CBSE Class 12 Mathematics Question Paper 2025 with Solution PDF-Set 65-7-1

Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions

(i) This question paper contains **38** questions. **All questions are compulsory.**

(ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.

(iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.

(iv) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.

(v) In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.

(vi) In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.

(vii) In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.

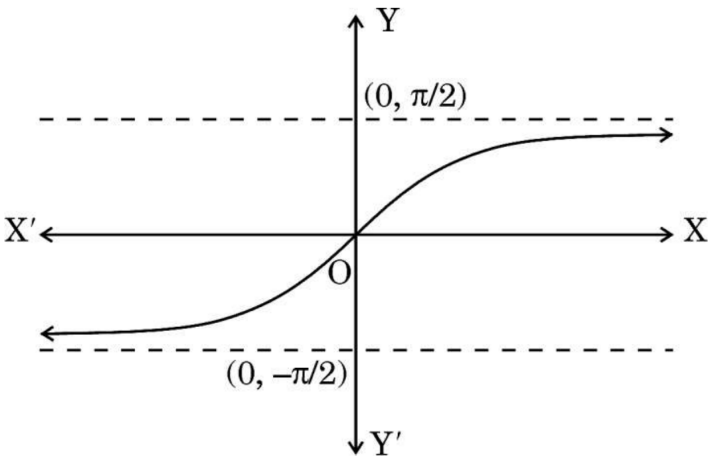
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculator is **not** allowed.

Section A (20 marks)

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The given graph illustrates: [1 mark]



(A) $y = \tan^{-1} x$

(B) $y = \text{cosec}^{-1} x$

(C) $y = \cot^{-1} x$

(D) $y = \sec^{-1} x$

Correct Answer: (A) $y = \tan^{-1} x$

Solution:

Concept:

- The principal inverse tangent has domain $\mathbb{R}$, range $(-\pi/2, \pi/2)$, and passes through the origin.
- Its derivative $1/(1+x^2) > 0$ makes its graph increasing everywhere.

Step 1: Identify the visible features The curve is continuous through $(0, 0)$, increases from left to right, and approaches the horizontal lines $y = -\pi/2$ and $y = \pi/2$ without reaching them. These are precisely the end limits of $\tan^{-1} x$.

Step 2: Eliminate the other functions The principal $\cot^{-1} x$ decreases and has range $(0, \pi)$. Both $\text{cosec}^{-1} x$ and $\sec^{-1} x$ require $|x| \geq 1$, so neither has a continuous graph through the origin. Thus options (B), (C) and (D) fail the displayed features.

Step 3: Conclusion The graph is $y = \tan^{-1} x$. Hence, the correct option is (A).

Quick Tip to Solve: Check the origin, direction of increase and both asymptotes before choosing an inverse function.

2. Domain of $f(x) = \cos^{-1} x + \sin x$ is: [1 mark]

(A) $\mathbb{R}$

(B) $(-1, 1)$

(C) $[-1, 1]$

(D) φ

Correct Answer: (C) $[-1, 1]$

Solution:

Concept:

- For a sum of real-valued functions, the domain is the intersection of their individual domains.
- The real function $\cos^{-1} x$ requires $-1 \le x \le 1$, whereas $\sin x$ is defined for every real x.

Step 1: Intersect the domains Both summands must exist at the same input. Therefore

$$D_f = [-1, 1] \cap \mathbb{R} = [-1, 1].$$

Adding an unrestricted sine term does not remove the restriction imposed by the inverse cosine.

Step 2: Check the endpoints and choices At $x = 1$, $\cos^{-1} 1 = 0$, and at $x = -1$, $\cos^{-1}(-1) = \pi$; the sine term is also finite at both inputs. Thus the endpoints are included, ruling out (B). Inputs outside this interval rule out (A), while the interval is nonempty, ruling out (D).

Step 3: Conclusion The domain is $[-1, 1]$. Hence, the correct option is (C).

Quick Tip to Solve: For a sum, intersect the domains; then check whether boundary inputs are allowed.

3. What is the total number of possible matrices of order 3×3 with each entry as $\sqrt{2}$ or $\sqrt{3}$? [1 mark]

(A) 9

(B) 512

(C) 615

(D) 64

Correct Answer: (B) 512

Solution:

Concept:

- A matrix of order $m \times n$ has mn distinct entry positions.
- If each position has k independent choices, the multiplication principle gives k^{mn} matrices.

Step 1: Count independent entry positions A 3×3 matrix contains $3 \cdot 3 = 9$ entries. Each entry can independently be either $\sqrt{2}$ or $\sqrt{3}$; no condition relates different entries or requires symmetry.

Step 2: Apply the multiplication principle There are two choices at each of nine positions, giving

$$2 \cdot 2 \cdots 2 = 2^9 = 512.$$

The number 9 counts positions rather than matrices. The choice 64 equals 2^6 and would count only

Quick Tip to Solve: Count all entry positions first, then raise the number of allowed values to that count.

six independent positions; 615 is not the product of the required choices.

Step 3: Conclusion The total number is 512. Hence, the correct option is (B).

Quick Tip to Solve: Count all entry positions first, then raise the number of allowed values to that count.

4. The matrix $A = \begin{pmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{pmatrix}$ is a/an: [1 mark]

(A) scalar matrix

(B) identity matrix

(C) null matrix

(D) symmetric matrix

Correct Answer: (D) symmetric matrix

Solution:

Concept:

- A real square matrix is symmetric when $A^T = A$.
- A scalar matrix is diagonal with all diagonal entries equal; the identity has every diagonal entry 1, and the null matrix has every entry zero.

Step 1: Compare the matrix with its transpose Transposition interchanges off-diagonal positions. Every off-diagonal entry here is zero, so those pairs agree; diagonal entries stay in their original positions. Consequently $A^T = A$, and the matrix is symmetric.

Step 2: Check the other offered classifications The diagonal entries $\sqrt{3}, \sqrt{2}, \sqrt{5}$ are unequal, so the matrix is not scalar. They are not all 1, so it is not the identity matrix; they are nonzero, so it is not the null matrix. It is also diagonal, but that classification is not one of the offered answers.

Step 3: Conclusion The matrix is symmetric. Hence, the correct option is (D).

Quick Tip to Solve: Every real diagonal matrix is symmetric, but it is scalar only if its diagonal entries agree.

5. If A and B are two square matrices each of order 3 with $|A| = 3$ and $|B| = 5$, then $|2AB|$ is: [1 mark]

(A) 30

(B) 120

(C) 15

(D) 225

Correct Answer: (B) 120

Solution:

Concept:

- For square matrices of the same order, $|AB| = |A||B|$.

Quick Tip to Solve: The scalar multiplier is raised to the matrix order before multiplying the determinant.

- For a matrix M of order n, $|kM| = k^n|M|$, since all n rows acquire the factor k.

Step 1: Evaluate the determinant of the product Using the given determinants,

$$|AB| = |A||B| = 3 \cdot 5 = 15.$$

This calculation uses the product rule and does not require knowing the individual entries of either matrix.

Step 2: Account for multiplication by 2 The product AB is of order 3, so multiplying the entire matrix by 2 multiplies its determinant by 2^3. Hence

$$|2AB| = 2^3|AB| = 8 \cdot 15 = 120.$$

The value 30 uses only one factor 2; 15 omits scaling; 225 incorrectly squares the product determinant.

Step 3: Conclusion $|2AB| = 120$. Hence, the correct option is (B).

Quick Tip to Solve: The scalar multiplier is raised to the matrix order before multiplying the determinant.

6. Let A be a square matrix of order 3. If $|A| = 5$, then $|\operatorname{adj} A|$ is: [1 mark]

(A) 5

(B) 125

(C) 25

(D) -5

Correct Answer: (C) 25

Solution:

Concept:

- For a square matrix A of order n, $A \operatorname{adj} A = |A|I_n$.
- Taking determinants gives $|A||\operatorname{adj} A| = |A|^n$; when $|A| \neq 0$, divide to obtain $|\operatorname{adj} A| = |A|^{n-1}$.

Step 1: Apply the adjoint determinant identity Here the order is $n = 3$, and $|A| = 5 \neq 0$, so division by $|A|$ is valid. Thus

$$|\operatorname{adj} A| = |A|^{3-1} = 5^2.$$

The exponent is one less than the order, because one factor $|A|$ is already on the left of the determinant identity.

Step 2: Evaluate and distinguish the options The resulting value is $5^2 = 25$. The value 5 misses the exponent; 125 equals 5^3 before division by 5; and -5 has neither the correct sign nor magnitude. Only option (C) agrees with the adjoint identity.

Step 3: Conclusion $|\operatorname{adj} A| = 25$. Hence, the correct option is (C).

Quick Tip to Solve: For the determinant of the adjoint, use the exponent order minus one, not the order.

7. If $\begin{bmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{bmatrix} = \begin{bmatrix} x+3 & 12 \\ 0 & 35 \end{bmatrix}$**, then the value of** $(x-y)$ **is:** [1 mark]

(A) 2 or 10

(B) -2 or 10

(C) 2 or -10

(D) -2 or -10

Correct Answer: (B) -2 or 10

Solution:

Concept:

- Equal matrices have equal corresponding entries, so each matching position gives an equation.
- An equation $y^2 = a^2$, for positive a, has the two real solutions $y = a$ and $y = -a$.

Step 1: Find the value of x from corresponding entries Comparing the upper-left entries gives $2x - 1 = x + 3$, hence $x = 4$. The upper-right entries independently give $3x = 12$, confirming the same value. The lower-left entries are both zero and impose no further restriction.

Step 2: Find both possible differences The lower-right entries give $y^2 - 1 = 35$, so $y^2 = 36$ and $y = 6$ or $y = -6$. Therefore

$$x - y = 4 - 6 = -2 \quad \text{or} \quad 4 - (-6) = 10.$$

Option (B) contains both values. Options (A) and (C) have the wrong sign for the first value, and (D) has the wrong sign for the second.

Step 3: Conclusion $x - y = -2$ or 10. Hence, the correct option is (B).

Quick Tip to Solve: For equality of matrices, compare entries directly and retain both roots of a squared variable.

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8. If $f(x) = \begin{cases} 1, & \text{if } x \leq 3 \\ ax + b, & \text{if } 3 < x < 5 \\ 7, & \text{if } 5 \leq x \end{cases}$ **is continuous in** $\mathbb{R}$**, then the values of** a **and** b **are:** [1 mark]

(A) $a = 3,\ b = -8$

(B) $a = 3,\ b = 8$

(C) $a = -3,\ b = -8$

(D) $a = -3,\ b = 8$

Correct Answer: (A) $a = 3,\ b = -8$

Solution:

Concept:

- Continuity at a joining point requires the two one-sided limits to equal the function value.
- Each polynomial or constant branch is continuous within its own interval; only the joining points need checking.

Quick Tip to Solve: At each boundary equate the middle expression to the constant branch; subtract the two equations first.

Step 1: Match the branches at the first join At $x = 3$, the function value and left-hand limit are both 1. The right-hand limit of the middle branch is $3a + b$. Therefore continuity requires $3a + b = 1$.

Step 2: Match the second join and solve At $x = 5$, the value and right-hand limit are 7, while the left-hand limit is $5a + b$. Thus $5a + b = 7$. Subtracting the first equation gives $2a = 6$, so $a = 3$, and then $b = 1 - 9 = -8$. Option (B) gives the wrong intercept, while (C) and (D) give the wrong slope; none satisfies both joins.

Step 3: Conclusion $a = 3,\ b = -8$. Hence the correct option is (A).

Quick Tip to Solve: At each boundary equate the middle expression to the constant branch; subtract the two equations first.

9. If $f(x) = -2x^8$, then the correct statement is: [1 mark]

(A) $f'(\frac{1}{2}) = f'(-\frac{1}{2})$
(B) $f'(\frac{1}{2}) = -f'(-\frac{1}{2})$
(C) $-f'(\frac{1}{2}) = f'(-\frac{1}{2})$
(D) $f'(\frac{1}{2}) = -f(-\frac{1}{2})$
Correct Answer: (B) $f'(\frac{1}{2}) = -f'(-\frac{1}{2})$

Solution:

Concept:

- The power rule states that $\frac{d}{dx}x^n = nx^{n-1}$.
- An odd power changes sign under $x \mapsto -x$, so the derivative of an even power is odd.

Step 1: Differentiate and evaluate For $f(x) = -2x^8$, $f'(x) = -16x^7$. Therefore $f'(\frac{1}{2}) = -\frac{1}{8}$ and $f'(-\frac{1}{2}) = \frac{1}{8}$. These two derivative values are nonzero and opposite in sign, rather than equal.

Step 2: Check every printed comparison Option (A) is false because $-\frac{1}{8} \neq \frac{1}{8}$. Option (B) is true since $-\frac{1}{8} = -\frac{1}{8}$. Option (C) is also true: multiplying the equality in (B) by -1 gives precisely (C). For (D), $f(-\frac{1}{2}) = -\frac{1}{128}$, so $-f(-\frac{1}{2}) = \frac{1}{128} \neq -\frac{1}{8}$. The printed options thus contain two equivalent correct comparisons.

Step 3: Conclusion Hence a correct choice is option (B). The equivalent printed comparison is discussed above.

Quick Tip to Solve: For an even power function, first recognize that its derivative is odd; also compare options for equivalent equalities.

10. A spherical ball has a variable diameter $\frac{5}{2}(3x+1)$. The rate of change of its volume w.r.t. x, when $x=1$, is: [1 mark]

(A) 225π

(B) 300π

(C) 375π

(D) 125π

Correct Answer: (C) 375π

Solution:

Concept:

- For a sphere of radius r, volume is $V=\frac{4}{3}\pi r^3$.
- The chain rule gives $\frac{dV}{dx}=4\pi r^2\frac{dr}{dx}$; the radius is half the diameter.

Step 1: Convert diameter into radius The printed diameter is $d=\frac{5}{2}(3x+1)$, so $r=\frac{5}{4}(3x+1)$. Hence $\frac{dr}{dx}=\frac{15}{4}$. When $x=1$, the diameter is 10 and the radius is 5. Using the diameter directly in the volume formula would incorrectly multiply the volume by eight.

Step 2: Apply the chain rule Now

$$\left.\frac{dV}{dx}\right|_{x=1}=4\pi(5)^2\left(\frac{15}{4}\right)=375\pi.$$

Options (A), (B) and (D) do not equal this product. The derivative must include both the squared radius and its rate of change, rather than merely differentiating the diameter.

Step 3: Conclusion The required rate of change is 375π. Hence the correct option is (C).

Quick Tip to Solve: Halve the given diameter before applying the chain rule, and substitute the value of x only after differentiating.

11. If $f:\mathbb{R}\to\mathbb{R}$ is defined as $f(x)=2x-\sin x$, then f is: [1 mark]

(A) a decreasing function

(B) an increasing function

(C) maximum at $x=\frac{\pi}{2}$

(D) maximum at $x=0$

Correct Answer: (B) an increasing function

Solution:

Concept:

- A differentiable function whose derivative is positive throughout an interval is strictly increasing there.
- For every real x, $-1\le\cos x\le 1$.

Quick Tip to Solve: Bound the trigonometric part of the derivative before trying to solve an equation for critical points.

Step 1: Find the derivative and its lower bound Differentiating $f(x) = 2x - \sin x$ gives $f\prime(x) = 2 - \cos x$. Since the largest possible value of $\cos x$ is 1, $f\prime(x) \geq 2 - 1 = 1 > 0$ for every real x. In particular, the derivative never vanishes or becomes negative.

Step 2: Determine the behaviour on the entire domain For any $a < b$, the mean value theorem gives $f(b) - f(a) = f\prime(c)(b - a) > 0$ for some $c \in (a, b)$. Thus the function is increasing on all of $\mathbb{R}$, so option (A) is false. It cannot have a maximum at $\pi/2$ or at 0, since values immediately to the right of either point are larger; hence (C) and (D) are false.

Step 3: Conclusion f is an increasing function on $\mathbb{R}$. Hence the correct option is (B).

Quick Tip to Solve: Bound the trigonometric part of the derivative before trying to solve an equation for critical points.

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12. $\int \frac{e^{9\log x} - e^{8\log x}}{e^{6\log x} - e^{5\log x}}\, dx$ **is equal to:** [1 mark]

(A) $x + C$

(B) $\frac{x^2}{2} + C$

(C) $\frac{x^4}{4} + C$

(D) $\frac{x^3}{3} + C$

Correct Answer: (C) $\frac{x^4}{4} + C$

Solution:

Concept:

- For $x > 0$, $e^{k\log x} = x^k$, where $\log$ denotes the natural logarithm.
- The power rule for integration is $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$.

Step 1: Simplify the integrand The numerator is $x^9 - x^8 = x^8(x - 1)$, and the denominator is $x^6 - x^5 = x^5(x - 1)$. On the domain of the original integrand, $x > 0$ and $x \neq 1$, so the common nonzero factor can be cancelled. The quotient becomes x^3, not x or x^2.

Step 2: Integrate and verify the choices Therefore

$$\int \frac{e^{9\log x} - e^{8\log x}}{e^{6\log x} - e^{5\log x}}\, dx = \int x^3\, dx = \frac{x^4}{4} + C.$$

Differentiating the proposed antiderivatives in (A), (B), (C) and (D) gives respectively $1, x, x^3, x^2$; only (C) reproduces the simplified integrand. The constant is understood on each interval of the original domain.

Step 3: Conclusion The integral is $\frac{x^4}{4} + C$. Hence the correct option is (C).

Quick Tip to Solve: Convert every exponential of a logarithm into a power, factor the difference, and cancel before integrating.

13. For a function $f(x)$, which of the following holds true? [1 mark]

(A) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(B) $\int_{-a}^{a} f(x)\,dx = 0$, if f is an even function

(C) $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if f is an odd function

(D) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx - \int_0^a f(2a+x)\,dx$

Correct Answer: (A) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

Solution:

Concept:

- Changing variables in a definite integral requires changing both the differential and the limits.
- On symmetric limits, an odd function integrates to zero, whereas an even function gives twice the integral over the positive half.

Step 1: Prove the reflection identity Put $t = a+b-x$ in $\int_a^b f(a+b-x)\,dx$. Then $dx = -dt$, and the limits $x = a, b$ correspond to $t = b, a$. Thus

$$\int_a^b f(a+b-x)\,dx = -\int_b^a f(t)\,dt = \int_a^b f(t)\,dt,$$

proving option (A) whenever the displayed integrals exist.

Step 2: Reject the remaining identities Option (B) incorrectly assigns the odd-function property to an even function: for $f(x) = 1$, its integral is $2a$, not zero. Option (C) incorrectly assigns the even-function property to an odd function: take $f(x) = x$ and $a = 1$. The left side is zero and the right side is 1. For (D), take $f(x) = 1$, $a = 1$: its left side is 2, but its right side is $1 - 1 = 0$.

Step 3: Conclusion The universally valid identity is option (A).

Quick Tip to Solve: Under reflection x becomes a+b-x; the reversal of limits cancels the minus sign in the differential.

14. $\int \frac{e^x}{\sqrt{4-e^{2x}}}\,dx$ **is equal to:** [1 mark]

(A) $\frac{1}{2}\cos^{-1}(e^x)+C$

(B) $\frac{1}{2}\sin^{-1}(e^x)+C$

(C) $\frac{e^x}{2}+C$

(D) $\sin^{-1}(\frac{e^x}{2})+C$

Correct Answer: (D) $\sin^{-1}(\frac{e^x}{2})+C$

Solution:

Concept:

- The substitution $u=e^x$ gives $du=e^x\,dx$.
- The standard formula is $\int \frac{du}{\sqrt{a^2-u^2}}=\sin^{-1}(u/a)+C$ for $a>0$.

Step 1: Make the exponential substitution Let $u=e^x$. The integral becomes

$$\int \frac{du}{\sqrt{4-u^2}}.$$

Here the constant in the standard form is $a=2$, so the argument of the inverse sine must be $u/2$. The real integrand is defined when $e^x<2$, or $x<\log 2$.

Step 2: Integrate and check the printed options Using the formula and restoring u, we obtain $\sin^{-1}(e^x/2)+C$. Its derivative is

$$\frac{e^x/2}{\sqrt{1-e^{2x}/4}}=\frac{e^x}{\sqrt{4-e^{2x}}}.$$

Options (A) and (B) use the wrong inverse-trigonometric argument and prefactor; (A) also has a negative derivative where defined. The derivative of (C) is just $e^x/2$, which omits the square-root denominator.

Step 3: Conclusion The integral is $\sin^{-1}(e^x/2)+C$. Hence the correct option is (D).

Quick Tip to Solve: Use $u=e^x$ to absorb the numerator, then read the square of the inverse-sine scale from the denominator.

15. A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{i}+15\hat{j}+6\hat{k}$ and the other is along the vector $2\hat{i}+10\hat{j}+\lambda\hat{k}$, then the value of λ is: [1 mark]

(A) 6

(B) 1

(C) $\frac{1}{4}$

(D) 4

Correct Answer: (D) 4

Solution:

Concept:

- Two nonzero vectors are parallel if one is a scalar multiple of the other.
- All corresponding components must have the same multiplier.

Step 1: Find the common multiplier Write $2\hat{i}+10\hat{j}+\lambda\hat{k}=t(3\hat{i}+15\hat{j}+6\hat{k})$. The first component gives $t=2/3$; the second confirms $10/15=2/3$.

Step 2: Check the third component and the options The third component must therefore satisfy $\lambda=6(2/3)=4$. For options (A), (B) and (C), the third-component ratios would be 1, $1/6$ and $1/24$, respectively, none matching $2/3$. Only option (D) preserves parallelism.

Step 3: Conclusion Hence, $\boxed{\lambda=4}$, and the correct option is (D).

Quick Tip to Solve: Find the multiplier from a known component, then apply it to the unknown component.

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16. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ for any two vectors, then vectors $\vec{a}$ and $\vec{b}$ are: [1 mark]

(A) orthogonal vectors

(B) parallel to each other

(C) unit vectors

(D) collinear vectors

Correct Answer: (A) orthogonal vectors

Solution:

Concept:

- The square of the magnitude is $|\vec{v}|^2=\vec{v}\cdot\vec{v}$.
- Orthogonality is characterised by $\vec{a}\cdot\vec{b}=0$.

Step 1: Expand the two squared magnitudes Squaring the given equality gives

$$|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}.$$

Squaring is valid because magnitudes are nonnegative.

Step 2: Interpret the resulting condition Cancelling the common terms gives $4\vec{a}\cdot\vec{b}=0$. For nonzero

Quick Tip to Solve: Square equal vector magnitudes and subtract: the mixed dot-product terms reveal orthogonality.

vectors, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, so $\theta = 90°$. No condition forces either magnitude to equal one. Parallel or collinear nonzero vectors have a nonzero dot product, so options (B), (C) and (D) do not follow.

Step 3: Conclusion The vectors are orthogonal. Hence, the correct option is (A).

Quick Tip to Solve: Square equal vector magnitudes and subtract: the mixed dot-product terms reveal orthogonality.

17. If $P(A) = \frac{1}{7}$, $P(B) = \frac{5}{7}$ and $P(A \cap B) = \frac{4}{7}$, then $P(\overline{A} \mid B)$ is: [1 mark]

(A) $\frac{6}{7}$

(B) $\frac{3}{4}$

(C) $\frac{4}{5}$

(D) $\frac{1}{5}$

Correct Answer: (D) $\frac{1}{5}$

Solution:

Concept:

- For $P(B) > 0$, $P(A \mid B) = P(A \cap B)/P(B)$.
- Within B, A and $\overline{A}$ partition the event, so their conditional probabilities sum to one.

Step 1: Calculate the conditional complement Using the printed values,

$$P(\overline{A} \mid B) = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{5/7 - 4/7}{5/7} = \frac{1}{5}.$$

Option (C), $4/5$, instead calculates $P(A \mid B)$. Options (A) and (B) do not equal the required ratio.

Step 2: Check the printed data The stated data contain an inconsistency: $P(A \cap B) = 4/7 > P(A) = 1/7$, although an intersection is a subset of A. Thus no actual pair of events satisfies all three values. Nevertheless, the requested conditional-complement calculation from the printed $P(B)$ and $P(A \cap B)$ gives the listed option $1/5$.

Step 3: Conclusion The intended answer from the printed conditional-probability calculation is $\boxed{1/5}$, option (D).

Quick Tip to Solve: For a complemented event conditioned on B, subtract the intersection probability from P(B) first.

18. A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is: [1 mark]

(A) $\frac{2}{13}$

(B) $\frac{3}{26}$

(C) $\frac{19}{26}$

(D) $\frac{3}{13}$

Correct Answer: (B) $\frac{3}{26}$

Solution:

Concept:

- Independent events satisfy $P(H \cap F) = P(H)P(F)$.
- A standard deck has twelve face cards: jack, queen and king in each of four suits.

Step 1: Find the individual probabilities For the coin, $P(H) = 1/2$. There are $3 \times 4 = 12$ face cards among 52 cards, so $P(F) = 12/52 = 3/13$. Aces are not face cards.

Step 2: Use independence and check the options The coin outcome does not affect which card is selected. Therefore

$$P(H \cap F) = \frac{1}{2} \times \frac{3}{13} = \frac{3}{26}.$$

Option (D) omits the coin probability. Option (A) uses an incorrect face-card count; option (C) is the complement of $7/26$, not the probability of this intersection.

Step 3: Conclusion Hence, the required probability is $\boxed{3/26}$, and the correct option is (B).

Quick Tip to Solve: Count exactly twelve face cards, then multiply their probability by one-half for the head.

19. Assertion (A): $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ **is continuous at** $x = 0$.

Reason (R): When $x \to 0$, $\sin \frac{1}{x}$ **is a finite value between** -1 **and** 1. [1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- Continuity at zero requires $\lim_{x \to 0} f(x) = f(0)$.
- The squeeze theorem applies when the absolute value of a function is bounded by a quantity tending to zero.

Step 1: Judge the assertion For $x \neq 0$, $|x \sin(1/x)| \leq |x|$. Since $|x| \to 0$, the squeeze theorem gives $\lim_{x \to 0} x \sin(1/x) = 0 = f(0)$. Thus Assertion (A) is true.

Step 2: Judge the reason For every nonzero x, $-1 \leq \sin(1/x) \leq 1$, so the sine value remains finite and bounded as x approaches zero. Reason (R) is true in this boundedness sense; it does not claim that sine has a limit.

Step 3: Check the explanatory link This very bound gives $|x \sin(1/x)| \leq |x|$, proving the assertion by squeezing. The oscillation of sine does not prevent the product from tending to zero because its amplitude is multiplied by x. Thus the reason correctly explains the assertion.

Step 4: Conclusion Both statements are true, and Reason (R) correctly explains Assertion (A). The correct option is (A).

Quick Tip to Solve: Bound the oscillating sine by one; the factor x then forces the product to approach zero.

20. Assertion (A): Set of values of $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is a null set.

Reason (R): $\sec^{-1} x$ is defined for $x \in \mathbb{R} - (-1, 1)$. [1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- The real domain of $\sec^{-1} x$ is $(-\infty, -1] \cup [1, \infty)$.
- Where secant is defined, $|\sec\theta| = 1/|\cos\theta| \geq 1$.

Step 1: Judge the assertion The printed argument satisfies $0 < \sqrt{3}/2 < 1$. If a real angle satisfied $\sec\theta = \sqrt{3}/2$, then $\cos\theta = 2/\sqrt{3} > 1$, which is impossible for a real angle. Hence the set of real values is empty, and Assertion (A) is true.

Step 2: Judge the reason Reason (R) states the correct domain, $\mathbb{R} - (-1, 1)$. This includes the endpoints -1 and 1 but excludes every number strictly between them. Thus Reason (R) is true.

Step 3: Check the explanatory link Since $\sqrt{3}/2 \in (-1, 1)$, it is excluded by the domain stated in the reason. The domain restriction directly explains why the inverse secant has no real value at this argument; therefore the reason explains the assertion.

Step 4: Conclusion Both statements are true, and Reason (R) correctly explains Assertion (A). The correct option is (A).

Quick Tip to Solve: Check the input against the inverse-secant domain before trying to identify an angle.

Section B (10 marks)

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Let $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$, where $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Discuss the bijectivity of the function. [2 marks]

Solution:

Concept:

- A bijective function is both one-one (injective) and onto (surjective).
- To prove onto, solve $y = f(x)$ for each arbitrary y in the codomain and verify that the resulting x belongs to the domain.

Quick Tip to Solve: Write the function as 1+1/(x-3); it immediately shows why the output cannot equal one.

Step 1: Check the function is well defined For $x \in A$, $x \neq 3$, so the denominator is nonzero. Also

$$f(x) = 1 + \frac{1}{x-3} \neq 1.$$

Hence every output belongs to B.

Step 2: Prove that the function is one-one Let $x_1, x_2 \in A$ and suppose $f(x_1) = f(x_2)$. Cross-multiplying the nonzero denominators gives

$$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3).$$

Expansion and cancellation yield $-3x_1 - 2x_2 = -3x_2 - 2x_1$, so $x_1 = x_2$. Thus f is injective.

Step 3: Prove that the function is onto For any $y \in B$, we have $y \neq 1$. Solving $y = (x-2)/(x-3)$ gives

$$x = \frac{3y-2}{y-1} = 3 + \frac{1}{y-1}.$$

Therefore $x \neq 3$, so $x \in A$, and substituting this value gives $f(x) = y$. Every element of B has a preimage, proving surjectivity.

Step 4: Conclusion The function is both one-one and onto, and therefore $\boxed{f : A \to B \text{ is bijective}}$.

Quick Tip to Solve: Write the function as 1+1/(x-3); it immediately shows why the output cannot equal one.

22. If $A = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$, then show that $A^2 - 4A + 7I = 0$. [2 marks]

Solution:

Concept:

- Matrix multiplication uses row-by-column products; each entry of A^2 is the dot product of a row of A with a column of A.
- The identity matrix is $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$; scalar multiples and sums of matrices are calculated entry by entry.

Step 1: Calculate the square For the given matrix, multiply in the stated order:

$$A^2 = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 4-3 & 6+6 \\ -2-2 & -3+4 \end{pmatrix} = \begin{pmatrix} 1 & 12 \\ -4 & 1 \end{pmatrix}.$$

In particular, the negative entry in the second row changes the diagonal products.

Step 2: Substitute into the polynomial The remaining terms are

$$4A = \begin{pmatrix} 8 & 12 \\ -4 & 8 \end{pmatrix}, \qquad 7I = \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}.$$

Quick Tip to Solve: Keep the identity contribution on the diagonal only; check all four entries separately.

Hence

$$A^2 - 4A + 7I = \begin{pmatrix} 1-8+7 & 12-12 \\ -4+4 & 1-8+7 \end{pmatrix}.$$

Both diagonal entries and both off-diagonal entries vanish.

Step 3: Conclusion Therefore $\boxed{A^2 - 4A + 7I = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}}$, as required.

Quick Tip to Solve: Keep the identity contribution on the diagonal only; check all four entries separately.

23(a). Differentiate $\frac{5^x}{x^5}$ with respect to x. [2 marks]

Solution:

Concept:

- The exponential derivative is $\frac{d}{dx}a^x = a^x \ln a$, where $a > 0$ is constant.
- The product rule is $(uv)\prime = u\prime v + uv\prime$; also $\frac{d}{dx}x^n = nx^{n-1}$.

Step 1: Rewrite and differentiate the factors Let $y = \frac{5^x}{x^5} = 5^x x^{-5}$, with $x \neq 0$. This separates the exponential factor from the reciprocal power. Taking $u = 5^x$ and $v = x^{-5}$, their derivatives are

$$u\prime = 5^x \ln 5, \qquad v\prime = -5x^{-6}.$$

The exponent in the numerator is the variable, so the exponential differentiation formula is needed.

Step 2: Apply the product rule and simplify Using both terms of the product rule gives

$$\begin{aligned} \frac{dy}{dx} &= (5^x \ln 5)x^{-5} + 5^x(-5x^{-6}) \\ &= \frac{5^x \ln 5}{x^5} - \frac{5 \cdot 5^x}{x^6} \\ &= \frac{5^x}{x^6}(x \ln 5 - 5). \end{aligned}$$

The common denominator is x^6; therefore the first numerator acquires a factor of x. The restriction $x \neq 0$ remains inherited from the original expression.

Step 3: Conclusion The required derivative is $\boxed{\frac{5^x(x \ln 5 - 5)}{x^6}}$, for $x \neq 0$.

Quick Tip to Solve: Rewrite the quotient as an exponential times a negative power before differentiating.

OR

23(b). **If $-2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.** [2 marks]

Solution:

Concept:

- For an implicit relation, regard y as a function of x and use the chain rule: $\frac{d}{dx}y^3 = 3y^2\frac{dy}{dx}$.
- The product rule gives $\frac{d}{dx}(xy) = y + x\frac{dy}{dx}$, since both factors depend on x.

Step 1: Differentiate the relation Start from $-2x^2 - 5xy + y^3 = 76$. The derivative of the constant on the right is zero. Differentiating each term with respect to x yields

$$-4x - 5\left(y + x\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0.$$

The term $-5y$ comes from differentiating the factor x in $-5xy$, and must be retained.

Step 2: Collect the derivative terms Expand the product-rule term and place all terms containing the derivative together:

$$-4x - 5y - 5x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0,$$
$$(3y^2 - 5x)\frac{dy}{dx} = 4x + 5y.$$

Divide by $3y^2 - 5x$ wherever this coefficient is nonzero. This expresses the tangent slope in terms of the coordinates of a point on the given curve.

Step 3: Conclusion Therefore $\boxed{\frac{dy}{dx} = \frac{4x + 5y}{3y^2 - 5x}}$, wherever $3y^2 - 5x \neq 0$.

Quick Tip to Solve: For the mixed term xy, write its full product-rule derivative before collecting dy/dx.

24. **In a Linear Programming Problem, the objective function $Z = 5x + 4y$ needs to be maximised under constraints $3x + y \leq 6$, $x \leq 1$, $x, y \geq 0$. Express the LPP on the graph and shade the feasible region and mark the corner points.** [2 marks]

Solution:

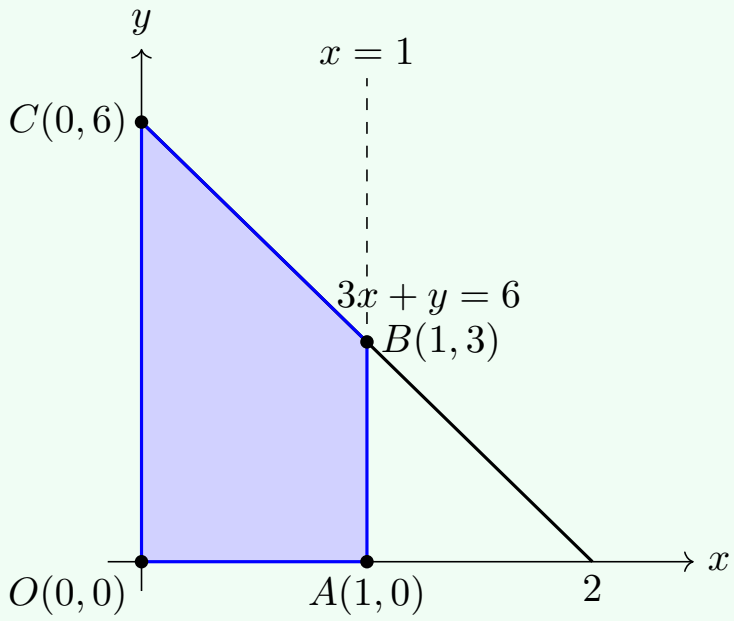
Concept:

- A feasible region is the common intersection of all constraint half-planes, including $x \geq 0$ and $y \geq 0$.
- Boundary intersections that satisfy every inequality are corner points; for a bounded feasible region, a linear objective attains its maximum at a corner.

Step 1: Locate the boundary lines The line $3x + y = 6$ passes through $(0, 6)$ and $(2, 0)$. Since $(0, 0)$ satisfies $3x + y \leq 6$, take the half-plane on or below this line. The inequality $x \leq 1$ selects the side on or to the left of the vertical line $x = 1$. Non-negativity restricts both of these half-planes to the first quadrant.

Quick Tip to Solve: Intersect x=1 with the sloping line first; the x-axis intercept (2,0) violates x<=1.

Step 2: Mark and shade their intersection The boundary $x = 1$ meets the sloping line where $3+y = 6$, giving $(1, 3)$. On the coordinate axes, the allowed endpoints are $(1, 0)$ and $(0, 6)$, together with the origin. Thus shade the closed quadrilateral with successive vertices $O(0, 0), A(1, 0), B(1, 3), C(0, 6)$. Every edge is included because all the inequalities are non-strict.



Feasible region and its corner points

Step 3: Check the objective at the corners For completeness, $Z = 5x + 4y$ has values $0, 5, 17, 24$ at O, A, B, C, respectively. The corner-point comparison identifies the largest objective value on the shaded region.

Step 4: Conclusion The feasible region is the shaded quadrilateral $OABC$, with corners $\boxed{(0, 0), (1, 0), (1, 3), (0, 6)}$. Its maximum objective value is 24 at $(0, 6)$.

Quick Tip to Solve: Intersect x=1 with the sloping line first; the x-axis intercept (2,0) violates x<=1.

25(a). 10 **identical blocks are marked with '0' on two of them, '1' on three of them, '2' on four of them and '3' on one of them and put in a box. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean.** [2 marks]

Solution:

Concept:

- For equally likely block selections, $P(X = x) = \dfrac{\text{number of blocks labelled } x}{\text{total number of blocks}}$.
- The mean of a discrete random variable is $E(X) = \sum_x xP(X = x)$; probabilities must be nonnegative and sum to one.

Step 1: Construct the probability distribution There are $2 + 3 + 4 + 1 = 10$ blocks in all, and each has the same chance of selection. The possible values of X are $0, 1, 2, 3$. Dividing the count for each label

Quick Tip to Solve: Use label times its frequency, then divide the total by ten to check the mean.

by ten gives

x	0	1	2	3
$P(X=x)$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{1}{10}$

Their sum is $(2+3+4+1)/10 = 1$, so this is a complete probability distribution.

Step 2: Calculate the weighted mean Multiply each value by its probability, including the zero-labelled blocks:

$$E(X) = 0\left(\frac{2}{10}\right) + 1\left(\frac{3}{10}\right) + 2\left(\frac{4}{10}\right) + 3\left(\frac{1}{10}\right)$$
$$= \frac{0+3+8+3}{10} = \frac{14}{10} = \frac{7}{5}.$$

The calculation is a frequency-weighted average of the labels, so values appearing on more blocks contribute more heavily.

Step 3: Conclusion The distribution is $P(X=0) = \frac{1}{5}$, $P(X=1) = \frac{3}{10}$, $P(X=2) = \frac{2}{5}$, $P(X=3) = \frac{1}{10}$, and the mean is $\boxed{\frac{7}{5} = 1.4}$.

Quick Tip to Solve: Use label times its frequency, then divide the total by ten to check the mean.

OR

25(b). In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village? [2 marks]

Solution:

Concept:

- For two events A and B, the addition rule is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. The intersection is subtracted because it is counted in both event totals.
- When an individual is selected uniformly from a population of N, an event containing n individuals has probability n/N.

Step 1: Define the events and count the overlap Let A be the event that the chosen person is a woman and B the event that the person works outside the village. There are 4000 women and 3000 outside workers among 8000 people. The women who work outside belong to both events. Their number is

$$n(A \cap B) = \frac{30}{100} \times 4000 = 1200.$$

The percentage is taken of the women, not of the whole village.

Step 2: Apply the addition rule The word either includes people belonging to both groups, so the

Quick Tip to Solve: First calculate 30 percent of the 4000 women; subtract that overlap before dividing by 8000.

required event is the union. Hence

$$n(A \cup B) = 4000 + 3000 - 1200 = 5800,$$
$$P(A \cup B) = \frac{5800}{8000} = \frac{29}{40}.$$

Subtracting the overlap once leaves every eligible person counted exactly once.

Step 3: Conclusion The required probability is $\boxed{\frac{29}{40}}$, or 0.725.

Quick Tip to Solve: First calculate 30 percent of the 4000 women; subtract that overlap before dividing by 8000.

Section C (18 marks)

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26(a). Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 4x^3 - 5$, $\forall x \in \mathbb{R}$ is one-one and onto.

[3 marks]

Solution:

Concept:

- A function is one-one if equal outputs force equal inputs: $f(a) = f(b) \Rightarrow a = b$.
- A function $f : \mathbb{R} \to \mathbb{R}$ is onto if every real target y is the image of some real input. Every real number has a unique real cube root.

Step 1: Prove that the function is one-one Let $a, b \in \mathbb{R}$ and suppose $f(a) = f(b)$. Using the given formula,

$$4a^3 - 5 = 4b^3 - 5,$$
$$4a^3 = 4b^3,$$
$$a^3 = b^3.$$

The real cube function is strictly increasing, so equality of cubes gives $a = b$. Equivalently, taking the unique real cube root of both sides gives this result directly. Thus two distinct real inputs cannot have the same image, proving injectivity on the entire domain.

Step 2: Prove that the function is onto Choose an arbitrary $y \in \mathbb{R}$ in the codomain. To obtain an input whose image is y, solve

$$4x^3 - 5 = y \quad \Longrightarrow \quad x^3 = \frac{y+5}{4} \quad \Longrightarrow \quad x = \sqrt[3]{\frac{y+5}{4}}.$$

This input is real for every real y, including when $y + 5$ is negative. Substituting it gives

$$f\left(\sqrt[3]{\frac{y+5}{4}}\right) = 4\left(\frac{y+5}{4}\right) - 5 = y.$$

Quick Tip to Solve: For a cubic of this form, use equality of cubes for injectivity and solve for the real cube root for surjectivity.

Therefore every element of the specified codomain has a preimage in the specified domain, proving surjectivity.

Step 3: Conclusion The function $f(x) = 4x^3 - 5$ is both one-one and onto; hence it is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

Quick Tip to Solve: For a cubic of this form, use equality of cubes for injectivity and solve for the real cube root for surjectivity.

OR

26(b). Let R be a relation defined on a set $\mathbb{N}$ of natural numbers such that $R = \{(x, y) : xy$ is a square of a natural number, $x, y \in \mathbb{N}\}$. Determine if the relation R is an equivalence relation.

[3 marks]

Solution:

Concept:

- A relation is an equivalence relation precisely when it is reflexive, symmetric and transitive on its stated set.
- A positive integer is a perfect square if and only if the exponent of each prime in its prime factorisation is even. Prime exponents add under multiplication.

Step 1: Check reflexivity Take any $x \in \mathbb{N}$. The product $xx = x^2$ is the square of the natural number x. Hence $(x, x) \in R$ for every natural number, so the relation is reflexive.

Step 2: Check symmetry Suppose $(x, y) \in R$. By definition, xy is a square of a natural number. Since $yx = xy$, the product yx is that same square. Thus $(y, x) \in R$, and the relation is symmetric.

Step 3: Check transitivity by prime exponents Suppose $(x, y) \in R$ and $(y, z) \in R$. For any prime p, let its exponents in x, y, z be a, b, c, respectively; an absent prime has exponent zero. Since xy and yz are perfect squares, $a + b$ and $b + c$ are even. Therefore

$$a + c = (a + b) + (b + c) - 2b$$

is even. This holds for every prime, so every exponent in the natural number xz is even. Consequently xz is the square of a natural number and $(x, z) \in R$. This establishes transitivity without assuming that dividing two integer squares always yields a natural-number square.

Step 4: Conclusion All three properties hold. Therefore $\boxed{R \text{ is an equivalence relation on } \mathbb{N}}$.

Quick Tip to Solve: For transitivity, compare the parity of each prime exponent in xy and yz; the repeated exponent of y cancels modulo 2.

27(a). **Let $2x+5y-1=0$ and $3x+2y-7=0$ represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.** [3 marks]

Solution:

Concept:

- The point common to two lines satisfies both linear equations simultaneously. Write the system as $AX = B$, where X is the column of unknown coordinates.
- If $\det A \neq 0$, then A^{-1} exists and the unique solution is $X = A^{-1}B$. For a two by two matrix, interchange diagonal entries and negate off-diagonal entries to form its adjugate.

Step 1: Express the two paths as a matrix equation Rearranging the equations gives $2x + 5y = 1$ and $3x + 2y = 7$. Thus

$$\underbrace{\begin{pmatrix} 2 & 5 \\ 3 & 2 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 1 \\ 7 \end{pmatrix}}_{B}.$$

The constant terms are 1 and 7, since the negative constants in the printed equations move to the right-hand side.

Step 2: Find the inverse and solve The determinant is

$$\det A = 2 \times 2 - 5 \times 3 = 4 - 15 = -11 \neq 0.$$

Hence the inverse exists, and

$$A^{-1} = \frac{1}{-11} \begin{pmatrix} 2 & -5 \\ -3 & 2 \end{pmatrix}.$$

Multiplying by the constants column gives

$$X = A^{-1}B = \frac{1}{-11} \begin{pmatrix} 2 & -5 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$
$$= \frac{1}{-11} \begin{pmatrix} 2 - 35 \\ -3 + 14 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}.$$

Step 3: Verify both path equations For $x = 3$ and $y = -1$, the first left-hand side is $2(3)+5(-1)-1 = 0$, and the second is $3(3) + 2(-1) - 7 = 0$. Therefore the computed coordinates lie on both paths. The nonzero determinant also ensures that this common point is unique.

Step 4: Conclusion The point common to the paths of the ants is $\boxed{(3, -1)}$.

Quick Tip to Solve: Keep the order of x and y fixed in the matrix, and check the final coordinate pair in both original equations.

OR

27(b). A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each such subject book is ₹150 (Chemistry), ₹175 (Physics) and ₹180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is ₹35,000, what profit did he earn after the sale of two days? [3 marks]

Solution:

Concept:

- A quantity matrix multiplied by a price column gives sales revenue for each day. Subject order must agree in both matrices.
- Profit equals total selling price minus total cost price. Add the daily revenues before subtracting the combined cost.

Step 1: Form the matrices Take the columns in the order Chemistry, Physics, Maths and the rows in the order day I, day II. The quantity matrix and price column are

$$Q = \begin{pmatrix} 50 & 60 & 35 \\ 40 & 45 & 50 \end{pmatrix}, \qquad P = \begin{pmatrix} 150 \\ 175 \\ 180 \end{pmatrix}.$$

Each entry of Q counts books, while each corresponding entry of P gives the selling price of one book in rupees. The matrix product is defined because the three columns of Q match the three rows of P, and the result has one revenue entry for each day.

Step 2: Calculate the daily and total revenue Multiplying each row by the price column gives

$$QP = \begin{pmatrix} 50(150) + 60(175) + 35(180) \\ 40(150) + 45(175) + 50(180) \end{pmatrix} = \begin{pmatrix} 24300 \\ 22875 \end{pmatrix}.$$

Thus day I contributes Rs. 24300 and day II contributes Rs. 22875. Adding using a row matrix,

$$\begin{pmatrix} 1 & 1 \end{pmatrix} QP = 24300 + 22875 = 47175.$$

This is the combined selling price of all the books sold over the two days.

Step 3: Deduct the cost price The given cost price is for all the books together, so it is subtracted once:

$$\text{Profit} = 47175 - 35000 = 12175.$$

Since the selling price exceeds the cost price, the result is a profit. As an independent check on the multiplication, the two days together contain 90 Chemistry, 105 Physics and 85 Maths books. Their total value is $90(150) + 105(175) + 85(180) = 47175$, agreeing with the matrix result.

Step 4: Conclusion Total sale in two days is $\boxed{\text{Rs. } 47175}$, and profit is $\boxed{\text{Rs. } 12175}$.

Quick Tip to Solve: Keep the same subject order in both matrices and subtract the combined cost only once.

28. Differentiate $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{3} - 1\right)\right\}}$ with respect to x. [3 marks]

Solution:

Concept:

- The chain rule differentiates a composite function by multiplying the derivatives of its successive layers.
- For natural logarithms, $\frac{d}{du}\log u = 1/u$, and $\frac{d}{du}\sqrt{u} = 1/(2\sqrt{u})$.

Step 1: Identify the nested functions The printed expression is

$$y = \sqrt{\log\{\sin(x^3/3 - 1)\}}.$$

Put $u = x^3/3 - 1$, $v = \sin u$ and $w = \log v$, so that $y = \sqrt{w}$. The order of the layers matters: the outer square root applies to the logarithm, while the sine applies to the entire polynomial expression $x^3/3 - 1$. We differentiate each layer before multiplying, keeping its own argument unchanged.

Step 2: Differentiate each layer The successive derivatives are

$$\frac{du}{dx} = x^2, \qquad \frac{dv}{du} = \cos u, \qquad \frac{dw}{dv} = \frac{1}{v}, \qquad \frac{dy}{dw} = \frac{1}{2\sqrt{w}}.$$

In particular, the constant -1 contributes zero, and differentiating $x^3/3$ gives x^2, so there is no extra factor of three. The derivative of sine introduces cosine of the same argument.

Step 3: Combine by the chain rule Multiplying these factors and substituting the original expressions gives the formal derivative

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2\sqrt{w}} \cdot \frac{1}{v} \cdot \cos u \cdot x^2 \\ &= \frac{x^2 \cot(x^3/3 - 1)}{2\sqrt{\log\{\sin(x^3/3 - 1)\}}}.\end{aligned}$$

Over the real numbers, however, the printed function has no open interval of definition: for positive sine, $\log(\sin u) \le 0$, with equality only when $\sin u = 1$. Thus the real square root exists only at isolated points, where the displayed denominator is zero. Consequently an ordinary real derivative is not defined; the expression above is the formal chain-rule result for the printed question.

Step 4: Conclusion The formal answer is $\boxed{\frac{x^2 \cot(x^3/3 - 1)}{2\sqrt{\log\{\sin(x^3/3 - 1)\}}}}$. The printed function has no ordinary real derivative, since its real domain contains only isolated points.

Quick Tip to Solve: Work from the outer square root inward, and keep the full sine argument x cubed over three minus one at every layer.

29. Amongst all pairs of positive integers with product as 289, find which of the two numbers add up to the least. [3 marks]

Solution:

Concept:

- If two positive numbers have fixed product 289, writing them as x and $289/x$ reduces their sum to a function of one variable.
- A stationary point is a minimum if the derivative changes from negative to positive. A minimum over positive real numbers also applies to integers when its pair is integral.

Step 1: Express the sum as a function Let the first positive number be x. Since the product is 289, the second must be $289/x$, and hence

$$S(x) = x + \frac{289}{x}, \qquad x > 0.$$

We first minimize over all positive real numbers. This larger domain includes every positive integer pair, so if its minimizing pair consists of integers, it also solves the stated problem. The condition $x > 0$ is essential both for the product relation and for the derivative sign analysis.

Step 2: Locate and classify the stationary point Differentiation gives

$$S\prime(x) = 1 - \frac{289}{x^2}.$$

At a stationary point, $x^2 = 289$, so positivity selects $x = 17$. For $0 < x < 17$, $x^2 < 289$ and therefore $S\prime(x) < 0$; for $x > 17$, $S\prime(x) > 0$. Thus the sum decreases up to 17 and increases thereafter. Equivalently, $S\prime\prime(x) = 578/x^3 > 0$ throughout the positive domain, confirming the minimum.

Step 3: Apply the integer condition and check The second number at the minimum is $289/17 = 17$, so both numbers are positive integers, and the minimum sum is $17 + 17 = 34$. Directly, $289 = 17^2$, so the unordered positive integer factor pairs are $(1, 289)$ and $(17, 17)$. Their sums are 290 and 34, respectively. This confirms that the continuous minimum is feasible in the required integer domain and that no other integer pair gives a smaller sum.

Step 4: Conclusion The two numbers are $\boxed{17 \text{ and } 17}$, and their least possible sum is $\boxed{34}$.

Quick Tip to Solve: For a fixed positive product, test the equal pair first; here its value 17 is integral, so it is feasible.

30. In the Linear Programming Problem for objective function $Z = 18x + 10y$ subject to constraints $4x + y \geq 20$, $2x + 3y \geq 30$, $x, y \geq 0$, find the minimum value of Z. [3 marks]

Solution:

Concept:

- A feasible point satisfies all the constraints together. For a linear objective, finite optimal values occur at boundary vertices, but an unbounded feasible region requires a check that the proposed minimum is global.

Quick Tip to Solve: For an unbounded feasible region, certify a minimum by writing the objective as a positive combination of the given constraint expressions.

- A nonnegative linear combination of lower-bound constraints gives a lower bound on the objective. Equality holds when each constraint with a positive multiplier holds with equality.

Step 1: Find the intersection of the constraint boundaries The two boundary equations are $4x+y=20$ and $2x+3y=30$. From the first, $y=20-4x$. Substitute this into the second equation:

$$2x+3(20-4x)=30 \implies -10x=-30 \implies x=3.$$

Therefore $y=20-12=8$. The point $(3,8)$ is in the first quadrant and satisfies both constraints with equality.

Step 2: Compare the feasible vertices On the y-axis, the inequalities require $y \geq 20$, giving the boundary vertex $(0,20)$. On the x-axis, they require $x \geq 15$, giving $(15,0)$. The objective values at these vertices and the intersection are

$$\begin{aligned} Z(0,20) &= 200, \\ Z(3,8) &= 18(3)+10(8)=134, \\ Z(15,0) &= 270. \end{aligned}$$

The smallest candidate value is thus 134.

Step 3: Establish the minimum over the entire unbounded region Multiply $4x+y \geq 20$ by $17/5$ and $2x+3y \geq 30$ by $11/5$, both positive numbers. Adding gives

$$18x+10y \geq \frac{17}{5}(20)+\frac{11}{5}(30)=134.$$

This proves that no feasible point, including points on the unbounded rays, can have an objective value below 134. Equality is achieved at the feasible point already found.

Step 4: Conclusion The minimum value is $\boxed{Z_{\min}=134}$, attained at $\boxed{(x,y)=(3,8)}$.

Quick Tip to Solve: For an unbounded feasible region, certify a minimum by writing the objective as a positive combination of the given constraint expressions.

31(a). The scalar product of the vector $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$ with a unit vector along sum of vectors $\vec{b}=2\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c}=\lambda\hat{i}-2\hat{j}-3\hat{k}$ is equal to 1. Find the value of λ. [3 marks]

Solution:

Concept:

- For a nonzero vector $\vec{v}$, the unit vector along it is $\hat{v}=\vec{v}/|\vec{v}|$. The modulus in the denominator is always positive.
- The scalar product is obtained by multiplying corresponding components and adding. Squaring an equation can introduce an extraneous root, so the result must be checked in the original equation.

Quick Tip to Solve: Normalise the sum vector first, and check any root after squaring against the positive square-root side of the original equation.

Step 1: Form the sum and its unit vector The sum of the given vectors is

$$\vec{b}+\vec{c}=(\lambda+2)\hat{i}-6\hat{j}+2\hat{k}.$$

Its modulus is

$$|\vec{b}+\vec{c}|=\sqrt{(\lambda+2)^2+(-6)^2+2^2}=\sqrt{(\lambda+2)^2+40}.$$

This is positive for every real λ, because the expression under the square root is at least 40. Hence the required unit vector is well defined and equals

$$\hat{u}=\frac{(\lambda+2)\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{(\lambda+2)^2+40}}.$$

Step 2: Apply the scalar-product condition Using $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$, the given condition becomes

$$\vec{a}\cdot\hat{u}=\frac{(\lambda+2)+6+4}{\sqrt{(\lambda+2)^2+40}}=1.$$

Thus $\lambda+12=\sqrt{(\lambda+2)^2+40}$, which also requires $\lambda+12>0$. Squaring both sides and expanding,

$$\lambda^2+24\lambda+144=\lambda^2+4\lambda+44,$$
$$20\lambda=-100,$$
$$\lambda=-5.$$

Step 3: Check the value before accepting it At $\lambda=-5$, the sum vector is $-3\hat{i}-6\hat{j}+2\hat{k}$, whose modulus is $\sqrt{9+36+4}=7$. Its scalar product with $\vec{a}$, before normalising, is $-3+6+4=7$. Therefore the required product with its unit vector is exactly $7/7=1$, so the root satisfies the original unsquared equation.

Step 4: Conclusion The required value is $\boxed{\lambda=-5}$.

Quick Tip to Solve: Normalise the sum vector first, and check any root after squaring against the positive square-root side of the original equation.

OR

31(b). Find the shortest distance between the lines:

$\vec{r}=(2\hat{i}-\hat{j}+3\hat{k})+\lambda(\hat{i}-2\hat{j}+3\hat{k})$

$\vec{r}=(\hat{i}+4\hat{k})+\mu(3\hat{i}-6\hat{j}+9\hat{k})$ [3 marks]

Solution:

Concept:

- Lines whose direction vectors are scalar multiples are parallel. The usual skew-line formula with the cross product of the two direction vectors cannot be used because that cross product is zero.

Quick Tip to Solve: Check whether the direction vectors are proportional before choosing a distance formula; here the selected joining vector is already perpendicular.

- For parallel lines with common direction $\vec{d}$ and points A, B, the shortest distance is $D = |\overrightarrow{AB} \times \vec{d}|/|\vec{d}|$, the length of the component of $\overrightarrow{AB}$ perpendicular to the lines.

Step 1: Identify points and the common direction From the first line take $A = (2, -1, 3)$ and direction $\vec{d} = \hat{i} - 2\hat{j} + 3\hat{k}$. From the second line take $B = (1, 0, 4)$. Its direction is $3\hat{i} - 6\hat{j} + 9\hat{k} = 3\vec{d}$, so the lines are parallel. The joining vector is

$$\overrightarrow{AB} = \vec{B} - \vec{A} = -\hat{i} + \hat{j} + \hat{k}.$$

The shortest joining segment must be perpendicular to their common direction.

Step 2: Calculate the perpendicular component using a cross product Compute

$$\begin{aligned}\overrightarrow{AB} \times \vec{d} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} \\ &= (3+2)\hat{i} - (-3-1)\hat{j} + (2-1)\hat{k} \\ &= 5\hat{i} + 4\hat{j} + \hat{k}.\end{aligned}$$

Consequently,

$$|\overrightarrow{AB} \times \vec{d}| = \sqrt{25+16+1} = \sqrt{42}, \qquad |\vec{d}| = \sqrt{1+4+9} = \sqrt{14}.$$

The nonzero cross product also confirms that the two parallel lines are distinct.

Step 3: Obtain and verify the shortest distance The area of the parallelogram formed by $\overrightarrow{AB}$ and $\vec{d}$ equals base $|\vec{d}|$ times perpendicular separation. Hence

$$D = \frac{\sqrt{42}}{\sqrt{14}} = \sqrt{3}.$$

As a check, $\overrightarrow{AB} \cdot \vec{d} = -1 - 2 + 3 = 0$, so the chosen joining vector itself is perpendicular to the lines. Its length is $\sqrt{1+1+1} = \sqrt{3}$, agreeing with the formula.

Step 4: Conclusion The shortest distance between the lines is $\boxed{\sqrt{3} \text{ units}}$.

Quick Tip to Solve: Check whether the direction vectors are proportional before choosing a distance formula; here the selected joining vector is already perpendicular.

Section D (20 marks)

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32(a). Find: $\int \dfrac{x^2+1}{(x^2+2)(2x^2+1)}\,dx$. [5 marks]

Solution:

Concept:

- Partial fractions express a rational function as a sum of simpler functions. Constant numerators suffice here because the given numerator and denominator factors involve only even powers of x.
- Use $\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a}\tan^{-1}(x/a) + C$, where $a > 0$, and C is an arbitrary constant.

Step 1: Form the partial-fraction identity Let

$$\frac{x^2+1}{(x^2+2)(2x^2+1)} = \frac{A}{x^2+2} + \frac{B}{2x^2+1}.$$

Multiplying through by the common denominator gives

$$x^2+1 = A(2x^2+1) + B(x^2+2) = (2A+B)x^2 + A + 2B.$$

This is a polynomial identity for every real x, so the coefficients of equal powers must agree.

Step 2: Solve for the coefficients Comparing coefficients gives $2A+B = 1$ and $A+2B = 1$. Subtracting yields $A = B$; substituting into either equation yields $3A = 1$. Thus $A = B = 1/3$, and

$$I = \frac{1}{3}\int \frac{dx}{x^2+2} + \frac{1}{3}\int \frac{dx}{2x^2+1}.$$

As a check, adding the two fractions produces numerator $(2x^2+1+x^2+2)/3 = x^2+1$, exactly as required.

Step 3: Apply the inverse-tangent formula with the correct scaling Since $x^2+2 = x^2+(\sqrt{2})^2$, the first integral is

$$\int \frac{dx}{x^2+2} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right).$$

For the second, put $u = \sqrt{2}x$, so $dx = du/\sqrt{2}$. Consequently,

$$\int \frac{dx}{2x^2+1} = \frac{1}{\sqrt{2}}\int \frac{du}{1+u^2} = \frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}x).$$

Each expression must now be multiplied by its partial-fraction coefficient $1/3$. The factor $1/\sqrt{2}$ in the second integral compensates for the derivative of $\sqrt{2}x$. Since the two integration constants add to an arbitrary constant, only one C is needed. Both original quadratic factors are strictly positive for real x, so the result is valid on the whole real line.

Quick Tip to Solve: Subtract the two coefficient equations first: their symmetry gives equal coefficients immediately; retain the substitution factor in each inverse tangent.

Step 4: Conclusion Therefore,

$$\boxed{I = \frac{1}{3\sqrt{2}}\left[\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \tan^{-1}(\sqrt{2}x)\right] + C.}$$

Quick Tip to Solve: Subtract the two coefficient equations first: their symmetry gives equal coefficients immediately; retain the substitution factor in each inverse tangent.

OR

32(b). Evaluate: $\displaystyle\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}\,dx.$ [5 marks]

Solution:

Concept:

- Reflection of limits gives $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. If $g(a-x) = g(x)$, adding the two forms of $\int_0^a xg(x)\,dx$ replaces the weight x by $a/2$.
- Trigonometric simplification gives $\tan x/(\sec x+\tan x) = \sin x/(1+\sin x)$. The simplified expression has a continuous extension at $x = \pi/2$.

Step 1: Simplify the quotient Multiplying the numerator and denominator by $\cos x$ wherever they are defined gives

$$\frac{x\tan x}{\sec x + \tan x} = \frac{x\sin x}{1+\sin x}.$$

On $[0, \pi]$, the denominator $1 + \sin x$ is positive. Although the original secant and tangent expressions are undefined at $\pi/2$, their quotient has the finite limit obtained from this simplified expression. Thus we use the continuous extension to evaluate the integral.

Step 2: Reflect the integral about the midpoint Write $I = \int_0^{\pi} x\sin x/(1+\sin x)\,dx$. Substitution $x = \pi - t$ and $\sin(\pi - t) = \sin t$ gives

$$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\sin x}\,dx.$$

Adding the original and reflected forms yields

$$2I = \pi\int_0^{\pi} \frac{\sin x}{1+\sin x}\,dx = \pi\left[\pi - J\right], \qquad J = \int_0^{\pi} \frac{dx}{1+\sin x}.$$

Step 3: Evaluate the remaining elementary integral A convenient primitive that remains finite throughout the interval is $-\cos x/(1+\sin x)$. Indeed, by the quotient rule,

$$\frac{d}{dx}\left(-\frac{\cos x}{1+\sin x}\right) = \frac{\sin x(1+\sin x) + \cos^2 x}{(1+\sin x)^2} = \frac{1}{1+\sin x}.$$

Quick Tip to Solve: Simplify secant and tangent into sine first, then reflect about $\pi/2$; use the primitive $-\cos x/(1+\sin x)$ to avoid an artificial singularity.

Consequently,

$$J = \left[-\frac{\cos x}{1+\sin x} \right]_0^{\pi} = 1 - (-1) = 2.$$

Substitute this into the reflected identity $2I = \pi(\pi - J)$.

Step 4: Conclusion Hence the required value is

$$\boxed{I = \frac{\pi}{2}(\pi - 2) = \frac{\pi^2}{2} - \pi.}$$

Quick Tip to Solve: Simplify secant and tangent into sine first, then reflect about $\pi/2$; use the primitive $-\cos x/(1+\sin x)$ to avoid an artificial singularity.

33. A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration. [5 marks]

Solution:

Concept:

- A circle centred at the origin with radius 8 satisfies $x^2+y^2 = 64$, with upper boundary $y = \sqrt{64 - x^2}$.
- A line through the origin inclined at angle θ has slope $\tan\theta$. Area is the integral of the upper boundary minus the lower boundary.

Step 1: Locate the scratch and its intersection with the circle Use the centre of the table as origin and the lines dividing the quadrants as coordinate axes. Since the inclination is $\pi/4$, the scratch has equation $y = x\tan(\pi/4) = x$. Substituting this into the circle gives

$$2x^2 = 64 \quad \Longrightarrow \quad x = 4\sqrt{2}$$

in the first quadrant. Hence the intersection is $P = (4\sqrt{2}, 4\sqrt{2})$. The circle meets the positive x-axis at $A = (8, 0)$. The desired region is bounded by the segment from the origin to P, the arc from P to A, and the segment of the x-axis returning to the origin.

Step 2: Split the area at the intersection For $0 \le x \le 4\sqrt{2}$, a vertical strip of the required region has height x, because the scratch is its upper boundary. For $4\sqrt{2} \le x \le 8$, the circular arc is its upper boundary and the height is $\sqrt{64 - x^2}$. The lower boundary is $y = 0$ in both intervals. Thus the area S equals

$$S = \int_0^{4\sqrt{2}} x\,dx + \int_{4\sqrt{2}}^{8} \sqrt{64 - x^2}\,dx.$$

The change of boundary is essential: integrating only the circular arc from zero to eight would give the whole first quadrant, including the region above the scratch.

Quick Tip to Solve: Find the line-circle intersection first, then switch the upper boundary from $y = x$ to the circular arc at $x = 4\sqrt{2}$.

Step 3: Evaluate the two integrals The first integral gives $[x^2/2]_0^{4\sqrt{2}} = 16$. For the circular arc, use

$$\int \sqrt{64 - x^2}\, dx = \frac{x}{2}\sqrt{64 - x^2} + 32 \sin^{-1}\left(\frac{x}{8}\right).$$

At the upper limit $x = 8$, the first term vanishes and the inverse sine is $\pi/2$, so this expression is 16π. At the lower limit $x = 4\sqrt{2}$, the square root is $4\sqrt{2}$ and the inverse sine is $\pi/4$, so the expression is $16 + 8\pi$. Consequently,

$$\begin{aligned} S &= 16 + \left(16\pi - 16 - 8\pi\right) \\ &= 8\pi. \end{aligned}$$

The result also agrees with the fact that the angle between the bounding radii is one eighth of a full revolution.

Step 4: Conclusion The required area, obtained by integration, is $\boxed{8\pi \text{ cm}^2}$.

Quick Tip to Solve: Find the line-circle intersection first, then switch the upper boundary from $y = x$ to the circular arc at $x = 4\sqrt{2}$.

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34. Solve the differential equation $\dfrac{dy}{dx} = \cos x - 2y$. [5 marks]

Solution:

Concept:

- A first-order linear equation has form $y' + P(x)y = Q(x)$. Its integrating factor is $I(x) = e^{\int P(x)\, dx}$.
- Multiplication by the integrating factor makes the left side a product derivative: $(Iy)' = IQ$. Integrating introduces one arbitrary constant.

Step 1: Put the equation in linear form and find the integrating factor Move the term involving y to the left to write the given equation as

$$\frac{dy}{dx} + 2y = \cos x.$$

Comparing with $y' + P(x)y = Q(x)$, we identify $P(x) = 2$ and $Q(x) = \cos x$. The integrating factor is therefore

$$I(x) = e^{\int 2\, dx} = e^{2x}.$$

A nonzero constant multiplier in the integrating factor is unnecessary, since it cancels in the solution. This equation is linear even though the independent-variable term is trigonometric.

Step 2: Convert to a product derivative and integrate Multiplying every term by e^{2x} yields

$$e^{2x}\frac{dy}{dx} + 2e^{2x}y = e^{2x}\cos x.$$

Quick Tip to Solve: Move $-2y$ to the left before identifying $P(x)$; the integrating factor is e^{2x}, while the complementary term is Ce^{-2x}.

By the product rule, the left side is $\dfrac{d}{dx}(ye^{2x})$. Integrating both sides with respect to x gives

$$ye^{2x} = \int e^{2x}\cos x\,dx + C,$$

where C is an arbitrary real constant. No initial condition is supplied, so this constant must be retained.

Step 3: Evaluate the exponential-trigonometric integral Let $J = \int e^{2x}\cos x\,dx$ and $K = \int e^{2x}\sin x\,dx$. Integration by parts gives

$$J = \frac{1}{2}e^{2x}\cos x + \frac{1}{2}K, \qquad K = \frac{1}{2}e^{2x}\sin x - \frac{1}{2}J.$$

Substituting the second expression into the first,

$$\frac{5}{4}J = \frac{1}{2}e^{2x}\cos x + \frac{1}{4}e^{2x}\sin x.$$

Hence $J = \dfrac{e^{2x}}{5}(2\cos x + \sin x)$, with any additive integration constant absorbed into C. Substitution into the preceding equation and division by the positive factor e^{2x} give

$$y = \frac{2\cos x + \sin x}{5} + Ce^{-2x}.$$

Step 4: Verify the expression in the original equation Differentiation gives $y' = (-2\sin x + \cos x)/5 - 2Ce^{-2x}$. On the other hand,

$$\cos x - 2y = \frac{\cos x - 2\sin x}{5} - 2Ce^{-2x},$$

which is the same derivative. Thus the family satisfies the original differential equation for every real value of C.

Step 5: Conclusion The general solution is $\boxed{y = \dfrac{2\cos x + \sin x}{5} + Ce^{-2x}}$, where $C \in \mathbb{R}$.

Quick Tip to Solve: Move $-2y$ to the left before identifying $P(x)$; the integrating factor is e^{2x}, while the complementary term is Ce^{-2x}.

35(a). Find the point Q on the line $\dfrac{2x+4}{6} = \dfrac{y+1}{2} = \dfrac{-2z+6}{-4}$ at a distance of $3\sqrt{2}$ from the point $P(1,2,3)$. [5 marks]

Solution:

Concept:

- In a symmetric equation of a line, setting each ratio equal to a parameter t gives the coordinates of every point on that line.

Quick Tip to Solve: Simplify each coordinate equation before applying distance; retain both roots of the resulting quadratic.

- The distance between $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ satisfies $PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$.
- A prescribed distance from a fixed point may give two points on a line. Every real parameter root must therefore be retained and checked.

Step 1: Write a general point of the printed line Let the common value of the three ratios be t. Then

$$\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4} = t.$$

Solving the three linear equations separately gives

$$2x + 4 = 6t, \quad y + 1 = 2t, \quad -2z + 6 = -4t,$$

and hence $x = 3t-2,\ y = 2t-1,\ z = 2t+3$. Thus the required point has the form $Q(3t-2, 2t-1, 2t+3)$. In particular, the direction ratios are $(3, 2, 2)$; the coefficient of z must be handled before reading them.

Step 2: Apply the given distance condition Since $P = (1, 2, 3)$ and $PQ = 3\sqrt{2}$, the square of the distance is 18. Subtracting the coordinates of P from those of Q, we obtain

$$\begin{aligned} (3t-3)^2 + (2t-3)^2 + (2t)^2 &= 18, \\ 9t^2 - 18t + 9 + 4t^2 - 12t + 9 + 4t^2 &= 18, \\ 17t^2 - 30t &= 0, \\ t(17t - 30) &= 0. \end{aligned}$$

Therefore the two permissible parameters are $t = 0$ and $t = 30/17$. Both are real, so both correspond to points at the required distance.

Step 3: Substitute and verify both points For $t = 0$, the point is $Q_1 = (-2, -1, 3)$. Its displacement from P is $(-3, -3, 0)$, whose squared length is $9 + 9 = 18$. For $t = 30/17$, the point is

$$Q_2 = \left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right).$$

Its displacement from P is $(39/17, 9/17, 60/17)$, and

$$PQ_2^2 = \frac{39^2 + 9^2 + 60^2}{17^2} = \frac{5202}{289} = 18.$$

Each point also belongs to the line because it was obtained by substituting a common parameter into all three coordinate equations.

Step 4: Conclusion The required points are $\boxed{(-2, -1, 3)}$ and $\boxed{\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)}$, each at distance $3\sqrt{2}$ from P.

Quick Tip to Solve: Simplify each coordinate equation before applying distance; retain both roots of the resulting quadratic.

OR

35(b). Find the image of the point $(-1, 5, 2)$ in the line $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$. Find the length of the line segment joining the points (given point and the image point). [5 marks]

Solution:

Concept:

- Reflection of a point A in a line means that the foot H of the perpendicular from A to the line is the midpoint of AA', where A' is the image.
- If $\mathbf{d}$ is a direction vector of the line, perpendicularity gives $(\mathbf{H} - \mathbf{A}) \cdot \mathbf{d} = 0$.
- Once the foot is known, $\mathbf{A}' = 2\mathbf{H} - \mathbf{A}$, and the distance between the point and its image is $AA' = 2AH$.

Step 1: Parametrize the given line Denote the given point by $A = (-1, 5, 2)$. Set the common ratio in the printed line equal to t:

$$\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3} = t.$$

Hence $x = 2+t,\ y = 2t,\ z = 2-3t$. A general point of the line is therefore $H = (2+t, 2t, 2-3t)$, and a direction vector is $\mathbf{d} = (1, 2, -3)$. In particular, the first direction component is 1, since $2x - 4 = 2t$ must first be divided by 2.

Step 2: Locate the perpendicular foot For H to be the foot of the perpendicular from A, the vector

$$\overrightarrow{AH} = (3 + t, 2t - 5, -3t)$$

must be perpendicular to $\mathbf{d}$. Taking their scalar product gives

$$\begin{aligned}(3+t) + 2(2t-5) + (-3)(-3t) &= 0,\\ 3 + t + 4t - 10 + 9t &= 0,\\ 14t - 7 &= 0.\end{aligned}$$

Thus $t = 1/2$, and the perpendicular foot is $H = (5/2, 1, 1/2)$. As a check, its three ratios in the original equation are each $1/2$, and $\overrightarrow{AH} = (7/2, -4, -3/2)$ has zero scalar product with $(1, 2, -3)$.

Step 3: Reflect the point through the foot Since H is the midpoint of AA', the image coordinates are

$$\begin{aligned}A' &= 2H - A\\ &= (5, 2, 1) - (-1, 5, 2)\\ &= (6, -3, -1).\end{aligned}$$

Indeed, the midpoint of $(-1, 5, 2)$ and $(6, -3, -1)$ is $(5/2, 1, 1/2)$, confirming the reflection condition. The joining segment is perpendicular to the line because it is parallel to AH.

Step 4: Compute the length of the joining segment The coordinate differences between the point and its image are $7, -8, -3$. Therefore

$$AA' = \sqrt{7^2 + (-8)^2 + (-3)^2} = \sqrt{49 + 64 + 9} = \sqrt{122}.$$

Quick Tip to Solve: Find the perpendicular foot using a dot product, then double its coordinates and subtract the given point.

Equivalently, $AH^2 = (7/2)^2 + (-4)^2 + (-3/2)^2 = 61/2$, so $2AH = 2\sqrt{61/2} = \sqrt{122}$, giving the same length independently.

Step 5: Conclusion The image is $\boxed{(6, -3, -1)}$, and the length of the segment joining the given point to its image is $\boxed{\sqrt{122}}$ units.

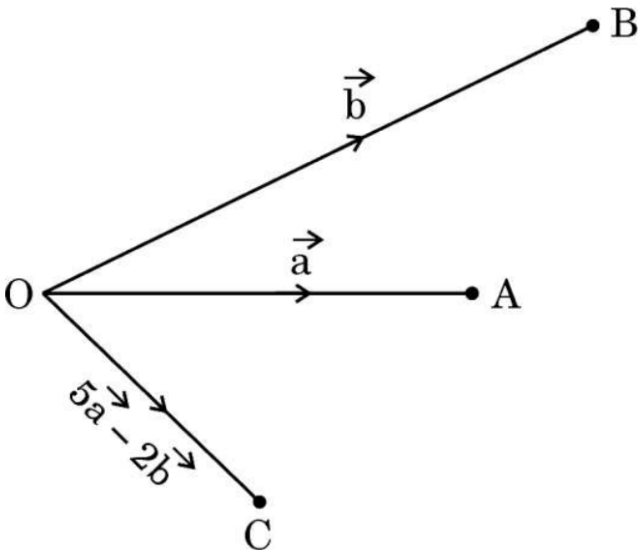
Quick Tip to Solve: Find the perpendicular foot using a dot product, then double its coordinates and subtract the given point.

Section E (12 marks)

This section comprises 3 case study based questions of 4 marks each.

36. Case Study - 1: Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his predecided destination, following straight paths from A to C and B to C in such a way that $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = 5\vec{a} - 2\vec{b}$ respectively.

Based upon the above information, answer the following questions: [4 marks]



36(i). Complete the given figure to explain their entire movement plan along the respective vectors. [1 mark]

Solution:

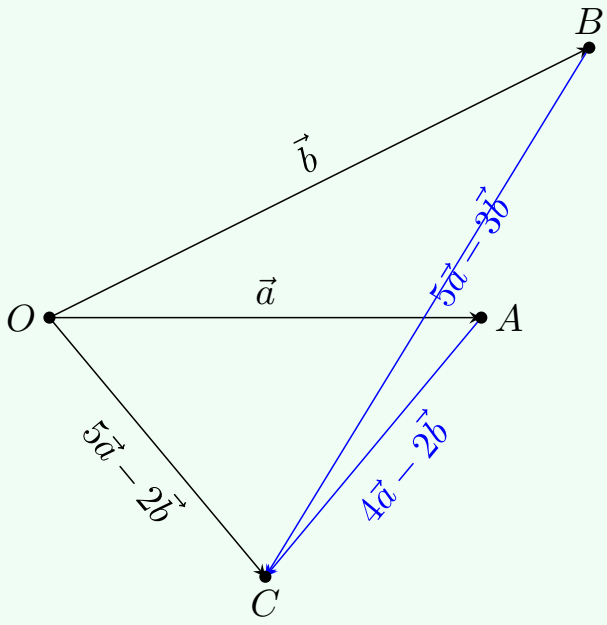
Concept:

- A directed segment represents a journey from its initial point to its terminal point.
- For position vectors measured from O, $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$.

Step 1: Retain the initial journeys The three arrows from O terminate at A, B and C, respectively. Keep their labels $\vec{a}$, $\vec{b}$ and $5\vec{a} - 2\vec{b}$. These are the separate outward journeys of the three friends.

Quick Tip to Solve: Put arrowheads towards C on both added segments, since the friends meet there.

Step 2: Add the journeys to the meeting point Join A to C and B to C, with both arrowheads pointing towards C. Label these arrows $\overrightarrow{AC} = 4\vec{a} - 2\vec{b}$ and $\overrightarrow{BC} = 5\vec{a} - 3\vec{b}$. No return arrow from C is needed, because C is the common meeting destination.



Completed movement plan (schematic)

Step 3: Conclusion The completed plan consists of $O \to A \to C$, $O \to B \to C$, and $O \to C$, as shown.

Quick Tip to Solve: Put arrowheads towards C on both added segments, since the friends meet there.

36(ii). Find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$. [1 mark]

Solution:

Concept:

- A displacement vector is terminal position vector minus initial position vector.
- The position vectors here are $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, and $\overrightarrow{OC} = 5\vec{a} - 2\vec{b}$.

Step 1: Find the displacement from A Using O as the common origin,

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (5\vec{a} - 2\vec{b}) - \vec{a} = 4\vec{a} - 2\vec{b}.$$

The subtraction removes the part of the displacement already covered in reaching A.

Step 2: Find the displacement from B Similarly,

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = (5\vec{a} - 2\vec{b}) - \vec{b} = 5\vec{a} - 3\vec{b}.$$

Reversing the order of subtraction would describe travel away from C, contrary to the planned meeting.

Quick Tip to Solve: For an arrow PQ, always subtract the position of P from the position of Q.

Step 3: Conclusion $\boxed{\overrightarrow{AC} = 4\vec{a} - 2\vec{b}, \quad \overrightarrow{BC} = 5\vec{a} - 3\vec{b}}$.

Quick Tip to Solve: For an arrow PQ, always subtract the position of P from the position of Q.

36(iii)(a). If $\vec{a} \cdot \vec{b} = 1$, distance of O to A is 1 km and that from O to B is 2 km, then find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Also, find $|\vec{a} \times \vec{b}|$. [2 marks]

Solution:

Concept:

- The dot product satisfies $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, where θ is the angle between the vectors.
- The magnitude of the cross product is $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$.

Step 1: Determine the angle The given distances give $|\vec{a}| = 1$ and $|\vec{b}| = 2$, in kilometres. Therefore

$$1 = (1)(2)\cos\theta, \qquad \cos\theta = \frac{1}{2}.$$

The angle between two nonzero vectors lies in $[0, \pi]$, so this fixes $\theta = \pi/3$, or 60°. The dot product is positive, consistent with this acute angle.

Step 2: Calculate the cross product magnitude Now substitute the same lengths and the angle into the cross product formula:

$$|\vec{a} \times \vec{b}| = (1)(2)\sin 60^\circ = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}.$$

As a check, the identity $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$ gives $4 - 1 = 3$. The cross product magnitude has units of squared length.

Step 3: Conclusion $\boxed{\theta = 60^\circ, \quad |\vec{a} \times \vec{b}| = \sqrt{3}\,\text{km}^2}$.

Quick Tip to Solve: After using the dot product, obtain the cross product magnitude from 4 minus 1 under a square root.

OR

36(iii)(b). If $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a unit vector perpendicular to $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$. [2 marks]

Solution:

Concept:

- The cross product of two nonparallel vectors is perpendicular to both.
- A unit vector along a nonzero vector $\vec{n}$ is $\vec{n}/|\vec{n}|$; its negative is also a unit vector.

Quick Tip to Solve: Cancel the common factor 2 before normalising the cross product to simplify arithmetic.

Step 1: Form the two vectors Adding and subtracting the given components gives

$$\vec{u} = \vec{a} + \vec{b} = 2\hat{i} + 3\hat{k}, \qquad \vec{v} = \vec{a} - \vec{b} = 2\hat{i} - 2\hat{j} + 5\hat{k}.$$

These are the vectors to which the requested direction must be perpendicular.

Step 2: Find and normalise their cross product Compute

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 3 \\ 2 & -2 & 5 \end{vmatrix} = 6\hat{i} - 4\hat{j} - 4\hat{k} = 2(3\hat{i} - 2\hat{j} - 2\hat{k}).$$

Its magnitude is $\sqrt{36 + 16 + 16} = 2\sqrt{17}$. Dividing by this positive magnitude gives $(3\hat{i} - 2\hat{j} - 2\hat{k})/\sqrt{17}$. For verification, its numerator has dot products $6 - 6 = 0$ with $\vec{u}$ and $6 + 4 - 10 = 0$ with $\vec{v}$; its squared length is $9 + 4 + 4 = 17$.

Step 3: Conclusion One required unit vector is $\boxed{\dfrac{3\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{17}}}$. Its negative is also valid.

Quick Tip to Solve: Cancel the common factor 2 before normalising the cross product to simplify arithmetic.

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37. Case Study - 2: Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.

A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, $\dfrac{dV}{dt} = kS$ is the differential equation, where V is the volume, S is the surface area and t is the time in hours.

Based upon the above information, answer the following questions: [4 marks]

37(i). Write the order and degree of the given differential equation. [1 mark]

Solution:

Concept:

- The order is the order of the highest derivative appearing in a differential equation.
- The degree is the power of the highest-order derivative when the equation is polynomial in its derivatives.

Step 1: Inspect the derivative order In $dV/dt = kS$, the only derivative is the first derivative of volume with respect to time. There is no second or higher derivative, so the highest derivative order is one. The dependence of surface area on the tablet radius does not introduce a higher derivative.

Step 2: Inspect the derivative power Rewrite the equation as $dV/dt - kS = 0$. The derivative dV/dt appears to the first power, without any radical or fractional power of a derivative. Hence the equation is polynomial in its derivative and its degree is one. The powers of radius in the volume or area formulas do not determine the degree.

Step 3: Conclusion $\boxed{\text{Order} = 1, \qquad \text{Degree} = 1}$.

Quick Tip to Solve: Count derivative order and derivative power separately; ignore powers of the radius.

37(ii). Substituting $V = \pi r^3$ and $S = 2\pi r^2$, we get the differential equation $\frac{dr}{dt} = \frac{2}{3}k$. Solve it, given that $r(0) = 5$ mm. [1 mark]

Solution:

Concept:

- A constant derivative $dr/dt = c$ integrates to $r = ct + C$, where C is an integration constant.
- The initial condition fixes C by substituting the stated time and radius.

Step 1: Integrate the stated equation Using the differential equation printed in the question,

$$\frac{dr}{dt} = \frac{2k}{3}, \qquad dr = \frac{2k}{3}\,dt.$$

Since k is constant, integration gives $r = \frac{2k}{3}t + C$. This describes a linear change of radius with time.

Step 2: Apply the initial radius At $t = 0$, the radius is 5 mm. Substitution gives $5 = 0 + C$, so $C = 5$. Thus the radius expressed in millimetres is $r(t) = 5 + \frac{2k}{3}t$, with t measured in hours. Differentiating this expression recovers the given equation, and putting $t = 0$ recovers the initial condition.

Step 3: Conclusion $\boxed{r(t) = 5 + \frac{2k}{3}t}$, with radius in millimetres and time in hours.

Quick Tip to Solve: For a constant rate, write radius as initial radius plus rate multiplied by time.

37(iii)(a). If it is given that $r = 3$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm. [2 marks]

Solution:

Concept:

- From the stated equation and $r(0) = 5$, the radius is $r(t) = 5 + \frac{2k}{3}t$.
- Use a measured radius to determine k, then set $r = 0$ to find the complete evaporation time.

Step 1: Use the radius after one hour At $t = 1$ hour, the radius is 3 mm. Substitution gives

$$3 = 5 + \frac{2k}{3}(1), \qquad -2 = \frac{2k}{3}, \qquad k = -3.$$

In the units used here, $k = -3\,\text{mm}\,\text{h}^{-1}$. The negative sign is required because the tablet shrinks. Consequently the given radius equation becomes $r(t) = 5 - 2t$; the radius decreases by 2 mm each hour.

Step 2: Determine the time at zero radius Set the radius equal to zero in this linear solution:

$$0 = 5 - 2t, \qquad 2t = 5, \qquad t = \frac{5}{2}\ \text{hours}.$$

This is the elapsed time measured from $t = 0$, not additional time after the observation at one hour. The radius remains nonnegative on $0 \le t \le 5/2$; the evaporation model ends at the zero-radius instant.

Quick Tip to Solve: Check the sign of k: a falling radius must give a negative slope in the stated equation.

Step 3: Conclusion $\boxed{k = -3\,\text{mm h}^{-1}, \qquad t = \frac{5}{2}\,\text{hours} = 2.5\,\text{hours}}$.

Quick Tip to Solve: Check the sign of k: a falling radius must give a negative slope in the stated equation.

OR

37(iii)(b). If it is given that $r = 1$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm. [2 marks]

Solution:

Concept:

- The differential equation supplied in part (ii) is $\frac{dr}{dt} = \frac{2k}{3}$, where r is the radius in millimetres and t is time in hours. A constant derivative gives a linear function of time.
- The initial condition $r(0) = 5$ fixes the integration constant. Complete evaporation corresponds to $r = 0$; reduction of radius requires a negative value of k.

Step 1: Use the solution with the initial condition Integrating the differential equation stated in part (ii), we obtain

$$r = \frac{2k}{3}t + C.$$

At $t = 0$, the radius is $5\,\text{mm}$, so $C = 5$. Therefore the relation to use in this alternative is

$$r = 5 + \frac{2k}{3}t.$$

The same initial condition applies to both alternatives of part (iii).

Step 2: Substitute the given radius after one hour Here $r = 1\,\text{mm}$ when $t = 1\,\text{hour}$. Substitution gives

$$1 = 5 + \frac{2k}{3}, \qquad \frac{2k}{3} = -4, \qquad k = -6.$$

Thus the radius equation becomes $r = 5 - 4t$. The negative slope agrees with the shrinking tablet. To find the instant when no radius remains, substitute $r = 0$:

$$0 = 5 - 4t \quad \Longrightarrow \quad t = \frac{5}{4}\,\text{hours}.$$

This is the time measured from the original instant $t = 0$, rather than the additional time after the one-hour observation.

Step 3: Conclusion Hence $\boxed{k = -6\,\text{mm h}^{-1}}$, using the supplied differential equation, and the tablet disappears at $\boxed{t = \frac{5}{4}\,\text{hours} = 1\,\text{hour}\,15\,\text{minutes}}$.

Quick Tip to Solve: Use the supplied radius equation from part (ii); the fall from 5 mm to 1 mm in one hour immediately gives its slope as -4 mm per hour.

38. Case Study - 3: Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let A_1: People with good health,

A_2: People with average health,

and A_3: People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25%, 35% and 50%, respectively.

Based upon the above information, answer the following questions: [4 marks]

38(i). A person was tested randomly. What is the probability that he/she has contracted the disease? [2 marks]

Solution:

Concept:

- Let D denote the event that a randomly selected person has contracted the disease. The categories A_1, A_2, A_3 are mutually exclusive and exhaustive.
- The total probability theorem gives $P(D) = \sum_{i=1}^{3} P(A_i)P(D \mid A_i)$. Each conditional disease probability must be weighted by the probability of belonging to its category.

Step 1: Record the category probabilities Since the person is selected randomly from the 1000 people,

$$P(A_1) = \frac{700}{1000} = \frac{7}{10}, \quad P(A_2) = \frac{200}{1000} = \frac{1}{5}, \quad P(A_3) = \frac{100}{1000} = \frac{1}{10}.$$

These probabilities add to one. The corresponding disease probabilities given in the passage are $P(D \mid A_1) = \frac{1}{4}$, $P(D \mid A_2) = \frac{7}{20}$ and $P(D \mid A_3) = \frac{1}{2}$.

Step 2: Apply the total probability theorem A person can contract the disease while belonging to any one of the three disjoint categories. Therefore,

$$\begin{aligned} P(D) &= \frac{7}{10} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{7}{20} + \frac{1}{10} \cdot \frac{1}{2} \\ &= \frac{7}{40} + \frac{7}{100} + \frac{1}{20} \\ &= \frac{35 + 14 + 10}{200} = \frac{59}{200}. \end{aligned}$$

Equivalently, the three categories contribute 175, 70 and 50 affected people per 1000, giving 295 affected people altogether.

Step 3: Conclusion The required probability is $\boxed{\frac{59}{200} = 0.295}$, or $\boxed{29.5\%}$.

Quick Tip to Solve: Weight each disease rate by its category size; a simple average of the three percentages would ignore the unequal groups.

38(ii). Given that the person has not contracted the disease, what is the probability that the person is from category A_2? [2 marks]

Solution:

Concept:

- Write D for contracting the disease and $\overline{D}$ for not contracting it. Within each category, $P(\overline{D} \mid A_i) = 1 - P(D \mid A_i)$.
- Bayes' theorem gives $P(A_2 \mid \overline{D}) = \dfrac{P(A_2)P(\overline{D} \mid A_2)}{P(\overline{D})}$. The denominator includes disease-free people from all three categories.

Step 1: Find the probability of remaining disease-free The category probabilities are $P(A_1) = 0.7$, $P(A_2) = 0.2$ and $P(A_3) = 0.1$. Taking complements of the given conditional disease probabilities gives

$$P(\overline{D} \mid A_1) = 0.75, \quad P(\overline{D} \mid A_2) = 0.65, \quad P(\overline{D} \mid A_3) = 0.50.$$

By total probability,

$$\begin{aligned} P(\overline{D}) &= (0.7)(0.75) + (0.2)(0.65) + (0.1)(0.50) \\ &= 0.525 + 0.130 + 0.050 = 0.705. \end{aligned}$$

Step 2: Condition on the person being disease-free The probability that a person belongs to the average-health category and has not contracted the disease is

$$P(A_2 \cap \overline{D}) = P(A_2)P(\overline{D} \mid A_2) = (0.2)(0.65) = 0.13.$$

Consequently, Bayes' theorem yields

$$P(A_2 \mid \overline{D}) = \frac{0.13}{0.705} = \frac{130}{705} = \frac{26}{141}.$$

As a count check, 130 of the 705 disease-free people belong to category A_2, so their fraction is the same ratio.

Step 3: Conclusion Given that the person has not contracted the disease, the probability of belonging to category A_2 is $\boxed{\dfrac{26}{141}}$, approximately 0.1844.

Quick Tip to Solve: First replace every disease percentage by its complement, then divide the disease-free A2 contribution by the total disease-free probability.