

Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

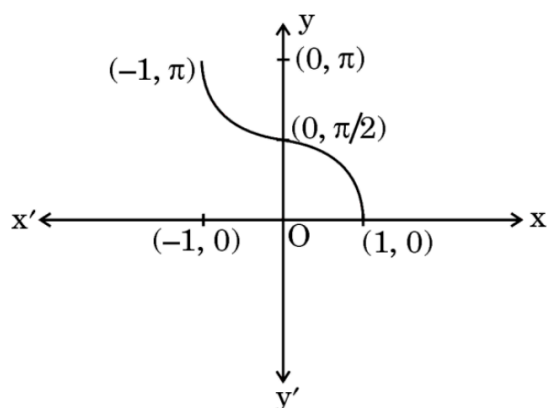
Maximum Marks: 80

General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

Section A

1. The following graph represents:



- (A) $y = \cos^{-1} x$ (B) $y = \sec^{-1} x$
(C) $y = \sin^{-1} x$ (D) $y = \tan^{-1} x$

2. Let $A = [a_{ij}]$ be a 2×2 matrix whose elements are given by $a_{ij} = \frac{(i+2j)^2}{3}$. Then A' is given by:

- (A) $\begin{pmatrix} 12 & \frac{25}{3} & \frac{16}{3} & 3 \end{pmatrix}$ (B) $\begin{pmatrix} 12 & \frac{16}{3} & \frac{25}{3} & 3 \end{pmatrix}$
(C) $\begin{pmatrix} 3 & \frac{25}{3} & \frac{16}{3} & 12 \end{pmatrix}$ (D) $\begin{pmatrix} 3 & \frac{16}{3} & \frac{25}{3} & 12 \end{pmatrix}$

3. The greatest integer function, $f(x) = [x]$, $0 < x < 3$ is not differentiable at how many points?

- (A) At only one point (B) At only two points
(C) At no point (D) At three points

4. The area of the region bounded by the curve $y = x$ and x -axis, between $x = 0$ and $x = 2$ is:

- (A) 2 sq. units (B) $\frac{1}{2}$ sq. unit
(C) 1 sq. unit (D) 4 sq. units

5. If $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is:

- (A) 0 (B) -2
(C) -1 (D) 2

6. A cylindrical tank is being filled with sand at a rate of $314 \text{ m}^3/\text{h}$. If the radius of the tank is 10 m, then the height of sand in the tank increases at the rate of:

- (A) 1.1 m/h (B) 1 m/h
(C) $\frac{\pi}{2}$ m/h (D) 2 m/h

7. The principal value of $\sec^{-1}(\sqrt{2}) + 2 \operatorname{cosec}^{-1}(-\sqrt{2})$ is:

- (A) $-\frac{\pi}{2}$ (B) $\frac{\pi}{4}$
(C) $-\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

8. If points $(2, 3)$, $(0, 4)$ and $(p, 2)$ are collinear, then the value of p is:

- (A) $\frac{4}{3}$ (B) $-\frac{3}{4}$
(C) 4 (D) -4

9. Differential of e^{e^x} with respect to x is:

- (A) $\log x$ (B) e^x
(C) $e^x \cdot e^{e^x}$ (D) $(e^x)^2$

10. $\int \frac{dx}{\sqrt{25 - 16x^2}}$ is equal to:

- (A) $\frac{1}{5} \sin^{-1} 4x + C$ (B) $\frac{1}{25} \sin^{-1} 16x + C$
(C) $\frac{1}{4} \sin^{-1} \frac{4x}{5} + C$ (D) $\frac{1}{16} \sin^{-1} \frac{4x}{5} + C$

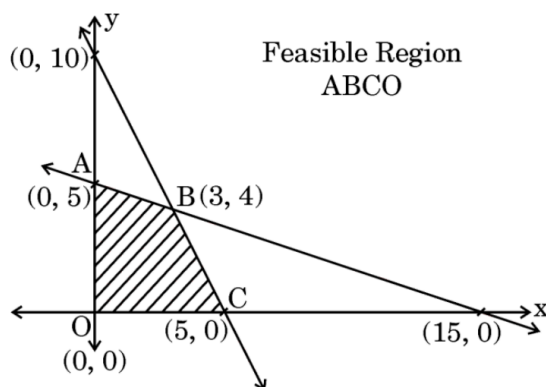
11. If $\int_0^1 \frac{dx}{e^x + e^{-x}} = \tan^{-1} e + k$, then the value of k is:

- (A) e (B) $\frac{\pi}{4}$
(C) 0 (D) $-\frac{\pi}{4}$

12. The sum of the order and the degree of the differential equation $y = x \frac{dy}{dx} + 3 \left[2 - \left(\frac{dy}{dx} \right)^2 \right]$ is:

- (A) not defined (B) 3
(C) 1 (D) 2

13. In the graph, the feasible region representing the Linear Programming Problem for maximising objective function $Z = px + qy$, $p, q > 0$ is shaded. If all points on segment AB give max (Z), then which of the following is true? (Feasible region ABCO with vertices $A(0, 5)$, $B(3, 4)$, $C(5, 0)$, $O(0, 0)$, and the boundary line through $(0, 10)$ and $(15, 0)$.)



- (A) $p = 2q$ (B) $p = 3q$
(C) $q = 3p$ (D) $q = 2p$

14. Three points $A(0, 1, 1)$, $B(2, 0, -1)$ and $C(1, 0, 3)$ form $\triangle ABC$. The ar($\triangle ABC$) is:

- (A) $\sqrt{58}$ sq. units (B) $\sqrt{53}$ sq. units
(C) $\frac{\sqrt{53}}{2}$ sq. units (D) $\sqrt{11}$ sq. units

15. If $(3\hat{i} - 2\hat{j} + 5\hat{k}) \times (4\hat{i} + p\hat{j} + q\hat{k}) = \vec{0}$, then the values of p and q are:

- (A) $p = -\frac{2}{3}$, $q = \frac{5}{3}$ (B) $p = -\frac{8}{3}$, $q = \frac{20}{3}$
(C) $p = \frac{20}{3}$, $q = -\frac{8}{3}$ (D) $p = 0$, $q = 0$

16. Which of the following is *not* a Linear Differential Equation?

- (A) $(1 + x^2) dy + 2xy dx = \cot x dx$ (B) $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$
(C) $x(1 + y^2) dx - y(1 + x^2) dy = 0$ (D) $y dx - (x + 3y^2) dy = 0$

17. The region represented by the system of inequations $3x + y \geq 3$, $2x - y \geq -5$, $x, y \geq 0$ is:

- (A) unbounded in 1st quadrant (B) bounded in 1st quadrant
(C) unbounded in 2nd quadrant (D) bounded in 2nd quadrant

18. The probability that a particular item is available in three shops A , B and C is $\frac{4}{5}$, $\frac{3}{4}$ and $\frac{2}{3}$ respectively. If a person visits all the three shops to buy the item, then what is the probability that it will be available in at least one of the shops?

- (A) $\frac{59}{60}$ (B) 1
(C) $\frac{1}{60}$ (D) $\frac{9}{60}$

19. **Assertion (A):** If A and B are two square matrices such that AB and BA are defined, then it is not necessary that $AB = BA$.

Reason (R): Product of two diagonal matrices of same order is commutative.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** A function $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = x^3 + 2$, $\forall x \in \mathbb{N}$ is one-one but not onto.

Reason (R): Since $\forall y \in \mathbb{N}$ (Codomain), there does not exist $x = (y - 2)^{1/3}$ in $\mathbb{N}$ (Domain) such that $f(x) = x^3 + 2 = y$.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Section B

21. (a) Show that the function $f(x) = \begin{cases} \frac{\cos x}{-x + \frac{\pi}{2}}, & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

OR

- (b) Find whether the function $f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$ at $x = 2$ is differentiable or not.

22. (a) Let two rods placed on the ground be represented by vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$. Find a vector representing a flag-post of height 5 m that has to be erected perpendicular to both the rods.

OR

- (b) A unit vector $\vec{a}$ is such that it makes an angle $\frac{\pi}{4}$ with x -axis, $\frac{\pi}{3}$ with y -axis and an acute angle θ with z -axis. Find θ and the components of $\vec{a}$.

23. Find the interval(s) for which the function $f(x) = \frac{x}{3} + \frac{3}{x}$, $x \neq 0$ is increasing.

24. If in a parallelogram $PQRS$, $\vec{PQ} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{QR} = \hat{i} - 2\hat{j} + \hat{k}$, then find the unit vectors parallel to the diagonals $\vec{PR}$ and $\vec{SQ}$.

25. Evaluate: $\tan^{-1} \left(-\frac{1}{\sqrt{3}} \right) + \cot^{-1} \left(\frac{1}{\sqrt{3}} \right) + \tan^{-1} \left(\sin \left(-\frac{\pi}{2} \right) \right) + \tan^{-1} \left(\tan \frac{2\pi}{3} \right)$

Section C

26. (a) Find a point on the line $\frac{x-2}{3} = \frac{y-1}{-2} = \frac{z-3}{2}$ at a distance of $\sqrt{2}$ units from the point $(1, 2, 3)$.

OR

(b) Find the shortest distance between the lines $\vec{r} = (4 + \lambda)\hat{i} + (2\lambda - 1)\hat{j} - 3\hat{k}$ and $\vec{r} = (1 + 2\mu)\hat{i} + (2 - 5\mu)\hat{k} + (4\mu - 1)\hat{j}$.

27. If $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$, then compute $A^2 - 5A + 4I$.

28. In a school, the probability of holding a debate competition is $\frac{1}{3}$ and that of a quiz competition is $\frac{2}{3}$. In the two participating teams, A has 4 girls and 6 boys and B has 7 girls and 3 boys. If a debate competition is held, the students are selected from team A and for the quiz competition they are selected from team B . If only two students are to be chosen from the teams, then find the probability that one will be a girl and the other a boy.

29. Solve the following Linear Programming Problem graphically:

$$\text{Maximize } Z = 20x + 10y$$

subject to constraints

$$x + 2y \leq 28$$

$$3x + y \leq 24$$

$$x \geq 2$$

$$x, y \geq 0$$

30. (a) Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. A function $f : A \rightarrow B$ is defined by $f(x) = \frac{x-2}{x-3}$. Find whether f is one-one and onto.

OR

(b) Let n be a fixed positive integer. A relation R is defined in set $\mathbb{Z}$ such that $R = \{(x, y) : (x - y) \text{ is divisible by } n, x, y \in \mathbb{Z}\}$. Determine if R is an equivalence relation.

31. (a) If $xy = e^{x-y}$, then find $\frac{dy}{dx}$.

OR

(b) Differentiate $\tan^{-1} \left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right)$ with respect to $\cos^{-1} x^2$.

Section D

32. (a) Find: $\int \frac{x \, dx}{(x-1)(x^2+4)}$

OR

(b) Evaluate: $\int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} \, dx$

33. Using integration, find the area of the region bounded by $y = 5x + 4$, $y = 0$, $x = -1$ and $x = 1$.

34. (a) Solve the differential equation $ye^y \, dx = (y^3 + 2xe^y) \, dy$, when $y(0) = 1$.

OR

(b) Find the general solution of the differential equation $(x^3 - 3xy^2) \, dx = (y^3 - 3x^2y) \, dy$.

35. Find the length of the perpendicular drawn from the point $P(1, 2, 3)$ to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also, find the equation of the perpendicular line joining P and the foot of the perpendicular.

36. Case Study - 1

A carpenter needs to design a wooden box in the shape of a cuboid such that the sum of its length and breadth is 3 cm more than its height. Twice of its length, thrice of its breadth and its height add up to 10 cm. Its breadth added to 7 times its height is 1 cm less than 3 times its length.

On the basis of the above information, answer the following questions:

- Write the equations representing the various dimensions and express them as the matrix equation $AX = B$. (1 mark)
- Find if A^{-1} exists. Justify your answer. (1 mark)
- (a) Find A^{-1} . (2 marks)

OR

- (b) Find $A^2 + 7I$. (2 marks)

37. Case Study - 2

An NGO organises a charity event in which they decide to distribute woollen caps to protect children from winter. The caps to be distributed are in three separate boxes, Box I has 30 red caps, Box II has 20 red and 10 green caps, and Box III has 30 green caps. The probability that a Box i is selected and a cap picked out is $\frac{i}{6}$, where $i = 1, 2, 3$.

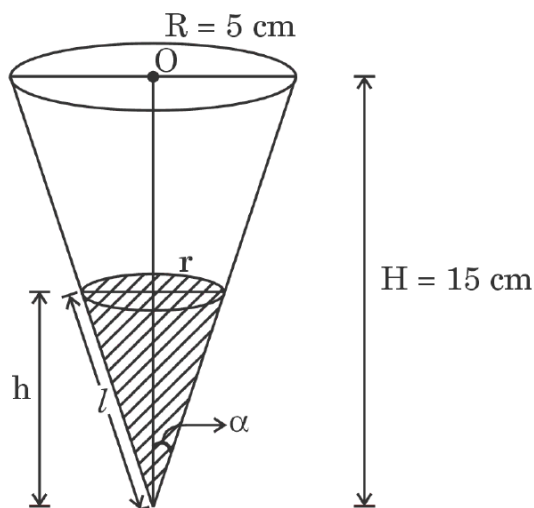
Based on the above information, answer the following questions:

A person selects a cap.

- What is the probability that he selects a red cap? (2 marks)
- If he selects a green cap, what is the probability that the cap has come from Box II? (2 marks)

38. Case Study - 3

At a birthday party, children are being served orange juice in conical cups, as shown in the figure.



Each cup is 15 cm deep and has a radius 5 cm. The juice is being poured into this cup at a rate of $0.1 \text{ cm}^3/\text{s}$.

On the basis of the above information, answer the following questions:

- Establish a relation between the height h of the juice in the cup and radius r of the surface of the juice in the cup, if the semi-vertical angle of the cone is α . (1 mark)
- At what rate is the juice level in the cup rising when the juice is 6 cm deep? (1 mark)
- (a) When the juice is 6 cm deep, then find at what rate is the upper surface area of juice increasing? (2 marks)

OR

- (b) When the juice is 6 cm deep, then find the rate at which the wetted surface area of the cup is increasing. (2 marks)

- End of Question Paper -