

CBSE Class 12 Mathematics Question Paper 2025 with Solution PDF-Set 65-2-3

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 hours

Maximum Marks: 80

General Instructions

- (i) This Question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.
- (iii) In **Section A** – Question Number **1 to 18** are Multiple Choice Questions (MCQs) type and Question Number **19 & 20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B** – Question Number **21 to 25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C** – Question Number **26 to 31** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D** – Question Number **32 to 35** are Long Answer (LA) type questions, carrying **5** marks each.
- (vii) In **Section E** – Question Number **36 to 38** are case study based questions, carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, **2** questions in Section – **D** and **2** questions in Section – **E**.
- (ix) Use of calculator is **NOT** allowed.

Section A (20 marks)

(This section comprises of 20 multiple choice questions (MCQs) of 1 mark each.)

1. If $\vec{p}$ and $\vec{q}$ are unit vectors, then which of the following values of $\vec{p} \cdot \vec{q}$ is not possible? [1 mark]

- (A) $-\frac{1}{2}$
- (B) $\frac{1}{\sqrt{2}}$
- (C) $\frac{\sqrt{3}}{2}$
- (D) $\sqrt{3}$

Correct Answer: (D) $\sqrt{3}$

Solution:

Concept:

- For vectors making an angle θ , $\vec{p} \cdot \vec{q} = |\vec{p}||\vec{q}|\cos\theta$.
- Unit vectors have magnitude one, so their scalar product belongs to $[-1, 1]$.

Step 1: Apply the unit-vector condition Since $|\vec{p}| = |\vec{q}| = 1$, the dot product equals $\cos\theta$, where $0 \leq \theta \leq \pi$. Hence its absolute value cannot exceed one. This bound also follows from the Cauchy-Schwarz inequality.

Step 2: Check each printed value Option (A), $-1/2$, occurs at $\theta = 2\pi/3$; option (B), $1/\sqrt{2}$, occurs at $\theta = \pi/4$; and option (C), $\sqrt{3}/2$, occurs at $\theta = \pi/6$. All lie within the permitted interval. But $\sqrt{3} > 1$, so option (D) violates the bound.

Step 3: Conclusion The impossible value is $\sqrt{3}$. Hence, the correct option is (D).

Quick Tip to Solve: For unit vectors, immediately compare each proposed dot product with the interval $[-1, 1]$.

2. Which of the following can be both a symmetric and skew-symmetric matrix? [1 mark]

- (A) Unit Matrix
- (B) Diagonal Matrix
- (C) Null Matrix
- (D) Row Matrix

Correct Answer: (C) Null Matrix

Solution:

Concept:

- A symmetric matrix satisfies $A^T = A$; a skew-symmetric matrix satisfies $A^T = -A$.
- The null matrix O has every entry zero and satisfies $-O = O$.

Step 1: Impose both transpose conditions If a real square matrix is simultaneously symmetric and skew-symmetric, then $A = A^T = -A$. Adding A to both sides gives $2A = O$, and therefore every

Quick Tip to Solve: Equate the two transpose expressions: $A = -A$ immediately forces every real entry to be zero.

entry of A must be zero. Thus the two conditions together determine the matrix completely.

Step 2: Verify the options The null matrix obeys $O^T = O = -O$. The unit matrix has nonzero diagonal entries, so it cannot be skew-symmetric. A general diagonal matrix is symmetric but its nonzero diagonal entries prevent skew-symmetry. A row matrix is generally not square, so it cannot have both transpose properties. A zero diagonal matrix is precisely the null matrix already named in option (C).

Step 3: Conclusion Only the Null Matrix has both properties. Hence, the correct option is (C).

Quick Tip to Solve: Equate the two transpose expressions: $A = -A$ immediately forces every real entry to be zero.

3. If $\int_0^a x \, dx \leq \frac{a}{2} + 6$, then

[1 mark]

- (A) $-4 \leq a \leq 3$
- (B) $a \geq 4, a \leq -3$
- (C) $-3 \leq a \leq 4$
- (D) $-3 \leq a \leq 0$

Correct Answer: (C) $-3 \leq a \leq 4$

Solution:

Concept:

- The antiderivative of x is $x^2/2$, so $\int_0^a x \, dx = a^2/2$ for any real a .
- A quadratic with positive leading coefficient is nonpositive between its real roots, including the roots.

Step 1: Evaluate and simplify the inequality The printed condition gives

$$\frac{a^2}{2} \leq \frac{a}{2} + 6.$$

Multiplication by the positive number 2 preserves the inequality, yielding $a^2 - a - 12 \leq 0$. Factor the left-hand side as $(a - 4)(a + 3)$.

Step 2: Determine the permitted interval The roots are -3 and 4 . Between them the factor $a - 4$ is nonpositive and $a + 3$ is nonnegative, so their product is nonpositive. Outside that interval both factors have the same sign and their product is positive. Equality is allowed, so both endpoints belong. Option (A) has incorrect endpoints, option (B) gives the exterior region, and option (D) omits valid positive values.

Step 3: Conclusion Thus $-3 \leq a \leq 4$. Hence, the correct option is (C).

Quick Tip to Solve: After integrating, factor the quadratic and retain the endpoints because the printed inequality includes equality.

4. If A and B are square matrices of same order, then $(AB^T - BA^T)$ is a

[1 mark]

- (A) symmetric matrix
- (B) skew-symmetric matrix
- (C) null matrix
- (D) unit matrix

Correct Answer: (B) skew-symmetric matrix

Solution:

Concept:

- Transposition reverses product order: $(XY)^T = Y^T X^T$, and $(A^T)^T = A$.
- A real square matrix M is skew-symmetric exactly when $M^T = -M$.

Step 1: Transpose the given expression Write $M = AB^T - BA^T$. Using linearity of transpose and the product rule,

$$\begin{aligned} M^T &= (AB^T)^T - (BA^T)^T \\ &= BA^T - AB^T. \end{aligned}$$

Both products are defined because A and B are square matrices of the same order.

Step 2: Identify the universal property The last expression equals $-M$, which is precisely the skew-symmetry condition. The matrix need not be symmetric: that would additionally require $M = O$. It can vanish for some choices, such as $A = B$, but need not vanish for all choices. A nonzero unit matrix cannot satisfy $I^T = -I$. Thus the property guaranteed for arbitrary permitted matrices is the one in option (B).

Step 3: Conclusion $AB^T - BA^T$ is skew-symmetric. Hence, the correct option is (B).

Quick Tip to Solve: Transpose the entire difference and reverse each product; matching its negative establishes skew-symmetry.

5. The value of $\cos\left(\frac{\pi}{6} + \cot^{-1}(-\sqrt{3})\right)$ is

[1 mark]

- (A) -1
- (B) $\frac{-\sqrt{3}}{2}$
- (C) 0
- (D) 1

Correct Answer: (A) -1

Solution:

Concept:

- The principal range of $\cot^{-1} x$ is $(0, \pi)$.
- $\cot(\pi/6) = \sqrt{3}$, while $\cot(\pi - \theta) = -\cot \theta$.

Step 1: Choose the principal inverse-cotangent value Let $\alpha = \cot^{-1}(-\sqrt{3})$. Since cotangent is

Quick Tip to Solve: For a negative inverse-cotangent argument, use the second-quadrant principal angle before adding the other angle.

negative in the second quadrant within its principal range, $\alpha = \pi - \pi/6 = 5\pi/6$. The angle $-\pi/6$ has the same cotangent but lies outside the stipulated principal range and cannot be used here.

Step 2: Evaluate the cosine and compare options Substitute the principal value into the printed expression:

$$\cos\left(\frac{\pi}{6} + \frac{5\pi}{6}\right) = \cos \pi = -1.$$

The values $-\sqrt{3}/2$, 0, and 1 would correspond to different resulting angles, so they do not agree with this exact sum. In particular, the negative sign inside the inverse cotangent changes its quadrant.

Step 3: Conclusion The value is $\boxed{-1}$. Hence, the correct option is (A).

Quick Tip to Solve: For a negative inverse-cotangent argument, use the second-quadrant principal angle before adding the other angle.

6. If p and q are respectively the order and degree of the differential equation $\frac{d}{dx} \left(\left(\frac{dy}{dx} \right)^3 \right) = 0$, then $(p - q)$ is [1 mark]

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Concept:

- The order is the order of the highest derivative present in a differential equation.
- The degree is the exponent of the highest-order derivative when the equation is polynomial in its derivatives.

Step 1: Expand the derivative By the chain rule,

$$\frac{d}{dx} \left(\left(\frac{dy}{dx} \right)^3 \right) = 3 \left(\frac{dy}{dx} \right)^2 \frac{d^2y}{dx^2}.$$

Thus the equation becomes $3(y')^2 y'' = 0$, which is already polynomial in the derivatives. No radicals or fractional derivative powers need to be eliminated.

Step 2: Read the order and degree The highest derivative is d^2y/dx^2 , so $p = 2$. Its exponent is one, so $q = 1$. The square on the first derivative does not determine the degree, and the original cube does not determine the order. Therefore $p - q = 2 - 1$; options (A), (C), and (D) cannot match these definitions.

Step 3: Conclusion $\boxed{p - q = 1}$. Hence, the correct option is (B).

Quick Tip to Solve: Expand the outer derivative first; degree refers to the highest-order derivative, not the largest exponent anywhere.

7. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval

[1 mark]

- (A) $(0, 2)$
- (B) $(-\infty, 2]$
- (C) $[1, 2]$
- (D) $[2, \infty)$

Correct Answer: (D) $[2, \infty)$

Solution:

Concept:

- A differentiable function is increasing where its derivative is positive.
- For $f(x) = x^2 - 4x + 6$, $f'(x) = 2x - 4$.

Step 1: Find the sign of the derivative Differentiate to obtain $f'(x) = 2(x - 2)$. This derivative is negative for $x < 2$, zero at $x = 2$, and positive for $x > 2$. Thus the turning point separates a decreasing branch on the left from an increasing branch on the right.

Step 2: Check the offered intervals Including the endpoint 2 is valid: if $2 \leq u < v$, then

$$f(v) - f(u) = (v - u)(v + u - 4) > 0,$$

since $v - u > 0$ and $v + u > 4$. Hence the function increases throughout $[2, \infty)$. Options (A) and (C) lie to the left of the turning point, while option (B) describes the entire decreasing branch.

Step 3: Conclusion The increasing interval is $[2, \infty)$. Hence, the correct option is (D).

Quick Tip to Solve: Locate the zero of the derivative and select the interval on which its sign is positive; a boundary stationary point is allowed.

8. The line $x = 1 + 5\mu$, $y = -5 + \mu$, $z = -6 - 3\mu$ passes through which of the following point?

[1 mark]

- (A) $(1, -5, 6)$
- (B) $(1, 5, 6)$
- (C) $(1, -5, -6)$
- (D) $(-1, -5, 6)$

Correct Answer: (C) $(1, -5, -6)$

Solution:

Concept:

- A parametric line has the form $(x, y, z) = \vec{a} + \mu\vec{b}$, where $\vec{a}$ is a point and $\vec{b}$ is its direction vector.
- A point lies on the line only if the same parameter value satisfies all three coordinate equations.

Step 1: Use the simplest parameter value Putting $\mu = 0$ in $x = 1 + 5\mu$, $y = -5 + \mu$, $z = -6 - 3\mu$ gives $x = 1$, $y = -5$, and $z = -6$. The constant terms therefore identify a point of the line directly, without needing a distance calculation.

Step 2: Check the alternatives consistently Options (A), (C), and (D) have $y = -5$, which forces

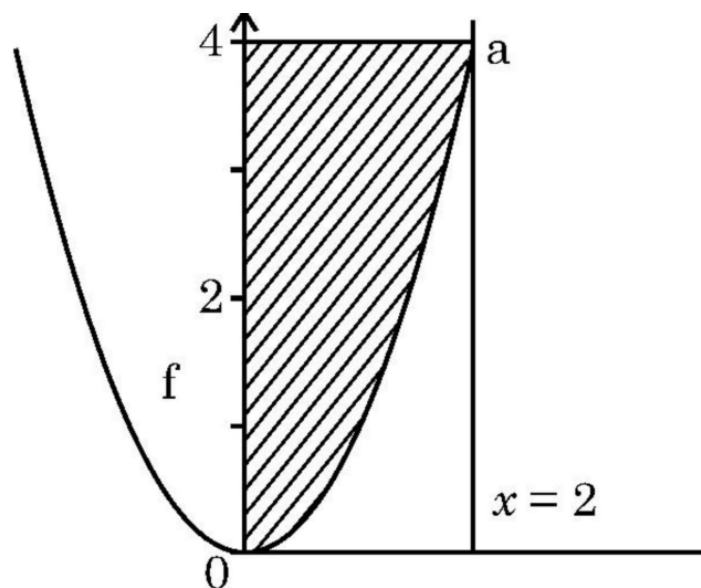
Quick Tip to Solve: Set the line parameter to zero to read a point from the constants, then verify every coordinate with that same parameter.

$\mu = 0$; the corresponding coordinates must then be $(1, -5, -6)$, eliminating (A) and (D). For option (B), $y = 5$ would require $\mu = 10$, giving $x = 51$ rather than 1. Only option (C) satisfies all three equations simultaneously.

Step 3: Conclusion The line passes through $(1, -5, -6)$. Hence, the correct option is (C).

Quick Tip to Solve: Set the line parameter to zero to read a point from the constants, then verify every coordinate with that same parameter.

9. The area of the shaded region (figure) represented by the curves $y = x^2$, $0 \leq x \leq 2$ and the y -axis is given by [1 mark]



- (A) $\int_0^2 x^2 dx$
 (B) $\int_0^2 \sqrt{y} dy$
 (C) $\int_0^4 x^2 dx$
 (D) $\int_0^4 \sqrt{y} dy$

Correct Answer: (D) $\int_0^4 \sqrt{y} dy$

Solution:

Concept:

- Area using horizontal strips is $A = \int (x_{\text{right}} - x_{\text{left}}) dy$.
- On the nonnegative branch of $y = x^2$, the curve has equation $x = \sqrt{y}$.

Step 1: Read the shaded boundaries The shaded region lies between the y -axis, the right branch of the parabola, and the horizontal line through $(2, 4)$. Thus its horizontal strips run from $x = 0$ to

Quick Tip to Solve: For shading next to the y -axis, use horizontal strips and convert the curve to x in terms of y .

$x = \sqrt{y}$, with $0 \leq y \leq 4$.

Step 2: Compare all four integrals The required integral is $\int_0^4 \sqrt{y} dy$. Option (A) measures the region below the curve and above the x -axis. Option (B) stops at $y = 2$ and omits part of the shading. Option (C) uses an incorrect horizontal range as well as vertical strips. The shaded area evaluates to $[\frac{2}{3}y^{3/2}]_0^4 = \frac{16}{3}$ square units.

Step 3: Conclusion The area is $\int_0^4 \sqrt{y} dy$. Hence, the correct option is (D).

Quick Tip to Solve: For shading next to the y -axis, use horizontal strips and convert the curve to x in terms of y .

10. If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(\bar{E}/\bar{F})$ is

[1 mark]

- (A) $\frac{P(\bar{E})}{P(\bar{F})}$
- (B) $1 - P(\bar{E}/F)$
- (C) $1 - P(E/F)$
- (D) $\frac{1 - P(E \cup F)}{P(\bar{F})}$

Correct Answer: (D) $\frac{1 - P(E \cup F)}{P(\bar{F})}$

Solution:

Concept:

- Conditional probability: $P(A/B) = P(A \cap B)/P(B)$, provided $P(B) > 0$.
- De Morgan's law gives $\bar{E} \cap \bar{F} = \overline{E \cup F}$.

Step 1: Check the conditioning event Since $P(F) \neq 1$, we have $P(\bar{F}) = 1 - P(F) > 0$. Thus the requested conditional probability is defined, and its numerator is the probability that neither event occurs.

Step 2: Apply complements and compare options

$$P(\bar{E}/\bar{F}) = \frac{P(\bar{E} \cap \bar{F})}{P(\bar{F})} = \frac{1 - P(E \cup F)}{P(\bar{F})}.$$

Option (A) replaces the intersection by the entire event $\bar{E}$, which need not be contained in $\bar{F}$. Options (B) and (C) condition on F , whereas the question conditions on $\bar{F}$; complementing the numerator event cannot change the conditioning event.

Step 3: Conclusion $P(\bar{E}/\bar{F}) = \frac{1 - P(E \cup F)}{P(\bar{F})}$. Hence, the correct option is (D).

Quick Tip to Solve: Keep the conditioning event fixed, then rewrite the intersection of complements using De Morgan's law.

11. The probability distribution of a random variable X is given by:

X	-4	-3	-2	-1	0
$P(X)$	0.1	0.2	0.3	0.2	0.2

Then $E(X)$ of distribution is

[1 mark]

- (A) -1.8
- (B) -1
- (C) 1
- (D) 1.8

Correct Answer: (A) -1.8

Solution:

Concept:

- For a discrete random variable, $E(X) = \sum_i x_i P(X = x_i)$.
- The probabilities must sum to 1, and the mean is a probability-weighted value.

Step 1: Check the distribution The listed values are -4, -3, -2, -1, 0, with probabilities 0.1, 0.2, 0.3, 0.2, 0.2, respectively. Their sum is $0.1 + 0.2 + 0.3 + 0.2 + 0.2 = 1$, so these weights form a valid probability distribution.

Step 2: Compute the weighted sum

$$\begin{aligned} E(X) &= (-4)(0.1) + (-3)(0.2) + (-2)(0.3) \\ &\quad + (-1)(0.2) + (0)(0.2) \\ &= -0.4 - 0.6 - 0.6 - 0.2 + 0 = -1.8. \end{aligned}$$

The positive options (C) and (D) are impossible because every possible value is nonpositive. Option (B), -1, is within the range of the values but does not equal their weighted mean.

Step 3: Conclusion $E(X) = -1.8$. Hence, the correct option is (A).

Quick Tip to Solve: Multiply each value by the probability directly below it; a mean of nonpositive values cannot be positive.

12. If projection of $\vec{a} = a\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units, then a is

[1 mark]

- (A) -13
- (B) -5
- (C) 13
- (D) 5

Correct Answer: (D) 5

Solution:

Concept:

- The scalar projection of $\vec{a}$ on $\vec{b}$ is $(\vec{a} \cdot \vec{b})/|\vec{b}|$.
- The dot product multiplies corresponding components and adds the results.

Step 1: Find the dot product and magnitude For $\vec{a} = (a, 1, 4)$ and $\vec{b} = (2, 6, 3)$,

$$\vec{a} \cdot \vec{b} = 2a + 6 + 12 = 2a + 18, \quad |\vec{b}| = \sqrt{2^2 + 6^2 + 3^2} = 7.$$

The denominator uses the vector onto which the projection is taken.

Step 2: Use the given projection

$$\frac{2a + 18}{7} = 4 \implies 2a + 18 = 28 \implies a = 5.$$

Checking the alternatives, $a = -13, -5, 13$ gives projections $-8/7, 8/7, 44/7$, respectively; none equals 4. For $a = 5$, the dot product is 28, giving exactly the stated projection.

Step 3: Conclusion $a = 5$. Hence, the correct option is (D).

Quick Tip to Solve: For projection on $\vec{b}$, divide by $|\vec{b}|$, not by the magnitude of the projected vector.

13. The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point $(4, -3, 7)$ is

[1 mark]

- (A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$
- (B) $x = 3t + 4, y = t + 3, z = 2t + 7$
- (C) $x = 3t + 4, y = t - 3, z = 2t + 7$
- (D) $x = 3t + 4, y = -t + 3, z = 2t + 7$

Correct Answer: (C) $x = 3t + 4, y = t - 3, z = 2t + 7$

Solution:

Concept:

- A line through $\vec{r}_0$ parallel to $\vec{d}$ has equation $\vec{r} = \vec{r}_0 + t\vec{d}$.
- The coefficients of the parameter are the direction ratios; the constants give a point on the line.

Quick Tip to Solve: Check both the parameter coefficients and the point obtained at $t = 0$ before choosing a line equation.

Step 1: Construct the required line The point is $(4, -3, 7)$ and the direction vector is $(3, 1, 2)$. Hence

$$\vec{r} = (4, -3, 7) + t(3, 1, 2),$$

which produces $x = 4 + 3t$, $y = -3 + t$, and $z = 7 + 2t$. Setting $t = 0$ verifies the specified point.

Step 2: Check the alternatives Option (A) has direction ratios $(4, -3, 7)$, which are not proportional to $(3, 1, 2)$. Option (B) has the correct direction but $x = 4$ forces $t = 0$, giving $y = 3$ instead of -3 , so it misses the point. Option (D) has direction $(3, -1, 2)$, which is not parallel to the given vector. Only option (C) has both the required point and direction.

Step 3: Conclusion The line is $x = 3t + 4$, $y = t - 3$, $z = 2t + 7$. Hence, the correct option is (C).

Quick Tip to Solve: Check both the parameter coefficients and the point obtained at $t = 0$ before choosing a line equation.

14. If a line makes angles of $\frac{3\pi}{4}$, $\frac{\pi}{3}$ and θ with the positive directions of x , y and z -axis respectively, then θ is [1 mark]

- (A) $-\frac{\pi}{3}$ only
- (B) $\frac{\pi}{3}$ only
- (C) $\frac{\pi}{6}$
- (D) $\pm \frac{\pi}{3}$

Correct Answer: $\theta = \pi/3$ or $2\pi/3$. None of the printed options states the complete valid answer; the item is ambiguous.

Solution:

Concept:

- Direction angles satisfy $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$.
- Angles with positive coordinate axes lie in $[0, \pi]$; either sign of a direction cosine is possible.

Step 1: Find the third squared direction cosine With $\alpha = 3\pi/4$ and $\beta = \pi/3$,

$$\cos^2 \theta = 1 - \cos^2 \frac{3\pi}{4} - \cos^2 \frac{\pi}{3} = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.$$

Thus $\cos \theta = \pm \frac{1}{2}$; taking the square root requires both signs.

Step 2: Apply the angle range and test the options In $[0, \pi]$, the two possibilities are $\theta = \pi/3$ and $\theta = 2\pi/3$. Both produce legitimate direction cosines with the two specified angles. Option (A) gives a negative direction angle. Option (B) includes the valid acute possibility but its word "only" excludes the equally valid obtuse one. Option (C) gives squared cosine $3/4$, not $1/4$. Option (D) contains a negative angle and omits $2\pi/3$.

Step 3: Conclusion $\theta = \pi/3$ or $2\pi/3$. None of the printed options states the complete valid answer; the item is ambiguous.

Quick Tip to Solve: After finding a squared direction cosine, retain both signs and use the direction-angle range $[0, \pi]$.

15. A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct? [1 mark]

- (A) The objective function maximizes the difference of the profit earned from products X and Y.
- (B) The objective function measures the total production of products X and Y.
- (C) The objective function maximizes the combined profit earned from selling X and Y.
- (D) The objective function ensures the company produces more of product X than product Y.

Correct Answer: (C) The objective function maximizes the combined profit earned from selling X and Y.

Solution:

Concept:

- An objective function represents the quantity to maximize or minimize.
- If per-unit profits are 5 and 7, selling x and y units yields combined profit $Z = 5x + 7y$.

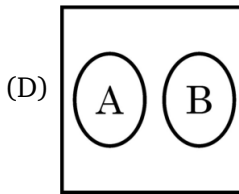
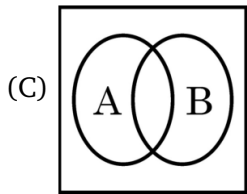
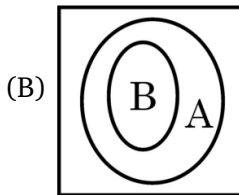
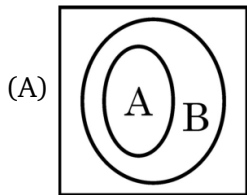
Step 1: Interpret each term The contribution from product X is $5x$, and that from product Y is $7y$. Adding them gives total profit earned from both products. Therefore the stated business goal uses Z to compare the combined profit for feasible production or sales choices.

Step 2: Compare the statements Option (A) describes a difference, which would involve subtraction rather than the displayed sum. Option (B) concerns total units, represented by $x + y$, rather than monetary profit with unequal unit coefficients. Option (D) would require a constraint such as $x > y$; the objective function alone imposes no such relation. Option (C) correctly describes maximizing the combined profit subject to the feasible constraints.

Step 3: Conclusion The objective function maximizes the combined profit from selling X and Y. Hence, the correct option is (C).

Quick Tip to Solve: Read the coefficients as per-unit contributions; constraints control quantities while the objective measures profit.

16. If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set A and B ? [1 mark]



Correct Answer: (B)



Solution:

Concept:

- Differentiability at a point implies continuity at that point.
- The converse need not hold: a continuous function may have a corner and fail to be differentiable.

Step 1: Establish the inclusion Every function belonging to B , the set of differentiable functions, also belongs to A , the set of continuous functions on the same domain. Hence $B \subseteq A$. This direction follows from the theorem that existence of the derivative ensures continuity.

Step 2: Show why the reverse inclusion fails The function $f(x) = |x|$ is continuous at 0, but its left derivative is -1 and its right derivative is 1 , so it is not differentiable there. Thus A contains functions outside B . Option (A) reverses the inclusion, option (C) allows differentiable functions outside the continuous set, and option (D) makes the sets disjoint. The diagram in option (B) correctly places B wholly inside A .

Step 3: Conclusion $B \subset A$: the set of differentiable functions lies inside the set of continuous functions. Hence, the correct option is (B).

Quick Tip to Solve: Use $f(x) = |x|$ at the origin to remember why continuity alone does not guarantee differentiability.

17. Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify $4AB + 3(AB + BA) - 4BA$, where A and B are both matrices of order 2×2 . It is known that $A \neq B \neq I$ and $A^{-1} \neq B$. Their answers are given as: Abhay: $6AB$, Bina: $7AB - BA$, Chhaya: $8AB$, Devesh: $7BA - AB$. Who answered it correctly? [1 mark]

- (A) Abhay
- (B) Bina
- (C) Chhaya
- (D) Devesh

Correct Answer: (B) Bina

Solution:

Concept:

- Matrix multiplication distributes over addition: $3(AB + BA) = 3AB + 3BA$.
- Like matrix products can be combined, but AB and BA must be retained separately unless commutativity is established.

Step 1: Expand the bracket Using distributivity, the given expression becomes

$$4AB + 3AB + 3BA - 4BA.$$

The order of the factors inside each product stays unchanged.

Step 2: Collect terms and compare the answers The coefficients of AB add to $4 + 3 = 7$, while those of BA add to $3 - 4 = -1$. Thus the simplified expression is $7AB - BA$. Abhay and Chhaya incorrectly reduce it to a single multiple of AB ; Devesh reverses the roles of the two products. The stated matrix conditions do not justify exchanging AB with BA .

Step 3: Conclusion $7AB - BA$. Bina answered correctly, so the correct option is (B).

Quick Tip to Solve: Group AB terms and BA terms separately; never exchange matrix factors without an explicit commutativity condition.

18. If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct? [1 mark]

- (A) $A = B$
- (B) $AB = BA$
- (C) $A = 0$ or $B = 0$
- (D) $A = I$ or $B = I$

Correct Answer: (B) $AB = BA$

Solution:

Concept:

- For square matrices of equal order, distributivity applies but generally $AB \neq BA$.

Quick Tip to Solve: Expand the matrix factors in order: the leftover cross-term $AB - BA$ immediately reveals the required condition.

- The scalar difference-of-squares factorisation holds for matrices precisely when the two matrices commute.

Step 1: Expand in the given order Without changing any factor order,

$$(A - B)(A + B) = A^2 + AB - BA - B^2.$$

The two cross-products do not automatically cancel for matrices.

Step 2: Use the identity and assess the alternatives Equating this expansion to the given $A^2 - B^2$ and subtracting the square terms gives $AB - BA = 0$, hence $AB = BA$. Equality, a zero matrix and an identity matrix are not necessary. For example, $A = 2I$ and $B = 3I$ satisfy the identity while $A \neq B$, both are nonzero, and neither equals I . This example excludes options (A), (C) and (D).

Step 3: Conclusion $AB = BA$; the correct option is (B).

Quick Tip to Solve: Expand the matrix factors in order: the leftover cross-term $AB-BA$ immediately reveals the required condition.

19. Assertion (A): Every point of the feasible region of a Linear Programming Problem is an optimal solution.

Reason (R): The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true but Reason (R) is false.
- (D) Assertion (A) is false but Reason (R) is true.

Correct Answer: (D) Assertion (A) is false but Reason (R) is true.

Solution:

Concept:

- A feasible point satisfies all constraints; an optimal point gives the required maximum or minimum of the objective function.
- The corner-point theorem directs the search for an attained finite optimum to the corner points of the feasible region.

Step 1: Judge the assertion The assertion is false. Feasibility alone does not ensure optimality. For example, on $0 \leq x \leq 1, 0 \leq y \leq 1$, maximising $Z = x + y$ gives $Z = 2$ at $(1, 1)$, whereas the feasible point $(0, 0)$ gives $Z = 0$.

Step 2: Judge the reason In the intended corner-point theorem sense, the reason is true: an attained finite optimum is obtained at one or more corner points. If adjacent corners have the same optimal value, their joining edge also consists of optimal points; the theorem still supplies optimal corners.

Quick Tip to Solve: Distinguish satisfying the constraints from optimising the objective; test two feasible points to disprove "every point".

Step 3: Determine the relationship The reason describes where the objective should be checked, whereas the assertion incorrectly declares every feasible point optimal. Thus the assertion is false and the reason is true in the standard CBSE corner-point interpretation; the alternatives requiring a true assertion are ruled out.

Step 4: Conclusion Assertion is false and Reason is true in the intended corner-point theorem sense. The correct option is (D).

Quick Tip to Solve: Distinguish satisfying the constraints from optimising the objective; test two feasible points to disprove "every point".

20. Assertion (A): $A = \text{diag } [3 \ 5 \ 2]$ is a scalar matrix of order 3×3 .

Reason (R): If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true but Reason (R) is false.
(D) Assertion (A) is false but Reason (R) is true.

Correct Answer: (D) Assertion (A) is false but Reason (R) is true.

Solution:

Concept:

- A diagonal matrix has zero entries outside its principal diagonal.
- A scalar matrix has equal principal-diagonal entries and is written kI , where k is a scalar and I is the identity matrix.

Step 1: Judge the assertion The principal diagonal of $A = \text{diag}[3 \ 5 \ 2]$ contains 3, 5, 2. Since these values are unequal, A cannot be written as kI_3 for a single scalar k . It is diagonal but not scalar, so the assertion is false.

Step 2: Judge the reason The reason states the defining equal-entry property of a scalar diagonal matrix in the intended textbook sense. For a nonzero scalar matrix, its nonzero diagonal entries are the same scalar. Thus the reason is true; the zero scalar matrix is also included in the definition kI with $k = 0$.

Step 3: Select the assertion-reason combination The true equal-diagonal criterion rejects the asserted example because its diagonal entries differ. Both options (A) and (B) require a true assertion, and (C) also requires a true assertion, so none of them applies.

Step 4: Conclusion Assertion is false but Reason is true. Hence, the correct option is (D).

Quick Tip to Solve: For a scalar matrix inspect only the principal diagonal: all diagonal entries must equal the same scalar.

Section B (10 marks)

(This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.)

21. Find the values of 'a' for which $f(x) = \sin x - ax + b$ is increasing on $\mathbb{R}$.

[2 marks]

Solution:

Concept:

- For a differentiable nondecreasing function on $\mathbb{R}$, its derivative must be nonnegative everywhere.
- $\frac{d}{dx} \sin x = \cos x$, and $-1 \leq \cos x \leq 1$ for all real x .

Step 1: Differentiate and impose the condition The constant b differentiates to zero, so

$$f'(x) = \cos x - a.$$

To be increasing throughout $\mathbb{R}$, the slope cannot be negative at any real point. Hence $\cos x - a \geq 0$, or $a \leq \cos x$, for every x .

Step 2: Find the limiting value and verify sufficiency The least possible value of $\cos x$ is -1 , attained at odd multiples of π . Therefore the necessary condition is $a \leq -1$. Conversely, if $a \leq -1$, then $f'(x) = \cos x - a \geq -1 - a \geq 0$. For $a < -1$, the derivative is strictly positive everywhere. At $a = -1$, it is $1 + \cos x$, which vanishes only at isolated odd multiples of π and is positive between them; its integral over every nonempty interval is positive. Thus the endpoint value also makes f strictly increasing.

Step 3: Conclusion The required values are $a \leq -1$, independently of b .

Quick Tip to Solve: For a derivative of the form $\cos x - a$, use the minimum $\cos x = -1$; isolated zero slopes do not prevent strict increase.

22. Find: $\int 2x^3 e^{x^2} dx$.

[2 marks]

Solution:

Concept:

- Use $t = x^2$, so $dt = 2x dx$ and $2x^3 dx = t dt$.
- Integration by parts gives $\int u dv = uv - \int v du$; apply it to a polynomial multiplied by an exponential.

Step 1: Convert the integral using substitution Write the integrand as $x^2 e^{x^2} (2x dx)$. Setting $t = x^2$ therefore gives

$$\int 2x^3 e^{x^2} dx = \int t e^t dt.$$

This choice absorbs the factor $2x$ exactly and leaves only a linear factor in the new variable.

Step 2: Integrate by parts and restore the variable Choose $u = t$, $dv = e^t dt$, so $du = dt$ and $v = e^t$.

Quick Tip to Solve: Split $2x^3$ into x^2 times $2x$; substitute the exponent first, then integrate the remaining $t e^t$ by parts.

Then

$$\int te^t dt = te^t - \int e^t dt = (t-1)e^t + C.$$

Restoring $t = x^2$ yields $(x^2 - 1)e^{x^2} + C$. As a check, its derivative is $2xe^{x^2} + (x^2 - 1)2xe^{x^2} = 2x^3e^{x^2}$, exactly the printed integrand. The arbitrary constant C is required for an indefinite integral.

Step 3: Conclusion $\int 2x^3e^{x^2} dx = (x^2 - 1)e^{x^2} + C.$

Quick Tip to Solve: Split $2x$ cubed into x squared times $2x$; substitute the exponent first, then integrate the remaining $t e$ to the t by parts.

23(a). If $x = e^{x/y}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x \log x}$.

[2 marks]

Solution:

Concept:

- Taking natural logarithms of $x = e^{x/y}$ gives $\log x = x/y$, with $x > 0$, $y \neq 0$ and $x \neq 1$.
- The product rule is $(uv)' = u'v + uv'$, and $(\log x)' = 1/x$.

Step 1: Take logarithms and remove the quotient From the printed relation $x = e^{x/y}$, take logarithms on both sides to obtain

$$\log x = \frac{x}{y}, \quad y \log x = x.$$

The exponential relation ensures $x > 0$; its exponent requires $y \neq 0$. Also, $x = 1$ cannot satisfy the equation because it would give $0 = 1/y$. These restrictions justify the divisions used below.

Step 2: Differentiate implicitly and isolate the derivative Treat y as a differentiable function of x . Applying the product rule to $y \log x = x$ gives

$$\frac{dy}{dx} \log x + \frac{y}{x} = 1.$$

Move y/x to the right and divide by $\log x$:

$$\frac{dy}{dx} = \frac{1 - y/x}{\log x} = \frac{x - y}{x \log x}.$$

The division is valid because $x > 0$ and $x \neq 1$. Equivalently, solving the logarithmic relation gives $y = x / \log x$, whose quotient-rule derivative agrees with this result.

Step 3: Conclusion Hence $\frac{dy}{dx} = \frac{x - y}{x \log x}$, as required.

Quick Tip to Solve: Take logarithms, multiply by y , and differentiate $y \log x = x$ by the product rule to avoid a nested quotient.

23(b). If $f(x) = \begin{cases} 2x - 3, & -3 \leq x \leq -2 \\ x + 1, & -2 < x \leq 0 \end{cases}$, check the differentiability of $f(x)$ at $x = -2$. [2 marks]

Solution:

Concept:

- Differentiability at an interior point implies continuity there. A jump between the two one-sided limits therefore rules out differentiability.
- The derivative at a , when it exists, is the finite two-sided limit $\lim_{h \rightarrow 0} [f(a+h) - f(a)]/h$.

Step 1: Check the value and continuity The printed first branch includes $x = -2$. Hence $f(-2) = 2(-2) - 3 = -7$. Approaching through that branch gives $\lim_{x \rightarrow -2^-} f(x) = -7$, whereas the second branch gives $\lim_{x \rightarrow -2^+} f(x) = (-2) + 1 = -1$. These limits are unequal, so the function is discontinuous at the joining point.

Step 2: Examine the derivative definition For $h < 0$, the difference quotient is $[2(-2+h) - 3 - (-7)]/h = 2$. For $h > 0$, it is $[(-2+h) + 1 - (-7)]/h = (6+h)/h$, which diverges as $h \rightarrow 0^+$. Merely comparing the branch slopes 2 and 1 would ignore the jump in the function values; the derivative must use the actual value $f(-2) = -7$.

Step 3: Conclusion Therefore f is not differentiable at $x = -2$, since it is discontinuous there and its right-hand derivative does not exist as a finite number.

Quick Tip to Solve: Check continuity at the joining point before comparing slopes; a jump alone settles non-differentiability.

24. If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, then evaluate $|\vec{a} + 2\vec{b}|$. [2 marks]

Solution:

Concept:

- For any vector $\vec{v}$, $|\vec{v}|^2 = \vec{v} \cdot \vec{v}$. Expand the dot product before taking the non-negative square root.
- The dot product is distributive and symmetric: $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.

Step 1: Expand the squared magnitude Let $\vec{v} = \vec{a} + 2\vec{b}$. Then

$$\begin{aligned} |\vec{v}|^2 &= (\vec{a} + 2\vec{b}) \cdot (\vec{a} + 2\vec{b}) \\ &= |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{a} + 4|\vec{b}|^2 \\ &= |\vec{a}|^2 + 4\vec{a} \cdot \vec{b} + 4|\vec{b}|^2. \end{aligned}$$

Both cross terms must be included; the coefficient of the given dot product is 4.

Step 2: Substitute the given data Using $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, we obtain

$$|\vec{v}|^2 = 2^2 + 4(4) + 4(3^2) = 4 + 16 + 36 = 56.$$

Quick Tip to Solve: Square the vector expression with a dot product; a coefficient 2 creates a cross-term coefficient 4.

The supplied scalar product is consistent with these lengths since $4 \leq 2 \cdot 3$. A magnitude cannot be negative, so only the positive square root is admissible.

Step 3: Conclusion

$$|\vec{a} + 2\vec{b}| = \sqrt{56} = 2\sqrt{14}.$$

Quick Tip to Solve: Square the vector expression with a dot product; a coefficient 2 creates a cross-term coefficient 4.

25(a). A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$. [2 marks]

Solution:

Concept:

- If α, β, γ are angles with the positive coordinate axes, the direction cosines satisfy $l^2 + m^2 + n^2 = 1$, where $l = \cos \alpha, m = \cos \beta, n = \cos \gamma$.
- A vector of magnitude r and direction cosines l, m, n is $r(l\hat{i} + m\hat{j} + n\hat{k})$.

Step 1: Use the equal-angle condition The three equal angles have a common cosine, say c . Thus $l = m = n = c$, and the direction-cosine identity gives $3c^2 = 1$, or $c = \pm 1/\sqrt{3}$. The signs must be the same for all three components because the angles, rather than just their absolute cosines, are equal. Both an acute common angle and an obtuse common angle meet the printed condition.

Step 2: Find the components Multiply each direction cosine by the given magnitude $5\sqrt{3}$. Each component is $(5\sqrt{3})c = \pm 5$, with a common sign. The two candidates have length $\sqrt{25 + 25 + 25} = 5\sqrt{3}$, and each makes equal angles with the three positive axes. If acute angles are intended, the positive candidate alone is selected; no acute-angle restriction is printed.

Step 3: Conclusion

$$\vec{a} = 5(\hat{i} + \hat{j} + \hat{k}) \quad \text{or} \quad \vec{a} = -5(\hat{i} + \hat{j} + \hat{k}).$$

For the usual acute-angle choice, $\vec{a} = 5\hat{i} + 5\hat{j} + 5\hat{k}$.

Quick Tip to Solve: Equal angles give equal direction cosines; keep the common negative sign unless acute angles are specified.

25(b). If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $QR = \frac{3}{2}QP$. [2 marks]

Solution:

Concept:

- The displacement from Q to P is $\vec{QP} = \vec{\alpha} - \vec{\beta}$, where $\vec{\alpha}$ and $\vec{\beta}$ are their position vectors.
- For collinear points on the same directed ray, displacement vectors scale by the ratio of their lengths. Add displacement to the starting position vector.

Step 1: Translate the directed length condition The phrase QP produced places R beyond P on the ray starting at Q. Consequently Q, P, R occur in that order, and $\vec{QR}$ has the same direction as $\vec{QP}$. Since $QR = \frac{3}{2}QP$, we have

$$\vec{QR} = \frac{3}{2}(\vec{\alpha} - \vec{\beta}).$$

Also $PR = QR - QP = \frac{1}{2}QP$, confirming that the division is external.

Step 2: Add the displacement to the starting position If $\vec{r}$ denotes the position vector of R, then

$$\begin{aligned}\vec{r} &= \vec{\beta} + \vec{QR} \\ &= \vec{\beta} + \frac{3}{2}(\vec{\alpha} - \vec{\beta}) \\ &= \frac{3}{2}\vec{\alpha} - \frac{1}{2}\vec{\beta}.\end{aligned}$$

This formula assigns R to the extension beyond P; reversing the subtraction in $\vec{QP}$ would incorrectly place it on the opposite ray.

Step 3: Conclusion

$$\boxed{\vec{OR} = \frac{3\vec{\alpha} - \vec{\beta}}{2}}.$$

Quick Tip to Solve: For QP produced, start at Q and add a positive multiple of P minus Q to avoid an external-ratio sign error.

Section C (18 marks)

(This section comprises of 6 Short Answer (SA) type questions of 3 marks each.)

26(a). If $y = \log \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2$, then show that $x(x+1)^2y_2 + (x+1)^2y_1 = 2$. [3 marks]

Solution:

Concept:

- For the real expression containing $\sqrt{x}$ and $1/\sqrt{x}$, work on $x > 0$. Use logarithm laws to simplify before differentiating.
- Here $y_1 = dy/dx$, $y_2 = d^2y/dx^2$, and $d(\log u)/dx = u'/u$ for a positive differentiable function u .

Step 1: Simplify the logarithm The square is inside the logarithm. Since $x > 0$,

$$\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = \left(\frac{x+1}{\sqrt{x}}\right)^2 = \frac{(x+1)^2}{x}.$$

Therefore

$$y = \log \frac{(x+1)^2}{x} = 2 \log(x+1) - \log x.$$

Both logarithm arguments are positive on the stated domain, so these laws and all subsequent differentiations are valid.

Step 2: Find the first and second derivatives Differentiating term by term gives

$$y_1 = \frac{2}{x+1} - \frac{1}{x} = \frac{x-1}{x(x+1)}.$$

Differentiate the unsimplified sum of reciprocal terms once more, which avoids a longer quotient-rule calculation:

$$y_2 = -\frac{2}{(x+1)^2} + \frac{1}{x^2}.$$

In particular, the derivative of $-1/x$ is positive; this sign matters for the requested identity.

Step 3: Substitute and simplify the left-hand side Using the reciprocal forms of the derivatives,

$$\begin{aligned} x(x+1)^2 y_2 &= -2x + \frac{(x+1)^2}{x}, \\ (x+1)^2 y_1 &= 2(x+1) - \frac{(x+1)^2}{x}. \end{aligned}$$

Adding cancels the two rational terms exactly, leaving

$$-2x + 2(x+1) = 2.$$

This calculation applies for every $x > 0$, where the original function and both derivatives are defined.

Step 4: Conclusion Hence

$$x(x+1)^2 y_2 + (x+1)^2 y_1 = 2.$$

Quick Tip to Solve: Expand the logarithm into two simple logarithms and retain reciprocal derivative forms to make cancellation visible.

26(b). If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, $-1 < x < 1$, $x \neq y$, then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.

[3 marks]

Solution:**Concept:**

- Squaring an equality gives a necessary algebraic relation. Factor the resulting expression before dividing, and use the given condition $x \neq y$.
- For a differentiable function $y = y(x)$, the product rule gives $\frac{d}{dx}(xy) = y + x\frac{dy}{dx}$. The restriction $-1 < x < 1$ ensures $1+x > 0$.

Step 1: Remove the radicals and factor The printed equation implies

$$x\sqrt{1+y} = -y\sqrt{1+x}.$$

Squaring both sides gives $x^2(1+y) = y^2(1+x)$. Hence

$$\begin{aligned} x^2 - y^2 + x^2y - xy^2 &= 0, \\ (x-y)(x+y) + xy(x-y) &= 0, \\ (x-y)(x+y+xy) &= 0. \end{aligned}$$

Since $x \neq y$, the first factor is nonzero. Therefore $x+y+xy=0$. This division is justified specifically by the stated hypothesis; it does not discard any permitted solution.

Step 2: Express the permitted branch and differentiate Rearranging gives $y(1+x) = -x$, so

$$y = -\frac{x}{1+x}, \quad 1+y = \frac{1}{1+x}.$$

Both denominators are nonzero because $x > -1$. This branch satisfies the original radical equation: its two terms become $x/\sqrt{1+x}$ and $-x/\sqrt{1+x}$, which cancel. Thus squaring has not led us to use an extraneous branch. Now apply the quotient rule:

$$\frac{dy}{dx} = -\frac{(1+x) - x}{(1+x)^2}.$$

Equivalently, differentiating $x+y+xy=0$ gives $(1+x)y' = -(1+y)$, and substituting $1+y = 1/(1+x)$ yields the same expression.

Step 3: Conclusion

$$\boxed{\frac{dy}{dx} = -\frac{1}{(1+x)^2}}$$

Thus the required identity holds at every point allowed by the given conditions.

Quick Tip to Solve: After squaring, factor out $x-y$ first; the given inequality eliminates that factor and leaves a simple rational expression for y .

27. Let R be a relation on set of real numbers $\mathbb{R}$ defined as $R = \{(x, y) : x - y + \sqrt{3} \text{ is an irrational number, } x, y \in \mathbb{R}\}$. Verify R for reflexivity, symmetry and transitivity. [3 marks]

Solution:

Concept:

- Reflexivity requires $(x, x) \in R$ for every real x ; symmetry requires $(x, y) \in R \Rightarrow (y, x) \in R$.
- Transitivity requires $(x, y), (y, z) \in R \Rightarrow (x, z) \in R$. A single explicit counterexample disproves a universal implication. Here $\sqrt{3}$ is irrational, whereas 0 is rational.

Step 1: Verify reflexivity for every real number For an arbitrary real number x , substitute $y = x$ in the defining expression:

$$x - x + \sqrt{3} = \sqrt{3}.$$

This is irrational for every real x , independently of whether x itself is rational. Thus every pair (x, x) belongs to R , so R is reflexive.

Step 2: Disprove symmetry Take $x = \sqrt{3}$ and $y = 0$. Then

$$x - y + \sqrt{3} = 2\sqrt{3},$$

which is irrational, so $(\sqrt{3}, 0) \in R$. But in the reversed pair,

$$y - x + \sqrt{3} = 0 - \sqrt{3} + \sqrt{3} = 0,$$

which is rational. Hence $(0, \sqrt{3}) \notin R$. The required implication fails, and R is not symmetric.

Step 3: Disprove transitivity Choose $x = 0$, $y = \sqrt{3}/2$ and $z = \sqrt{3}$. The two expressions

$$x - y + \sqrt{3} = \frac{\sqrt{3}}{2}, \quad y - z + \sqrt{3} = \frac{\sqrt{3}}{2}$$

are both irrational. Therefore $(x, y) \in R$ and $(y, z) \in R$. However,

$$x - z + \sqrt{3} = 0 - \sqrt{3} + \sqrt{3} = 0$$

is rational, so $(x, z) \notin R$. This disproves transitivity. In particular, adding irrational quantities does not guarantee an irrational result, so no general closure argument can replace this test.

Step 4: Conclusion Therefore, the relation is reflexive, but neither symmetric nor transitive.

Quick Tip to Solve: Use multiples of $\sqrt{3}$ so that the added $\sqrt{3}$ cancels in the reversed pair or in the final pair.

28. Solve the following linear programming problem graphically:

Minimise $Z = 2x + y$ **subject to the constraints:**

$$3x + y \geq 9$$

$$x + y \geq 7$$

$$x + 2y \geq 8$$

$$x, y \geq 0$$

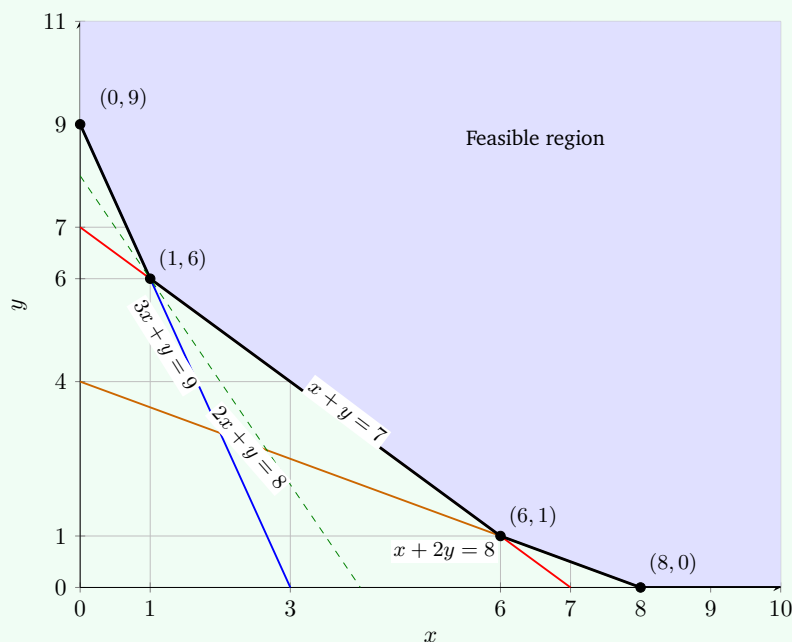
[3 marks]

Solution:

Concept:

- Each linear inequality gives a closed half-plane. The feasible region is their intersection with the first quadrant $x \geq 0, y \geq 0$.
- Evaluate the linear objective $Z = 2x + y$ at the feasible corner points. For an unbounded region, also establish that no feasible point can give a smaller value.

Step 1: Draw the constraint lines and feasible region Draw $3x + y = 9$ through $(0, 9), (3, 0)$; $x + y = 7$ through $(0, 7), (7, 0)$; and $x + 2y = 8$ through $(0, 4), (8, 0)$. The origin violates each inequality, so select the half-plane away from the origin for each line. Restrict the drawing to the first quadrant. Their common region lies above the lower boundary shown in the graph and extends indefinitely upward and to the right. All boundary lines are included because the inequalities are non-strict.



The shaded unbounded region satisfies all constraints; its lower boundary has four corner points.

Step 2: Find the feasible corner points On the y -axis, the constraints require $y \geq 9$, giving the corner $A = (0, 9)$. Solving $3x + y = 9$ and $x + y = 7$ gives $2x = 2$, so $B = (1, 6)$; it also satisfies $x + 2y = 13 \geq 8$. Solving $x + y = 7$ and $x + 2y = 8$ gives $C = (6, 1)$, which satisfies $3x + y = 19 \geq 9$. On the x -axis, all constraints hold for $x \geq 8$, giving $D = (8, 0)$. The intersection of the first and third lines is $(2, 3)$, but $2 + 3 < 7$, so it is outside the feasible region and must be excluded.

Quick Tip to Solve: Exclude intersections that violate another constraint; here $(2, 3)$ fails $x + y \geq 7$, while averaging the first two constraints proves the lower bound.

Step 3: Evaluate the objective and confirm the minimum The objective values at the four corners are

Corner point	$Z = 2x + y$
(0, 9)	9
(1, 6)	8
(6, 1)	13
(8, 0)	16

The lowest corner value is 8. To confirm this on the entire unbounded region, combine two constraints:

$$2x + y = \frac{(3x + y) + (x + y)}{2} \geq \frac{9 + 7}{2} = 8.$$

Equality requires both $3x + y = 9$ and $x + y = 7$, whose unique intersection is (1, 6). It satisfies the third constraint and non-negativity. Graphically, the objective line $2x + y = 8$ supports the feasible region at B; moving it toward smaller objective values leaves the region.

Step 4: Conclusion

$$Z_{\min} = 8 \text{ at } (x, y) = (1, 6).$$

Quick Tip to Solve: Exclude intersections that violate another constraint; here (2, 3) fails $x + y \geq 7$, while averaging the first two constraints proves the lower bound.

29(a). A die with number 1 to 6 is biased such that $P(2) = \frac{3}{10}$ and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the dice, if the dice is thrown twice. [3 marks]

Solution:

Concept:

- Let X be the number of occurrences of face 2 in two throws. Each throw is an independent Bernoulli trial with success probability $p = P(2) = \frac{3}{10}$.
- For a binomial random variable, $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$, where n is the number of independent trials and r counts successes.
- The mean of a discrete random variable is $E(X) = \sum_r r P(X = r)$; for a binomial distribution this is np .

Step 1: Identify success and failure A success means that the number on the die is 2, so $p = 3/10$. All five other faces together give failure probability $q = 1 - p = 7/10$. Their individual probabilities are equal, namely $(7/10)/5 = 7/50$, but their identities do not affect the count of twos. Since the die is thrown twice, X can take the values 0, 1, 2.

Quick Tip to Solve: Count appearances of face 2, rather than averaging the numerical values on the die; use np with $n = 2$.

Step 2: Construct the distribution Independence of successive throws gives

$$\begin{aligned}P(X = 0) &= \left(\frac{7}{10}\right)^2 = \frac{49}{100}, \\P(X = 1) &= 2 \left(\frac{3}{10}\right) \left(\frac{7}{10}\right) = \frac{42}{100}, \\P(X = 2) &= \left(\frac{3}{10}\right)^2 = \frac{9}{100}.\end{aligned}$$

The factor 2 in the middle line counts the two possible orders: success followed by failure, or failure followed by success. These probabilities sum to 1.

Step 3: Evaluate the expectation Weight each possible count by its probability:

$$E(X) = 0 \left(\frac{49}{100}\right) + 1 \left(\frac{42}{100}\right) + 2 \left(\frac{9}{100}\right) = \frac{60}{100}.$$

This also agrees with the binomial formula $np = 2(3/10)$. The expectation is a long-run average count and need not itself be an integer.

Step 4: Conclusion The mean number of times face 2 appears in two throws is $\boxed{\frac{3}{5} = 0.6}$.

Quick Tip to Solve: Count appearances of face 2, rather than averaging the numerical values on the die; use np with $n = 2$.

OR

29(b). Two dice are thrown. Defined are the following two events A and B :

$A = \{(x, y) : x + y = 9\}$, $B = \{(x, y) : x \neq 3\}$, where (x, y) denote a point in the sample space.

Check if events A and B are independent or mutually exclusive.

[3 marks]

Solution:

Concept:

- Two fair dice give 36 equally likely ordered pairs (x, y) , with each coordinate in $\{1, 2, 3, 4, 5, 6\}$.
- Events are independent exactly when $P(A \cap B) = P(A)P(B)$.
- Events are mutually exclusive exactly when $A \cap B = \emptyset$; for equally likely finite outcomes this means $P(A \cap B) = 0$.

Step 1: Count the outcomes of each event The ordered pairs with sum 9 are

$$A = \{(3, 6), (4, 5), (5, 4), (6, 3)\}.$$

Hence $P(A) = 4/36 = 1/9$. Event B excludes first coordinate 3. There are five allowed values of x and six values of y , so B contains $5 \times 6 = 30$ outcomes and $P(B) = 30/36 = 5/6$. The restriction applies to the first die only; an outcome with second coordinate 3 can belong to B .

Quick Tip to Solve: List the four ordered pairs with sum 9, then remove only the pair whose first coordinate equals 3.

Step 2: Find the intersection Of the four outcomes in A , only $(3, 6)$ has first coordinate 3. Thus

$$A \cap B = \{(4, 5), (5, 4), (6, 3)\}, \quad P(A \cap B) = \frac{3}{36} = \frac{1}{12}.$$

In particular, the intersection is nonempty: it is possible for the sum to be 9 and the first die to be different from 3 in the same throw.

Step 3: Apply the independence test The product of the marginal probabilities is

$$P(A)P(B) = \frac{1}{9} \cdot \frac{5}{6} = \frac{5}{54}.$$

In a common denominator, $P(A \cap B) = 9/108$ whereas $P(A)P(B) = 10/108$, so they are unequal. Equivalently, $P(B | A) = 3/4$, which differs from $P(B) = 5/6$. Knowledge of A therefore changes the probability of B .

Step 4: Conclusion Events A and B are neither independent nor mutually exclusive.

Quick Tip to Solve: List the four ordered pairs with sum 9, then remove only the pair whose first coordinate equals 3.

30(a). Solve the differential equation $2(y + 3) - xy \frac{dy}{dx} = 0$; **given** $y(1) = -2$.

[3 marks]

Solution:

Concept:

- A separable differential equation can be integrated after placing all y -dependent factors with dy and all x -dependent factors with dx .
- Use $y/(y + 3) = 1 - 3/(y + 3)$, together with $\int du/u = \ln |u|$.
- The initial condition determines the integration constant and selects the local solution branch through the prescribed point.

Step 1: Separate the variables The given equation is

$$xy \frac{dy}{dx} = 2(y + 3).$$

Near the initial point $(1, -2)$, both x and $y + 3$ are nonzero. Dividing by $x(y + 3)$ gives

$$\frac{y}{y + 3} dy = \frac{2}{x} dx.$$

The division excludes the constant solution $y = -3$, but that solution cannot satisfy the supplied condition $y(1) = -2$.

Quick Tip to Solve: Before integrating, divide y by $y + 3$ to get $1 - 3/(y + 3)$; substitute the initial point only after integration.

Step 2: Integrate both sides Rewrite the left side before integration:

$$\int \left(1 - \frac{3}{y+3}\right) dy = \int \frac{2}{x} dx,$$
$$y - 3 \ln |y+3| = 2 \ln |x| + C.$$

Absolute values are required by the logarithmic antiderivative. The relation is an implicit family of solutions on intervals where the separation is valid.

Step 3: Use the initial condition and verify Substitute $x = 1$, $y = -2$. Since $\ln 1 = 0$,

$$-2 - 3 \ln |-2+3| = 2 \ln |1| + C \implies C = -2.$$

To check the result, differentiate the implicit relation:

$$\left(1 - \frac{3}{y+3}\right) \frac{dy}{dx} = \frac{2}{x}.$$

The bracket equals $y/(y+3)$; multiplying by $x(y+3)$ recovers $xy \, dy/dx = 2(y+3)$. At the initial point its coefficient is nonzero, so the relation defines the required local branch.

Step 4: Conclusion The particular solution is $y - 3 \ln |y+3| = 2 \ln |x| - 2$, taking the branch through $(1, -2)$.

Quick Tip to Solve: Before integrating, divide y by $y+3$ to get $1 - 3/(y+3)$; substitute the initial point only after integration.

OR

30(b). Solve the following differential equation: $(1+x^2) \frac{dy}{dx} + 2xy = 4x^2$.

[3 marks]

Solution:

Concept:

- A first-order linear equation has the form $dy/dx + P(x)y = Q(x)$. Its integrating factor is $I(x) = e^{\int P(x) dx}$.
- Multiplication by the integrating factor converts the left side to the product derivative $d(Iy)/dx$.
- The arbitrary integration constant remains undetermined because no initial condition is given.

Step 1: Put the equation in linear form The given equation is

$$(1+x^2) \frac{dy}{dx} + 2xy = 4x^2.$$

Since $1+x^2 > 0$ for every real x , divide throughout by this expression to obtain

$$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}.$$

Thus $P(x) = 2x/(1+x^2)$ and $Q(x) = 4x^2/(1+x^2)$.

Quick Tip to Solve: Recognize $(1+x^2)y$ as the product: its derivative is exactly $(1+x^2)y' + 2xy$.

Step 2: Determine the integrating factor Using the substitution $u = 1 + x^2$, $du = 2x dx$,

$$\int P(x) dx = \int \frac{2x}{1+x^2} dx = \ln(1+x^2).$$

We may choose

$$I(x) = e^{\ln(1+x^2)} = 1 + x^2.$$

A nonzero constant multiple of this factor produces the same family of solutions, so no extra constant is needed in the integrating factor.

Step 3: Integrate the product derivative Multiply the linear equation by $1 + x^2$:

$$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2.$$

The product rule identifies the left side exactly as

$$\frac{d}{dx}[(1+x^2)y] = 4x^2.$$

Integrating yields

$$(1+x^2)y = \frac{4}{3}x^3 + C.$$

Differentiating this relation returns the original equation, so it verifies the result for every real x . Since the problem supplies no initial value, C is an arbitrary real constant.

Step 4: Conclusion The general solution is $y = \frac{\frac{4}{3}x^3 + C}{1+x^2}$, where C is an arbitrary constant.

Quick Tip to Solve: Recognize $(1+x^2)y$ as the product: its derivative is exactly $(1+x^2)y' + 2xy$.

31. If $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$, then find the values of a and b .

[3 marks]

Solution:

Concept:

- The power rule gives $\int_a^b x^n dx = (b^{n+1} - a^{n+1})/(n+1)$, for $n \neq -1$.
- Equal fourth powers of real numbers imply equal magnitudes, so $b = a$ or $b = -a$. A nonzero integral rules out equal limits.

Step 1: Evaluate the first integral By the power rule,

$$\int_a^b x^3 dx = \left[\frac{x^4}{4} \right]_a^b = \frac{b^4 - a^4}{4} = 0.$$

Hence $b^4 = a^4$. Factoring gives $(b-a)(b+a)(b^2+a^2) = 0$, so for real limits either $b = a$ or $b = -a$. The possibility that both limits are zero is already included in these cases.

Quick Tip to Solve: Obtain symmetric limits from equal fourth powers, then use the even-power integral to determine their magnitude and order.

Step 2: Use the second integral to fix the limits Equal limits would make the second integral zero, contradicting its given value $2/3$. Consequently $b = -a$. Evaluating the second integral gives

$$\frac{b^3 - a^3}{3} = \frac{2}{3} \implies b^3 - a^3 = 2.$$

Substitute $b = -a$: $-2a^3 = 2$, so $a^3 = -1$. Taking the real cube root yields $a = -1$, and thus $b = 1$. The reversed order cannot satisfy the positive integral of the nonnegative function x^2 .

Step 3: Verify both given conditions With these limits, direct substitution gives $(1^4 - (-1)^4)/4 = 0$, in agreement with the first integral. Equivalently, the odd function x^3 integrates to zero on this symmetric interval. For the even power, $(1^3 - (-1)^3)/3 = 2/3$, which verifies the second condition. Both equations are therefore satisfied by the same ordered pair, and the alternatives considered above have been excluded.

Step 4: Conclusion The required limits are $a = -1, b = 1$.

Quick Tip to Solve: Obtain symmetric limits from equal fourth powers, then use the even-power integral to determine their magnitude and order.

Section D (20 marks)

(This section comprises of 4 Long Answer (LA) type questions of 5 marks each.)

32(a). Find the shortest distance between the lines:

$$\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3} \text{ and } \frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}.$$

[5 marks]

Solution:

Concept:

- A line in symmetric form $(x - x_0)/l = (y - y_0)/m = (z - z_0)/n$ passes through (x_0, y_0, z_0) and has direction vector (l, m, n) .
- For nonparallel lines through points with position vectors $\mathbf{a}_1, \mathbf{a}_2$, and direction vectors $\mathbf{b}_1, \mathbf{b}_2$, their shortest distance is $d = |(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)| / |\mathbf{b}_1 \times \mathbf{b}_2|$.
- The cross product is perpendicular to both line directions. Projecting any connecting vector onto this common normal removes its components along the two lines.

Step 1: Identify points and direction vectors From the first line, choose $A = (-1, 1, 9)$ and $\mathbf{b}_1 = (2, 1, -3)$. From the second, choose $B = (3, -15, 9)$ and $\mathbf{b}_2 = (2, -7, 5)$. In particular, $y + 15 = y - (-15)$, so the second point has a negative y -coordinate. The connecting vector is

$$\overrightarrow{AB} = \mathbf{a}_2 - \mathbf{a}_1 = (4, -16, 0).$$

The direction vectors are not scalar multiples, so the nonparallel-line distance formula applies.

Quick Tip to Solve: Retain the sign of the point coordinate from $y + 15$; use the scalar triple product and take its absolute value only in the distance formula.

Step 2: Compute the common normal Taking the cross product in the stated order,

$$\begin{aligned}\mathbf{b}_1 \times \mathbf{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 2 & -7 & 5 \end{vmatrix} \\ &= (5 - 21)\hat{i} - (10 + 6)\hat{j} + (-14 - 2)\hat{k} \\ &= -16\hat{i} - 16\hat{j} - 16\hat{k}.\end{aligned}$$

Its magnitude is $\sqrt{(-16)^2 + (-16)^2 + (-16)^2} = 16\sqrt{3}$. As a check, its dot products with both $(2, 1, -3)$ and $(2, -7, 5)$ vanish, confirming perpendicularity to both lines.

Step 3: Project the connecting vector onto the normal The scalar triple product is

$$(4, -16, 0) \cdot (-16, -16, -16) = -64 + 256 + 0 = 192.$$

For arbitrary points on the two lines, their connecting vector differs from $\overrightarrow{AB}$ only by multiples of the line direction vectors. Those additions have zero projection onto the common normal. Thus this normal projection is fixed, and its magnitude is the separation of the lines. Using the formula,

$$d = \frac{|192|}{16\sqrt{3}} = \frac{12}{\sqrt{3}} = 4\sqrt{3}.$$

The nonzero scalar triple product also shows that the lines are noncoplanar, so this is a positive distance between skew lines rather than zero at an intersection.

Step 4: Conclusion The shortest distance between the given lines is $4\sqrt{3}$ units.

Quick Tip to Solve: Retain the sign of the point coordinate from $y + 15$; use the scalar triple product and take its absolute value only in the distance formula.

OR

32(b). Find the image A' of the point $A(2, 1, 2)$ in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA' . Find the foot of perpendicular from point A on the line l . [5 marks]

Solution:

Concept:

- For reflection of a point A in a line, its foot H on the line is the midpoint of the segment joining A to its image A' . Thus, $\vec{OA'} = 2\vec{OH} - \vec{OA}$.
- A perpendicular displacement has zero scalar product with the direction vector of the line: $\overrightarrow{AH} \cdot \vec{d} = 0$.
- A line through a point with position vector $\vec{a}$ and direction vector $\vec{v}$ has equation $\vec{r} = \vec{a} + t\vec{v}$, where $t \in \mathbb{R}$.

Step 1: Express a general point on the given line The given line passes through $B(4, 2, 2)$ and has direction vector $\vec{d} = (1, -1, -1)$. Let the perpendicular from $A(2, 1, 2)$ meet it at H . Since H lies on l ,

Quick Tip to Solve: Find the foot using a zero scalar product first; then double its coordinates and subtract the original point to get the image.

write

$$H = (4 + \lambda, 2 - \lambda, 2 - \lambda).$$

The displacement from A to this point is therefore

$$\overrightarrow{AH} = (2 + \lambda, 1 - \lambda, -\lambda).$$

Step 2: Apply the perpendicularity condition As $AH \perp l$, its direction is perpendicular to $\vec{d}$. Hence

$$\begin{aligned}(2 + \lambda) - (1 - \lambda) - (-\lambda) &= 0, \\ 1 + 3\lambda &= 0, \\ \lambda &= -\frac{1}{3}.\end{aligned}$$

Substitution into the coordinates of H gives the foot

$$H = \left(\frac{11}{3}, \frac{7}{3}, \frac{7}{3}\right).$$

In particular, $\overrightarrow{AH} = (5/3, 4/3, 1/3)$, whose scalar product with $(1, -1, -1)$ is $(5 - 4 - 1)/3 = 0$, checking the perpendicularity.

Step 3: Use the midpoint property to obtain the image Under reflection in l , the point H remains fixed and the perpendicular displacement reverses direction. Consequently, H bisects AA' , so

$$\begin{aligned}A' &= 2H - A \\ &= \left(\frac{22}{3} - 2, \frac{14}{3} - 1, \frac{14}{3} - 2\right) \\ &= \left(\frac{16}{3}, \frac{11}{3}, \frac{8}{3}\right).\end{aligned}$$

The displacements $\overrightarrow{AH}$ and $\overrightarrow{HA'}$ are equal, as required for the image in the line.

Step 4: Write the equation of the joining line A direction vector of AA' is

$$A' - A = \left(\frac{10}{3}, \frac{8}{3}, \frac{2}{3}\right) = \frac{2}{3}(5, 4, 1).$$

Thus, using A as a point on the line, its vector and Cartesian forms are

$$\begin{aligned}\vec{r} &= 2\hat{i} + \hat{j} + 2\hat{k} + t(5\hat{i} + 4\hat{j} + \hat{k}), \quad t \in \mathbb{R}, \\ \frac{x-2}{5} &= \frac{y-1}{4} = \frac{z-2}{1}.\end{aligned}$$

The parameter values $t = 1/3$ and $t = 2/3$ give H and A' , respectively, confirming that both lie on this joining line.

Quick Tip to Solve: Find the foot using a zero scalar product first; then double its coordinates and subtract the original point to get the image.

Step 5: Conclusion The image and the foot of the perpendicular are respectively

$$A'(16/3, 11/3, 8/3), \quad H(11/3, 7/3, 7/3).$$

The line joining the point and its image is

$$\vec{r} = 2\hat{i} + \hat{j} + 2\hat{k} + t(5\hat{i} + 4\hat{j} + \hat{k}).$$

Quick Tip to Solve: Find the foot using a zero scalar product first; then double its coordinates and subtract the original point to get the image.

33. Find: $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx.$

[5 marks]

Solution:

Concept:

- Substitution converts an integral into a simpler variable: if $u = g(x)$, then $du = g'(x) dx$.
- For $a > 0$, $\int \frac{du}{u^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$, where C is an arbitrary constant.
- On intervals where the given integrand is real and defined, $\tan x > 0$. Hence $\sqrt{\cot x} = 1/\sqrt{\tan x}$, taking the nonnegative square roots.

Step 1: Introduce the square-root substitution Denote the required integral by I and put $t = \sqrt{\tan x}$. Then $t > 0$, $\tan x = t^2$, and differentiation gives

$$\sec^2 x dx = 2t dt.$$

Since $\sec^2 x = 1 + \tan^2 x = 1 + t^4$, we have

$$dx = \frac{2t}{1 + t^4} dt, \quad \sqrt{\cot x} = \frac{1}{t}.$$

Therefore both square-root terms become rational functions of the same variable:

$$I = \int \left(t + \frac{1}{t}\right) \frac{2t}{1 + t^4} dt = \int \frac{2(t^2 + 1)}{t^4 + 1} dt.$$

Step 2: Reduce the quartic denominator to a quadratic Divide the numerator and denominator of the integrand by t^2 :

$$I = \int \frac{2(1 + t^{-2})}{t^2 + t^{-2}} dt.$$

Now choose $u = t - 1/t$. Its differential exactly matches the numerator because

$$du = (1 + t^{-2}) dt.$$

Quick Tip to Solve: After putting t equal to the square root of $\tan x$, use $u=t-1/t$; its derivative matches the numerator after division by t squared.

Also, squaring the substitution gives

$$u^2 = t^2 - 2 + t^{-2}, \quad t^2 + t^{-2} = u^2 + 2.$$

Thus the transformed integral is the standard inverse-tangent integral

$$I = 2 \int \frac{du}{u^2 + 2}.$$

Step 3: Integrate and restore the original variable Use $a = \sqrt{2}$ in the standard formula. Then

$$I = \sqrt{2} \tan^{-1} \left(\frac{u}{\sqrt{2}} \right) + C.$$

Since $u = \sqrt{\tan x} - \sqrt{\cot x}$, this yields an antiderivative directly in terms of x . The constant is arbitrary on each interval on which the integrand is defined.

Step 4: Verify the antiderivative by differentiation To check the substitutions, observe that

$$\frac{dt}{dx} = \frac{1+t^4}{2t}, \quad \frac{du}{dx} = (1+t^{-2}) \frac{1+t^4}{2t}.$$

Differentiating $F = \sqrt{2} \tan^{-1}(u/\sqrt{2})$ gives

$$\begin{aligned} F' &= \frac{2}{u^2 + 2} \frac{du}{dx} \\ &= \frac{2t^2}{1+t^4} \frac{(1+t^{-2})(1+t^4)}{2t} \\ &= t + \frac{1}{t}. \end{aligned}$$

This equals the original integrand $\sqrt{\tan x} + \sqrt{\cot x}$, confirming the coefficient and the sign in the answer.

Step 5: Conclusion

$$\int (\sqrt{\tan x} + \sqrt{\cot x}) dx = \sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}} \right) + C.$$

Quick Tip to Solve: After putting t equal to the square root of $\tan x$, use $u=t-1/t$; its derivative matches the numerator after division by t squared.

34. Using integration, find the area of the region bounded by the line $y = 5x + 2$, the x -axis and the ordinates $x = -2$ and $x = 2$. [5 marks]

Solution:

Concept:

- The geometrical area between $y = f(x)$ and the x -axis on $[a, b]$ is $\int_a^b |f(x)| dx$. Unlike a signed integral, every part of an area must be non-negative.
- If the curve crosses the x -axis inside the interval, split the integral at the zero of $f(x)$. Below the axis use $-f(x)$, and above the axis use $f(x)$.
- Here an antiderivative of $5x + 2$ is $F(x) = \frac{5}{2}x^2 + 2x$. Areas are expressed in square units because no particular length unit is specified.

Step 1: Locate the crossing and determine the signs The line meets the x -axis where its ordinate is zero:

$$5x + 2 = 0 \implies x = -\frac{2}{5}.$$

This point lies strictly between the bounding ordinates $x = -2$ and $x = 2$. At the left endpoint the ordinate is -8 , while at the right endpoint it is 12 . Since the line is increasing, it stays below the axis on $[-2, -\frac{2}{5}]$ and above the axis on $[-\frac{2}{5}, 2]$. Therefore the enclosed region has two portions whose positive areas must be added.

Step 2: Express the geometrical area as two integrals The vertical distance from the line to the axis is $-(5x + 2)$ on the left portion and $5x + 2$ on the right portion. Thus, writing the required area as S ,

$$S = - \int_{-2}^{-2/5} (5x + 2) dx + \int_{-2/5}^2 (5x + 2) dx.$$

Integrating without splitting would allow the area below the axis to cancel part of the area above it, and would not give the geometrical area requested.

Step 3: Evaluate the portion below the axis Using $F(x) = \frac{5}{2}x^2 + 2x$, we have

$$F(-2) = 10 - 4 = 6, \quad F\left(-\frac{2}{5}\right) = \frac{2}{5} - \frac{4}{5} = -\frac{2}{5}.$$

Hence the positive area of the left portion is

$$S_1 = -[F(x)]_{-2}^{-2/5} = -\left(-\frac{2}{5} - 6\right) = \frac{32}{5}.$$

Step 4: Evaluate the portion above the axis and add At the right endpoint, $F(2) = 10 + 4 = 14$. Therefore

$$S_2 = [F(x)]_{-2/5}^2 = 14 - \left(-\frac{2}{5}\right) = \frac{72}{5}.$$

Quick Tip to Solve: First solve $5x + 2 = 0$; split at $-2/5$ and reverse the sign of the integral on the portion below the axis.

Adding these two positive contributions gives

$$S = S_1 + S_2 = \frac{32}{5} + \frac{72}{5} = \frac{104}{5}.$$

As a numerical check, the two portions have horizontal widths $8/5$ and $12/5$, and endpoint heights 8 and 12. Their triangle areas agree with the two integrals.

Step 5: Conclusion The required area bounded by the line, the x -axis and the two ordinates is $\frac{104}{5}$ square units, or 20.8 square units.

Quick Tip to Solve: First solve $5x + 2 = 0$; split at $-2/5$ and reverse the sign of the integral on the portion below the axis.

35(a). Given $A = \begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}$, find AB . Hence, solve the system of linear equations:

$$x - y + z = 4$$

$$x - 2y - 2z = 9$$

$$2x + y + 3z = 1$$

[5 marks]

Solution:

Concept:

- In matrix multiplication, entry (i, j) of AB is the dot product of row i of A with column j of B . The order of multiplication matters.
- A system of linear equations can be written as $BX = C$, where B is the coefficient matrix, X the column of unknowns and C the constant column.
- If $AB = kI$, with $k \neq 0$, then $B^{-1} = A/k$. Left-multiplying $BX = C$ by A gives $kX = AC$, which directly yields the solution.

Step 1: Multiply each row of A by the columns of B Using the printed matrices, the nine entries of the product are

$$AB = \begin{pmatrix} -4 + 4 + 8 & 4 - 8 + 4 & -4 - 8 + 12 \\ -7 + 1 + 6 & 7 - 2 + 3 & -7 - 2 + 9 \\ 5 - 3 - 2 & -5 + 6 - 1 & 5 + 6 - 3 \end{pmatrix}.$$

For example, the first entry uses the first column $(1, 1, 2)^T$ of B , and the second entry uses its second column $(-1, -2, 1)^T$. This keeps the row-by-column order explicit and prevents confusing AB with BA .

Quick Tip to Solve: Recognise B as the coefficient matrix; once $AB = 8I$ is known, multiply the constant column by A and divide by 8.

Step 2: Simplify the matrix product Evaluating the dot products gives

$$AB = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix} = 8I_3.$$

Every off-diagonal entry vanishes and each diagonal entry equals 8. Therefore $\frac{1}{8}A$ is an inverse of the square matrix B . In particular, B is invertible and the given equations have a unique solution.

Step 3: Write the equations in matrix form With the unknowns in the order x, y, z , put

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad C = \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}.$$

The coefficient rows of the three printed equations are exactly the rows of B . Hence the system is $BX = C$. Multiplying both sides on the left by A gives

$$ABX = AC \implies 8X = AC, \quad X = \frac{1}{8}AC.$$

The matrix A must act on the left so that the already calculated product AB appears.

Step 4: Compute the unknowns and verify them Substitute the constant column:

$$AC = \begin{pmatrix} -4(4) + 4(9) + 4(1) \\ -7(4) + 1(9) + 3(1) \\ 5(4) - 3(9) - 1(1) \end{pmatrix} = \begin{pmatrix} 24 \\ -16 \\ -8 \end{pmatrix}.$$

Consequently $X = (3, -2, -1)^T$. Checking in the original equations gives

$$\begin{aligned} 3 - (-2) + (-1) &= 4, \\ 3 - 2(-2) - 2(-1) &= 9, \\ 2(3) + (-2) + 3(-1) &= 1. \end{aligned}$$

All three constants are reproduced, confirming both the coefficient order and the arithmetic.

Step 5: Conclusion $AB = 8I_3$. The unique solution of the system is $x = 3, y = -2, z = -1$.

Quick Tip to Solve: Recognise B as the coefficient matrix; once $AB = 8I$ is known, multiply the constant column by A and divide by 8.

35(b). If $A = \begin{pmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{pmatrix}$, then find A^{-1} . Hence, solve the system of linear equations:

$$x - 2y = 10$$

$$2x - y - z = 8$$

$$-2y + z = 7$$

[5 marks]

Solution:

Concept:

- A square matrix A is invertible when $|A| \neq 0$, and $A^{-1} = \text{adj}(A)/|A|$.
- The cofactor C_{ij} is $(-1)^{i+j}$ times the minor obtained by deleting row i and column j . The adjugate is the transpose of the cofactor matrix.
- For a system $BX = D$, where X is the column of unknowns, $X = B^{-1}D$. Here its coefficient matrix is $B = A^T$, so $B^{-1} = (A^{-1})^T$.

Step 1: Check that the matrix is invertible Expanding along the first row gives

$$|A| = 1\{(-1)(1) - (-2)(-1)\} - 2\{(-2)(1) - (-2)(0)\} = -3 + 4 = 1.$$

Since the determinant is nonzero, the inverse exists and the associated system has a unique solution.

Step 2: Obtain the cofactors and inverse Computing each signed minor gives

$$C_{11} = -3, \quad C_{12} = 2, \quad C_{13} = 2,$$

$$C_{21} = -2, \quad C_{22} = 1, \quad C_{23} = 1,$$

$$C_{31} = -4, \quad C_{32} = 2, \quad C_{33} = 3.$$

For example, $C_{31} = \begin{vmatrix} 2 & 0 \\ -1 & -2 \end{vmatrix} = -4$ and $C_{32} = -\begin{vmatrix} 1 & 0 \\ -2 & -2 \end{vmatrix} = 2$. Therefore

$$A^{-1} = \frac{\text{adj}(A)}{1} = \begin{pmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{pmatrix}.$$

The transpose in the adjugate is essential: the displayed inverse is not the cofactor matrix itself.

Step 3: Write the given equations in matrix form In the order x, y, z , the printed equations have coefficient matrix

$$B = \begin{pmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{pmatrix} = A^T.$$

Quick Tip to Solve: Compare the system's coefficient matrix with the supplied matrix before multiplying: here it is the transpose of A , so transpose the calculated inverse too.

Thus $BX = D$, with $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $D = \begin{pmatrix} 10 \\ 8 \\ 7 \end{pmatrix}$. Using the inverse just found,

$$X = (A^{-1})^T D = \begin{pmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{pmatrix} \begin{pmatrix} 10 \\ 8 \\ 7 \end{pmatrix}.$$

Step 4: Multiply and check all three equations Row-by-column multiplication yields

$$x = -30 + 16 + 14 = 0,$$

$$y = -20 + 8 + 7 = -5,$$

$$z = -40 + 16 + 21 = -3.$$

Substitution checks the constants independently: $0 - 2(-5) = 10$, $2(0) - (-5) - (-3) = 8$, and $-2(-5) + (-3) = 7$. Each equation is satisfied.

Step 5: Conclusion $A^{-1} = \begin{pmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{pmatrix}$, and the unique solution is $\boxed{x = 0, y = -5, z = -3}$.

Quick Tip to Solve: Compare the system's coefficient matrix with the supplied matrix before multiplying: here it is the transpose of A, so transpose the calculated inverse too.

Section E (12 marks)

(This section comprises of 3 case study based questions of 4 marks each.)

36. A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$.

Based on the above, answer the following:

[4 marks]

36(i). How many relations can be there from S to J?

[1 mark]

Solution:

Concept:

- A relation from S to J is any subset of the Cartesian product $S \times J$, whose elements are ordered speaker-judge pairs.
- A set containing n elements has 2^n subsets, since each element can independently be included or excluded.

Step 1: Count the available ordered pairs There are four speakers and three judges. Each speaker can be paired with any of the three judges, so $|S \times J| = |S| |J| = 4 \times 3 = 12$.

Step 2: Count all subsets of the Cartesian product For each of the twelve ordered pairs, choose

Quick Tip to Solve: For all relations, multiply the two set sizes first and use that product as the exponent of 2; reserve the power of the codomain size for functions.

whether it belongs to the relation. These independent binary choices give $2^{12} = 4096$ relations, including the empty relation and the whole Cartesian product. The question asks for relations, so we count all subsets; 3^4 would instead count functions assigning exactly one judge to every speaker.

Step 3: Conclusion The number of relations from S to J is $2^{4 \times 3} = 4096$.

Quick Tip to Solve: For all relations, multiply the two set sizes first and use that product as the exponent of 2; reserve the power of the codomain size for functions.

36(ii). A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$. Check if it is bijective. [1 mark]

Solution:

Concept:

- A function is one-one if distinct domain elements have distinct images.
- A function is onto if every codomain element is an image; bijective means both one-one and onto.

Step 1: Check whether the function is one-one Here $f(S_2) = J_2$ and $f(S_3) = J_2$. Since $S_2 \neq S_3$, two distinct speakers have the same judge as their image. Thus f is not one-one.

Step 2: Check whether the function is onto The images of S_1, S_2, S_3, S_4 are J_1, J_2, J_2, J_3 , respectively. Hence the range is $\{J_1, J_2, J_3\} = J$, so every judge is an image and f is onto. Being onto alone does not meet both conditions for a bijection.

Step 3: Conclusion The function is onto but not one-one, and hence it is not bijective.

Quick Tip to Solve: To disprove injectivity, find two different inputs with the same output; here use S_2 and S_3 .

36(iii)(a). How many one-one functions can be there from set S to set J ? [2 marks]

Solution:

Concept:

- An injective (one-one) function assigns different images to every pair of different inputs.
- For finite sets, an injection from S to J can exist only if $|S| \leq |J|$; this follows from the pigeonhole principle.

Step 1: Count the elements in the two sets The domain is $S = \{S_1, S_2, S_3, S_4\}$, containing four speakers, while the codomain is $J = \{J_1, J_2, J_3\}$, containing three judges. Each function must assign exactly one judge to each of the four speakers. A one-one function would require all four assigned judges to be distinct.

Step 2: Apply the pigeonhole principle There are only three possible images, so after three distinct judges have been assigned to three speakers, no unused judge remains for the fourth. In any assignment at least two speakers therefore have the same image. This contradicts the requirement for a one-one function, irrespective of which judges are assigned first. Since $4 > 3$, no injection from S to J exists.

Quick Tip to Solve: Compare the set sizes before counting injections: a larger finite domain cannot inject into a smaller codomain.

Step 3: Conclusion The number of one-one functions from S to J is $\boxed{0}$.

Quick Tip to Solve: Compare the set sizes before counting injections: a larger finite domain cannot inject into a smaller codomain.

OR

36(iii)(b). Another student considers a relation $R_1 = \{(S_1, S_2), (S_2, S_4)\}$ in set S . Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. [2 marks]

Solution:

Concept:

- A relation on S is reflexive if (x, x) belongs to it for every $x \in S$.
- Symmetry requires (y, x) whenever (x, y) is present. A single pair without its reverse disproves symmetry.

Step 1: Identify all missing diagonal pairs The set is $S = \{S_1, S_2, S_3, S_4\}$ and the given relation is $R_1 = \{(S_1, S_2), (S_2, S_4)\}$. Neither of these is a diagonal pair. Reflexivity requires the four pairs $(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)$. Thus every one of these must be added; fewer than four additions would leave at least one element unrelated to itself.

Step 2: Verify that symmetry still fails After adding just those four diagonal pairs, every element is related to itself, so the enlarged relation is reflexive. It still contains (S_1, S_2) without (S_2, S_1) , and (S_2, S_4) without (S_4, S_2) . In particular, the first missing reverse pair already proves that the enlarged relation is not symmetric. No additional pair is needed to secure this property.

Step 3: Conclusion Add exactly $\{(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)\}$. The minimum number of ordered pairs to add is $\boxed{4}$.

Quick Tip to Solve: For reflexivity add only missing diagonal pairs, then retain an off-diagonal pair whose reverse is absent to keep the relation non-symmetric.

37. Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated).

Based on the above, answer the following:

[4 marks]

37(i)(a). What is the probability that a randomly selected car is an electric car?

[2 marks]

Solution:

Concept:

- Let A, B, C denote manufacture by Amber, Bonzi and Comet respectively. These events are mutually exclusive and exhaustive, with probabilities 0.60, 0.30, 0.10.

Quick Tip to Solve: Weight each electric fraction by the corresponding market share before adding the three contributions.

- For an electric car, event E , the law of total probability is $P(E) = \sum_{X=A,B,C} P(X)P(E | X)$.

Step 1: Record the conditional electric-car probabilities The electric percentages refer to each manufacturer's own production, not to all cars in the market. Thus

$$P(E | A) = 0.20, \quad P(E | B) = 0.10, \quad P(E | C) = 0.05.$$

To obtain each manufacturer's contribution to the total electric-car share, multiply its market share by its conditional electric fraction.

Step 2: Combine the three disjoint contributions An electric car comes from exactly one of these manufacturers. Hence the required weighted sum is

$$\begin{aligned} P(E) &= (0.60)(0.20) + (0.30)(0.10) + (0.10)(0.05) \\ &= 0.120 + 0.030 + 0.005. \end{aligned}$$

The unweighted average of the three electric percentages would be inappropriate because their production shares differ.

Step 3: Conclusion The probability that a randomly selected car is electric is $0.155 = \frac{31}{200}$, or 15.5%.

Quick Tip to Solve: Weight each electric fraction by the corresponding market share before adding the three contributions.

OR

37(i)(b). What is the probability that a randomly selected car is a petrol car?

[2 marks]

Solution:

Concept:

- Let A, B, C denote manufacture by Amber, Bonzi and Comet respectively. These events are mutually exclusive and exhaustive, with probabilities 0.60, 0.30, 0.10.
- Petrol and electric cars exhaust the stated types. If F denotes a petrol car, then $P(F | X) = 1 - P(E | X)$ for each manufacturer X .

Step 1: Find each conditional petrol-car fraction The electric fractions within Amber, Bonzi and Comet production are 0.20, 0.10, 0.05. Their complementary petrol fractions are therefore

$$P(F | A) = 0.80, \quad P(F | B) = 0.90, \quad P(F | C) = 0.95.$$

These conditional probabilities must be weighted by the production shares; adding or averaging them without weights ignores the different numbers of cars produced.

Quick Tip to Solve: For the petrol alternative, subtract each electric percentage from 100 percent, then weight by market share.

Step 2: Apply total probability to petrol cars Since the manufacturers form a partition of all cars,

$$\begin{aligned}P(F) &= (0.60)(0.80) + (0.30)(0.90) + (0.10)(0.95) \\&= 0.480 + 0.270 + 0.095.\end{aligned}$$

As a check, the electric fraction is $0.120 + 0.030 + 0.005 = 0.155$, so the complementary petrol fraction is $1 - 0.155$, agreeing with this weighted calculation.

Step 3: Conclusion The probability of a petrol car is $0.845 = \frac{169}{200}$, or 84.5%.

Quick Tip to Solve: For the petrol alternative, subtract each electric percentage from 100 percent, then weight by market share.

37(ii). A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet? [1 mark]

Solution:

Concept:

- Bayes' theorem reverses the conditioning: $P(C | E) = P(C)P(E | C)/P(E)$, where C denotes Comet and E an electric car.
- The total electric probability includes the contributions of all three manufacturers.

Step 1: Find the electric-car population From the given market shares and conditional electric fractions,

$$P(E) = 0.60(0.20) + 0.30(0.10) + 0.10(0.05) = 0.155.$$

Step 2: Condition on the car being electric The joint probability of a Comet electric car is $0.10(0.05) = 0.005$. Restricting the sample space to electric cars gives

$$P(C | E) = \frac{0.005}{0.155} = \frac{5}{155}.$$

The given 0.05 is instead the probability of electric status conditional on Comet manufacture.

Step 3: Conclusion The required probability is $\frac{1}{31}$.

Quick Tip to Solve: Use the Comet electric contribution as numerator and the total electric contribution as denominator.

37(iii). A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi? [1 mark]

Solution:

Concept:

- Let A, B, C denote Amber, Bonzi and Comet, and E electric status. Manufacture by Amber and Bonzi are mutually exclusive events.
- Conditional addition gives $P(A \cup B | E) = [P(A \cap E) + P(B \cap E)]/P(E)$.

Step 1: Compute the relevant joint probabilities The contributions of Amber and Bonzi electric cars are $0.60(0.20) = 0.120$ and $0.30(0.10) = 0.030$. Including Comet gives

$$P(E) = 0.120 + 0.030 + 0.10(0.05) = 0.155.$$

Step 2: Add the two disjoint electric contributions No car is manufactured by both Amber and Bonzi, so there is no overlap to subtract. Within the electric-car sample space,

$$P(A \cup B | E) = \frac{0.120 + 0.030}{0.155} = \frac{150}{155}.$$

Using $0.60 + 0.30$ alone would ignore the information that the selected car is electric.

Step 3: Conclusion The required probability is $\boxed{\frac{30}{31}}$.

Quick Tip to Solve: After conditioning on electric status, Amber or Bonzi is the complement of Comet: subtract the Comet conditional probability from one.

38. A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres.

Based on the above, answer the following:

[4 marks]

38(i). Find the intervals on which $f(x)$ is increasing or decreasing, $x \in [0, \pi]$.

[2 marks]

Solution:

Concept:

- A differentiable function increases where its derivative is positive and decreases where its derivative is negative.
- Use the product rule for $f(x) = e^x \sin x$; since $e^x > 0$, the derivative's sign depends only on the trigonometric factor.

Step 1: Differentiate and locate the stationary point By the product rule,

$$f'(x) = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x).$$

Write $\sin x + \cos x = \sqrt{2} \sin(x + \pi/4)$. Thus $f'(x) = 0$ requires $x + \pi/4 = k\pi$. The only such point in

Quick Tip to Solve: Replace sine plus cosine by a shifted sine; its first zero in the given domain is at three pi over four.

$[0, \pi]$ is $x = 3\pi/4$. The exponential factor never vanishes.

Step 2: Examine the sign on either side For $0 < x < 3\pi/4$, the angle $x + \pi/4$ lies in $(\pi/4, \pi)$, where sine is positive. Hence $f'(x) > 0$. For $3\pi/4 < x < \pi$, the angle lies in $(\pi, 5\pi/4)$, where sine is negative, giving $f'(x) < 0$. These two intervals exhaust the interior of the stated domain except the stationary point.

Step 3: Conclusion f is strictly increasing on $[0, 3\pi/4]$ and strictly decreasing on $[3\pi/4, \pi]$. The corresponding derivative-sign intervals are $(0, 3\pi/4)$ and $(3\pi/4, \pi)$.

Quick Tip to Solve: Replace sine plus cosine by a shifted sine; its first zero in the given domain is at three pi over four.

38(ii). Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion. [2 marks]

Solution:

Concept:

- An interior critical point occurs where $f'(x) = 0$ or where the derivative does not exist. This function is differentiable everywhere.
- At a stationary point c , $f''(c) < 0$ proves a local maximum, while $f''(c) > 0$ proves a local minimum. An inflexion point requires a change in concavity.

Step 1: Determine the critical point in the interval Differentiation gives $f'(x) = e^x(\sin x + \cos x)$. Since e^x is positive, setting this equal to zero gives $\sin(x + \pi/4) = 0$. In the interior of $[0, \pi]$, its sole solution is $c = 3\pi/4$. There are no points where the derivative is undefined.

Step 2: Apply the second derivative test Differentiating once more,

$$f''(x) = e^x(\sin x + \cos x) + e^x(\cos x - \sin x) = 2e^x \cos x.$$

Therefore $f''(3\pi/4) = -\sqrt{2}e^{3\pi/4} < 0$. This agrees with the change of f' from positive to negative at this point. For completeness, concavity changes at $x = \pi/2$, where f'' changes from positive to negative; however, $f'(\pi/2) = e^{\pi/2} \neq 0$, so that inflexion point is not a stationary critical point. The endpoints are boundaries rather than interior stationary points.

Step 3: Conclusion The sole interior critical point $x = 3\pi/4$ is a local maximum, with value $e^{3\pi/4}/\sqrt{2}$. There is no interior local minimum. The point $(\pi/2, e^{\pi/2})$ is an inflexion point, but is not a critical point of f .

Quick Tip to Solve: Classify a stationary point using the second derivative there; do not confuse a zero of the second derivative with a zero of the first.