

CBSE Class 12 Physics Question Paper 2026 with Solution PDF-Set 55/1/2



Question Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours

Maximum Marks: 70

General Instructions

- (i) This question paper contains **33** questions. **All** questions are **compulsory**.
- (ii) Question paper is divided into **FIVE** sections – Sections **A, B, C, D** and **E**.
- (iii) In **Section A** : Question numbers **1** to **16** are Multiple Choice (MCQ) type questions. Each question carries 1 mark.
- (iv) In **Section B** : Question numbers **17** to **21** are Very Short Answer (VSA) type questions. Each question carries 2 marks.
- (v) In **Section C** : Question numbers **22** to **28** are Short Answer (SA) type questions. Each question carries 3 marks.
- (vi) In **Section D** : Question numbers **29** & **30** are Case Study-Based questions. Each question carries 4 marks.
- (vii) In **Section E** : Question numbers **31** to **33** are Long Answer (LA) type questions. Each question carries 5 marks.
- (viii) There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
- (ix) Kindly note that there is a separate question paper for Visually Impaired candidates.
- (x) Use of calculator is **NOT** allowed.

You may use the following values of physical constants wherever necessary :

$$c = 3 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ J s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\text{Mass of electron } (m_e) = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Mass of neutron} = 1.675 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23} \text{ per gram mole}$$

$$\text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Section A (16 marks)

1. In a region electric field is given by $\vec{E} = 4x \hat{i}$ N/C. The potential difference between points A ($x = 1$ m) and B ($x = 3$ m), ($V_A - V_B$) is: [1 mark]

- (A) -16 V
- (B) 16 V
- (C) -8 V
- (D) 8 V

Correct Answer: (B) 16 V

Solution:

Concept:

- Electric field points towards decreasing potential: $E_x = -dV/dx$, where V is potential and x is position.
- For a varying field, $V_B - V_A = -\int_{x_A}^{x_B} E_x dx$; the required reverse difference has the opposite sign.

Step 1: Integrate the non-uniform field The field $E_x = 4x$ changes along the path, so use its integral rather than its value at one endpoint. With $x_A = 1$ m and $x_B = 3$ m,

$$V_B - V_A = -\int_1^3 4x dx = -[2x^2]_1^3 = -2(9 - 1) = -16 \text{ V}.$$

Step 2: Check the requested order and options The question asks for $V_A - V_B$, so reverse the sign of the calculated difference. Option (A) would answer $V_B - V_A$. Options (C) and (D) have the wrong magnitude: integrating $4x$ gives $2x^2$, and evaluating both limits gives a magnitude of 16 V, not 8 V.

Step 3: Conclusion $V_A - V_B = +16$ V. Hence, the correct option is (B).

Quick Tip to Solve: Write the requested potential order before integrating; motion along the positive electric field must lower potential.

2. In an unbiased p-n junction, at equilibrium, which of the following statements is true? [1 mark]

- (A) Diffusion current is zero but drift current exists.
- (B) Diffusion current exists but drift current is zero.
- (C) Diffusion and drift currents are equal and opposite.
- (D) Both the diffusion and drift currents exist but are unequal.

Correct Answer: (C) Diffusion and drift currents are equal and opposite.

Solution:

Concept:

- Diffusion current is caused by the concentration gradient of majority charge carriers across the p-n junction.
- The built-in electric field across the depletion layer drives a drift current opposite to the diffusion current. At equilibrium the net current is zero.

Step 1: Identify both current mechanisms Electrons diffuse from the n side towards the p side, and holes diffuse in the opposite direction. Their diffusion establishes the depletion region and its electric

field. Thermally generated minority carriers are swept across the junction by this field, producing the drift current. Thus an unbiased junction can have both component currents even though no external current flows.

Step 2: Apply the equilibrium condition Choose the diffusion-current direction as positive. In equilibrium,

$$I_{\text{net}} = I_{\text{diff}} + I_{\text{drift}} = 0.$$

The components therefore have equal magnitudes and opposite directions. Options (A) and (B) incorrectly remove one component; option (D) would give a nonzero net current and cannot describe equilibrium.

Step 3: Conclusion Diffusion and drift currents are equal and opposite. Hence, the correct option is (C).

Quick Tip to Solve: In an equilibrium junction, zero net current means cancellation of drift and diffusion currents, not absence of either mechanism.

3. A copper wire is stretched to increase its length by 1%. Then the change in its resistance is close to: [1 mark]

- (A) 1%
- (B) 4%
- (C) -4%
- (D) 2%

Correct Answer: (D) 2%

Solution:

Concept:

- The resistance of a uniform wire is $R = \rho L/A$, where ρ is resistivity, L is length and A is cross-sectional area.
- For stretching at unchanged temperature with constant volume $\mathcal{V} = AL$, resistivity stays fixed and $R = \rho L^2/\mathcal{V}$, so $R \propto L^2$.

Step 1: Include the decrease in cross-sectional area After a 1% increase in length, $L' = 1.01L$. Conservation of the wire's volume gives $A'L' = AL$, hence $A' = A/1.01$. Both the greater length and the smaller area increase resistance.

Step 2: Calculate the fractional resistance change Using the same resistivity before and after stretching,

$$\frac{R'}{R} = \frac{L'}{L} \frac{A}{A'} = (1.01)^2 = 1.0201.$$

Thus $\Delta R/R = 0.0201$, or a 2.01% increase, close to 2%. Option (A) neglects the area change; (B) overestimates the increase, and (C) has the wrong sign.

Step 3: Conclusion The resistance increases by approximately 2%. Hence, the correct option is (D).

Quick Tip to Solve: For a slightly stretched wire of fixed volume, double the percentage increase in length to estimate the percentage increase in resistance.

4. Four independent waves are expressed as:

(i) $y_1 = A_1 \sin \omega t$ (ii) $y_2 = A_2 \sin 2\omega t$

(iii) $y_3 = A_3 \cos \omega t$ (iv) $y_4 = A_4 \sin(\omega t + \pi/3)$

The interference between two of these waves is possible in:

[1 mark]

- (A) (i) and (iii) only
(B) (iii) and (iv) only
(C) (i), (iii) and (iv) only
(D) All of them

Correct Answer: (C) (i), (iii) and (iv) only

Solution:

Concept:

- Sustained interference requires waves of the same frequency with a constant phase difference; their amplitudes need not be equal.
- A cosine is a phase-shifted sine: $\cos \omega t = \sin(\omega t + \pi/2)$. Here ω denotes angular frequency.

Step 1: Compare frequencies and phases Rewrite wave (iii) as $y_3 = A_3 \sin(\omega t + \pi/2)$. Waves (i), (iii) and (iv) then all have angular frequency ω , with respective phases 0, $\pi/2$ and $\pi/3$. Each pair among these has a time-independent phase difference: $\pi/2$, $\pi/3$ or $\pi/6$ in magnitude.

Step 2: Exclude the wave with a different frequency Wave (ii) has angular frequency 2ω , so its phase difference from any of the other three changes with time. It therefore does not give a sustained interference pattern with them. Options (A) and (B) each omit other eligible pairs; option (D) incorrectly includes wave (ii). The stated harmonic expressions identify all three same-frequency waves as eligible.

Step 3: Conclusion Interference is possible between pairs selected from (i), (iii) and (iv) only. Hence, the correct option is (C).

Quick Tip to Solve: Convert cosine to sine with a constant phase shift, then compare the coefficients of time; unequal amplitudes do not exclude interference.

5. A concave lens of focal length 40 cm is coaxially in contact with two convex lenses, each of focal length 20 cm, on each side. The focal length of the combination is:

[1 mark]

- (A) zero
(B) $\frac{20}{3}$ cm
(C) $-\frac{20}{3}$ cm
(D) $\frac{40}{3}$ cm

Correct Answer: (D) $\frac{40}{3}$ cm

Solution:

Concept:

- For thin lenses coaxially in contact, their powers add: $1/F = 1/f_1 + 1/f_2 + 1/f_3$, where F is the combination's focal length.

- In air, a convex lens has positive focal length and a concave lens has negative focal length under the Cartesian sign convention.

Step 1: Assign signs to all three focal lengths There is one concave lens between two convex lenses. Therefore the concave lens has $f_1 = -40$ cm, while each convex lens has focal length $+20$ cm. Both convex lenses must be included in the sum of powers.

Step 2: Add the reciprocal focal lengths Using centimetres consistently,

$$\frac{1}{F} = -\frac{1}{40} + \frac{1}{20} + \frac{1}{20} = \frac{3}{40} \text{ cm}^{-1}.$$

The total power is positive, so the combination is converging. Option (A) does not follow from this nonzero reciprocal; (B) would require twice the calculated power, and (C) has both an incorrect sign and magnitude.

Step 3: Conclusion The focal length is $F = +\frac{40}{3}$ cm. Hence, the correct option is (D).

Quick Tip to Solve: Count every lens and attach the concave lens's negative sign before adding reciprocal focal lengths; never add focal lengths directly.

6. The distance-of-closest-approach for an alpha particle is ' d ' when it moves head-on with speed v towards a target nucleus. If the alpha particle is replaced by a proton moving with the same speed, the new distance-of-closest-approach will be: [1 mark]

- (A) $2d$
 (B) d
 (C) $\frac{d}{2}$
 (D) $\frac{d}{\sqrt{2}}$

Correct Answer: (A) $2d$

Solution:

Concept:

- In a head-on approach to a heavy target nucleus, the projectile's initial kinetic energy becomes electrostatic potential energy at the turning point: $\frac{1}{2}mv^2 = kQq/r$.
- Here m and q are the projectile mass and charge, Q is the target charge, $k = 1/(4\pi\epsilon_0)$, and r is the distance of closest approach. At fixed speed and target, $r \propto q/m$.

Step 1: Compare the alpha particle and proton An alpha particle has charge $q_\alpha = 2e$ and mass approximately $m_\alpha = 4m_p$. A proton has charge $q_p = e$ and mass m_p . The question fixes their speed, so their kinetic energies are different: the proton has one quarter of the alpha particle's initial kinetic energy.

Step 2: Form the distance ratio Solving energy conservation gives $r = 2kQq/(mv^2)$. The common target charge and speed cancel in the ratio:

$$\frac{r_p}{d} = \frac{q_p/m_p}{q_\alpha/m_\alpha} = \frac{e/m_p}{2e/(4m_p)} = 2.$$

Option (C) would follow from incorrectly keeping kinetic energy fixed while halving charge. Options

(B) and (D) likewise do not satisfy the required charge-to-mass ratio at the same speed.

Step 3: Conclusion The proton's distance of closest approach is $2d$. Hence, the correct option is (A).

Quick Tip to Solve: Check what is held fixed: at equal speed, closest approach scales as charge divided by mass; at equal kinetic energy, it scales as charge.

7. Electromagnetic waves used for purification of water are:

[1 mark]

- (A) X-rays
- (B) Ultraviolet rays
- (C) Infrared rays
- (D) Ultrasonic rays

Correct Answer: (B) Ultraviolet rays

Solution:

Concept:

- Ultraviolet radiation is part of the electromagnetic spectrum, at frequencies above visible violet light.
- Germicidal ultraviolet radiation damages the genetic material of microorganisms, preventing their reproduction and enabling disinfection of water.

Step 1: Identify the relevant electromagnetic application In the standard electromagnetic-spectrum applications taught here, ultraviolet rays are used to disinfect water by inactivating microorganisms. This germicidal action is the intended meaning of purification in the question; it does not describe removing dissolved salts or other chemical impurities.

Step 2: Compare the remaining options X-rays are associated chiefly with internal imaging and material examination, rather than the standard water-disinfection application asked here. Infrared radiation primarily produces heating and is not the required germicidal band. Ultrasonic waves are mechanical sound waves and are not electromagnetic waves at all, so option (D) also fails the category stated in the question.

Step 3: Conclusion Ultraviolet rays are used for the stated purification of water. Hence, the correct option is (B).

Quick Tip to Solve: Connect water disinfection with germicidal ultraviolet radiation, and reject ultrasonic waves immediately when the question specifies electromagnetic waves.

8. A conducting wire connects two charged metallic spheres A and B of radii r_1 and r_2 respectively. The distance between the spheres is very large compared to their radii. The ratio of electric fields, (E_A/E_B) at the surfaces of spheres A and B will be: [1 mark]

- (A) $\frac{r_1}{r_2}$
(B) $\frac{r_2}{r_1}$
(C) $\frac{r_1^2}{r_2^2}$
(D) $\frac{r_2^2}{r_1^2}$

Correct Answer: (B) $\frac{r_2}{r_1}$

Solution:

Concept:

- Conductors joined by a conducting wire reach electrostatic equilibrium at a common potential.
- For an isolated conducting sphere of radius r and charge Q , $V = kQ/r$ and the surface field is $E = kQ/r^2 = V/r$, where $k = 1/(4\pi\epsilon_0)$.

Step 1: Apply equality of potentials The spheres are far apart, so each may be treated as an isolated sphere. Charge flows through the connecting wire until $V_A = V_B$. If the final charges are Q_A and Q_B ,

$$\frac{kQ_A}{r_1} = \frac{kQ_B}{r_2} \Rightarrow \frac{Q_A}{Q_B} = \frac{r_1}{r_2}.$$

Equal potential does not mean equal charge when the radii differ.

Step 2: Compare the surface fields Divide the field magnitudes and use the charge ratio:

$$\frac{E_A}{E_B} = \frac{Q_A}{Q_B} \frac{r_2^2}{r_1^2} = \frac{r_1}{r_2} \frac{r_2^2}{r_1^2} = \frac{r_2}{r_1}.$$

Option (A) gives the charge ratio instead of the field ratio. Options (C) and (D) incorrectly retain squared radii; the redistribution of charge removes one power. The smaller sphere has the stronger surface field.

Step 3: Conclusion The required ratio is $E_A/E_B = r_2/r_1$. Hence, the correct option is (B).

Quick Tip to Solve: For connected spheres, first set their potentials equal and use $E = V/r$: the surface field varies inversely with radius.

9. A straight conductor lies along x -axis. It carries a current of 10 A along $+x$ direction. The magnetic field $\vec{B}$ due to 1 cm segment of this conductor, centred at the origin, at a point $(0, 1 \text{ m}, 0)$ is: [1 mark]

- (A) $(1 \text{ nT}) \hat{j}$
- (B) $(10 \text{ nT}) \hat{k}$
- (C) $-(10 \text{ nT}) \hat{k}$
- (D) $-(1 \text{ nT}) \hat{j}$

Correct Answer: (B) $(10 \text{ nT}) \hat{k}$

Solution:

Concept:

- The Biot-Savart law for a short current element is $\Delta \vec{B} = (\mu_0/4\pi) I \Delta \vec{\ell} \times \hat{r}/r^2$, where I is current and r is the distance to the observation point.
- For a segment much shorter than its distance from the point, use $\Delta B = (\mu_0/4\pi) I \Delta \ell \sin \theta / r^2$; the cross product fixes the direction.

Step 1: Determine the field direction The current element points along $+x$, while the observation point is on $+y$. Thus $\theta = 90^\circ$ and

$$\hat{i} \times \hat{j} = \hat{k}.$$

The magnetic field points along $+z$. Options (A) and (D) incorrectly place the field along the displacement direction; option (C) reverses the right-hand-rule direction.

Step 2: Substitute the short-segment values Here $\Delta \ell = 1 \text{ cm} = 10^{-2} \text{ m}$, $I = 10 \text{ A}$, and $r = 1 \text{ m}$. Since $\Delta \ell / r = 0.01$, the short-element approximation is appropriate:

$$\Delta B = 10^{-7} \frac{10 \times 10^{-2} \times 1}{1^2} = 10^{-8} \text{ T} = 10 \text{ nT}.$$

This is the contribution of the specified centimetre-long segment, so the formula for the entire infinite wire must not be used.

Step 3: Conclusion The field is $\vec{B} = (10 \text{ nT}) \hat{k}$. Hence, the correct option is (B).

Quick Tip to Solve: Use $\hat{i} \times \hat{j} = \hat{k}$ for the direction, then convert the segment's centimetre length to metres before applying Biot-Savart.

10. The angular width of interference fringes in Young's double-slit experiment depends on: [1 mark]

- (A) distance between the slits and the screen only
- (B) wavelength of light used only
- (C) both wavelength of light used and the slits separation
- (D) slits separation only

Correct Answer: (C) both wavelength of light used and the slits separation

Solution:

Concept:

- In Young's double-slit experiment, the linear fringe width is $\beta = \lambda D / d$, where λ is wavelength, D is the screen distance, and d is the slit separation.
- For small angles, angular fringe width is the angle between adjacent bright fringes, $\Delta \theta = \beta / D$.

Step 1: Express the angular fringe width Divide the linear distance between successive bright fringes by the distance from the slits to the screen:

$$\Delta\theta = \frac{\beta}{D} = \frac{\lambda D/d}{D} = \frac{\lambda}{d}.$$

The factor D cancels. Moving the screen changes the linear width but does not change the angular width in the usual small-angle treatment.

Step 2: Identify both dependencies Increasing λ increases the angular fringe width; increasing d decreases it. Thus both wavelength and slit separation matter. Option (A) incorrectly uses screen distance, while options (B) and (D) each omit one of the two quantities in λ/d .

Step 3: Conclusion The angular fringe width depends on both the wavelength and the slit separation. Hence, the correct option is (C).

Quick Tip to Solve: To turn the linear fringe-width formula into the angular one, divide by the screen distance and cancel D .

11. An electromagnetic wave passes from vacuum into a dielectric medium with relative electrical permittivity $(3/2)$ and relative magnetic permeability $(8/3)$. Then, its: [1 mark]

- (A) wavelength is doubled and frequency remains unchanged.
- (B) wavelength is doubled and frequency is halved.
- (C) wavelength is halved and frequency remains unchanged.
- (D) wavelength and frequency both will remain unchanged.

Correct Answer: (C) wavelength is halved and frequency remains unchanged.

Solution:

Concept:

- The speed of an electromagnetic wave in a medium is $v = c/\sqrt{\epsilon_r \mu_r}$, where c is its vacuum speed and ϵ_r, μ_r are the relative permittivity and permeability.
- Frequency f remains unchanged on transmission across a stationary boundary; the wavelength is $\lambda = v/f$.

Step 1: Calculate the speed in the medium Use both specified material properties; the relative permeability is not unity here:

$$\epsilon_r \mu_r = \frac{3}{2} \times \frac{8}{3} = 4, \quad v = \frac{c}{\sqrt{4}} = \frac{c}{2}.$$

The wave therefore travels at half its vacuum speed after entering the dielectric.

Step 2: Apply the boundary condition for frequency The source fixes the oscillation frequency, so $f_{\text{medium}} = f_{\text{vacuum}} = f$. If the vacuum wavelength is $\lambda_0 = c/f$, then

$$\lambda_{\text{medium}} = \frac{c/2}{f} = \frac{\lambda_0}{2}.$$

Options (A) and (B) incorrectly double the wavelength, and (B) also changes frequency. Option (D) ignores the reduction of speed and wavelength.

Step 3: Conclusion The wavelength is halved and the frequency remains unchanged. Hence, the

correct option is (C).

Quick Tip to Solve: Calculate $\sqrt{\epsilon_r \mu_r}$ first; divide both speed and wavelength by this factor while keeping frequency fixed.

12. A resistor and an inductor of negligible resistance are connected in series to a 20 V ac source. If the voltage across the resistor is 12 V, the voltage across the inductor will be: [1 mark]

- (A) 6 V
- (B) 8 V
- (C) 10 V
- (D) 16 V

Correct Answer: (D) 16 V

Solution:

Concept:

- In a series circuit containing a resistor and an ideal inductor, the same alternating current flows through both components.
- The resistor voltage V_R is in phase with current, while the inductor voltage V_L leads current by 90° . Their phasors are perpendicular, so $V^2 = V_R^2 + V_L^2$, where V is the source voltage.

Step 1: Use perpendicular voltage phasors Since the inductor's resistance is negligible, its voltage has no in-phase resistive component. The total voltage is therefore the vector sum of V_R and V_L , not their arithmetic sum. The given voltages can be treated consistently as rms values.

Step 2: Calculate the inductor voltage Rearrange the phasor relation and substitute $V = 20\text{ V}$ and $V_R = 12\text{ V}$:

$$V_L = \sqrt{V^2 - V_R^2} = \sqrt{20^2 - 12^2}\text{ V} = \sqrt{256}\text{ V} = 16\text{ V}.$$

The tempting 8 V in option (B) comes from ordinary subtraction, which neglects the phase difference. Neither 6 V nor 10 V satisfies $12^2 + V_L^2 = 20^2$.

Step 3: Conclusion The voltage across the inductor is 16 V. Hence, the correct option is (D).

Quick Tip to Solve: For an ideal series RL circuit, form a right triangle of voltage phasors; here 12, 16, 20 is a scaled 3, 4, 5 triangle.

13. For question number 13 to 16, two statements are given – one labelled as Assertion (A) and the other labelled as Reason (R). Select the correct answer to these questions from the options (A), (B), (C) and (D) as given below:

Reason (A): In Bohr model of hydrogen atom, the energy levels are discrete and quantised.

Reason (R): In a hydrogen atom, the electrostatic force on the electron provides the necessary centripetal force to it to revolve around the nucleus. [1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Solution:

Concept:

- Bohr's quantisation condition is $m_e v r = n \hbar$, where $n = 1, 2, 3, \dots$, m_e is electron mass, v its speed and r its orbital radius.
- In hydrogen, electrostatic attraction supplies centripetal force, $ke^2/r^2 = m_e v^2/r$. Quantised angular momentum together with this force law gives $E_n = -13.6/n^2$ eV.

Step 1: Evaluate statement A The first statement, although labelled Reason (A) in the printed English question, is the assertion under the common instructions. It is true: Bohr's hydrogen energies are -13.6 eV, -3.4 eV, and so on for integer n . The electron cannot occupy energies continuously between the allowed levels.

Step 2: Evaluate the reason The reason is also true within Bohr's model. The attractive electrostatic force between proton and electron acts toward the nucleus and supplies the centripetal force needed for the assumed circular orbit:

$$\frac{ke^2}{r^2} = \frac{m_e v^2}{r}.$$

Step 3: Check the explanatory link The classical force equation by itself permits a continuous range of radii and energies. The discrete levels arise only when Bohr's additional condition $m_e v r = n \hbar$ is imposed. Thus the reason describes the force maintaining the orbit but does not explain quantisation. Option (A) claims an explanation that is absent; options (C) and (D) incorrectly reject true statements.

Step 4: Conclusion Both the assertion and the reason are true, but the reason does not correctly explain the assertion. Hence, the correct option is (B).

Quick Tip to Solve: Separate the classical orbit condition from Bohr's integer angular-momentum condition: only the latter restricts the allowed energy levels.

14. Reason (A): The mass of a nucleus is less than the sum of the masses of the constituent nucleons.

Reason (R): Energy is absorbed when the nucleons are bound together to form a nucleus. [1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution:

Concept:

- A bound nucleus has mass defect $\Delta m = Zm_p + Nm_n - M$, where Z and N are its proton and neutron counts, m_p and m_n are free nucleon masses, and M is the nuclear mass.
- Its binding energy is $B = \Delta mc^2 > 0$. This energy is released on formation from free nucleons and must be supplied to separate the bound nucleus into free nucleons.

Step 1: Evaluate statement A The first statement is the assertion under the common instructions, although the English paper labels it Reason (A). It is true. Binding lowers the system's total rest energy, giving

$$M = Zm_p + Nm_n - \frac{B}{c^2} < Zm_p + Nm_n.$$

Thus the bound nucleus has less mass than its separate constituent nucleons.

Step 2: Evaluate the reason The reason is false because it reverses the energy transfer. When initially free nucleons form a bound nucleus, energy is released. Energy must instead be absorbed when the nucleus is completely broken into separate nucleons.

Step 3: Check the explanatory link A false statement about absorption during formation cannot explain the true mass deficit. The correct explanation is release of binding energy, corresponding to the decrease Δmc^2 in rest energy. Options (A) and (B) wrongly treat the reason as true, and option (D) also wrongly treats the assertion as false.

Step 4: Conclusion The assertion is true, but the reason is false. Hence, the correct option is (C).

Quick Tip to Solve: Remember the direction of binding-energy transfer: assembling a bound nucleus releases energy; separating it into free nucleons requires energy.

15. Assertion (A): All atoms have a net magnetic moment.

Reason (R): A current loop does not always behave as a magnetic dipole.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (D) Both Assertion (A) and Reason (R) are false.

Solution:

Concept:

- The net magnetic moment of an atom is the vector sum of the magnetic moments associated with its electrons; orbital motion and electron spin contribute to this sum.
- A current loop acts as a magnetic dipole with magnetic moment $\mu = IA$, where I is current and A is its directed area.

Step 1: Judge the Assertion Electron magnetic moments can cancel in an atom. In atoms with completely filled electron shells, the paired spin contributions and orbital contributions cancel, giving zero net permanent magnetic moment. Therefore, the word “all” makes Assertion (A) false.

Step 2: Judge the Reason A current-carrying loop has a magnetic moment and behaves as a magnetic dipole; its magnetic field has the dipolar form at distances large compared with the loop size. Thus the claim that it does not always behave as a magnetic dipole is false in the stated current-loop model.

Step 3: Check the explanatory relationship Both statements are false, so Reason (R) cannot correctly explain Assertion (A). Options (A) and (B) require both statements to be true, while option (C) requires a true Assertion. Each is ruled out by the cancellation of atomic magnetic moments.

Step 4: Conclusion Both the Assertion and the Reason are false. Hence, the correct option is (D).

Quick Tip to Solve: Test the word “all” using a filled-shell atom: cancellation of electron moments gives a direct counterexample.

16. Assertion (A): If accelerated electrons are passed through a narrow slit, a diffraction pattern is observed.

Reason (R): Electrons behave as both particles and waves.

[1 mark]

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Both Assertion (A) and Reason (R) are false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Concept:

- An electron of momentum p has de Broglie wavelength $\lambda = h/p$, where h is Planck's constant.
- Diffraction is the spreading and interference of waves at an aperture. It is appreciable when the aperture width is comparable to the wavelength.

Step 1: Judge the Assertion Accelerated electrons have momentum and an associated de Broglie wavelength. On passing through a suitably narrow slit, their matter waves diffract, producing a distribution of maxima and minima in the electron detection intensity. Assertion (A) is therefore true under the stated narrow-slit conditions.

Step 2: Judge the Reason Electrons show particle properties, such as localized detection and definite charge, as well as wave properties, such as interference and diffraction. Hence Reason (R), which states that electrons behave as both particles and waves, is true.

Step 3: Establish the explanation It is the wave aspect of electron motion that accounts for the diffraction pattern in the Assertion. Therefore, the Reason supplies the physical explanation, ruling out option (B), which denies that link. Options (C) and (D) are also excluded because the Reason is true.

Step 4: Conclusion The Assertion and the Reason are true, and the Reason correctly explains the Assertion. Hence, the correct option is (A).

Quick Tip to Solve: When a question describes electron diffraction, connect the observation directly to the electron's de Broglie wave.

Section B (10 marks)

17(a). A beam of light consisting of two wavelengths 400 nm and 600 nm is used to illuminate a single slit of width 1 mm. Find the least distance of the point from the central maximum where the dark fringes due to both wavelengths coincide on the screen placed 1.5 m from the slit. [2 marks]

Solution:

Concept:

- For a single slit of width a , a dark fringe of order m satisfies $a \sin \theta = m\lambda$, where $m = 1, 2, 3, \dots$ and λ is the wavelength.
- At small angles, its distance from the central maximum is $y_m = m\lambda D/a$, where D is the slit-to-screen distance. Coincident minima therefore require equal values of $m\lambda$.

Step 1: Find the least coincident orders Let m_1 and m_2 denote the dark-fringe orders for 400 nm and 600 nm, respectively. At the same point on the screen,

$$m_1(400) = m_2(600) \Rightarrow 2m_1 = 3m_2.$$

The smallest positive integer solution is $m_1 = 3$, $m_2 = 2$. Hence the third minimum of the shorter wavelength coincides with the second minimum of the longer wavelength. Zero order is excluded because the centre is bright, not dark.

Step 2: Calculate the screen distance Convert $a = 1 \text{ mm} = 10^{-3} \text{ m}$, use $D = 1.5 \text{ m}$, and substitute either wavelength with its order:

$$\begin{aligned} y &= \frac{3(400 \times 10^{-9})(1.5)}{10^{-3}} \text{ m} \\ &= 1.8 \times 10^{-3} \text{ m}. \end{aligned}$$

Using $m_2\lambda_2 = 2(600 \text{ nm})$ gives the same result. Here $y/D = 1.2 \times 10^{-3}$, so the small-angle approximation is appropriate.

Step 3: Conclusion The least distance is $y = 1.8 \text{ mm}$, on either side of the central maximum.

Quick Tip to Solve: Reduce the wavelengths to the ratio 2 : 3; the first common minimum has the reversed order ratio 3 : 2.

17(b). In a Young's double-slit experimental set-up with slit separation 0.6 mm a beam of light consisting of two wavelengths 440 nm and 660 nm is used to obtain interference pattern on a screen kept 1.5 m in front of the slits. Find the least distance of the point from the central maximum where the bright fringes due to both the wavelengths coincide. [2 marks]

Solution:

Concept:

- In Young's double-slit experiment, a bright fringe of order m is at $y_m = m\lambda D/d$, where d is slit separation, D is screen distance and λ is wavelength.
- Bright fringes for two wavelengths coincide when $m_1\lambda_1 = m_2\lambda_2$. To find the first coincidence away from the central maximum, use the smallest positive integer orders.

Step 1: Identify the first noncentral coincidence For $\lambda_1 = 440$ nm and $\lambda_2 = 660$ nm, the coincidence condition is

$$m_1(440) = m_2(660) \Rightarrow 2m_1 = 3m_2.$$

Thus the least positive orders are $m_1 = 3$ and $m_2 = 2$. The central bright fringes already coincide at $m_1 = m_2 = 0$; the requested distance from the central maximum is understood as the next coincidence away from it.

Step 2: Substitute the slit separation and screen distance With $d = 0.6$ mm $= 0.6 \times 10^{-3}$ m and $D = 1.5$ m,

$$\begin{aligned} y &= \frac{3(440 \times 10^{-9})(1.5)}{0.6 \times 10^{-3}} \text{ m} \\ &= 3.3 \times 10^{-3} \text{ m.} \end{aligned}$$

As a check, the fringe widths are 1.1 mm and 1.65 mm. Their third and second multiples, respectively, give the same screen position.

Step 3: Conclusion The least nonzero distance of coincident bright fringes from the central maximum is **3.3 mm**, on either side.

Quick Tip to Solve: Exclude the common central fringe, then use orders 3 and 2 for wavelengths in the ratio 2 : 3.

18. Find the ratio $\frac{\lambda_\alpha}{\lambda_p}$ of de Broglie wavelength λ_α associated with an alpha particle to de Broglie wavelength λ_p associated with a proton if both are moving with the (a) same velocity (b) same kinetic energy. [2 marks]

Solution:

Concept:

- The de Broglie wavelength is $\lambda = h/p$, where h is Planck's constant and p is momentum. For nonrelativistic motion, $p = mv$ and $p = \sqrt{2mK}$, with mass m , speed v and kinetic energy K .
- An alpha particle contains two protons and two neutrons, so its mass is approximately $m_\alpha = 4m_p$. This standard mass approximation is sufficient for the requested ratios.

Step 1: Compare wavelengths at the same velocity For part (a), both particles have the same speed v ,

so their momenta are proportional to their masses. Using $\lambda = h/(mv)$,

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{h/(m_\alpha v)}{h/(m_p v)} = \frac{m_p}{m_\alpha} = \frac{1}{4}.$$

Thus the heavier alpha particle has four times the proton's momentum and one quarter of its wavelength.

Step 2: Compare wavelengths at the same kinetic energy For part (b), the common kinetic energy K gives $p = \sqrt{2mK}$, so wavelength is inversely proportional to the square root of mass:

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{h/\sqrt{2m_\alpha K}}{h/\sqrt{2m_p K}} = \sqrt{\frac{m_p}{m_\alpha}} = \frac{1}{2}.$$

At equal kinetic energy the alpha particle has twice the proton's momentum. Particle charge does not enter either comparison because neither condition specifies acceleration through a common potential difference.

Step 3: Conclusion (a) $\lambda_\alpha/\lambda_p = 1/4$; (b) $\lambda_\alpha/\lambda_p = 1/2$.

Quick Tip to Solve: At equal speed use the inverse mass ratio; at equal kinetic energy use its square root. Here $m_\alpha/m_p = 4$.

19. What is the order of magnitude of drift velocity of electrons in a conductor? Deduce the relation between the current flowing through a conductor and drift velocity of electrons in it. [2 marks]

Solution:

Concept:

- Drift velocity is the small average velocity acquired by free electrons in a conductor under an applied electric field; it is distinct from their random thermal motion.
- Electric current is the rate of flow of charge, $I = \Delta Q/\Delta t$. Let n be the number of free electrons per unit volume, A the cross-sectional area, and e the magnitude of the electron charge.

Step 1: State the usual scale of drift velocity In an ordinary metallic conductor carrying current, the electron drift speed is typically of the order of 10^{-4} m s^{-1} . It is not a universal fixed value: it depends on the material, current and cross-sectional area. The electron drift direction is opposite to the electric field and hence opposite to conventional current.

Step 2: Count the charge crossing a section If the magnitude of drift velocity is v_d , electrons drift through a distance $v_d \Delta t$ in time Δt . The volume whose electrons cross a chosen section in this time is $Av_d \Delta t$. Therefore,

$$\begin{aligned} N_e &= nAv_d \Delta t, \\ \Delta Q &= N_e e = neAv_d \Delta t, \\ I &= \frac{\Delta Q}{\Delta t} = neAv_d. \end{aligned}$$

Here I and v_d are magnitudes; the vector current density is $\vec{J} = -ne\vec{v}_d$, since electrons have negative charge.

Step 3: Conclusion The usual order of electron drift speed is 10^{-4} m s^{-1} , and its relation to the current magnitude is $I = neAv_d$.

Quick Tip to Solve: Imagine a cylinder of length $v_d \Delta t$: its volume times ne gives the charge crossing the section, so dividing by time immediately gives $I = neAv_d$.

20. A wire of length L is bent round into (i) a square coil having N turns and (ii) a circular coil having N turns. The coil in both cases is free to turn about a vertical axis coinciding with the plane of the coil, in a uniform, horizontal magnetic field and carry the same currents. Find the ratio of the maximum value of the torque acting on the square coil to that on the circular coil. [2 marks]

Solution:

Concept:

- The torque on a plane coil of N turns carrying current I in a uniform magnetic field B is $\tau = NIAB \sin \theta$, where A is the area of one turn and θ is the angle between the coil's normal and the field.
- Maximum torque is $\tau_{\max} = NIAB$. Each coil uses the same total wire length L , so the perimeter of each of its N turns is L/N .

Step 1: Find the area of one square turn Let the side of a square turn be a . Neglecting the length of connecting leads, the total wire length gives

$$4Na = L, \quad a = \frac{L}{4N}, \quad A_s = a^2 = \frac{L^2}{16N^2}.$$

The maximum torque on the square coil is consequently $\tau_{s,\max} = NIBL^2/(16N^2)$.

Step 2: Find the circular area and compare the torques For a circular turn of radius r , the wire length gives

$$2\pi rN = L, \quad r = \frac{L}{2\pi N}, \quad A_c = \pi r^2 = \frac{L^2}{4\pi N^2}.$$

The vertical rotation axis lies in the coil's plane and the field is horizontal. The coil can therefore reach the orientation with its normal perpendicular to the field, giving the stated maximum in either shape. Since N , I and B are common, they cancel from the ratio:

$$\frac{\tau_{s,\max}}{\tau_{c,\max}} = \frac{A_s}{A_c} = \frac{L^2/(16N^2)}{L^2/(4\pi N^2)} = \frac{\pi}{4}.$$

Step 3: Conclusion The required maximum-torque ratio, square coil to circular coil, is $\pi : 4$. The circular coil has the larger torque because it encloses the larger area for the same wire length and number of turns.

Quick Tip to Solve: With equal current, field and number of turns, compare only the area of one turn; first divide the total wire length by N to get that turn's perimeter.

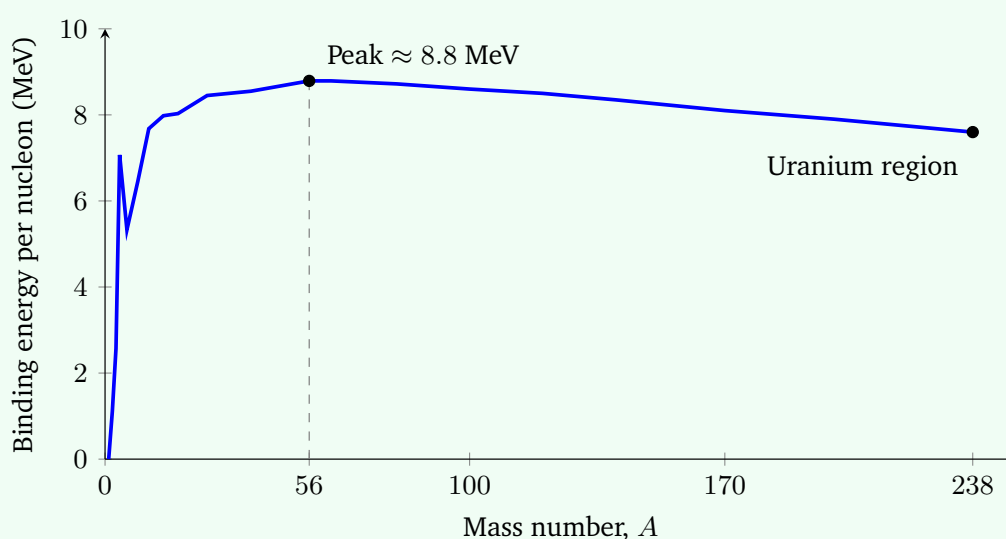
21. Draw a graph showing variation of binding energy per nucleon as a function of mass number A . The binding energy per nucleon for heavy nuclei ($A > 170$) decreases with increase in mass number. Explain its significance. [2 marks]

Solution:

Concept:

- Binding energy per nucleon is B/A , where B is the total binding energy and A the mass number. A larger B/A means that the nucleons are, on average, more tightly bound.
- If a nuclear reaction increases the total binding energy, the corresponding decrease in rest energy is released: $Q = B_{\text{products}} - B_{\text{initial}}$.

Step 1: Sketch the binding-energy curve The binding energy per nucleon rises rapidly, with irregularities, for light nuclei. It reaches a broad maximum of about 8.8 MeV near $A \simeq 56$, in the iron–nickel region. It then decreases gradually for heavier nuclei, reaching about 7.6 MeV near uranium. The sketch shows the overall trend; the peak identifies the region of greatest binding per nucleon.



Schematic binding energy per nucleon versus mass number (selected light-nucleus irregularities shown).

Step 2: Interpret the decrease for heavy nuclei A heavy nucleus with $A > 170$ has a smaller binding energy per nucleon than medium-mass nuclei. When it undergoes fission into suitable lighter nuclei, the products move towards the higher part of the curve. Their nucleons become more tightly bound, so the final total binding energy is greater and energy is released. The decrease therefore explains why fission of heavy nuclei can supply energy. It indicates energetic favourability, while whether and how rapidly fission occurs also depends on the fission barrier.

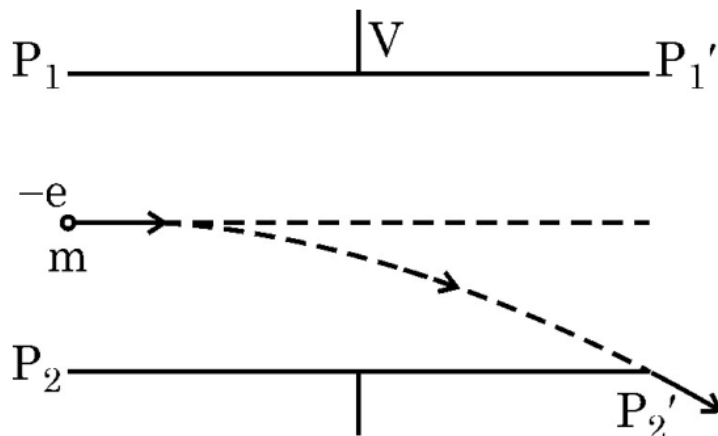
Step 3: Conclusion The curve peaks near $A \simeq 56$ and falls slowly for heavy nuclei. This fall makes fission of suitable heavy nuclei into more tightly bound, medium-mass products an energy-releasing process.

Quick Tip to Solve: For fission, compare the heights of the initial nucleus and the fragments on the B/A curve: moving towards a higher binding energy per nucleon releases energy.

Section C (21 marks)

22. Figure shows a narrow beam of electrons entering with a velocity of 3×10^7 m/s, symmetrically through the space between two parallel horizontal plates P_1P_1' and P_2P_2' kept 2 cm apart.

If each plate is 3 cm long, calculate the potential difference V applied between the plates so that the beam just strikes the end P_2' . [3 marks]



Solution:

Concept:

- Between parallel plates separated by d , the approximately uniform electric field has magnitude $E = V/d$, neglecting edge effects. An electron of charge $-e$ and mass m_e has acceleration magnitude $a = eE/m_e$, opposite to the field.
- There is no horizontal electric force, so the horizontal velocity remains u . With zero initial vertical velocity, the vertical displacement in transit time t is $y = \frac{1}{2}at^2$.

Step 1: Determine the required displacement and flight time The beam enters midway between the plates. To just reach the lower plate at its far end, the downward displacement must be half the plate separation, while the horizontal distance equals the plate length:

$$\begin{aligned} d &= 2 \text{ cm} = 2.0 \times 10^{-2} \text{ m}, \\ y &= \frac{d}{2} = 1.0 \times 10^{-2} \text{ m}, \\ \ell &= 3 \text{ cm} = 3.0 \times 10^{-2} \text{ m}, \\ t &= \frac{\ell}{u} = \frac{3.0 \times 10^{-2}}{3.0 \times 10^7} = 1.0 \times 10^{-9} \text{ s}. \end{aligned}$$

The horizontal speed, rather than the changing total speed, determines this transit time.

Step 2: Calculate the transverse acceleration and potential difference Taking downward displacement as positive, the initial vertical velocity is zero. Thus the required acceleration magnitude is

$$a = \frac{2y}{t^2} = \frac{2(1.0 \times 10^{-2})}{(1.0 \times 10^{-9})^2} = 2.0 \times 10^{16} \text{ m s}^{-2}.$$

Use $m_e = 9.1 \times 10^{-31} \text{ kg}$ and $e = 1.6 \times 10^{-19} \text{ C}$. From $m_e a = eV/d$,

$$\begin{aligned} V &= \frac{m_e a d}{e} \\ &= \frac{(9.1 \times 10^{-31})(2.0 \times 10^{16})(2.0 \times 10^{-2})}{1.6 \times 10^{-19}} \\ &= 2.275 \times 10^3 \text{ V}. \end{aligned}$$

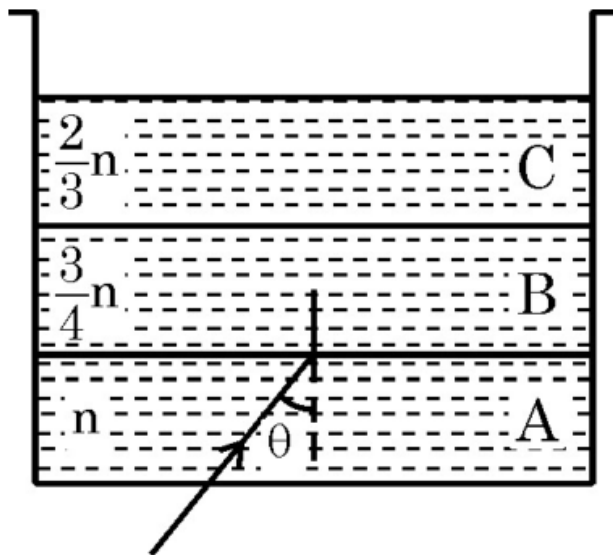
Step 3: Choose the plate polarity The electron must accelerate downward towards $P_2P'_2$. Since its charge is negative, the electric field must point upward. Therefore the lower plate $P_2P'_2$ must be at a higher potential than the upper plate $P_1P'_1$. This polarity bends the beam in the direction shown in the figure.

Step 4: Conclusion The required potential difference has magnitude $V = 2.275 \times 10^3 \text{ V} \approx 2.28 \text{ kV}$, with the lower plate positive relative to the upper plate.

Quick Tip to Solve: For a beam entering midway, set $y = d/2$ and $t = \ell/u$; combining them with $a = eV/(m_e d)$ gives $V = m_e u^2 d^2 / (e \ell^2)$.

23. (a) State the two conditions under which total internal reflection occurs.

(b) A transparent container contains layers of three immiscible transparent liquids A, B and C of refractive indices n , $\frac{3}{4}n$ and $\frac{2}{3}n$, respectively. A laser beam is incident at the interface between A and B at an angle θ as shown in figure. Prove that the beam does not enter region C at all for $\sin \theta \geq \frac{2}{3}$. [3 marks]



Solution:

Concept:

- Snell's law is $n_1 \sin i = n_2 \sin r$, with angles measured from the normal and n_1, n_2 the refractive indices.
- For light incident from a denser medium on a rarer medium, the critical angle satisfies $\sin i_c = n_2/n_1$. Total internal reflection requires $i > i_c$; at equality the limiting refracted ray runs along the

boundary.

- For parallel interfaces the normals are parallel, so the refraction angle in a layer equals its incidence angle at the next interface.

Step 1: State the conditions in part (a) Total internal reflection occurs when (i) light travels from an optically denser medium towards an optically rarer medium, and (ii) the angle of incidence in the denser medium is greater than the critical angle for that pair of media. At the critical angle itself, refraction is grazing; this limiting case is not called total internal reflection.

Step 2: Check whether the ray can enter liquid B The printed refractive indices are $n_A = n$, $n_B = 3n/4$ and $n_C = 2n/3$, so each successive liquid is optically rarer. At the A–B interface,

$$\sin i_{c,AB} = \frac{n_B}{n_A} = \frac{3}{4}.$$

If $\sin \theta > 3/4$, the beam is totally internally reflected in A and cannot reach C. If $\sin \theta = 3/4$, the refracted ray is along the A–B boundary, so it also does not reach the B–C boundary. It remains to examine $2/3 \leq \sin \theta < 3/4$.

Step 3: Apply Snell's law at the parallel interfaces In the remaining range the beam enters B at angle r . Since the interfaces are parallel, its incidence angle at B–C is also r . Therefore,

$$n \sin \theta = \frac{3n}{4} \sin r \quad \Rightarrow \quad \sin r = \frac{4}{3} \sin \theta \geq \frac{8}{9}.$$

The B–C critical angle satisfies

$$\sin i_{c,BC} = \frac{n_C}{n_B} = \frac{2n/3}{3n/4} = \frac{8}{9}.$$

Thus, for $2/3 < \sin \theta < 3/4$, the ray undergoes total internal reflection at B–C. At $\sin \theta = 2/3$, it is incident at the critical angle and the limiting refracted ray travels along B–C, without entering the interior of C.

Step 4: Conclusion Total internal reflection needs denser-to-rarer incidence at an angle greater than the critical angle. Considering both interfaces and their grazing limits, the beam does not enter region C for $\boxed{\sin \theta \geq 2/3}$.

Quick Tip to Solve: First check total internal reflection at A–B. For a transmitted ray, parallel interfaces conserve $n \sin i$, and the final grazing limit is $\sin \theta = n_C/n_A$.

24(a). (a) Using Gauss's law, deduce an expression for electric field at a point due to a uniformly charged infinite plane thin sheet.

(b) Two large thin plane sheets, each having surface charge density σ , are held close and parallel to each other in air. What is the net electric field at a point (i) inside and (ii) outside, the sheets?

[3 marks]

Solution:

Concept:

- Gauss's law is $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$, where Q_{enc} is the charge inside the closed Gaussian surface and ϵ_0 is the permittivity of free space.
- Plane symmetry makes the field of a uniformly charged infinite sheet normal to the sheet, with

equal magnitude on either side. The fields of several sheets add vectorially.

Step 1: Choose a Gaussian pillbox for part (a) Let the sheet carry uniform surface charge density σ , and first take $\sigma > 0$. Choose a short cylindrical Gaussian surface with its axis normal to the sheet and its two flat faces, each of area S , on opposite sides. Symmetry makes the field perpendicular to the sheet and directed outwards on both sides; any tangential component would select a preferred direction within the uniform plane. The field is parallel to the curved wall, so that wall contributes zero flux. At each flat face it is parallel to the outward area vector, and the two equal face fluxes add.

Step 2: Evaluate the enclosed charge and flux The pillbox encloses the sheet area S , hence $Q_{\text{enc}} = \sigma S$. Applying Gauss's law gives

$$ES + ES = \frac{\sigma S}{\epsilon_0} \Rightarrow E = \frac{\sigma}{2\epsilon_0}.$$

This field is independent of distance from the ideal infinite sheet. In general its magnitude is $|\sigma|/(2\epsilon_0)$, directed away from a positive sheet and towards a negative sheet.

Step 3: Superpose the fields for part (b) Both sheets have the same signed charge density σ . Treat them as infinite planes, neglecting edge effects, and use ϵ_0 for air. Each produces field magnitude $|\sigma|/(2\epsilon_0)$.

(i) Inside, between the sheets: the two fields have equal magnitudes and opposite directions, so

$$E_{\text{inside}} = 0.$$

(ii) Outside the sheets: the fields point in the same direction and add, so

$$E_{\text{outside}} = \frac{|\sigma|}{2\epsilon_0} + \frac{|\sigma|}{2\epsilon_0} = \frac{|\sigma|}{\epsilon_0}.$$

For positive sheets the outside field points away from the pair on either side; for negative sheets it points towards the pair.

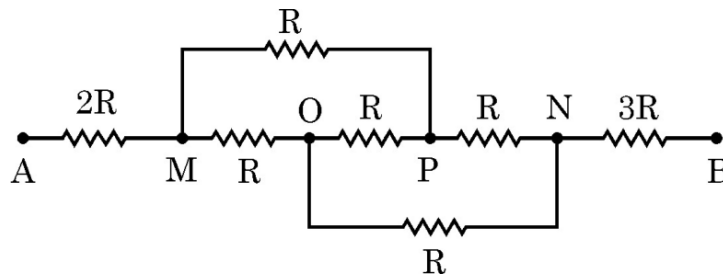
Step 4: Conclusion A single infinite sheet produces $E = |\sigma|/(2\epsilon_0)$. For two sheets with equal charge density, $E_{\text{inside}} = 0$ and $E_{\text{outside}} = |\sigma|/\epsilon_0$, normal to the sheets.

Quick Tip to Solve: Draw the direction of each sheet's field before adding: equal like-charged sheets cancel between them and reinforce outside them.

24(b). (a) Obtain the condition of balance of a Wheatstone bridge.

(b) Find net resistance of the network of resistors connected between A and B, as shown in figure.

[3 marks]



Solution:

Concept:

- A Wheatstone bridge is balanced when its two junctions connected by the galvanometer have equal potential, so the galvanometer current is zero.
- Ohm's law is $V = IR$. Series resistances add, while parallel resistances obey $1/R_{eq} = 1/R_1 + 1/R_2$.

Step 1: Define the four arms for part (a) Consider a bridge supplied across terminals L and H. One branch is L–X–H, with successive resistances R_1, R_2 ; the other is L–Y–H, with successive resistances R_3, R_4 . A galvanometer joins X and Y. At balance, its current is zero and $V_X = V_Y$. Let I_1 flow along the first branch and I_2 along the second; each branch current is unchanged at its intermediate junction because no current enters the galvanometer.

Step 2: Derive the balance condition Equal potentials at X and Y mean equal voltage drops from L to the two junctions and also from the junctions to H. Thus,

$$I_1 R_1 = I_2 R_3, \quad I_1 R_2 = I_2 R_4.$$

Dividing the first equation by the second eliminates the currents:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad \text{or} \quad R_1 R_4 = R_2 R_3.$$

Conversely, this ratio makes the junction potentials equal, giving zero galvanometer current. It is the Wheatstone bridge balance condition.

Step 3: Identify the balanced bridge in part (b) In the printed network, the bridge is between M and N. Its two paths are M–O–N and M–P–N. Each path contains two resistors of resistance R . The resistor O–P joins the two intermediate junctions. Since

$$\frac{R_{MO}}{R_{ON}} = \frac{R}{R} = 1 = \frac{R_{MP}}{R_{PN}},$$

the bridge is balanced: $V_O = V_P$, and no current flows through the O–P resistor. It therefore has no effect on the equivalent resistance between M and N. The remaining paths each have resistance $2R$ and are in parallel:

$$R_{MN} = \frac{(2R)(2R)}{2R + 2R} = R.$$

Step 4: Add the outer series resistors The resistance $2R$ between A and M and the resistance $3R$

between N and B carry the full terminal current, so they are in series with the equivalent R_{MN} :

$$R_{AB} = 2R + R_{MN} + 3R = 2R + R + 3R = 6R.$$

Step 5: Conclusion The Wheatstone balance condition is $R_1/R_2 = R_3/R_4$, for the arms defined above. The given network has $R_{AB} = 6R$, because its O–P resistor carries zero current.

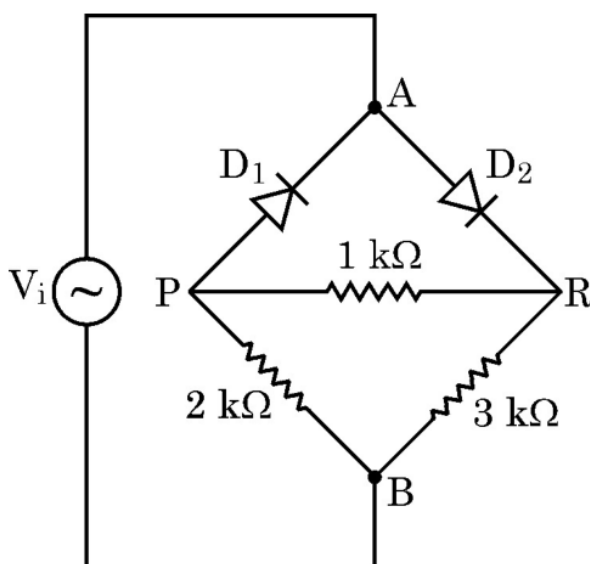
Quick Tip to Solve: Trace the two paths M–O–N and M–P–N before combining resistors; their equal arm ratios make the central O–P resistor inactive.

25. An ac voltage $V_i = 12 \sin(100\pi t)$ V is applied between points A and B in a network of two ideal diodes and three resistors as shown in figure.

During the positive half-cycle of the input voltage V_i supplied to the network:

- Identify which of the two diodes will conduct and why?
- Redraw an equivalent circuit diagram to show the flow of current.
- Calculate the output voltage drops V_o across the three resistors when the input voltage attains its peak value.

[3 marks]



Solution:

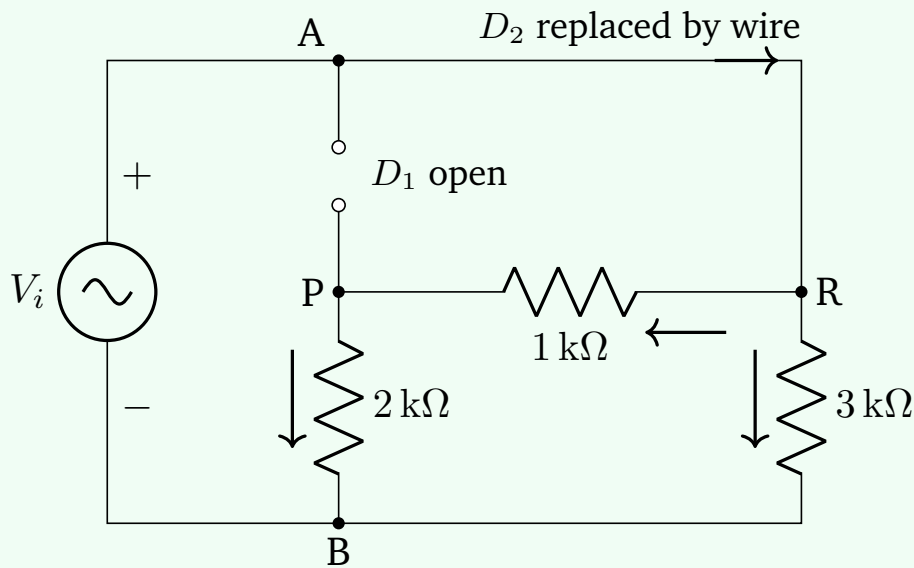
Concept:

- An ideal diode conducts with zero voltage drop when its anode is at a higher potential than its cathode; a reverse-biased diode behaves as an open circuit.
- Ohm's law gives the voltage drop across a resistor as $V = IR$, where I is its current and R its resistance. Series resistors carry the same current, while parallel branches have the same terminal voltage.

Step 1: (a) Determine the conducting diode During the positive half-cycle, $V_A > V_B$. In the printed circuit, the anode of D_2 is towards A and its cathode is towards R, so D_2 is forward biased and conducts. The cathode of D_1 is towards A and its anode is towards P; it is reverse biased and does not conduct. Consequently, A and R are at the same potential, while the direct A–P branch is interrupted.

Step 2: (b) Replace the diodes by their ideal equivalents Replace D_2 by a wire and D_1 by an open

gap, as in the diagram below. Current flows from A to R, where it divides into two paths: R to B through $3\text{ k}\Omega$, and R to P to B through the series combination $1\text{ k}\Omega + 2\text{ k}\Omega$. Both paths return to the negative source terminal B. Thus the source sees two parallel branches, each of resistance $3\text{ k}\Omega$.



Equivalent circuit for the positive half-cycle; arrows show conventional current.

Step 3: (c) Calculate the drops at the positive peak From $V_i = 12 \sin(100\pi t)\text{ V}$, the positive peak input is 12 V . Taking $V_B = 0$, we have $V_A = V_R = 12\text{ V}$. The $3\text{ k}\Omega$ resistor is directly across R and B, so its drop is 12 V . The current in the other branch is

$$I_{RPB} = \frac{12}{(1 + 2) \times 10^3} = 4 \times 10^{-3}\text{ A} = 4\text{ mA}.$$

Hence, using the direction of conventional current for each voltage drop,

$$V_R - V_P = I_{RPB}(1 \times 10^3) = 4\text{ V},$$

$$V_P - V_B = I_{RPB}(2 \times 10^3) = 8\text{ V},$$

$$V_R - V_B = 12\text{ V}.$$

The series drops add to $4 + 8 = 12\text{ V}$, as required. Also $V_P = 8\text{ V} < V_A$, confirming that D_1 remains reverse biased. Each parallel branch draws 4 mA , giving a total source current of 8 mA at the peak.

Step 4: Conclusion D_2 conducts and D_1 is reverse biased. At the positive peak, the output voltage drops across $1\text{ k}\Omega$, $2\text{ k}\Omega$ and $3\text{ k}\Omega$ are respectively 4 V , 8 V , 12 V . Their higher-potential ends are R, P and R, respectively.

Quick Tip to Solve: Short the conducting diode and open the reverse-biased diode first; then spot that the $1\text{ k}\Omega$ and $2\text{ k}\Omega$ resistors form one voltage-divider branch across the full input.

26. A $12.0\ \mu\text{F}$ capacitor is charged to a potential difference of $150\ \text{V}$. The terminals of the charged capacitor are then connected to those of an uncharged $6.0\ \mu\text{F}$ capacitor. Calculate final potential difference across and charge on, each capacitor. [3 marks]

Solution:

Concept:

- The magnitude of charge on either plate of a capacitor is $Q = CV$, where C is its capacitance and V its potential difference.
- When two capacitors are connected terminal to terminal, they are in parallel and acquire the same final potential difference. The total charge on each connected set of plates is conserved in the absence of an external source.

Step 1: Find the charge initially stored Let $C_1 = 12.0\ \mu\text{F}$ be the initially charged capacitor and $C_2 = 6.0\ \mu\text{F}$ the initially uncharged capacitor. The first capacitor is charged to $V_0 = 150\ \text{V}$, so its initial plate-charge magnitude is

$$Q_0 = C_1 V_0 = (12.0 \times 10^{-6})(150) = 1.80 \times 10^{-3}\ \text{C}.$$

The initial charge on the second capacitor is zero. On connecting the capacitors, charge flows through the wires until the potential differences become equal. No battery is connected during this redistribution.

Step 2: Apply charge conservation to obtain the common potential Denote the final common potential difference by V_f . The final positive plate charges are $Q_{1f} = C_1 V_f$ and $Q_{2f} = C_2 V_f$. Conservation of charge on the joined positive-side plates gives

$$\begin{aligned} Q_{1f} + Q_{2f} &= Q_0, \\ (C_1 + C_2)V_f &= C_1 V_0, \\ V_f &= \frac{1.80 \times 10^{-3}}{(12.0 + 6.0) \times 10^{-6}} = 100\ \text{V}. \end{aligned}$$

The negative-side plates carry equal and opposite total charge. The common potential is lower than the original $150\ \text{V}$ because the original charge is now distributed over a larger total capacitance.

Step 3: Calculate the final charge on each capacitor Substitute the same final voltage into $Q = CV$ separately for the two capacitors:

$$\begin{aligned} Q_{1f} &= (12.0 \times 10^{-6})(100) = 1.20 \times 10^{-3}\ \text{C} = 1200\ \mu\text{C}, \\ Q_{2f} &= (6.0 \times 10^{-6})(100) = 6.0 \times 10^{-4}\ \text{C} = 600\ \mu\text{C}. \end{aligned}$$

The check $1200 + 600 = 1800\ \mu\text{C}$ reproduces the initial charge. The final charges are in the ratio $12.0 : 6.0 = 2 : 1$; connection in parallel equalises potential difference, while charge is proportional to capacitance.

Step 4: Conclusion Both capacitors have final potential difference $100\ \text{V}$. The final charge magnitudes are $1.20 \times 10^{-3}\ \text{C}$ on the $12.0\ \mu\text{F}$ capacitor and $6.0 \times 10^{-4}\ \text{C}$ on the $6.0\ \mu\text{F}$ capacitor. The two plates of each capacitor carry charges of these magnitudes with opposite signs.

Quick Tip to Solve: For charge sharing with an initially uncharged capacitor, use $V_f = C_1 V_0 / (C_1 + C_2)$; then find each charge from its own capacitance times this common voltage.

27. A semiconductor has equal electron and hole concentration of $3 \times 10^8 \text{ m}^{-3}$. On doping with a certain impurity, the hole concentration increases to $6 \times 10^{10} \text{ m}^{-3}$.

(a) What type of semiconductor is obtained on doping?

(b) Calculate the new electron concentration of the semiconductor.

(c) How does the energy gap of semiconductor change with doping? Draw the energy band diagram for it.

[3 marks]

Solution:

Concept:

- In an intrinsic semiconductor, electron and hole densities are equal: $n = p = n_i$. Acceptor doping creates holes, giving a p-type semiconductor.
- At the same temperature and in thermal equilibrium, the mass-action law is $np = n_i^2$, where n and p are the electron and hole number densities.
- An acceptor introduces an energy level E_A just above the valence-band edge E_V . The small energy $E_A - E_V$ enables valence electrons to occupy acceptor states and leave holes behind.

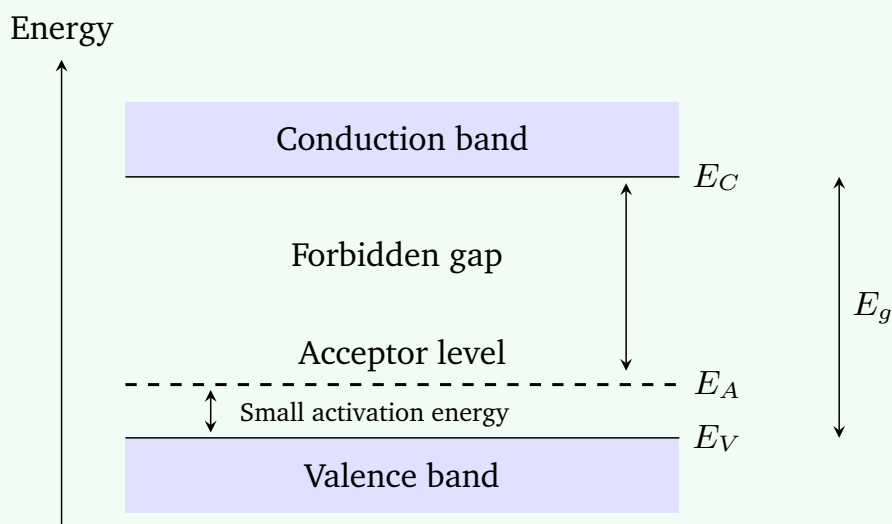
Step 1: (a) Identify the majority carriers The printed initial concentration is $n_i = 3 \times 10^8 \text{ m}^{-3}$. Doping raises the hole concentration to $p = 6 \times 10^{10} \text{ m}^{-3}$, so holes become the majority carriers and electrons the minority carriers. The impurity is therefore an acceptor, such as a trivalent atom in silicon or germanium, and the semiconductor is p-type.

Step 2: (b) Apply the mass-action law At unchanged temperature, use the intrinsic density before doping rather than equating the new electron and hole densities:

$$\begin{aligned} n &= \frac{n_i^2}{p} \\ &= \frac{(3 \times 10^8)^2}{6 \times 10^{10}} \text{ m}^{-3} \\ &= \frac{9 \times 10^{16}}{6 \times 10^{10}} \text{ m}^{-3} = 1.5 \times 10^6 \text{ m}^{-3}. \end{aligned}$$

The product of the resulting densities is $9 \times 10^{16} \text{ m}^{-6}$, equal to n_i^2 , as required.

Step 3: (c) Explain the impurity energy level Acceptor doping introduces a level just above the valence band, inside the forbidden gap. Hence the effective energy needed to create mobile holes is reduced to the small separation $E_A - E_V$; a valence electron can reach this level without crossing the full intrinsic gap. The host band gap $E_g = E_C - E_V$ itself remains essentially unchanged for ordinary dilute doping. Thus the reduction refers to the carrier-activation energy, with the additional acceptor level shown in the diagram.



Energy bands of the p-type semiconductor; the acceptor level lies just above the valence band.

Step 4: Conclusion (a) The semiconductor is **p-type**. (b) Its new electron concentration is $1.5 \times 10^6 \text{ m}^{-3}$. (c) Acceptor states lie just above the valence band, reducing the energy needed to generate holes; the host band separation is essentially unchanged.

Quick Tip to Solve: Square the printed intrinsic density before dividing by the new hole density; show the acceptor level close to the valence band for p-type doping.

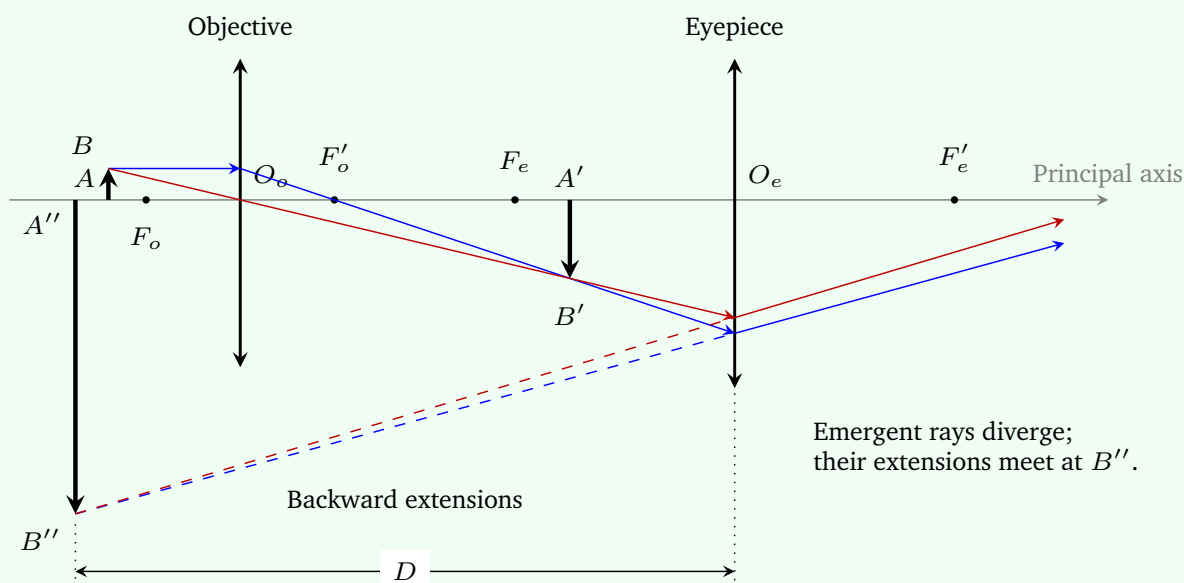
28. Draw a labelled ray diagram showing the formation of image by a compound microscope when final image is formed at least distance of distinct vision. Derive an expression for its magnifying power for this case. [3 marks]

Solution:

Concept:

- A compound microscope combines the objective's linear magnification $m_o = v_o/u_o$ with the eyepiece's angular magnification M_e : $M = m_o M_e$. Here u_o, v_o are signed object and image distances from the objective.
- For a thin converging lens, $1/f = 1/v - 1/u$. The eyepiece acts as a simple microscope, with its final virtual image at $v_e = -D$, where D is the least distance of distinct vision.

Step 1: Construct the two successive images Place the small object AB just beyond the objective's first focus F_o . The objective forms a real, enlarged and inverted image $A'B'$. Arrange this intermediate image between the eyepiece's first focus F_e and its optical centre O_e . The eyepiece then forms an enlarged virtual image $A''B''$ at distance D from it, taking the eye close to the eyepiece. Dashed lines are backward extensions of the emergent rays; their intersection locates the final image, which is inverted relative to AB .



Compound microscope adjusted for a virtual final image at distance D ; distances are schematic.

Step 2: Derive the eyepiece magnification Let $x = |u_e|$ be the distance of the intermediate image from the eyepiece. Using $u_e = -x$, $v_e = -D$ and focal length f_e ,

$$\frac{1}{f_e} = -\frac{1}{D} + \frac{1}{x} \Rightarrow \frac{D}{x} = 1 + \frac{D}{f_e}.$$

If the intermediate image height has magnitude h' , the small angle subtended through the eyepiece is $\beta \simeq h'/x$. Seen directly at the near point, it would subtend $\alpha' \simeq h'/D$. Therefore

$$M_e = \frac{\beta}{\alpha'} = \frac{D}{x} = 1 + \frac{D}{f_e}.$$

Step 3: Combine the two magnifications For original object height h , the objective gives $|m_o| = h'/h = v_o/|u_o|$. Comparing the final viewing angle with h/D , the angle of the original object seen directly at the near point, gives

$$|M| = \frac{v_o}{|u_o|} \left(1 + \frac{D}{f_e} \right).$$

Since the object is close to the objective focus, $|u_o| \simeq f_o$. With $v_o \simeq L$, where L is the effective tube length in the usual microscope approximation,

$$|M| \simeq \frac{L}{f_o} \left(1 + \frac{D}{f_e} \right).$$

Step 4: Conclusion For the final image at the least distance of distinct vision, the magnifying power has magnitude $|M| \simeq (L/f_o)(1 + D/f_e)$. The signed magnification is negative because the final image is inverted: $M \simeq -(L/f_o)(1 + D/f_e)$.

Quick Tip to Solve: For the near-point image use the eyepiece factor $1 + D/f_e$; place the intermediate image inside the eyepiece focal length in the ray diagram.

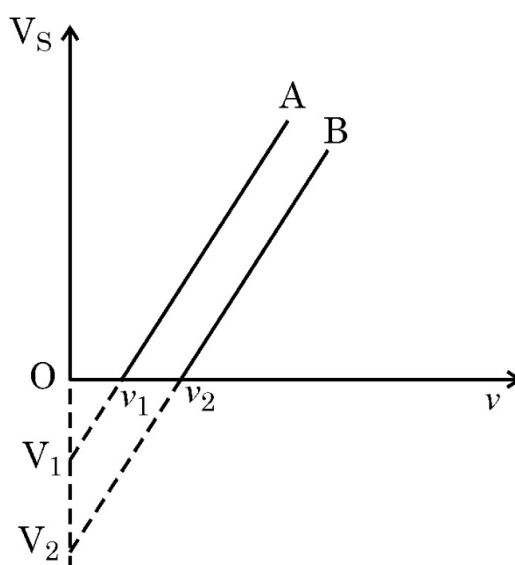
Section D (8 marks)

29. A researcher performs an experiment on photo-electric effect using two metals A and B with unknown work functions. She illuminates the surfaces of A and B with monochromatic radiation of various frequencies and records the value of corresponding stopping potentials (V_s). The graph shows the variation of stopping potential (V_s) with the frequency of incident radiation (ν) for metals A and B.

[4 marks]

29(i). From the graph, the work functions of A and B are (h is Planck's constant and e value of charge on an electron):

[1 mark]



- (A) ν_1 and ν_2
- (B) V_1 and V_2
- (C) $h\nu_1$ and $h\nu_2$
- (D) $\frac{h\nu_1}{e}$ and $\frac{h\nu_2}{e}$

Correct Answer: (C) $h\nu_1$ and $h\nu_2$

Solution:

Concept:

- Einstein's photoelectric equation is $eV_s = h\nu - \phi$, where ϕ is the work function and e is the positive magnitude of the electron charge.
- At the threshold frequency ν_0 , the maximum photoelectron kinetic energy and stopping potential vanish, giving $\phi = h\nu_0$.

Step 1: Read the frequency intercepts On the frequency axis, $V_s = 0$. The line for metal A meets this axis at ν_1 , and the line for B at ν_2 . These intercepts are the threshold frequencies, so substituting them into Einstein's equation gives

$$\phi_A = h\nu_1, \quad \phi_B = h\nu_2.$$

Step 2: Check the units of the alternatives Work function is an energy. Option (A) gives frequencies, whereas (B) gives potentials. Option (D) divides each energy by charge, again giving a potential rather than an energy. Only option (C) expresses the two threshold photon energies.

Step 3: Conclusion The work functions of A and B are $h\nu_1$ and $h\nu_2$, respectively. Hence, the correct

option is (C).

Quick Tip to Solve: Locate each line's frequency-axis intercept, then multiply that threshold frequency by Planck's constant to obtain its work function.

29(ii). For radiation of frequency $\nu > \nu_2$ incident on the surfaces of A and B, the maximum kinetic energy of ejected electron is: [1 mark]

- (A) greater for metal A because it has a smaller work function.
- (B) greater for metal B because it has a larger work function.
- (C) greater for metal B because it has higher threshold frequency.
- (D) the same for both metal A and metal B because it is independent of work functions of metals.

Correct Answer: (A) greater for metal A because it has a smaller work function.

Solution:

Concept:

- The maximum kinetic energy is $K_{\max} = h\nu - \phi$; at a common incident frequency, the smaller work function leaves more photon energy for the electron.
- Work function and threshold frequency are related by $\phi = h\nu_0$, with h Planck's constant.

Step 1: Compare the threshold frequencies The graph shows $\nu_1 < \nu_2$. Thus metal A has the smaller work function, $\phi_A = h\nu_1 < \phi_B = h\nu_2$. The given frequency exceeds ν_2 , so both metals emit photoelectrons.

Step 2: Subtract the two kinetic energies For the same incident radiation,

$$\begin{aligned} K_A - K_B &= (h\nu - h\nu_1) - (h\nu - h\nu_2) \\ &= h(\nu_2 - \nu_1) > 0. \end{aligned}$$

Options (B) and (C) reverse the effect: a larger work function or threshold consumes more of the photon energy. Option (D) is false because the maximum kinetic energy explicitly depends on work function.

Step 3: Conclusion The maximum kinetic energy is greater for metal A because its work function is smaller. Hence, the correct option is (A).

Quick Tip to Solve: At a fixed incident frequency, subtract the work functions from the same photon energy; the lower threshold gives the faster photoelectrons.

29(iii). If the intensity of the incident radiation for both metals A and B, is doubled keeping its frequency constant, then: [1 mark]

- (A) the slope of the parallel lines will increase.
- (B) the slope of the parallel lines will decrease.
- (C) the threshold frequencies for both A and B will decrease.
- (D) the slope of the parallel lines will not change but more electrons will be emitted per second.

Correct Answer: (D) the slope of the parallel lines will not change but more electrons will be emitted per second.

Solution:

Concept:

- Einstein's photoelectric equation is $eV_s = h\nu - \phi$, where V_s is stopping potential, h is Planck's constant, e is the magnitude of electron charge and ϕ is the work function.
- At a fixed frequency, intensity determines the number of incident photons per second, while each photon still has energy $h\nu$.

Step 1: Check the slope and threshold Rearranging the photoelectric equation gives

$$V_s = \frac{h}{e}\nu - \frac{\phi}{e}.$$

Thus the slope is h/e for either metal and its threshold frequency is ϕ/h . Neither quantity depends on intensity, so changing intensity cannot produce the changes suggested in (A), (B) or (C).

Step 2: Determine the emission rate Doubling the intensity at the same frequency doubles the incident photon rate on a given illuminated area. In the photoemission regime, this increases the number of electrons emitted per second. It does not raise the maximum kinetic energy of an individual electron, so the stopping potential at that frequency also stays unchanged.

Step 3: Conclusion The slopes remain unchanged, while more photoelectrons are emitted per second. Hence, the correct option is (D).

Quick Tip to Solve: Keep frequency and intensity separate: frequency sets photon energy; intensity at fixed frequency sets photon number.

29(iv)(a). The threshold frequency for a metal surface is ν_0 . If the radiation of frequency $3\nu_0$ illuminates the surface, the maximum kinetic energy (KE) of photoelectrons is E_1 . If the frequency were increased to $6\nu_0$, the maximum KE of the photoelectrons becomes E_2 . Then $\frac{E_1}{E_2}$ equals: [1 mark]

- (A) 1/3
- (B) 1/2
- (C) 2/5
- (D) 3/4

Correct Answer: (C) 2/5

Solution:

Concept:

- The work function of a metal is $\phi = h\nu_0$, where ν_0 is its threshold frequency and h is Planck's constant.
- The maximum kinetic energy of a photoelectron is $K_{\max} = h\nu - \phi = h(\nu - \nu_0)$, where ν is the incident frequency.

Step 1: Subtract the work function at each frequency The metal is unchanged, so the same work function must be subtracted from both photon energies:

$$\begin{aligned} E_1 &= h(3\nu_0 - \nu_0) = 2h\nu_0, \\ E_2 &= h(6\nu_0 - \nu_0) = 5h\nu_0. \end{aligned}$$

Both incident frequencies exceed threshold, so both kinetic energies are positive.

Step 2: Form the energy ratio The common factor $h\nu_0$ cancels when the energies are divided:

$$\frac{E_1}{E_2} = \frac{2h\nu_0}{5h\nu_0} = \frac{2}{5}.$$

The tempting value 1/2 compares photon frequencies alone and ignores the work function. Values 1/3 and 3/4 likewise do not follow from the available excess energies.

Step 3: Conclusion The ratio of maximum kinetic energies is $E_1/E_2 = 2/5$. Hence, the correct option is (C).

Quick Tip to Solve: For frequencies $a\nu_0$ and $b\nu_0$, use the kinetic-energy ratio $(a - 1)/(b - 1)$, not a/b .

29(iv)(b). Let m be the slope of the graph line for metal B. If e is the value of electron charge, then Planck's constant ' h ' is given by: [1 mark]

- (A) me
 (B) $\frac{1}{me}$
 (C) $\frac{m}{e}$
 (D) $\frac{e}{m}$

Correct Answer: (A) me

Solution:

Concept:

- The stopping potential satisfies $eV_s = h\nu - \phi$, where h is Planck's constant, e is the positive magnitude of electron charge and ϕ is the metal's work function.
- In a graph of V_s against incident frequency ν , the slope is the coefficient of ν in the straight-line equation.

Step 1: Identify the coefficient of frequency Divide Einstein's equation by e to obtain

$$V_s = \frac{h}{e}\nu - \frac{\phi}{e}.$$

Comparing with $y = mx + c$ shows that the slope for metal B is $m = h/e$. Its work function changes the intercept, not this slope.

Step 2: Rearrange for Planck's constant Multiplication by e gives $me = h$. As a unit check, the slope has units Vs, so me has units VCs = Js, as required. The alternatives $1/(me)$, m/e and e/m result from incorrect rearrangements and do not have the required dimensions.

Step 3: Conclusion Planck's constant is $h = me$. Hence, the correct option is (A).

Quick Tip to Solve: Read the vertical axis first: the slope of stopping potential versus frequency is h/e , while that of kinetic energy versus frequency is h .

30. A galvanometer is used to detect or/and measure small currents in an electrical circuit. It essentially works on the fact that a current-carrying coil experiences a deflecting torque when placed in a magnetic field. This deflection in the coil can be measured and it is related to the current flowing in the coil, the number of turns in the coil, area of the coil and the magnetic field. A hair spring attached to the coil provides a counter torque and helps in measuring the deflection. A galvanometer can be converted to an ammeter or a voltmeter of desired range by using suitable resistances. [4 marks]

30(i). The torque on the coil remains constant irrespective of the coil's orientation during rotation due to: [1 mark]

- (A) use of soft iron core which increases the magnetic field.
- (B) radial magnetic field
- (C) hair spring which provides the counter torque
- (D) eddy current in the iron core which causes damping.

Correct Answer: (B) radial magnetic field

Solution:

Concept:

- A coil with N turns, area A and current I experiences magnetic torque $\tau = NIAB \sin \alpha$, where α is the angle between its normal and the magnetic field B .
- In a moving-coil galvanometer, a radial field keeps the plane of the coil parallel to the local magnetic field throughout its permitted rotation.

Step 1: Apply the radial-field geometry Because the field lies in the coil's plane, the coil's normal remains perpendicular to it. Therefore $\alpha = 90^\circ$ and

$$\tau = NIAB \sin 90^\circ = NIAB.$$

At a fixed current, the magnetic deflecting torque is consequently independent of the coil's angular deflection.

Step 2: Distinguish the other components' roles Increasing field strength with a soft iron core increases the magnitude of torque, but field strength alone does not remove the angular factor. The hair spring supplies restoring torque, which grows with deflection. Eddy currents provide damping during motion. These roles do not explain an orientation-independent magnetic torque.

Step 3: Conclusion The radial magnetic field keeps the deflecting torque constant for a given current. Hence, the correct option is (B).

Quick Tip to Solve: For a galvanometer's radial field, use $\tau = NIAB$; the angle between the coil normal and field stays 90° .

30(ii). The best way to increase current sensitivity of a galvanometer is by:

[1 mark]

- (A) increasing number of turns of the coil
- (B) increasing area of coil and magnetic field strength
- (C) decreasing area of coil and magnetic field strength
- (D) increasing torsional constant of the hair spring

Correct Answer: Both (A) and (B) increase current sensitivity; the printed data do not determine a unique best change.

Solution:

Concept:

- In a radial field, equilibrium of the magnetic and spring torques gives $NIAB = k\theta$. Here N is the number of turns, A is coil area, B is field strength and k is the spring's torsional constant.
- Current sensitivity is angular deflection per unit current, $S_I = \theta/I = NAB/k$.

Step 1: Test each proposed change With other quantities fixed, increasing N , as in (A), increases S_I . Increasing both A and B , as in (B), also increases it. Decreasing both A and B , as in (C), decreases sensitivity, and increasing k , as in (D), also decreases sensitivity.

Step 2: Check whether a unique best change follows The paper does not specify the sizes of the increases or constraints on coil construction. For example, doubling N doubles sensitivity, whereas increasing A and B by ten percent each multiplies it by only 1.21. Larger increases in A and B could instead give the larger gain. Therefore the word “best” cannot establish a unique comparison. Extra coil resistance from more turns concerns voltage sensitivity and does not negate the increase in current sensitivity.

Step 3: Conclusion Both (A) and (B) increase current sensitivity. The printed question has no unique best option without further constraints; (C) and (D) decrease sensitivity.

Quick Tip to Solve: Write $S_I = NAB/k$ before judging the options; a claim of the best improvement also requires the sizes of the proposed changes.

30(iii)(a). A moving coil galvanometer has a coil with area of cross-section $4.0 \times 10^{-3} \text{ m}^2$ and number of turns 50. The coil is rotating in a magnetic field of 0.25 T. The torque acting on the coil when a current of 5 A passes through it is:

[1 mark]

- (A) 1.0 N m
- (B) 2.0 N m
- (C) 0.50 N m
- (D) 0.25 N m

Correct Answer: (D) 0.25 N m

Solution:

Concept:

- A coil's magnetic torque is $\tau = NIAB \sin \alpha$, where N is the number of turns, I is current, A is area and α is the angle between the coil normal and magnetic field B .
- A moving-coil galvanometer uses a radial magnetic field, keeping $\alpha = 90^\circ$; hence its deflecting torque is $\tau = NIAB$.

Step 1: Identify the values in SI units The given quantities are $N = 50$, $A = 4.0 \times 10^{-3} \text{ m}^2$, $B = 0.25 \text{ T}$ and $I = 5 \text{ A}$. The area is already in square metres, so no area conversion is needed. The radial-field condition removes any dependence on the coil's deflection angle.

Step 2: Calculate the magnetic torque Substitute into the galvanometer torque expression:

$$\begin{aligned}\tau &= 50 \times 5 \times (4.0 \times 10^{-3}) \times 0.25 \\ &= 0.25 \text{ N m.}\end{aligned}$$

As a quick arithmetic check, $NA = 0.20 \text{ m}^2$ and $IB = 1.25 \text{ A T}$. Their product rules out 1.0, 2.0 and 0.50 N m.

Step 3: Conclusion The torque acting on the coil is 0.25 N m. Hence, the correct option is (D).

Quick Tip to Solve: Multiply the turns and area first: here $50 \times 4.0 \times 10^{-3} = 0.20$, which keeps the power of ten under control.

OR

30(iii)(b). A galvanometer coil has a resistance of 15Ω and the meter shows full scale deflection for a current of 3 mA . The value of resistance required to convert it into a voltmeter of range (0-12 V) is: [1 mark]

- (A) 4015Ω
- (B) 3985Ω
- (C) 415Ω
- (D) 385Ω

Correct Answer: (B) 3985Ω

Solution:

Concept:

- A galvanometer becomes a voltmeter by adding a high resistance R in series with its coil resistance G .
- At full-scale voltage V , the current is limited to the full-scale galvanometer current I_g , so $V = I_g(G + R)$.

Step 1: Find the required total resistance Here $G = 15 \Omega$, $I_g = 3 \text{ mA} = 3 \times 10^{-3} \text{ A}$ and $V = 12 \text{ V}$. The total voltmeter resistance must therefore be

$$G + R = \frac{V}{I_g} = \frac{12}{3 \times 10^{-3}} = 4000 \Omega.$$

The conversion from milliamperes to amperes is essential for the resistance to be in ohms.

Step 2: Subtract the resistance already present Since the coil already supplies 15Ω , only the remaining resistance must be added:

$$R = 4000 - 15 = 3985 \Omega.$$

The value 4015Ω incorrectly adds the coil resistance to the required total; 415Ω and 385Ω are too small and would allow excessive current at 12 V .

Step 3: Conclusion A resistance of 3985Ω must be connected in series with the galvanometer. Hence,

the correct option is (B).

Quick Tip to Solve: For a voltmeter, first calculate the total V/I_g , then subtract the coil resistance to obtain the extra series resistance.

30(iv). A galvanometer with coil of resistance $20\ \Omega$ shows full scale deflection for a current of $5\ \text{mA}$. To convert it into an ammeter of range $(0\text{--}10\ \text{A})$, a resistance of: [1 mark]

- (A) $0.05\ \Omega$ should be connected in series with it
- (B) $0.05\ \Omega$ should be connected in parallel with it
- (C) $0.01\ \Omega$ should be connected in parallel with it
- (D) $0.01\ \Omega$ should be connected in series with it

Correct Answer: (C) $0.01\ \Omega$ should be connected in parallel with it

Solution:

Concept:

- A galvanometer is converted to an ammeter by connecting a small shunt resistance S in parallel with its coil resistance G .
- At the maximum measured current I , the galvanometer carries I_g and the shunt carries $I - I_g$. Equal branch voltages give $I_g G = (I - I_g)S$.

Step 1: Find the current diverted through the shunt The given coil resistance is $G = 20\ \Omega$, with $I_g = 5\ \text{mA} = 0.005\ \text{A}$. For an ammeter range of $10\ \text{A}$, the shunt must carry

$$I - I_g = 10 - 0.005 = 9.995\ \text{A}.$$

The voltage across both parallel branches at full scale is $I_g G = 0.100\ \text{V}$.

Step 2: Calculate the shunt resistance and connection Using the common branch voltage,

$$S = \frac{I_g G}{I - I_g} = \frac{0.005 \times 20}{9.995} = 0.010005 \dots \Omega \approx 0.01\ \Omega.$$

A series resistance would not divert current away from the delicate coil, eliminating (A) and (D). A $0.05\ \Omega$ parallel shunt would carry only $2\ \text{A}$ at the coil's full-scale voltage, so (B) cannot provide the stated range.

Step 3: Conclusion A resistance of approximately $0.01\ \Omega$ must be connected in parallel with the galvanometer. Hence, the correct option is (C).

Quick Tip to Solve: For a large ammeter range, estimate $S \approx I_g G / I$, then check that the small resistor is connected in parallel.

Section E (15 marks)

31(a). (a) State Faraday's law of electromagnetic induction.

(b) Derive an expression for the self-inductance of an air-filled long solenoid of length l and cross-sectional area A having N turns.

(c) A conducting rod of length 50 cm, with one end pivoted, is rotated with angular speed of 60 rpm in a uniform magnetic field of 4.0 mT directed perpendicular to the plane of rotation of rod. Find the emf induced in the rod. [5 marks]

Solution:

Concept:

- Faraday's law connects induced emf to changing magnetic flux linkage: $\varepsilon = -d(N\Phi)/dt$, where Φ is the flux through each of N turns. The minus sign expresses Lenz's law.
- Self-inductance L is defined by the flux linkage produced by the coil's own current: $N\Phi = LI$. For a long air-filled solenoid, $B = \mu_0(N/l)I$, where l is its length and I is its current.
- A rotating rod has different speeds at different distances from the pivot: $v = \omega r$. An element dr has motional emf $d\varepsilon = B\omega r dr$ when the field is perpendicular to its plane of rotation.

Step 1: (a) State Faraday's law Whenever the magnetic flux linked with a circuit changes, an emf is induced in the circuit. Its magnitude equals the rate of change of magnetic flux linkage. For a coil with N fixed turns, each linked by flux Φ ,

$$\varepsilon = -\frac{d(N\Phi)}{dt} = -N\frac{d\Phi}{dt}.$$

The induced emf acts in a direction such that the current it would produce opposes the change of flux responsible for it. This opposition is represented by the negative sign; a current actually flows only if the conducting circuit is closed.

Step 2: (b) Calculate the flux linkage of the solenoid Let a current I flow through the N closely wound turns. The number of turns per unit length is $n = N/l$. For a long solenoid, neglect end effects and take its internal field as uniform and parallel to its axis. With air having permeability approximately μ_0 ,

$$B = \mu_0 n I = \frac{\mu_0 N I}{l}.$$

The field is perpendicular to the area of each turn, so the magnetic flux through one turn is $\Phi = BA$. Assuming all the turns link this flux, the total flux linkage is

$$N\Phi = NBA = N \left(\frac{\mu_0 N I}{l} \right) A = \frac{\mu_0 N^2 A}{l} I.$$

Step 3: (b) Apply the definition of self-inductance For a fixed solenoid geometry in air, the flux linkage is proportional to its own current. Dividing the expression above by I ,

$$L = \frac{N\Phi}{I} = \frac{\mu_0 N^2 A}{l}.$$

Here L is measured in henry, A in square metres and l in metres. Combining this result with Faraday's law also gives the self-induced emf $\varepsilon = -L dI/dt$. The factor N^2 arises because the turns determine both the field strength and the number of flux linkages.

Step 4: (c) Integrate the emf along the rotating rod Denote the rod length by a , to distinguish it from the solenoid length. At distance r from the pivot, an element dr moves tangentially with speed ωr . Since the field is normal to the plane of rotation, the magnetic force on positive charges is along the rod. The emf magnitudes of all elements add:

$$|\varepsilon| = \int_0^a B\omega r \, dr = \frac{1}{2}B\omega a^2.$$

The factor $1/2$ is essential: the speed increases from zero at the pivot to ωa at the free end, so the tip speed cannot be used for the whole rod.

Step 5: (c) Convert the data and substitute The given values in SI units are $a = 50 \text{ cm} = 0.50 \text{ m}$ and $B = 4.0 \text{ mT} = 4.0 \times 10^{-3} \text{ T}$. A speed of 60 revolutions per minute corresponds to one revolution per second:

$$\omega = \frac{60 \times 2\pi}{60} = 2\pi \text{ rad s}^{-1}.$$

Therefore,

$$\begin{aligned} |\varepsilon| &= \frac{1}{2}(4.0 \times 10^{-3})(2\pi)(0.50)^2 \text{ V} \\ &= \pi \times 10^{-3} \text{ V} \approx 3.14 \times 10^{-3} \text{ V}. \end{aligned}$$

The sense of rotation and the direction of the perpendicular field are not specified, so the polarity of the two ends cannot be assigned. The magnitude is determined by the given data.

Step 6: Conclusion Faraday's law is $\varepsilon = -d(N\Phi)/dt$. The self-inductance of the long air-filled solenoid is $L = \mu_0 N^2 A/l$. The emf induced between the rod's ends has magnitude $|\varepsilon| \approx 3.14 \text{ mV}$ (about 3.1 mV to two significant figures).

Quick Tip to Solve: For a rod rotating about one end, use $B\omega a^2/2$, and convert revolutions per minute to radians per second with $\omega = 2\pi f/60$ when f is the numerical rpm value.

OR

31(b). (a) Draw a labelled diagram of a step-up transformer. State the principle on which it works and obtain the ratio of secondary voltage to primary voltage in terms of number of turns and currents in the two coils.

(b) The ratio of the number of turns in the primary to the secondary of an ideal transformer is 1 : 5. If 5 kW power at 200 V is supplied to the primary, find (i) current in the primary, and (ii) output voltage. [5 marks]

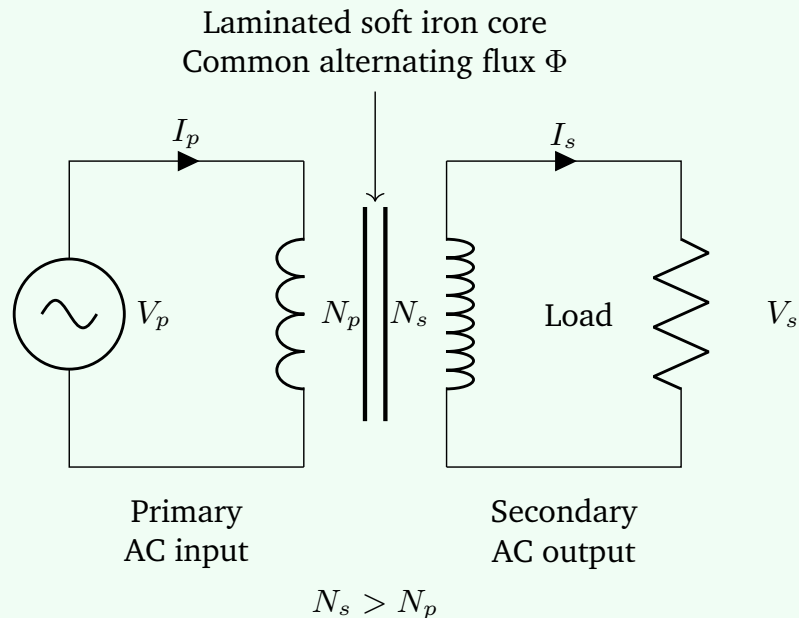
Solution:

Concept:

- A transformer works on mutual induction: an alternating current in the primary produces a changing magnetic flux in the core, which induces an emf in the secondary.
- Faraday's law gives $e = -N d\Phi/dt$, where N is the number of turns and Φ is the magnetic flux through each turn. In an ideal transformer, the same flux links both coils.
- An ideal transformer has no power loss. For the usual ideal load calculation, $V_p I_p = V_s I_s$, where V_p , V_s and I_p , I_s are the primary and secondary rms voltages and currents.

Step 1: (a) Draw and identify the step-up transformer The primary and secondary are insulated

coils coupled by a common laminated soft iron core. The primary is connected to an alternating-voltage source and the secondary supplies the load. A step-up transformer has more secondary turns than primary turns, $N_s > N_p$. The core provides a path for the magnetic flux linking the coils; its laminations reduce eddy-current losses. The diagram is a circuit representation, with the two parallel lines denoting the magnetic core.



Step-up transformer: $N_s > N_p$, $V_s > V_p$, and $I_s < I_p$.

Step 2: State the principle and obtain the voltage ratio The alternating primary current produces a time-varying flux $\Phi(t)$ in the core. This flux links the secondary and induces an emf in it by mutual induction. Assume negligible flux leakage and winding resistance. Choosing corresponding winding senses, the induced emfs are

$$e_p = -N_p \frac{d\Phi}{dt}, \quad e_s = -N_s \frac{d\Phi}{dt}.$$

The common flux derivative cancels when their magnitudes are divided. For sinusoidal flux $\Phi = \Phi_m \sin \omega t$, the rms induced emfs are $E_p = N_p \omega \Phi_m / \sqrt{2}$ and $E_s = N_s \omega \Phi_m / \sqrt{2}$. The primary induced emf opposes the applied voltage; their magnitudes are equal in the ideal limit. The secondary terminal voltage equals its induced emf, giving

$$\frac{V_s}{V_p} = \frac{E_s}{E_p} = \frac{N_s}{N_p}.$$

Thus increasing the number of secondary turns increases the output voltage. A steady direct current cannot maintain the changing flux needed for continuous transformer action.

Step 3: Relate the voltage ratio to the currents For an ideal transformer, input power equals output power. With the ideal load convention used here,

$$V_p I_p = V_s I_s \quad \Rightarrow \quad \frac{V_s}{V_p} = \frac{I_p}{I_s}.$$

Combining this with the turns ratio gives

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}.$$

Hence a step-up in voltage is accompanied by a step-down in current: the transformer transfers energy without increasing the power. The voltage ratio follows the turns ratio, whereas the secondary-to-primary current ratio is its reciprocal.

Step 4: (b)(i) Calculate the primary current The supplied power is $P_p = 5 \text{ kW} = 5000 \text{ W}$, and the primary voltage is $V_p = 200 \text{ V}$. Using the ideal-transformer power relation,

$$I_p = \frac{P_p}{V_p} = \frac{5000 \text{ W}}{200 \text{ V}} = 25 \text{ A}.$$

Step 5: (b)(ii) Calculate the output voltage The given primary-to-secondary turns ratio is $N_p : N_s = 1 : 5$, so the voltage multiplication factor is $N_s/N_p = 5$. Therefore,

$$V_s = \frac{N_s}{N_p} V_p = 5 \times 200 \text{ V} = 1000 \text{ V}.$$

As a consistency check, $I_s = I_p/5 = 5 \text{ A}$, and the output power is $1000 \times 5 = 5000 \text{ W}$, equal to the input power.

Step 6: Conclusion The transformer works on mutual induction, and for an ideal transformer

$$\boxed{\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}}.$$

For $N_p : N_s = 1 : 5$, the primary current is $\boxed{25 \text{ A}}$ and the output voltage is $\boxed{1000 \text{ V}}$.

Quick Tip to Solve: Write the turns ratio as N_s/N_p before multiplying the input voltage. Here voltage rises fivefold, so current falls fivefold; check that input and output powers agree.

- 32(a).** (a) An electric dipole consists of two point charges q and $-q$ separated by a distance $2a$. Derive an expression for the electric field $\vec{E}$ due to this dipole at a point distant r from the centre of the dipole on the equatorial plane. Write the expression for the electric field at a far off point, i.e. $r \gg a$.
 (b) A dipole is placed in x - y plane such that charges q and $-q$ are located at $x = a$ and $x = b$ respectively. There exists an electric field $\vec{E} = 2\hat{i} \text{ N/C}$ in the region. Calculate the force $\vec{F}$ and torque $\vec{\tau}$ experienced by the dipole. [5 marks]

Solution:

Concept:

- The dipole moment is $\vec{p} = q(2a)\hat{p}$, directed from the negative charge to the positive charge. The electric field is the vector sum of the fields of the two charges.
- Coulomb's law gives a point charge's field as $\vec{E} = kQ\vec{R}/R^3$, where $k = 1/(4\pi\epsilon_0)$ and $\vec{R}$ points from the charge to the observation point.
- A dipole in a uniform external field has zero net force and torque $\vec{\tau} = \vec{p} \times \vec{E}$. Its torque vanishes when its axis is parallel or antiparallel to the field.

Step 1: (a) Set up the equatorial geometry Put the dipole centre at O , the positive charge at $x = a$, and the negative charge at $x = -a$. Choose an equatorial point P at $(0, r)$. By rotational symmetry about the dipole axis this represents any point at distance r in the equatorial plane. The distance of P

from each charge is the same:

$$d = \sqrt{r^2 + a^2}, \quad \vec{p} = 2qa\hat{i}.$$

The field of the positive charge points away from it, whereas the field of the negative charge points towards it. Their components perpendicular to the dipole axis cancel; their axial components both point from the positive charge towards the negative charge.

Step 2: (a) Add the two electric-field vectors The displacement vectors from the two charges to P are $-a\hat{i} + r\hat{j}$ and $a\hat{i} + r\hat{j}$, respectively. Applying Coulomb's law separately gives

$$\begin{aligned}\vec{E}_+ &= \frac{kq(-a\hat{i} + r\hat{j})}{(r^2 + a^2)^{3/2}}, \\ \vec{E}_- &= \frac{-kq(a\hat{i} + r\hat{j})}{(r^2 + a^2)^{3/2}}, \\ \vec{E}_{\text{eq}} &= \vec{E}_+ + \vec{E}_- = -\frac{2kqa\hat{i}}{(r^2 + a^2)^{3/2}} = -\frac{\vec{p}}{4\pi\epsilon_0(r^2 + a^2)^{3/2}}.\end{aligned}$$

Thus the magnitude is $p/[4\pi\epsilon_0(r^2 + a^2)^{3/2}]$, and the direction is opposite to $\vec{p}$. The minus sign records this direction, rather than a negative field magnitude.

Step 3: (a) Take the far-field limit For $r \gg a$, factor r^3 out of the denominator. Since $a^2/r^2 \ll 1$, the remaining factor may be replaced by unity to leading order:

$$(r^2 + a^2)^{3/2} = r^3 \left(1 + \frac{a^2}{r^2}\right)^{3/2} \simeq r^3.$$

Therefore $\vec{E}_{\text{eq}} \simeq -\vec{p}/(4\pi\epsilon_0 r^3)$. The field falls as the inverse cube of distance and retains its direction opposite to the dipole moment.

Step 4: (b) Calculate the forces in the printed uniform field The printed field is $\vec{E} = 2\hat{i} \text{ N C}^{-1}$, independent of position, so both charges experience the same external field. Write $E_0 = 2 \text{ N C}^{-1}$. The individual forces and their sum are

$$\vec{F}_+ = qE_0\hat{i}, \quad \vec{F}_- = -qE_0\hat{i}, \quad \vec{F}_{\text{net}} = \vec{0}.$$

Their different x -coordinates do not change the field strength or prevent this cancellation.

Step 5: (b) Calculate the torque Taking the given positions $x = a$ and $x = b$ as the points on the x -axis, the dipole moment is $\vec{p} = q(a - b)\hat{i}$. Its direction is parallel or antiparallel to the external field, depending on the order of the charges. Hence

$$\vec{\tau} = \vec{p} \times \vec{E} = q(a - b)E_0(\hat{i} \times \hat{i}) = \vec{0}.$$

Equivalently, the two equal and opposite forces have the same line of action, so they form no couple. As the resultant force is zero, this torque is independent of the chosen origin. The result uses the usual reading that no transverse separation of the charges is specified.

Step 6: Conclusion (a) The exact equatorial field is

$$\boxed{\vec{E}_{\text{eq}} = -\frac{\vec{p}}{4\pi\epsilon_0(r^2 + a^2)^{3/2}}}, \quad \boxed{\vec{E}_{\text{eq}} \simeq -\frac{\vec{p}}{4\pi\epsilon_0 r^3} \quad (r \gg a)}.$$

Both point opposite to the dipole moment. (b) For the stated dipole along the x -axis in the uniform

field, $\vec{F} = \vec{0} \text{ N}$ and $\vec{\tau} = \vec{0} \text{ N m}$.

Quick Tip to Solve: On an equatorial line the transverse field components cancel and the axial components oppose the dipole moment. For part (b), first check that the printed field has no position factor before computing the two forces.

OR

32(b). (a) Two cells of emf E_1 and E_2 with internal resistances r_1 and r_2 respectively, are connected in parallel by connecting their positive terminals together and negative terminals together. Deduce an expression for equivalent emf and equivalent internal resistance of the combination.

(b) A parallel combination, as stated in (a) above, of two cells of emfs E and $3E$ and internal resistances R each is connected across a resistance $2R$. Find the current that flows through resistance $2R$. [5 marks]

Solution:

Concept:

- For a cell supplying current I_i , its terminal voltage is $V = E_i - I_i r_i$, where E_i is its emf and r_i its internal resistance. A negative I_i means current enters that cell's positive terminal.
- Cells connected in parallel have a common terminal voltage. Kirchhoff's junction rule gives the current supplied to the external circuit as $I = I_1 + I_2$.
- An equivalent cell must obey $V = E_{\text{eq}} - I r_{\text{eq}}$ for the same total current I . With a load resistance R_L , Ohm's law gives $I = E_{\text{eq}} / (R_L + r_{\text{eq}})$.

Step 1: (a) Define currents and the common terminal voltage Let A be the common positive terminal and B the common negative terminal. Define $V = V_A - V_B$, and let I_1 and I_2 be the currents delivered by the respective cells towards terminal A . Since both cell branches are joined to the same two terminals, Kirchhoff's voltage rule gives

$$V = E_1 - I_1 r_1 = E_2 - I_2 r_2.$$

Therefore

$$I_1 = \frac{E_1 - V}{r_1}, \quad I_2 = \frac{E_2 - V}{r_2}.$$

These equations remain valid even when the cells have unequal emfs and one cell receives current from the other; the sign of its current then accounts for that behaviour.

Step 2: (a) Form the equivalent terminal equation The current leaving the positive terminal for the load is the algebraic sum of the two branch currents. Substituting the preceding expressions and collecting the common voltage gives

$$\begin{aligned} I &= I_1 + I_2 \\ &= \frac{E_1}{r_1} + \frac{E_2}{r_2} - V \left(\frac{1}{r_1} + \frac{1}{r_2} \right), \\ V &= \frac{E_1/r_1 + E_2/r_2}{1/r_1 + 1/r_2} - \frac{I}{1/r_1 + 1/r_2}. \end{aligned}$$

Comparing this expression with $V = E_{\text{eq}} - Ir_{\text{eq}}$ identifies

$$E_{\text{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}, \quad r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2}.$$

The equivalent emf is a conductance-weighted mean of the emfs; the equivalent internal resistance is the parallel combination of the two internal resistances. In an open external circuit $I = 0$, so the terminal voltage equals E_{eq} , even if a circulating current exists between unequal cells.

Step 3: (b) Substitute the two specified cells Here $E_1 = E$, $E_2 = 3E$, and $r_1 = r_2 = R$. Equal internal resistances give equal weights to the two emfs. Thus

$$E_{\text{eq}} = \frac{ER + (3E)R}{R + R} = 2E,$$

$$r_{\text{eq}} = \frac{R \cdot R}{R + R} = \frac{R}{2}.$$

The external resistance $2R$ is in series with the internal resistance $R/2$ of the equivalent cell. Therefore the total resistance in the equivalent circuit is $2R + R/2 = 5R/2$.

Step 4: (b) Calculate and verify the load current Applying Ohm's law to the complete equivalent circuit gives

$$I = \frac{E_{\text{eq}}}{2R + r_{\text{eq}}} = \frac{2E}{2R + R/2} = \frac{4E}{5R}.$$

As a check, the common terminal voltage is $V = I(2R) = 8E/5$. Hence the individual branch currents are

$$I_1 = \frac{E - 8E/5}{R} = -\frac{3E}{5R}, \quad I_2 = \frac{3E - 8E/5}{R} = \frac{7E}{5R}.$$

Their sum is $4E/(5R)$, as required. The negative first current shows that the cell of emf E receives current; the cell of emf $3E$ supplies both this current and the load current.

Step 5: Conclusion For the parallel combination with like terminals joined,

$$E_{\text{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}, \quad r_{\text{eq}} = \frac{r_1 r_2}{r_1 + r_2}.$$

In part (b), the current through $2R$ is $I = 4E/(5R)$, in amperes when E is in volts and R in ohms. It flows through the external resistance from the common positive terminal to the common negative terminal.

Quick Tip to Solve: For two cells with equal internal resistance connected in parallel, average their emfs and halve the internal resistance. Add the load resistance only after finding this equivalent cell.

33(a). (a) Using the relation for refraction at a curved spherical surface, derive the expression for lens maker's formula.

(b) Three lenses L_1 , L_2 and L_3 , each of focal length 40 cm, are placed coaxially. The distance between L_1 and L_2 and between L_2 and L_3 are 120 cm and 20 cm respectively. An object is kept at a distance of 80 cm to the left of lens L_1 . Find the distance of the final image formed from the object. [5 marks]

Solution:

Concept:

- For paraxial refraction at a spherical surface, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, where n_1, n_2 are the refractive indices of the incident and refracting media, and u, v, R are signed distances from the surface pole.
- For a thin lens, the surface separation is neglected. With light travelling from left to right, distances to the right are positive. The thin-lens formula is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.
- In successive refraction, the image formed by one lens supplies the object for the next. An object at the first focus of a converging lens produces a parallel emergent beam; a further converging lens brings that beam to its second focus.

Step 1: (a) Apply spherical refraction at the first surface Consider a thin lens of refractive index μ in air. Let its two signed radii of curvature be R_1 and R_2 . The object distance is u , and let v' be the image distance that the first surface alone would produce. Refraction from air into the lens gives

$$\frac{\mu}{v'} - \frac{1}{u} = \frac{\mu - 1}{R_1}.$$

This relation uses the paraxial approximation: rays make small angles with the principal axis.

Step 2: Apply refraction at the second surface and add The first surface's intermediate image acts as the object for the second surface. Since the lens is thin, its signed object distance at that surface is also v' . If the final image distance is v , refraction from the lens into air gives

$$\frac{1}{v} - \frac{\mu}{v'} = \frac{1 - \mu}{R_2}.$$

Adding the two surface equations eliminates the intermediate image distance:

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

To identify the focal length, take an object at infinity, so $1/u \rightarrow 0$ and $v = f$. Thus the lens maker's formula for a thin lens in air is

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

The radii retain their Cartesian signs; for a biconvex lens with light incident from the left, $R_1 > 0$ and $R_2 < 0$.

Step 3: (b) Locate the image formed by the first lens Take the optical centre of L_1 as $x = 0$. The object is at $x = -80$ cm, L_2 is at $x = 120$ cm, and L_3 is at $x = 140$ cm. Each focal length is +40 cm. For L_1 , $u_1 = -80$ cm, so

$$\begin{aligned} \frac{1}{v_1} &= \frac{1}{f_1} + \frac{1}{u_1} \\ &= \frac{1}{40} - \frac{1}{80} = \frac{1}{80} \text{ cm}^{-1}, \\ v_1 &= +80 \text{ cm}. \end{aligned}$$

The first image is real and lies at $x = 80$ cm, before the second lens.

Step 4: Determine the beam emerging from the second lens The first image is $120 - 80 = 40$ cm to the left of L_2 , so it is a real object for L_2 , with $u_2 = -40$ cm. Hence

$$\frac{1}{v_2} = \frac{1}{40} + \frac{1}{-40} = 0.$$

Thus L_2 produces an image at infinity: the rays incident on L_3 are parallel. Propagation through the 20 cm gap does not change their parallel character.

Step 5: Locate the final image and measure from the original object For the parallel incident beam, $1/u_3 = 0$. The third lens therefore forms the final image at its second focal plane:

$$v_3 = f_3 = +40 \text{ cm}.$$

Its coordinate is $x_I = 140 + 40 = 180$ cm. The required separation is measured from the original object at $x_O = -80$ cm, giving

$$x_I - x_O = 180 - (-80) = 260 \text{ cm}.$$

Step 6: Conclusion (a) The lens maker's formula is $\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$ for a thin lens in air.
(b) The final image is real, 40 cm to the right of L_3 , and 260 cm from the original object.

Quick Tip to Solve: After each lens, locate its image on one common coordinate axis before finding the next object distance. Here L_1 forms its image exactly one focal length before L_2 , so the last lens receives a parallel beam.

OR

33(b). (a) Draw a ray diagram to show the image formation by a concave mirror when the object is kept between its focus and the centre of curvature. Using this diagram, derive the mirror formula.

(b) A concave mirror produces a two times magnified virtual image of an object kept 10 cm in front of it. Calculate the focal length of the mirror. [5 marks]

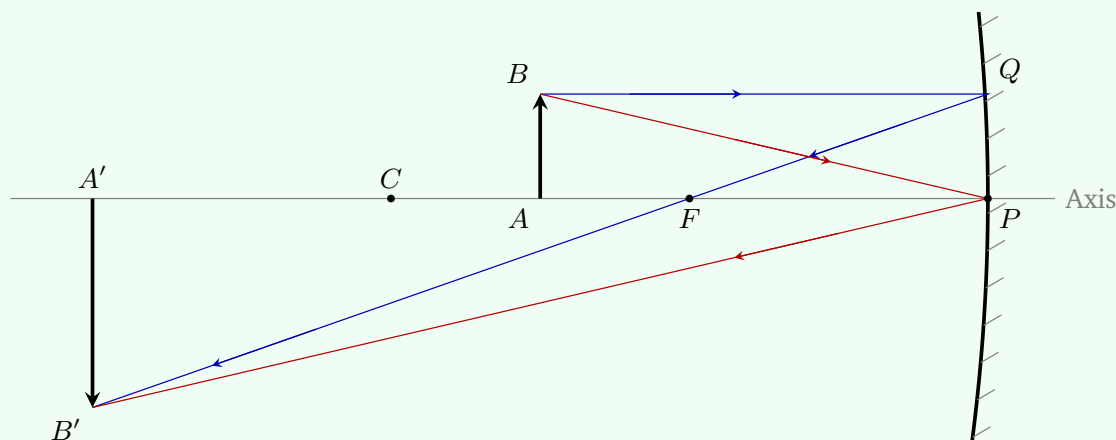
Solution:

Concept:

- For a spherical mirror of small aperture, paraxial rays obey the laws of reflection. A ray parallel to the principal axis of a concave mirror reflects through its principal focus, while a ray striking the pole has equal angles of incidence and reflection.
- Use the Cartesian sign convention: the pole is the origin, and distances measured in the direction of incident light are positive. For an object in front of a concave mirror, $u < 0$ and $f < 0$; a real image has $v < 0$, whereas a virtual image has $v > 0$.
- Transverse magnification is $m = h_i/h_o = -v/u$, where h_o and h_i are the signed object and image heights. A virtual image of a real object in a concave mirror is erect, so its magnification is positive.

Step 1: (a) Construct the ray diagram Place the upright object AB between the focus F and centre of curvature C , with A on the principal axis and B at its top. Draw BQ parallel to the axis; after reflection it passes through F . Draw a second ray BP to the pole P ; it reflects below the axis with its

angle of reflection equal to its angle of incidence. Their intersection is B' , and A' is its perpendicular projection onto the axis. Thus the image $A'B'$ lies beyond C and is real, inverted and enlarged. The diagram uses the paraxial approximation in which the small separation between the mirror surface at Q and the pole's transverse plane is neglected.



Paraxial ray construction: object between C and F , image beyond C . P is the pole and Q the point of reflection of the parallel ray.

Step 2: Relate the object and image heights at the pole First work with positive lengths: let $AP = U$, $A'P = V$, $FP = F_0$, $AB = h$ and $A'B' = H$. The right triangles ABP and $A'B'P$ are similar because the incident and reflected pole rays make equal angles with the principal axis. Therefore,

$$\frac{H}{h} = \frac{V}{U}.$$

Here H denotes the magnitude of the inverted image height; its signed height is $-H$. This distinction avoids confusing the geometrical length ratio with the negative signed magnification of a real image.

Step 3: Use the ray through the focus and obtain the formula In the small-aperture approximation, the height QP is h . The right triangles QPF and $B'A'F$ are similar, since the reflected ray QB' passes through F . As $A'F = V - F_0$, their corresponding sides give

$$\frac{H}{h} = \frac{V - F_0}{F_0}.$$

Equating the two height ratios and rearranging,

$$\begin{aligned} \frac{V}{U} &= \frac{V - F_0}{F_0}, \\ VF_0 &= UV - UF_0, \\ F_0(U + V) &= UV, \\ \frac{1}{F_0} &= \frac{1}{U} + \frac{1}{V}. \end{aligned}$$

For the arrangement shown, $u = -U$, $v = -V$ and $f = -F_0$. Substituting these signed distances gives the mirror formula

$$\boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}.$$

The formula applies to paraxial imaging by a spherical mirror when all distances are used with the same Cartesian sign convention.

Step 4: (b) Determine the virtual image distance The object is 10 cm in front of the mirror, so $u = -10$ cm. The virtual image is erect and twice the object height, hence $m = +2$. Use the signed magnification equation:

$$2 = -\frac{v}{-10 \text{ cm}} \Rightarrow v = +20 \text{ cm}.$$

The positive image distance correctly places this virtual image behind the mirror. This numerical arrangement has its object between the pole and focus; it is a different object position from the diagram requested in part (a).

Step 5: Calculate the focal length with signed distances Substitute the object and virtual image distances into the derived formula:

$$\begin{aligned}\frac{1}{f} &= \frac{1}{-10 \text{ cm}} + \frac{1}{20 \text{ cm}} \\ &= -\frac{1}{20} \text{ cm}^{-1}, \\ f &= -20 \text{ cm}.\end{aligned}$$

The negative sign agrees with a concave mirror whose focus is in front of it. Also, $|u| < |f|$, so the object is inside the focal length, as required for the stated enlarged virtual image.

Step 6: Conclusion (a) The image is real, inverted and enlarged beyond C , and the mirror formula is $\boxed{1/f = 1/u + 1/v}$. (b) The concave mirror has $\boxed{f = -20 \text{ cm}}$, i.e. a focal-length magnitude of 20 cm.

Quick Tip to Solve: For a magnified virtual image in a concave mirror, take m positive and u negative. Find $v = -mu$ first; its positive sign is a useful check before substituting into the mirror formula.