

CBSE Class 12 Physics Question Paper 2026 with Solution PDF-Set 55/3/3



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** sections – **Sections A, B, C, D and E**.
- (iii) In **Section A** – Questions no. **1 to 16** are Multiple Choice type questions. Each question carries **1** mark.
- (iv) In **Section B** – Questions no. **17 to 21** are Very Short Answer type questions. Each question carries **2** marks.
- (v) In **Section C** – Questions no. **22 to 28** are Short Answer type questions. Each question carries **3** marks.
- (vi) In **Section D** – Questions no. **29 and 30** are Case Study-based questions. Each question carries **4** marks.
- (vii) In **Section E** – Questions no. **31 to 33** are Long Answer type questions. Each question carries **5** marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculators is **not** allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Section A (16 marks)

1. A plane electromagnetic wave travels through a medium and the magnetic field associated with it is given by $B = 5 \times 10^{-8} \sin(3 \times 10^{10} t - 150 x)$ T, where x is in metres and t is in seconds. The velocity of the wave is: [1 mark]

- (A) $2.0 \times 10^8 \text{ m s}^{-1}$
- (B) $4.5 \times 10^7 \text{ m s}^{-1}$
- (C) $3.5 \times 10^7 \text{ m s}^{-1}$
- (D) $2.5 \times 10^8 \text{ m s}^{-1}$

Correct Answer: (A) $2.0 \times 10^8 \text{ m s}^{-1}$

Solution:

Concept:

- A wave written as $B = B_m \sin(\omega t - kx)$ has angular frequency ω , wave number k , and phase speed $v = \omega/k$.
- The phase $\omega t - kx$ stays constant for a point travelling with the wave; this sign combination describes propagation along the positive x -direction.

Step 1: Identify the phase coefficients Comparing the given expression with the standard travelling-wave equation gives

$$\omega = 3 \times 10^{10} \text{ rad s}^{-1}, \quad k = 150 \text{ rad m}^{-1}.$$

The coefficient $5 \times 10^{-8} \text{ T}$ is the magnetic-field amplitude and does not enter the speed calculation.

Step 2: Divide angular frequency by wave number

$$v = \frac{\omega}{k} = \frac{3 \times 10^{10}}{150} = 2.0 \times 10^8 \text{ m s}^{-1}.$$

This is the value in (A). The values in (B), (C), and (D) do not equal the ratio of the two phase coefficients. The speed need not be the vacuum value because the wave travels through a medium.

Step 3: Conclusion The wave velocity is $2.0 \times 10^8 \text{ m s}^{-1}$ along the positive x -direction. Hence, the correct option is (A).

Quick Tip to Solve: In a sine wave, divide the coefficient of time by the coefficient of position; ignore the amplitude when finding speed.

2. A ray of monochromatic light travelling in air is incident on a glass slab and is partly reflected and partly refracted. Both the reflected and refracted lights will have: [1 mark]

- (A) same wavelength
- (B) same frequency
- (C) same intensity
- (D) same speed

Correct Answer: (B) same frequency

Solution:

Concept:

- The frequency ν of monochromatic light is set by its source and remains unchanged during reflection and refraction at a stationary boundary.
- Wave speed, frequency and wavelength obey $v = \nu\lambda$; a change in medium changes speed and hence wavelength, while frequency stays fixed.

Step 1: Compare the frequencies The reflected and refracted waves arise from the same incident oscillation at the air-glass boundary. They therefore oscillate at the same frequency as the incident wave:

$$\nu_{\text{reflected}} = \nu_{\text{refracted}} = \nu_{\text{incident}}$$

Step 2: Check the remaining properties The reflected light travels in air, whereas the refracted light travels in glass and has a smaller speed. Since $\lambda = v/\nu$, their wavelengths differ as well, ruling out (A) and (D). The incident energy is divided between reflection and transmission; no condition requires equal intensities, so (C) is not generally true.

Step 3: Conclusion Both reflected and refracted lights have the same frequency. Hence, the correct option is (B).

Quick Tip to Solve: At a stationary optical boundary, keep frequency fixed first, then use the new speed to determine the wavelength.

3. When the forward bias voltage in a semiconductor diode is changed from 0.8 V to 1.0 V, the forward current changes by 2.0 mA. The forward bias resistance of the diode will be: [1 mark]

- (A) 200 Ω
- (B) 175 Ω
- (C) 100 Ω
- (D) 125 Ω

Correct Answer: (C) 100 Ω

Solution:

Concept:

- The forward dynamic resistance of a diode over a small operating interval is $r_f = \Delta V / \Delta I$, the change in voltage divided by the corresponding change in current.
- A diode has a nonlinear current-voltage characteristic; its incremental resistance uses differences rather than an isolated voltage-to-current ratio.

Step 1: Calculate the voltage and current changes The given voltages specify the endpoints of the

forward-bias interval. Thus,

$$\Delta V = 1.0 - 0.8 = 0.2 \text{ V}, \quad \Delta I = 2.0 \times 10^{-3} \text{ A}.$$

Convert milliamperes to amperes before evaluating resistance in ohms.

Step 2: Evaluate the incremental resistance

$$r_f = \frac{0.2}{2.0 \times 10^{-3}} = 100 \, \Omega.$$

This matches (C). The resistances in (A), (B), and (D), multiplied by the stated current change, would give voltage changes of 0.400, 0.350, and 0.250 V, respectively, rather than 0.200 V.

Step 3: Conclusion The forward bias resistance over the given interval is $100 \, \Omega$. Hence, the correct option is (C).

Quick Tip to Solve: For two diode operating voltages and a current change, use the voltage difference in the numerator and convert mA to A.

4. A square loop of side L lies in the x - y plane in a magnetic field $\vec{B} = B_0(2\hat{i} + 3\hat{j} + 4\hat{k})$, where B_0 is a constant. The magnetic flux through the loop is: [1 mark]

- (A) $\sqrt{29} B_0 L^2$
- (B) $4B_0 L^2$
- (C) $3B_0 L^2$
- (D) $2B_0 L^2$

Correct Answer: (B) $4B_0 L^2$

Solution:

Concept:

- Magnetic flux through a plane surface in a uniform field is $\Phi_B = \vec{B} \cdot \vec{A}$, where $\vec{A}$ is perpendicular to the surface and has magnitude equal to its area.
- A square of side L has area L^2 . For a surface in the x - y plane, its area vector is along the z -axis.

Step 1: Choose the area vector Taking the surface normal along positive z , write $\vec{A} = L^2\hat{k}$. The field components along $\hat{i}$ and $\hat{j}$ lie within the loop's plane, so they contribute no flux through it.

Step 2: Form the scalar product

$$\Phi_B = B_0(2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot L^2\hat{k} = 4B_0 L^2.$$

Option (A) incorrectly uses the entire field magnitude $\sqrt{29}B_0$, without projecting onto the normal. Options (C) and (D) use the in-plane y - and x -components respectively. Reversing the chosen normal changes the signed flux, while its magnitude remains the same.

Step 3: Conclusion The magnetic flux magnitude is $4B_0 L^2$, in webers when SI units are used. Hence, the correct option is (B).

Quick Tip to Solve: For a loop in the x - y plane, select only the $\hat{k}$ coefficient of the magnetic field and multiply it by the area.

5. An electric dipole with dipole moment $\vec{P} = (2.54 \times 10^{-28} \text{ C} \cdot \text{m})(2.00\hat{i} + 3.00\hat{j})$ is placed in an electric field $\vec{E} = (1000\hat{i}) \text{ N/C}$. An external agent turns the dipole until its electric dipole moment is $\vec{P} = (2.54 \times 10^{-28} \text{ C} \cdot \text{m})(-3.00\hat{i} + 2.00\hat{j})$. The work done by the agent is: [1 mark]

- (A) $2.54 \times 10^{-25} \text{ J}$
 (B) $1.27 \times 10^{-24} \text{ J}$
 (C) $1.02 \times 10^{-23} \text{ J}$
 (D) $2.29 \times 10^{-26} \text{ J}$

Correct Answer: (B) $1.27 \times 10^{-24} \text{ J}$

Solution:

Concept:

- The potential energy of an electric dipole in a uniform electric field is $U = -\vec{P} \cdot \vec{E}$, where $\vec{P}$ is its dipole moment.
- For rotation without a net change in kinetic energy, the external agent's work is $W_{\text{ext}} = U_f - U_i$. The work done by the electric field has the opposite sign.

Step 1: Calculate the initial and final energies Only the x -component of the dipole moment contributes because $\vec{E}$ is along $\hat{i}$. Therefore,

$$U_i = -(2.54 \times 10^{-28})(2.00)(1000) = -5.08 \times 10^{-25} \text{ J},$$

$$U_f = -(2.54 \times 10^{-28})(-3.00)(1000) = +7.62 \times 10^{-25} \text{ J}.$$

The y -components have zero scalar product with the given field.

Step 2: Take the change in potential energy

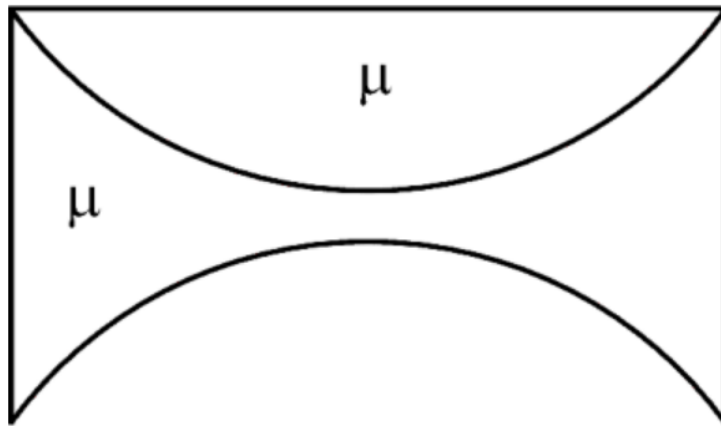
$$W_{\text{ext}} = [7.62 - (-5.08)] \times 10^{-25} = 1.27 \times 10^{-24} \text{ J}.$$

The positive sign shows that the agent supplies energy. The result depends on the change of the x -component from 2.00 to -3.00 , a difference of five times the common factor. Options (A), (C), and (D) do not give this energy difference.

Step 3: Conclusion The work done by the external agent is $+1.27 \times 10^{-24} \text{ J}$. Hence, the correct option is (B).

Quick Tip to Solve: For a field along x , use $W_{\text{ext}} = E_x(P_{ix} - P_{fx})$; the perpendicular dipole components do not contribute.

6. A thin plano-convex lens and a thin equi-concave lens are kept coaxially in contact as shown in the figure. Assuming both the lenses are made of glass of refractive index μ , and R is the radius of curvature of each curved surface, the focal length of the combination is: [1 mark]



- (A) $\frac{R}{\mu - 1}$
 (B) $-\frac{R}{\mu - 1}$
 (C) $\frac{2R}{\mu - 1}$
 (D) $-\frac{2R}{\mu - 1}$

Correct Answer: (B) $-\frac{R}{\mu - 1}$

Solution:

Concept:

- For a thin lens in air, the lens-maker formula is $1/f = (\mu - 1)(1/R_1 - 1/R_2)$, with signed radii R_1, R_2 and refractive index μ .
- Thin lenses in contact have additive powers: $1/F = 1/f_1 + 1/f_2$, where F is the focal length of the combination.

Step 1: Find the two lens powers The plano-convex lens has one plane surface and one convex surface of radius R , so its positive power is $(\mu - 1)/R$. The equi-concave lens has two equally curved concave surfaces, giving negative power $-2(\mu - 1)/R$.

$$\frac{1}{f_1} = \frac{\mu - 1}{R}, \quad \frac{1}{f_2} = -\frac{2(\mu - 1)}{R}.$$

Step 2: Add the powers and invert

$$\frac{1}{F} = \frac{\mu - 1}{R} - \frac{2(\mu - 1)}{R} = -\frac{\mu - 1}{R}, \quad F = -\frac{R}{\mu - 1}.$$

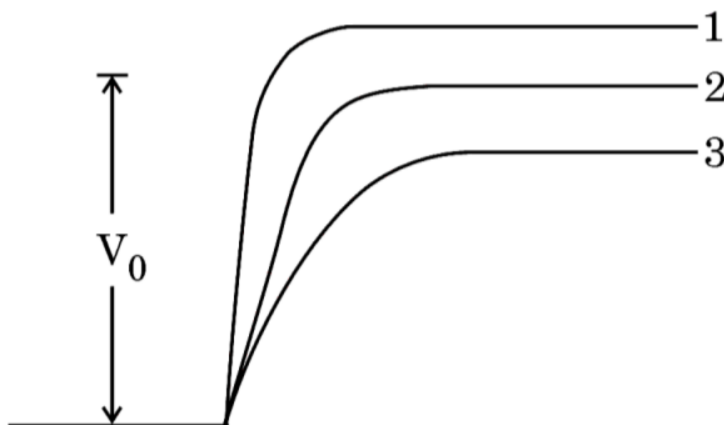
The negative lens is stronger, so the combination diverges; this rules out the positive answers (A) and

(C). Option (D) has an extra factor of two. The common curved interface is between identical glass, consistent with an effective plano-concave lens.

Step 3: Conclusion The focal length of the combination is $F = -R/(\mu - 1)$. Hence, the correct option is (B).

Quick Tip to Solve: Add lens powers, not focal lengths: one convex curvature contributes $+1$ and two concave curvatures contribute -2 in units of $(\mu - 1)/R$.

7. If the given figure, V_0 is the potential barrier across a p-n junction in an unbiased condition. Which of the following statements is correct? [1 mark]



- (A) Curves 1 and 3 both correspond to forward biasing of p-n junction.
(B) Curve 3 corresponds to forward biasing and Curve 1 corresponds to reverse biasing.
(C) Curve 1 corresponds to forward biasing and Curve 3 corresponds to reverse biasing.
(D) Curves 1 and 3 both correspond to reverse biasing.

Correct Answer: (B) Curve 3 corresponds to forward biasing and Curve 1 corresponds to reverse biasing.

Solution:

Concept:

- Forward bias opposes a p-n junction's built-in electric field and lowers its potential barrier from V_0 to approximately $V_0 - V_F$, where V_F is the applied forward voltage.
- Reverse bias reinforces the junction field and raises the barrier to approximately $V_0 + V_R$, where V_R is the magnitude of the reverse voltage.

Step 1: Compare the curves with the unbiased barrier Curve 2 represents the unbiased barrier V_0 . Curve 1 reaches a higher plateau than curve 2, while curve 3 reaches a lower plateau. Thus curve 1 shows an increased barrier and curve 3 a reduced barrier.

Step 2: Identify the corresponding bias conditions A reduced barrier is produced by forward bias, so curve 3 corresponds to forward biasing. The increased barrier of curve 1 corresponds to reverse biasing. Option (C) interchanges these assignments; (A) and (D) incorrectly assign the same bias to

curves whose barrier changes are opposite.

Step 3: Conclusion Curve 3 represents forward biasing and curve 1 represents reverse biasing. Hence, the correct option is (B).

Quick Tip to Solve: Compare each plateau with the unbiased level: a lower junction barrier means forward bias and a higher barrier means reverse bias.

8. If r_1 and r_2 are the radii of atomic nuclei of mass numbers 64 and 27 respectively, then the value of $\left(\frac{r_1}{r_2}\right)$ is: [1 mark]

- (A) 1
- (B) $\frac{4}{3}$
- (C) $\frac{3}{4}$
- (D) $\frac{27}{64}$

Correct Answer: (B) $\frac{4}{3}$

Solution:

Concept:

- The nuclear radius is $r = r_0 A^{1/3}$, where A is the mass number and r_0 is the common nuclear-radius constant.
- Taking a ratio cancels r_0 : radius scales with the cube root of mass number, while nuclear volume scales with mass number.

Step 1: Form the radius ratio The first nucleus has $A_1 = 64$ and the second has $A_2 = 27$. Apply the radius law in the requested order:

$$\frac{r_1}{r_2} = \frac{r_0(64)^{1/3}}{r_0(27)^{1/3}}.$$

Step 2: Evaluate the cube roots and compare the options Since $64 = 4^3$ and $27 = 3^3$, the numerator and denominator reduce to 4 and 3. The first nucleus must be larger, so the ratio exceeds unity. Option (A) would require equal mass numbers; (C) reverses the requested radius ratio; (D) is the inverse mass-number ratio without taking cube roots.

Step 3: Conclusion $r_1/r_2 = 4/3$. Hence, the correct option is (B).

Quick Tip to Solve: For nuclear-radius ratios, take the cube roots of the two mass numbers first and retain their stated order.

9. A ray of yellow light undergoes total internal reflection when it is incident at the interface of two media. This ray is successively replaced by the ray of blue, green and red lights. Which of the following statements is true if the angle of incidence is same in all cases? [1 mark]

- (A) Total internal reflection occurs for blue light only.
- (B) Total internal reflection occurs for red light only.
- (C) Total internal reflection occurs for both green and blue lights.
- (D) None of them will undergo total internal reflection.

Correct Answer: (C) Total internal reflection occurs for both green and blue lights.

Solution:

Concept:

- Total internal reflection requires incidence from the optically denser to the rarer medium and $i > C$, where i is the incidence angle and C the critical angle.
- If μ is the refractive index of the denser medium relative to the rarer one, $\sin C = 1/\mu$. Under the usual normal-dispersion assumption, shorter-wavelength light has a larger μ and a smaller critical angle.

Step 1: Order the critical angles Blue and green have shorter wavelengths than yellow, whereas red has a longer wavelength. The usual dispersion ordering therefore gives

$$C_{\text{blue}} < C_{\text{green}} < C_{\text{yellow}} < C_{\text{red}}.$$

Yellow already undergoes total internal reflection, so the given angle satisfies $i > C_{\text{yellow}}$.

Step 2: Apply the same incidence angle to each colour The same i necessarily exceeds both C_{green} and C_{blue} , so both colours undergo total internal reflection. It need not exceed the larger red critical angle; red is not guaranteed to undergo it. Option (A) wrongly excludes green, (B) wrongly excludes blue and green, and (D) contradicts the smaller critical angles of blue and green.

Step 3: Conclusion Total internal reflection occurs for both green and blue lights. Hence, the correct option is (C).

Quick Tip to Solve: From yellow undergoing total internal reflection, move toward shorter wavelengths: their smaller critical angles also satisfy the same incidence angle.

10. The ratio of the de Broglie wavelengths associated with the electron revolving in the first and third orbits in hydrogen atom is: [1 mark]

- (A) 2
- (B) $\frac{1}{2}$
- (C) 3
- (D) $\frac{1}{3}$

Correct Answer: (D) $\frac{1}{3}$

Solution:

Concept:

- The de Broglie wavelength of an electron is $\lambda = h/p = h/(mv)$, where h is Planck's constant and p its momentum.
- In the Bohr model of hydrogen, $r_n = a_0 n^2$ and the standing-wave condition is $2\pi r_n = n\lambda_n$, where n is the orbit number and a_0 the first-orbit radius.

Step 1: Find the dependence on orbit number Substitute the Bohr radius into the standing-wave condition:

$$\lambda_n = \frac{2\pi r_n}{n} = \frac{2\pi a_0 n^2}{n} = 2\pi a_0 n.$$

Thus, the electron wavelength is proportional to n , even though the orbital radius is proportional to n^2 .

Step 2: Compare the first and third orbits For $n = 1$, the wavelength is $2\pi a_0$; for $n = 3$, it is $6\pi a_0$. The requested ratio is first to third, so the smaller wavelength is the numerator. Option (C) reverses that ratio, while (A) and (B) use a factor of two that does not correspond to the given orbit numbers.

Step 3: Conclusion $\lambda_1/\lambda_3 = 1/3$. Hence, the correct option is (D).

Quick Tip to Solve: For two Bohr orbits of the same hydrogen atom, electron wavelength ratios equal orbit-number ratios, not radius ratios.

11. Radiation of wavelength 200 nm is incident on a photosensitive surface of work function 4.2 eV. The kinetic energy of fastest photoelectrons emitted from this surface will be close to: [1 mark]

- (A) 3.5 eV
- (B) 3.0 eV
- (C) 2.5 eV
- (D) 2.0 eV

Correct Answer: (D) 2.0 eV

Solution:

Concept:

- A photon of wavelength λ has energy $E = hc/\lambda$, where h is Planck's constant and c is the speed of light.
- Einstein's photoelectric equation gives the maximum electron kinetic energy as $K_{\max} = hc/\lambda - \phi$, where ϕ is the surface work function.

Step 1: Calculate the incident photon energy Using $hc \simeq 1240 \text{ eV nm}$ lets us retain the given wavelength in nanometres:

$$E = \frac{1240 \text{ eV nm}}{200 \text{ nm}} = 6.20 \text{ eV}.$$

This exceeds $\phi = 4.2 \text{ eV}$, so photoemission is possible.

Step 2: Subtract the energy needed to escape The fastest electron uses the work-function energy to leave the surface and retains the remaining photon energy as kinetic energy:

$$K_{\text{max}} = 6.20 \text{ eV} - 4.2 \text{ eV} = 2.0 \text{ eV}.$$

Options (A), (B) and (C) all assign more kinetic energy than remains after overcoming the work function, and are therefore inconsistent with energy conservation.

Step 3: Conclusion The kinetic energy of the fastest photoelectrons is approximately 2.0 eV . Hence, the correct option is (D).

Quick Tip to Solve: With wavelength in nanometres, calculate $1240/\lambda$ directly in electronvolts and then subtract the work function in electronvolts.

12. The phase difference between the two superimposing waves that give rise to a bright spot in a Young's double-slit experiment is (n is an integer): [1 mark]

- (A) $2n\pi$
- (B) $2n\pi + \frac{\pi}{4}$
- (C) $2n\pi + \frac{\pi}{2}$
- (D) $2n\pi + \pi$

Correct Answer: (A) $2n\pi$

Solution:

Concept:

- A bright fringe in Young's double-slit experiment is an interference maximum: the two coherent waves arrive in phase and reinforce one another.
- Phase difference δ and path difference Δ are related by $\delta = 2\pi\Delta/\lambda$, where λ is the wavelength. Constructive interference requires $\Delta = n\lambda$, with integer n .

Step 1: Convert the bright-fringe path condition to phase Substitute an integral number of wavelengths into the phase relation:

$$\delta = \frac{2\pi}{\lambda}(n\lambda) = 2n\pi.$$

Each complete 2π cycle leaves the waves in phase; $n = 0$ gives the central bright fringe.

Step 2: Test the remaining phase differences The intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$, so it is maximum when $\cos \delta = 1$. The extra $\pi/4$ in (B) and $\pi/2$ in (C) give intermediate intensities, not bright-fringe maxima. The extra π in (D) gives $\cos \delta = -1$, the destructive-interference condition and

a dark fringe for equal slit intensities.

Step 3: Conclusion The required phase difference is $2n\pi$, where n is an integer. Hence, the correct option is (A).

Quick Tip to Solve: Reduce each proposed phase difference modulo 2π : a bright-fringe maximum has zero remainder, whereas a dark fringe has remainder π .

13. Assertion (A): Nuclear forces are always attractive.

Reason (R): The nuclear force between protons and neutrons in a nucleus is a weak force.

Select the correct answer from the codes (A), (B), (C) and (D) given below:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (D) Both Assertion (A) and Reason (R) are false.

Solution:

Concept:

- The nuclear force is a very strong, short-range interaction between nucleons; its attractive part binds protons and neutrons into a nucleus.
- The force depends on nucleon separation: it is attractive at ordinary nuclear separations but strongly repulsive at very small separations. This repulsive core prevents the nucleus from collapsing.

Step 1: Evaluate the assertion Assertion (A) says that nuclear forces are *always* attractive. The repulsive interaction at very small nucleon separations is a counterexample. Nuclear attraction binds the nucleus, but it does not describe the force at every separation. Thus Assertion (A) is false.

Step 2: Evaluate the reason Reason (R) describes the binding force between protons and neutrons as weak. This nuclear binding force is a manifestation of the strong interaction, sufficiently strong at nuclear distances to bind nucleons despite proton-proton electrical repulsion. The weak interaction is a different interaction, involved in processes such as beta decay. Thus Reason (R) is false.

Step 3: Check the explanatory relation A false reason cannot explain the false claim that the nuclear force is always attractive. Codes (A) and (B) require both statements to be true, and code (C) requires a true assertion; none matches the two truth values found here.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: Test the word “always” against the nuclear repulsive core, and distinguish the strong binding interaction from the weak interaction responsible for beta decay.

14. Assertion (A): Photoelectric current depends upon the intensity of the incident radiation.

Reason (R): Stopping potential is independent of the intensity of the incident radiation.

Select the correct answer from the codes (A), (B), (C) and (D) given below:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- At a fixed frequency above threshold, greater incident intensity supplies more photons per unit time and hence more emitted electrons per unit time. Photoelectric current therefore increases with intensity for fixed collection conditions.
- The stopping potential V_0 obeys $eV_0 = K_{\max} = h\nu - \phi$, where e is the elementary charge, ν the frequency and ϕ the work function. It measures the maximum energy per electron.

Step 1: Evaluate the assertion For the same frequency and photosensitive surface, higher intensity increases the rate at which photons strike the surface. More electrons are emitted and collected per second, so the photocurrent depends on intensity. Assertion (A) is true under the usual photoelectric experimental conditions.

Step 2: Evaluate the reason Increasing intensity at fixed frequency does not change the energy $h\nu$ of an individual photon. Thus $K_{\max}$ and the retarding potential required to stop the fastest electrons remain unchanged. Reason (R), that stopping potential is independent of intensity, is true.

Step 3: Check whether the reason explains the assertion The reason concerns the maximum energy of each emitted electron. The assertion concerns the number of electrons collected per unit time, which is explained by photon flux. Therefore, the true reason does not explain the assertion. Code (A) incorrectly asserts an explanatory link, while codes (C) and (D) incorrectly label at least one true statement false.

Step 4: Conclusion Both statements are true, but the reason is not the correct explanation of the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Separate electron count from electron energy: intensity controls photocurrent, while frequency controls stopping potential for a fixed surface.

15. Assertion (A): In a Wheatstone bridge circuit, if we interchange the position of the cell and the galvanometer, the balance condition $\frac{P}{Q} = \frac{R}{S}$ remains unchanged.

Reason (R): $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{Q}{S} = \frac{P}{R}$ so balance condition remains same.

Select the correct answer from the codes (A), (B), (C) and (D) given below:

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- A Wheatstone bridge is balanced when the galvanometer junctions have equal potential and no current flows through the galvanometer.
- For bridge arms P, Q, R, S , balance requires $P/Q = R/S$, or equivalently $PS = QR$.

Step 1: Test the assertion Initially, the two potential-divider branches contain P, Q and R, S . After exchanging the cell and galvanometer diagonals, the divider branches contain P, R and Q, S . Zero galvanometer current then requires $P/R = Q/S$. This is again $PS = QR$, so the same resistor values balance the bridge. Assertion (A) is true.

Step 2: Test the reason Cross-multiplication gives

$$\frac{P}{Q} = \frac{R}{S} \Rightarrow PS = QR \Rightarrow \frac{Q}{S} = \frac{P}{R}.$$

Thus the algebraic equality stated in Reason (R) is true.

Step 3: Check the explanatory link The transformed ratio is precisely the balance condition after the two diagonals are interchanged. The reason therefore explains the assertion. Option (B) wrongly denies this link, while (C) and (D) wrongly classify true statements as false.

Step 4: Conclusion Both Assertion (A) and Reason (R) are true, and Reason (R) correctly explains Assertion (A). Hence, the correct option is (A).

Quick Tip to Solve: Reduce either bridge ratio to $PS = QR$; the same cross-product condition survives interchange of the cell and galvanometer.

16. Assertion (A): The cylindrical soft iron core in a moving coil galvanometer only makes the magnetic field radial and does not affect the strength of the magnetic field.

Reason (R): In a moving coil galvanometer, the plane of the coil is always perpendicular to the magnetic field.

Select the correct answer from the codes (A), (B), (C) and (D) given below:

[1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (D) Both Assertion (A) and Reason (R) are false.

Solution:

Concept:

- Soft iron has high magnetic permeability. In a moving-coil galvanometer, its cylindrical core strengthens the magnetic field and, together with the shaped pole pieces, makes the field radial.
- The magnetic torque is $\tau = NIAB \sin \theta$, where θ is the angle between the coil's normal and the magnetic field, N is the number of turns, I the current and A the coil area.

Step 1: Test the assertion The core increases the magnetic field strength because of its high permeability. Thus it does more than help produce a radial field. The words “only” and “does not affect the strength” make Assertion (A) false.

Step 2: Test the reason In the radial-field arrangement, the magnetic field lies in the plane of the coil at every operating deflection. The coil's normal is therefore perpendicular to the field, so $\theta = 90^\circ$ and $\tau = NIAB$. The reason confuses the plane with its normal: the plane is parallel to the field, not perpendicular. Reason (R) is false.

Step 3: Assess the explanation and the codes A false reason cannot explain the stated assertion. Both statements are false, which excludes (A) and (B), where both would be true, and (C), where the assertion would be true.

Step 4: Conclusion Both the assertion and the reason are false. Hence, the correct option is (D).

Quick Tip to Solve: For a radial-field galvanometer, remember: field in the coil's plane, normal at 90° , and soft iron strengthens the field.

Section B (10 marks)

17. Write any two points of difference between intrinsic and extrinsic semiconductors.

[2 marks]

Solution:

Concept:

- An intrinsic semiconductor is a pure semiconductor; an extrinsic semiconductor is obtained by deliberately adding a small, controlled amount of suitable impurity to the pure material. This addition is called doping.
- Let n and p be the electron and hole number densities. Thermal generation in an intrinsic semiconductor produces electron-hole pairs, so $n = p$.

Step 1: Compare purity and preparation First difference: An intrinsic semiconductor is chemically pure silicon or germanium, without intentional doping. An extrinsic semiconductor is doped: a pentavalent donor produces an n-type material, while a trivalent acceptor produces a p-type material. Thus purity and controlled impurity addition distinguish their preparation.

Step 2: Compare charge-carrier concentrations Second difference: In an intrinsic semiconductor, the electron and hole concentrations are equal, $n = p = n_i$, where n_i is the intrinsic carrier concentration. In an extrinsic semiconductor, these concentrations are unequal under normal doped operation: $n > p$ in n-type material, and $p > n$ in p-type material. Electrons are the majority carriers in n-type material; holes are the majority carriers in p-type material.

Step 3: Conclusion Intrinsic semiconductors are pure and have equal electron and hole concentrations; extrinsic semiconductors are deliberately doped and have majority and minority carriers with unequal concentrations.

Quick Tip to Solve: Give paired contrasts under the same headings: purity first, then $n = p$ versus majority/minority carriers.

18. Explain the terms mass defect and binding energy. How are they related?

[2 marks]

Solution:

Concept:

- A bound nucleus has a smaller rest mass than the sum of the rest masses of its constituent free protons and neutrons.
- Einstein's mass-energy equivalence relates a mass difference Δm to energy Δmc^2 , where c is the speed of light in vacuum.

Step 1: Define the mass defect The mass defect of a nucleus is the difference between the sum of the masses of its free constituent nucleons and the actual nuclear mass. For a nucleus with mass number A , atomic number Z and nuclear mass M ,

$$\Delta m = Zm_p + (A - Z)m_n - M.$$

Here m_p and m_n are the free proton and neutron masses, respectively; Z protons and $A - Z$ neutrons make up the nucleus. The mass M is the nuclear mass, excluding orbital electrons.

Step 2: Define binding energy and connect the terms The binding energy B is the minimum energy required to separate a nucleus completely into its constituent free protons and neutrons. The same

amount of energy is released when those free nucleons combine to form the nucleus. This released energy corresponds to the reduction in rest mass, hence

$$B = \Delta m c^2.$$

If Δm is measured in kilograms and c in m s^{-1} , then B is obtained in joules.

Step 3: Conclusion Mass defect is the shortfall of nuclear mass relative to the total free-nucleon mass. Binding energy is the energy needed to separate the nucleus into free nucleons, and $B = \Delta m c^2$.

Quick Tip to Solve: Subtract nuclear mass from the sum of free-nucleon masses; this keeps Δm and the required separation energy positive.

19. Light of frequency 5.0×10^{14} Hz is incident on a metal surface. If the maximum speed of photoelectrons emitted is $6.63 \times 10^5 \text{ m s}^{-1}$, calculate the threshold frequency for the surface. [2 marks]

Solution:

Concept:

- Einstein's photoelectric equation is $h\nu = \phi + K_{\text{max}}$, where h is Planck's constant, ν the incident frequency, ϕ the work function and K_{max} the maximum photoelectron kinetic energy.
- At the threshold frequency ν_0 , electrons just escape with zero maximum kinetic energy, so $\phi = h\nu_0$. Also, $K_{\text{max}} = \frac{1}{2}m_e v_{\text{max}}^2$, where m_e is the electron mass.

Step 1: Calculate the maximum kinetic energy Use $m_e = 9.1 \times 10^{-31} \text{ kg}$ and $v_{\text{max}} = 6.63 \times 10^5 \text{ m s}^{-1}$:

$$\begin{aligned} K_{\text{max}} &= \frac{1}{2}(9.1 \times 10^{-31})(6.63 \times 10^5)^2 \\ &= 2.00004 \times 10^{-19} \text{ J.} \end{aligned}$$

This energy is the part of the photon energy left after overcoming the metal's work function.

Step 2: Obtain the threshold frequency Substitute $\phi = h\nu_0$ in Einstein's equation and rearrange. With $h = 6.63 \times 10^{-34} \text{ J s}$ and $\nu = 5.0 \times 10^{14} \text{ Hz}$,

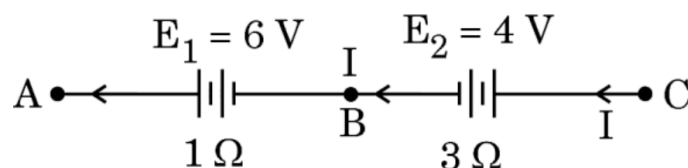
$$\begin{aligned} \nu_0 &= \nu - \frac{K_{\text{max}}}{h} \\ &= 5.0 \times 10^{14} - \frac{2.00004 \times 10^{-19}}{6.63 \times 10^{-34}} \\ &= (5.0 - 3.01665) \times 10^{14} \text{ Hz} \\ &= 1.98335 \times 10^{14} \text{ Hz.} \end{aligned}$$

As a check, the threshold lies below the incident frequency, consistent with the observed emission.

Step 3: Conclusion The threshold frequency is approximately $\nu_0 = 2.0 \times 10^{14} \text{ Hz}$, rounded to two significant figures.

Quick Tip to Solve: Use $\nu_0 = \nu - m_e v_{\text{max}}^2 / (2h)$: square the speed, then subtract the kinetic-energy frequency from the incident frequency.

20(a). In the given figure, a steady current I flows through the circuit when points A and C are connected by a wire of negligible resistance. Find the potential difference between points B and C . [2 marks]



Solution:

Concept:

- Kirchhoff's loop rule states that the algebraic sum of potential changes round a closed circuit is zero. Opposing cells supply a net emf equal to the difference of their emfs.
- A cell being charged has terminal voltage $V = E + Ir$, where E is its emf, r its internal resistance and I the current entering its positive terminal.

Step 1: Determine the loop current from the cell polarities The long plate of the 6 V cell is towards A , whereas the long plate of the 4 V cell is towards C . Thus the cells oppose one another. Joining A and C with an ideal wire gives $V_A = V_C$. The stronger cell drives current in the direction shown, from C through B to A . Applying the loop rule gives

$$6 - 4 - I(1 + 3) = 0, \quad I = \frac{2}{4} = 0.5 \text{ A.}$$

Step 2: Find the potential change from C to B Across the 4 V cell, the current enters the positive terminal at the C side, so this cell is being charged. Moving from C to B , the emf produces a fall of 4 V and the internal resistance produces a further fall of Ir_2 . Therefore,

$$V_B - V_C = -E_2 - Ir_2 = -4 - (0.5)(3) = -5.5 \text{ V.}$$

As a check, the discharging 6 V cell has terminal voltage $V_A - V_B = 6 - (0.5)(1) = 5.5 \text{ V}$, consistent with $V_A = V_C$.

Step 3: Conclusion The potential difference has magnitude 5.5 V, with C at the higher potential:

$$V_B - V_C = -5.5 \text{ V.}$$

Quick Tip to Solve: Mark each cell's positive plate first. Here current enters the positive terminal of the weaker cell, so its terminal voltage is $E + Ir$.

20(b). A battery of emf 21 V and internal resistance $3\ \Omega$ is connected to a resistor. If the current in the circuit is 3 A, find:

(i) the resistance of the resistor.

(ii) the terminal voltage of the battery.

[2 marks]

Solution:

Concept:

- For a battery of emf E and internal resistance r supplying current I through an external resistor R , the total circuit resistance is $R + r$, so $E = I(R + r)$.
- The terminal voltage of a discharging battery is $V = E - Ir$. It also equals IR , the potential difference across the external resistor.

Step 1: Calculate the external resistance The given values are $E = 21\text{ V}$, $r = 3\ \Omega$ and $I = 3\text{ A}$. The emf drives current through both the external and internal resistances in series. Hence, for part (i),

$$R + r = \frac{E}{I} = \frac{21}{3} = 7\ \Omega, \quad R = 7 - 3 = 4\ \Omega.$$

The value $7\ \Omega$ is the total resistance; the requested resistor excludes the battery's internal resistance.

Step 2: Calculate the terminal voltage For part (ii), the battery supplies energy to the circuit and loses a potential Ir inside itself. The voltage available across its terminals is therefore

$$V = E - Ir = 21 - (3)(3) = 12\text{ V}.$$

The same result follows from the external resistor: $V = IR = (3)(4) = 12\text{ V}$. Thus the emf is shared between a 9 V internal drop and the terminal voltage.

Step 3: Conclusion (i) The resistance of the resistor is $4\ \Omega$. (ii) The terminal voltage of the battery is 12 V .

Quick Tip to Solve: First subtract the internal resistance from E/I ; then check the terminal voltage by both $E - Ir$ and IR .

21. In Young's double slit experiment, the central maximum is bright in the interference pattern obtained on a screen. What will happen when:

(a) light waves emitted out of slits S_1 and S_2 have an initial phase difference of π radian?

(b) one of the slits is closed?

Justify your answers.

[2 marks]

Solution:

Concept:

- In Young's double slit experiment, the total phase difference is the initial phase difference plus the phase difference due to the path difference. At the geometrical centre, the path difference is zero.
- For equal slit intensities I_0 , the resultant intensity is $I = 4I_0 \cos^2(\delta/2)$, where δ is the total phase difference. Interference requires two coherent waves to overlap.

Step 1: Introduce the initial phase difference For part (a), the waves reaching the original central

position have zero path difference, but now have an initial phase difference of π . Their displacements are opposite, so they interfere destructively:

$$I_{\text{centre}} = 4I_0 \cos^2\left(\frac{\pi}{2}\right) = 0.$$

The original central bright fringe becomes dark. Adding π to the phase at every point interchanges the positions of bright and dark fringes. Thus the pattern shifts by half a fringe width; its fringe width is unchanged.

Step 2: Close one slit For part (b), only one wave reaches each screen point, so the interference term from the two slits disappears. Consequently, the alternate bright and dark interference fringes vanish. For identical slits, the intensity at the centre is now I_0 , one-quarter of the original two-slit central maximum $4I_0$. With finite-width slits, the remaining illumination has the single-slit diffraction envelope, rather than the two-slit interference fringes.

Step 3: Conclusion (a) The original central maximum becomes a dark fringe because the total phase difference there is π . (b) Closing either slit removes the two-slit interference pattern because only one wave remains.

Quick Tip to Solve: At the centre, set the path difference to zero and use the initial phase directly. Closing a slit removes the second interfering wave.

Section C (21 marks)

22. (a) Depict the variation of electric field ($\vec{E}$) and magnetic field ($\vec{B}$) with respect to the direction of propagation of an electromagnetic wave. Write their two important characteristics.

(b) Show that $\frac{1}{\sqrt{\epsilon_0\mu_0}}$ gives the velocity of an electromagnetic wave in free space.

[3 marks]

Solution:

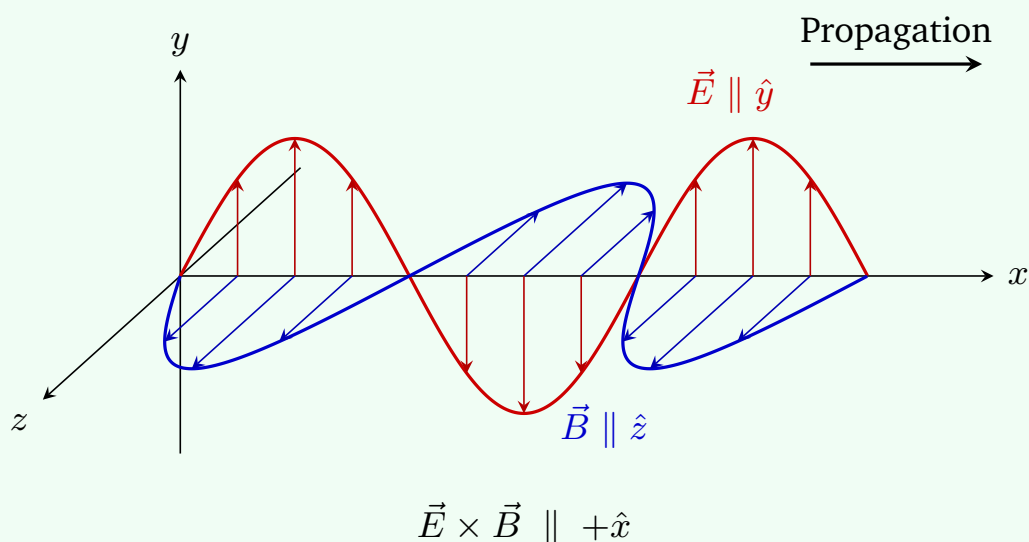
Concept:

- An electromagnetic wave consists of time-varying electric and magnetic fields. In free space these fields sustain one another through Faraday's law and the Ampere-Maxwell law.
- The propagation direction is along $\vec{E} \times \vec{B}$. The phase speed of a sinusoidal wave is $v = \omega/k$, where ω is angular frequency and $k = 2\pi/\lambda$ is wave number.

Step 1: Depict the field variation For part (a), choose propagation along $+x$, the electric field along y , and the magnetic field along z . One possible plane electromagnetic wave is

$$\begin{aligned}\vec{E}(x, t) &= E_0 \sin(kx - \omega t) \hat{y}, \\ \vec{B}(x, t) &= B_0 \sin(kx - \omega t) \hat{z}.\end{aligned}$$

Here E_0 and B_0 are the respective field amplitudes. The sketch shows their sinusoidal spatial variation at a fixed instant. The two field scales are independent.



A snapshot of the perpendicular, in-phase fields; the wave travels along $+x$.

Step 2: State two important characteristics First, the fields are mutually perpendicular and each is perpendicular to the propagation direction: $\vec{E} \perp \vec{B} \perp \hat{x}$. The wave is therefore transverse. Second, the fields oscillate in phase with the same frequency and wavelength: at any position their maxima, minima and zeros occur together. In free space their amplitudes satisfy $E_0 = cB_0$.

Step 3: Establish the free-space wave speed For part (b), applying Faraday's law and the Ampere-Maxwell law to these sinusoidal fields gives, respectively,

$$kE_0 = \omega B_0, \quad kB_0 = \mu_0 \varepsilon_0 \omega E_0.$$

Multiplying and cancelling the nonzero amplitudes yields $k^2 = \mu_0 \varepsilon_0 \omega^2$. Therefore,

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}.$$

Using $\varepsilon_0 = 8.854 \times 10^{-12} \text{ F m}^{-1}$ and $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$,

$$v = \frac{1}{\sqrt{(8.854 \times 10^{-12})(4\pi \times 10^{-7})}} \simeq 3.00 \times 10^8 \text{ m s}^{-1} = c.$$

Also, $[\varepsilon_0 \mu_0] = \text{s}^2 \text{m}^{-2}$, so the reciprocal square root has the required unit of speed.

Step 4: Conclusion The electric and magnetic fields are transverse and in phase, as depicted. Their free-space propagation speed is $c = 1/\sqrt{\varepsilon_0 \mu_0} \simeq 3.00 \times 10^8 \text{ m s}^{-1}$, directed along $\vec{E} \times \vec{B}$.

Quick Tip to Solve: Use $\hat{y} \times \hat{z} = \hat{x}$ to fix the propagation direction. Draw both fields with the same phase, even though their amplitudes use different scales.

23. A ray of light is incident at angle of 45° on one face of a prism with an equilateral triangular base. If it passes symmetrically through the prism, find the:

- (a) angle of minimum deviation for the prism, and
(b) refractive index of the material of the prism.

[3 marks]

Solution:

Concept:

- For a prism of angle A , the internal refraction angles satisfy $r_1 + r_2 = A$, and the total deviation is $\delta = i + e - A$. Here i and e are the angles of incidence and emergence measured from the normals.
- Symmetric passage through a prism in air is the minimum-deviation condition: $i = e$ and $r_1 = r_2 = A/2$. Snell's law at the first face gives the refractive index $\mu = \sin i / \sin r_1$.

Step 1: Identify the prism angle and use symmetry The cross-section is an equilateral triangle, so the angle between the two refracting faces is $A = 60^\circ$. The given incidence angle is $i = 45^\circ$. Since the ray passes symmetrically through the prism, the emergence angle equals the incidence angle and the two internal refraction angles are equal:

$$e = 45^\circ, \quad r_1 = r_2 = \frac{60^\circ}{2} = 30^\circ.$$

It is the internal angles that add to the prism angle; the incidence angle is measured outside the prism and is not $A/2$.

Step 2: Calculate the minimum deviation for part (a) Add the deviations at the two surfaces. Each surface changes the direction by the difference between the external and internal angles. Therefore,

$$\begin{aligned} \delta_{\min} &= (i - r_1) + (e - r_2) \\ &= i + e - A \\ &= 45^\circ + 45^\circ - 60^\circ = 30^\circ. \end{aligned}$$

The subscript indicates minimum deviation because the path is symmetric, as specified in the question.

Step 3: Apply Snell's law for part (b) Taking the surrounding medium as air, with refractive index approximately unity, use the first refracting face:

$$\begin{aligned} \mu &= \frac{\sin i}{\sin r_1} = \frac{\sin 45^\circ}{\sin 30^\circ} \\ &= \frac{1/\sqrt{2}}{1/2} = \sqrt{2} \approx 1.414. \end{aligned}$$

This also agrees with the minimum-deviation formula $\mu = \sin[(A + \delta_{\min})/2] / \sin(A/2)$. A refractive index is a ratio, so it has no unit.

Step 4: Conclusion (a) The angle of minimum deviation is 30° . (b) The refractive index of the prism material is $\mu = \sqrt{2} \approx 1.414$.

Quick Tip to Solve: The phrase “passes symmetrically” immediately gives $e = i$ and $r = A/2$; then use $\delta_{\min} = 2i - A$ and $\mu = \sin i / \sin(A/2)$.

24. With the help of circuit diagrams, briefly explain the forward biasing and the reverse biasing of a p-n junction diode. [3 marks]

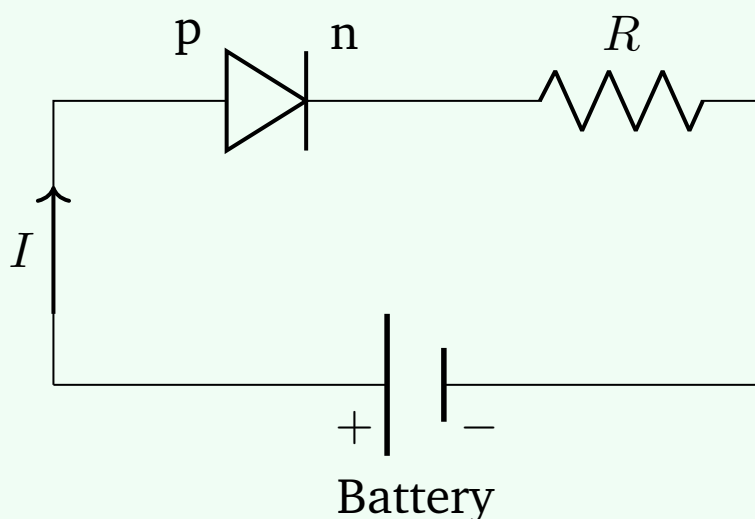
Solution:

Concept:

- A p-n junction has a depletion region containing immobile ions and a built-in potential barrier. Biasing means applying an external dc voltage across its p and n sides.
- Forward bias opposes the junction barrier and promotes majority-carrier diffusion. Reverse bias reinforces the barrier; before breakdown, only a small current due to minority carriers flows.

Step 1: Connect and explain forward bias Connect the p side (anode) to the positive terminal of the battery and the n side (cathode, shown by the bar of the diode symbol) to the negative terminal. A series resistor R limits the current. The applied electric field opposes the built-in field, reducing the potential barrier and narrowing the depletion region.

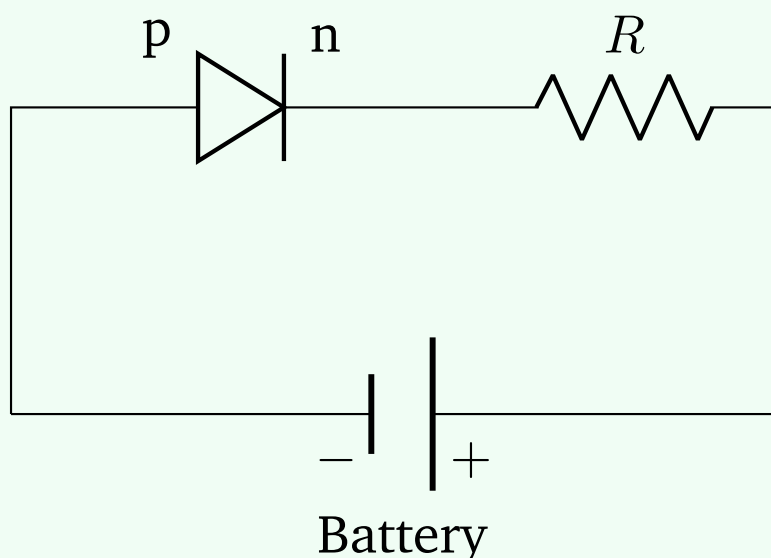
Holes from the p region and electrons from the n region are pushed towards the junction. Majority carriers can then cross it much more easily. An appreciable forward current flows once the applied forward voltage is sufficiently large; the conventional current through the diode is from p to n. Thus, in forward conduction the junction offers low resistance.



Forward bias: p is connected to positive and n to negative; I is conventional current.

Step 2: Reverse the connections and explain reverse bias Connect the p side to the negative terminal and the n side to the positive terminal of the battery. The applied electric field now reinforces the built-in field, increasing the potential barrier and widening the depletion region. Majority carriers are pulled away from the junction, so their current is strongly suppressed and the junction offers very high resistance.

A very small reverse current still flows because thermally generated minority carriers are swept across the junction: electrons on the p side move towards the n side, and holes on the n side move towards the p side. Its conventional direction through the diode is from n to p. This is the reverse saturation current; the explanation applies below the reverse breakdown voltage.



Reverse bias: p is connected to negative and n to positive.

Step 3: Conclusion Forward bias means $p \rightarrow +$ and $n \rightarrow -$: a narrower depletion layer and appreciable majority-carrier current. Reverse bias means $p \rightarrow -$ and $n \rightarrow +$: a wider depletion layer and only a small minority-carrier current before breakdown.

Quick Tip to Solve: Identify the cathode by the diode's bar: connecting that n side to the battery's negative terminal gives forward bias, while connecting it to positive gives reverse bias.

25. (a) Establish the relation between drift velocity of electrons (v_d) and electric current (I) in a conductor.

(b) How is v_d affected when the length of the conductor is doubled, keeping the voltage applied across the conductor constant? [3 marks]

Solution:

Concept:

- Electric current is the rate of charge flow, $I = \Delta Q / \Delta t$. Let n be the number of free electrons per unit volume, A the conductor's cross-sectional area, e the magnitude of the electronic charge and v_d the magnitude of electron drift velocity.
- In the relaxation-time model, $v_d = eE\tau/m$, where E is the electric field, τ is the mean relaxation time and m is the electron mass. For a uniform conductor of length L across voltage V , $E = V/L$.

Step 1: Count the charge crossing a section for part (a) Consider a uniform conductor carrying a steady current. In a time interval Δt , the net drift displacement of electrons is $v_d \Delta t$. The volume contributing to the net transport across a chosen cross-section is therefore $Av_d \Delta t$. With n free electrons in each unit volume, the number transported and the magnitude of their charge are

$$N = nAv_d \Delta t, \quad \Delta Q = Ne = nAev_d \Delta t.$$

Random thermal motion alone produces no net current; the small directed drift produces this net charge transfer.

Step 2: Obtain the current-drift relation Divide the charge magnitude by the time interval:

$$I = \frac{\Delta Q}{\Delta t} = \frac{nAev_d\Delta t}{\Delta t} = nAev_d.$$

Thus the current is proportional to the drift speed for fixed n and A . This is a relation between magnitudes. Electrons move opposite to the conventional current, so in vector notation the current density is $\vec{J} = -ne\vec{v}_d$, where $\vec{v}_d$ is the electron drift-velocity vector.

Step 3: Compare the drift speeds for part (b) At unchanged temperature and for the same material, τ remains constant. Substituting $E = V/L$ into the drift-speed expression gives

$$v_d = \frac{e\tau V}{mL}.$$

The voltage V is held fixed, so v_d is inversely proportional to L . On doubling the conductor's length, the field and the drift speed become

$$E' = \frac{V}{2L} = \frac{E}{2}, \quad v'_d = \frac{e\tau V}{2mL} = \frac{v_d}{2}.$$

The polarity has not changed, so only the magnitude decreases; the drift direction remains the same.

Step 4: Conclusion (a) The required relation is $I = nAev_d$, with conventional current opposite to electron drift. (b) At fixed applied voltage and unchanged material and temperature, doubling the length halves the drift speed: $v'_d = v_d/2$.

Quick Tip to Solve: For a change in conductor length at fixed voltage, write $E = V/L$ first and then use $v_d \propto E$; fixed voltage does not mean fixed electric field.

26(a). A circular coil of 30 turns and radius 8.0 cm carrying a current of 6 A is suspended vertically in a uniform horizontal magnetic field of 1.0 T. The field lines make an angle of 30° with the plane of the coil. Calculate the magnitude of the external torque that must be applied to prevent the coil from turning. What would happen if the circular coil is replaced by a planar coil of irregular shape that encloses the same area, keeping other parameters unchanged? [3 marks]

Solution:

Concept:

- A planar coil of N turns, current I and area A has magnetic moment $\mu = NIA\hat{n}$, where $\hat{n}$ is normal to its plane.
- In a uniform magnetic field $\mathbf{B}$, the torque is $\tau_{\text{mag}} = \mu \times \mathbf{B}$, with magnitude $NIAB \sin \theta$. Here θ is the angle between the coil's normal and the field.
- Rotational equilibrium requires zero net torque, so the applied external torque must be equal in magnitude and opposite in direction to the magnetic torque.

Step 1: Convert the radius and identify the correct angle The radius is $r = 8.0 \text{ cm} = 0.080 \text{ m}$, so the area enclosed by each turn is

$$A = \pi r^2 = \pi(0.080)^2 = 0.0064\pi \text{ m}^2.$$

The given 30° is measured from the plane of the coil. Since the area vector is perpendicular to that

plane, the angle needed in the torque formula is $\theta = 90^\circ - 30^\circ = 60^\circ$.

Step 2: Calculate the balancing external torque Substitute $N = 30$, $I = 6 \text{ A}$, $B = 1.0 \text{ T}$, the area and the normal-field angle:

$$\begin{aligned} |\tau_{\text{ext}}| &= |\tau_{\text{mag}}| = NIAB \sin 60^\circ \\ &= 30(6)(0.0064\pi)(1.0) \frac{\sqrt{3}}{2} \\ &\simeq 3.13 \text{ N m}. \end{aligned}$$

The external torque acts against the field's tendency to rotate the coil's magnetic moment towards the field. Thus $\tau_{\text{ext}} = -\tau_{\text{mag}}$.

Step 3: Examine a planar coil of irregular shape For any planar closed coil in a uniform magnetic field, the magnetic moment depends on its enclosed area, number of turns and current; it does not depend on the outline of the coil. With the same area, orientation and other parameters, NIA and θ remain unchanged. Consequently, its magnetic torque and the required balancing external torque both retain their previous values.

Step 4: Conclusion The required external torque has magnitude 3.13 N m , opposite to the magnetic torque. Replacing the circular coil by the specified irregular planar coil leaves this torque unchanged.

Quick Tip to Solve: When the field's angle is given with the coil's plane, use $NIAB \cos \alpha$; here $\alpha = 30^\circ$. Use the enclosed area to compare different planar shapes.

OR

26(b). An alpha particle (mass $6.4 \times 10^{-27} \text{ kg}$ and charge $3.2 \times 10^{-19} \text{ C}$) having 8.0 MeV energy, enters a region of a uniform magnetic field of 0.5 T . If the field is directed perpendicular to the velocity of the particle, find the radius of the circular path described by the particle. Mention the condition under which the particle in this region (i) describes a helical path, and (ii) goes straight undeviated.

[3 marks]

Solution:

Concept:

- A charged particle moving in a magnetic field experiences $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. For perpendicular velocity and field, $|F| = |q|vB$ supplies the centripetal force mv^2/r .
- With kinetic energy $K = \frac{1}{2}mv^2$, the momentum is $mv = \sqrt{2mK}$, giving the circular-path radius $r = \sqrt{2mK}/(|q|B)$.
- A magnetic field changes the direction of motion but does no work. The component of velocity parallel to a uniform field stays constant, while a nonzero perpendicular component produces circular motion.

Step 1: Convert the kinetic energy to joules The given 8.0 MeV is the particle's kinetic energy. Since $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$,

$$K = 8.0 \times 10^6 \times 1.6 \times 10^{-19} = 1.28 \times 10^{-12} \text{ J}.$$

Use $m = 6.4 \times 10^{-27} \text{ kg}$, $q = 3.2 \times 10^{-19} \text{ C}$ and $B = 0.5 \text{ T}$. The initial velocity is perpendicular to the field, so the complete speed contributes to the circular motion.

Step 2: Equate magnetic and centripetal forces From $qvB = mv^2/r$, the radius is $r = mv/(qB)$.

Eliminate the speed using the kinetic energy:

$$\begin{aligned} r &= \frac{\sqrt{2mK}}{qB} \\ &= \frac{\sqrt{2(6.4 \times 10^{-27})(1.28 \times 10^{-12})}}{(3.2 \times 10^{-19})(0.5)} \\ &= \frac{\sqrt{1.6384 \times 10^{-38}}}{1.6 \times 10^{-19}} = \frac{1.28 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ m.} \end{aligned}$$

The magnetic force remains perpendicular to the velocity, keeping the speed and hence this radius constant throughout the circular path.

Step 3: State the conditions for the other paths (i) **Helical path:** The initial velocity must have both a nonzero component perpendicular to the field and a nonzero component parallel to it. Thus its angle with $\mathbf{B}$ is neither 0° , 90° nor 180° . Circular motion due to the perpendicular component combines with uniform motion along the field to produce a helix.

(ii) **Straight undeviated path:** The velocity must be parallel or antiparallel to the field, so $\mathbf{v} \times \mathbf{B} = 0$. There is then no magnetic force and the moving particle continues along a straight line.

Step 4: Conclusion The circular-path radius is $r = 0.80 \text{ m}$. A helix requires both parallel and perpendicular velocity components; straight undeviated motion requires velocity parallel or antiparallel to the magnetic field.

Quick Tip to Solve: Convert MeV to joules first and use $r = \sqrt{2mK}/(|q|B)$. For the path type, separate the velocity into components along and across the field.

27. State Faraday's law of electromagnetic induction. Briefly describe two methods for producing induced emf. [3 marks]

Solution:

Concept:

- Magnetic flux through a plane loop in a uniform field is $\Phi_B = BA \cos \theta$, where B is the field magnitude, A is the enclosed area and θ is the angle between the field and the area normal.
- Faraday's law relates induced emf to the rate of change of magnetic flux linkage: $\mathcal{E} = -N d\Phi_B/dt$ for N turns having the same flux through each turn. The negative sign expresses Lenz's law.

Step 1: State Faraday's law Whenever the magnetic flux linked with a circuit changes, an emf is induced in it. The magnitude of the induced emf equals the time rate of change of magnetic flux linkage:

$$|\mathcal{E}| = \left| \frac{d(N\Phi_B)}{dt} \right|.$$

For a fixed number of turns, the signed expression is $\mathcal{E} = -N d\Phi_B/dt$. Its direction is such that, if the circuit is closed, the induced current opposes the change of flux that produces it. An open circuit can have an induced emf even though no sustained current flows.

Step 2: Produce emf by moving a magnet relative to a coil Connect a coil to a sensitive galvanometer and move a bar magnet towards the coil along its axis. The magnetic field through the coil increases, changing its flux linkage and inducing an emf; the galvanometer shows a current. Pulling the magnet away reverses the flux change and therefore the induced emf. When both magnet and coil are

stationary, the flux is constant and no emf is induced. Faster relative motion gives a larger rate of flux change and hence a larger induced emf.

Step 3: Produce emf by changing the current in a nearby coil Place a primary coil connected to a battery, switch and rheostat near a stationary secondary coil connected to a galvanometer. Change the primary current with the rheostat, or make and break the primary circuit. The primary's changing current changes the magnetic field and the flux linked with the secondary, inducing an emf there. Once the primary current becomes steady, the secondary flux becomes constant and the induced emf vanishes. This method, called mutual induction, requires no mechanical motion of either coil.

Step 4: Conclusion Faraday's law is $\mathcal{E} = -d(N\Phi_B)/dt$. Two methods are relative motion of a magnet and coil, and changing the current in a nearby coil; both produce emf by changing the linked magnetic flux.

Quick Tip to Solve: For each proposed method, identify what changes the coil's flux. A stationary magnet or a steady nearby current supplies flux but does not by itself induce emf.

28. A deuterium nucleus and an alpha particle approach a target nucleus separately in head-on position. Find the ratio of their distance of closest approach to the nucleus, when both have:

- (a) the same velocity, and
- (b) the same momentum.

[3 marks]

Solution:

Concept:

- In a head-on approach to a heavy target nucleus, the incident nucleus momentarily stops at its distance of closest approach. Its initial kinetic energy K becomes electrostatic potential energy: $K = kZe q/r$, taking the initial separation to be large and neglecting target recoil. Here Ze is the target charge, q is the projectile charge and $k = 1/(4\pi\epsilon_0)$.
- For a deuterium nucleus, $q_d = e$ and $m_d \simeq 2u$; for an alpha particle, $q_\alpha = 2e$ and $m_\alpha \simeq 4u$, where u is the atomic mass unit. Thus $m_\alpha = 2m_d$.
- Non-relativistic kinetic energy can be expressed as $K = \frac{1}{2}mv^2$ for a specified speed or as $K = p^2/(2m)$ for a specified momentum.

Step 1: Express the closest-approach ratio Conservation of mechanical energy gives

$$K = \frac{kZe q}{r} \Rightarrow r = \frac{kZe q}{K}.$$

Both projectiles approach the same target, so the factors kZe cancel. Keeping the order deuterium to alpha throughout,

$$\frac{r_d}{r_\alpha} = \frac{q_d}{q_\alpha} \frac{K_\alpha}{K_d} = \frac{1}{2} \frac{K_\alpha}{K_d}.$$

Step 2: Apply the equal-velocity condition For part (a), both incident speeds are v , so kinetic energy is proportional to mass:

$$\frac{K_\alpha}{K_d} = \frac{\frac{1}{2}m_\alpha v^2}{\frac{1}{2}m_d v^2} = \frac{4u}{2u} = 2.$$

Substituting into the distance ratio gives $r_d/r_\alpha = \frac{1}{2} \times 2 = 1$. The alpha particle has twice the electrostatic repulsion factor but also twice the incident kinetic energy.

Step 3: Apply the equal-momentum condition For part (b), both momentum magnitudes are p , so kinetic energy is inversely proportional to mass:

$$\frac{K_{\alpha}}{K_d} = \frac{p^2/(2m_{\alpha})}{p^2/(2m_d)} = \frac{m_d}{m_{\alpha}} = \frac{1}{2}.$$

Hence the distance ratio becomes $r_d/r_{\alpha} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. At the same momentum the alpha particle has smaller kinetic energy as well as larger charge, and therefore turns back farther away.

Step 4: Conclusion The ratios of the closest distances, in the order deuterium nucleus : alpha particle, are $\boxed{1 : 1}$ for equal velocity and $\boxed{1 : 4}$ for equal momentum.

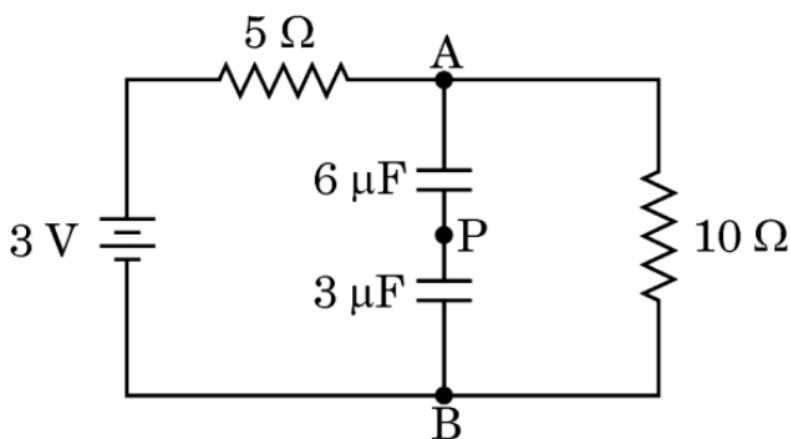
Quick Tip to Solve: Start with $r \propto q/K$: at fixed velocity use $r \propto q/m$, whereas at fixed momentum use $r \propto qm$. Keep the requested particle order unchanged.

Section D (8 marks)

Questions number 29 and 30 are Case Study-based questions. Read the following paragraphs and answer the questions that follow.

29. Capacitors are manufactured with certain standard capacitances and working voltages. However, these standard values may not be the ones that are actually needed in a particular application. Two or more capacitors can be grouped in series or in parallel to achieve desired capacitance and voltage. When connected in series, the total capacitance decreases while the voltage rating increases, whereas in parallel connections, the total capacitance increases and maintains the same voltage rating. A capacitor stores energy in the electric field between its plates and stored energy is proportional to the square of the voltage and capacitance $U = \frac{1}{2}CV^2$, where symbols have their usual meanings.

Two capacitors, one of $3\ \mu\text{F}$ and the other of $6\ \mu\text{F}$, are connected in series in the circuit as shown in the figure, for a long time. [4 marks]



29(i). The total capacitance of the circuit is:

[1 mark]

- (A) $6\ \mu\text{F}$
- (B) $3\ \mu\text{F}$
- (C) $9\ \mu\text{F}$
- (D) $2\ \mu\text{F}$

Correct Answer: (D) $2\ \mu\text{F}$

Solution:

Concept:

- Capacitors in series carry equal charge magnitudes, while their potential differences add. Thus $1/C_{\text{eq}} = 1/C_1 + 1/C_2$, where C_{eq} is the equivalent capacitance.
- For two series capacitors, $C_{\text{eq}} = C_1 C_2 / (C_1 + C_2)$, which is smaller than either individual capacitance.

Step 1: Identify the capacitor connection The $6\ \mu\text{F}$ and $3\ \mu\text{F}$ capacitors lie successively in the branch from A to B, with their common junction at P. They therefore form a series combination. The resistors determine the applied voltage in steady state but do not change this capacitance combination.

Step 2: Calculate and compare the options Substitute the two capacitances into the series formula:

$$C_{\text{eq}} = \frac{6 \times 3}{6 + 3} \mu\text{F} = \frac{18}{9} \mu\text{F} = 2 \mu\text{F}.$$

The value $9\ \mu\text{F}$ in (C) would be obtained for a parallel connection. Options (A) and (B) merely repeat individual capacitances; neither is smaller than both capacitors as a series equivalent must be.

Step 3: Conclusion The total capacitance is $2\ \mu\text{F}$. Hence, the correct option is (D).

Quick Tip to Solve: A series capacitance must be smaller than the smallest component; here it must be below $3\ \mu\text{F}$, immediately eliminating three options.

29(ii). The current in the $10\ \Omega$ resistor is:

[1 mark]

- (A) $0.3\ \text{A}$
- (B) $0.6\ \text{A}$
- (C) $0.2\ \text{A}$
- (D) 0

Correct Answer: (C) $0.2\ \text{A}$

Solution:

Concept:

- After a long time with a constant DC source, an ideal capacitor is fully charged and carries no steady current: $I_C = C\,dV_C/dt = 0$. Its branch acts as an open circuit.
- Resistors in a single conducting loop add in series, and Ohm's law gives $I = V/R_{\text{total}}$.

Step 1: Find the conducting path In steady state, no current flows through the middle capacitor branch. However, the outer loop remains complete through the $3\ \text{V}$ battery, the $5\ \Omega$ resistor and the $10\ \Omega$ resistor. The same current flows through both resistors.

Step 2: Apply Ohm's law to the loop The total resistance in the conducting loop is

$$R_{\text{total}} = 5 + 10 = 15\ \Omega, \quad I = \frac{3}{15} = 0.2\ \text{A}.$$

Option (A), $0.3\ \text{A}$, wrongly omits the $5\ \Omega$ resistor, and (B), $0.6\ \text{A}$, wrongly omits the $10\ \Omega$ resistor. Option (D) confuses the zero capacitor-branch current with the nonzero current in the complete outer loop.

Step 3: Conclusion The current in the $10\ \Omega$ resistor is $0.2\ \text{A}$. Hence, the correct option is (C).

Quick Tip to Solve: Replace fully charged capacitors by open branches, then trace the remaining closed path before adding its resistances.

29(iii). The potential difference between point A and B is:

[1 mark]

- (A) 2 V
- (B) 0.3 V
- (C) 0.2 V
- (D) 3 V

Correct Answer: (A) 2 V

Solution:

Concept:

- A fully charged ideal capacitor draws no steady DC current. The current in the external resistive loop is obtained after treating the capacitor branch as open.
- Components connected across the same two nodes have the same potential difference. The voltage between A and B equals the voltage across the $10\ \Omega$ resistor: $V_{AB} = IR$.

Step 1: Calculate the steady loop current The $5\ \Omega$ and $10\ \Omega$ resistors form the conducting loop with the 3 V battery. Consequently,

$$I = \frac{3}{5 + 10} = 0.2\text{ A.}$$

The positive terminal feeds the top of the circuit, so current flows from A to B through the $10\ \Omega$ resistor.

Step 2: Find the voltage across the shared nodes Since the $10\ \Omega$ resistor is connected directly between A and B ,

$$V_A - V_B = I(10\ \Omega) = 0.2 \times 10 = 2\text{ V.}$$

The full battery voltage 3 V, option (D), is not across these nodes because 1 V drops across the $5\ \Omega$ resistor. Option (C) repeats the numerical current with voltage units; option (B) also fails Ohm's law for the $10\ \Omega$ branch.

Step 3: Conclusion The potential difference is $V_A - V_B = 2\text{ V}$, with A at higher potential. Hence, the correct option is (A).

Quick Tip to Solve: Mark the two nodes first: the capacitor pair and the $10\ \Omega$ resistor share A and B , so use the resistor's voltage drop.

29(iv)(a). The value of charge on the plates of the $6\ \mu\text{F}$ capacitor is:

[1 mark]

- (A) $6\ \mu\text{C}$
- (B) $4\ \mu\text{C}$
- (C) $12\ \mu\text{C}$
- (D) $8\ \mu\text{C}$

Correct Answer: (B) $4\ \mu\text{C}$

Solution:

Concept:

- Series capacitors carry equal charge magnitudes Q . Their equivalent capacitance is $C_{\text{eq}} = C_1 C_2 / (C_1 + C_2)$, and $Q = C_{\text{eq}} V_{AB}$.
- In steady DC operation the capacitor branch carries no current. The resistors set V_{AB} , the voltage

across the series capacitor combination.

Step 1: Determine the voltage across the capacitor pair The conducting outer loop contains both resistors in series. Its current and the voltage across the $10\ \Omega$ resistor are

$$I = \frac{3}{5 + 10} = 0.2\text{ A}, \quad V_{AB} = I \times 10 = 2\text{ V}.$$

The capacitor pair shares the same endpoints A and B , so its total voltage is also 2 V .

Step 2: Use the common series charge First combine the two capacitances, then use their total voltage:

$$C_{\text{eq}} = \frac{6 \times 3}{6 + 3} = 2\ \mu\text{F}, \quad Q = C_{\text{eq}} V_{AB} = 2 \times 2 = 4\ \mu\text{C}.$$

Each capacitor has this charge magnitude on its plates. Option (A), $6\ \mu\text{C}$, wrongly uses the entire battery voltage across the pair; option (C), $12\ \mu\text{C}$, wrongly places all 2 V across the $6\ \mu\text{F}$ capacitor alone. Option (D), $8\ \mu\text{C}$, double-counts the common series charge.

Step 3: Conclusion The plates of the $6\ \mu\text{F}$ capacitor carry charges $+4\ \mu\text{C}$ and $-4\ \mu\text{C}$, of magnitude $4\ \mu\text{C}$ each. Hence, the correct option is (B).

Quick Tip to Solve: For series capacitors, multiply the equivalent capacitance by the voltage across the whole pair; this gives the charge on either capacitor.

OR

29(iv)(b). The wire between two capacitors is cut at point P . The current in the circuit will: [1 mark]

- (A) increase
- (B) decrease
- (C) remain the same
- (D) first increase then become stable

Correct Answer: (C) remain the same

Solution:

Concept:

- In a DC circuit after a long time, fully charged ideal capacitors carry no steady current. Opening a branch that already carries zero current does not change the steady conducting paths.
- The battery current in the remaining resistive loop is determined by Ohm's law, $I = V/(R_1 + R_2)$.

Step 1: Examine the circuit before cutting Before the wire is cut at P , the capacitor branch has already reached steady state, so its current is zero. The battery supplies only the outer loop containing $5\ \Omega$ and $10\ \Omega$ in series:

$$I_{\text{before}} = \frac{3}{5 + 10} = 0.2\text{ A}.$$

Step 2: Trace the path after cutting Cutting at P breaks only the middle capacitor branch. It leaves the battery and both resistors connected in exactly the same closed outer loop. Thus the resistance seen by the battery remains $15\ \Omega$, giving

$$I_{\text{after}} = \frac{3}{15} = 0.2\text{ A}.$$

Neither an increase nor a decrease is expected, so (A) and (B) fail. The capacitors were already at their steady voltages and no conducting path is newly created; option (D)'s initial increase is therefore also unsupported in the ideal circuit.

Step 3: Conclusion The current remains the same, 0.2 A . Hence, the correct option is (C).

Quick Tip to Solve: Before predicting a current change from cutting a wire, check whether that branch was carrying any current immediately before the cut.

30. A charged particle $+q$ in an electric field $\vec{E}$ experiences a force in the direction of the electric field. As a result, its kinetic energy changes. Similarly, the charged particle also experiences a force when it moves in a magnetic field $\vec{B}$. But this magnetic force is perpendicular to both velocity $\vec{v}$ of the charged particle and the magnetic field $\vec{B}$, so it cannot change the kinetic energy of the charged particle. Consider two charged particles 1 and 2 of masses m and $\frac{m}{2}$ having charges $-q$ and $+2q$ respectively. They are accelerated from rest through the same potential difference V and acquire kinetic energy K_1 and K_2 . Then they enter in a region of uniform magnetic field $\vec{B}$ perpendicular to their velocities. [4 marks]

30(i). The ratio of their kinetic energies $\left(\frac{K_1}{K_2}\right)$ is: [1 mark]

- (A) $\frac{1}{2}$
- (B) $\frac{1}{4}$
- (C) 4
- (D) 1

Correct Answer: (A) $\frac{1}{2}$

Solution:

Concept:

- The electric field's work becomes kinetic energy: a particle accelerated from rest through potential difference of magnitude V gains $K = |Q|V$, where Q is its charge.
- A magnetic force is perpendicular to velocity, so it does no work and leaves the kinetic energy unchanged.

Step 1: Find each kinetic energy Particle 1 has charge magnitude q and particle 2 has charge magnitude $2q$. Since each is accelerated from rest, the positive energy gains are

$$K_1 = qV, \quad K_2 = 2qV.$$

The negative sign of the first charge determines its direction of acceleration, not a negative kinetic energy.

Step 2: Compare the energy gains Dividing cancels the common potential difference:

$$\frac{K_1}{K_2} = \frac{qV}{2qV} = \frac{1}{2}.$$

The masses affect the resulting speeds but do not enter this energy ratio. Thus $1/4$ and 4 incorrectly involve additional factors, while 1 would require equal charge magnitudes.

Step 3: Conclusion The kinetic-energy ratio is $K_1/K_2 = 1/2$. Hence, the correct option is (A).

Quick Tip to Solve: For acceleration from rest through equal potential differences, take the ratio of charge magnitudes directly to obtain the kinetic-energy ratio.

30(ii). The ratio of the radii of the circular paths described by them $\left(\frac{r_1}{r_2}\right)$ is: [1 mark]

- (A) $\frac{1}{\sqrt{2}}$
- (B) $\sqrt{2}$
- (C) $\frac{1}{2}$
- (D) 2

Correct Answer: (D) 2

Solution:

Concept:

- For perpendicular entry into a uniform magnetic field, $|Q|vB = Mv^2/r$, so the orbit radius is $r = Mv/(|Q|B)$, where M is the particle's mass.
- Acceleration from rest gives $Mv^2/2 = |Q|V$. Combining the two relations gives $r = \sqrt{2MV/|Q|}/B$.

Step 1: Form the radius ratio The potential difference and magnetic-field magnitude are the same for both particles. Therefore,

$$\frac{r_1}{r_2} = \sqrt{\frac{m_1/|Q_1|}{m_2/|Q_2|}}.$$

Use charge magnitudes because radius is positive; the sign of charge controls the sense of turning.

Step 2: Substitute both masses and charges Here $m_1 = m$, $m_2 = m/2$, $|Q_1| = q$, and $|Q_2| = 2q$. Hence,

$$\frac{r_1}{r_2} = \sqrt{\frac{m/q}{(m/2)/(2q)}} = \sqrt{4} = 2.$$

Both the smaller mass and larger charge reduce the second radius. The options $1/\sqrt{2}$ and $\sqrt{2}$ omit one of these changes, while $1/2$ reverses the requested ratio.

Step 3: Conclusion The radii satisfy $r_1/r_2 = 2$. Hence, the correct option is (D).

Quick Tip to Solve: At fixed accelerating voltage and magnetic field, compare $\sqrt{m/|Q|}$; both the mass and the charge change in this problem.

30(iii). Suppose particles 1 and 2 enter the magnetic field $\vec{B} = B_0\hat{k}$ with velocities $\vec{v}_1 = v_1\hat{i}$ and $\vec{v}_2 = v_2\hat{i}$. Then: [1 mark]

- (A) both particles revolve clockwise
- (B) both particles revolve anticlockwise
- (C) particle 1 revolves clockwise while particle 2 revolves anticlockwise
- (D) particle 1 revolves anticlockwise while particle 2 revolves clockwise

Correct Answer: (D) particle 1 revolves anticlockwise while particle 2 revolves clockwise

Solution:

Concept:

- The magnetic force is $\vec{F} = Q(\vec{v} \times \vec{B})$. Reversing the charge sign reverses the force at a given velocity.
- In a right-handed Cartesian system, $\hat{i} \times \hat{k} = -\hat{j}$. Clockwise and anticlockwise are viewed from the positive z -side toward the xy -plane.

Step 1: Determine the initial forces Both velocities point along $+\hat{i}$, while the magnetic field points along $+\hat{k}$. Therefore,

$$\begin{aligned}\vec{F}_1 &= (-q)(-v_1 B_0 \hat{j}) = +qv_1 B_0 \hat{j}, \\ \vec{F}_2 &= (2q)(-v_2 B_0 \hat{j}) = -2qv_2 B_0 \hat{j}.\end{aligned}$$

Thus particle 1 initially bends toward positive y , and particle 2 toward negative y .

Step 2: Translate bending into rotation A path initially directed to the right and bending upward turns anticlockwise. A path initially directed to the right and bending downward turns clockwise. The two charges must rotate in opposite senses, ruling out (A) and (B); (C) reverses the force directions. The unequal speed magnitudes affect the radii, not these senses of revolution.

Step 3: Conclusion Particle 1 revolves anticlockwise and particle 2 revolves clockwise. Hence, the correct option is (D).

Quick Tip to Solve: First calculate $\hat{i} \times \hat{k} = -\hat{j}$ for a positive charge, then reverse the force for the negatively charged particle.

30(iv)(a). If period of revolution for particle 1 is 4 s, then for particle 2, the period will be: [1 mark]

- (A) 1 s
- (B) 2 s
- (C) 4 s
- (D) 8 s

Correct Answer: (A) 1 s

Solution:

Concept:

- The period of a charged particle moving perpendicular to a uniform magnetic field is $T = 2\pi M/(|Q|B)$, where M is its mass and $|Q|$ its charge magnitude.
- This follows from $T = 2\pi r/v$ and $r = Mv/(|Q|B)$. Speed cancels, so the period is independent of the accelerating voltage in this nonrelativistic treatment.

Step 1: Compare the mass-to-charge ratios Since the particles move in the same magnetic field,

$$\frac{T_2}{T_1} = \frac{m_2/|Q_2|}{m_1/|Q_1|} = \frac{(m/2)/(2q)}{m/q} = \frac{1}{4}.$$

Halving the mass halves the period, and doubling the charge magnitude halves it once again.

Step 2: Use the given first period With $T_1 = 4$ s, substitution gives

$$T_2 = \frac{T_1}{4} = \frac{4 \text{ s}}{4} = 1 \text{ s}.$$

The option 2 s includes only one of the two factors; 4 s ignores the changed mass-to-charge ratio, and 8 s predicts an increase despite both changes shortening the period.

Step 3: Conclusion Particle 2 has period $T_2 = 1$ s. Hence, the correct option is (A).

Quick Tip to Solve: In the same magnetic field, compare $m/|Q|$ directly: half the mass and twice the charge give one quarter of the period.

OR

30(iv)(b). If the value of momentum for particles 1 and 2 are p_1 and p_2 , then:

[1 mark]

- (A) $p_1 = \frac{p_2}{2}$
- (B) $p_1 = p_2$
- (C) $p_1 = 2p_2$
- (D) $p_1 = 4p_2$

Correct Answer: (B) $p_1 = p_2$

Solution:

Concept:

- For a nonrelativistic particle of mass M , kinetic energy and momentum magnitude satisfy $K = p^2/(2M)$, hence $p = \sqrt{2MK}$.
- Acceleration from rest through potential difference of magnitude V gives $K = |Q|V$. Therefore the momentum magnitude is $p = \sqrt{2M|Q|V}$.

Step 1: Evaluate the two momentum magnitudes For particle 1, the mass is m and charge magnitude is q . For particle 2, they are $m/2$ and $2q$. Thus,

$$p_1 = \sqrt{2mqV},$$
$$p_2 = \sqrt{2\left(\frac{m}{2}\right)(2q)V} = \sqrt{2mqV}.$$

The mass reduction exactly cancels the increase in charge magnitude in the product $M|Q|$.

Step 2: Compare the momenta Their ratio is

$$\frac{p_1}{p_2} = \sqrt{\frac{mq}{(m/2)(2q)}} = 1.$$

The printed scalar symbols denote momentum magnitudes. Equal accelerating voltages do not

generally imply equal momentum, but the equal mass-charge products do so here. The factors $1/2$, 2 , and 4 in the other options fail to include this cancellation.

Step 3: Conclusion The momentum magnitudes are equal: $p_1 = p_2$. Hence, the correct option is (B).

Quick Tip to Solve: For particles accelerated through the same voltage, compare $m|Q|$ under the square root; here $m|Q| = (m/2)(2q)$.

Section E (15 marks)

31(a). (i) A point object is kept in front of a convex spherical surface of radius of curvature R . Draw the ray diagram to show the formation of image and derive the relation between the object and image distance (u and v) in terms of refractive index n of the medium and R .

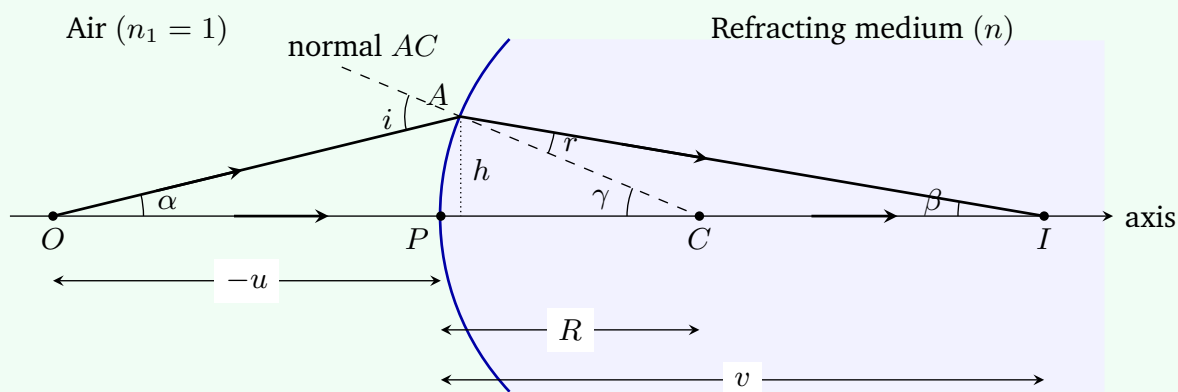
(ii) A convex lens of focal length of 20 cm is used to form the image of an object placed 30 cm away from the lens. Find the position and nature of the image formed. [5 marks]

Solution:

Concept:

- For refraction from air into a medium of refractive index n , Snell's law is $\sin i = n \sin r$, where i and r are measured from the normal. For paraxial rays, the small-angle approximation gives $i = nr$.
- Use the Cartesian sign convention: distances are measured from the pole or optical centre, positive along the incident light. Thus a real object on the left has $u < 0$; a convex refracting surface with its centre on the right has $R > 0$.
- A thin lens satisfies $1/f = 1/v - 1/u$, with signed focal length f , object distance u , image distance v , and transverse magnification $m = v/u$.

Step 1: (i) Construct the image at a convex refracting surface Take the object O in air and the refracting medium on the right. Let P be the pole and C the centre of curvature, so that $PC = R$. The figure shows a real-image configuration, with I on the principal axis. A ray OP along the axis enters normally and proceeds undeviated. An oblique paraxial ray OA meets the surface at A ; its normal is the radius AC . It bends towards this normal into the denser medium and meets the axial ray at I . The angles in the sketch are exaggerated for clarity; the derivation uses a small aperture. Let h be the perpendicular height of A above the axis.



Refraction at a convex spherical surface: $OP = -u$, $PI = v$, $PC = R$; the normal is AC .

Step 2: (i) Relate the angles using the ray geometry Let the positive small angles made with the principal axis by OA , AI and AC be α , β and γ , respectively. In the displayed real-image geometry, the angle of incidence is the sum $i = \alpha + \gamma$, whereas the refraction angle is $r = \gamma - \beta$. Applying the paraxial form of Snell's law,

$$\begin{aligned}\alpha + \gamma &= n(\gamma - \beta), \\ \alpha + n\beta &= (n - 1)\gamma.\end{aligned}$$

For rays close to the axis, the axial displacement of A from P is of second order and is neglected. Therefore the three small angles, measured in radians, satisfy

$$\alpha \simeq \frac{h}{OP}, \quad \beta \simeq \frac{h}{PI}, \quad \gamma \simeq \frac{h}{PC}.$$

Step 3: (i) Introduce signed distances and obtain the relation Substituting the geometrical expressions and cancelling the nonzero ray height gives

$$\frac{1}{OP} + \frac{n}{PI} = \frac{n - 1}{PC}.$$

Here OP , PI and PC have been positive lengths. With the Cartesian convention, $OP = -u$, $PI = v$ and $PC = R$, hence

$$\boxed{\frac{n}{v} - \frac{1}{u} = \frac{n - 1}{R}}.$$

This is the required paraxial relation for an object in air. If both media have refractive indices n_1 and n_2 , the corresponding relation is $n_2/v - n_1/u = (n_2 - n_1)/R$. The signed formula also covers a virtual image through a negative value of v .

Step 4: (ii) Calculate the image position for the convex lens A convex lens has positive focal length. The object is on its incident side, so the given data become $f = +20$ cm and $u = -30$ cm. Rearrange

the lens formula before substituting:

$$\begin{aligned}\frac{1}{v} &= \frac{1}{f} + \frac{1}{u} \\ &= \frac{1}{20} - \frac{1}{30} = \frac{3-2}{60} = \frac{1}{60} \text{ cm}^{-1}, \\ v &= +60 \text{ cm}.\end{aligned}$$

The positive image distance places the image on the opposite side of the lens from the object.

Step 5: (ii) Determine the nature and size of the image The transverse magnification is

$$m = \frac{v}{u} = \frac{+60}{-30} = -2.$$

The negative sign means that the image is inverted, while $|m| = 2$ means that its height is twice the object's height. Since the outgoing rays actually converge at the positive image distance, the image is real and can be obtained on a screen. This agrees with the ray-optics rule: an object between F and $2F$ of a convex lens forms a real, inverted, enlarged image beyond $2F$.

Step 6: Conclusion (i) For refraction from air at the convex spherical surface, $n/v - 1/u = (n-1)/R$.
(ii) The convex lens forms its image **60 cm** on the other side of the lens. It is real, inverted and twice the object's size, with $m = -2$.

Quick Tip to Solve: Write the signed values first: here $u = -30$ cm and $f = +20$ cm. The object lies between F and $2F$, so a positive image distance beyond 40 cm and $|m| > 1$ provide immediate checks.

OR

31(b). (i) Two thin converging lenses of focal length f_1 and f_2 are placed coaxially in contact. Derive expression for the focal length of the combination.

(ii) A beam of coherent light of wavelength 550 nm is incident normal to the plane of a pair of two slits S_1 and S_2 each of width 1.2×10^{-6} m separated by 1.1 mm. Dark and bright fringes are observed on a screen 2.2 m away from the plane of the slits.

Calculate:

(I) fringe width.

(II) distance of the second dark fringe from the central maximum.

(III) what will happen when the entire apparatus is immersed in water.

[5 marks]

Solution:

Concept:

- For a thin lens, the Cartesian sign convention gives $1/v - 1/u = 1/f$, where u and v are the signed object and image distances and f is its focal length.
- For two thin lenses in contact, the first lens's image acts as the second lens's object. Their common optical plane allows their lens equations to be added.
- Young's interference fringes have width $\beta = \lambda D/d$, where λ is the wavelength in the medium, D the screen distance and d the slit separation. The n th dark fringe is at $|y_n| = (n - \frac{1}{2})\beta$.
- In a medium of refractive index μ , light's frequency is unchanged while its wavelength becomes λ/μ ; hence fringe distances contract by the factor $1/\mu$.

Step 1: (i) Write the two lens equations Let an axial object be at signed distance u from the first lens. The first lens alone would form an image at v_1 . Its equation is

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}.$$

Since the two thin lenses are in contact, the distance of this intermediate image from the second lens is also v_1 . It acts as the object for the second lens; when it lies on the outgoing side, it is a virtual object with positive object distance. If the final image is at v ,

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}.$$

Step 2: (i) Eliminate the intermediate image distance Add the two equations. The intermediate terms cancel, giving

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

A single equivalent thin lens of focal length F , with the same object and final image positions, obeys $1/v - 1/u = 1/F$. Comparing the two equations yields

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}, \quad F = \frac{f_1 f_2}{f_1 + f_2}.$$

Both given lenses converge, so both focal lengths are positive; the combination converges more strongly than either lens separately.

Step 3: (ii)(I) Calculate the interference fringe width Convert the wavelength and slit separation to SI units:

$$\lambda = 550 \times 10^{-9} \text{ m}, \quad d = 1.1 \times 10^{-3} \text{ m}, \quad D = 2.2 \text{ m}.$$

The separation d , rather than each slit's width, determines interference fringe spacing. The individual slit width controls the diffraction envelope. Thus

$$\begin{aligned} \beta &= \frac{\lambda D}{d} = \frac{(550 \times 10^{-9})(2.2)}{1.1 \times 10^{-3}} \text{ m} \\ &= 1.1 \times 10^{-3} \text{ m} = 1.1 \text{ mm}. \end{aligned}$$

Step 4: (ii)(II) Locate the second dark fringe Destructive interference requires the path difference to be $(n - \frac{1}{2})\lambda$. Since the first dark fringe is half a fringe width from the central maximum, the second dark fringe is one and a half fringe widths away:

$$|y_2| = \left(2 - \frac{1}{2}\right) \beta = \frac{3}{2}(1.1 \text{ mm}) = 1.65 \text{ mm}.$$

There is a second dark fringe on each side of the centre, at equal distances from it.

Step 5: (ii)(III) Find the effect of immersion in water Uniform immersion leaves D and d fixed but reduces the light's wavelength to λ/μ_w , where μ_w is water's refractive index. Consequently,

$$\beta_w = \frac{\beta}{\mu_w}, \quad |y_{2,w}| = \frac{1.65}{\mu_w} \text{ mm}.$$

The fringes become closer together; the central maximum remains at the same position because the two optical paths there remain equal. Using the usual approximate value $\mu_w = 4/3$, the spacing

becomes 0.825 mm and the second dark fringe is 1.2375 mm, approximately 1.24 mm, from the centre.

Step 6: Conclusion (i) The combination has focal length $F = f_1 f_2 / (f_1 + f_2)$. (ii) The fringe width is 1.1 mm, and the second dark fringe lies 1.65 mm from the central maximum on either side. In water all fringe distances contract by $1/\mu_w$, with no displacement of the central maximum.

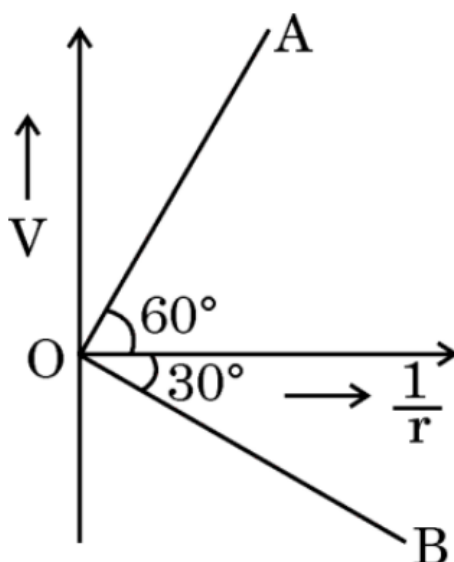
Quick Tip to Solve: Use slit separation, not slit width, in $\beta = \lambda D/d$; dark-fringe distances start at $\beta/2$, so the second is $3\beta/2$.

32(a). (i) In the figure, OA and OB show the variation of electric potential V at a point due to two point charges Q_1 and Q_2 with $\frac{1}{r}$ respectively. Here r represents the distance of the point from the two point charges.

(I) Identify the nature of the two charges Q_1 and Q_2 .

(II) What is the value of $\left(\frac{Q_1}{Q_2}\right)$? Justify your answer.

(ii) Two point charges $-2 \mu\text{C}$ and $5 \mu\text{C}$ are placed at $(-30 \text{ cm}, 0)$ and $(30 \text{ cm}, 0)$ respectively in an external electric field $\vec{E} = \frac{A}{x^2} \hat{i}$, where $A = 9 \times 10^5 \text{ N m}^2 \text{ C}^{-1}$. Find the electrostatic potential energy of this configuration. [5 marks]



Solution:

Concept:

- For a point charge Q , $V = kQ/r$, where $k = 1/(4\pi\epsilon_0)$. A plot of V against $1/r$ is a straight line with signed slope kQ .
- Electric field and potential satisfy $E_x = -dV/dx$. Integrating the specified external field determines its potential, after choosing a potential reference.
- The energy of two charges in a prescribed external field is $U = q_1 V_{\text{ext}}(x_1) + q_2 V_{\text{ext}}(x_2) + kq_1 q_2 / r_{12}$. The last term is their mutual interaction, counted once.

Step 1: (i)(I) Identify the signs of the charges In the graph, OA corresponds to Q_1 and slopes upwards from the origin, whereas OB corresponds to Q_2 and slopes downwards. Since $k > 0$, the slope of $V = kQ(1/r)$ has the sign of Q . Thus Q_1 is positive and Q_2 is negative. This also agrees directly with the positive potential on OA and the negative potential on OB .

Step 2: (i)(II) Compare the signed graph slopes Both straight lines use the same axes and scales, so their slope ratio equals the charge ratio. The angle of OA above the positive horizontal axis is 60° , whereas OB is 30° below it. Therefore its signed angle is -30° :

$$\frac{Q_1}{Q_2} = \frac{kQ_1}{kQ_2} = \frac{\tan 60^\circ}{\tan(-30^\circ)} = \frac{\sqrt{3}}{-1/\sqrt{3}} = -3.$$

The magnitude of Q_1 is three times that of Q_2 , and the negative ratio records their opposite signs.

Step 3: (ii) Obtain the external potential The printed field is $\vec{E} = (A/x^2)\hat{i}$. Use $E_x = -dV_{\text{ext}}/dx$:

$$\begin{aligned}\frac{dV_{\text{ext}}}{dx} &= -\frac{A}{x^2}, \\ V_{\text{ext}}(x) &= \frac{A}{x} + C.\end{aligned}$$

Choose zero potential as $|x|$ tends to infinity on each side of the singularity at $x = 0$; this sets $C = 0$. Preserve the sign of the coordinate in $V = A/x$, even though the field contains x^2 . At the two specified positions,

$$\begin{aligned}x_1 &= -0.30 \text{ m}, & V_1 &= -3.0 \times 10^6 \text{ V}, \\ x_2 &= +0.30 \text{ m}, & V_2 &= +3.0 \times 10^6 \text{ V}.\end{aligned}$$

Step 4: (ii) Add the two external-field energies The point charges are $q_1 = -2 \times 10^{-6} \text{ C}$ and $q_2 = 5 \times 10^{-6} \text{ C}$. The potential in these terms is due to the imposed external field alone:

$$\begin{aligned}U_{\text{ext}} &= q_1 V_1 + q_2 V_2 \\ &= (-2 \times 10^{-6})(-3.0 \times 10^6) \text{ J} \\ &\quad + (5 \times 10^{-6})(3.0 \times 10^6) \text{ J} \\ &= 6 + 15 = 21 \text{ J}.\end{aligned}$$

Both products are positive: the negative charge occupies a negative external potential, while the positive charge occupies a positive external potential.

Step 5: (ii) Include the mutual interaction energy The separation of the two charges is $r_{12} = 0.30 - (-0.30) = 0.60 \text{ m}$. Their mutual electrostatic potential energy is negative because their signs are opposite:

$$\begin{aligned}U_{12} &= \frac{kq_1 q_2}{r_{12}} \\ &= \frac{(9 \times 10^9)(-2 \times 10^{-6})(5 \times 10^{-6})}{0.60} \text{ J} \\ &= -0.15 \text{ J}.\end{aligned}$$

Add this pair energy only once to the external-field contribution:

$$U = U_{\text{ext}} + U_{12} = 21 - 0.15 = 20.85 \text{ J}.$$

Step 6: Conclusion (i) $Q_1 > 0, Q_2 < 0$, with $Q_1/Q_2 = -3$. (ii) With the external potential taken as $V_{\text{ext}} = A/x$, the configuration's total electrostatic potential energy is $+20.85 \text{ J}$.

Quick Tip to Solve: Keep the slope below the horizontal axis negative. For the energy calculation, use signed x in $V = A/x$, then add the attractive pair energy exactly once.

32(b). (i) Two infinitely long straight wires having linear charge densities $-\lambda$ and 3λ are held vertically parallel to each other, distance r apart in free space. Find the nature and magnitude of the force/length exerted by one wire on the other.

(ii) A small hollow conducting sphere of radius r_1 is given a charge Q . It is surrounded by a concentric conducting spherical shell of inner radius r_2 and outer radius r_3 , having charge $-3q$.

If a point charge $2q$ were kept at the centre, find:

(I) the electric flux through a concentric spherical Gaussian surface of radius x for (1) $x < r_1$, and (2) $r_1 < x < r_2$.

(II) electric field at a point distant x from the centre for (1) $x > r_3$, and (2) $r_1 < x < r_2$.

(III) surface charge density on the inner surface of (1) sphere, and (2) shell.

[5 marks]

Solution:

Concept:

- An infinite straight line of charge with linear density λ_s produces an electric field of magnitude $E = |\lambda_s|/(2\pi\epsilon_0 r)$ at perpendicular distance r . The force per unit length on another wire is $|\lambda_t|E$.
- Gauss's law gives the outward electric flux $\Phi = Q_{\text{enc}}/\epsilon_0$. For a spherically symmetric charge distribution, $\vec{E} = Q_{\text{enc}}\hat{r}/(4\pi\epsilon_0 x^2)$, where $\hat{r}$ points radially outward.
- The electric field is zero within a conductor in electrostatic equilibrium. Therefore, the charge on an inner conducting surface cancels the total charge in its cavity. For a uniform spherical surface, $\sigma = Q_s/(4\pi r^2)$.

Step 1: (i) Obtain the force per unit length between the wires Take the field produced by the wire carrying charge density 3λ at the position of the wire carrying $-\lambda$. Its magnitude is

$$E = \frac{3|\lambda|}{2\pi\epsilon_0 r}.$$

A length ℓ of the second wire has charge magnitude $|\lambda|\ell$, so

$$\frac{F}{\ell} = |\lambda|E = \frac{3\lambda^2}{2\pi\epsilon_0 r}.$$

The charges have opposite signs, hence each wire is pulled towards the other: the force is attractive. The two forces per unit length have equal magnitudes and opposite directions; the expression has SI units N m^{-1} .

Step 2: (ii) Establish the induced surface charges The small sphere retains its net charge Q , while the surrounding shell has the printed net charge $-3q$. Keep the uppercase Q distinct from the lowercase q . A Gaussian surface lying within the metal of the small sphere must enclose zero net charge. Thus its inner surface carries $-2q$; its outer surface consequently carries $Q + 2q$. Similarly, a Gaussian surface within the metal of the surrounding shell encloses the central charge, the small sphere and the shell's inner surface. Thus

$$2q + Q + Q_{\text{shell,in}} = 0, \quad Q_{\text{shell,in}} = -(Q + 2q).$$

The shell's outer surface has the remainder

$$Q_{\text{shell,out}} = -3q + (Q + 2q) = Q - q.$$

The concentric geometry and the central point charge make the induced charge uniform on each

spherical surface.

Step 3: (ii)(I) Calculate the two electric fluxes For $0 < x < r_1$, the Gaussian sphere lies inside the cavity and encloses only the central point charge $2q$. Charges on conducting surfaces outside it do not enter the enclosed charge. Therefore,

$$\Phi_{x < r_1} = \frac{2q}{\epsilon_0}.$$

For $r_1 < x < r_2$, the surface encloses both the point charge and the entire small sphere, whose net charge remains Q . Therefore,

$$\Phi_{r_1 < x < r_2} = \frac{Q + 2q}{\epsilon_0}.$$

These are signed outward fluxes, measured in $\text{N m}^2 \text{C}^{-1}$.

Step 4: (ii)(II) Determine the electric fields Spherical symmetry makes the field radial and constant in magnitude over a spherical Gaussian surface, so $E_r(4\pi x^2) = Q_{\text{enc}}/\epsilon_0$. Outside the whole system, $x > r_3$, the enclosed charge is

$$Q_{\text{enc}} = 2q + Q - 3q = Q - q, \quad \vec{E} = \frac{Q - q}{4\pi\epsilon_0 x^2} \hat{r}.$$

In the gap $r_1 < x < r_2$, the enclosed charge is $Q + 2q$, giving

$$\vec{E} = \frac{Q + 2q}{4\pi\epsilon_0 x^2} \hat{r}.$$

A positive numerator means an outward field; a negative numerator means an inward field. The magnitude is the absolute value of the coefficient of $\hat{r}$, in N C^{-1} .

Step 5: (ii)(III) Find the inner surface charge densities Divide each induced inner-surface charge by its spherical area. Using the stated radius r_1 for the small hollow sphere and inner radius r_2 for the shell,

$$\sigma_{\text{sphere, in}} = -\frac{2q}{4\pi r_1^2}, \quad \sigma_{\text{shell, in}} = -\frac{Q + 2q}{4\pi r_2^2}.$$

The density units are C m^{-2} . These charges ensure zero electric field in the conducting material, while the prescribed net charges determine the charges remaining on the outer surfaces.

Step 6: Conclusion (i) The force is attractive, with $F/\ell = 3\lambda^2/(2\pi\epsilon_0 r)$. (ii)(I) The requested fluxes are $2q/\epsilon_0$ and $(Q + 2q)/\epsilon_0$. (II) The fields are $(Q - q)\hat{r}/(4\pi\epsilon_0 x^2)$ outside and $(Q + 2q)\hat{r}/(4\pi\epsilon_0 x^2)$ in the gap. (III) The inner surface densities are $-2q/(4\pi r_1^2)$ and $-(Q + 2q)/(4\pi r_2^2)$, respectively.

Quick Tip to Solve: Work outward from the central charge: each inner conducting surface cancels its cavity charge, and the remaining charge goes to its outer surface. Keep the printed Q and q distinct.

33(a). (i) A light bulb and an open coil inductor are connected in series across an ac source of variable frequency. How will the glow of the bulb be affected when:

(I) an iron bar is inserted inside the coil, and

(II) the frequency of the source is decreased?

Justify your answers. Assume that in each above case other factors remain unchanged.

(ii) An ac voltage $V = 280 \sin(100\pi t)$ volt is connected across a series LCR circuit in which $R = 400 \, \Omega$, $L = \frac{5}{\pi} \text{ H}$ and $C = \frac{50}{\pi} \, \mu\text{F}$. Taking $\sqrt{2} = 1.4$, calculate:

(I) impedance of the circuit.

(II) rms value of current that flows in the circuit.

(III) power factor of the circuit.

[5 marks]

Solution:

Concept:

- An inductor offers inductive reactance $X_L = \omega L = 2\pi fL$, where L is inductance and f is frequency. For a series resistor and ideal inductor, $Z = \sqrt{R^2 + X_L^2}$ and $I_{\text{rms}} = V_{\text{rms}}/Z$.
- The bulb's glow depends on its heating power $P_b = I_{\text{rms}}^2 R_b$, where R_b is its resistance. At fixed source voltage, increasing the series impedance decreases the current and bulb power.
- In a series LCR circuit, capacitive reactance is $X_C = 1/(\omega C)$, impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and power factor is $\cos \phi = R/Z$. For sinusoidal voltage of peak value V_0 , $V_{\text{rms}} = V_0/\sqrt{2}$.

Step 1: (i)(I) Insert the iron bar into the coil An iron core increases the magnetic permeability in the coil and hence increases its inductance L . At unchanged frequency, $X_L = 2\pi fL$ increases. With the other factors held fixed, the total series impedance rises and the rms current through both the coil and bulb decreases:

$$L \uparrow \Rightarrow X_L \uparrow \Rightarrow Z \uparrow \Rightarrow I_{\text{rms}} \downarrow \Rightarrow P_b \downarrow.$$

Therefore, the bulb glows less brightly. The explanation uses the increase in inductive opposition to alternating current.

Step 2: (i)(II) Decrease the source frequency Consider this change separately, with inductance, source voltage and resistances held fixed as specified. Decreasing f decreases $X_L = 2\pi fL$, so the impedance of the series bulb-inductor circuit decreases. Its current increases, and the bulb dissipates more power:

$$f \downarrow \Rightarrow X_L \downarrow \Rightarrow Z \downarrow \Rightarrow I_{\text{rms}} \uparrow \Rightarrow P_b \uparrow.$$

Thus the bulb glows more brightly. This part concerns a series inductive circuit; the capacitor specified in part (ii) belongs to the separate numerical problem.

Step 3: (ii) Identify the voltage and compute both reactances Comparing $V = 280 \sin(100\pi t)$ with $V = V_0 \sin(\omega t)$ gives $V_0 = 280 \text{ V}$ and $\omega = 100\pi \text{ rad s}^{-1}$. The printed capacitance is $C = (50/\pi) \, \mu\text{F} = (50/\pi) \times 10^{-6} \text{ F}$. Hence

$$X_L = \omega L = (100\pi) \frac{5}{\pi} = 500 \, \Omega,$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(100\pi)(50/\pi) \times 10^{-6}} = 200 \, \Omega.$$

The net reactance is $X_L - X_C = 300 \, \Omega$, which is positive. Thus the circuit is inductive, and its current lags the applied voltage.

Step 4: (ii)(I) Calculate the impedance Resistance is perpendicular to net reactance in the impedance

triangle, so their contributions combine quadratically:

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{400^2 + 300^2} = \sqrt{250000} = 500 \Omega. \end{aligned}$$

The reactances subtract because inductive and capacitive voltage contributions have opposite phases relative to the current. Adding their magnitudes would give the wrong impedance.

Step 5: (ii)(II) Calculate the rms current The coefficient 280 is the peak voltage, so first convert it to an rms value using the approximation explicitly supplied:

$$V_{\text{rms}} = \frac{280}{\sqrt{2}} = \frac{280}{1.4} = 200 \text{ V}.$$

Now apply the ac form of Ohm's law:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{200}{500} = 0.40 \text{ A}.$$

Using $280/Z$ directly would instead give the peak current 0.56 A, which is not the rms value requested.

Step 6: (ii)(III) Determine the power factor For a series LCR circuit, the ratio of resistance to impedance is the cosine of the voltage-current phase difference:

$$\cos \phi = \frac{R}{Z} = \frac{400}{500} = 0.80.$$

Since $X_L > X_C$, current lags voltage, so this is a lagging power factor. It is dimensionless; the lagging description specifies the inductive nature that the positive cosine alone does not distinguish.

Step 7: Conclusion (i) Inserting the iron bar dims the bulb; decreasing frequency makes it brighter. (ii) The impedance is 500Ω , the rms current is 0.40 A, and the power factor is 0.80 lagging.

Quick Tip to Solve: Read ω from the sine argument and convert microfarads to farads before finding X_C . Here $R : X : Z = 400 : 300 : 500$ is a scaled 4 : 3 : 5 triangle; divide peak voltage by 1.4 for the requested rms current.

33(b). (i) State Lenz's law and explain that it follows the law of conservation of energy.

(ii) Write the dimensional formula for self-inductance.

The current in a coil changes from 8.0 A to 2.0 A in 0.6 s. If an average emf induced in the coil is 50 V, calculate the self-inductance of the coil. [5 marks]

Solution:

Concept:

- Lenz's law fixes the direction of induced current: its magnetic effect opposes the change in magnetic flux that produces it. The minus sign in Faraday's law, $\varepsilon = -d(N\Phi)/dt$, expresses this opposition, where $N\Phi$ is the flux linkage.
- Conservation of energy requires the electrical energy produced by induction to come from work done by an external agent or from energy already stored in the magnetic field.
- For a coil of constant self-inductance L , the average self-induced emf is $\varepsilon_{av} = -L\Delta I/\Delta t$, where I is the coil current. The magnitude relation is $|\varepsilon_{av}| = L|\Delta I|/\Delta t$.

Step 1: (i) State Lenz's law The direction of the induced current in a closed circuit is such that the magnetic field produced by it opposes the change in magnetic flux responsible for inducing the current. Thus, if the external flux increases, the induced field opposes that increase; if the external flux decreases, the induced field acts to maintain it. It is the change in flux that is opposed, rather than the original flux in every situation.

Step 2: (i) Connect the opposing effect to conservation of energy Consider a bar magnet moved towards a conducting coil. The increasing flux induces a current whose magnetic field repels the approaching magnet. An external agent must therefore do work against this repulsive force to continue the motion. When the magnet is withdrawn, the induced current reverses and the coil attracts it, again opposing the imposed motion. The external mechanical work supplies the energy of the induced current, which can be dissipated as heat in the coil's resistance.

If the induced current instead assisted the change, an approaching magnet would be accelerated towards the coil. Electrical energy and increasing kinetic energy would then be obtained without the corresponding external work or decrease of stored energy. This would violate conservation of energy. The opposition required by Lenz's law therefore ensures energy conservation.

Step 3: (ii) Derive the dimensional formula Rearrange the self-induction relation to write $L = |\varepsilon|/|dI/dt|$. Electric potential difference is energy per unit charge, so its dimensions are

$$[\varepsilon] = \frac{[ML^2T^{-2}]}{[AT]} = [ML^2T^{-3}A^{-1}].$$

Since $[dI/dt] = [AT^{-1}]$, division gives

$$[L] = \frac{[ML^2T^{-3}A^{-1}]}{[AT^{-1}]} = [ML^2T^{-2}A^{-2}].$$

Here M , L , T and A inside the dimensional brackets represent mass, length, time and electric current;

the symbol L outside the brackets denotes self-inductance. Its SI unit is the henry, $\text{H} = \text{V s A}^{-1}$.

Step 4: (ii) Substitute the current change and time interval The current falls from $I_i = 8.0 \text{ A}$ to $I_f = 2.0 \text{ A}$ in $\Delta t = 0.6 \text{ s}$. Thus,

$$\begin{aligned}\Delta I &= I_f - I_i = 2.0 - 8.0 = -6.0 \text{ A}, \\ \frac{\Delta I}{\Delta t} &= \frac{-6.0}{0.6} = -10 \text{ A s}^{-1}.\end{aligned}$$

The given average emf, 50 V , is a magnitude. Hence use the magnitude of the rate of change to calculate the positive inductance:

$$L = \frac{|\varepsilon_{\text{av}}| \Delta t}{|\Delta I|} = \frac{50 \times 0.6}{6.0} \text{ H} = 5 \text{ H}.$$

The negative current change means the induced emf acts to sustain the original current and oppose its decrease, consistently with Lenz's law.

Step 5: Conclusion Lenz's law requires the induced current to oppose the change in flux, so external work supplies the electrical energy produced and energy is conserved. The dimensional formula of self-inductance is $[ML^2T^{-2}A^{-2}]$, and the coil has self-inductance $[L = 5 \text{ H}]$.

Quick Tip to Solve: For a falling current, calculate inductance with $|\Delta I| = 8.0 - 2.0$; use the negative sign only to explain that the induced emf sustains the original current.