

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. $\int \frac{1}{1 + \cos x} dx$ is equal to

- (A) $\frac{1}{2} \tan \frac{x}{2} + C$ (B) $\frac{-1}{2} \cot \frac{x}{2} + C$
 (C) $-\cot \frac{x}{2} + C$ (D) $\tan \frac{x}{2} + C$

2. For $f(x) = x + \frac{9}{x}$, ($x \neq 0$)

- (A) local maximum value is 2 (B) local minimum value is -2
 (C) local maximum value is -2 (D) local minimum value $<$ local maximum value

3. Which of the following expressions will give the area of region bounded by the curve $y = x^2$ and line $y = 16$?

- (A) $\int_0^4 x^2 dx$ (B) $2 \int_0^4 x^2 dx$
 (C) $\int_0^{16} \sqrt{y} dy$ (D) $2 \int_0^{16} \sqrt{y} dy$

4. The general solution of the differential equation $x dy - y dx = 0$ is

- (A) $x^2 - y^2 = k$ (B) $xy = k$
 (C) $x = ky$ (D) $\log y + \log x = k$

5. The integrating factor of the differential equation $2x \frac{dy}{dx} - y = 3$ is

- (A) $\sqrt{x}$ (B) $\frac{1}{\sqrt{x}}$
 (C) e^x (D) e^{-x}

6. If $|\vec{a}| = 5$ and $-2 \leq \lambda \leq 1$, then the sum of greatest and the smallest value of $|\lambda \vec{a}|$ is

- (A) -5 (B) 5
 (C) 10 (D) 15

7. Vector of magnitude 3 making equal angles with x and y axes and perpendicular to z axis is

- (A) $\hat{i} + 2\sqrt{2}\hat{j}$ (B) $3\hat{k}$
 (C) $\frac{3\sqrt{2}}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}$ (D) $\sqrt{3}\hat{i} + \sqrt{3}\hat{j} + \sqrt{3}\hat{k}$

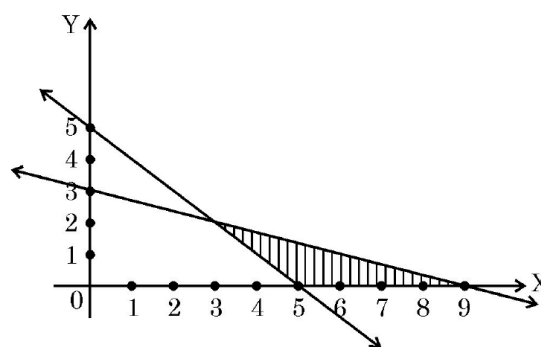
8. Direction cosines of line $x = y = 1 - z$ are

- (A) $1, 1, 1$ (B) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$
 (C) $0, 0, 1$ (D) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

9. In a linear programming problem, the linear function which has to be maximized or minimized is called

- (A) a feasible function (B) an objective function
 (C) an optimal function (D) a constraint

10. For the feasible region shown below, the non-trivial constraints of the linear programming problem are



- (A) $x + y \leq 5$, $x + 3y \leq 9$ (B) $x + y \leq 5$, $x + 3y \geq 9$
 (C) $x + y \geq 5$, $x + 3y \leq 9$ (D) $x + y \geq 5$, $3x + y \leq 9$

11. For two events A and B such that $P(A) \neq 0$ and $P(B) \neq 1$, $P(A'/B') =$

- (A) $1 - P(A/B)$ (B) $1 - P(A'/B)$
(C) $\frac{1 - P(A \cap B)}{P(B')}$ (D) $\frac{1 - P(A \cup B)}{P(B')}$

12. A relation R on set $A = \{1, 2, 3\}$ defined as $R = \{(1, 2), (2, 1), (2, 2)\}$ is

- (A) Reflexive only (B) Reflexive and Transitive
(C) Symmetric and Transitive (D) Symmetric only

13. If A and B are square matrices of same order, then which of the following statements is/are always true?

- (i) $(A + B)(A - B) = A^2 - B^2$
(ii) $AB = BA$
(iii) $(A + B)^2 = A^2 + AB + BA + B^2$
(iv) $AB = 0 \Rightarrow A = 0$ or $B = 0$
(A) Only (i) and (iii) (B) Only (ii) and (iii)
(C) Only (iii) (D) Only (iii) and (iv)

14. If $A = \begin{pmatrix} 1 & a & b \\ -1 & 2 & c \\ 0 & 5 & 8 \end{pmatrix}$ is a symmetric matrix, then the value of $3a + b + c$ is

- (A) 2 (B) 6
(C) 4 (D) 0

15. If $A = \begin{pmatrix} \cos x & \frac{-\sin x}{\sqrt{2}} \\ \frac{\sin x}{\sqrt{2}} & \cos x \end{pmatrix}$ and $A + A' = \sqrt{2} I$, then value of $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is

- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$
(C) 0 (D) $\frac{\pi}{4}$

16. For a square matrix A , $(3A)^{-1}$ equals

- (A) $3A^{-1}$ (B) $9A^{-1}$
(C) $\frac{1}{3}A^{-1}$ (D) $\frac{1}{9}A^{-1}$

17. If $\begin{vmatrix} -1 & -2 & 5 \\ -2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = -86$, then the sum of all possible values of a is

- (A) 4 (B) 5
(C) -4 (D) 9

18. If $e^{x+y} = 3x$, then $\frac{dy}{dx}$ equals

- (A) $\frac{3}{e^{x+y}}$ (B) $\frac{1}{e^{x+y}}$
(C) $\frac{1 - e^{x+y}}{e^{x+y}}$ (D) $\frac{3 - e^{x+y}}{e^{x+y}}$

19. **Assertion (A):** A line can have direction cosines $\langle 1, 1, 1 \rangle$.

Reason (R): $\cos \theta = 1$ is possible for $\theta = 0$.

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

20. For two vectors $\vec{a}$ and $\vec{b}$:

Assertion (A): $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$

Reason (R): $|\vec{a} \times \vec{b}| = (\vec{a} \cdot \vec{b}) \tan \theta$, $\left(0 \leq \theta \leq \frac{\pi}{2}\right)$

- (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Section B

21. (a) Find the absolute maximum value of $f(x) = \cos x + \sin^2 x$, $x \in [0, \pi]$.

OR

(b) If the volume of a solid hemisphere increases at a uniform rate, prove that its surface area varies inversely as its radius.

22. If $\overrightarrow{AB} = \hat{j} + \hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$ represent the two vectors along the sides AB and AC of $\triangle ABC$, prove that the median $\overrightarrow{AD} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$, where D is midpoint of BC . Hence, find the length of median AD .

23. Find the co-ordinates of foot of perpendicular drawn from $(0, 0, 0)$ to line $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z-3}{-2}$.

24. (a) Check whether $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R}$ defined as $f(x) = \frac{x-2}{x-3}$ is onto or not.

OR

(b) Check whether $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$ (where $\mathbb{Z}$ is the set of integers) defined as $f(x, y) = (2y, 3x)$ is injective or not.

25. If $x = \sin t - \cos t$, $y = \sin t \cos t$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

Section C

26. If $\frac{d}{dx}(F(x)) = \frac{1}{e^x + 1}$, then find $F(x)$ given that $F(0) = \log \frac{1}{2}$.

27. (a) Solve the following differential equation:
 $x \frac{dy}{dx} = y - x \sin^2\left(\frac{y}{x}\right)$, given that $y(1) = \frac{\pi}{6}$.

OR

(b) Find the general solution of the differential equation: $y \log y \frac{dx}{dy} + x = 2 \log y$

28. Solve the following linear programming problem graphically:

Maximize $Z = 8000x + 12000y$

Subject to constraints

$$3x + 4y \leq 60$$

$$x + 3y \leq 30$$

$$x \geq 0, y \geq 0$$

29. (a) The probability of hitting the target by a trained sniper is three times the probability of not hitting the target on a stormy day due to high wind speed. The sniper fired two shots on the target on a stormy day when wind speed was very high. Find the probability that

(i) target is hit

(ii) atleast one shot misses the target.

OR

(b) Mother, Father and Son line up at random for a family picture. Let events E: Son on one end and F: Father in the middle. Find $P(E/F)$.

30. Find: $\int \frac{2x + 1}{\sqrt{6x + x^2}} dx$

31. (a) Evaluate: $\int_{\pi/12}^{5\pi/12} \frac{dx}{1 + \sqrt{\cot x}}$

OR

(b) Evaluate: $\int_{-\pi/6}^{\pi/2} (\sin |x| + \cos |x|) dx$

Section D

32. Find the domain of $p(x) = \sin^{-1}(1 - 2x^2)$.
Hence, find the value of x for which $p(x) = \frac{\pi}{2}$.
Also, write the range of $2p(x) + 5$.

33. A line passing through the points $A(1, 2, 3)$ and $B(5, 8, 11)$ intersects the line $\vec{r} = 4\hat{i} + \hat{j} + \lambda(5\hat{i} + 2\hat{j} + \hat{k})$. Find the co-ordinates of the point of intersection. Hence, write the equation of a line passing through the point of intersection and perpendicular to both the lines.

34. (a) If $P = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, find (QP) and hence solve the following system of equations using matrices:

$$x - y = 3, \quad 2x + 3y + 4z = 17, \quad y + 2z = 7$$

OR

(b) Obtain the value of $\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ in terms of x, y and z . Further, if $\Delta = 0$ and x, y, z are non-zero real numbers, prove that $x^{-1} + y^{-1} + z^{-1} = -1$.

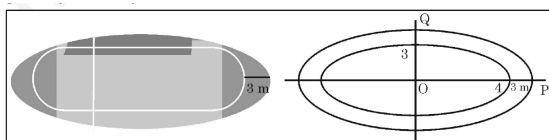
35. (a) Find the sub-interval of $(0, \pi)$ in which $f(x) = \tan^{-1}(\sin x - \cos x)$ is increasing and decreasing.

OR

(b) A rectangle of perimeter 24 cm is revolved along one of its sides to sweep out a cylinder of maximum volume. Find the dimensions of the rectangle.

Section E

36. A racing track is built around an elliptical ground whose equation is given by $9x^2 + 16y^2 = 144$. The width of the track is 3 m as shown below:



Based on given information, answer the following questions:

36(i). Express y as a function of x from the given equation of ellipse.

36(ii). Integrate the function obtained in (i) with respect to x .

36(iii). (a) Find the area of the region enclosed within the elliptical ground excluding the track using integration.

OR

(b) Write the co-ordinates of the points P and Q where the outer edge of the track cuts x axis and y axis in first quadrant and find the area of the triangle formed by points P , O , Q using integration.

37. Sports car racing is a form of motorsport which uses sports car prototypes. The competition is held on special tracks designed in various shapes. The equation of one such track is given as follows:

$$f(x) = \begin{cases} x^4 - 4x^2 + 4, & 0 \leq x < 3 \\ x^2 + 40, & x \geq 3 \end{cases}$$

Based on given information, answer the following questions:

37(i). Find $f'(x)$ for $0 \leq x < 3$.

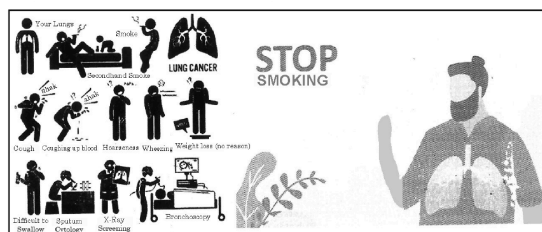
37(ii). Find $f'(4)$.

37(iii). (a) Test for continuity of $f(x)$ at $x = 3$.

OR

(b) Test for differentiability of $f(x)$ at $x = 3$.

38. Smoking increases the risk of lung problems.



A study revealed that 170 in 1000 males who smoke develop lung complications, while 120 out of 1000 females who smoke develop lung related problems. In a colony, 50 people were found to be smokers of which 30 are males. A person is selected at random from these 50 people and tested for lung related problems.

Based on the given information, answer the following questions:

38(i). What is the probability that selected person is a female?

38(ii). If a male person is selected, what is the probability that he will not be suffering from lung problems?

38(iii). (a) A person selected at random is detected with lung complications. Find the probability that selected person is a female.

OR

(b) A person selected at random is not having lung problems, find the probability that the person is a male.

- End of Question Paper -