

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. If $2 \cos^{-1} x = y$, then

- (A) $0 \leq y \leq \pi$ (B) $-\pi \leq y \leq \pi$
 (C) $0 \leq y \leq 2\pi$ (D) $-\pi \leq y \leq 0$

2. Which of the following cannot be the order of a row-matrix?

- (A) 2×1 (B) 1×2
 (C) 1×1 (D) $1 \times n$

3. Which of the following properties is/are true for two matrices of suitable orders?

- (i) $(A + B)' = A' + B'$
 (ii) $(A - B)' = B' - A'$
 (iii) $(AB)' = A'B'$
 (iv) $(kAB)' = kB'A'$ (k is a scalar)
 (A) (i) only (B) (i), (ii) and (iii)
 (C) (i) and (ii) (D) (i) and (iv)

4. If $A_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ and $A_2 = \begin{pmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{pmatrix}$,

then

- (A) $A_1 = 2A_2$ (B) $A_1 = -2A_2$
 (C) $A_1 = A_2$ (D) $A_1 = -A_2$

5. One of the values of x for which

$$\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1 \text{ is}$$

- (A) 0 (B) $\frac{\pi}{4}$
 (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

6. If A and B are skew symmetric matrices of same order, then which of the following matrices is also skew symmetric?

- (A) AB (B) $AB + BA$
 (C) $(A + B)^2$ (D) $A - B$

7. The least value of $f(x) = x^3 - 12x$, $x \in [0, 3]$ is

- (A) -16 (B) -9
 (C) 0 (D) 16

8. If $\int \frac{3ax}{b^2 + c^2x^2} dx = A \log |b^2 + c^2x^2| + K$, then the value of A is

- (A) $3a$ (B) $\frac{3a}{2b^2}$
 (C) $\frac{3a}{b^2c^2}$ (D) $\frac{3a}{2c^2}$

9. The value of $\int_{-1}^1 \frac{x^3}{x^2 + 2|x| + 1} dx$ is

- (A) 0 (B) $\log 2$
 (C) $2 \log 2$ (D) $\frac{1}{2} \log 2$

10. The area bounded by the curve $y = x|x|$, x -axis and the ordinates $x = -1$ and $x = 1$ is given by

- (A) 0 (B) $\frac{1}{3}$
 (C) $\frac{2}{3}$ (D) $\frac{4}{3}$

11. The integrating factor of differential equation $R \frac{dx}{dy} + Px = Q$ where P, Q, R are functions of y is

- (A) $e^{\int P dy}$ (B) $e^{\int \frac{P}{R} dy}$
 (C) $e^{\int \frac{P}{R} dx}$ (D) $e^{\int P dx}$

12. The order and degree of the differential equation $\frac{d}{dx} \left(\frac{d^2y}{dx^2} \right) = 0$ respectively are

- (A) 0, 1 (B) 1, 1
 (C) 2, 1 (D) 1, not defined

13. The value of p for which vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is

- (A) 0 (B) 1
(C) $\frac{5}{2}$ (D) $-\frac{5}{2}$

14. The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear, is

- (A) 8 (B) -8
(C) 2 (D) $\frac{5}{3}$

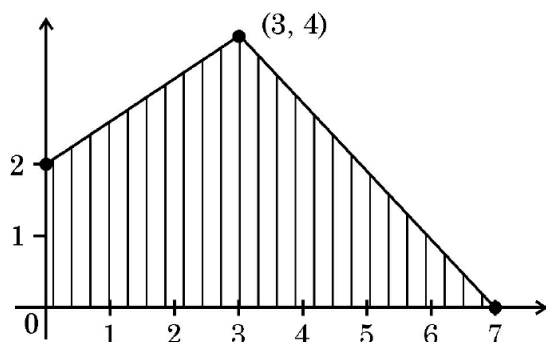
15. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then the value of $|\vec{a} \cdot \vec{b}|$ is

- (A) $6\sqrt{3}$ (B) $8\sqrt{3}$
(C) $12\sqrt{3}$ (D) $3\sqrt{12}$

16. The length of perpendicular drawn from point $(2, 5, 7)$ on line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ is

- (A) 2 (B) 5
(C) $\sqrt{74}$ (D) $\sqrt{78}$

17. The feasible region of a linear programming problem with objective function $Z = 5x + 7y$ is shown below:



The maximum value of Z - minimum value of Z is

- (A) 8 (B) 29
(C) 35 (D) 43

18. The degree of an objective function of a linear programming problem is

- (A) 0 (B) 1
(C) 2 (D) Any natural number

19. **Assertion (A):** In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is $\frac{2}{3}$.

Reason (R): For any two events A and B ,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true and Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is true.

20. **Assertion (A):** Lines given by $x = py + q$, $z = ry + s$ and $x = p'y + q'$, $z = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$.

Reason (R): Two lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are perpendicular to each other if $\vec{b}_1 \cdot \vec{b}_2 = 0$.

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true and Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is true.

Section B

21(a). Check whether function $f(x)$ defined as

$$f(x) = \begin{cases} \frac{|x-3|}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$$

is continuous at $x = 3$ or not.

OR

21(b). If $3(x^2 + y^2) = 4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{3}{2}\right)$.

22. A room freshener bottle in the shape of an inverted cone sprays the perfume at regular intervals such that volume of the perfume in the bottle decreases at the steady rate of $1 \text{ mm}^3/\text{min}$. Find the rate at which level of perfume is dropping at an instant when level of perfume in the bottle is 10 mm , if the semi-vertical angle of conical bottle is $\frac{\pi}{3}$.

23. Find the vector of magnitude 14 in the direction of $\vec{QP}$, where P and Q are the points $(1, 3, 2)$ and $(-1, 0, 8)$ respectively.

24. Vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths.

25(a). Simplify: $\tan^{-1}\left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right)$, $0 < x < \frac{\pi}{4}$.

OR

25(b). Evaluate: $\cot\left[\sin^{-1} 1 - \cos^{-1}\left(-\frac{1}{2}\right)\right]$.

Section C

26. Evaluate: $\int_0^1 x \tan^{-1} x \, dx$.

27(a). Find: $\int x^2 e^x \, dx$.

OR

27(b). Find: $\int \frac{x^2}{(x^2 + 9)(x^2 + 16)} \, dx$.

28. If $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1 + \sin x}$ and $I_2 = \int_{-1/2}^{1/2} |x| \, dx$, then show that $I_1 - 4I_2 = 0$.

29(a). Find the general solution of the following differential equation: $x^2 \frac{dy}{dx} = x^2 + xy + y^2$.

OR

29(b). Find the particular solution of the differential equation $xy \frac{dy}{dx} = (x + 2)(y + 2)$, given that $y(1) = -1$.

30. Solve the following linear programming problem graphically:

Minimize $Z = 13x - 15y$

Subject to constraints

$$x + y \leq 7,$$

$$2x - 3y + 6 \geq 0,$$

$$x \geq 0, y \geq 0.$$

31(a). Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball.

OR

31(b). The probability of simultaneous occurrence of atleast one of the two events X and Y is a . If the probability that exactly one of the events X, Y occurs is b , prove that $P(X') + P(Y') = 2 - 2a + b$.

Section D

32(a). A relation R is defined on $\mathbb{Z}$, the set of integers, as $R = \{(x, y) : |x - y| \text{ is divisible by a prime number } p, x, y \in \mathbb{Z}\}$. Check whether R is an equivalence relation or not.

OR

32(b). A function $f : \mathbb{R} - \left\{\frac{3}{5}\right\} \rightarrow \mathbb{R} - \left\{\frac{3}{5}\right\}$ is defined as $f(x) = \frac{3x+2}{5x-3}$. Show that f is one-one and onto.

33(a). If $A = \begin{pmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{pmatrix}$, find A^{-1} and use it to solve the following system of equations: $2y + z = 7$, $2x - y - z = 8$, $x - 2y = 10$.

OR

33(b). If $\begin{pmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{pmatrix}$ is a singular matrix, then find all values of x where $x \in \left[0, \frac{\pi}{2}\right]$.

34. If $x = \cos t$, $y = \cos mt$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$.

35. Check whether the lines given by $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ are parallel or not. If parallel, find the distance between them, otherwise find their point of intersection, if the lines are intersecting.

Section E

36(i). An online delivery company in a city has 5000 subscribers and collects annual subscription fees of Rs.300 per subscriber for unlimited free deliveries. The company wishes to increase the annual subscription fee. It is predicted that, for every increase of Rs.1, ten subscribers will discontinue. Assume that the company increased the annual fee by Rs. x .

How many subscribers will discontinue after an increase of Rs. x in annual fee?

36(ii). If $R(x)$ denotes the total revenue collected after the increase of Rs. x in subscription fee, express $R(x)$ as a function of x .

36(iii)(a). Find the value of x for which $R(x)$ is maximum.

OR

36(iii)(b). Find the sub-intervals of $(0, 5000)$ in which $R(x)$ is increasing and decreasing.

37(i). In an online jackpot, there is one first prize of Rs.3,00,000, two second prizes of Rs.2,00,000 each and three third prizes of Rs.50,000 each. A total of 1,00,000 jackpot tickets each costing Rs.100 were sold thereby raising a fund of Rs.1,00,00,000. Rohan bought one ticket.

What are the possible amounts, the person can win?

37(ii)(a). What is the probability that the person wins atleast Rs.2,00,000?

OR

37(ii)(b). What is the probability that the person does not win any amount?

37(iii). In another jackpot, Rohan also bought a ticket having a prize money of Rs.5,00,000. The chances of winning the jackpot are 1 in 1,00,000. Find the probability that on exactly one of tickets he wins the jackpot.

38(i). Roundabouts are often made on busy roads to ease the traffic and avoid red lights. One such round-about is made such that equation representing its boundary is given by $C_1 : x^2 + y^2 = 64$. There is a circular pond with a fountain in the middle of the roundabout whose equation is given by $C_2 : x^2 + y^2 = 4$.

Represent the given equations C_1 and C_2 with the help of a diagram.

38(ii). Express y as a function of x , ($y = f(x)$), for both C_1 and C_2 .

38(iii)(a). Using integration find the area of region covered by the roundabout.

OR

38(iii)(b). Using integration, find the area of region covered by circular pond.

- End of Question Paper -