that shortcut is inappropriate.

Step 3: Conclusion The refractive index is approximately 1.414, closest to 1.4. Hence, the correct option is (B).

Quick Tip to Solve: For a 60° prism use the sine formula; reserve $D_m = (n - 1)A$ for a prism explicitly described as thin.

30(iv)(b). The angle of minimum deviation for a ray of light incident on a thin prism, made of crown glass ($n = 1.52$) is D_m . If the prism was made of dense flint glass ($n = 1.62$) instead of crown glass, the angle of minimum deviation will: [1 mark]

- (A) decrease by 4%
- (B) increase by 4%
- (C) decrease by 19%
- (D) increase by 19%

Correct Answer: (D) increase by 19%

Solution:

Concept:

- For a thin prism, $D_m = (n - 1)A$, where n is refractive index and A the prism angle.
- For fixed A , deviation is proportional to $n - 1$.

Step 1: Compare the deviations for the two glasses For crown glass, $D_m = (1.52 - 1)A = 0.52A$. For dense flint glass, the new deviation D'_m is

$$D'_m = (1.62 - 1)A = 0.62A, \quad \frac{D'_m}{D_m} = \frac{0.62}{0.52}.$$

The higher index increases deviation, ruling out (A) and (C).

Step 2: Calculate the percentage increase The change must be compared with the original crown-glass deviation:

$$\frac{D'_m - D_m}{D_m} \times 100\% = \frac{0.62 - 0.52}{0.52} \times 100\% \simeq 19.23\%.$$

Thus the increase is approximately 19%, ruling out the 4% in (B). Comparing indices directly would use the wrong baseline: deviation depends on $n - 1$.

Step 3: Conclusion The minimum deviation increases by approximately 19%. Hence, the correct option is (D).

Quick Tip to Solve: For a change of glass in the same thin prism, divide Δn by the original $n - 1$ to obtain the fractional change in deviation.

Section E (15 marks)

31(a). (i) With the help of a labelled diagram, explain the principle of working of a moving coil galvanometer. Write the purpose of using (i) radial magnetic field, and (ii) soft iron core, in it.

(ii) Define current sensitivity of a galvanometer. "Increasing the current sensitivity may not necessarily increase the voltage sensitivity." Give reason. [5 marks]

Solution:

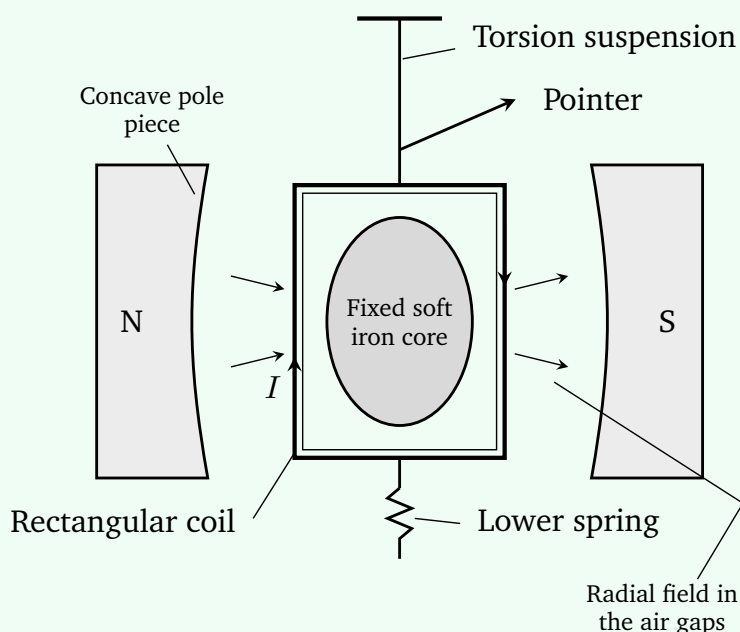
Concept:

- A current-carrying coil in a magnetic field experiences a torque. For a coil of N turns and area A , $\tau_m = NIAB \sin \alpha$, where α is the angle between its normal and the field B .
- The suspension provides a restoring torque $\tau_r = k\theta$, where k is the torsional constant and θ is the

angular deflection in radians.

- Current sensitivity is deflection per unit current, $S_I = \theta/I$. Voltage sensitivity is deflection per unit applied voltage, $S_V = \theta/V$; the coil resistance G gives $V = IG$.

Step 1: (i) Identify the construction and working principle A light rectangular coil of many turns is suspended between the concave pole pieces of a strong permanent magnet. A cylindrical soft iron core is placed inside the coil. A fine torsion suspension and a lower spring carry current to and from the coil and provide the restoring torque. A pointer or mirror indicates its angular deflection. The diagram is a schematic view: the coil rotates about the vertical suspension axis while the core remains fixed. When current flows, the magnetic forces on the two opposite vertical sides of the coil form a couple. Thus the coil turns, twisting the suspension. It comes to rest when the opposing elastic torque balances the magnetic torque. This conversion of a small current into an observable angular deflection is the principle of the moving coil galvanometer.



Moving coil galvanometer: coil, fixed core, concave pole pieces, suspension and indicating pointer (schematic).

Step 2: (i) Explain the radial magnetic field and obtain the deflection The shaped pole pieces and cylindrical core create a radial magnetic field in the air gap. At every working position of the coil, its plane remains parallel to the local field at its active sides. Its normal is therefore perpendicular to the field, so the torque is maximum for a given current and does not acquire a deflection-dependent sine factor:

$$\tau_m = NIAB \sin 90^\circ = NIAB.$$

At equilibrium the magnetic torque equals the restoring torque:

$$NIAB = k\theta \Rightarrow \theta = \frac{NAB}{k}I.$$

Hence the deflection is directly proportional to current, producing a uniform scale. Reversing the current reverses the magnetic couple and therefore the deflection.

Step 3: (i) State the purpose of the soft iron core Soft iron has high magnetic permeability. The core concentrates and strengthens the magnetic field in the narrow air gap, increasing the torque and deflection produced by a small current. Its cylindrical shape, together with the concave pole faces, also helps make the field radial. Thus the radial geometry gives a linear current scale, while the

increased field strength improves sensitivity.

Step 4: (ii) Define and calculate current sensitivity The current sensitivity of a galvanometer is the angular deflection produced per unit current through its coil. From the equilibrium relation,

$$S_I = \frac{\theta}{I} = \frac{NAB}{k}.$$

Its SI unit is rad A^{-1} . A greater number of turns, larger coil area, stronger magnetic field, or smaller torsional constant increases current sensitivity when the other quantities remain fixed.

Step 5: (ii) Relate voltage sensitivity to coil resistance If V is the potential difference across a coil of resistance G , then $I = V/G$. Substitution gives

$$\theta = \frac{NAB}{kG}V, \quad S_V = \frac{\theta}{V} = \frac{NAB}{kG} = \frac{S_I}{G}.$$

Consequently, a change that raises S_I may also raise G , so the ratio S_I/G need not increase. For example, increasing the number of turns using the same wire and approximately the same area per turn increases both N and wire length. Since resistance is proportional to wire length, both S_I and G increase in approximately the same ratio; voltage sensitivity is then approximately unchanged. An increase of current sensitivity increases voltage sensitivity only if it is not offset by the change in resistance.

Step 6: Conclusion The galvanometer works by magnetic torque balanced by elastic restoring torque, giving $\theta = NIAB/k$. The radial field ensures a uniform scale, and the soft iron core strengthens and helps radialise the field. Current sensitivity is $S_I = NAB/k$, but voltage sensitivity is $S_V = S_I/G$; an accompanying increase in coil resistance can prevent any increase in voltage sensitivity.

Quick Tip to Solve: First write $k\theta = NIAB$, then divide by I or by $V = IG$. Always check the coil resistance before comparing voltage sensitivities.

OR

31(b). (i) (I) Write Ampere's circuital law in mathematical form and explain the terms used.

(II) As the current carrying solenoid is made longer, the magnetic field produced outside it approaches zero. Why?

(III) A flexible loop of irregular shape carrying current when located in an external magnetic field, changes to a circular shape. Give reason.

(ii) A galvanometer of resistance G is converted into a voltmeter to measure up to V volts, by connecting a resistance R_1 in series with the coil. If R_1 is replaced by R_2 , then it can only measure up to $\frac{V}{2}$ volt. Find the value of the resistance R_3 (in terms of R_1 and R_2) needed to convert it into a voltmeter that can read up to $2V$.

[5 marks]

Solution:

Concept:

- Ampere's circuital law relates the circulation of a steady magnetic field around a closed path to the net current enclosed by that path.
- Magnetic fields obey superposition. For an ideal infinitely long solenoid the exterior fields of its turns cancel, while the interior fields reinforce.

- A current element in a magnetic field experiences $d\vec{F} = I d\vec{\ell} \times \vec{B}$. A planar current loop has magnetic moment $\vec{m} = IA\hat{n}$ and magnetic potential energy $U = -\vec{m} \cdot \vec{B}$.
- A galvanometer of coil resistance G and full-scale current I_g , with series resistance R , has voltmeter range $V_{\max} = I_g(G + R)$.

Step 1: (i)(I) State Ampere's circuital law and identify the terms For steady currents in free space, the line integral of the magnetic field around a closed path is

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}.$$

Here $\vec{B}$ is the magnetic field at an element of the closed path C , $d\vec{\ell}$ is the directed infinitesimal path element, and μ_0 is the permeability of free space. The symbol I_{enc} denotes the algebraic sum of the currents passing through a surface bounded by C . The positive normal to that surface and the direction of traversal of C are linked by the right-hand rule; oppositely directed currents therefore enter with opposite signs.

Step 2: (i)(II) Explain the weak exterior field of a long solenoid Each turn produces a magnetic field, and the resultant field is their vector sum. Inside a long solenoid, the axial contributions of neighbouring turns reinforce. Outside it, the contributions largely cancel; the remaining external field is mainly associated with the ends. As the solenoid is lengthened, a point outside near its central region lies farther from both ends, so these end effects decrease. In the infinitely long idealisation the cancellation outside is complete. A finite solenoid can still have an appreciable field near an end, so the zero-field statement refers to the long-solenoid approximation away from the ends.

Step 3: (i)(III) Explain why the flexible loop becomes circular The magnetic force on every small current-carrying segment is perpendicular to that segment. The loop tends to orient its magnetic moment along the external field. In this stable orientation the forces on its elements act outward in its plane and tend to increase its enclosed area. Equivalently, with $\vec{m}$ parallel to $\vec{B}$, the energy $U = -IAB$ decreases as the area A increases. The wire length, hence the perimeter, remains fixed. Among plane figures with a given perimeter, a circle encloses the largest area; the flexible loop therefore tends to take a circular shape.

Step 4: (ii) Use the two stated voltmeter ranges The full-scale current I_g and coil resistance G are properties of the same galvanometer and remain unchanged when the series resistor changes. Hence

$$\begin{aligned} V &= I_g(G + R_1), \\ \frac{V}{2} &= I_g(G + R_2). \end{aligned}$$

Divide the first equation by the second and rearrange:

$$\begin{aligned} G + R_1 &= 2(G + R_2), \\ G &= R_1 - 2R_2. \end{aligned}$$

In particular, physical positive coil resistance requires $R_1 > 2R_2$, consistent with the larger series resistance giving the larger range.

Step 5: (ii) Set up the doubled-range condition For the desired range, the total resistance must permit the same full-scale current at $2V$:

$$2V = I_g(G + R_3).$$

Comparing this with $V = I_g(G + R_1)$ gives

$$\begin{aligned}G + R_3 &= 2(G + R_1), \\R_3 &= G + 2R_1.\end{aligned}$$

The resistance being doubled is the total resistance, including the galvanometer coil; doubling only the external resistor would not give the required range.

Step 6: Conclusion Ampere's law is $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$. Exterior-field cancellation explains the long-solenoid limit, and the loop becomes circular to maximise area at fixed perimeter in its stable magnetic orientation. Substituting $G = R_1 - 2R_2$, the required series resistance is $R_3 = 3R_1 - 2R_2$.

Quick Tip to Solve: For different ranges of the same galvanometer, keep I_g fixed and take ratios of $G + R$. A doubled voltmeter range requires twice the total resistance, including G .

32(a). (i) Explain with the help of a labelled ray diagram the formation of final image by an astronomical telescope at infinity. Write the expression for its magnifying power.

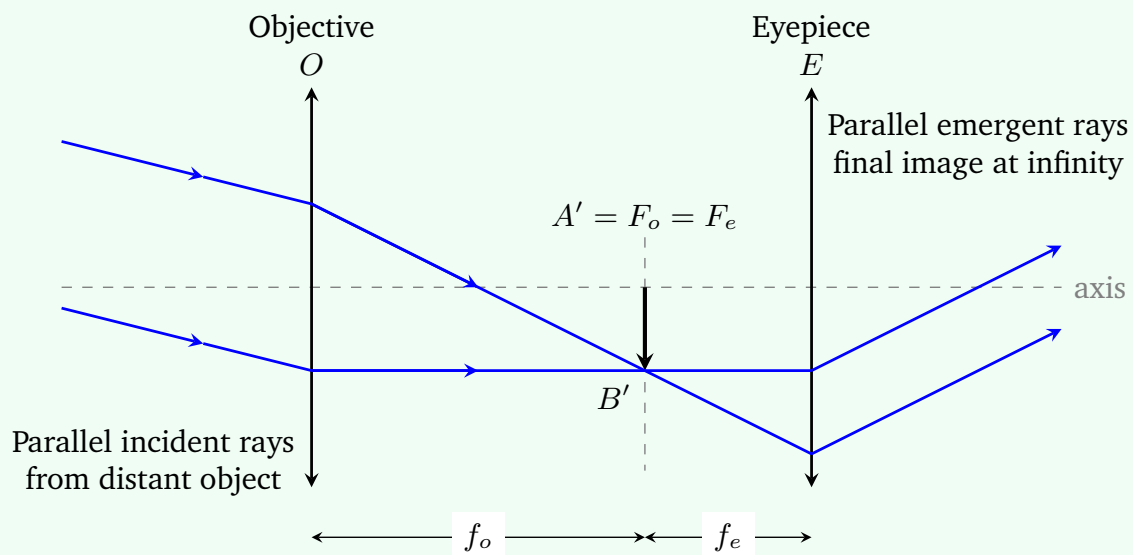
(ii) The total magnification produced by a compound microscope is 20. The magnification produced by the eyepiece is 5. When the microscope is focussed on a certain object, the distance between the objective and eyepiece is observed to be 14 cm. Calculate the focal lengths of the objective and the eyepiece. (Given that the least distance of distinct vision = 25 cm) [5 marks]

Solution:

Concept:

- In a telescope in normal adjustment, the objective's real image lies in the first focal plane of the eyepiece, so the final image is at infinity. Let their positive focal lengths be f_o and f_e .
- Angular magnification is $M = \beta/\alpha$, where α and β are the small angles subtended at the unaided eye and through the instrument, respectively.
- For a compound microscope, $|M| = |m_o|m_e$, with objective linear magnification $m_o = v_o/u_o$. Each lens obeys $1/f = 1/v - 1/u$, using the Cartesian sign convention.
- An eyepiece forming its final virtual image at the near point has $m_e = 1 + D/f_e$; for a final image at infinity, $m_e = D/f_e$. Here $D = 25$ cm.

Step 1: (i) Form the telescope image in normal adjustment Parallel rays from a distant object enter the large-aperture objective. It forms a real, inverted and diminished image $A'B'$ in its second focal plane. Place the eyepiece so that this plane is also its first focal plane: $F_o = F_e = A'$. The eyepiece acts as a magnifier; rays from each point of the intermediate image emerge parallel, giving a final virtual image at infinity. The final image is inverted relative to the distant object, and the eye observes it without accommodation. The separation of the two lenses is $f_o + f_e$. The drawing traces rays from one off-axis point of the distant object.



Astronomical telescope in normal adjustment: the intermediate image lies in the common focal plane, and the emerging rays are parallel.

Step 2: (i) Obtain the angular magnifying power If the intermediate image height has magnitude h , the small-angle approximation gives $|\alpha| \simeq h/f_o$ and $|\beta| \simeq h/f_e$. The image is inverted, so the signed angular magnification is

$$M = \frac{\beta}{\alpha} = -\frac{f_o}{f_e}, \quad |M| = \frac{f_o}{f_e}.$$

Thus a long-focal-length objective and a short-focal-length eyepiece provide high magnification.

Step 3: (ii) Find the eyepiece focal length for near-point viewing Taking the conventional near-point setting for this numerical problem, the final virtual image is at $v_e = -D = -25$ cm. The given eyepiece magnification then gives

$$5 = 1 + \frac{25}{f_e} \Rightarrow f_e = 6.25 \text{ cm}.$$

To locate the intermediate image, apply the eyepiece lens formula:

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{6.25} = -\frac{1}{5} \text{ cm}^{-1}.$$

Hence $u_e = -5$ cm: the intermediate image is 5 cm to the left of the eyepiece.

Step 4: (ii) Locate the object and calculate the objective focal length The lenses are separated by 14 cm, so the objective forms its real image at

$$v_o = 14 - 5 = 9 \text{ cm}.$$

Since the objective produces an inverted real image, its signed magnification is negative:

$$|m_o| = \frac{20}{5} = 4, \quad m_o = -4 = \frac{9}{u_o} \Rightarrow u_o = -2.25 \text{ cm}.$$

Therefore,

$$\frac{1}{f_o} = \frac{1}{9} - \frac{1}{-2.25} = \frac{1+4}{9} = \frac{5}{9} \text{ cm}^{-1}, \quad f_o = 1.8 \text{ cm}.$$

The object lies beyond the objective focus, consistently producing a magnified real intermediate image.

Step 5: (ii) State the viewing-condition assumption The printed numerical question does not

explicitly specify where the microscope's final image is formed. The near-point assumption above gives $f_e = 6.25$ cm. If normal adjustment with final image at infinity is intended instead, $m_e = D/f_e = 5$ gives $f_e = 5$ cm; then $u_e = -f_e = -5$ cm, and the same calculation gives $f_o = 1.8$ cm. Stating the viewing condition is therefore necessary to distinguish the eyepiece answers.

Step 6: Conclusion (i) In normal adjustment the telescope forms its final image at infinity, with $M = -f_o/f_e$ and lens separation $f_o + f_e$. (ii) For near-point viewing, $f_o = 1.8$ cm, $f_e = 6.25$ cm. If the microscope is in normal adjustment, $f_e = 5$ cm instead, while f_o is unchanged.

Quick Tip to Solve: Divide the microscope's total magnification by the eyepiece magnification first. Find the eyepiece object distance before subtracting it from the lens separation; specify near-point or infinity viewing before choosing the eyepiece formula.

OR

32(b). (i) Two coherent light waves, each of intensity I_0 superpose each other and produce interference pattern on a screen. Obtain the expression for the resultant intensity at a point where the phase difference between the waves is ϕ . Write its maximum and minimum possible values.

(ii) In a single slit diffraction experiment, the aperture of the slit is 3 mm and the separation between the slit and the screen is 1.5 m. A monochromatic light of wavelength 600 nm is normally incident on the slit. Calculate the distance of (I) first order minimum, and (II) second order maximum, from the centre of the screen.

[5 marks]

Solution:

Concept:

- The superposition principle adds electric fields. Intensity is proportional to the time average of the square of the resultant electric field; equal individual intensities imply equal amplitudes.
- In single-slit diffraction, minima satisfy $a \sin \theta = n\lambda$, where a is slit width and λ is wavelength. For a distant screen, the distance from its centre is $y \simeq D\theta$.
- Secondary maxima lie approximately halfway between successive minima: $a \sin \theta \simeq (n + \frac{1}{2})\lambda$, with $n = 1, 2, \dots$. This is the usual school-level approximation, not an exact maximum condition.

Step 1: (i) Add the two coherent electric fields Let the fields at the chosen point be $E_1 = E_0 \cos \omega t$ and $E_2 = E_0 \cos(\omega t + \phi)$. The fixed phase difference ϕ permits a stable interference pattern. Using the sum of cosines,

$$E = E_1 + E_2 = 2E_0 \cos \frac{\phi}{2} \cos \left(\omega t + \frac{\phi}{2} \right).$$

If k is the common proportionality constant between intensity and squared amplitude, then $I_0 = kE_0^2$. Thus

$$I = k \left(2E_0 \cos \frac{\phi}{2} \right)^2 = 4I_0 \cos^2 \frac{\phi}{2} = 2I_0(1 + \cos \phi).$$

Adding intensities alone would miss the interference term $2I_0 \cos \phi$.

Step 2: (i) Determine the extreme intensities Constructive interference occurs when $\phi = 2m\pi$, so the amplitudes reinforce and $I_{\max} = 4I_0$. Destructive interference occurs when $\phi = (2m + 1)\pi$, so the equal amplitudes cancel and $I_{\min} = 0$, where m is an integer. Across the pattern, the average intensity remains $2I_0$; interference redistributes energy.

Step 3: (ii)(I) Locate the first minimum Convert the data to SI units: $a = 3 \times 10^{-3}$ m, $D = 1.5$ m,

and $\lambda = 600 \times 10^{-9} \text{ m}$. The first minimum corresponds to $a \sin \theta_1 = \lambda$. Since the angle is small,

$$y_{\min,1} \simeq \frac{D\lambda}{a} = \frac{1.5(600 \times 10^{-9})}{3 \times 10^{-3}} = 3.0 \times 10^{-4} \text{ m} = 0.30 \text{ mm}.$$

An identical minimum lies on the other side of the central maximum.

Step 4: (ii) (II) Locate the second secondary maximum The central maximum has order zero. The first secondary maximum lies between the first and second minima, while the second lies between the second and third minima. Using their approximate midpoint positions,

$$y_{\max,2} \simeq \frac{5D\lambda}{2a} = \frac{5}{2}(0.30 \text{ mm}) = 0.75 \text{ mm}.$$

This is the standard approximate answer. For comparison, the exact diffraction envelope has maxima satisfying $\tan \beta = \beta$, where $\beta = \pi a \sin \theta / \lambda$; its second secondary maximum is at $\beta \simeq 7.725$, giving about 0.738 mm.

Step 5: Conclusion (i) $I = 4I_0 \cos^2(\phi/2)$, with $I_{\max} = 4I_0$ and $I_{\min} = 0$. (ii) The first minimum is 0.30 mm and the second secondary maximum is approximately 0.75 mm from the centre, on either side.

Quick Tip to Solve: Calculate $D\lambda/a$ once. It gives the first minimum; multiply by $5/2$ for the approximate second secondary maximum, counting the central maximum as order zero.

33(a). (i) A parallel plate capacitor with plate area A and plate separation d has a capacitance C_0 . A slab of dielectric constant K having area A and thickness $\left(\frac{d}{4}\right)$ is inserted in the capacitor, parallel to the plates. Find the new value of its capacitance.

(ii) You are provided with a large number of $1 \mu\text{F}$ identical capacitors and a power supply of 1200 V. The dielectric medium used in each capacitor can withstand up to 200 V only. Find the minimum number of capacitors and their arrangement, required to build a capacitor system of equivalent capacitance of $2 \mu\text{F}$ for use with this supply. [5 marks]

Solution:

Concept:

- A parallel-plate capacitor in air has capacitance $C_0 = \epsilon_0 A/d$, where A is plate area and d is separation, neglecting fringing fields.
- Dielectric and air layers stacked along the field act as capacitors in series. Their voltage drops add while the free charge on the plates is the same.
- For n identical capacitors of capacitance C in series, $C_s = C/n$ and the applied voltage divides equally. Parallel branches have the same voltage and their capacitances add.

Step 1: (i) Find the voltage across the air and dielectric layers The printed slab thickness is $t = d/4$. It occupies the full plate area, leaving a total air thickness $d - t = 3d/4$. Let the free charge on either plate have magnitude Q . The field in air is $E_a = Q/(\epsilon_0 A)$, whereas the field inside the dielectric is reduced to $E_K = Q/(K\epsilon_0 A)$. Consequently,

$$V = E_a(d - t) + E_K t = \frac{Q}{\epsilon_0 A} \left(d - t + \frac{t}{K} \right).$$

This expression is independent of where the slab is placed between the plates, since only the total air thickness enters the voltage drop.

Step 2: (i) Express the new capacitance in terms of the original value By the definition $C' = Q/V$, substituting $t = d/4$ gives

$$\begin{aligned} C' &= \frac{\epsilon_0 A}{3d/4 + d/(4K)} \\ &= \frac{4K\epsilon_0 A}{d(3K+1)} = \frac{4K}{3K+1} C_0. \end{aligned}$$

As a check, $K = 1$ gives $C' = C_0$, as expected for no dielectric contrast. Even for very large K , the remaining air layer limits the capacitance to $4C_0/3$.

Step 3: (ii) Determine the number needed in each series branch Each capacitor may have at most 200 V across it. For a series branch of n identical capacitors connected across 1200 V, equal capacitance ensures equal voltage division:

$$\frac{1200}{n} \leq 200 \Rightarrow n \geq 6.$$

Use six capacitors in every branch. Their combined capacitance is

$$C_s = \frac{1 \mu\text{F}}{6} = \frac{1}{6} \mu\text{F}.$$

Each individual capacitor then supports exactly 200 V, its stated maximum.

Step 4: (ii) Combine enough branches in parallel and verify the minimum If m such branches are connected in parallel, their capacitances add. To obtain the specified equivalent capacitance,

$$m \left(\frac{1}{6} \mu\text{F} \right) = 2 \mu\text{F} \Rightarrow m = 12, \quad N = mn = 12 \times 6 = 72.$$

Minimality can also be checked using energy. A system of equivalent capacitance $2 \mu\text{F}$ across 1200 V stores 1.44 J. Each $1 \mu\text{F}$ capacitor at its allowed maximum voltage stores at most 0.020 J. Thus every permissible arrangement needs at least $1.44/0.020 = 72$ capacitors. The proposed arrangement attains this lower bound.

Step 5: Conclusion (i) The new capacitance is $C' = \frac{4K}{3K+1} C_0$. (ii) The minimum is 72 capacitors: connect 12 parallel branches, each containing 6 capacitors in series. The result is $2 \mu\text{F}$, with 200 V across each capacitor at the specified supply voltage.

Quick Tip to Solve: Treat dielectric thickness as t/K when finding the effective plate separation. For a capacitor bank, satisfy the voltage limit with series units first, then restore the desired capacitance using parallel branches.

33(b). (i) An electric dipole of dipole moment $\vec{p}$ consists of point charges q and $-q$, separated by $2a$. Derive an expression for electric potential in terms of its dipole moment at a point at a distance x ($\gg a$) from its centre and lying (I) along its axis, and (II) along its bisector line.

(ii) An electric dipole of dipole moment $\vec{p} = (0.8\hat{i} + 0.6\hat{j}) \times 10^{-29}$ C m is placed in an electric field $\vec{E} = 1.0 \times 10^7 \hat{k}$ V/m. Calculate the magnitude of the torque acting on it and the angle it makes with the x -axis, at this instant. [5 marks]

Solution:

Concept:

- Electric potential is a scalar. Taking zero potential at infinity, a point charge Q produces $V = kQ/r$, where $k = 1/(4\pi\epsilon_0)$ and r is its distance from the observation point. Add the signed potentials of the two charges.
- A dipole has moment $\vec{p}$ directed from $-q$ to $+q$, with magnitude $p = 2aq$. In the far field its potential is $V = kp \cos \theta / r^2$, where θ is measured from $\vec{p}$.
- The torque on a dipole in a uniform field is $\vec{\tau} = \vec{p} \times \vec{E}$. Its direction follows the right-hand rule and its magnitude is $pE \sin \phi$, where ϕ is the angle between the dipole and the field.

Step 1: (i)(I) Add the potentials on the dipole axis Place $-q$ at coordinate $-a$ and $+q$ at $+a$. First choose the axial observation point a distance $x > a$ from the centre on the positive-charge side. Its distances from $+q$ and $-q$ are $x - a$ and $x + a$, respectively. By superposition,

$$\begin{aligned} V_{\text{axial}} &= k \left(\frac{q}{x-a} - \frac{q}{x+a} \right) \\ &= kq \frac{(x+a) - (x-a)}{x^2 - a^2} = \frac{k(2aq)}{x^2 - a^2} = \frac{kp}{x^2 - a^2}. \end{aligned}$$

For $x \gg a$, the ratio a^2/x^2 is negligible, so the far-field expression becomes $V_{\text{axial}} \simeq kp/x^2$. At the corresponding point on the negative-charge side, the distances to the charges interchange and the potential is $-kp/(x^2 - a^2) \simeq -kp/x^2$. Specifying the side is essential because potential has a sign even though x denotes a positive distance.

Step 2: (i)(II) Use equal distances on the perpendicular bisector At a point a distance x from the centre on the perpendicular bisector, both charges are at the same distance $r = \sqrt{x^2 + a^2}$. Their signed potentials therefore cancel:

$$V_{\text{equatorial}} = \frac{kq}{\sqrt{x^2 + a^2}} - \frac{kq}{\sqrt{x^2 + a^2}} = 0.$$

This result is exact; it does not need the far-field approximation. It also agrees with the dipole expression because the perpendicular bisector corresponds to $\theta = 90^\circ$. Zero potential here does not imply zero electric field: the fields are vectors and their axial components add.

Step 3: (ii) Evaluate the torque vector and its magnitude The dipole lies in the xy -plane and the field is along $+z$, so they are perpendicular. Use $\hat{i} \times \hat{k} = -\hat{j}$ and $\hat{j} \times \hat{k} = \hat{i}$:

$$\begin{aligned} \vec{\tau} &= [(0.8\hat{i} + 0.6\hat{j}) \times \hat{k}] 10^{-29} \times 10^7 \\ &= (0.6\hat{i} - 0.8\hat{j}) \times 10^{-22} \text{ N m}, \\ |\vec{\tau}| &= \sqrt{0.6^2 + (-0.8)^2} \times 10^{-22} = 1.0 \times 10^{-22} \text{ N m}. \end{aligned}$$

As a check, $|\vec{p}| = \sqrt{0.8^2 + 0.6^2} \times 10^{-29} = 1.0 \times 10^{-29} \text{ C m}$, giving the same magnitude from $pE \sin 90^\circ$.

Step 4: (ii) Determine the angle of the torque with the x-axis Let α be the angle between $\vec{\tau}$ and the positive x -axis. The directional cosine is

$$\cos \alpha = \frac{\tau_x}{|\vec{\tau}|} = \frac{0.6 \times 10^{-22}}{1.0 \times 10^{-22}} = 0.6, \quad \alpha = \cos^{-1}(0.6) = 53.1^\circ.$$

Since $\tau_y < 0$, the torque points into the fourth quadrant: it is directed 53.1° below the positive x -axis in the xy -plane. Its counterclockwise azimuth is equivalently 306.9° ; this is the torque direction at the stated instant.

Step 5: Conclusion For $x \gg a$, $V_{\text{axial}} = \pm p/(4\pi\epsilon_0 x^2)$, with the positive sign on the $+q$ side, while $V_{\text{equatorial}} = 0$. The torque is $(0.6\hat{i} - 0.8\hat{j}) \times 10^{-22} \text{ N m}$, of magnitude $1.0 \times 10^{-22} \text{ N m}$, making 53.1° with the positive x -axis toward negative y .

Quick Tip to Solve: Add signed scalar potentials before making the far-field approximation. For a field along $+z$, use $\vec{\tau} = E(p_y\hat{i} - p_x\hat{j})$; the component signs immediately fix the quadrant.