

CBSE Class 12 Mathematics Question Paper 2025 with Solution PDF-Set 65-2-2

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 80

General Instructions

- (i) This Question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.
- (iii) In **Section A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) type and Question Number **19 & 20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.
- (vii) In **Section E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, **2** questions in Section – **D** and **2** questions in Section – **E**.
- (ix) Use of calculator is **NOT** allowed.

Section A (20 marks)

(This section comprises of 20 multiple choice questions (MCQs) of 1 mark each.)

1. The values of x for which the angle between the vectors $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$ is obtuse, is [1 mark]

- (A) 0 or $\frac{1}{2}$
(B) $x > \frac{1}{2}$
(C) $\left(0, \frac{1}{2}\right)$
(D) $\left[0, \frac{1}{2}\right]$

Correct Answer: (C) $\left(0, \frac{1}{2}\right)$

Solution:

Concept:

- For nonzero vectors, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$; an obtuse angle requires $\vec{a} \cdot \vec{b} < 0$.
- A product with two distinct real roots changes sign at each root; strict inequalities exclude the roots.

Step 1: Calculate the scalar product Both vectors are nonzero for every real x , since their fixed components include 1 and 7. Therefore their angle is defined. Multiplying corresponding components gives

$$\vec{a} \cdot \vec{b} = (2x^2)(7) + (4x)(-2) + (1)(x) = 14x^2 - 7x = 7x(2x - 1).$$

Step 2: Use the strict sign condition The roots are 0 and $\frac{1}{2}$. Between them, $x > 0$ and $2x - 1 < 0$, so the product is negative. Outside them it is positive. At either endpoint the vectors are perpendicular, excluding (A) and the closed interval (D); (B) gives a positive scalar product.

Step 3: Conclusion The required values satisfy $0 < x < \frac{1}{2}$. Hence, the correct option is (C).

Quick Tip to Solve: For an obtuse angle, factor the dot product and select its negative interval; never include a zero of the dot product.

2. If a line makes angles of $\frac{3\pi}{4}$, $\frac{\pi}{3}$ and θ with the positive directions of x , y and z -axis respectively, then θ is [1 mark]

- (A) $-\frac{\pi}{3}$ only
(B) $\frac{\pi}{3}$ only
(C) $\frac{\pi}{6}$
(D) $\pm\frac{\pi}{3}$

Correct Answer: The valid angles are $\pi/3$ and $2\pi/3$; none of the printed options states both correctly.

Solution:

Concept:

- The direction cosines of a line satisfy $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$.
- An angle with a positive coordinate axis lies in $[0, \pi]$; solving for a squared cosine can give both an acute and an obtuse angle.

Step 1: Find the remaining direction cosine Substitute the two specified angles into the direction-cosine identity:

$$\cos^2 \theta = 1 - \cos^2 \frac{3\pi}{4} - \cos^2 \frac{\pi}{3} = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.$$

Thus $\cos \theta = \pm\frac{1}{2}$; neither sign is excluded by the question.

Step 2: Compare both valid angles with the options In $[0, \pi]$, these values give $\theta = \pi/3$ or $2\pi/3$. Option (B) says $\pi/3$ only and omits the valid obtuse angle. Options (A) and (D) include the negative angle $-\pi/3$, which is not a direction angle with a positive axis. Option (C) has $\cos^2(\pi/6) = 3/4$, contrary to the identity.

Step 3: Conclusion θ may be $\pi/3$ or $2\pi/3$. No printed option gives the complete correct answer.

Quick Tip to Solve: After obtaining a squared direction cosine, keep both signs and find the angles in the interval from zero to π .

3. $\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to [1 mark]

- (A) $\tan x + \cot x + C$
(B) $(\tan x + \cot x)^2 + C$
(C) $\tan x - \cot x + C$
(D) $(\tan x - \cot x)^2 + C$

Correct Answer: (C) $\tan x - \cot x + C$

Solution:

Concept:

- Use $\sin^2 x + \cos^2 x = 1$ to split the integrand into standard trigonometric terms.
- The standard antiderivatives are $\int \sec^2 x \, dx = \tan x + C$ and $\int \csc^2 x \, dx = -\cot x + C$.

Quick Tip to Solve: Split the reciprocal of $\sin^2 x \cos^2 x$ using the identity for their sum, and remember the minus sign in the integral of cosecant squared.

Step 1: Split the reciprocal product On any interval where $\sin x \cos x \neq 0$, insert the Pythagorean identity in the numerator:

$$\frac{1}{\sin^2 x \cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} = \sec^2 x + \csc^2 x.$$

Step 2: Integrate and check the alternatives Integrating the two terms gives

$$\int (\sec^2 x + \csc^2 x) dx = \tan x - \cot x + C.$$

Differentiating (A) gives $\sec^2 x - \csc^2 x$, with the wrong sign. Squaring in (B) or (D) introduces an extra factor by the chain rule, so those expressions do not have the required derivative.

Step 3: Conclusion The integral is $\tan x - \cot x + C$. Hence, the correct option is (C).

Quick Tip to Solve: Split the reciprocal of $\sin^2 x \cos^2 x$ using the identity for their sum, and remember the minus sign in the integral of cosecant squared.

4. Let P be a skew-symmetric matrix of order 3. If $\det(P) = \alpha$, then $(2025)^\alpha$ is [1 mark]

- (A) 0
- (B) 1
- (C) 2025
- (D) $(2025)^3$

Correct Answer: (B) 1

Solution:

Concept:

- A skew-symmetric matrix satisfies $P^T = -P$, and transposition preserves its determinant.
- For a matrix of order n , $\det(-P) = (-1)^n \det(P)$. Thus an odd-order skew-symmetric matrix has zero determinant.

Step 1: Apply the determinant identities Here P has order 3. Taking determinants on both sides of $P^T = -P$ yields

$$\det(P) = \det(P^T) = \det(-P) = (-1)^3 \det(P) = -\det(P).$$

Hence $2\det(P) = 0$, so the given exponent is $\alpha = 0$.

Step 2: Evaluate the power and rule out distractors The base 2025 is nonzero, so its zeroth power equals one. Option (A) confuses the zero determinant with the value of the exponential expression. Option (C) would require exponent 1, and option (D) exponent 3; the matrix order is not the determinant.

Step 3: Conclusion $(2025)^\alpha = 2025^0 = 1$. Hence, the correct option is (B).

Quick Tip to Solve: For any odd-order skew-symmetric matrix, write its determinant as zero first, then evaluate the expression that uses it.

5. The principal value of $\sin^{-1}\left(\cos \frac{43\pi}{5}\right)$ is

[1 mark]

- (A) $\frac{-7\pi}{5}$
- (B) $\frac{-\pi}{10}$
- (C) $\frac{\pi}{10}$
- (D) $\frac{3\pi}{5}$

Correct Answer: (B) $\frac{-\pi}{10}$

Solution:

Concept:

- Cosine has period 2π , so whole multiples of 2π may be removed from its argument.
- The principal range of $\sin^{-1}$ is $[-\pi/2, \pi/2]$; use $\cos t = \sin(\pi/2 - t)$.

Step 1: Reduce the cosine angle Subtract 8π , which is four complete periods:

$$\cos \frac{43\pi}{5} = \cos \left(8\pi + \frac{3\pi}{5} \right) = \cos \frac{3\pi}{5}.$$

This cosine is negative because its reduced angle lies in the second quadrant.

Step 2: Convert to a sine in the principal range Using the complementary-angle identity,

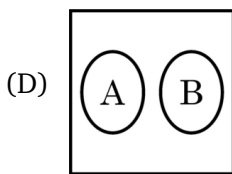
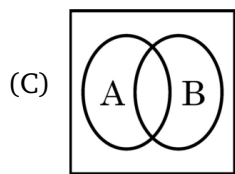
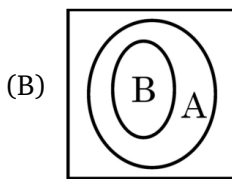
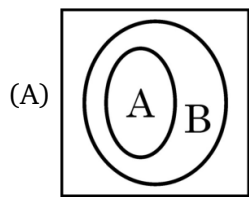
$$\cos \frac{3\pi}{5} = \sin \left(\frac{\pi}{2} - \frac{3\pi}{5} \right) = \sin \left(-\frac{\pi}{10} \right).$$

The angle $-\pi/10$ lies in the principal range, so inverse sine returns it. Options (A) and (D) lie outside this range; (C) has the wrong sign for the negative input.

Step 3: Conclusion The principal value is $-\pi/10$. Hence, the correct option is (B).

Quick Tip to Solve: Reduce the cosine angle modulo 2π before converting to sine, and check that the final angle belongs to the inverse-sine principal range.

6. If A denotes the set of continuous functions and B denotes the set of differentiable functions, then which of the following depicts the correct relation between set A and B ? [1 mark]



Correct Answer: (B)

Solution:

Concept:

- Every function differentiable on a given domain is continuous there; hence the differentiable functions form a subset of the continuous functions.
- Continuity alone need not imply differentiability: $f(x) = |x|$ is continuous on $\mathbb{R}$ but is not differentiable at $x = 0$.

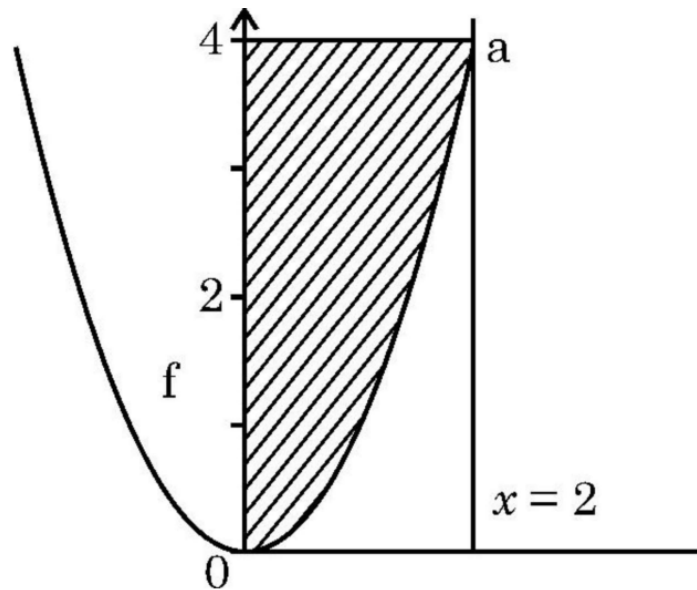
Step 1: Apply the implication between the two properties Let $f \in B$. By definition, f is differentiable, and differentiability implies continuity at each point of differentiability. Consequently $f \in A$, establishing $B \subseteq A$. The reverse implication fails for $|x|$, whose left and right derivatives at zero are -1 and 1 .

Step 2: Match the inclusion with the printed diagrams The correct diagram must put the entire set B inside A , with room in A for continuous functions that are not differentiable. Printed option (B) does this. Option (A) reverses the inclusion; (C) wrongly allows differentiable functions outside A ; (D) wrongly makes the sets disjoint.

Step 3: Conclusion B is a proper subset of A , shown in option (B). Hence, the correct option is (B).

Quick Tip to Solve: Read the letters inside the nested ovals carefully: the smaller oval must be B , because differentiability implies continuity.

7. The area of the shaded region (figure) represented by the curves $y = x^2$, $0 \leq x \leq 2$ and the y -axis is given by [1 mark]



- (A) $\int_0^2 x^2 dx$
 (B) $\int_0^2 \sqrt{y} dy$
 (C) $\int_0^4 x^2 dx$
 (D) $\int_0^4 \sqrt{y} dy$

Correct Answer: (D) $\int_0^4 \sqrt{y} dy$

Solution:

Concept:

- For horizontal strips, an area is $\int_a^b (x_{\text{right}} - x_{\text{left}}) dy$, where a, b are the lower and upper heights.
- On the nonnegative branch of $y = x^2$, the horizontal coordinate is $x = \sqrt{y}$.

Step 1: Read the shaded region and its limits The printed shading lies between the y -axis on the left and the right-hand branch of the parabola on the right, capped at the height of the point with $x = 2$. That height is $y = 2^2 = 4$, and the bottom is the origin. Thus the vertical limits are 0 and 4.

Step 2: Form the horizontal-strip integral At a fixed height y , the strip runs from $x = 0$ to $x = \sqrt{y}$, giving width $\sqrt{y}$. Therefore its area element is $\sqrt{y} dy$. Option (A) instead computes the area below the parabola; (B) stops at the wrong height, and (C) incorrectly extends the x -range to 4.

Step 3: Conclusion The shaded area is $\int_0^4 \sqrt{y} dy$ (equal to $16/3$ square units). Hence, the correct option is (D).

Quick Tip to Solve: For shading against the y -axis, take horizontal strips; convert the endpoint $x = 2$ into the upper height $y = 4$.

8. Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify $4AB + 3(AB + BA) - 4BA$, where A and B are both matrices of order 2×2 . It is known that $A \neq B \neq I$ and $A^{-1} \neq B$.

Their answers are:

Abhay: $6AB$

Bina: $7AB - BA$

Chhaya: $8AB$

Devesh: $7BA - AB$

Who answered it correctly?

[1 mark]

(A) Abhay

(B) Bina

(C) Chhaya

(D) Devesh

Correct Answer: (B) Bina

Solution:

Concept:

- Matrix multiplication distributes over addition, so $3(AB + BA) = 3AB + 3BA$.
- Matrix products generally do not commute: AB and BA must be collected as separate terms unless equality is established.

Step 1: Expand the bracket Apply scalar distribution without changing either product order:

$$4AB + 3(AB + BA) - 4BA = 4AB + 3AB + 3BA - 4BA.$$

All additions are defined because both products are matrices of the same order 2×2 .

Step 2: Collect like matrix products Combine the terms containing AB and then those containing BA :

$$(4 + 3)AB + (3 - 4)BA = 7AB - BA.$$

This is Bina's expression. Abhay's $6AB$ assumes $AB = BA$, which is not given. Chhaya's $8AB$ is not the general simplification, and Devesh reverses the roles of the two products. Special matrix choices do not justify those expressions as general identities.

Step 3: Conclusion The simplified expression is $7AB - BA$, so Bina answered correctly. Hence, the correct option is (B).

Quick Tip to Solve: Collect AB and BA exactly like two different algebraic terms; the supplied restrictions do not authorize interchanging them.

9. If p and q are respectively the order and degree of the differential equation $\frac{d}{dx} \left(\left(\frac{dy}{dx} \right)^3 \right) = 0$, then $(p - q)$ is [1 mark]

- (A) 0
- (B) 1
- (C) 2
- (D) 3

Correct Answer: (B) 1

Solution:

Concept:

- The order is the order of the highest derivative present after differentiating the stated expression.
- The degree is the power of the highest-order derivative in a polynomial differential equation; it is not the total degree of all derivative factors.

Step 1: Expand the derivative using the chain rule Treat dy/dx as a function of x . The given equation becomes

$$\frac{d}{dx} \left[\left(\frac{dy}{dx} \right)^3 \right] = 3 \left(\frac{dy}{dx} \right)^2 \frac{d^2y}{dx^2} = 0.$$

This is already polynomial in the derivatives, and the highest derivative is the second derivative.

Step 2: Identify order and degree separately The second derivative occurs to the first power, so $p = 2$ and $q = 1$. The cube in the original expression does not make the degree three, because it belongs to the expression before differentiation. The factor involving the first derivative does not alter the power of the highest derivative. Thus options (A), (C) and (D) do not match the difference.

Step 3: Conclusion $p - q = 2 - 1 = 1$. Hence, the correct option is (B).

Quick Tip to Solve: Expand any outer differentiation before counting order or degree; read the exponent of the highest derivative only.

10. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval [1 mark]

- (A) $(0, 2)$
- (B) $(-\infty, 2]$
- (C) $[1, 2]$
- (D) $[2, \infty)$

Correct Answer: (D) $[2, \infty)$

Solution:

Concept:

- A differentiable function is increasing where its derivative is positive.
- The sign of $f'(x)$ determines whether function values rise or fall as x increases.

Quick Tip to Solve: Factor the derivative as $2(x - 2)$; positive values occur to the right of 2.

Step 1: Differentiate the function For $f(x) = x^2 - 4x + 6$, differentiate term by term to obtain

$$f'(x) = 2x - 4 = 2(x - 2).$$

The derivative vanishes at $x = 2$, which separates the two possible signs.

Step 2: Check the intervals For $x > 2$, the derivative is positive; for $x < 2$, it is negative. Hence the minimum at the endpoint $x = 2$ may be included in the increasing interval. Options (A), (B) and (C) contain only points to the left of this minimum, where the function is decreasing.

Step 3: Conclusion The function is increasing on $[2, \infty)$. Hence, the correct option is (D).

Quick Tip to Solve: Factor the derivative as $2(x - 2)$; positive values occur to the right of 2.

11. In the following probability distribution, the value of p is:

X	0	1	2	3
$P(X)$	p	p	0.3	$2p$

[1 mark]

- (A) $\frac{7}{40}$
(B) $\frac{1}{10}$
(C) $\frac{9}{35}$
(D) $\frac{1}{4}$

Correct Answer: (A) $\frac{7}{40}$

Solution:

Concept:

- The probabilities in a discrete probability distribution sum to 1.
- Each probability must also lie between 0 and 1.

Step 1: Add the given probabilities The four distinct values $X = 0, 1, 2, 3$ have probabilities $p, p, 0.3, 2p$. Using the total probability condition gives

$$p + p + 0.3 + 2p = 1, \quad 4p = 0.7.$$

Step 2: Solve and check the choices Dividing by 4 gives $p = 0.175 = 7/40$. The probabilities then are 0.175, 0.175, 0.3, 0.35, all valid. Options (B), (C) and (D) give totals 0.7, 93/70 and 1.3, respectively, so none defines this distribution.

Step 3: Conclusion $p = \frac{7}{40}$. Hence, the correct option is (A).

Quick Tip to Solve: Combine the unknown probabilities first: $p + p + 2p = 4p$, leaving 0.7 for their total.

12. If $\vec{PQ} \times \vec{PR} = 4\hat{i} + 8\hat{j} - 8\hat{k}$, then the area ($\triangle PQR$) is

[1 mark]

- (A) 2 sq units
- (B) 4 sq units
- (C) 6 sq units
- (D) 12 sq units

Correct Answer: (C) 6 sq units

Solution:

Concept:

- The magnitude $|\vec{PQ} \times \vec{PR}|$ is the area of the parallelogram formed by the two vectors.
- The triangle on the same two sides has half the parallelogram area.

Step 1: Calculate the cross-product magnitude The mutually perpendicular unit vectors allow the magnitude to be obtained by squaring the components and adding:

$$|4\hat{i} + 8\hat{j} - 8\hat{k}| = \sqrt{4^2 + 8^2 + (-8)^2} = \sqrt{144} = 12.$$

Step 2: Convert to the triangle area The requested figure is a triangle, so multiply this magnitude by $1/2$. The value 12 in option (D) is the parallelogram area, while 2 and 4 do not follow from the magnitude of the given vector.

Step 3: Conclusion $\text{Area}(\triangle PQR) = \frac{1}{2}(12) = 6$ square units. Hence, the correct option is (C).

Quick Tip to Solve: Find the magnitude of the whole cross product, then halve it; no single component gives the area.

13. If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(\bar{E}/\bar{F})$ is

[1 mark]

- (A) $\frac{P(\bar{E})}{P(\bar{F})}$
- (B) $1 - P(\bar{E}/F)$
- (C) $1 - P(E/F)$
- (D) $\frac{1 - P(E \cup F)}{P(\bar{F})}$

Correct Answer: (D) $\frac{1 - P(E \cup F)}{P(\bar{F})}$

Solution:

Concept:

- Conditional probability is $P(A | B) = P(A \cap B)/P(B)$, provided $P(B) > 0$.
- De Morgan's law gives $\bar{E} \cap \bar{F} = \overline{E \cup F}$.

Step 1: Apply the definition to the complements Since $P(F) \neq 1$, the denominator $P(\bar{F}) = 1 - P(F)$

Quick Tip to Solve: Keep the conditioning event $\bar{F}$ unchanged in the denominator, then use De Morgan's law above it.

is positive. Thus

$$P(\bar{E} | \bar{F}) = \frac{P(\bar{E} \cap \bar{F})}{P(\bar{F})}.$$

Step 2: Express the intersection using a union By the complement rule, $P(\bar{E} \cap \bar{F}) = 1 - P(E \cup F)$. Option (A) replaces the required intersection by $\bar{E}$ alone. Options (B) and (C) condition on F , not on $\bar{F}$; they therefore do not give the required general formula.

Step 3: Conclusion

$$P(\bar{E} | \bar{F}) = \frac{1 - P(E \cup F)}{P(\bar{F})}.$$

Hence, the correct option is (D).

Quick Tip to Solve: Keep the conditioning event $\bar{F}$ unchanged in the denominator, then use De Morgan's law above it.

14. Which of the following can be both a symmetric and skew-symmetric matrix?

[1 mark]

- (A) Unit Matrix
- (B) Diagonal Matrix
- (C) Null Matrix
- (D) Row Matrix

Correct Answer: (C) Null Matrix

Solution:

Concept:

- A symmetric matrix satisfies $A^T = A$, where A^T denotes its transpose.
- A skew-symmetric matrix satisfies $A^T = -A$; both definitions concern square matrices.

Step 1: Impose both conditions If the same matrix is symmetric and skew-symmetric, then

$$A = A^T = -A \implies 2A = O.$$

Every entry is real, so every entry must vanish.

Step 2: Identify the matrix type The null matrix satisfies both transpose conditions. A unit matrix is not skew-symmetric. A diagonal matrix with any nonzero diagonal entry fails skew-symmetry; the only diagonal matrix that works is precisely the null matrix. A general row matrix is not square and cannot satisfy these transpose equalities.

Step 3: Conclusion The only matrix satisfying both conditions is the null matrix O . Hence, the correct option is (C).

Quick Tip to Solve: Equate the two transpose expressions: $A = -A$ immediately forces every entry to be zero.

15. The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point $(4, -3, 7)$ is [1 mark]

(A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$

(B) $x = 3t + 4, y = t + 3, z = 2t + 7$

(C) $x = 3t + 4, y = t - 3, z = 2t + 7$

(D) $x = 3t + 4, y = -t + 3, z = 2t + 7$

Correct Answer: (C) $x = 3t + 4, y = t - 3, z = 2t + 7$

Solution:

Concept:

- A line through (x_0, y_0, z_0) with direction vector (a, b, c) has equations $x = x_0 + at, y = y_0 + bt, z = z_0 + ct$.
- The constant terms locate a point on the line, while coefficients of the common parameter give its direction ratios.

Step 1: Insert the point and direction Use the point $(4, -3, 7)$ and the direction ratios $(3, 1, 2)$. This yields

$$x = 4 + 3t, \quad y = -3 + t, \quad z = 7 + 2t.$$

At $t = 0$ all three coordinates agree with the specified point.

Step 2: Check the other choices Option (A) has direction ratios $(4, -3, 7)$, which are not proportional to $(3, 1, 2)$. In (B), $x = 4$ forces $t = 0$ and then $y = 3$, so the required point is absent. Option (D) has direction ratios $(3, -1, 2)$, so it is not parallel to the given vector.

Step 3: Conclusion The line is $x = 3t + 4, y = t - 3, z = 2t + 7$. Hence, the correct option is (C).

Quick Tip to Solve: Check constants and parameter coefficients separately: point $(4, -3, 7)$, direction $(3, 1, 2)$.

16. If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct? [1 mark]

(A) $A = B$

(B) $AB = BA$

(C) $A = O$ or $B = O$

(D) $A = I$ or $B = I$

Correct Answer: (B) $AB = BA$

Solution:

Concept:

- Matrix multiplication is distributive, but in general $AB \neq BA$.
- The scalar difference-of-squares factorisation extends to matrices exactly when the mixed products cancel.

Quick Tip to Solve: Expand without changing multiplication order; the extra term $AB - BA$ determines whether the identity holds.

Step 1: Expand in the specified order Preserve the order of each matrix product while distributing:

$$(A - B)(A + B) = A^2 + AB - BA - B^2.$$

Replacing BA by AB before using the hypothesis would assume what needs to be proved.

Step 2: Use the stated equality Comparing this expansion with $A^2 - B^2$ gives $AB - BA = O$. The stronger restrictions in (A), (C) and (D) are unnecessary. For example, $A = 2I$ and $B = 3I$ satisfy the given identity although they are unequal and neither is the zero or identity matrix.

Step 3: Conclusion The matrices commute: $AB = BA$. Hence, the correct option is (B).

Quick Tip to Solve: Expand without changing multiplication order; the extra term $AB - BA$ determines whether the identity holds.

17. The line $x = 1 + 5\mu$, $y = -5 + \mu$, $z = -6 - 3\mu$ passes through which of the following point? [1 mark]

- (A) $(1, -5, 6)$
- (B) $(1, 5, 6)$
- (C) $(1, -5, -6)$
- (D) $(-1, -5, 6)$

Correct Answer: (C) $(1, -5, -6)$

Solution:

Concept:

- A point lies on a parametric line only when one common parameter value satisfies all three coordinates.
- Setting the parameter to zero directly supplies the base point of a line.

Step 1: Evaluate the coordinates at zero For $\mu = 0$, the equations give

$$x = 1 + 5(0) = 1, \quad y = -5 + 0 = -5, \quad z = -6 - 3(0) = -6.$$

Thus a listed candidate is obtained directly.

Step 2: Reject inconsistent coordinates In (A) and (B), $x = 1$ forces $\mu = 0$, which gives $z = -6$, not 6; (B) also has the wrong sign of y . In (D), $y = -5$ forces $\mu = 0$, contradicting its $x = -1$. No different parameter can repair these simultaneous requirements.

Step 3: Conclusion The line passes through $(1, -5, -6)$. Hence, the correct option is (C).

Quick Tip to Solve: Try parameter zero first; then use one coordinate to fix the parameter when eliminating the other options.

18. A factory produces two products X and Y . The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct? [1 mark]

- (A) The objective function maximizes the difference of the profit earned from products X and Y .
- (B) The objective function measures the total production of products X and Y .
- (C) The objective function maximizes the combined profit earned from selling X and Y .
- (D) The objective function ensures the company produces more of product X than product Y .

Correct Answer: (C) The objective function maximizes the combined profit earned from selling X and Y .

Solution:

Concept:

- An objective function expresses the quantity to be maximised or minimised in terms of the decision variables.
- If unit profits are 5 and 7, selling x and y units produces respective profits $5x$ and $7y$.

Step 1: Interpret the sum The positive terms in $Z = 5x + 7y$ add the profits from both products. Maximising Z therefore seeks the largest combined profit among the feasible choices of x and y .

Step 2: Distinguish the other interpretations A difference of profits would require subtraction, so (A) does not match. Total units sold would be $x + y$, not the weighted profit sum, ruling out (B). The objective does not impose a constraint such as $x > y$, so it cannot ensure the production comparison asserted in (D).

Step 3: Conclusion The objective function maximises the combined profit from selling X and Y . Hence, the correct option is (C).

Quick Tip to Solve: Read each coefficient as profit per unit; addition combines profits, while production restrictions belong to constraints.

19. Assertion (A): $A = \text{diag}[3 \ 5 \ 2]$ is a scalar matrix of order 3×3 .

Reason (R): If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true but Reason (R) is false.
- (D) Assertion (A) is false but Reason (R) is true.

Correct Answer: No option is strictly correct as worded: both statements are false. Option (D) is the intended answer if the reason means that all diagonal entries are equal.

Solution:

Concept:

- A scalar matrix has the form kI_n : all diagonal entries equal the same scalar k , and every off-diagonal entry is zero.
- A diagonal matrix need not be scalar. Equality of only its non-zero entries does not guarantee equality of all its diagonal entries.

Step 1: Evaluate the assertion The diagonal entries of $A = \text{diag}(3, 5, 2)$ are 3, 5, 2. They are unequal, so no scalar k satisfies $A = kI_3$. Thus Assertion (A) is false, even though the matrix is diagonal and has order 3×3 .

Step 2: Test the reason as printed Consider $B = \text{diag}(3, 3, 0)$. It is diagonal and all its non-zero elements equal 3, yet it is not scalar because its diagonal entries are not all equal. Thus Reason (R), read literally, is false. The correct definition requires all diagonal entries to be equal, including any zero entries.

Step 3: Compare the truth values with the choices A false assertion cannot be explained by the reason. Options (A), (B) and (C) require a true assertion, so they fail. Option (D) requires a true reason; it becomes the intended choice only if the reason is understood as referring to equality of all diagonal entries.

Step 4: Conclusion As printed, both statements are false and none of the four options represents that combination. Under the intended definition of a scalar matrix, the expected option is (D).

Quick Tip to Solve: To test whether a diagonal matrix is scalar, compare every diagonal entry, including zeros; do not compare only its non-zero entries.

20. Assertion (A): Every point of the feasible region of a Linear Programming Problem is an optimal solution.

Reason (R): The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true but Reason (R) is false.
- (D) Assertion (A) is false but Reason (R) is true.

Correct Answer: No option is strictly correct: both statements are false as written. Option (D) is intended if the reason is read as the usual corner-point theorem, without the restrictive word 'only'.

Solution:

Concept:

- A feasible solution satisfies every constraint. An optimal solution additionally maximizes or minimizes the objective function over the feasible region.
- For a bounded polygonal feasible region, an optimum occurs at a corner point. If the objective is constant along an optimal edge, every point of that edge is also optimal.

Step 1: Evaluate the assertion Take the feasible square $0 \leq x \leq 1$, $0 \leq y \leq 1$, and maximize $Z = x$. The feasible point $(0, 0)$ has $Z = 0$, whereas $(1, 0)$ has $Z = 1$. Consequently, every feasible point need not be optimal, and Assertion (A) is false.

Step 2: Examine the word only in the reason In this same example the maximum value is 1, attained at every point $(1, y)$ with $0 \leq y \leq 1$. In particular, $(1, \frac{1}{2})$ is optimal but is not a corner point. Thus the claim that optimal solutions exist only at corner points is false. The corner-point theorem guarantees an optimal corner point; it does not exclude other optimal points.

Step 3: Test the explanation and the options The assertion is false, so the reason cannot correctly explain it. Options (A), (B) and (C) incorrectly require a true assertion. Option (D) is the expected choice only when the reason is understood as the usual corner-point theorem and its restrictive word 'only' is disregarded.

Step 4: Conclusion Both statements are false when read literally, so none of the printed options is strictly correct. The intended answer under the usual corner-point interpretation is (D).

Quick Tip to Solve: Distinguish 'an optimum occurs at a corner' from 'only corners are optimal'; equal optimal values at adjacent corners make their whole edge optimal.

Section B (10 marks)

(This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.)

21. Find the values of 'a' for which $f(x) = \sin x - ax + b$ is increasing on $\mathbb{R}$.

[2 marks]

Solution:

Concept:

- A differentiable function is increasing on an interval if its derivative is non-negative throughout that interval. Here f is differentiable for every real x .
- The range of $\cos x$ is $[-1, 1]$; its minimum value -1 occurs at $x = (2n + 1)\pi$, where n is an integer.

Step 1: Differentiate the given function For $f(x) = \sin x - ax + b$, the constants a and b do not depend on x . Thus

$$f'(x) = \cos x - a.$$

The additive constant b shifts the graph vertically and therefore has no effect on whether the function increases.

Step 2: Impose the condition at every real point We require $\cos x - a \geq 0$ for all $x \in \mathbb{R}$. Since the smallest value of $\cos x$ is -1 , the smallest derivative is $-1 - a$. Hence

$$-1 - a \geq 0 \iff a \leq -1.$$

This condition is also necessary: if $a > -1$, the derivative is negative at $x = \pi$ and in a neighbourhood of that point. At the boundary $a = -1$, the derivative $1 + \cos x$ vanishes only at isolated points and is positive elsewhere, so the function is still strictly increasing.

Step 3: Conclusion The required values are $a \leq -1$, with no restriction on the constant b .

Quick Tip to Solve: Use the minimum of $\cos x$, not its maximum, because the derivative must stay non-negative for every real x .

22. Evaluate: $\int_0^\pi \frac{\sin 2px}{\sin x} dx, p \in \mathbb{N}$.

[2 marks]

Solution:

Concept:

- Reflection about the midpoint gives $\int_0^a f(x) dx = \int_0^a f(a - x) dx$, provided the integral exists.
- For a natural number p , $\sin(2p\pi - \theta) = -\sin \theta$, while $\sin(\pi - x) = \sin x$. Thus the given integrand changes sign under $x \mapsto \pi - x$.

Step 1: Establish the integral and apply reflection Write

$$I = \int_0^\pi \frac{\sin(2px)}{\sin x} dx.$$

Quick Tip to Solve: Before expanding $\sin(2px)$, replace x by $\pi - x$; an integrand that reverses sign has zero integral over this symmetric interval.

The zeros of the denominator at the endpoints are removable: the integrand tends to $2p$ as $x \rightarrow 0^+$ and to $-2p$ as $x \rightarrow \pi^-$. Hence the integral exists. Replacing x by $\pi - x$ gives

$$I = \int_0^\pi \frac{\sin\{2p(\pi - x)\}}{\sin(\pi - x)} dx.$$

Step 2: Use the integer value of p Since p is a natural number, $\sin(2p\pi) = 0$ and $\cos(2p\pi) = 1$. Therefore,

$$\sin(2p\pi - 2px) = -\sin(2px).$$

Together with $\sin(\pi - x) = \sin x$, this yields

$$I = - \int_0^\pi \frac{\sin(2px)}{\sin x} dx = -I.$$

Adding the original and reflected expressions gives $2I = 0$; the contributions on the two halves of the interval cancel.

Step 3: Conclusion

$$\int_0^\pi \frac{\sin(2px)}{\sin x} dx = 0 \quad (p \in \mathbb{N}).$$

Quick Tip to Solve: Before expanding $\sin(2px)$, replace x by $\pi - x$; an integrand that reverses sign has zero integral over this symmetric interval.

23(a). If $x = e^{x/y}$, then prove that $\frac{dy}{dx} = \frac{x - y}{x \log x}$.

[2 marks]

Solution:

Concept:

- Take natural logarithms to turn the exponential equation into an algebraic relation involving x and y . Since $x = e^{x/y}$, we have $x > 0$ and $y \neq 0$.
- For a differentiable function $y(x)$, the product rule gives $\frac{d}{dx}(y \log x) = y' \log x + \frac{y}{x}$.

Step 1: Rewrite the exponential equation Taking natural logarithms on both sides of $x = e^{x/y}$ gives

$$\log x = \frac{x}{y}, \quad y \log x = x.$$

In this equation $x = 1$ is impossible, since it would give $0 = 1/y$. Consequently $\log x \neq 0$, so division by $\log x$ in the following calculation is legitimate.

Step 2: Differentiate and rearrange Differentiating $y \log x = x$ with respect to x , using the product rule on the left-hand side, gives

$$\frac{dy}{dx} \log x + \frac{y}{x} = 1.$$

Quick Tip to Solve: After taking logarithms, multiply by y before differentiating; the product rule then avoids a quotient of implicit functions.

Move the second term to the right and divide by $\log x$:

$$\frac{dy}{dx} = \frac{1 - y/x}{\log x} = \frac{x - y}{x \log x}.$$

The expression retains y , as required in the printed statement; equivalently one may substitute $y = x / \log x$ after differentiating.

Step 3: Conclusion Hence, $\boxed{\frac{dy}{dx} = \frac{x - y}{x \log x}}$, as required.

Quick Tip to Solve: After taking logarithms, multiply by y before differentiating; the product rule then avoids a quotient of implicit functions.

OR

23(b). If

$$f(x) = \begin{cases} 2x - 3, & -3 \leq x \leq -2, \\ x + 1, & -2 < x \leq 0. \end{cases}$$

Check the differentiability of $f(x)$ at $x = -2$.

[2 marks]

Solution:

Concept:

- Differentiability at an interior point requires continuity there. Check the value and the two one-sided limits before comparing derivatives.
- The derivative at $x = c$ is obtained from $\lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$; the same value $f(c)$ must be used on both sides.

Step 1: Test continuity at the joining point The first branch includes $x = -2$, whereas the second branch applies only for $x > -2$. Hence

$$f(-2) = 2(-2) - 3 = -7.$$

The one-sided limits are

$$\lim_{x \rightarrow -2^-} f(x) = -7, \quad \lim_{x \rightarrow -2^+} f(x) = -2 + 1 = -1.$$

Because these limits are unequal, the function has a jump discontinuity at $x = -2$.

Step 2: Check the derivative definition directly For $h < 0$, $f(-2+h) = -7 + 2h$, so the left-hand difference quotient is $2h/h = 2$. For $h > 0$, the right branch gives $f(-2+h) = -1 + h$, and therefore

$$\frac{f(-2+h) - f(-2)}{h} = \frac{6+h}{h} = \frac{6}{h} + 1.$$

This tends to $+\infty$ as $h \rightarrow 0^+$, rather than a finite right derivative. Merely differentiating the right

Quick Tip to Solve: At a piecewise boundary, check continuity first and use the actual function value in each one-sided difference quotient.

branch to obtain slope 1 would overlook the discontinuity and the actual value at the joining point.

Step 3: Conclusion The function is discontinuous, and therefore not differentiable at $x = -2$.

Quick Tip to Solve: At a piecewise boundary, check continuity first and use the actual function value in each one-sided difference quotient.

24. Let $\vec{p} = 2\hat{i} - 3\hat{j} - \hat{k}$, $\vec{q} = -3\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + 2\hat{k}$. Express $\vec{r}$ in the form of $\vec{r} = \lambda\vec{p} + \mu\vec{q}$, and hence find the values of λ and μ . [2 marks]

Solution:

Concept:

- Two vectors are equal exactly when their coefficients of $\hat{i}$, $\hat{j}$, $\hat{k}$ are equal.
- In a linear combination $\lambda\vec{p} + \mu\vec{q}$, multiply each component of $\vec{p}$ and $\vec{q}$ by its respective scalar before adding.

Step 1: Compare the three components Substitute the given vectors into the required expression:

$$\lambda\vec{p} + \mu\vec{q} = (2\lambda - 3\mu)\hat{i} + (-3\lambda + 4\mu)\hat{j} + (-\lambda + \mu)\hat{k}.$$

Equating this with $\vec{r} = \hat{i} + \hat{j} + 2\hat{k}$ gives

$$2\lambda - 3\mu = 1, \quad -3\lambda + 4\mu = 1, \quad -\lambda + \mu = 2.$$

Step 2: Solve and check consistency The third equation gives $\mu = \lambda + 2$. Substitute this into the first:

$$2\lambda - 3(\lambda + 2) = 1 \Rightarrow \lambda = -7, \quad \mu = -5.$$

The unused second equation gives $-3(-7) + 4(-5) = 21 - 20 = 1$, so the same scalars satisfy all three components. This check ensures that $\vec{r}$ actually belongs to the plane spanned by the two given vectors.

Step 3: Conclusion The required expression is $\vec{r} = -7\vec{p} - 5\vec{q}$; hence $\lambda = -7, \mu = -5$.

Quick Tip to Solve: Use the simplest component equation first, here $-\lambda + \mu = 2$, and verify the remaining component after solving for the two scalars.

25(a). A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$. [2 marks]

Solution:

Concept:

- The direction cosines l, m, n of a vector are the cosines of its angles with the positive coordinate axes and satisfy $l^2 + m^2 + n^2 = 1$.
- For magnitude a , the vector is $a(\hat{i}l + \hat{j}m + \hat{k}n)$. Equal angles imply equal direction cosines, including

Quick Tip to Solve: Equal angles force equal direction cosines, so first use $3l^2 = 1$; retain both common signs unless the angles are stated to be acute.

their signs.

Step 1: Determine the common direction cosine Let the common angle with each axis be θ . Then $l = m = n = \cos \theta$. The direction-cosine identity gives

$$3 \cos^2 \theta = 1 \Rightarrow \cos \theta = \pm \frac{1}{\sqrt{3}}.$$

All three direction cosines have the same sign: choosing their signs independently would give different angles with the axes.

Step 2: Multiply by the given magnitude Using $|\vec{a}| = 5\sqrt{3}$, each Cartesian component equals

$$5\sqrt{3} \cos \theta = \pm 5.$$

Thus there are two opposite directions satisfying the stated condition. If the common angle is acute, all components are $+5$; if it is obtuse, all are -5 . Each vector has magnitude $\sqrt{25 + 25 + 25} = 5\sqrt{3}$, as required. The question does not restrict the common angle to be acute.

Step 3: Conclusion $\vec{a} = \pm 5(\hat{i} + \hat{j} + \hat{k})$, with the same sign on all components. For equal acute angles, $\vec{a} = 5\hat{i} + 5\hat{j} + 5\hat{k}$.

Quick Tip to Solve: Equal angles force equal direction cosines, so first use $3l^2 = 1$; retain both common signs unless the angles are stated to be acute.

OR

25(b). If $\vec{a}$ and $\vec{b}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $\vec{QR} = \frac{3}{2}\vec{QP}$. [2 marks]

Solution:

Concept:

- If the position vectors of points P and Q are $\vec{a}$ and $\vec{b}$, then $\vec{QP} = \vec{a} - \vec{b}$.
- For collinear points on the ray QP, directed displacement vectors have the same direction. Position vectors obey $\vec{OR} = \vec{OQ} + \vec{QR}$, where O is the origin.

Step 1: Express the displacement from Q to R R lies on QP produced beyond P. Therefore $\vec{QR}$ points in the same direction as $\vec{QP}$. The given length relation can consequently be written as the vector relation

$$\vec{QR} = \frac{3}{2}\vec{QP} = \frac{3}{2}(\vec{a} - \vec{b}).$$

The factor exceeds one, confirming that R lies beyond P and not between Q and P.

Quick Tip to Solve: For a point on QP produced, begin with $\vec{OR} = \vec{OQ} + t\vec{QP}$; here $t = 3/2$, which avoids reversing an external-division ratio.

Step 2: Add the position vector of Q Starting at the origin, first move to Q and then from Q to R:

$$\begin{aligned}\vec{OR} &= \vec{b} + \frac{3}{2}(\vec{a} - \vec{b}) \\ &= \frac{3}{2}\vec{a} - \frac{1}{2}\vec{b}.\end{aligned}$$

As a check, $\vec{PR} = \vec{OR} - \vec{a} = \frac{1}{2}(\vec{a} - \vec{b})$, so PR is half of QP in the same direction; hence QR is indeed three halves of QP.

Step 3: Conclusion The position vector of R is $\boxed{\frac{3\vec{a} - \vec{b}}{2}}$.

Quick Tip to Solve: For a point on QP produced, begin with $\vec{OR} = \vec{OQ} + t\vec{QP}$; here $t = 3/2$, which avoids reversing an external-division ratio.

Section C (18 marks)

(This section comprises of 6 Short Answer (SA) type questions of 3 marks each.)

26(a). If $y = \log \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2$, then show that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.

[3 marks]

Solution:

Concept:

- Logarithm laws give $\log(A/B) = \log A - \log B$ and $\log A^2 = 2 \log A$ for positive A, B . Here the square roots and denominator require $x > 0$.
- The chain rule gives $\frac{d}{dx} \log u = \frac{u'}{u}$. We use $y_1 = dy/dx$ and $y_2 = d^2y/dx^2$.

Step 1: Simplify the logarithm before differentiating Combine the two terms inside the parentheses over the common denominator $\sqrt{x}$:

$$\sqrt{x} + \frac{1}{\sqrt{x}} = \frac{x+1}{\sqrt{x}}.$$

Since $x > 0$, all arguments in the following logarithms are positive, and

$$y = \log \frac{(x+1)^2}{x} = 2 \log(x+1) - \log x.$$

This form reduces the calculation to derivatives of two elementary logarithms.

Step 2: Calculate the first and second derivatives Differentiate each logarithm with respect to x :

$$y_1 = \frac{2}{x+1} - \frac{1}{x}.$$

Differentiate once more. The derivative of $(x+1)^{-1}$ is $-(x+1)^{-2}$, while the derivative of $-x^{-1}$ is $+x^{-2}$. Hence

$$y_2 = -\frac{2}{(x+1)^2} + \frac{1}{x^2}.$$

Quick Tip to Solve: Rewrite the squared radical sum as $(x+1)^2/x$ first; logarithm laws then remove the need for a lengthy chain-rule calculation.

Step 3: Substitute into the required expression Keep the two weighted derivatives separate initially so that each denominator cancels explicitly:

$$x(x+1)^2 y_2 = -2x + \frac{(x+1)^2}{x},$$

$$(x+1)^2 y_1 = 2(x+1) - \frac{(x+1)^2}{x}.$$

On adding, the fractional terms cancel. The remaining linear terms give $-2x + 2(x+1) = 2$, independent of x .

Step 4: Conclusion Therefore $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$, as required.

Quick Tip to Solve: Rewrite the squared radical sum as $(x+1)^2/x$ first; logarithm laws then remove the need for a lengthy chain-rule calculation.

OR

26(b). If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, $-1 < x < 1$, $x \neq y$, then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$. [3 marks]

Solution:

Concept:

- An implicit relation can sometimes be factored into an explicit expression for y . The condition $x \neq y$ permits division by $x - y$.
- The quotient rule is $\frac{d}{dx}(u/v) = \frac{vu' - uv'}{v^2}$, where $v \neq 0$. Any equation obtained by squaring should be checked in the original radical relation.

Step 1: Eliminate the square roots Move one radical term to the other side and square:

$$x\sqrt{1+y} = -y\sqrt{1+x} \Rightarrow x^2(1+y) = y^2(1+x).$$

Expand and collect the terms on the left:

$$x^2 - y^2 + x^2 y - x y^2 = 0.$$

Both pairs contain the common factor $x - y$, so

$$(x - y)(x + y + xy) = 0.$$

Step 2: Use the given inequality and check the branch Since $x \neq y$, the first factor is nonzero. Thus

$$x + y + xy = 0 \Rightarrow y = -\frac{x}{1+x}.$$

The condition $-1 < x < 1$ ensures $1 + x > 0$. For this expression, $1 + y = 1/(1+x) > 0$, and

Quick Tip to Solve: After squaring, factor out $x - y$ before differentiating; the given condition removes that factor and exposes the simple relation $y = -x/(1+x)$.

substitution in the original left-hand side gives

$$\frac{x}{\sqrt{1+x}} - \frac{x}{1+x} \sqrt{1+x} = 0.$$

Hence this branch satisfies the unsquared equation. At $x = 0$, it gives $y = 0$, which is excluded by $x \neq y$; the derivative is required on the permitted part of the branch.

Step 3: Differentiate the explicit expression Apply the quotient rule to $y = -x/(1+x)$:

$$\begin{aligned} \frac{dy}{dx} &= \frac{(1+x)(-1) - (-x)(1)}{(1+x)^2} \\ &= \frac{-(1+x) + x}{(1+x)^2}. \end{aligned}$$

The terms involving x cancel in the numerator, leaving the constant -1 .

Step 4: Conclusion Therefore $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$, as required.

Quick Tip to Solve: After squaring, factor out $x - y$ before differentiating; the given condition removes that factor and exposes the simple relation $y = -x/(1+x)$.

27. Prove that $f : \mathbb{N} \rightarrow \mathbb{N}$ defined as $f(x) = ax + b$ ($a, b \in \mathbb{N}$) is one-one but not onto.

[3 marks]

Solution:

Concept:

- A function is one-one if equal images imply equal inputs: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.
- A function onto $\mathbb{N}$ must attain every natural number. Here $\mathbb{N} = \{1, 2, 3, \dots\}$, so $a \geq 1$ and $b \geq 1$.

Step 1: Establish that the rule maps natural numbers to natural numbers For every $x \in \mathbb{N}$, the product ax and the sum $ax + b$ are positive integers. Thus the stated rule does define a function from $\mathbb{N}$ to $\mathbb{N}$. We now examine injectivity and surjectivity separately, using the positivity of the given constants.

Step 2: Compare two inputs with equal images Let $x_1, x_2 \in \mathbb{N}$ and suppose their images are equal. Substituting the definition gives

$$\begin{aligned} f(x_1) = f(x_2) &\Rightarrow ax_1 + b = ax_2 + b \\ &\Rightarrow a(x_1 - x_2) = 0 \\ &\Rightarrow x_1 = x_2, \end{aligned}$$

where the final implication is valid because $a \geq 1$, so $a \neq 0$. Hence two distinct natural numbers cannot have the same image.

Step 3: Find a natural number outside the range The smallest input is 1, and multiplication by the positive number a preserves inequalities. Therefore every image satisfies

$$f(x) = ax + b \geq a + b \geq 2.$$

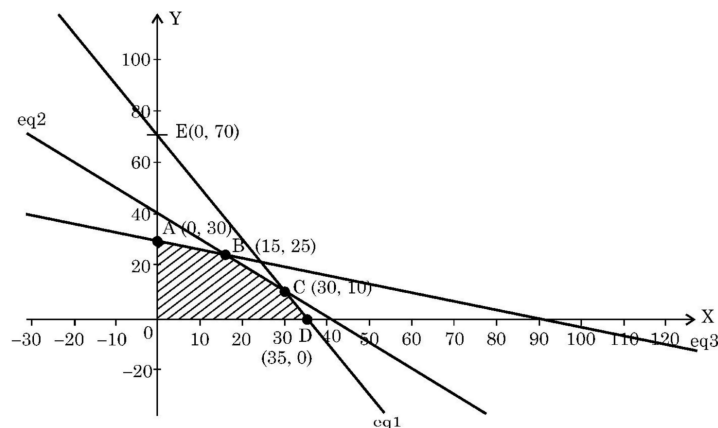
Quick Tip to Solve: Use $a \neq 0$ for injectivity and the minimum value $f(1) = a + b \geq 2$ to disprove surjectivity immediately.

Consequently 1, although in the codomain $\mathbb{N}$, is not the image of any input. This single missing codomain element disproves surjectivity; no listing of the entire range is needed.

Step 4: Conclusion Equal images force equal inputs, but 1 is absent from the range. Thus $f(x) = ax + b$ is one-one but not onto $\mathbb{N}$.

Quick Tip to Solve: Use $a \neq 0$ for injectivity and the minimum value $f(1) = a + b \geq 2$ to disprove surjectivity immediately.

28. The feasible region along with corner points for a linear programming problem are shown in the graph. Write all the constraints for the given linear programming problem. [3 marks]



Solution:

Concept:

- Each straight boundary line gives a linear equation. Two labelled points determine the equation, and a point in the shaded region determines the inequality sign.
- Solid boundary lines are included in the feasible region, so the constraints use $\leq$ or $\geq$. A region in the first quadrant also requires $x \geq 0$ and $y \geq 0$.

Step 1: Find the line through A and B From the graph, $A = (0, 30)$ and $B = (15, 25)$. The slope of AB is

$$m_{AB} = \frac{25 - 30}{15 - 0} = -\frac{1}{3}.$$

Therefore $y - 30 = -x/3$, giving the boundary equation $x + 3y = 90$. The origin belongs to the shaded region and satisfies $0 < 90$, so its side of this boundary is $x + 3y \leq 90$.

Step 2: Find the line through B and C Using $B = (15, 25)$ and $C = (30, 10)$, the slope is

$$m_{BC} = \frac{10 - 25}{30 - 15} = -1.$$

Hence $y - 25 = -(x - 15)$, or $x + y = 40$. Testing the shaded point $(0, 0)$ again selects $x + y \leq 40$.

Step 3: Find the line through C and D and include the axes The coordinates $C = (30, 10)$ and

Quick Tip to Solve: Use the labelled adjacent vertices to form each boundary equation, then test the origin to choose the shaded half-plane.

$D = (35, 0)$ give slope $(0 - 10)/(35 - 30) = -2$. Thus

$$y - 10 = -2(x - 30) \Rightarrow 2x + y = 70.$$

This also agrees with the labelled intercept $E = (0, 70)$. The shaded side contains the origin, giving $2x + y \leq 70$. Finally, the region lies on or to the right of the y -axis and on or above the x -axis, giving the two non-negativity constraints.

Step 4: Conclusion All the constraints are

$$x + 3y \leq 90, \quad x + y \leq 40, \quad 2x + y \leq 70, \quad x \geq 0, \quad y \geq 0.$$

Quick Tip to Solve: Use the labelled adjacent vertices to form each boundary equation, then test the origin to choose the shaded half-plane.

29(a). Solve the differential equation $2(y + 3) - xy \frac{dy}{dx} = 0$; given $y(1) = -2$.

[3 marks]

Solution:

Concept:

- In a separable differential equation, rearrange to place all functions of y with dy , and all functions of x with dx , then integrate.
- Use $\frac{y}{y+3} = 1 - \frac{3}{y+3}$ and $\int du/u = \ln|u| + C$. An initial condition fixes the integration constant.

Step 1: Separate the variables The equation and the initial condition are

$$xy \frac{dy}{dx} = 2(y + 3), \quad y(1) = -2.$$

Near the specified point, $x \neq 0$ and $y + 3 \neq 0$, so division gives

$$\frac{y}{y+3} dy = \frac{2}{x} dx.$$

The constant function $y = -3$, excluded by this division, does satisfy the differential equation, but it does not satisfy the given initial condition.

Step 2: Integrate both sides Rewrite the fraction so that its integral is immediate:

$$\int \left(1 - \frac{3}{y+3}\right) dy = 2 \int \frac{dx}{x},$$
$$y - 3 \ln|y+3| = 2 \ln|x| + C.$$

The absolute values make the logarithmic antiderivatives valid wherever their arguments are nonzero.

Quick Tip to Solve: Divide y by $y + 3$ before integrating; the initial values make both logarithms zero and give $C = -2$ immediately.

Step 3: Apply the initial condition Substitute $x = 1$ and $y = -2$. Since $\ln 1 = 0$,

$$-2 - 3 \ln |-2 + 3| = 2 \ln 1 + C \Rightarrow C = -2.$$

The required local branch passes through $(1, -2)$, where $x > 0$ and $y+3 > 0$. As a check, differentiating the implicit relation gives

$$\left(1 - \frac{3}{y+3}\right) \frac{dy}{dx} = \frac{2}{x},$$

which rearranges to the original equation.

Step 4: Conclusion The particular solution, on the branch through $(1, -2)$, is

$$y - 3 \ln |y + 3| = 2 \ln |x| - 2.$$

Quick Tip to Solve: Divide y by $y + 3$ before integrating; the initial values make both logarithms zero and give $C = -2$ immediately.

OR

29(b). Solve the following differential equation: $(1 + x^2) \frac{dy}{dx} + 2xy = 4x^2$.

[3 marks]

Solution:

Concept:

- A first-order linear differential equation has the form $\frac{dy}{dx} + P(x)y = Q(x)$. Its integrating factor is $I(x) = e^{\int P(x) dx}$.
- Multiplication by the integrating factor turns the left side into $\frac{d}{dx}(Iy)$; integration then gives $Iy = \int IQ dx + C$.

Step 1: Put the equation into linear form Since $1 + x^2 > 0$ for every real x , divide the given equation by $1 + x^2$:

$$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}.$$

Thus $P(x) = 2x/(1+x^2)$ and $Q(x) = 4x^2/(1+x^2)$. The dependent variable y occurs only to the first power, as required for this method.

Step 2: Calculate the integrating factor Substitute $u = 1 + x^2$, so $du = 2x dx$, to obtain

$$\int \frac{2x}{1+x^2} dx = \ln(1+x^2).$$

An arbitrary nonzero constant multiplier in the integrating factor can be omitted. Consequently

$$I(x) = e^{\ln(1+x^2)} = 1 + x^2.$$

Quick Tip to Solve: Recognize $(1+x^2)y' + 2xy$ directly as the derivative of $(1+x^2)y$; this gives the integrating-factor result in one step.

Step 3: Integrate the resulting product derivative Multiply the linear equation by $1 + x^2$. By the product rule,

$$(1 + x^2) \frac{dy}{dx} + 2xy = \frac{d}{dx} [(1 + x^2)y] = 4x^2.$$

Integration with respect to x gives

$$(1 + x^2)y = \int 4x^2 dx = \frac{4}{3}x^3 + C.$$

Differentiating this relation recovers the equation exactly. No initial condition is specified, so the arbitrary real constant remains in the general solution.

Step 4: Conclusion The general solution is

$$y = \frac{\frac{4}{3}x^3 + C}{1 + x^2}, \quad C \in \mathbb{R}.$$

Quick Tip to Solve: Recognize $(1 + x^2)y' + 2xy$ directly as the derivative of $(1 + x^2)y$; this gives the integrating-factor result in one step.

30(a). A die with number 1 to 6 is biased such that $P(2) = \frac{3}{10}$ and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the dice, if the dice is thrown twice. [3 marks]

Solution:

Concept:

- In independent repetitions, the number X of successes follows the binomial distribution: $P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}$, where n is the number of trials and p is the success probability in each trial.
- The mean of a discrete random variable is $E(X) = \sum_r r P(X = r)$. For a binomial variable this simplifies to $E(X) = np$.

Step 1: Define success and its probability Let X be the number of appearances of the face numbered 2 in two throws. A success is obtaining 2, so $p = 3/10$. A failure means obtaining any of the other five faces, whose combined probability is

$$1 - p = 1 - \frac{3}{10} = \frac{7}{10}.$$

Each of those five faces has probability $7/50$, but their individual probabilities are not needed because only appearances of 2 are counted. Successive throws are independent and the success probability remains fixed.

Step 2: Form the probability distribution There may be no success, exactly one success, or two successes. For exactly one success there are two orders: success followed by failure, or failure followed

Quick Tip to Solve: For the expected number of appearances of one chosen face, use np directly: here $n = 2$ and $p = 3/10$, regardless of how the remaining probability is shared.

by success. Hence

$$\begin{aligned}P(X = 0) &= \left(\frac{7}{10}\right)^2 = \frac{49}{100}, \\P(X = 1) &= 2 \left(\frac{3}{10}\right) \left(\frac{7}{10}\right) = \frac{42}{100}, \\P(X = 2) &= \left(\frac{3}{10}\right)^2 = \frac{9}{100}.\end{aligned}$$

The three probabilities sum to 1, as required.

Step 3: Calculate the expected count Multiply each possible count by its probability and add:

$$E(X) = 0 \left(\frac{49}{100}\right) + 1 \left(\frac{42}{100}\right) + 2 \left(\frac{9}{100}\right) = \frac{60}{100}.$$

This also agrees with the binomial formula $np = 2(3/10)$. The mean counts appearances of 2; it is not the average of the face values.

Step 4: Conclusion The mean number of times 2 appears in two throws is $\boxed{\frac{3}{5} = 0.6}$.

Quick Tip to Solve: For the expected number of appearances of one chosen face, use np directly: here $n = 2$ and $p = 3/10$, regardless of how the remaining probability is shared.

OR

30(b). Two dice are thrown. Defined are the following two events A and B :

$A = \{(x, y) : x + y = 9\}$, $B = \{(x, y) : x \neq 3\}$, where (x, y) denote a point in the sample space.

Check if events A and B are independent or mutually exclusive.

[3 marks]

Solution:

Concept:

- For two fair dice the sample space consists of 36 equally likely ordered pairs (x, y) , where x is the first die's score and y is the second die's score. Thus an event's probability is its number of outcomes divided by 36.
- Events are independent exactly when $P(A \cap B) = P(A)P(B)$. They are mutually exclusive when $A \cap B = \emptyset$; the two conditions test different properties.

Step 1: List the outcomes of the first event The sum must be 9, and each score lies between 1 and 6. The possible ordered pairs are therefore

$$A = \{(3, 6), (4, 5), (5, 4), (6, 3)\}, \quad P(A) = \frac{4}{36} = \frac{1}{9}.$$

Order matters because $(4, 5)$ and $(5, 4)$ describe different outcomes of the two dice.

Step 2: Count the second event and the intersection For B , the first score may be 1, 2, 4, 5 or 6, while

Quick Tip to Solve: List A first and delete the pair with first coordinate 3 to get $A \cap B$; then check the product rule and the empty-intersection rule separately.

the second score may be any of the six numbers. Thus

$$P(B) = \frac{5 \times 6}{36} = \frac{5}{6}.$$

Remove (3, 6) from A because its first score is 3. The other three outcomes satisfy both conditions:

$$A \cap B = \{(4, 5), (5, 4), (6, 3)\}, \quad P(A \cap B) = \frac{3}{36} = \frac{1}{12}.$$

Step 3: Apply both event tests Compare the intersection probability with the product:

$$P(A)P(B) = \frac{1}{9} \times \frac{5}{6} = \frac{5}{54} = \frac{10}{108}, \quad P(A \cap B) = \frac{1}{12} = \frac{9}{108}.$$

The two probabilities are unequal, so the independence condition fails. Also, the intersection contains three outcomes, showing that the two events can occur together. Consequently, the condition for mutual exclusivity fails as well.

Step 4: Conclusion The events A and B are neither independent nor mutually exclusive.

Quick Tip to Solve: List A first and delete the pair with first coordinate 3 to get $A \cap B$; then check the product rule and the empty-intersection rule separately.

31. f and g are continuous functions on interval $[a, b]$. Given that $f(a-x) = f(x)$ and $g(x) + g(a-x) = a$, show that $\int_0^a f(x)g(x)dx = \frac{a}{2} \int_0^a f(x)dx$. [3 marks]

Solution:

Concept:

- Reflection of the variable in a definite integral gives $\int_0^a h(x)dx = \int_0^a h(a-x)dx$. It follows from the substitution $x = a - t$ and reversal of the transformed limits.
- Linearity allows integrals with the same limits to be added inside a single integral, and a constant factor can be taken outside. Use the two given functional identities after reflection.

Step 1: Reflect the integration variable Denote the integral on the left by

$$I = \int_0^a f(x)g(x)dx.$$

Put $x = a - t$, so $dx = -dt$. When $x = 0$, $t = a$, and when $x = a$, $t = 0$. Reversing the limits removes the minus sign:

$$\begin{aligned} I &= - \int_a^0 f(a-t)g(a-t)dt \\ &= \int_0^a f(a-t)g(a-t)dt. \end{aligned}$$

Rename the dummy variable t as x .

Step 2: Use the symmetry of the first function The given identity $f(a-x) = f(x)$ changes the

Quick Tip to Solve: When a problem pairs x with $a-x$, reflect the integral and add: the symmetric factor stays unchanged and the other two factors combine into the given constant.

reflected expression into

$$I = \int_0^a f(x)g(a-x) dx.$$

This has the same factor $f(x)$ as the original integral, but replaces $g(x)$ by $g(a-x)$.

Step 3: Add the two expressions Add the original integral and its reflected expression, keeping the same limits:

$$\begin{aligned} 2I &= \int_0^a f(x)\{g(x) + g(a-x)\} dx \\ &= \int_0^a a f(x) dx \\ &= a \int_0^a f(x) dx. \end{aligned}$$

Use $g(x) + g(a-x) = a$, then take the constant a outside the integral.

Step 4: Conclusion Dividing by 2 proves the required identity:

$$\boxed{\int_0^a f(x)g(x) dx = \frac{a}{2} \int_0^a f(x) dx.}$$

Quick Tip to Solve: When a problem pairs x with $a-x$, reflect the integral and add: the symmetric factor stays unchanged and the other two factors combine into the given constant.

Section D (20 marks)

(This section comprises of 4 Long Answer (LA) type questions of 5 marks each.)

32(a). Find the shortest distance between the lines:

$$\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3} \text{ and } \frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}.$$

[5 marks]

Solution:

Concept:

- A line in symmetric form gives a point $\vec{a}$ and a direction vector $\vec{b}$; its vector form is $\vec{r} = \vec{a} + t\vec{b}$.
- For two non-parallel lines, $\vec{n} = \vec{b}_1 \times \vec{b}_2$ is perpendicular to both. Their shortest distance is $d = |(\vec{a}_2 - \vec{a}_1) \cdot \vec{n}|/|\vec{n}|$, the absolute projection of a joining vector onto this common normal.

Step 1: Identify points and direction vectors Read each numerator carefully: $x+1 = x - (-1)$ and $y+15 = y - (-15)$. Thus we may take points $A(-1, 1, 9)$ and $B(3, -15, 9)$ on the respective lines, with

$$\vec{b}_1 = (2, 1, -3), \quad \vec{b}_2 = (2, -7, 5), \quad \vec{AB} = (4, -16, 0).$$

Since these direction vectors are not proportional, the parallel-line distance formula does not apply.

Step 2: Calculate the common normal Expand the cross product with the negative sign in the middle

Quick Tip to Solve: Here the cross product is a multiple of $(1, 1, 1)$; use the simpler unit normal $(1, 1, 1)/\sqrt{3}$ and project $(4, -16, 0)$ onto it.

cofactor:

$$\begin{aligned}\vec{n} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 2 & -7 & 5 \end{vmatrix} \\ &= (5 - 21)\hat{i} - (10 + 6)\hat{j} + (-14 - 2)\hat{k} \\ &= -16\hat{i} - 16\hat{j} - 16\hat{k}, \\ |\vec{n}| &= \sqrt{256 + 256 + 256} = 16\sqrt{3}.\end{aligned}$$

Its nonzero magnitude confirms that the common-normal direction is well defined.

Step 3: Project the joining vector onto the normal The scalar triple product is

$$\vec{AB} \cdot \vec{n} = 4(-16) + (-16)(-16) + 0(-16) = 192.$$

Hence the distance calculation gives

$$d = \frac{|192|}{16\sqrt{3}} = \frac{12}{\sqrt{3}}.$$

The triple product is nonzero, so the lines are skew and do not intersect. The absolute value ensures a positive distance irrespective of the chosen order of the two lines.

Step 4: Verify using points on the common perpendicular As a check, the first line at parameter $t = 2$ gives $P = (3, 3, 3)$; the second at parameter $s = -2$ gives $Q = (-1, -1, -1)$. Thus

$$\vec{PQ} = (-4, -4, -4), \quad \vec{PQ} \cdot \vec{b}_1 = 0, \quad \vec{PQ} \cdot \vec{b}_2 = 0.$$

Therefore this segment is perpendicular to both lines. Its length $\sqrt{16 + 16 + 16}$ agrees with the projection calculation, providing an independent check of the signs and coordinates.

Step 5: Conclusion The shortest distance between the two lines is $4\sqrt{3}$ units.

Quick Tip to Solve: Here the cross product is a multiple of $(1, 1, 1)$; use the simpler unit normal $(1, 1, 1)/\sqrt{3}$ and project $(4, -16, 0)$ onto it.

OR

32(b). Find the image A' of the point $A(2, 1, 2)$ in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA' . Find the foot of perpendicular from point A on the line l . [5 marks]

Solution:

Concept:

- The perpendicular foot H lies on the given line and satisfies $\vec{AH} \cdot \vec{b} = 0$, where $\vec{b}$ is the line's direction vector.
- The line of reflection bisects AA' perpendicularly. Thus $\vec{OA'} = 2\vec{OH} - \vec{OA}$; the joining line has direction $\vec{AA'}$.

Step 1: Find the perpendicular foot The direction vector of l is $\vec{b} = (1, -1, -1)$. Let $H = (4 +$

Quick Tip to Solve: Find the foot using one dot product, then double its coordinates and subtract the original point to obtain the image.

$\lambda, 2 - \lambda, 2 - \lambda$ be the foot on l . Subtracting $A = (2, 1, 2)$ gives $\overrightarrow{AH} = (2 + \lambda, 1 - \lambda, -\lambda)$. Apply perpendicularity:

$$\begin{aligned}\overrightarrow{AH} \cdot \vec{b} &= (2 + \lambda) - (1 - \lambda) + \lambda = 0, \\ 1 + 3\lambda &= 0 \Rightarrow \lambda = -\frac{1}{3}.\end{aligned}$$

Substitute into the point on l :

$$H = \left(4 - \frac{1}{3}, 2 + \frac{1}{3}, 2 + \frac{1}{3}\right) = \left(\frac{11}{3}, \frac{7}{3}, \frac{7}{3}\right).$$

Step 2: Use the midpoint relation to find the image Let $A' = (x', y', z')$. Since H is the midpoint of AA' , double its coordinates and subtract those of A :

$$(x', y', z') = 2\left(\frac{11}{3}, \frac{7}{3}, \frac{7}{3}\right) - (2, 1, 2) = \left(\frac{16}{3}, \frac{11}{3}, \frac{8}{3}\right).$$

Step 3: Form and verify the joining line Subtracting the coordinates of A from A' gives

$$\overrightarrow{AA'} = \left(\frac{10}{3}, \frac{8}{3}, \frac{2}{3}\right) = \frac{2}{3}(5, 4, 1).$$

A direction vector is therefore $(5, 4, 1)$. The joining line through A has parametric form $(x, y, z) = (2 + 5\mu, 1 + 4\mu, 2 + \mu)$. It contains H at $\mu = 1/3$ and A' at $\mu = 2/3$. Also, $(5, 4, 1) \cdot (1, -1, -1) = 0$, verifying perpendicularity to l .

Step 4: Conclusion The image is $A' = (16/3, 11/3, 8/3)$, the foot is $H = (11/3, 7/3, 7/3)$, and the joining line is

$$\frac{x-2}{5} = \frac{y-1}{4} = \frac{z-2}{1}.$$

Quick Tip to Solve: Find the foot using one dot product, then double its coordinates and subtract the original point to obtain the image.

33. Find: $\int \frac{5x}{(x+1)(x^2+9)} dx.$

[5 marks]

Solution:

Concept:

- A proper rational function with a linear factor and an irreducible quadratic factor is decomposed as a constant over the linear factor plus a linear numerator over the quadratic factor.
- Use $\int f'(x)/f(x) dx = \ln|f(x)| + C$ and $\int dx/(x^2 + a^2) = a^{-1} \tan^{-1}(x/a) + C$, where $a > 0$.

Step 1: Set up the partial fractions The numerator has degree smaller than the denominator, so

Quick Tip to Solve: Set $x = -1$ after clearing denominators to obtain $A = -1/2$ immediately; then compare coefficients to find the linear numerator over $x^2 + 9$.

polynomial division is unnecessary. Since $x^2 + 9$ has no real linear factors, write

$$\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+D}{x^2+9}.$$

Multiplying by the common denominator gives the polynomial identity

$$\begin{aligned} 5x &= A(x^2+9) + (Bx+D)(x+1) \\ &= (A+B)x^2 + (B+D)x + (9A+D). \end{aligned}$$

Step 2: Determine the coefficients Equating coefficients of like powers of x , we obtain

$$A+B=0, \quad B+D=5, \quad 9A+D=0.$$

The first and third equations give $B = -A$ and $D = -9A$. Substitution in the second yields $-10A = 5$. Therefore,

$$A = -\frac{1}{2}, \quad B = \frac{1}{2}, \quad D = \frac{9}{2}.$$

Thus the original integrand separates into three elementary terms:

$$\frac{5x}{(x+1)(x^2+9)} = -\frac{1}{2(x+1)} + \frac{x}{2(x^2+9)} + \frac{9}{2(x^2+9)}.$$

Step 3: Integrate the elementary terms Denote the required integral by I . By linearity,

$$I = -\frac{1}{2} \int \frac{dx}{x+1} + \frac{1}{2} \int \frac{x dx}{x^2+9} + \frac{9}{2} \int \frac{dx}{x^2+9}.$$

In the middle integral, $u = x^2 + 9$ gives $du = 2x dx$, so its contribution is $\frac{1}{4} \ln(x^2 + 9)$. In the last integral, use $a = 3$: multiplication by $9/2$ gives $\frac{3}{2} \tan^{-1}(x/3)$. The first term contributes $-\frac{1}{2} \ln|x+1|$. Since $x^2 + 9 > 0$, only the linear factor needs absolute-value signs. Combine the integration constants into one constant C , on any interval excluding $x = -1$.

Step 4: Conclusion The required integral is

$$\boxed{-\frac{1}{2} \ln|x+1| + \frac{1}{4} \ln(x^2+9) + \frac{3}{2} \tan^{-1}\left(\frac{x}{3}\right) + C}.$$

Quick Tip to Solve: Set $x = -1$ after clearing denominators to obtain $A = -1/2$ immediately; then compare coefficients to find the linear numerator over $x^2 + 9$.

34(a). Given $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB . Hence, solve the system of linear equations:

$$x - y + z = 4,$$

$$x - 2y - 2z = 9,$$

$$2x + y + 3z = 1.$$

[5 marks]

Solution:

Concept:

- In a matrix product AB , the entry in row i , column j is the dot product of row i of A with column j of B .
- A linear system with coefficient matrix B is written $BX = D$, where X is the column of unknowns and D is the column of constants. If $AB = kI$ with $k \neq 0$, multiplying by A gives $kX = AD$.

Step 1: Multiply the given matrices Use the rows of A and the columns of B in their printed order. Each entry is obtained by multiplying corresponding elements and adding:

$$AB = \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}.$$

Simplifying the nine entries yields

$$AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I_3,$$

where I_3 is the identity matrix of order three. Thus $A/8$ is the inverse of B ; in particular the given system has a unique solution.

Step 2: Express the system as a matrix equation The coefficients of x, y, z in the first, second and third equations are precisely the three rows of B . Therefore, with the unknowns in the order x, y, z ,

$$BX = D, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad D = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}.$$

Premultiply both sides by A . Associativity of matrix multiplication and the product already calculated give

$$A(BX) = AD \implies (AB)X = AD \implies 8X = AD.$$

The order matters: premultiplication puts the product AB next to the vector of unknowns.

Step 3: Evaluate the column of unknowns Calculate the product on the right using the same

Quick Tip to Solve: Recognise B as the coefficient matrix first. Once $AB = 8I_3$ is found, compute $AD/8$ directly instead of finding another inverse by cofactors.

row-by-column rule:

$$\begin{aligned} AD &= \begin{bmatrix} -4(4) + 4(9) + 4(1) \\ -7(4) + 1(9) + 3(1) \\ 5(4) - 3(9) - 1(1) \end{bmatrix} \\ &= \begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}. \end{aligned}$$

Dividing each entry of $8X = AD$ by eight gives

$$X = \frac{1}{8} \begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}.$$

Step 4: Verify all three original equations Substitute the entries of X back into the printed system:

$$\begin{aligned} x - y + z &= 3 - (-2) + (-1) = 4, \\ x - 2y - 2z &= 3 - 2(-2) - 2(-1) = 9, \\ 2x + y + 3z &= 2(3) + (-2) + 3(-1) = 1. \end{aligned}$$

Each computed left-hand side equals the required right-hand side, confirming the matrix calculation and the signs of all three unknowns.

Step 5: Conclusion

$$AB = 8I_3, \quad x = 3, \quad y = -2, \quad z = -1.$$

Quick Tip to Solve: Recognise B as the coefficient matrix first. Once $AB = 8I_3$ is found, compute $AD/8$ directly instead of finding another inverse by cofactors.

34(b). If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find A^{-1} . Hence, solve the system of linear equations:

$$\begin{aligned} x - 2y &= 10, \\ 2x - y - z &= 8, \\ -2y + z &= 7. \end{aligned}$$

[5 marks]

Solution:**Concept:**

- For a square matrix A , the inverse exists when $|A| \neq 0$, and $A^{-1} = \frac{\text{adj } A}{|A|}$. The adjugate is the transpose of the cofactor matrix.
- In a matrix equation $BX = C$, multiplication by B^{-1} gives $X = B^{-1}C$. Also, $(A^T)^{-1} = (A^{-1})^T$.

Step 1: Calculate the determinant Expanding the given matrix along its first row,

$$|A| = 1\{(-1)(1) - (-2)(-1)\} - 2\{(-2)(1) - (-2)(0)\} = -3 + 4 = 1.$$

Thus the matrix is nonsingular and its inverse exists. Notice that the coefficient matrix of the printed system is the transpose of A , so the transpose will be needed when applying this inverse.

Step 2: Find cofactors and the inverse The signed minors give the cofactor matrix

$$C_A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}.$$

For example, $C_{12} = -\{(-2)(1) - (-2)(0)\} = 2$, $C_{23} = -\{1(-1) - 2(0)\} = 1$, and $C_{31} = 2(-2) - 0(-1) = -4$. Transposing the complete cofactor matrix and dividing by the determinant yields

$$A^{-1} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}.$$

As a check, multiplying A by this matrix produces the identity: its first row gives $(1, 0, 0)$, its second row gives $(0, 1, 0)$, and its third row gives $(0, 0, 1)$.

Step 3: Express the equations in matrix form The equations $x - 2y = 10$, $2x - y - z = 8$ and $-2y + z = 7$ become

$$A^T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}, \quad A^T = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}.$$

Quick Tip to Solve: Compare the coefficients row by row before using the inverse: here the equation matrix is A^T , so use $(A^{-1})^T$.

Therefore

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = (A^{-1})^T \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}.$$

Step 4: Multiply and check the solution Computing the three entries gives

$$x = -30 + 16 + 14 = 0,$$

$$y = -20 + 8 + 7 = -5,$$

$$z = -40 + 16 + 21 = -3.$$

Substitution into the original equations verifies all three: $0 - 2(-5) = 10$, $2(0) - (-5) - (-3) = 8$, and $-2(-5) + (-3) = 7$. Since the coefficient matrix is invertible, these values give the unique solution.

Step 5: Conclusion $A^{-1} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, and the system has solution $x = 0, y = -5, z = -3$.

Quick Tip to Solve: Compare the coefficients row by row before using the inverse: here the equation matrix is A^T , so use $(A^{-1})^T$.

35. Using integration, find the area of the region bounded by the line $y = 5x + 2$, the x -axis and the ordinates $x = -2$ and $x = 2$. [5 marks]

Solution:

Concept:

- The geometric area between a curve $y = f(x)$ and the x -axis from a to b is $\int_a^b |f(x)| dx$, so each contribution must be non-negative.
- If the curve crosses the axis inside the interval, split the integral at the crossing and use the upper boundary minus the lower boundary on each subinterval.

Step 1: Find the crossing and the sign of the line The line meets the x -axis when

$$5x + 2 = 0 \implies x = -\frac{2}{5}.$$

This value lies between the two ordinates $x = -2$ and $x = 2$. On $[-2, -2/5]$ the line is below the axis, since $5x + 2 \leq 0$; on $[-2/5, 2]$ it is above the axis. These signs determine the two positive area integrals.

Step 2: Set up the required area using integration On the left interval the upper boundary is $y = 0$, so its vertical height above the line is $-(5x + 2)$. On the right interval the upper boundary is the line and the lower boundary is $y = 0$. Consequently,

$$A = -\int_{-2}^{-2/5} (5x + 2) dx + \int_{-2/5}^2 (5x + 2) dx.$$

Quick Tip to Solve: Locate the zero of the line before integrating; split there and reverse the sign only on the interval where the line lies below the axis.

A single signed integral would subtract the left area from the right area, instead of adding them.

Step 3: Evaluate the contribution below the axis An antiderivative is $F(x) = \frac{5}{2}x^2 + 2x$. At the relevant limits,

$$F(-2) = 10 - 4 = 6, \quad F\left(-\frac{2}{5}\right) = \frac{2}{5} - \frac{4}{5} = -\frac{2}{5}.$$

Thus the left area is

$$\mathcal{A}_1 = -[F(x)]_{-2}^{-2/5} = -\left(-\frac{2}{5} - 6\right) = \frac{32}{5}.$$

The minus sign converts the negative signed integral into a positive area.

Step 4: Evaluate the contribution above the axis At the right endpoint, $F(2) = 10 + 4 = 14$. Therefore

$$\mathcal{A}_2 = [F(x)]_{-2/5}^2 = 14 - \left(-\frac{2}{5}\right) = \frac{72}{5}.$$

Adding the two non-negative contributions gives

$$\mathcal{A} = \frac{32}{5} + \frac{72}{5} = \frac{104}{5}.$$

Both contributions are positive, as required for geometric area.

Step 5: Conclusion The required area is $\boxed{\frac{104}{5} \text{ square units}}$.

Quick Tip to Solve: Locate the zero of the line before integrating; split there and reverse the sign only on the interval where the line lies below the axis.

Section E (12 marks)

(This section comprises of 3 case study based questions of 4 marks each.)

36. Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated).

Based on the above, answer the following:

[4 marks]

36(i)(a). What is the probability that a randomly selected car is an electric car?

[2 marks]

Solution:

Concept:

- Let A , B and C denote manufacture by Amber, Bonzi and Comet, respectively, and let E denote selection of an electric car.
- The manufacturers form a mutually exclusive and exhaustive partition, so the total probability is $P(E) = P(A)P(E | A) + P(B)P(E | B) + P(C)P(E | C)$.

Step 1: Identify the market shares and conditional probabilities A random car belongs to Amber,

Quick Tip to Solve: Multiply each manufacturer's market share by its electric fraction before adding the three contributions.

Bonzi or Comet with probabilities $P(A) = 0.60$, $P(B) = 0.30$, and $P(C) = 0.10$. Within these manufacturers' outputs, the electric proportions give $P(E | A) = 0.20$, $P(E | B) = 0.10$, and $P(E | C) = 0.05$. These proportions refer to each manufacturer's own production, so they must be weighted by the corresponding market shares.

Step 2: Add the probabilities of the three electric-car routes The three disjoint possibilities are an electric car from Amber, an electric car from Bonzi, and an electric car from Comet. Applying the multiplication rule within each route and then adding,

$$\begin{aligned} P(E) &= (0.60)(0.20) + (0.30)(0.10) + (0.10)(0.05) \\ &= 0.120 + 0.030 + 0.005 = 0.155. \end{aligned}$$

A simple average of the three electric proportions would incorrectly treat the manufacturers as having equal market shares.

Step 3: Conclusion The probability that the randomly selected car is electric is $\frac{31}{200} = 0.155$, or 15.5%.

Quick Tip to Solve: Multiply each manufacturer's market share by its electric fraction before adding the three contributions.

OR

36(i)(b). What is the probability that a randomly selected car is a petrol car?

[2 marks]

Solution:

Concept:

- The law of total probability weights each manufacturer's conditional probability by its market share. Let A, B, C denote cars manufactured by Amber, Bonzi and Comet, respectively.
- Petrol and electric cars are complementary categories in the given production data. If E denotes electric and P denotes petrol, then $P(P | A) = 1 - P(E | A)$, and similarly for the other manufacturers.

Step 1: Find each manufacturer's petrol fraction The market shares are $P(A) = 0.60$, $P(B) = 0.30$ and $P(C) = 0.10$. Subtract each electric fraction from one:

$$\begin{aligned} P(P | A) &= 1 - 0.20 = 0.80, \\ P(P | B) &= 1 - 0.10 = 0.90, \\ P(P | C) &= 1 - 0.05 = 0.95. \end{aligned}$$

These fractions describe petrol cars within the individual manufacturers' production, not within the whole market.

Step 2: Weight the petrol fractions by market share Every car belongs to exactly one manufacturer,

Quick Tip to Solve: Weight each electric fraction by its market share to get 0.155; the petrol probability is its complement $1 - 0.155$.

so the three disjoint contributions can be added:

$$\begin{aligned}P(P) &= P(A)P(P | A) + P(B)P(P | B) + P(C)P(P | C) \\&= 0.60(0.80) + 0.30(0.90) + 0.10(0.95) \\&= 0.48 + 0.27 + 0.095 = 0.845.\end{aligned}$$

Step 3: Conclusion The probability that a randomly selected car is petrol is $0.845 = \frac{169}{200}$.

Quick Tip to Solve: Weight each electric fraction by its market share to get 0.155; the petrol probability is its complement $1 - 0.155$.

36(ii). A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet? [1 mark]

Solution:

Concept:

- Let A, B, C denote Amber, Bonzi and Comet, and let E denote an electric car. The total probability of E is the sum of the manufacturers' electric-car contributions.
- Bayes' theorem gives $P(C | E) = P(C)P(E | C)/P(E)$: the electric Comet contribution is divided by the entire electric-car share.

Step 1: Calculate the electric-car share Multiply each market share by its conditional electric fraction:

$$P(E) = 0.60(0.20) + 0.30(0.10) + 0.10(0.05) = 0.155.$$

Step 2: Condition on the car being electric Comet contributes $P(C \cap E) = 0.10(0.05) = 0.005$ of all cars. Therefore,

$$P(C | E) = \frac{0.005}{0.155} = \frac{1}{31}.$$

The denominator is the electric-car share because the selected car is already known to be electric.

Step 3: Conclusion The required probability that the electric car was manufactured by Comet is $\frac{1}{31}$.

Quick Tip to Solve: For a manufacturer given an electric car, divide that manufacturer's electric contribution by the sum of all three electric contributions.

36(iii). A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi? [1 mark]

Solution:

Concept:

- Let A, B, C represent the manufacturers Amber, Bonzi and Comet, and E an electric car. Manufacturer events are mutually exclusive, so their contributions add.
- Conditional probability gives $P(A \cup B | E) = P((A \cup B) \cap E) / P(E)$.

Step 1: Calculate the electric-car contributions Multiply each manufacturer's market share by its electric fraction:

$$P(A \cap E) = 0.60(0.20) = 0.12,$$

$$P(B \cap E) = 0.30(0.10) = 0.03,$$

$$P(C \cap E) = 0.10(0.05) = 0.005.$$

Hence the total electric share is $P(E) = 0.155$.

Step 2: Restrict the electric cars to Amber or Bonzi A car cannot be manufactured by both Amber and Bonzi, so the favourable electric share is $0.12 + 0.03 = 0.15$. Thus

$$P(A \cup B | E) = \frac{0.15}{0.155} = \frac{30}{31}.$$

Step 3: Conclusion The probability that the electric car was manufactured by Amber or Bonzi is $\boxed{\frac{30}{31}}$.

Quick Tip to Solve: Since Amber, Bonzi and Comet exhaust the market, subtract the conditional Comet probability $1/31$ from one.

37. A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres.

Based on the above, answer the following:

[4 marks]

37(i). Find the intervals on which $f(x)$ is increasing or decreasing, $x \in [0, \pi]$.

[2 marks]

Solution:

Concept:

- The sign of $f'(x)$ determines monotonicity: a positive derivative gives increase, while a negative derivative gives decrease.
- By the product rule, for $f(x) = e^x \sin x$, $f'(x) = e^x(\sin x + \cos x)$. The factor e^x is positive for every real x .

Step 1: Find the stationary point in the stated interval Using $\sin x + \cos x = \sqrt{2} \sin(x + \pi/4)$, we have

$$f'(x) = \sqrt{2}e^x \sin\left(x + \frac{\pi}{4}\right).$$

Quick Tip to Solve: Factor out the positive e^x and rewrite $\sin x + \cos x$ as a shifted sine to read the sign on $[0, \pi]$.

The zero condition is $x + \pi/4 = n\pi$. In $[0, \pi]$, only $x = 3\pi/4$ satisfies it; the other values lie outside the interval.

Step 2: Determine the derivative signs For $0 < x < 3\pi/4$, the angle $x + \pi/4$ lies between $\pi/4$ and π , where sine is positive. Hence $f'(x) > 0$. For $3\pi/4 < x < \pi$, that angle lies between π and $5\pi/4$, where sine is negative, so $f'(x) < 0$. At $3\pi/4$ the derivative is zero; this isolated zero does not prevent strict monotonicity on either closed segment. The endpoint values $f(0) = f(\pi) = 0$ also agree with an increase followed by a decrease.

Step 3: Conclusion f is increasing on $[0, 3\pi/4]$ and decreasing on $[3\pi/4, \pi]$; the derivative has the respective signs on the interiors.

Quick Tip to Solve: Factor out the positive e^x and rewrite $\sin x + \cos x$ as a shifted sine to read the sign on $[0, \pi]$.

37(ii). Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion. [2 marks]

Solution:

Concept:

- An interior critical point occurs where $f'(x) = 0$ or the derivative does not exist. The function $e^x \sin x$ is differentiable everywhere.
- At a stationary point c , $f''(c) < 0$ implies a local maximum and $f''(c) > 0$ implies a local minimum. A point of inflexion requires a change in concavity.

Step 1: Identify the critical point From the product rule,

$$f'(x) = e^x(\sin x + \cos x) = \sqrt{2}e^x \sin(x + \pi/4).$$

Solving $f'(x) = 0$ on $[0, \pi]$ gives the single interior critical point $c = 3\pi/4$. There are no points where the derivative fails to exist.

Step 2: Apply the second derivative test Differentiating once more gives

$$f''(x) = e^x(\sin x + \cos x) + e^x(\cos x - \sin x) = 2e^x \cos x.$$

Therefore

$$f''(3\pi/4) = -\sqrt{2}e^{3\pi/4} < 0.$$

Equivalently, f' changes from positive to negative there. The concavity is negative on both sides sufficiently near this point, so it is not an inflexion point. The maximum value is $f(3\pi/4) = e^{3\pi/4}/\sqrt{2}$. Although concavity changes at $x = \pi/2$, that point is not critical, since $f'(\pi/2) = e^{\pi/2} \neq 0$.

Step 3: Conclusion The only interior critical point, $x = 3\pi/4$, is a local maximum. There is no local minimum or point of inflexion among the critical points.

Quick Tip to Solve: Differentiate twice and substitute the stationary value; an inflexion point need not be a critical point, so keep the two conditions separate.

38. A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$.

Based on the above, answer the following:

[4 marks]

38(i). How many relations can be there from S to J ?

[1 mark]

Solution:

Concept:

- A relation from S to J is any subset of the Cartesian product $S \times J$.
- A set containing n elements has 2^n subsets, including the empty set and the whole set.

Step 1: Count the possible ordered pairs There are four speakers and three judges. Each speaker can be paired with each judge, giving

$$|S \times J| = |S| |J| = 4 \times 3 = 12.$$

Step 2: Count all subsets of the product For each of these twelve ordered pairs, a relation either includes the pair or excludes it. These independent choices give 2^{12} subsets. The question asks for relations in general, so we count all subsets; the rule of exactly one image per speaker would instead count functions.

Step 3: Conclusion The number of relations from S to J is $2^{12} = 4096$.

Quick Tip to Solve: For relations use $2^{|S||J|}$; reserve $|J|^{|S|}$ for functions with exactly one image per speaker.

38(ii). A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$. Check if it is bijective.

[1 mark]

Solution:

Concept:

- A function is one-one if distinct domain elements always have distinct images.
- A function is onto if its range equals its codomain; a bijection must be both one-one and onto.

Step 1: Test whether the function is one-one From the given ordered pairs, $f(S_2) = J_2 = f(S_3)$, although $S_2 \neq S_3$. Thus two distinct speakers have the same judge, so the function fails the one-one condition.

Step 2: Test whether the function is onto Judge J_1 is the image of S_1 , judge J_2 is the image of S_2 and S_3 , and judge J_3 is the image of S_4 . Therefore the range is $\{J_1, J_2, J_3\} = J$, and the function is onto.

Step 3: Conclusion The given function is onto but not one-one. Hence it is not bijective.

Quick Tip to Solve: First look for a repeated second component: the repeated J_2 immediately rules out bijectivity.

38(iii)(a). How many one-one functions can be there from set S to set J ?

[2 marks]

Solution:

Concept:

- A one-one function maps every pair of distinct domain elements to distinct codomain elements.
- For finite sets, a one-one function from S to J can exist only if $|S| \leq |J|$.

Step 1: Compare the sizes of the two sets The domain $S = \{S_1, S_2, S_3, S_4\}$ has four speakers, while the codomain $J = \{J_1, J_2, J_3\}$ has only three judges. A one-one assignment would require four different judges, one for each speaker. Hence the necessary condition $|S| \leq |J|$ is not satisfied.

Step 2: Apply the distinct-image requirement Assigning different judges to the first three speakers uses all three members of J . The fourth speaker still requires an image because every domain element must be assigned exactly one judge, but there is no unused judge available. Reusing a judge gives two distinct speakers the same image and violates injectivity. This reasoning applies to every possible ordering of assignments, so no assignment can be one-one.

Step 3: Conclusion The number of one-one functions from S to J is 0.

Quick Tip to Solve: Compare set sizes before counting: four domain elements cannot have distinct images in a codomain of three elements.

OR

38(iii)(b). Another student considers a relation $R_1 = \{(S_1, S_2), (S_2, S_4)\}$ in set S . Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric.

[2 marks]

Solution:

Concept:

- A relation on a set S is reflexive if (x, x) belongs to it for every $x \in S$. Thus every element must be related to itself.
- A relation is symmetric if (x, y) in the relation always implies (y, x) is also present. A single pair whose reverse is absent is enough to disprove symmetry.

Step 1: Identify all missing diagonal pairs The given set is $S = \{S_1, S_2, S_3, S_4\}$, while the given relation contains only (S_1, S_2) and (S_2, S_4) . Neither is a diagonal pair. Reflexivity therefore requires adding

$$D = \{(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)\}.$$

All four pairs are compulsory: omitting even one would leave the corresponding element unrelated to itself. In particular, (S_3, S_3) is required even though S_3 does not occur in the original relation.

Step 2: Check that symmetry still fails Let the enlarged relation be $R'_1 = R_1 \cup D$. It contains all the required diagonal pairs, so it is reflexive on S . However, $(S_1, S_2) \in R'_1$ and $(S_2, S_1) \notin R'_1$; hence it is not symmetric. The newly added diagonal pairs are their own reverses and do not supply the missing reverse of this off-diagonal pair.

Quick Tip to Solve: Add only the missing diagonal pairs for reflexivity, then keep one off-diagonal pair without its reverse to ensure symmetry fails.

Step 3: Conclusion The minimum addition is 4 ordered pairs:

$$(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4).$$

The resulting relation is reflexive but not symmetric.

Quick Tip to Solve: Add only the missing diagonal pairs for reflexivity, then keep one off-diagonal pair without its reverse to ensure symmetry fails.