CBSE Class 12 Mathematics Question Paper 2025 with Solution PDF-Set 65-1-2

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions

(i) This Question paper contains **38** questions. **All** questions are **compulsory**.

(ii) Question paper is divided into **FIVE** Sections – Section **A**, **B**, **C**, **D** and **E**.

(iii) In Section **A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.

(iv) In Section **B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.

(v) In Section **C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.

(vi) In Section **D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.

(vii) In Section **E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.

(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, **2** questions in Section – **D** and **2** questions in Section – **E**.

(ix) Use of calculator is **NOT** allowed.

Section A (20 marks)

*This section comprises of **20** Multiple Choice Questions (MCQs) of **1** mark each.*

1. If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P(E/\overline{F})$ is equal to: [1 mark]

(A) $\frac{1}{6}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{7}{9}$

Correct Answer: (C) $\frac{2}{3}$

Solution:

Concept:

- Independent events satisfy $P(E \cap F) = P(E)P(F)$. Independence also holds between E and $\overline{F}$.
- Conditional probability is $P(E \mid G) = P(E \cap G)/P(G)$, provided $P(G) > 0$.

Step 1: Find the probability involving the complement Here $P(\overline{F}) = 1 - 3/7 = 4/7$. Also $P(E \cap F) = (2/3)(3/7) = 2/7$, so $P(E \cap \overline{F}) = P(E) - P(E \cap F) = 2/3 - 2/7 = 8/21$.

Step 2: Use the conditional-probability formula Therefore

$$P(E \mid \overline{F}) = \frac{8/21}{4/7} = \frac{8}{21}\frac{7}{4} = \frac{2}{3}.$$

Conditioning on an independent event leaves the probability unchanged. Option (A) is not this ratio, option (B) is not $P(E)$, and option (D) is likewise different from the computed conditional probability.

Step 3: Conclusion $P(E/\overline{F}) = \frac{2}{3}$. Hence, the correct option is (C).

Quick Tip to Solve: For independent events, conditioning on either the event or its complement leaves the other probability unchanged.

2. If $\vec{\alpha} = \hat{i} - 4\hat{j} + 9\hat{k}$ and $\vec{\beta} = 2\hat{i} - 8\hat{j} + \lambda\hat{k}$ are two mutually parallel vectors, then λ is equal to: [1 mark]

(A) -18

(B) 18

(C) $-\dfrac{34}{9}$

(D) $\dfrac{34}{9}$

Correct Answer: (B) 18

Solution:

Concept:

- Nonzero parallel vectors are scalar multiples: $\vec{\beta} = t\vec{\alpha}$.
- Each corresponding component must have the same multiplier t.

Step 1: Determine the scalar multiplier Comparing the coefficients of $\hat{i}$ gives $2 = t(1)$, so $t = 2$. The $\hat{j}$ components confirm this: $-8 = 2(-4)$. Thus the two already known components consistently give the same positive multiplier.

Step 2: Compare the remaining component The $\hat{k}$ coefficient must satisfy $\lambda = 2(9) = 18$. Option (A) would reverse only the last component and cannot be parallel. Options (C) and (D) do not have the required ratio $\lambda/9 = 2$; they can arise from using an unrelated dot-product condition.

Step 3: Conclusion $\lambda = 18$. Hence, the correct option is (B).

Quick Tip to Solve: For parallel vectors, obtain the multiplier from a known nonzero component before solving for the unknown component.

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3. $\displaystyle\int \frac{1 - 2\sin x}{\cos^2 x}\,dx$ is equal to: [1 mark]

(A) $\tan x - 2\sec x + C$

(B) $-\tan x + 2\sec x + C$

(C) $-\tan x - 2\sec x + C$

(D) $\tan x + 2\sec x + C$

Correct Answer: (A) $\tan x - 2\sec x + C$

Solution:

Concept:

- $\sec^2 x = 1/\cos^2 x$ and $\sec x \tan x = \sin x/\cos^2 x$.
- $\int \sec^2 x\,dx = \tan x + C$ and $\int \sec x \tan x\,dx = \sec x + C$.

Step 1: Rewrite the integrand On any interval where $\cos x \neq 0$,

$$\frac{1 - 2\sin x}{\cos^2 x} = \sec^2 x - 2\sec x \tan x.$$

This form identifies both standard derivative patterns and preserves the minus sign in the numerator.

Quick Tip to Solve: Split a numerator over cosine squared into secant squared and secant times tangent, then check each sign.

Step 2: Integrate and check the option signs Integrating term by term gives $\tan x - 2\sec x + C$. Its derivative is $\sec^2 x - 2\sec x \tan x$, exactly the given integrand. Option (B) changes both signs, option (C) has the wrong sign for $\sec^2 x$, and option (D) changes the negative sine term to a positive one.

Step 3: Conclusion $\int \frac{1 - 2\sin x}{\cos^2 x}\, dx = \tan x - 2\sec x + C$. The correct option is (A).

Quick Tip to Solve: Split a numerator over cosine squared into secant squared and secant times tangent, then check each sign.

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4. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = \sqrt{37}$, $|\vec{b}| = 3$ and $|\vec{c}| = 4$, then angle between $\vec{b}$ and $\vec{c}$ is [1 mark]

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Correct Answer: (C) $\frac{\pi}{3}$

Solution:

Concept:

- For vectors $\vec{b}, \vec{c}$, $|\vec{b} + \vec{c}|^2 = |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c}$.
- If θ is their angle, $\vec{b} \cdot \vec{c} = |\vec{b}||\vec{c}| \cos\theta$, with $0 \le \theta \le \pi$.

Step 1: Use the zero-sum relation From $\vec{a}+\vec{b}+\vec{c} = \vec{0}$, we have $\vec{b}+\vec{c} = -\vec{a}$. Taking squared magnitudes gives

$$37 = 9 + 16 + 2\vec{b} \cdot \vec{c},$$

hence $\vec{b} \cdot \vec{c} = 6$. Negating a vector does not change its magnitude.

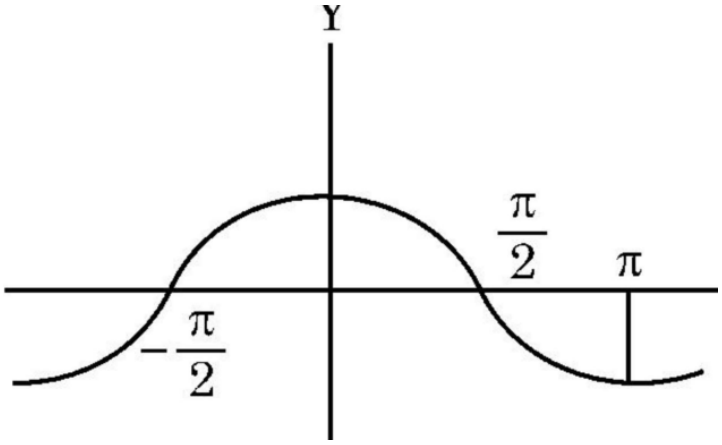
Step 2: Calculate the angle Now $\cos\theta = 6/(3 \times 4) = 1/2$. In the standard range for the angle between two vectors this gives $\theta = \pi/3$. The cosines of options (A), (B) and (D) are respectively $\sqrt{3}/2$, $1/\sqrt{2}$ and 0, none of which equals the required $1/2$.

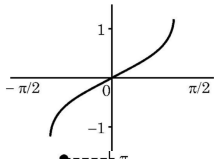
Step 3: Conclusion The angle between $\vec{b}$ and $\vec{c}$ is $\pi/3$. The correct option is (C).

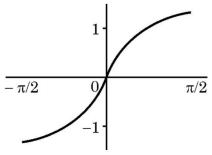
Quick Tip to Solve: When three vectors sum to zero, square the sum of the two whose angle is required and use the third magnitude.

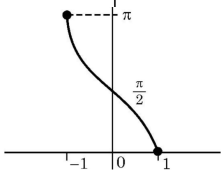
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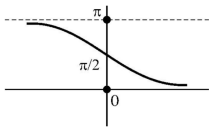
5. The graph of a trigonometric function is as shown. Which of the following will represent the graph of its inverse? [1 mark]

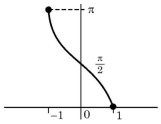


(A) 

(B) 

(C) 

(D) 

Correct Answer: (C) 

Solution:

Concept:

- The given curve is $y = \cos x$: it is highest at $x = 0$, vanishes at $\pm\pi/2$, and reaches its minimum at $x = \pi$.
- The principal inverse cosine uses $\cos : [0, \pi] \to [-1, 1]$; its inverse has domain $[-1, 1]$ and range $[0, \pi]$.

Step 1: Find the inverse graph characteristics Restrict cosine to its principal interval $[0, \pi]$, where it decreases and is one-to-one. Interchanging input and output gives $y = \cos^{-1} x$. Its key points are $(-1, \pi)$, $(0, \pi/2)$ and $(1, 0)$, and it decreases throughout its domain.

Step 2: Compare all four printed graphs Option (C) shows precisely these three points with the finite endpoints at $x = -1$ and $x = 1$. Options (A) and (B) are increasing curves through the origin, so neither represents inverse cosine. Option (D), although decreasing, extends beyond the required finite domain and does not have the two endpoint coordinates of inverse cosine.

Step 3: Conclusion The inverse is $y = \cos^{-1} x$, represented by option (C).

Quick Tip to Solve: To identify inverse cosine, check its decreasing curve and the three points (-1, pi), (0, pi/2), and (1, 0).

6. If A is a square matrix of order 3 such that $\det(A) = 9$, then $\det(9\,A^{-1})$ is equal to [1 mark]

(A) 9

(B) 9^2

(C) 9^3

(D) 9^4

Correct Answer: (B) 9^2

Solution:

Concept:

- For an $n \times n$ matrix, $\det(kA) = k^n \det A$, because every row is multiplied by k.
- For an invertible matrix, $\det(A^{-1}) = 1/\det A$. Here $\det A = 9 \neq 0$, so the inverse exists.

Step 1: Apply the scalar-multiple rule The matrix A^{-1} also has order 3. Consequently

$$\det(9A^{-1}) = 9^3 \det(A^{-1}).$$

The exponent is the matrix order, not the value of its determinant, since scaling affects all three rows.

Step 2: Substitute the determinant of the inverse Using $\det(A^{-1}) = 1/9$, we obtain $9^3/9 = 9^2 = 81$. Option (A) omits a factor from the scalar rule. Option (C) forgets the inverse determinant factor, and option (D) multiplies by $\det A$ instead of dividing by it.

Step 3: Conclusion $\det(9A^{-1}) = 9^2 = 81$. Hence, the correct option is (B).

Quick Tip to Solve: When a scalar multiplies an inverse matrix, first raise the scalar to the matrix order, then divide by the original determinant.

7. If $f(x) = |x| + |x - 1|$, then which of the following is correct? [1 mark]

(A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$.

(B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$.

(C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.

(D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.

Correct Answer: (C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.

Solution:

Concept:

- Absolute-value functions are continuous everywhere, and sums of continuous functions remain continuous.
- Differentiability at a point requires equal finite left and right derivatives.

Step 1: Check continuity at both points Both $|x|$ and $|x - 1|$ are continuous for every real x. Their sum is therefore continuous at $x = 0$ and $x = 1$, with $f(0) = f(1) = 1$. Thus any option asserting lack of continuity is false.

Quick Tip to Solve: For sums of absolute values, split at each zero of the expressions inside the moduli and compare adjacent slopes.

Step 2: Compare the one-sided slopes The piecewise formula is

$$f(x) = \begin{cases} 1 - 2x, & x < 0, \\ 1, & 0 \le x \le 1, \\ 2x - 1, & x > 1. \end{cases}$$

At 0, the left and right derivatives are -2 and 0. At 1, they are 0 and 2. Hence differentiability fails at both points. Option (A) wrongly claims differentiability, while (B) and (D) wrongly deny continuity.

Step 3: Conclusion f is continuous but not differentiable at $x = 0$ and $x = 1$. The correct option is (C).

Quick Tip to Solve: For sums of absolute values, split at each zero of the expressions inside the moduli and compare adjacent slopes.

8. Which of the following is not a homogeneous function of x and y? [1 mark]

(A) $y^2 - xy$

(B) $x - 3y$

(C) $\sin^2 \frac{y}{x} + \frac{y}{x}$

(D) $\tan x - \sec y$

Correct Answer: (D) $\tan x - \sec y$

Solution:

Concept:

- A function is homogeneous of degree r if $f(tx, ty) = t^r f(x, y)$ for a common scale factor t.
- A function of the ratio y/x is homogeneous of degree zero on its domain.

Step 1: Check the algebraic and ratio functions For option (A), $(ty)^2 - (tx)(ty) = t^2(y^2 - xy)$, so its degree is 2. For (B), $tx - 3ty = t(x - 3y)$, giving degree 1. In (C), the ratio $ty/(tx) = y/x$ is unchanged, so the whole expression is homogeneous of degree zero.

Step 2: Check the trigonometric expression In (D), replacing both arguments by their scaled values gives $\tan(tx) - \sec(ty)$. Trigonometric functions of individual variables do not scale as a fixed power of t. For example, at $y = 0$, the expression becomes $\tan x - 1$, which has a nonzero constant term and variable-dependent trigonometric terms, so no fixed degree works.

Step 3: Conclusion $\tan x - \sec y$ is not homogeneous. Hence, the correct option is (D).

Quick Tip to Solve: Scale x and y together; ratios stay unchanged, whereas trigonometric arguments usually change.

9. Let A be a matrix of order $m \times n$ and B is a matrix such that A^TB and BA^T are defined. Then, the order of B is: [1 mark]

(A) $m \times m$

(B) $n \times n$

(C) $m \times n$

(D) $n \times m$

Correct Answer: (C) $m \times n$

Solution:

Concept:

- If A has order $m \times n$, its transpose A^T has order $n \times m$.
- A product exists when the number of columns of its first matrix equals the number of rows of its second matrix.

Step 1: Use the first product Let the order of B be $r \times s$. Since A^TB is defined, the m columns of A^T must match the r rows of B. Thus $r = m$. This condition alone does not determine the columns of B.

Step 2: Use the second product For BA^T, the s columns of B must match the n rows of A^T, so $s = n$. Together the two conditions give B order $m \times n$. Options (A), (B) and (D) do not meet both conditions for general m, n; accidental equality when $m = n$ does not change the general result.

Step 3: Conclusion The order of B is $m \times n$. Hence, the correct option is (C).

Quick Tip to Solve: Write the transpose order first, then impose each product compatibility condition separately.

10. If the feasible region of a linear programming problem with objective function $Z = ax + by$ is bounded, then which of the following is correct? [1 mark]

(A) It will only have a maximum value.

(B) It will only have a minimum value.

(C) It will have both maximum and minimum values.

(D) It will have neither maximum nor minimum value.

Correct Answer: (C) It will have both maximum and minimum values.

Solution:

Concept:

- The feasible region of an LPP is the intersection of its linear constraint half-planes.
- A linear objective $Z = ax + by$ is continuous; on a nonempty closed bounded feasible region it attains both extrema.

Step 1: Apply the bounded-region result A bounded feasible region lies within a finite part of the plane. Since the usual non-strict linear inequalities define a closed feasible region, the region is compact. The continuous objective function therefore has both a greatest and a least value over the feasible points.

Step 2: Compare the alternatives For a polygonal feasible region these extreme values can be found

Quick Tip to Solve: For a bounded feasible polygon, compare objective values at all corners to find both extrema.

by evaluating Z at its corner points. If an objective line is parallel to an edge, an extreme value may occur at multiple points, but it still exists. Thus (A) and (B) wrongly exclude one extreme, and (D) wrongly excludes both; (C) states the correct bounded-region conclusion.

Step 3: Conclusion It will have both maximum and minimum values. Hence, the correct option is (C).

Quick Tip to Solve: For a bounded feasible polygon, compare objective values at all corners to find both extrema.

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11. If $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$**, then** A^{-1} **is** [1 mark]

(A) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

(C) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Correct Answer: (D) $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Solution:

Concept:

- The inverse of a nonsingular diagonal matrix is obtained by taking reciprocals of its diagonal entries.
- A matrix B is the inverse of A when $AB = BA = I$.

Step 1: Take reciprocals of the diagonal entries The given diagonal entries are $-1, 1, 1$, all nonzero. Their reciprocals are again $-1, 1, 1$. Therefore the candidate inverse is $\mathrm{diag}(-1, 1, 1)$, exactly the given matrix.

Step 2: Verify the inverse and reject other signs

$$A^2 = \mathrm{diag}((-1)^2, 1^2, 1^2) = I_3.$$

Hence $A^{-1} = A$. In (A) the last two product diagonal entries would be -1; in (B) all three would be

Quick Tip to Solve: For a diagonal matrix with entries only 1 and -1, squaring gives the identity immediately.

-1; and in (C) the second would be -1. These products cannot equal the identity.

Step 3: Conclusion $A^{-1} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Hence, the correct option is (D).

Quick Tip to Solve: For a diagonal matrix with entries only 1 and -1, squaring gives the identity immediately.

12. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is [1 mark]

(A) xe^x

(B) $\frac{e^x}{x}$

(C) $\frac{x}{e^x}$

(D) $xe^{1/x}$

Correct Answer: (B) $\frac{e^x}{x}$

Solution:

Concept:

- A linear differential equation has the form $dy/dx + P(x)y = Q(x)$.
- Its integrating factor is $e^{\int P(x)\,dx}$, up to a nonzero constant multiplier.

Step 1: Collect every term involving y The right side equals $1/x + y/x$. Bringing y/x to the left gives

$$\frac{dy}{dx} + \left(1 - \frac{1}{x}\right) y = \frac{1}{x}.$$

Thus $P(x) = 1 - 1/x$, rather than simply 1; the equation is considered on an interval not containing zero.

Step 2: Integrate the coefficient

$$\text{I.F.} = e^{\int (1-1/x)\,dx} = e^{x - \ln|x|} = \frac{e^x}{|x|}.$$

On any such interval, e^x/x differs only by a constant sign and is a valid integrating factor. Options (A), (C) and (D) do not have logarithmic derivative $1 - 1/x$, so they fail the required condition.

Step 3: Conclusion The integrating factor is e^x/x. Hence, the correct option is (B).

Quick Tip to Solve: Move all y terms to the left before identifying P(x); otherwise the integrating factor is wrong.

13. Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = \hat{j} - 2\hat{i}$. Then which of the following is true? [1 mark]

(A) $a_{12} > 0$

(B) All $a_{ij} < 0$

(C) $a_{13} + a_{31} = -6$

(D) $a_{23} > a_{32}$

Correct Answer: (D) $a_{23} > a_{32}$

Solution:

Concept:

- In $a_{ij} = j - 2i$, i is the row number and j is the column number.
- Substitute each pair of indices separately before testing the inequalities.

Step 1: Test the first three statements For (A), $a_{12} = 2 - 2(1) = 0$, so it is not positive. For (B), $a_{13} = 3 - 2(1) = 1 > 0$, contradicting the claim that all entries are negative. For (C), $a_{31} = 1 - 2(3) = -5$, so $a_{13} + a_{31} = 1 - 5 = -4$, not -6.

Step 2: Compare the remaining entries

$$a_{23} = 3 - 2(2) = -1, \qquad a_{32} = 2 - 2(3) = -4.$$

Since $-1 > -4$, the inequality in (D) is true. Notice that interchanging the indices changes which number is multiplied by 2; treating a_{23} and a_{32} as equal would give an incorrect comparison.

Step 3: Conclusion $a_{23} > a_{32}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the first subscript as the row index: multiply it by 2 before subtracting from the second.

14. The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is: [1 mark]

(A) 0

(B) 2

(C) 4

(D) 5

Correct Answer: (C) 4

Solution:

Concept:

- A continuous function on a closed interval attains its absolute maximum and minimum.
- Compare values at interior critical points and at both endpoints to find the absolute maximum.

Step 1: Find the interior critical point For $f(x) = x^3 - 3x + 2$, $f\prime(x) = 3x^2 - 3 = 3(x^2 - 1)$. The stationary points satisfy $x = \pm 1$, but only $x = 1$ belongs to $[0, 2]$. The derivative exists throughout the interval, so there are no additional interior candidates.

Quick Tip to Solve: For a closed interval, always evaluate both endpoints as well as the critical points.

Step 2: Compare all candidate values

$$f(0) = 2, \qquad f(1) = 1 - 3 + 2 = 0, \qquad f(2) = 8 - 6 + 2 = 4.$$

The greatest of these values is 4. Option (A) gives the minimum, (B) gives the left-endpoint value, and (D) exceeds every candidate value. An endpoint can provide the absolute maximum even though the derivative there is not zero.

Step 3: Conclusion The absolute maximum value is 4, attained at $x = 2$. Hence, the correct option is (C).

Quick Tip to Solve: For a closed interval, always evaluate both endpoints as well as the critical points.

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15. If $\int \frac{2^{1/x}}{x^2}\,dx = k \cdot 2^{1/x} + C$, then k is equal to [1 mark]

(A) $\frac{-1}{\log 2}$

(B) $-\log 2$

(C) -1

(D) $\frac{1}{2}$

Correct Answer: (A) $\frac{-1}{\log 2}$

Solution:

Concept:

- Use a substitution whose derivative matches the factor $1/x^2$.
- For $a > 0$, $a \neq 1$, $\int a^u\,du = a^u/\ln a + C$.

Step 1: Make the reciprocal substitution Put $u = 1/x$, with $x \neq 0$. Then $du = -dx/x^2$, so

$$\int \frac{2^{1/x}}{x^2}\,dx = -\int 2^u\,du.$$

The minus sign comes from differentiating the reciprocal and must be retained.

Step 2: Integrate and identify the coefficient

$$-\int 2^u\,du = -\frac{2^u}{\ln 2} + C = -\frac{2^{1/x}}{\ln 2} + C.$$

Comparing with $k\,2^{1/x} + C$ gives $k = -1/\ln 2$. Differentiating the proposed form yields $-k \ln 2\, 2^{1/x}/x^2$; only option (A) makes its coefficient 1. Options (B), (C) and (D) have incorrect factors or signs.

Step 3: Conclusion $k = -\frac{1}{\log 2}$, where log denotes the natural logarithm. Hence, the correct option is (A).

Quick Tip to Solve: When the exponent is 1/x and the denominator is x squared, substitute u=1/x and keep the minus sign.

16. If A and B are invertible matrices, then which of the following is not correct? [1 mark]

(A) $(A+B)^{-1} = B^{-1} + A^{-1}$

(B) $(AB)^{-1} = B^{-1}A^{-1}$

(C) $\text{adj}(A) = |A|\,A^{-1}$

(D) $|A|^{-1} = |A^{-1}|$

Correct Answer: (A) $(A+B)^{-1} = B^{-1} + A^{-1}$

Solution:

Concept:

- For invertible square matrices, $(AB)^{-1} = B^{-1}A^{-1}$; inversion reverses the order of multiplication.
- The adjugate identity is $A\,\text{adj}(A) = |A|I$, and $|A^{-1}| = 1/|A|$.

Step 1: Test the sum formula Take $A = B = I$, where I is the identity matrix. Then $(A+B)^{-1} = (2I)^{-1} = \frac{1}{2}I$, whereas $B^{-1} + A^{-1} = 2I$. These are unequal, so option (A) is not a valid identity. Also, invertibility of A and B alone does not even guarantee invertibility of their sum.

Step 2: Verify the other identities For (B), multiplication gives $(AB)(B^{-1}A^{-1}) = I$, proving the product rule. For (C), multiply $A\,\text{adj}(A) = |A|I$ by A^{-1} to obtain $\text{adj}(A) = |A|A^{-1}$. For (D), take determinants in $AA^{-1} = I$: $|A||A^{-1}| = 1$. Hence all three are correct.

Step 3: Conclusion The statement that is not correct is $(A+B)^{-1} = B^{-1} + A^{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: Test a claimed sum identity with $A = B = I$; inverses do not distribute over addition.

17. The corner points of the feasible region in graphical representation of an L.P.P. are $(2, 72), (15, 20)$ and $(40, 15)$. If $Z = 18x + 9y$ be the objective function, then [1 mark]

(A) Z is maximum at $(2, 72)$, minimum at $(15, 20)$

(B) Z is maximum at $(15, 20)$, minimum at $(40, 15)$

(C) Z is maximum at $(40, 15)$, minimum at $(15, 20)$

(D) Z is maximum at $(40, 15)$, minimum at $(2, 72)$

Correct Answer: (C) Z is maximum at $(40, 15)$, minimum at $(15, 20)$

Solution:

Concept:

- A linear objective function reaches its finite extrema at corner points of the feasible region.
- Here $Z = 18x + 9y$; substitute each listed vertex and compare the resulting values.

Step 1: Evaluate at all three vertices

$$Z(2, 72) = 18(2) + 9(72) = 36 + 648 = 684,$$
$$Z(15, 20) = 18(15) + 9(20) = 270 + 180 = 450,$$
$$Z(40, 15) = 18(40) + 9(15) = 720 + 135 = 855.$$

Each value uses both coordinates; the vertex with the greatest y-coordinate need not give the largest

Quick Tip to Solve: Compare $2x + y$ at each vertex first, since $Z = 9(2x + y)$ has the same ordering.

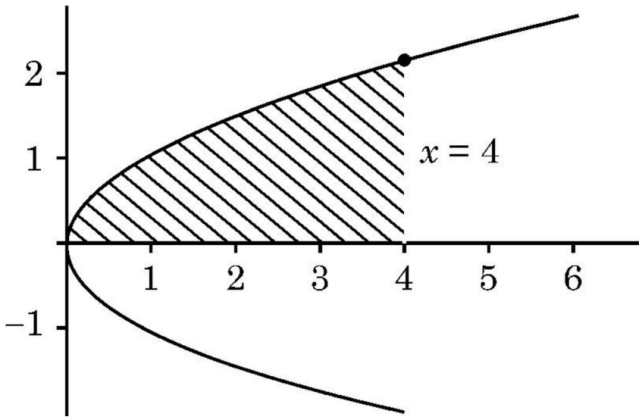
objective value.

Step 2: Compare the values and options The ordering is $450 < 684 < 855$. Thus the minimum occurs at $(15, 20)$, while the maximum occurs at $(40, 15)$. Option (A) incorrectly places the maximum at $(2, 72)$; (B) reverses the actual extremal vertices, and (D) incorrectly places the minimum at $(2, 72)$.

Step 3: Conclusion $Z_{\max} = 855$ at $(40, 15)$, and $Z_{\min} = 450$ at $(15, 20)$. Hence, the correct option is (C).

Quick Tip to Solve: Compare $2x + y$ at each vertex first, since $Z = 9(2x + y)$ has the same ordering.

18. The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x-axis is given by [1 mark]



(A) $\int_0^4 x\,dx$

(B) $\int_0^2 y^2\,dy$

(C) $2\int_0^4 \sqrt{x}\,dx$

(D) $\int_0^4 \sqrt{x}\,dx$

Correct Answer: (D) $\int_0^4 \sqrt{x}\,dx$

Solution:

Concept:

- For vertical strips, area is $\int_a^b (y_{\text{upper}} - y_{\text{lower}})\,dx$.
- The shaded part is above the x-axis, so $y^2 = x$ gives its upper boundary $y = \sqrt{x}$.

Step 1: Identify the strip and limits The official figure shades only the first-quadrant part of the parabola, between $x = 0$ and $x = 4$. A vertical strip has height $\sqrt{x} - 0$ and width dx. Hence its elemental area is $dA = \sqrt{x}\,dx$, not $x\,dx$.

Step 2: Form the integral and test alternatives Therefore $A = \int_0^4 \sqrt{x}\,dx$. Option (A) uses the wrong

Quick Tip to Solve: Use upper minus lower for vertical strips; the unshaded lower branch contributes no area here.

upper boundary. With horizontal strips, the right boundary is $x = 4$, so the correct equivalent integral is $\int_0^2 (4 - y^2)\, dy$, not option (B). Option (C) doubles the required area by including the symmetric lower region, which is not shaded. The area evaluates to $\left[\frac{2}{3}x^{3/2}\right]_0^4 = \frac{16}{3}$.

Step 3: Conclusion The shaded area is $\int_0^4 \sqrt{x}\, dx = \frac{16}{3}$ square units. Hence, the correct option is (D).

Quick Tip to Solve: Use upper minus lower for vertical strips; the unshaded lower branch contributes no area here.

19. Assertion (A): Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x - 5,\ \forall x \in Z$ is bijective.

Reason (R): A function is bijective if it is both surjective and injective. [1 mark]

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (D) Assertion (A) is false, but Reason (R) is true.

Solution:

Concept:

- A function is injective if equal outputs force equal inputs, and surjective if every codomain element has a preimage.
- A bijection must satisfy both conditions on the stated domain and codomain.

Step 1: Judge the assertion If $f(x_1) = f(x_2)$, then $3x_1 - 5 = 3x_2 - 5$, so $x_1 = x_2$. Thus f is injective. However, the integer 0 has no integer preimage: $3x - 5 = 0$ requires $x = 5/3 \notin Z$. Consequently $f : Z \to Z$ is not surjective and the assertion that it is bijective is false.

Step 2: Judge the reason The reason states the definition of a bijective function: it is both injective and surjective. This statement is true; neither property alone is sufficient.

Step 3: Check the logical relationship Although the defining criterion in the reason is true, the given map fails its surjectivity requirement. Thus both statements cannot be true as required in options (A) and (B). Option (C) also fails because the assertion is false and the reason is true.

Step 4: Conclusion The assertion is false, but the reason is true. Hence, the correct option is (D).

Quick Tip to Solve: For an integer-valued linear map, test one integer output such as 0 before claiming it is onto.

20. Assertion (A): $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$ **is continuous at** $x = 5$ **for** $k = \frac{5}{2}$.

Reason (R): For a function f to be continuous at $x = a$, $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$. [1 mark]

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (D) Assertion (A) is false, but Reason (R) is true.

Solution:

Concept:

- Continuity at $x = a$ requires $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.
- At a piecewise junction, evaluate each one-sided limit using its own branch.

Step 1: Judge the assertion The branch $3x - 8$ includes $x = 5$, so $f(5) = 3(5) - 8 = 7$, and the left-hand limit is also 7. The right-hand limit is $2k$. At the asserted value $k = 5/2$, this is 5, which differs from 7. The function is therefore discontinuous at 5, making the assertion false.

Step 2: Judge the reason The equality of the two one-sided limits with the function value is precisely the criterion for continuity at an interior point. Thus the reason is true.

Step 3: Check the relationship Applying the reason would require $2k = 7$, giving $k = 7/2$, rather than the asserted $5/2$. Options (A) and (B) require a true assertion, and option (C) requires a false reason; none matches these evaluations.

Step 4: Conclusion The assertion is false, but the reason is true. Hence, the correct option is (D).

Quick Tip to Solve: At $x = 5$, include equality in the left branch and compare 7 directly with $2k$.

Section B (10 marks)

*This section comprises of **5** Very Short Answer (VSA) type questions of **2** marks each.*

21(a). Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ **and** $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$**. Determine the angle formed between the kite strings. Assume there is no slack in the strings.** [2 marks]

Solution:

Concept:

- For nonzero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, where $0 \leq \theta \leq \pi$.
- The dot product is the sum of componentwise products; a vector magnitude is the square root of the sum of its squared components.

Quick Tip to Solve: Keep the sign of the j-component in the dot product, but square it when finding the magnitude.

Step 1: Find the dot product The strings are taut, so the specified vectors give their directions. Using $\vec{a} = (3, 1, 2)$ and $\vec{b} = (2, -2, 4)$,

$$\vec{a} \cdot \vec{b} = 3(2) + 1(-2) + 2(4) = 6 - 2 + 8 = 12.$$

The negative middle component must retain its sign; it reduces the dot product.

Step 2: Find the magnitudes and apply the angle formula

$$|\vec{a}| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{14}, \qquad |\vec{b}| = \sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{24} = 2\sqrt{6}.$$

Hence

$$\cos\theta = \frac{12}{\sqrt{14}\sqrt{24}} = \frac{12}{4\sqrt{21}} = \frac{3}{\sqrt{21}} = \sqrt{\frac{3}{7}}.$$

The positive cosine confirms that the angle between these directed vectors is acute; the inverse cosine specifies it uniquely in the standard interval.

Step 3: Conclusion The angle between the kite strings is $\theta = \cos^{-1}\left(\sqrt{\frac{3}{7}}\right) \approx 49.1°$.

Quick Tip to Solve: Keep the sign of the j-component in the dot product, but square it when finding the magnitude.

OR

21(b). Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A(2, 1, 3) and B(8, −1, 0) respectively. [2 marks]

Solution:

Concept:

- The displacement vector is $\overrightarrow{AB} = \vec{OB} - \vec{OA}$.
- A vector of magnitude m opposite to a nonzero vector $\vec{v}$ is $-m\vec{v}/|\vec{v}|$; dividing by the magnitude first gives a unit vector.

Step 1: Calculate the displacement and its length From $A(2, 1, 3)$ to $B(8, -1, 0)$, subtract corresponding coordinates:

$$\overrightarrow{AB} = (8 - 2)\hat{i} + (-1 - 1)\hat{j} + (0 - 3)\hat{k} = 6\hat{i} - 2\hat{j} - 3\hat{k}.$$

Its magnitude is

$$|\overrightarrow{AB}| = \sqrt{6^2 + (-2)^2 + (-3)^2} = \sqrt{36 + 4 + 9} = 7.$$

The direction opposite to AB is the direction of BA, so all three component signs must reverse.

Step 2: Scale the opposite unit vector The unit vector opposite to $\overrightarrow{AB}$ is $\frac{-6\hat{i}+2\hat{j}+3\hat{k}}{7}$. Multiplying it by 21 gives

$$\vec{u} = 21\frac{-6\hat{i} + 2\hat{j} + 3\hat{k}}{7} = -18\hat{i} + 6\hat{j} + 9\hat{k}.$$

As a check, $|\vec{u}| = \sqrt{324 + 36 + 81} = 21$, and $\vec{u} = -3\overrightarrow{AB}$; the negative scalar confirms the required

Quick Tip to Solve: Since $|AB| = 7$ and the target length is 21, multiply AB by -3 directly.

opposite direction.

Step 3: Conclusion The required vector is $\boxed{-18\hat{i}+6\hat{j}+9\hat{k}}$, with magnitude 21 units.

Quick Tip to Solve: Since $|AB|=7$ and the target length is 21, multiply AB by -3 directly.

22. Find the values of a for which $f(x)=x^2-2ax+b$ is an increasing function for $x>0$. [2 marks]

Solution:

Concept:

- For a differentiable function, a positive derivative on an interval ensures that the function is strictly increasing on that interval.
- The constants a and b do not depend on x. The derivative of $x^2-2ax+b$ is $2x-2a$.

Step 1: Differentiate and establish the condition

$$f\prime(x)=2x-2a=2(x-a).$$

For increasing behaviour throughout $x>0$, the slope must not become negative anywhere in this interval. Thus we need $x-a\geq 0$ for every positive x, rather than merely at one selected value of x.

Step 2: Check both possible ranges of the parameter If $a\leq 0$, then for every $x>0$, $x-a>0$; hence $f\prime(x)>0$ and the function is strictly increasing. The endpoint $a=0$ is included because then $f\prime(x)=2x>0$ on the stated open interval. If $a>0$, choose $x=a/2>0$. Then $f\prime(a/2)=-a<0$, and in fact the derivative is negative throughout $(0,a)$. Therefore such a value of a cannot satisfy the requirement. The constant b only shifts the graph vertically and imposes no restriction.

Step 3: Conclusion

$$\boxed{a\leq 0}.$$

Quick Tip to Solve: Check the derivative arbitrarily close to the open endpoint x=0; a positive a always creates a decreasing interval.

23(a). Differentiate $2^{\cos^2 x}$ w.r.t. $\cos^2 x$. [2 marks]

Solution:

Concept:

- When differentiating with respect to a function, regard that function as the independent variable. Here set $u=\cos^2 x$.
- For a constant base $c>0$, $\dfrac{d}{du}(c^u)=c^u\ln c$. This follows from $c^u=e^{u\ln c}$.

Step 1: Identify the variable of differentiation Let $y=2^{\cos^2 x}$ and write $u=\cos^2 x$. The question asks for dy/du, so the expression to differentiate is simply $y=2^u$. There is no need to differentiate $\cos^2 x$

Quick Tip to Solve: Replace the entire expression after w.r.t. by u first; this prevents an unnecessary derivative of cos squared x.

with respect to x, since x is not the requested variable.

Step 2: Differentiate the exponential and substitute back Using the exponential representation and the chain rule with respect to u,

$$\frac{dy}{du} = \frac{d}{du}\left(e^{u \ln 2}\right) = e^{u \ln 2} \ln 2 = 2^u \ln 2.$$

Substituting $u = \cos^2 x$ restores the original notation. The factor $\ln 2$ is essential because the base is 2, whereas an exponential with base e would have no such extra factor. No factor $-2 \sin x \cos x$ belongs in this derivative; that factor would appear only in dy/dx.

Step 3: Conclusion

$$\boxed{\frac{d\left(2^{\cos^2 x}\right)}{d(\cos^2 x)} = 2^{\cos^2 x} \ln 2}.$$

Quick Tip to Solve: Replace the entire expression after w.r.t. by u first; this prevents an unnecessary derivative of cos squared x.

OR

23(b). **If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.** [2 marks]

Solution:

Concept:

- Implicit differentiation treats y as a function of x, so $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$.
- For a differentiable expression v, $\frac{d}{dx}(\tan^{-1} v) = \frac{v'}{1+v^2}$. The parameter a is constant.

Step 1: Differentiate both sides with respect to x From $\tan^{-1}(x^2 + y^2) = a^2$, apply the chain rule to the left side and use $\frac{d}{dx}(a^2) = 0$:

$$\frac{2x + 2y\frac{dy}{dx}}{1 + (x^2 + y^2)^2} = 0.$$

Both the derivative of x^2 and the implicit derivative of y^2 must be retained in the numerator.

Step 2: Isolate the derivative The denominator $1 + (x^2 + y^2)^2$ is strictly positive for real x, y, so multiplying by it gives

$$2x + 2y\frac{dy}{dx} = 0, \qquad y\frac{dy}{dx} = -x.$$

At a point where $y \neq 0$, division by y gives the required derivative. As a check, taking the tangent of the original relation gives $x^2 + y^2 = \tan(a^2)$ whenever the relation is satisfied; differentiating this constant-radius equation yields the same result. At $y = 0$, division by y is unavailable and this finite slope formula does not apply.

Quick Tip to Solve: The inverse tangent denominator is always positive, so after differentiating only its numerator needs to equal zero.

Step 3: Conclusion

$$\boxed{\frac{dy}{dx} = -\frac{x}{y}} \qquad (y \neq 0).$$

Quick Tip to Solve: The inverse tangent denominator is always positive, so after differentiating only its numerator needs to equal zero.

24. Evaluate: $\sin^{-1}\left[\sin\frac{3\pi}{5}\right]$. [2 marks]

Solution:

Concept:

- The principal value of $\sin^{-1} t$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
- The supplementary-angle identity is $\sin(\pi - \theta) = \sin\theta$. It lets us replace an angle outside the principal interval by an angle with the same sine inside that interval.

Step 1: Locate the given angle The angle $3\pi/5$ satisfies

$$\frac{\pi}{2} < \frac{3\pi}{5} < \pi.$$

Thus it lies in the second quadrant, outside the principal interval of inverse sine. Consequently, directly cancelling $\sin^{-1}$ and $\sin$ to obtain $3\pi/5$ would give a value outside the permitted range.

Step 2: Use the corresponding principal angle The supplementary first-quadrant angle is

$$\pi - \frac{3\pi}{5} = \frac{2\pi}{5}.$$

By the sine identity, $\sin(3\pi/5) = \sin(2\pi/5)$. Also $0 < 2\pi/5 < \pi/2$, so this angle belongs to the principal interval where inverse sine reverses sine. Therefore

$$\sin^{-1}\left[\sin\frac{3\pi}{5}\right] = \sin^{-1}\left[\sin\frac{2\pi}{5}\right] = \frac{2\pi}{5}.$$

The sine is positive here, so the required principal angle is positive.

Step 3: Conclusion

$$\boxed{\frac{2\pi}{5}}.$$

Quick Tip to Solve: For an angle between pi/2 and pi, subtract it from pi before applying the inverse sine principal-value rule.

25. The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Find the area of the parallelogram. [2 marks]

Solution:

Concept:

- If $\vec{u}, \vec{v}$ are adjacent side vectors of a parallelogram, its area is $|\vec{u} \times \vec{v}|$.
- For diagonal vectors $\vec{a} = \vec{u} + \vec{v}$ and $\vec{b} = \vec{u} - \vec{v}$, $|\vec{a} \times \vec{b}| = 2|\vec{u} \times \vec{v}|$. Thus area equals half the magnitude of the cross product of the diagonals.

Step 1: Calculate the cross product of the diagonals The given vectors are $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Expanding the determinant,

$$\begin{aligned}\vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} \\ &= \hat{i}(1-3) - \hat{j}(-2-1) + \hat{k}(6+1) \\ &= -2\hat{i} + 3\hat{j} + 7\hat{k}.\end{aligned}$$

In particular the middle component is positive because the cofactor sign reverses -3.

Step 2: Find its magnitude and use the diagonal-area relation The magnitude is

$$|\vec{a} \times \vec{b}| = \sqrt{(-2)^2 + 3^2 + 7^2} = \sqrt{62}.$$

To verify the factor of one-half, expand $(\vec{u} + \vec{v}) \times (\vec{u} - \vec{v}) = -2(\vec{u} \times \vec{v})$. Taking magnitudes removes the minus sign. Since the given vectors are diagonals rather than adjacent sides, the parallelogram area is

$$A = \frac{1}{2}|\vec{a} \times \vec{b}| = \frac{\sqrt{62}}{2}.$$

Reversing a diagonal direction would change the cross-product sign but leave this area unchanged.

Step 3: Conclusion The area of the parallelogram is $\boxed{\frac{\sqrt{62}}{2}}$ square units.

Quick Tip to Solve: First identify whether vectors are sides or diagonals: for diagonals, halve the magnitude of their cross product.

Section C (18 marks)

*This section comprises of **6** Short Answer (SA) type questions of **3** marks each.*

26(a). Verify that lines given by $\vec{r} = (1-\lambda)\hat{i} + (\lambda-2)\hat{j} + (3-2\lambda)\hat{k}$ and $\vec{r} = (\mu+1)\hat{i} + (2\mu-1)\hat{j} - (2\mu+1)\hat{k}$ are skew lines. Hence, find the shortest distance between the lines. [3 marks]

Solution:

Concept:

- For a line $\vec{r} = \vec{a} + t\vec{b}$, $\vec{a}$ is the position vector of a point and $\vec{b}$ is its direction vector.
- Two lines are skew when they are neither parallel nor intersecting. Their shortest distance is $d = \dfrac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$.

Step 1: Identify the points and directions Writing the lines in standard vector form gives

$$\vec{a}_1 = (1, -2, 3), \quad \vec{b}_1 = (-1, 1, -2),$$
$$\vec{a}_2 = (1, -1, -1), \quad \vec{b}_2 = (1, 2, -2).$$

The direction vectors are not proportional: their first components would require a multiplier of -1, while their second components would require a multiplier of 2. Therefore, the lines are not parallel.

Step 2: Check that the lines do not intersect At a possible intersection, the first two coordinates must satisfy

$$1 - \lambda = 1 + \mu, \qquad \lambda - 2 = 2\mu - 1.$$

Hence $\lambda = -\mu$, and substitution gives $\mu = -\frac{1}{3}$, $\lambda = \frac{1}{3}$. The third coordinate on the first line is then $3 - 2\lambda = \frac{7}{3}$, whereas on the second it is $-2\mu - 1 = -\frac{1}{3}$. These are unequal, so no point satisfies all three coordinate equations. Together with non-parallelism, this verifies that the lines are skew.

Step 3: Calculate the perpendicular separation The common perpendicular has direction

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{i} - 4\hat{j} - 3\hat{k}.$$

Also, $\vec{a}_2 - \vec{a}_1 = (0, 1, -4)$, so its scalar product with the cross product is $0 - 4 + 12 = 8$. The magnitude of the cross product is $\sqrt{4 + 16 + 9} = \sqrt{29}$. Thus the distance formula gives $d = \frac{|8|}{\sqrt{29}}$.

Step 4: Conclusion The lines are skew, and their shortest distance is $\boxed{\dfrac{8}{\sqrt{29}} \text{ units}}$.

Quick Tip to Solve: Check all three coordinate equations when testing intersection; satisfying only two does not prove that the lines meet.

OR

26(b). During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder. [3 marks]

Solution:

Concept:

- If a point W divides BF internally in the ratio $BW : WF = m : n$, its position vector is $\vec{W} = \dfrac{n\vec{B} + m\vec{F}}{m+n}$.
- For collinear points, the ratio of the segment lengths can also be read from their displacement vectors when those vectors point in the same direction.

Step 1: Find the two displacement vectors The position vectors correspond to $B = (2, 8, 0)$, $W = (6, 12, 0)$ and $F = (12, 18, 0)$. Subtracting the endpoint coordinates gives

$$\overrightarrow{BW} = \vec{W} - \vec{B} = 4\hat{i} + 4\hat{j},$$
$$\overrightarrow{WF} = \vec{F} - \vec{W} = 6\hat{i} + 6\hat{j}.$$

Both are positive multiples of $\hat{i} + \hat{j}$. Thus the three positions are collinear, and the wicketkeeper lies between the bowler and the leg slip fielder. The division is therefore internal.

Step 2: Compute the ratio and verify it The corresponding lengths are

$$BW = \sqrt{4^2 + 4^2} = 4\sqrt{2}, \qquad WF = \sqrt{6^2 + 6^2} = 6\sqrt{2}.$$

Therefore $BW : WF = 4\sqrt{2} : 6\sqrt{2} = 2 : 3$. For an independent section-formula check, let the ratio be $m : n$. The first coordinate gives

$$6 = \frac{2n + 12m}{m+n} \quad \Longrightarrow \quad 6m + 6n = 2n + 12m \quad \Longrightarrow \quad 4n = 6m.$$

Hence $m : n = 2 : 3$. The second coordinate confirms this: $\frac{3(8)+2(18)}{2+3} = 12$, while the first is $\frac{3(2)+2(12)}{5} = 6$. Both reproduce the wicketkeeper position exactly.

Step 3: Conclusion The wicketkeeper divides the segment internally in the ratio $\boxed{BW : WF = 2 : 3}$.

Quick Tip to Solve: Write the ratio as bowler-to-wicketkeeper : wicketkeeper-to-fielder before substituting into the section formula.

27. Solve the following linear programming problem graphically:

Maximise $Z = 20x + 30y$

Subject to the constraints:

$x + y \leq 80$

$2x + 3y \geq 100$

$x \geq 14$

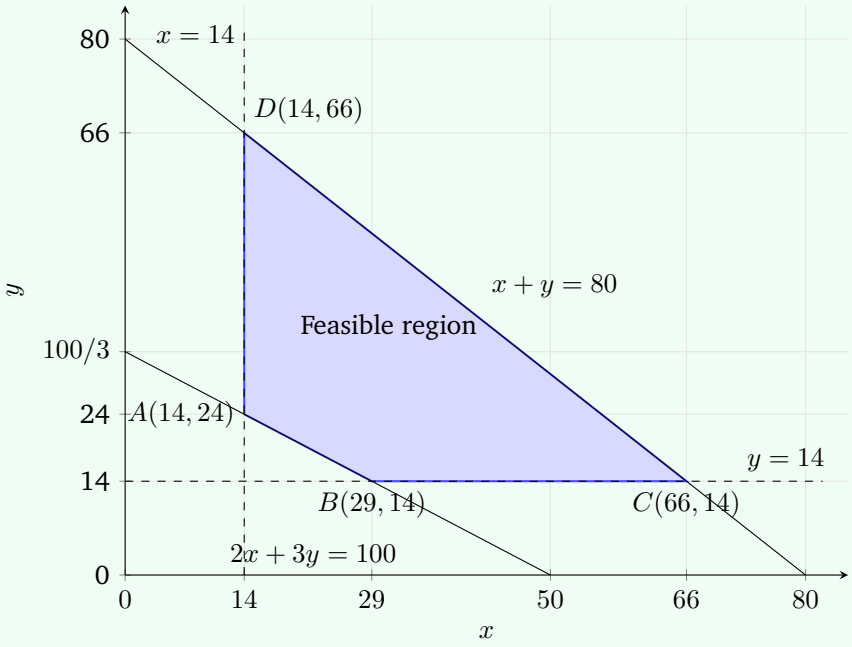
$y \geq 14$ [3 marks]

Solution:

Concept:

- Each linear inequality represents a half-plane. The feasible region is the common part of all the required half-planes, including their boundary lines.
- A linear objective on a non-empty bounded polygon attains its maximum at a vertex. Here $Z = 20x + 30y$, where x, y are the decision variables.

Step 1: Plot the boundaries and shade their common region The line $x + y = 80$ passes through $(80, 0)$ and $(0, 80)$; retain the side on or below this line. The line $2x + 3y = 100$ passes through $(50, 0)$ and $(0, \frac{100}{3})$; retain the side on or above it. Finally, retain the right side of $x = 14$ and the upper side of $y = 14$. The shaded quadrilateral in the graph is the intersection of all four half-planes. All edges belong to the feasible region because the inequalities are non-strict.



Feasible region $ABCD$ for the given constraints

Step 2: Determine the feasible vertices On $x = 14$, the lower sloping boundary gives $28 + 3y = 100$, so $A = (14, 24)$. On $y = 14$, it gives $2x + 42 = 100$, so $B = (29, 14)$. The upper boundary $x + y = 80$ meets $y = 14$ at $C = (66, 14)$, and meets $x = 14$ at $D = (14, 66)$. The point $(14, 14)$ is excluded because $2(14) + 3(14) = 70 < 100$. Thus the feasible region is the bounded polygon $ABCD$.

Quick Tip to Solve: On the boundary x+y=80, replace y by 80-x to see immediately which endpoint makes the objective largest.

Step 3: Evaluate the objective at each corner Substitute the four vertices into the objective:

$$Z(A) = 20(14) + 30(24) = 1000,$$
$$Z(B) = 20(29) + 30(14) = 1000,$$
$$Z(C) = 20(66) + 30(14) = 1740,$$
$$Z(D) = 20(14) + 30(66) = 2260.$$

The vertex theorem applies because the polygon is bounded and non-empty. As a check along its upper edge, $y = 80 - x$ gives $Z = 2400 - 10x$, which decreases as x increases; the upper-left corner is consequently the maximizing corner.

Step 4: Conclusion The maximum value is $\boxed{Z_{\max} = 2260}$, attained at $\boxed{(x, y) = (14, 66)}$.

Quick Tip to Solve: On the boundary x+y=80, replace y by 80-x to see immediately which endpoint makes the objective largest.

28. The area of an expanding rectangle is increasing at the rate of $48\,\text{cm}^2/\text{s}$. The length of the rectangle is always square of its breadth. At what rate the length of rectangle increasing at an instant, when breadth $= 4.5\,\text{cm}$? [3 marks]

Solution:

Concept:

- For a rectangle, area is $A = lb$, where l is its length and b its breadth. Here the given numerical relation is $l = b^2$.
- When dimensions depend on time t, differentiate their relation with respect to time using the chain rule: $\frac{d}{dt}(b^n) = nb^{n-1}\frac{db}{dt}$.

Step 1: Express the area in one variable The length and breadth do not change independently: at every instant the question gives $l = b^2$. Substituting into the area formula therefore gives

$$A = lb = b^2 b = b^3.$$

We are given $\frac{dA}{dt} = 48\,\text{cm}^2/\text{s}$, and must obtain the length rate at the instant $b = 4.5\,\text{cm}$.

Step 2: Find the breadth rate at the specified instant Differentiate the area expression with respect to time, remembering that breadth itself is a function of time:

$$\frac{dA}{dt} = 3b^2\frac{db}{dt}.$$

At the given instant,

$$48 = 3(4.5)^2\frac{db}{dt}, \qquad \frac{db}{dt} = \frac{48}{3(4.5)^2} = \frac{64}{81}\,\text{cm/s}.$$

The positive sign agrees with an expanding rectangle.

Quick Tip to Solve: Eliminate length from the area first; combining the two derivatives directly gives $dl/dt = 2(dA/dt)/(3b)$.

Step 3: Relate the length rate to the breadth rate Now differentiate the given relation $l = b^2$:

$$\frac{dl}{dt} = 2b\frac{db}{dt}.$$

Substituting the breadth and the breadth rate at the same instant gives

$$\frac{dl}{dt} = 2(4.5)\frac{64}{81} = \frac{64}{9}\ \text{cm/s}.$$

Both dimensions have positive rates, consistent with the stated increase in area.

Step 4: Conclusion The length is increasing at $\boxed{\frac{64}{9}\ \text{cm/s}}$, approximately $7.11\ \text{cm/s}$.

Quick Tip to Solve: Eliminate length from the area first; combining the two derivatives directly gives $dl/dt = 2(dA/dt)/(3b)$.

29(a). The probability distribution for the number of students being absent in a class on a Saturday is as follows:

X	0	2	4	5
P(X)	p	2p	3p	p

Where X is the number of students absent.

(i) Calculate p. (1 mark)

(ii) Calculate the mean of the number of absent students on Saturday. (2 marks) [3 marks]

Solution:

Concept:

- A discrete probability distribution satisfies $\sum_x P(X = x) = 1$, with each probability non-negative.
- The mean, or expected value, is $E(X) = \sum_x xP(X = x)$; it is a weighted average of the possible numbers of absent students.

Step 1: Part (i): normalise the probabilities The possible values of the number X of absent students are $0, 2, 4, 5$, with probabilities $p, 2p, 3p, p$, respectively. These four mutually exclusive possibilities cover the distribution, so their probabilities must add to one:

$$p + 2p + 3p + p = 1 \quad \Longrightarrow \quad 7p = 1 \quad \Longrightarrow \quad p = \frac{1}{7}.$$

Thus the probabilities in order are $1/7, 2/7, 3/7, 1/7$. Each lies between zero and one, and their sum is one, so this value produces a valid distribution.

Step 2: Part (ii): calculate the probability-weighted mean Multiply each possible absence count by its own probability, then add the products. In particular, the count zero contributes zero even though its

Quick Tip to Solve: First normalise the probability row, then multiply corresponding entries of the two rows; do not average the listed counts directly.

probability is nonzero:

$$\begin{aligned} E(X) &= 0(p) + 2(2p) + 4(3p) + 5(p) \\ &= 0 + 4p + 12p + 5p \\ &= 21p = 21\left(\frac{1}{7}\right) = 3. \end{aligned}$$

This calculation weights the larger count four more heavily because it has probability $3/7$. An ordinary unweighted average of $0, 2, 4, 5$ would be inappropriate because those counts are not equally likely. The mean describes the average absence count over repeated Saturdays, rather than requiring every Saturday to have exactly that many absentees.

Step 3: Conclusion $\boxed{p = \frac{1}{7}}$, and the mean number of absent students on Saturday is $\boxed{3}$.

Quick Tip to Solve: First normalise the probability row, then multiply corresponding entries of the two rows; do not average the listed counts directly.

OR

29(b). For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test. [3 marks]

Solution:

Concept:

- If M and F are mutually exclusive and exhaustive categories, then $P(D) = P(M)P(D \mid M) + P(F)P(D \mid F)$.
- Here D is distinction in the test; $P(D \mid M)$ and $P(D \mid F)$ are conditional distinction probabilities within the respective categories.

Step 1: Identify the category probabilities A candidate is chosen uniformly at random from all 3000 applicants. Two thirds are female, giving $3000(2/3) = 2000$ female applicants. The remaining 1000 are male. Thus

$$P(F) = \frac{2000}{3000} = \frac{2}{3}, \qquad P(M) = \frac{1000}{3000} = \frac{1}{3}.$$

The given distinction probabilities are $P(D \mid M) = 0.4 = 2/5$ and $P(D \mid F) = 0.35 = 7/20$. The category proportions are necessary because the two groups have different sizes.

Step 2: Combine the two disjoint routes to distinction A randomly selected applicant can have a distinction either as a male applicant or as a female applicant. These events are disjoint, so use total

Quick Tip to Solve: Weight each distinction probability by its own applicant proportion; a simple average of 0.4 and 0.35 ignores the unequal group sizes.

probability:

$$\begin{aligned} P(D) &= P(M)P(D \mid M) + P(F)P(D \mid F) \\ &= \frac{1}{3} \cdot \frac{2}{5} + \frac{2}{3} \cdot \frac{7}{20} \\ &= \frac{2}{15} + \frac{7}{30} = \frac{11}{30}. \end{aligned}$$

As a numerical check, weighting the distinction rates by the group sizes gives $1000(0.4) + 2000(0.35) = 1100$; dividing by 3000 again gives $11/30$. The result lies between 0.35 and 0.4, as a weighted average should.

Step 3: Conclusion The required probability is $\boxed{\frac{11}{30}}$, approximately 0.3667.

Quick Tip to Solve: Weight each distinction probability by its own applicant proportion; a simple average of 0.4 and 0.35 ignores the unequal group sizes.

30(a). Find: $\displaystyle\int \frac{\cos 2x}{(\sin x + \cos x)^2}\, dx$ [3 marks]

Solution:

Concept:

- The double-angle identity factors the numerator: $\cos 2x = \cos^2 x - \sin^2 x = (\cos x - \sin x)(\cos x + \sin x)$.
- For $u = \sin x + \cos x$, the differential is $du = (\cos x - \sin x)\, dx$, and $\int du/u = \ln|u| + C$.

Step 1: Factor the numerator and simplify the integrand Let

$$I = \int \frac{\cos 2x}{(\sin x + \cos x)^2}\, dx.$$

Use the difference-of-squares form of the double-angle identity:

$$\cos 2x = (\cos x - \sin x)(\cos x + \sin x).$$

Since $\cos x + \sin x$ equals $\sin x + \cos x$, cancellation of one factor yields

$$I = \int \frac{\cos x - \sin x}{\sin x + \cos x}\, dx.$$

This cancellation is valid on any interval where the original denominator is nonzero.

Step 2: Use the denominator as the substitution variable Set $u = \sin x + \cos x$. Its derivative is exactly the remaining numerator:

$$du = (\cos x - \sin x)\, dx.$$

Therefore

$$I = \int \frac{du}{u} = \ln|u| + C.$$

Quick Tip to Solve: Factor cos 2x before choosing a substitution; one denominator factor cancels and leaves its derivative in the numerator.

The absolute value is needed because the nonzero quantity u may be negative. Returning to x gives $\ln|\sin x+\cos x|+C$, with an arbitrary integration constant.

Step 3: Check by differentiating On an interval where $\sin x+\cos x\neq 0$,

$$\frac{d}{dx}\ln|\sin x+\cos x|=\frac{\cos x-\sin x}{\sin x+\cos x}=\frac{\cos 2x}{(\sin x+\cos x)^2}.$$

The derivative recovers the original integrand, confirming both the substitution and the sign.

Step 4: Conclusion $\boxed{\int\frac{\cos 2x}{(\sin x+\cos x)^2}\,dx=\ln|\sin x+\cos x|+C}$.

Quick Tip to Solve: Factor cos 2x before choosing a substitution; one denominator factor cancels and leaves its derivative in the numerator.

OR

30(b). Evaluate: $\int_0^{\pi/2}\frac{5\sin x+3\cos x}{\sin x+\cos x}\,dx$ [3 marks]

Solution:

Concept:

- For a continuous function on $[0,a]$, reflection gives $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$.
- Complementary angles satisfy $\sin(\pi/2-x)=\cos x$ and $\cos(\pi/2-x)=\sin x$. Adding the reflected integrand to the original often cancels their common denominator.

Step 1: Define the integral and check continuity Write

$$I=\int_0^{\pi/2}\frac{5\sin x+3\cos x}{\sin x+\cos x}\,dx.$$

Sine and cosine are nonnegative throughout this interval and never vanish simultaneously. Thus $\sin x+\cos x>0$, the quotient is continuous, and the reflection property can be used without introducing an improper integral.

Step 2: Apply the complementary-angle substitution Put $x=\pi/2-t$, with $dx=-dt$. When $x=0$, $t=\pi/2$; when $x=\pi/2$, $t=0$. Reversing the new limits removes the negative sign. Therefore

$$I=\int_0^{\pi/2}\frac{5\cos t+3\sin t}{\cos t+\sin t}\,dt.$$

Since the integration variable is a dummy variable, rename t as x before adding the two expressions.

Step 3: Add the integrands and evaluate Both representations have identical limits and denominators,

Quick Tip to Solve: For limits 0 and pi/2, reflect x to pi/2-x so sine and cosine swap; here the two numerators add to eight times the denominator.

so

$$2I = \int_0^{\pi/2} \frac{8\sin x + 8\cos x}{\sin x + \cos x}\, dx$$
$$= 8\int_0^{\pi/2} dx = 8\,[x]_0^{\pi/2} = 4\pi.$$

Cancelling the denominator is valid everywhere on the interval because it is strictly positive. Finally divide this equality by 2 to obtain the desired integral.

Step 4: Conclusion The value of the definite integral is $\boxed{2\pi}$.

Quick Tip to Solve: For limits 0 and pi/2, reflect x to pi/2-x so sine and cosine swap; here the two numerators add to eight times the denominator.

31. Sketch the graph of $y = |x + 3|$ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration. [3 marks]

Solution:

Concept:

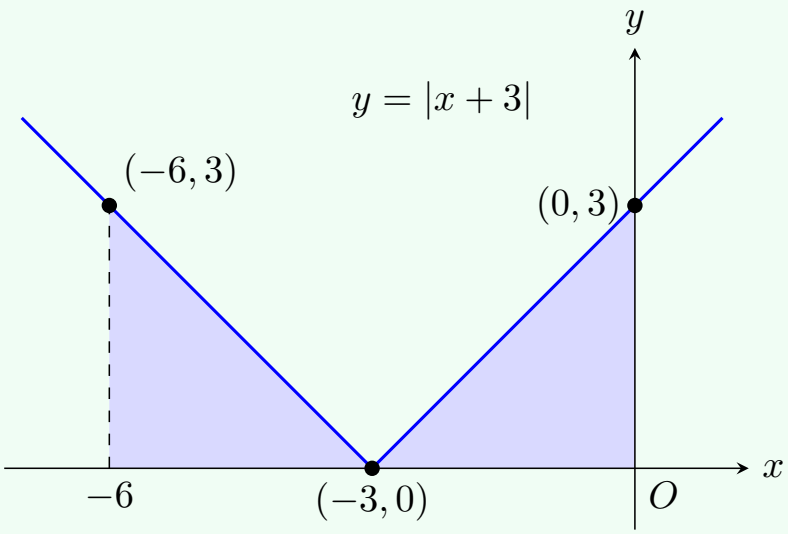
- The modulus function satisfies $|u| = u$ for $u \geq 0$, and $|u| = -u$ for $u < 0$. Its graph consists of two straight-line rays meeting where $u = 0$.
- The area between a nonnegative curve $y = f(x)$ and the x-axis on $[a, b]$ is $A = \int_a^b f(x)\, dx$. Split the integral where the formula for the curve changes.

Step 1: Express the curve piecewise and sketch it The expression $x+3$ changes sign at $x = -3$. Hence

$$y = |x + 3| = \begin{cases} -x - 3, & x < -3, \\ x + 3, & x \geq -3. \end{cases}$$

The left ray has slope -1, and the right ray has slope 1. They meet at the vertex $(-3, 0)$, forming a V-shaped graph above the x-axis. At the specified boundaries, $x = -6$ and $x = 0$, the ordinate is 3. The shaded region lies below these line segments and above the x-axis, bounded by the two vertical lines.

Quick Tip to Solve: Split the modulus integral at x=-3, its zero; both halves are nonnegative and have equal areas because the boundaries are equally far from the vertex.



Graph of $y = |x + 3|$ with the required region shaded.

Step 2: Set up the area integral on both sides of the vertex Since $y \geq 0$, no signed-area correction is needed. However, the modulus must be removed using the appropriate formula on each interval:

$$A = \int_{-6}^{0} |x + 3|\, dx = \int_{-6}^{-3} (-x - 3)\, dx + \int_{-3}^{0} (x + 3)\, dx.$$

Using the expression $x + 3$ on the entire interval would incorrectly subtract the left-hand area, because $x + 3$ is negative there.

Step 3: Evaluate the two integrals The antiderivatives give

$$\begin{aligned} A &= \left[-\frac{x^2}{2} - 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0} \\ &= \left(\frac{9}{2} - 0\right) + \left(0 + \frac{9}{2}\right) = 9. \end{aligned}$$

Each side therefore contributes $9/2$ square units. As a check on the integration, the two shaded triangles each have base 3 and height 3.

Step 4: Conclusion The graph is a V with vertex $(-3, 0)$, and the required area is $\boxed{9 \text{ square units}}$.

Quick Tip to Solve: Split the modulus integral at x=-3, its zero; both halves are nonnegative and have equal areas because the boundaries are equally far from the vertex.

Section D (20 marks)

*This section comprises of **4** Long Answer (LA) type questions of **5** marks each.*

32(a). **Differentiate** $\tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right)$ **w.r.t.** $\cos^{-1}(2x\sqrt{1-x^2})$, $x \in \left(\dfrac{1}{\sqrt{2}}, 1\right)$. [5 marks]

Solution:

Concept:

- For functions $u = u(x)$ and $v = v(x)$, $du/dv = (du/dx)/(dv/dx)$, provided $dv/dx \neq 0$.
- The substitution $x = \cos\theta$ simplifies the radical. Principal ranges must be checked: $\tan^{-1}$ takes values in $(-\pi/2, \pi/2)$, and $\cos^{-1}$ in $[0, \pi]$.

Step 1: Translate the domain into an angle interval Let $u = \tan^{-1}(\sqrt{1-x^2}/x)$ and $v = \cos^{-1}(2x\sqrt{1-x^2})$. Put $x = \cos\theta$. The given interval $1/\sqrt{2} < x < 1$ implies $0 < \theta < \pi/4$, because cosine is decreasing on $(0, \pi/2)$. Hence $\sin\theta > 0$, and the positive square root is $\sqrt{1-x^2} = \sin\theta$.

Step 2: Simplify the first function We obtain

$$u = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = \tan^{-1}(\tan\theta) = \theta.$$

This cancellation is justified because θ is inside the principal range of inverse tangent. Thus $du/d\theta = 1$. Checking this range avoids silently replacing an inverse function by the wrong branch.

Step 3: Simplify the second function with the correct branch The double-angle identity gives

$$v = \cos^{-1}(\sin 2\theta) = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right).$$

Since $0 < \theta < \pi/4$, the angle $\pi/2 - 2\theta$ lies in $(0, \pi/2)$, a subset of the principal range $[0, \pi]$ of inverse cosine. Therefore $v = \pi/2 - 2\theta$, and $dv/d\theta = -2$. The negative sign follows from this decreasing linear expression and must be retained.

Step 4: Apply differentiation with respect to another function Since the denominator is nonzero, the chain rule yields

$$\frac{du}{dv} = \frac{du/d\theta}{dv/d\theta} = \frac{1}{-2}.$$

As an independent check, eliminating θ from the simplified expressions gives $v = \pi/2 - 2u$. Its differential relation is $dv = -2\,du$, agreeing with the quotient above. All the simplifications hold throughout the open interval in the question; neither endpoint is needed.

Step 5: Conclusion The derivative of the first given function with respect to the second is $\boxed{-\dfrac{1}{2}}$.

Quick Tip to Solve: Use $x = \cos\theta$ and translate the interval before cancelling inverse trigonometric functions; the branch determines the sign.

OR

32(b). Find $\frac{dy}{dx}$, if $y = x^{\tan x} + \frac{\sqrt{x^2+1}}{2}$. [5 marks]

Solution:

Concept:

- A variable base raised to a variable exponent is differentiated using logarithmic differentiation: for $z = x^{g(x)}$, write $\log z = g(x) \log x$, with $x > 0$.
- The product rule gives $(fg)' = f'g + fg'$, and the chain rule gives $\frac{d}{dx}\sqrt{h(x)} = h'(x)/(2\sqrt{h(x)})$ where the square root is positive.
- Here $\frac{d}{dx}\tan x = \sec^2 x$ and $\frac{d}{dx}\log x = 1/x$. A sum is differentiated term by term.

Step 1: Separate the variable-power term Write $y = z + w$, where $z = x^{\tan x}$ and $w = \frac{1}{2}\sqrt{x^2+1}$. For the real logarithmic calculation, take $x > 0$, at points where $\tan x$ is defined, namely $\cos x \neq 0$. The constant-exponent power rule is insufficient because both base and exponent vary.

Step 2: Take logarithms and apply the product rule Since $z > 0$, taking natural logarithms yields

$$\log z = (\tan x) \log x.$$

Differentiating both sides with respect to x, including both factors on the right, gives

$$\frac{1}{z}\frac{dz}{dx} = \sec^2 x \log x + \frac{\tan x}{x}.$$

Both product-rule terms are essential: one differentiates the exponent and the other differentiates the logarithm of the base.

Step 3: Restore the original power and differentiate the radical Substituting $z = x^{\tan x}$, we obtain

$$\frac{dz}{dx} = x^{\tan x}\left(\sec^2 x \log x + \frac{\tan x}{x}\right).$$

Independently, the chain rule applied to $w = \frac{1}{2}(x^2+1)^{1/2}$ gives

$$\frac{dw}{dx} = \frac{1}{2}\cdot\frac{1}{2}(x^2+1)^{-1/2}\cdot 2x = \frac{x}{2\sqrt{x^2+1}}.$$

The factor $1/2$ outside the square root remains throughout this differentiation. Also $x^2 + 1 > 0$, so the radical's denominator never vanishes.

Step 4: Combine the derivatives of the two terms By the sum rule, $dy/dx = dz/dx + dw/dx$. Add the derivative of the variable-power term to the derivative of the radical.

Step 5: Conclusion

$$\boxed{\frac{dy}{dx} = x^{\tan x}\left(\sec^2 x \log x + \frac{\tan x}{x}\right) + \frac{x}{2\sqrt{x^2+1}}.}$$

Quick Tip to Solve: For $x^{\tan x}$, take logarithms first and differentiate both factors of $(\tan x)\log x$; keep the radical's outside factor $1/2$.

33. Find the absolute maximum and absolute minimum of function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$. [5 marks]

Solution:

Concept:

- A polynomial is continuous everywhere. The extreme value theorem guarantees that a continuous function on a closed bounded interval attains an absolute maximum and an absolute minimum.
- An interior extremum of a differentiable function must satisfy $f'(x) = 0$. To find absolute extrema, compare the original function at these stationary points and at both endpoints.

Step 1: Identify every possible location of an extremum Here $f(x) = 2x^3 - 15x^2 + 36x + 1$ is continuous on $[1, 5]$ and differentiable at every interior point. Its absolute extrema therefore occur either at the endpoints $x = 1, 5$ or at an interior point where its derivative vanishes. There are no points where the derivative is undefined. Endpoint values are essential because the condition $f'(x) = 0$ applies only to an interior extremum, and need not hold at a boundary of the interval.

Step 2: Find the interior stationary points Differentiating the polynomial term by term yields

$$\begin{aligned} f'(x) &= 6x^2 - 30x + 36 \\ &= 6(x^2 - 5x + 6) \\ &= 6(x - 2)(x - 3). \end{aligned}$$

Thus $f'(x) = 0$ gives $x = 2$ and $x = 3$, both inside the specified interval. The derivative is positive on $(1, 2)$, negative on $(2, 3)$ and positive on $(3, 5)$. The function therefore increases, decreases, and then increases. Consequently, 2 is a local maximum point and 3 is a local minimum point; these classifications alone do not determine the absolute extrema.

Step 3: Evaluate and compare all candidate values Substitute each candidate into the original function:

$$\begin{aligned} f(1) &= 2 - 15 + 36 + 1 = 24, \\ f(2) &= 16 - 60 + 72 + 1 = 29, \\ f(3) &= 54 - 135 + 108 + 1 = 28, \\ f(5) &= 250 - 375 + 180 + 1 = 56. \end{aligned}$$

This list contains both endpoints and every interior stationary point, so it covers every possible location of an absolute extremum. Comparing the values gives $24 < 28 < 29 < 56$. In particular, the local maximum value at 2 is smaller than the right-endpoint value, while the local minimum value at 3 is larger than the left-endpoint value. This explains why merely using the second derivative at the stationary points would not suffice.

Step 4: Conclusion The absolute maximum is $\boxed{56}$, attained at $\boxed{x = 5}$, and the absolute minimum is $\boxed{24}$, attained at $\boxed{x = 1}$, on $[1, 5]$.

Quick Tip to Solve: Put both endpoints and every interior root of the derivative in one candidate list, then evaluate the original function at each point.

34. A school wants to allocate students into three clubs: Sports, Music and Drama, under following conditions:

The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club.The number of students in Music club should be 20 more than half the number of students in Sports club.The total number of students to be allocated in all three clubs are 180.

Find the number of students allocated to different clubs, using matrix method. [5 marks]

Solution:

Concept:

- Translate each allocation condition into a linear equation using one variable for each club. Write the system as $AX = B$, where A is the coefficient matrix, X the column of unknown numbers and B the constant column.
- Elementary row operations on the augmented matrix $[A \mid B]$ preserve the solution set. Reducing it to triangular form allows the unknowns to be found by back-substitution, which is a matrix method of solving the system.

Step 1: Translate the club conditions into equations Let x, y, z be the numbers of students allocated to Sports, Music and Drama, respectively. Sports has as many students as the other two clubs together, so $x = y + z$. Music has 20 more students than half the Sports allocation, so $y = x/2 + 20$. The total allocation is 180. Thus the three equations are

$$\begin{aligned} x - y - z &= 0, \\ -x + 2y &= 40, \\ x + y + z &= 180. \end{aligned}$$

Multiplying the Music equation by 2 avoids fractions without changing its meaning.

Step 2: Express the system in matrix form Keeping the unknowns in the same order, Sports, Music and Drama, in each row, the system is

$$\underbrace{\begin{pmatrix} 1 & -1 & -1 \\ -1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 0 \\ 40 \\ 180 \end{pmatrix}}_{B}.$$

A zero in the second row records that the Music condition does not involve Drama directly. We solve this matrix equation by elementary row operations on its augmented matrix.

Step 3: Reduce the augmented matrix to triangular form Starting from $[A \mid B]$, apply $R_2 \leftarrow R_2 + R_1$ and $R_3 \leftarrow R_3 - R_1$:

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ -1 & 2 & 0 & 40 \\ 1 & 1 & 1 & 180 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 40 \\ 0 & 2 & 2 & 180 \end{array}\right].$$

Quick Tip to Solve: Fix the variable order before writing the coefficient matrix, and carry every row operation through the constant column to avoid changing the system.

Next, apply $R_3 \leftarrow R_3 - 2R_2$:

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 40 \\ 0 & 0 & 4 & 100 \end{array}\right].$$

Every operation was applied to the constant column as well as the coefficients. All three diagonal coefficients are nonzero, so this triangular system determines a unique allocation.

Step 4: Back-substitute and check the allocation The last row gives $4z = 100$, hence $z = 25$. The second row gives $y - z = 40$, so $y = 40 + 25 = 65$. The first row gives $x - y - z = 0$, hence $x = 65 + 25 = 90$. These are nonnegative whole numbers as required for student counts. To check the original conditions: $90 = 65 + 25$, $65 = 90/2 + 20$, and $90 + 65 + 25 = 180$. Thus all three allocation requirements are satisfied simultaneously.

Step 5: Conclusion The school should allocate $\boxed{90}$ students to Sports, $\boxed{65}$ students to Music and $\boxed{25}$ students to Drama.

Quick Tip to Solve: Fix the variable order before writing the coefficient matrix, and carry every row operation through the constant column to avoid changing the system.

35(a). Find the image A' of the point A(1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A'. [5 marks]

Solution:

Concept:

- Reflection in a line makes the perpendicular foot H the midpoint of AA'. Hence $\vec{OA'} = 2\vec{OH} - \vec{OA}$.
- The perpendicular-foot condition is $\overrightarrow{AH} \cdot \mathbf{d} = 0$, where $\mathbf{d}$ is the line's direction vector. The difference of two points supplies a direction vector for their joining line.

Step 1: Parameterise the given line Put the common ratio equal to t. A general point is $H = (t, 1 + 2t, 2 + 3t)$, and the direction vector is $\mathbf{d} = (1, 2, 3)$. We choose the value of t that makes AH perpendicular to the given line. This point is the centre of reflection for this particular point A, since the original point and its image lie at equal distances on opposite sides of H.

Step 2: Find the perpendicular foot Since $A = (1, 6, 3)$, the vector $\overrightarrow{AH} = (t - 1, 2t - 5, 3t - 1)$. Therefore

$$\begin{aligned} (t-1) + 2(2t-5) + 3(3t-1) &= 0, \\ 14t - 14 &= 0, \\ t &= 1. \end{aligned}$$

Thus $H = (1, 3, 5)$. To check the construction, $\overrightarrow{AH} = (0, -3, 2)$ and its dot product with $(1, 2, 3)$ is $-6 + 6 = 0$. Moreover H has common ratio 1 in the given line, so both conditions defining the perpendicular foot hold.

Quick Tip to Solve: Obtain the perpendicular foot with a dot product, then use twice the foot minus the original point to reflect it.

Step 3: Reflect the point and form the joining line By the midpoint relation,

$$\begin{aligned} A' &= 2H - A \\ &= 2(1,3,5) - (1,6,3) \\ &= (1,0,7). \end{aligned}$$

The vector $A' - A = (0,-6,4)$ is parallel to $(0,-3,2)$. The joining line through A therefore has parametric equations $x = 1,\ y = 6 - 3s,\ z = 3 + 2s$, with $s \in \mathbb{R}$. The values $s = 0, 1, 2$ give A, H, A', respectively, confirming that the foot bisects the joining segment. The zero first direction component requires $x = 1$ to be written separately; a symmetric fraction with denominator zero would be invalid.

Step 4: Conclusion The image is $\boxed{A' = (1,0,7)}$. The joining line is $\boxed{x = 1, \quad \frac{y-6}{-3} = \frac{z-3}{2}}$.

Quick Tip to Solve: Obtain the perpendicular foot with a dot product, then use twice the foot minus the original point to reflect it.

OR

35(b). Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point Q(2, 4, −1) is 7 units. Also, find the equation of line joining P and Q. [5 marks]

Solution:

Concept:

- Setting the common ratio in a symmetric line equal to a real parameter gives the coordinates of every point on that line.
- The squared distance between (x,y,z) and (a,b,c) is $(x-a)^2 + (y-b)^2 + (z-c)^2$. A direction vector for the line joining two points is their coordinate difference.

Step 1: Express the coordinates of the unknown point Let the common ratio be t. The given line yields

$$x = t - 5, \qquad y = 4t - 3, \qquad z = 6 - 9t.$$

Thus $P = (t-5, 4t-3, 6-9t)$. This parameterisation preserves the negative sign of the third direction component. Since $Q = (2,4,-1)$, the coordinate differences $P - Q$ are $(t-7, 4t-7, 7-9t)$.

Step 2: Impose the distance condition and solve The required distance is 7 units, so its square is 49. Applying the three-dimensional distance formula gives

$$\begin{aligned} (t-7)^2 + (4t-7)^2 + (7-9t)^2 &= 49, \\ t^2 - 14t + 49 + 16t^2 - 56t + 49 & \\ + 81t^2 - 126t + 49 &= 49, \\ 98t^2 - 196t + 98 &= 0, \\ 98(t-1)^2 &= 0. \end{aligned}$$

Quick Tip to Solve: Parameterise the line first and square the distance equation; a repeated parameter root means only one point satisfies the distance.

Hence $t = 1$ is a repeated root; there is a single required point rather than two distinct points. Substitution gives $P = (-4, 1, -3)$. Directly checking the distance, $PQ = \sqrt{6^2 + 3^2 + 2^2} = \sqrt{49} = 7$, as required.

Step 3: Form the line joining the two points The vector from P to Q is

$$\overrightarrow{PQ} = (2 + 4, 4 - 1, -1 + 3) = (6, 3, 2).$$

Consequently the joining line through P has coordinates $(x, y, z) = (-4, 1, -3) + s(6, 3, 2)$, for $s \in \mathbb{R}$. Eliminating s gives the symmetric equation. At $s = 1$ the point is Q, so the equation contains both required points. As an additional geometric check, $(6, 3, 2) \cdot (1, 4, -9) = 6 + 12 - 18 = 0$. Thus PQ is perpendicular to the original line, explaining why the distance condition produces a repeated root at its nearest point to Q.

Step 4: Conclusion The required point is $\boxed{P = (-4, 1, -3)}$. The joining line is $\boxed{\frac{x+4}{6} = \frac{y-1}{3} = \frac{z+3}{2}}$.

Quick Tip to Solve: Parameterise the line first and square the distance equation; a repeated parameter root means only one point satisfies the distance.

Section E (12 marks)

*This section comprises of **3** case study/passage based questions of **4** marks each.*

36(i). A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively.

Based on the above information, answer the following: What is the probability that a customer after availing the loan will default on the loan repayment? [2 marks]

Solution:

Concept:

- Let F, L, V denote fixed, floating and variable interest rates, respectively, and let D denote default on repayment. These three loan categories are mutually exclusive and exhaustive.
- The law of total probability gives $P(D) = P(F)P(D \mid F) + P(L)P(D \mid L) + P(V)P(D \mid V)$.

Step 1: Identify the category and conditional probabilities The category probabilities are $P(F) = 0.10$, $P(L) = 0.20$, and $P(V) = 0.70$. The corresponding default probabilities are $P(D \mid F) = 0.05$, $P(D \mid L) = 0.03$, and $P(D \mid V) = 0.01$. These conditional probabilities cannot simply be added, since each applies only within its own loan category.

Step 2: Weight each default rate by its loan category Each product is the probability of both choosing

Quick Tip to Solve: Convert both sets of percentages to decimals before multiplying; weight each default rate by the proportion of customers in that category.

that category and defaulting. Hence

$$\begin{aligned} P(D) &= (0.10)(0.05) + (0.20)(0.03) + (0.70)(0.01) \\ &= 0.005 + 0.006 + 0.007 \\ &= 0.018. \end{aligned}$$

The sum accounts for every possible interest category exactly once.

Step 3: Conclusion The probability of default is $\boxed{0.018 = \frac{9}{500}}$, or 1.8%.

Quick Tip to Solve: Convert both sets of percentages to decimals before multiplying; weight each default rate by the proportion of customers in that category.

36(ii). (Continuing from the bank loan case study above.) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest?

[2 marks]

Solution:

Concept:

- Let V be the event of taking a variable-rate loan and D the event of defaulting. The required probability is $P(V \mid D)$, since default is already known.
- Bayes' theorem gives $P(V \mid D) = \dfrac{P(V)P(D \mid V)}{P(D)}$, where the denominator includes defaults from all loan categories.

Step 1: Calculate the total probability of default Using the mutually exclusive fixed, floating and variable categories,

$$\begin{aligned} P(D) &= (0.10)(0.05) + (0.20)(0.03) + (0.70)(0.01) \\ &= 0.005 + 0.006 + 0.007 = 0.018. \end{aligned}$$

This denominator represents all customers who default, irrespective of their type of interest.

Step 2: Condition on the observed default The probability of taking a variable-rate loan and defaulting is $P(V \cap D) = (0.70)(0.01) = 0.007$. Therefore,

$$P(V \mid D) = \frac{P(V \cap D)}{P(D)} = \frac{0.007}{0.018} = \frac{7}{18}.$$

Notice that 0.01 is the probability of default given a variable loan; it is the reverse conditional probability and is not the quantity asked for.

Step 3: Conclusion The probability that the defaulting customer availed a variable-rate loan is $\boxed{\frac{7}{18}}$.

Quick Tip to Solve: When the question says a customer has defaulted, restrict the denominator to all defaults and the numerator to variable-rate defaults.

37(i). A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections. Let the length of the side perpendicular to the partition be x metres and parallel to the partition be y metres.

Based on this information, write the equation for the total boundary material used in the boundary and parallel to the partition, in terms of x and y. [1 mark]

Solution:

Concept:

- A rectangle with side lengths x and y has outer perimeter $2x + 2y$.
- The partition is parallel to the side of length y, so its length is also y. Its material must be counted in addition to the outer boundary.

Step 1: Count the outer boundary There are two outer sides perpendicular to the partition, each of length x, and two outer sides parallel to it, each of length y. Thus the outer boundary uses $2x + 2y$ metres.

Step 2: Include the internal partition The one internal partition contributes a further y metres. The total material used is therefore $(2x + 2y) + y = 2x + 3y$ metres. Equating this total to the given available material supplies the constraint for the design.

Step 3: Conclusion The required equation is $\boxed{2x + 3y = 300}$, with $x, y > 0$.

Quick Tip to Solve: Count three lengths parallel to the partition: two outside edges and the single internal divider.

37(ii). (Continuing from the solar panel case study above.) Write the area of the solar panel as a function of x. [1 mark]

Solution:

Concept:

- The rectangular area is $A = xy$; an internal partition does not subtract any area in this model.
- The material constraint is $2x + 3y = 300$, because the partition has length y.

Step 1: Express the parallel side in terms of x From $2x + 3y = 300$, we obtain

$$y = \frac{300 - 2x}{3}.$$

Positive side lengths require $x > 0$ and $300 - 2x > 0$, so the physically meaningful domain is $0 < x < 150$.

Step 2: Substitute into the area formula Multiply the perpendicular side by the parallel side:

$$A(x) = x\left(\frac{300 - 2x}{3}\right) = 100x - \frac{2}{3}x^2.$$

This expresses the whole enclosed roof area using only the variable x, while respecting the material

Quick Tip to Solve: Eliminate y using the material constraint before differentiating; the partition changes the constraint but the area remains xy.

used by the divider.

Step 3: Conclusion $\boxed{A(x) = 100x - \frac{2}{3}x^2 \text{ m}^2}$, for $0 < x < 150$ when x is measured in metres.

Quick Tip to Solve: Eliminate y using the material constraint before differentiating; the partition changes the constraint but the area remains xy.

37(iii)(a). Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area. [2 marks]

Solution:

Concept:

- The total material gives $2x + 3y = 300$, so $A(x) = xy = 100x - \frac{2}{3}x^2$, with $0 < x < 150$.
- A differentiable function has an interior critical point where $A\prime(x) = 0$. If $A\prime\prime(x) < 0$ there, the point is a local maximum.

Step 1: Find the critical point Differentiate the area function:

$$A\prime(x) = 100 - \frac{4}{3}x.$$

Setting the derivative equal to zero gives

$$100 - \frac{4}{3}x = 0 \quad \Longrightarrow \quad x = 75.$$

The derivative exists throughout the domain, so this is the only critical value. It lies within $0 < x < 150$. The corresponding parallel side is $y = (300 - 2 \cdot 75)/3 = 50$ metres.

Step 2: Apply the second derivative test and evaluate the area The second derivative is

$$A\prime\prime(x) = -\frac{4}{3} < 0.$$

Therefore the critical point gives a maximum. In fact the area function is strictly concave throughout its domain, so this maximum is global. Its value is

$$A(75) = 100(75) - \frac{2}{3}(75)^2 = 7500 - 3750 = 3750 \text{ m}^2.$$

The dimensions also satisfy the material check $2(75) + 3(50) = 300$.

Step 3: Conclusion The only critical point is $x = 75$ m, with $y = 50$ m. The maximum area is $\boxed{3750 \text{ m}^2}$.

Quick Tip to Solve: After finding x from the zero derivative, check the constant negative second derivative and recover y from 2x+3y=300.

OR

37(iii)(b). Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition. [2 marks]

Solution:

Concept:

- Including the partition gives the constraint $2x + 3y = 300$, so $A(x) = 100x - \frac{2}{3}x^2$ for $0 < x < 150$.
- The first derivative test identifies a maximum when the derivative changes from positive to negative at a critical value.

Step 1: Find the stationary value The derivative of the area is

$$A\prime(x) = 100 - \frac{4}{3}x = \frac{4}{3}(75 - x).$$

Thus $A\prime(x) = 0$ at $x = 75$, which belongs to the physical domain. The derivative is defined everywhere in that domain, so there are no other interior critical values to test.

Step 2: Test the derivative sign and calculate the area For $0 < x < 75$, $75 - x > 0$, so $A\prime(x) > 0$ and the area increases. For $75 < x < 150$, $75 - x < 0$, so $A\prime(x) < 0$ and the area decreases. This change from increasing to decreasing proves that $x = 75$ gives the global maximum on the physical domain. At this value,

$$y = \frac{300 - 2(75)}{3} = 50 \text{ m}, \qquad A = 75 \cdot 50 = 3750 \text{ m}^2.$$

Both outer boundary and partition are included, as verified by $2(75) + 3(50) = 300$.

Step 3: Conclusion The maximum enclosed area is $\boxed{3750 \text{ m}^2}$, attained with $x = 75$ m and $y = 50$ m.

Quick Tip to Solve: Factor the derivative as (4/3)(75-x); its sign on either side of 75 immediately gives the required first derivative test.

38(i). A class teacher is keen to assess the learning of the concept of "relations" taught to her students. She writes the following five relations, each defined on the set A = {1, 2, 3}:

R_1 = {(2,3), (3,2)}

R_2 = {(1,2), (1,3), (3,2)}

R_3 = {(1,2), (2,1), (1,1)}

R_4 = {(1,1), (1,2), (3,3), (2,2)}

R_5 = {(1,1), (1,2), (3,3), (2,2), (2,1), (2,3), (3,2)}

Students are asked to answer the following questions about the above relations: Identify the relation which is reflexive and transitive but not symmetric. [1 mark]

Solution:

Concept:

- A relation on A is reflexive if (a, a) belongs to it for every $a \in A$.
- Symmetry requires (b, a) whenever (a, b) is present; transitivity requires (a, c) whenever (a, b) and

Quick Tip to Solve: After checking the diagonal pairs, test the reverse of the only non-diagonal pair in R_4.

(b, c) are present.

Step 1: Check reflexivity and transitivity For $R_4 = \{(1,1), (1,2), (3,3), (2,2)\}$, all three diagonal pairs are present, so it is reflexive on $A = \{1, 2, 3\}$. Its only non-diagonal pair is $(1, 2)$. Composing this pair with the available diagonal pairs gives $(1, 2)$ again; composing diagonal pairs gives the same diagonal pair. Thus every required composite pair is already in R_4, and it is transitive.

Step 2: Check failure of symmetry The pair $(1, 2)$ belongs to R_4, but its reverse $(2, 1)$ does not. Therefore R_4 is not symmetric. The other reflexive candidate, R_5, is symmetric and so does not meet the final condition.

Step 3: Conclusion The required relation is $\boxed{R_4}$: it is reflexive and transitive but not symmetric.

Quick Tip to Solve: After checking the diagonal pairs, test the reverse of the only non-diagonal pair in R_4.

38(ii). (Continuing from the relations case study above.) Identify the relation which is reflexive and symmetric but not transitive. [1 mark]

Solution:

Concept:

- A relation on A is reflexive if (a, a) belongs to it for every $a \in A$.
- Symmetry requires (b, a) whenever (a, b) is present; transitivity requires (a, c) whenever (a, b) and (b, c) are present.

Step 1: Check reflexivity and symmetry The relation R_5 contains $(1,1), (2,2), (3,3)$, so it is reflexive on A. Its non-diagonal pairs occur as reverse pairs: $(1,2), (2,1)$ and $(2,3), (3,2)$. Each diagonal pair reverses to itself. Consequently every pair has its reverse in R_5, making the relation symmetric.

Step 2: Find a counterexample to transitivity Both $(1, 2)$ and $(2, 3)$ belong to R_5. Transitivity would therefore require $(1, 3)$, but this pair is absent. This single failed composition proves that R_5 is not transitive. Although R_4 is reflexive, it lacks symmetry.

Step 3: Conclusion The required relation is $\boxed{R_5}$: it is reflexive and symmetric but not transitive.

Quick Tip to Solve: Use the chain $1 \to 2 \to 3$; the missing pair $(1, 3)$ immediately disproves transitivity.

38(iii)(a). Identify the relations which are symmetric but neither reflexive nor transitive. [2 marks]

Solution:

Concept:

- A relation on A is reflexive if (a, a) belongs to it for every $a \in A$.
- Symmetry requires (b, a) whenever (a, b) is present; transitivity requires (a, c) whenever (a, b) and (b, c) are present.

Step 1: Test the relation R1 In $R_1 = \{(2,3), (3,2)\}$, each pair has its reverse, so R_1 is symmetric. It is not reflexive because the diagonal pairs $(1,1), (2,2), (3,3)$ are absent. It is also not transitive: $(2, 3)$

Quick Tip to Solve: For a symmetric relation containing (a, b) and (b, a), check whether (a, a) and (b, b) exist.

and $(3,2)$ are present, but their required composite $(2,2)$ is absent.

Step 2: Test the relation R3 In $R_3 = \{(1,2),(2,1),(1,1)\}$, the first two pairs reverse one another and $(1,1)$ reverses to itself. Thus R_3 is symmetric. It is not reflexive on A, since $(2,2)$ and $(3,3)$ are missing. Further, $(2,1)$ and $(1,2)$ are present, while the required pair $(2,2)$ is missing, so it is not transitive.

Step 3: Exclude the remaining relations R_2 and R_4 are not symmetric, since they contain $(1,2)$ without $(2,1)$. The relation R_5 is reflexive, so it fails the requirement of being neither reflexive nor transitive.

Step 4: Conclusion The required relations are $\boxed{R_1 \text{ and } R_3}$. Both are symmetric, neither is reflexive, and neither is transitive.

Quick Tip to Solve: For a symmetric relation containing (a,b) and (b,a), check whether (a,a) and (b,b) exist.

OR

38(iii)(b). State the pairs which need to be added to relation R_2 to make it an equivalence relation.

[2 marks]

Solution:

Concept:

- An equivalence relation must be reflexive, symmetric and transitive on the whole set A.
- If two elements are connected by an equivalence relation, symmetry and transitivity put them in the same equivalence class; all ordered pairs within that class must occur.

Step 1: Add the pairs required by reflexivity and symmetry The given relation is $R_2 = \{(1,2),(1,3),(3,2)\}$. Reflexivity requires adding $(1,1),(2,2),(3,3)$. Symmetry then requires the reverses of the original pairs, namely $(2,1),(3,1),(2,3)$. These six pairs are all absent from the given relation.

Step 2: Verify that the completed relation is transitive After these additions, the relation contains every pair of $A \times A$, where $A = \{1,2,3\}$. Therefore, whenever (a,b) and (b,c) occur, (a,c) also occurs automatically. It is reflexive, symmetric and transitive, hence an equivalence relation. All six additions are necessary: the diagonal pairs are forced by reflexivity and the other three by symmetry, so none may be omitted.

Step 3: Conclusion The pairs to be added are

$$\boxed{\{(1,1),(2,2),(3,3),(2,1),(3,1),(2,3)\}}.$$

The resulting equivalence relation is $A \times A$.

Quick Tip to Solve: List missing diagonal pairs first, then missing reverse pairs, and finally verify transitivity of the completed set.