

## Question Paper

Conducted by the Central Board of Secondary Education (CBSE)



Time Allowed: 3 Hours

Maximum Marks: 80

### General Instructions

- (i) This question paper contains **38 questions**. All questions are compulsory.
- (ii) Section A: Questions 1 to 18 are multiple choice questions and Questions 19 and 20 are Assertion-Reason questions. Each carries 1 mark.
- (iii) Section B: Questions 21 to 25 carry 2 marks each.
- (iv) Section C: Questions 26 to 31 carry 3 marks each.
- (v) Section D: Questions 32 to 35 carry 5 marks each.
- (vi) Section E: Questions 36 to 38 carry 4 marks each. These are case study questions.
- (vii) Attempt one alternative where an internal choice is provided. Follow the instructions within each question.
- (viii) Use of calculators is not allowed.

## Section A

1. For any square matrix  $A$  with real entries, if  $A + A'$  is a symmetric matrix then:

- (A)  $(A - A')$  cannot be a skew symmetric matrix  
(B)  $(A - A')$  is a skew symmetric matrix  
(C)  $A$  is always a symmetric matrix  
(D)  $A$  is always a skew symmetric matrix

2. A matrix  $B = [b_{ij}]_{m \times m}$  is said to be a diagonal matrix, if:

- (A)  $b_{ij} = 0$ , when  $i = j$   
(B)  $b_{ij} = 1$ , when  $i = j$   
(C)  $b_{ij} = 1$ , when  $i \neq j$   
(D)  $b_{ij} = 0$ , when  $i \neq j$

3. If  $A$  is a non-singular matrix, then which of the following is *not* true?

- (A)  $\text{adj } A$  is singular  
(B)  $(\text{adj } A)^{-1} = \text{adj}(A^{-1})$   
(C)  $|A| \neq 0$   
(D)  $A^{-1}$  exists

4. If  $f(x) = \begin{cases} \frac{x^2 - 4x - 5}{x + 1}, & x \neq -1 \\ k, & x = -1 \end{cases}$  is continuous at  $x = -1$ , then the value of  $k$  is:

- (A) Any real value  
(B) 6  
(C) -1  
(D) -6

5. For the inverse trigonometric functions, which of the following Principal Value Branch is *not* correctly defined?

- (A)  $\tan^{-1} : \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$   
(B)  $\sec^{-1} : \mathbb{R} - (-1, 1) \rightarrow [0, \pi]$   
(C)  $\cot^{-1} : \mathbb{R} \rightarrow (0, \pi)$   
(D)  $\text{cosec}^{-1} : \mathbb{R} - (-1, 1) \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

6. Let  $A = \begin{pmatrix} 0 & 3 & 4 \\ 1 & 0 & 2 \end{pmatrix}$  and  $B = \begin{pmatrix} -3 & 0 & 1 \\ 2 & 4 & 0 \end{pmatrix}$ . If  $A + B + C = O$ , then matrix  $C$  is:

- (A)  $\begin{pmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{pmatrix}$   
(B)  $\begin{pmatrix} 3 & 3 & 5 \\ -3 & -4 & -2 \end{pmatrix}$   
(C)  $\begin{pmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{pmatrix}$   
(D)  $\begin{pmatrix} -3 & -3 & -5 \\ 3 & 4 & 2 \end{pmatrix}$

7. If vectors  $\vec{a} = 3\hat{i} + 2\hat{j} + \lambda\hat{k}$  and  $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ , represent the two strips of the Red Cross sign placed outside a doctor's clinic, then the value of  $\lambda$  is:

- (A) 1  
(B)  $\frac{5}{2}$   
(C)  $\frac{2}{5}$   
(D) 0

8. If  $3P(A) = P(B) = \frac{3}{5}$  and  $P(A|B) = \frac{2}{3}$ , then  $P(A \cup B)$  is:

- (A)  $\frac{3}{5}$   
(B)  $\frac{1}{5}$   
(C)  $\frac{2}{15}$   
(D)  $\frac{2}{5}$

9. If the area of  $\triangle ABC$  with vertices  $A(3, 1)$ ,  $B(-2, 1)$  and  $C(0, k)$  is 5 sq. units, then values of  $k$  are:

- (A) 3, 1  
(B) -1, 3  
(C) -1, 2  
(D) 0, 2

10.  $\int \frac{1}{1 - \cos x} dx$  is equal to:

- (A)  $2 \log \left| \sin \frac{x}{2} \right| + C$   
(B)  $\frac{1}{2} \log |\sin 2x| + C$   
(C)  $\log |1 - \cos 2x| + C$   
(D)  $\log |1 + \cos 2x| + C$

11. If  $\int_0^1 (kx - 4x + 1) dx = 0$ , then the value of  $k$  is:

- (A) 1  
(B) 0  
(C) 2  
(D) -1

12. If position vector  $\vec{p}$  of a point  $(24, n)$  is such that  $|\vec{p}| = 25$ , then the value of  $n$  is:

- (A)  $\pm 49$  (B)  $\pm 5$   
(C)  $\pm 1$  (D)  $\pm 7$

13. The order and degree of the differential

equation  $y = \left(\frac{d^2y}{dx^2}\right)^2 - \lambda \frac{dy}{dx}$  is:

- (A) Order  $= 2$ , Degree  $= 2$   
(B) Order  $= 2$ , Degree  $= 3$   
(C) Order  $= 1$ , Degree  $= 2$   
(D) Order  $= 2$ , Degree  $= 4$

14. The corner points of the feasible region determined by the system of linear constraints are  $(0, 0)$ ,  $(0, 40)$ ,  $(20, 40)$ ,  $(60, 20)$  and  $(60, 0)$ . If the objective function of an LPP is  $Z = 4x + 3y$ , then the maximum value is:

- (A) 200 (B) 300  
(C) 240 (D) 120

15. An ant is observed crawling on a sheet of paper along a straight line given by equation  $y = 2x - 4$ . Area of the surface covered by the ant bounded by  $y$ -axis,  $x$ -axis and  $x = 1$  is:

- (A) 1 sq. unit (B) 3 sq. units  
(C) 2 sq. units (D) 4 sq. units

16. The general solution for the differential

equation  $\frac{dy}{dx} = e^{3x-y}$  is:

- (A)  $3e^y = e^{3x} + C$  (B)  $\log(3x - y) = C$   
(C)  $e^{3x-y} = C$  (D)  $-e^y + 3e^{3x} = C$

17. Derivative of  $\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$ ,  $-\frac{\pi}{4} < x < \frac{\pi}{4}$ , with respect to  $x$  is:

- (A)  $-1$  (B)  $1$   
(C)  $\frac{\pi}{4}$  (D)  $-\frac{\pi}{4}$

18. Absolute minimum value of  $f(x) = (x - 2)^2 + 5$  in the interval  $[-3, 2]$  is:

- (A)  $-3$  (B)  $2$   
(C)  $5$  (D)  $30$

19. **Assertion (A):** A relation  $R$  on the set

$\{1, 2, 3\}$  defined as  $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$  is an equivalence relation.

**Reason (R):** A relation that is reflexive, symmetric and transitive is an equivalence relation.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).  
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).  
(C) Assertion (A) is true, but Reason (R) is false.  
(D) Assertion (A) is false, but Reason (R) is true.

20. **Assertion (A):** Consider a Linear Programming Problem with minimise  $Z = x + 2y$  subject to constraints  $2x + y \geq 3$ ,  $x + 2y \geq 6$ ,  $x, y \geq 0$  which gives minimum  $Z$  at infinitely many points. The corner points of feasible region are  $(0, 3)$  and  $(6, 0)$ .

**Reason (R):** If two corner points produce the same minimum value of the objective function, then every point on the line segment joining the points will give the same minimum value.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).  
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).  
(C) Assertion (A) is true, but Reason (R) is false.  
(D) Assertion (A) is false, but Reason (R) is true.

## Section B

21. Evaluate  $\tan \left[ \cos^{-1} \left( \tan \frac{3\pi}{4} \right) \right]$ .

22. Find the angle between the following pair of lines:

$$\frac{x-2}{3} = \frac{y+5}{2} = \frac{1-z}{-6} \text{ and } \frac{x-7}{1} = \frac{y}{2} = \frac{6-z}{-2}.$$

23. Determine the interval(s) in which  $f(x) = x^3 + \frac{3}{x}$ ,  $x \neq 0$  is increasing.

24. (a) Differentiate  $x^x$  with respect to  $x \log x$ .

OR

(b) If  $y = P \cos(\mu x) + Q \sin(\mu x)$ , then show that  $\frac{d^2 y}{dx^2} + \mu^2 y = 0$ .

25. (a) Three honey bees were found flying along the vectors  $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ ,  $\vec{b} = 4\hat{j} - 2\hat{k}$  and  $\vec{c} = 3\hat{i} + 2\hat{k}$  respectively. Find the value of  $\lambda$  such that the path for  $\vec{a} + \lambda\vec{b}$  is perpendicular to  $\vec{c}$ .

OR

(b) If  $A$ ,  $B$  and  $C$  be three non-collinear points such that  $\vec{AB} = \hat{i} + 2\hat{j} - \hat{k}$  and  $\vec{AC} = 2\hat{i} - 3\hat{j}$ , then find the area of  $\triangle ABC$ .

## Section C

**26.** The volume of a wooden block in the shape of a cube increases at a constant rate as the air becomes moist during the rainy season. Show that the rate of change of its surface area varies inversely as the length of edge of the cube.

**27. (a)** A die is rolled. Consider events:  $A = \{1, 2, 5\}$ ,  $B = \{3, 5\}$ ,  $C = \{2, 3, 4, 5\}$  and hence find:

(i)  $P(A|C)$  and  $P(C|A)$

(ii)  $P(A \cap B|C)$  and  $P(A \cup B|C)$

OR

**(b)** A box contains 6 cards numbered 1 to 6. A student is asked to pick up two cards, one by one after replacement and note down the numbers on the cards. Let  $A$  be the event of getting sum of the numbers on two cards as 10, and  $B$ , the event of a number other than 4 on the first card selected. Find  $P(A \text{ and } B)$  and find whether the events  $A$  and  $B$  are independent events or not.

**28.** Solve the differential equation  $(x - \sin y) dy + \tan y dx = 0$ .

**29. (a)** Let three toys  $A$ ,  $B$  and  $C$  be placed in the same straight line. If the position vectors of  $A$ ,  $B$  and  $C$  are  $55\hat{i} - 2\hat{j}$ ,  $5\hat{i} + 8\hat{j}$  and  $a\hat{i} - 52\hat{j}$  respectively, find the value of ' $a$ '.

OR

**(b)** If  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  are unit vectors, then prove that  $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 9$ .

**30. (a)** Find:  $\int \frac{x - \sin x}{1 - \cos x} dx$ .

OR

**(b)** Evaluate:  $\int_0^2 \frac{1}{\sqrt{x^2 + 2x + 3}} dx$ .

**31.** Solve the following Linear Programming Problem graphically:

Maximize  $Z = \frac{2x}{5} + \frac{3y}{10}$  subject to constraints

$$2x + y \leq 1000$$

$$x + y \leq 800$$

$$x, y \geq 0.$$

## Section D

**32.** Three students  $A$ ,  $B$  and  $C$  go to a book-store to buy art books, story books and puzzle solving books.  $A$  buys one of each type of book for a total of ₹ 21.  $B$  buys 4 art books, 3 story books and 2 puzzle solving books for ₹ 60.  $C$  buys 6 art books, 2 story books and 3 puzzle solving books and pays ₹ 10 more than  $B$ . Use matrix method to find the cost of each type of book.

**33. (a)** If  $y\sqrt{x^2+1} = \log(\sqrt{x^2+1} - x)$ , then show that  $(x^2+1)\frac{dy}{dx} + xy + 1 = 0$ .

OR

**(b)** Find the differential of  $x^{\cot x} + \frac{2x^2 - 3}{2x^2 - x + 2}$  with respect to  $x$ .

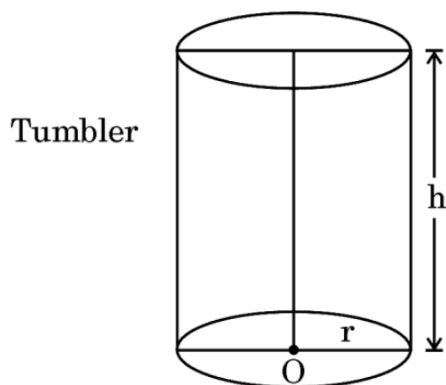
**34.** Sketch the graph defined by  $\left\{(x, y) : \frac{x^2}{25} + \frac{y^2}{25} = 1\right\}$ . Find the area of the region of minor segment cut off by the line  $x = \frac{5}{2}$ , using integration.

**35. (a)** Find the foot of the perpendicular from the point  $(0, 2, 3)$  on the line  $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$  and hence find the length of the perpendicular.

OR

**(b)** Find the value of  $p$  if the shortest distance between the lines  $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$  and  $\vec{r} = (p\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$  is  $\frac{3}{\sqrt{2}}$  units.

36. A company produces cylindrical tumblers, open from the top. Since they want uniformity in the product, they fix the surface area of the tumblers produced.



Based on the above information, answer the following questions:

If for a tumbler,  $V$  is its volume,  $h$  the height and  $r$  the radius of the circular base, then:

- (i) Differentiate its volume with respect to radius of the base, where the surface area is constant. [2]
- (ii) If the company wants to maximize the volume of each tumbler, then establish a relation between its height and the radius of the base. [2]

37. There are three types of vaccines  $A_1, A_2, A_3$ , available in the market to protect the population of the country from spread of certain infection. According to a survey conducted, it was found that 25% of the population was given Vaccine  $A_1$ , 35% of the population was given Vaccine  $A_2$  and 40% of the population was given Vaccine  $A_3$ . The survey also stated that the probabilities that Vaccines  $A_1, A_2$  and  $A_3$  would protect against the infection were 60%, 55% and 50% respectively.

Based on the above information, answer the following questions:

Find the probability that:

- (i) The person taking vaccine  $A_2$  will get infected. [1]
- (ii) If a person is chosen randomly, he/she will be protected from the infection. [1]
- (iii) (a) The person was given Vaccine  $A_1$ , given that the randomly chosen person is infected. [2]

OR

- (iii) (b) The person was given Vaccine  $A_3$ , given that the randomly chosen person is not infected. [2]

**38.** A school wants the students of class XII to do a project on 'Sustainability' keeping the world environment in mind. They select the student participants on the basis of an essay writing competition. 7 students out of 80 are selected for the project and are categorized into two sets such that:

Girl students belong to Set  $A = \{G_1, G_2, G_3, G_4\}$ ,

Boy students belong to Set  $B = \{B_1, B_2, B_3\}$ .

Based on the above information, answer the following questions:

(i) How many relations are possible from Set  $A \rightarrow$  Set  $B$ ? [1]

(ii) Let  $R$  be a relation from  $A \rightarrow B$  such that  $R =$

$\{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$ . Is  $R$  an injective function? Justify your answer. [1]

(iii) (a) Let the relation  $R$  from  $A \rightarrow A$  be such that  $R = \{(x, y), x, y \in A, x \text{ and } y \text{ are students from the same colony in the city}\}$ . Verify if  $R$  is an equivalence relation. [2]

OR

(iii) (b) Verify if any function  $f : B \rightarrow A$  is bijective. Give reason to support your answer. [2]

- End of Question Paper -